Publication V


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LONG-TERM VARIABILITY OF RADIO-BRIGHT BL LACERTAE OBJECTS

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Received 2008 October 19; accepted 2009 March 27; published 2009 May 6

ABSTRACT

Radio-bright BL Lacertae objects (BLOs) are typically very variable and exhibit prominent flaring. We use a sample of 24 BLOs, regularly monitored at Metsähovi Radio Observatory, to get a clear idea of their flaring behavior in the radio domain and to find possible commonalities in their variability patterns. Our goal was to compare the results given by computational timescales and the observed variability parameters determined directly from the flux curves. Also, we wanted to find out if the BLO flares adhere to the generalized shock model, which gives a schematic explanation for the physical process giving rise to the variability. We use long-term monitoring data from 4.8, 8, 14.5, 22, 37, 90, and 230 GHz, obtained mainly from the University of Michigan and Metsähovi Radio Observatories. The structure function, discrete correlation function, and Lomb–Scargle periodogram timescales, calculated in a previous study, are analyzed in more detail. Also, we determine flare durations, rise and decay times, and absolute and relative peak fluxes from the monitoring data. We find that radio-bright BLOs demonstrate a wide range of variability behavior, and few common denominators can be found. BLOs include sources with fast and strong variability, such as OJ 287, PKS 1749+096, and BL Lac, but also sources with more rolling fluctuations such as PKS 0735+178. The most extreme flares can last for up to 13 years or have peak fluxes of approximately 12 Jy in the observer’s frame. When the Doppler boosting effect is taken into account, the peak flux of a flare does not depend on the duration of the flare. A rough analysis of the times and peak flux evolution indicates that, typically, BLO flares in the mm–cm wavelengths are high peaking, i.e., are in the adiabatic stage. Thus, the results concur with the generalized shock model, which assigns shocks traveling in the jet as the main cause for active galactic nucleus variability. Comparing the computational timescales and the parameters obtained from the flux curve analysis (i.e., rise and decay times and intervals of the flares) reveals that they do have a significant correlation, albeit with large scatter.

Key words: BL Lacertae objects: general – galaxies: active – methods: statistical – radiation mechanisms: non-thermal – radio continuum: galaxies

Online-only material: extended figure, machine-readable and VO tables

1. INTRODUCTION

BL Lacertae objects (BLOs) are a relatively rare subclass of active galactic nuclei (AGNs). Traditionally the deﬁning properties of BLOs include a featureless optical spectrum, a ﬂat radio spectrum, and vigorous variability at all frequency bands (Stein et al. 1976; Kollgaard 1994; Jannuzi et al. 1994; Urry & Padovani 1995, and references therein). Most BLOs are thought to be highly beamed objects (Blandford & Königl 1979), which means that the relativistic jets emanating from the core are pointing very closely in our direction. This is partly the cause of the featureless optical spectrum; the nonthermal continuum emission from the jet can swamp the thermal emission, including the emission lines, from the host galaxy. However, in the case of less beamed objects, the lineless spectrum must be created by other mechanisms.

The ﬁrst BLO samples were discovered in either radio (Stickel et al. 1991, 1993) or X-ray surveys (Gioia et al. 1990; Stocke et al. 1991). Lately, surveys at different wavelengths (Londish et al. 2002) and the cross-correlation of existing radio and other waveband catalogs (Perlman et al. 1998; Caccianiga et al. 1999; Landt et al. 2001; Giommi et al. 2005; Turriziani et al. 2007; Plotkin et al. 2008) have produced a large number of new members to the class. Typically the selection criteria are different from those of the pioneering surveys, which has widened the deﬁnition of a BL Lac object; hence, the variability characteristics of many of the newest BLOs are unknown due to a lack of data.

In a recent paper (Nieppola et al. 2007), 37 GHz fractional variability indices from Metsähovi Radio Observatory data were calculated for 90 BLOs. All sources, for which even a crude estimation of variability could be calculated, exhibited an increase of 10% of the minimum ﬂux level at some point during the 3.5-year observation period. Almost half of them doubled their minimum ﬂux density. However, this sample of 90 sources was only one fourth of the full Metsähovi BLO sample; the rest are too faint to allow any variability analysis.

The variability of BLOs, like all ﬂaring AGNs, is thought to be caused by shocks forming and traveling in the jet. The origin and early development of these shocks are not yet very well understood. Once propagating downstream in the jet, their evolution is easier to model with Compton, synchrotron, and adiabatic losses (Marscher & Gear 1985; Hughes et al. 1989), Valtaoja et al. (1992) constructed a generalized view of the Marscher & Gear shock model, containing a general scenario of the AGN ﬂare behavior, to be used in comparison with observations. The generalized model describes how the shape of the shock spectrum remains unchanged as its peak moves from higher to lower frequencies. The evolutionary track of the shock consists of growth ($S_m \propto \nu_m^{b}$), plateau ($S_m \approx$ constant), and decay ($S_m \propto \nu_m^{a}$) stages, where $S_m$ and $\nu_m$ are the turnover ﬂux and frequency for the shock spectrum, and $a$ and $b$ are
model-dependent parameters. The flares can ideally be divided into two groups: (1) low-peaking flares, which will reach their maximum intensity at lower frequencies than the observing frequency, and (2) high-peaking flares, which have peaked at high frequencies relative to the observing frequency and are already decaying. The observing frequency, however, is not a constant quantity, but can be chosen freely. Thus the low- and high-peaking classes are not fixed either: the same flare can be low-peaking in one frequency band and high-peaking on another. Consequently, the classes are not separated, but rather opposite ends of a continuum of cases. Low- and high-peaking flares should be distinguishable from observations. For high-peaking flares the peak fluxes and time lags are strongly frequency-dependent, the highest observing frequency peaking first with the highest peak flux. For low-peaking flares the peaks are nearly simultaneous in all frequencies, and the peak flux of the flare is not significantly dependent on the frequency.

In this work, we will study the long-term radio variability of BL Lacertae objects at several frequencies. We will focus on a sample of 24 BLOs. The current number of definite and probable BLOs is over 1000 (Véron-Cetty & Véron 2006). This means that our sample is not a representative cross-section for the whole population, but rather represents only the rare, radio-luminous BLOs. The bases of our work are the extensive databases of Metsähovi and University of Michigan Radio Observatories at frequencies 4.8, 8, 14.5 GHz (UMRAO) and 22 and 37 GHz (Metsähovi).

We will study the variability of our BLO sample from two points of view: computational timescales obtained using statistical analysis methods and observed parameters of the flares, e.g., the duration, rise and decay times, and absolute and relative peak fluxes. The goal is to determine how well the computational timescales correspond to the behavior we observe in the source, as well as to gain a deeper understanding of what kind of variability can be expected from radio-luminous BLOs, when monitored for tens of years. We are also interested in how well the BLO flares adhere to the generalized shock model of Valtaoja et al. (1992), and whether the flares are mostly high- or low-peaking. A similar study is performed on a larger AGN sample, including radio-loud quasars, in an accompanying paper (Hovatta et al. 2008b).

In Section 2, we will present our sample and data. We briefly describe the methods we used in Section 3. In Sections 4 and 5, we discuss the timescales and the observed flux curves, respectively. In Section 6, we examine the correspondence between the timescales and observed flaring, and describe the behavior of the source sample individually. We will finish with a discussion and conclusions in Sections 7 and 8, respectively. Throughout this paper, we assume $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.27$, and $\Omega_\Lambda = 0.73$.

2. SAMPLE AND DATA

Our sample has been selected from the BLO sample monitored at the Metsähovi Radio Observatory for more than 20 years. The whole Metsähovi BLO sample comprises 398 sources, selected mainly from Véron-Cetty & Véron (2000). Most of the sources in the full sample are usually very faint or nondetectable (meaning S/N $\leq 4$) in the high radio frequencies (Nieppola et al. 2007). The sample in this study contains the very brightest sources with well-sampled flux curves. The selection criterion was ample data from a period of at least 10 years in at least two radio frequencies. The sampling has to be sufficient to determine the peaks of possible flares with adequate accuracy. There are 24 available sources, 13 of which had one or several significant flares during the observing period. Two of the BLOs are high-energy BLOs (HBLs), four intermediate BLOs (IBLs), and 18 low-energy BLOs (LBLs; for the basis of this classification, see Nieppola et al. 2006). The source sample is listed in Table 1. Columns 1 and 2 give alternative names for the source;Cols. (3) and (4) give the right ascension and declination, respectively; Col (5) gives the redshift; Col (6) gives the Doppler boosting factor of the source taken from Hovatta et al. (2009); and Col (7) gives the BLO class according to Nieppola et al. (2006). Column 8 indicates whether the source has been included in the flare analysis in this work, and Col (9) gives the reference for the redshift.

We used seven different frequency bands in the analysis, covering the radio domain quite extensively. Low-frequency data at 4.8, 8, and 14.5 GHz are from University of Michigan Radio Observatory. Details of the observing system and data reduction can be found in Aller et al. (1985). Data at 22, 37, and 87 GHz are from Metsähovi Radio Observatory (Salonen et al. 1987; Teräsranta et al. 1992, 1998, 2004, 2005; Nieppola et al. 2007). The data reduction is described in Teräsranta et al. (1998). The high-frequency data at 90 and 230 GHz were obtained at the Swedish-ESO Submillimetre Telescope (SEST) in La Silla, Chile, from 1987 to 2003 (Tornikoski et al. 1996, and some unpublished data), and also collected from the literature (Steppe et al. 1988, 1992, 1993; Reuter et al. 1997). The 87 GHz archival data from Metsähovi were combined with the 90 GHz data to form the 90 GHz flux curve.

3. METHODS

3.1. Timescales

The long-term timescales of our sample have been calculated in Hovatta et al. (2007), where our sample represented the BLO class in comparison with other AGN subgroups. In this paper, we report the individual timescales of the BLOs. The timescales have been calculated in three ways: using the structure function (SF), the discrete correlation function (DCF), and the Lomb–Scargle periodogram. DCF and periodogram may provide several timescales of different durations. In this paper, we will concentrate only on the most significant ones. The theoretical aspects of the methods are discussed in Hovatta et al. (2007) and the references therein. In the interest of comparing them with the observed flux curves, we have kept the computational timescales in the observer’s frame throughout the paper and have not performed any redshift or Doppler corrections on them.

The three methods ($T_{SF}$, $T_{DCF}$, and $T_P$) respond to flux variations of different scales. SF is the most sensitive, picking up the short timescales and the structure of the flares, such as the rise and decay times. DCF and L-S periodogram are meant to provide the timescale of longer variations, like the peak-to-peak intervals of major flares. The periodogram was originally developed to search for strict, sinusoidal periodicities. In this context, it is used to give a characteristic timescale, and a result from the periodogram analysis does not mean that the source is strictly periodic. The distinctions of SF, DCF, and periodogram are also thoroughly discussed in Hovatta et al. (2007).

3.2. Flare Parameters

For comparison with the computational timescales, we determined some flare parameters directly from the observations. We use the word “flare” to describe a separate period of heightened
We have not separated the individual shocks contributing to the flux density rise, so in some cases one flare may include several components but this is more pronounced in the low-frequency domain. The start and end times, and thus also the duration, of the flares are based on a careful visual estimation of each flux curve. Their error is dependent on the sampling frequency: with well-sampled flux curves the determination of the flares is more accurate. In 90 and 230 GHz, the sampling was often too poor to allow the definition of the flares, which is why these frequencies are, in many cases, at least partly excluded from the analysis. A flare was included in the analysis if it was discernible at least at two frequencies, one of them representing the most significant timescale. In the case of the periodogram, the highest peak after the DCF has been on the negative side, as the representative timescale. In the DCF analyses, we have chosen the first discernible peak, which lasted from the very beginning of monitoring in the late 1970s–early 1980s, and the decay stage has been recorded at a timescale that was longer than half of the total observing period. Some flares were not discernible at all frequencies, and outbursts of different magnitudes, which results in many different timescales obtained in DCF and periodogram analyses.

There are some sources which have only a lower limit of their SF, which is very faint and exhibit a relatively uneventful flux curve. In that case, the errors of the flux measurements are large compared to the flux densities. This leaves a considerable margin of error in the modest flux rises and falls, and adds to the uncertainties of the timescales. Second, some objects exhibit rapid variability and outbursts of different magnitudes, which results in many different timescales obtained in DCF and periodogram analyses. In the DCF analyses, we have chosen the first discernible peak, after the DCF has been on the negative side, as the representative timescale. In the case of the periodogram, the highest peak represents the most significant timescale. If such a peak occurred at a timescale that was longer than half of the total observing period, it was discarded to avoid spurious timescales. In some cases, the most significant DCF timescale is, for example, the period, it was discarded to avoid spurious timescales. In some cases, the highest peak after the DCF has been on the negative side, as the representative timescale. In the case of the periodogram, the highest peak represents the most significant timescale.
Table 2
The Most Significant Timescales Obtained for the Sample Sources Using the Structure Function ($T_{SF}$), the Discrete Correlation Function ($T_{DCF}$), and the Lomb–Scargle Periodogram ($T_P$)

<table>
<thead>
<tr>
<th>Source Alias</th>
<th>$v$ (GHz)</th>
<th>$T_{SF}$ (year)</th>
<th>$T_{DCF}$ (year)</th>
<th>$T_P$ (year)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>0109+224</td>
<td>4.8</td>
<td>1.358</td>
<td>1.848</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>0109+224</td>
<td>8</td>
<td>4.817</td>
<td>7.324</td>
<td>...</td>
<td>1</td>
</tr>
<tr>
<td>0109+224</td>
<td>14.5</td>
<td>1.078</td>
<td>5.818</td>
<td>...</td>
<td>1.2</td>
</tr>
<tr>
<td>0109+224</td>
<td>22</td>
<td>0.857</td>
<td>7.734</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>0109+224</td>
<td>37</td>
<td>1.918</td>
<td>2.943</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Notes. All time scales are in the observer's frame. 1, faint source; 2, multiple timescales; 3, exceptionally large errors in $T_{DCF}$. See the text for details. (This table is available in its entirety in machine-readable and Virtual Observatory (VO) forms in the online journal. A portion is shown here for guidance regarding its form and content.)

Studying the timescales of Table 2 more closely, we find that three objects, namely S5 0716+714, Mark 421, and PKS 1749+096, are among the five sources with the shortest timescales, independent of the method of calculation. Therefore, they can well be dubbed the BLOs with the fastest variability. Their shortest timescales vary from 70 days ($T_{SF}$ for S5 0716+714 at 14.5 GHz) to 2.8 years ($T_P$ for PKS 0754+100 at 37 GHz). It is noteworthy that Mark 421 has very low flux levels and the definition of the timescales suffers, as explained earlier. Other short timescale sources include OJ 287 (according to both SF and periodogram), PKS 0754+100, S4 0954+65, Mark 501, and 3C 371.0.

The objects 3C 446 and 1308+326 (according to both DCF and periodogram), OJ 425 (according to both SF and periodogram), as well as PKS 0735+178, ON 231, and 4C 14.60 seem to be good examples of sources with long timescales and slow variability. They have longest timescales ranging between 6.8 ($T_{SF}$ for ON 231 at 4.8 GHz) and 15.2 years ($T_P$ for 3C 446 at 8 GHz).

In general, the DCF and periodogram timescales have quite a good correspondence, and the SF timescales are clearly shorter (for more information on the correspondence of the timescales in general, see Hovatta et al. 2007). For some sources, however, the differences between the timescales obtained with the three methods can be large. Usually this is because the most significant timescale is defined differently for these methods. In most cases when the periodogram and DCF timescales differ significantly, a similar timescale to $T_P$ has been seen also in the DCF but is has not been the most significant one. In some cases, the different frequency bands have strikingly dissimilar values. This is typically due to the faintness of the source and the low amplitude variability, which can make the determination of a timescale a difficult task and overemphasize the influence of discrepant data points in the flux curve.

The mean values of $T_{DCF}$, $T_{SF}$, and $T_P$ for various AGN subgroups, including BLOs, are reported in Hovatta et al. (2007).

5. OBSERVED RADIO OUTBURSTS

5.1. Flare Morphology

There are 13 BLOs which exhibit significant flaring during our monitoring period. The flux curves of the flaring sources are available in Figure 2 (Figures 2.2–2.13 are available in the online version of the journal), where each flare, identified at 22 or 37 GHz, is marked. It is evident that the flux curves are very diverse in morphology.

designed to distinguish the shortest timescales of variability, lengthening the timescale of these minor variations. As a result, most of the SF timescales calculated for ON 231 give only lower limits. Another example of a lower limit is the 22 GHz SF timescale of OQ 530 (see for Figure 1 bottom panel for the flux curve at 22 GHz). Although the flux curves at 22 and 37 GHz are very similar, and time windows are comparable, the SF gives differing results. However, the structure of the SF is very complicated. At 22 GHz, hints of shorter timescales can be seen, but their plateaus in the SF are not clear enough to be picked as the representative timescale. Also, the 90 GHz DCF timescale of 1308+326 has exceptionally large errors, and the value should be treated with caution.
Figure 2. Flux curves of objects in the flare analysis. The peak of each flare included in the analysis is marked in the curve.

(An extended version of this figure is available in the online journal.)

5.1.1. Sample Means by Source and Frequency

The mean values of flare duration, rise time, decay time, absolute peak flux, and relative peak flux, determined as described in Section 3.2 are listed in Table 3 for each of the 13 sources.

The parameters have been calculated as an average for all frequency bands, and for 37 GHz separately for comparison. In one end, we have objects such as AO 0235+164, OJ 287, PKS 1749+096, and BL Lac itself with rapid and frequent fluctuations. In the other end of the range, we find PKS 0735+178, 1308+326, and 3C 446 which have flares that last for several years, with only a couple of them covered by the span of our observations. In fact, there is much doubt about the nature of the latter objects. 1308+326 and 3C 446 were originally included in our BLO master list because several authors have classified them as borderline cases between BLOs and quasars (Gabuzda et al. 1993; Falomo et al. 1994; Laurent-Muehleisen et al. 1999; Aller et al. 1999). Later, they have been listed as quasars in the Veron-Cetty & Veron Catalogs. While listed as BLO in Veron-Cetty & Veron (2006), PKS 0735+178 exhibits a similar type of radio flux curve.

The typical BLO flare has a measured peak flux well below 10 Jy, as seen in Table 3. The average relative peak fluxes are mostly below 5 Jy. The brightest flares in our sample are those of 3C 446, measured in both absolute and relative fluxes.

In Table 4, we present the minimum, maximum, mean, and median values of flare duration and absolute and relative peak fluxes for each frequency used in our analysis. We calculated the relative peak fluxes in two different ways, first by subtraction ($S_{\text{max}} - S_{\text{min}}$) and second by division ($S_{\text{max}}/S_{\text{min}}$). The latter can be considered as a variability index for each flare. In the discussion of shock models (e.g., Section 7) the first one is used. The duration and absolute peak flux of the flares are also tabulated in two different ways. We show the absolute duration in years and absolute peak flux in janskys for all the frequency bands and also the values of each individual flare normalized to the value at 22 GHz (in the calculation of the durations this was not possible for flare 2 of 1308+326 and flare 7 of BL Lac because their duration at 22 GHz could not be calculated).

There were 34 flares in total at 4.8 GHz, 38 at 8 GHz, 45 at 14.5, 22, and 37 GHz, 17 at 90 GHz, and 8 at 230 GHz. In duration, the difference between the minimum and maximum values is vast. The majority of the flares are relatively short in duration for radio band events, and sources PKS 0735+178 and 1308+326 alone have flares extending over six years, as we already learned from Table 3. This can be seen in the median values in Table 4, which are between 2.3 and 2.7 years. The duration of the flares changes little with frequency which can clearly be seen in the relative durations. The absolute median values are slightly longer at 4.8 GHz and 8 GHz than in the higher frequencies, but only by 0.4 years at most. However, at 90 and 230 GHz the sparser sampling of the flux curves does not allow as accurate determination of the flare duration as the frequent sampling of the lower frequencies. In reality, the mean duration of the 90 and 230 GHz flares may be slightly shorter.

The absolute peak flux exhibits a stronger correlation with frequency. The flare peak fluxes range between 0.7 and 12.1 Jy. The median values rise with frequency up to 5.1 Jy at 37 GHz which is also seen when the normalized absolute peak fluxes are studied. In 90 and 230 GHz, the median peaks are more moderate, 4.4 and 3.2 Jy, respectively, corresponding to 90% and 67% of the flux at 22 GHz. Also in this case, the sparse sampling of the highest frequencies has its effect: with more data points their median peak fluxes might be higher. The parameters of the relative flux behave roughly in the same manner. The relative fluxes of BLO flares range between 0.4 and 10.1 Jy. It is also seen that maximum fluxes are 1.4–18.5 times higher than minimum fluxes.
Table 3
The Number of Flares Included in the Analysis and the Mean Values of the Duration, Rise Time, Decay Time, Absolute Peak Flux and Relative Peak Flux for the 13 Flaring Sources in Our Sample

<table>
<thead>
<tr>
<th>Source</th>
<th>Frequency</th>
<th>Number of Flares</th>
<th>Duration (year)</th>
<th>Rise Time (year)</th>
<th>Decay Time (year)</th>
<th>Absolute Peak Flux (Jy)</th>
<th>Relative peak flux (Jy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2 0109+22</td>
<td>All 37 GHz</td>
<td>3</td>
<td>3.9</td>
<td>1.3</td>
<td>2.6</td>
<td>1.6</td>
<td>1.1</td>
</tr>
<tr>
<td>AO 0235+164</td>
<td>All 37 GHz</td>
<td>4</td>
<td>2.3</td>
<td>1.2</td>
<td>1.1</td>
<td>4.5</td>
<td>3.6</td>
</tr>
<tr>
<td>PKS 0422+004</td>
<td>All 37 GHz</td>
<td>2</td>
<td>3.3</td>
<td>1.5</td>
<td>1.7</td>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>S5 0716+714</td>
<td>All 37 GHz</td>
<td>2</td>
<td>3.0</td>
<td>1.2</td>
<td>1.8</td>
<td>4.4</td>
<td>4.1</td>
</tr>
<tr>
<td>PKS 0735+178</td>
<td>All 37 GHz</td>
<td>1</td>
<td>10.8</td>
<td>2.9</td>
<td>7.8</td>
<td>4.6</td>
<td>3.5</td>
</tr>
<tr>
<td>PKS 0754+100</td>
<td>All 37 GHz</td>
<td>2</td>
<td>3.4</td>
<td>1.5</td>
<td>2.0</td>
<td>2.3</td>
<td>1.3</td>
</tr>
<tr>
<td>OJ 287</td>
<td>All 37 GHz</td>
<td>9</td>
<td>1.4</td>
<td>0.7</td>
<td>0.7</td>
<td>4.9</td>
<td>2.7</td>
</tr>
<tr>
<td>1308+326</td>
<td>All 37 GHz</td>
<td>2</td>
<td>12.8</td>
<td>5.9</td>
<td>6.9</td>
<td>4.1</td>
<td>3.5</td>
</tr>
<tr>
<td>PKS 1413+135</td>
<td>All 37 GHz</td>
<td>2</td>
<td>4.8</td>
<td>2.2</td>
<td>2.5</td>
<td>3.0</td>
<td>2.2</td>
</tr>
<tr>
<td>PKS 1749+096</td>
<td>All 37 GHz</td>
<td>5</td>
<td>1.7</td>
<td>0.9</td>
<td>0.8</td>
<td>6.4</td>
<td>4.3</td>
</tr>
<tr>
<td>S5 2007+77</td>
<td>All 37 GHz</td>
<td>1</td>
<td>3.3</td>
<td>1.5</td>
<td>1.8</td>
<td>3.3</td>
<td>2.1</td>
</tr>
<tr>
<td>BL Lac</td>
<td>All 37 GHz</td>
<td>9</td>
<td>1.8</td>
<td>0.9</td>
<td>0.9</td>
<td>5.0</td>
<td>2.9</td>
</tr>
<tr>
<td>3C 446</td>
<td>All 37 GHz</td>
<td>3</td>
<td>5.8</td>
<td>2.7</td>
<td>3.1</td>
<td>7.8</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Note. The Parameters have been calculated for all frequency bands and separately for 37 GHz.

Table 4
Minimum, Maximum, Mean, and Median Values of Flare Duration and Absolute and Relative Peak Fluxes (both $S_{\text{max}} - S_{\text{min}}$ and $S_{\text{max}}/S_{\text{min}}$) for All Frequencies Used in the Analysis

<table>
<thead>
<tr>
<th>$\nu$ (GHz)</th>
<th>Duration (year)</th>
<th>Normalized Duration</th>
<th>$\nu$ (GHz)</th>
<th>Absolute Peak Flux (Jy)</th>
<th>Normalized Abs. Peak Flux</th>
<th>$\nu$ (GHz)</th>
<th>Relative Peak Flux (Jy)</th>
<th>Relative Peak Flux ($S_{\text{max}}/S_{\text{min}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8</td>
<td>0.6</td>
<td>12.7</td>
<td>3.5</td>
<td>2.7</td>
<td>0.4</td>
<td>2.2</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>8</td>
<td>0.7</td>
<td>12.8</td>
<td>3.4</td>
<td>2.7</td>
<td>0.4</td>
<td>3.2</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>14.5</td>
<td>0.3</td>
<td>12.4</td>
<td>3.0</td>
<td>2.4</td>
<td>0.7</td>
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<tr>
<td>22</td>
<td>0.3</td>
<td>13.0</td>
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<tr>
<td>37</td>
<td>0.4</td>
<td>13.2</td>
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<td>90</td>
<td>0.6</td>
<td>10.3</td>
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<tr>
<td>230</td>
<td>0.9</td>
<td>9.9</td>
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Note. Values are also shown for duration and absolute flux of the flares normalized to the values at 22 GHz.
5.1.2. Flare Intensity Versus Duration

In Figure 3, we have plotted the absolute peak flux, $S_o$, of each flare against the flare duration, $t_o$, using all frequencies (top panel) and 37 GHz only (bottom panel). The distribution of the data points seems to be bimodal, with a dividing line running from the upper left corner to the lower right. On a closer inspection, we find that the data points in the upper right are those of the quasar-like objects PKS 0735+178, 1308+326, and 3C 446 (see Section 5.1.1). Their long and intense outbursts thus clearly differ from the typical BLO flares. Only one flare of 3C 446 is more BLO-like, as can be seen in Figure 3.

We also find a distinct declining trend in both panels of Figure 3. It is evident in both the typical BLOs and quasarlike objects, but less so in the latter, however, due to the one weak and short flare of 3C 446. The correlation for the “genuine” BLOs is significant also according to the Spearman rank correlation test. When all frequencies are considered, the Spearman correlation coefficient is $\rho = -0.238$ and the probability of no correlation $P = 0.001$. The strength of the correlation is slightly distorted by the fact that all the frequencies are included in the calculation, and thus every flare is counted for more than once. We also checked its significance at 4.8, 8, 14.5, 22, and 37 GHz separately. In this case, the significance of the correlation seems to vanish.

A natural explanation for the negative trend observed in Figure 3 is the effect of Doppler boosting. In Equation (1), the Doppler boosting factor is defined by the Lorentz factor of the jet flow $\Gamma$, speed of the jet $\beta$, and the viewing angle to the line of sight of the observer $\theta$:

$$D = \frac{1}{\Gamma(1 - \beta \cos \theta)}.$$  (1)

As the boosting factor $D$ increases, the internal timescales of the source get shorter and flux levels become higher. To better investigate the intrinsic properties of the sources, we plotted the Doppler-corrected peak luminosity, $L_i$, of each flare against the Doppler-corrected duration, $t_i$, of the flare (Figure 4). The corrections and luminosity calculation were performed with equations (see, e.g., Kembhavi & Narlikar 1999, but note the typing error in their Equation (3.102))

$$t_i = \left(\frac{D_{\text{var}}}{1+z}\right) t_o$$  (2)

and

$$L_i = \left(\frac{1+z}{D_{\text{var}}}\right)^{3+\alpha} \frac{4\pi d_i^2}{1+z} S_o,$$  (3)

where $z$ stands for redshift and $d_i$ for the luminosity distance. The subscripts $i$ and $o$ denote intrinsic and observational quantities, respectively. The Doppler factors $D_{\text{var}}$ were taken from Hovatta et al. (2009), where they have been determined from our extensive database of total flux density observations at 22 and 37 GHz in the same manner as in Lähteenmäki & Valtaoja (1999). Applying an exponential fit to individual shock
Figure 5. Ratio of the decay ($\Delta t_D$) and rise ($\Delta t_R$) times of all flares plotted against the duration of the flares. All values are Doppler-corrected. In the top panel, all flares are included and in the bottom panel, the source-specific mean values are plotted. For clarity, also the line $\Delta t_D / \Delta t_R = 1$, where the rise and decay times are of equal length, is included.

components extracted from the flux curves gives the observed variability brightness temperature. Comparing it with the intrinsic brightness temperature gives the amount of boosting. In Equation (3), we have assumed an evolving feature in the jet, in which the intrinsic brightness temperature gives the amount of boosting. In variability brightness temperature. Comparing it with the intrinsic brightness temperature gives the observed variability brightness temperature.

In Figure 5, where the values have been corrected for relativistic boost according to Equation (2). In the top panel, all flares in all frequencies are included, and in the bottom panel only the source-specific mean values are plotted. In the top panel, the individual flares cover the available parameter space quite well. There are many flares in which the decay time is several times longer than the rise time and the mean ratio for all the flares is 1.61. The mean ratio for flares with decay time longer than the rise time is 2.32 while for sources with decay time shorter than the rise time the mean ratio is 0.67. In the case of the the source-specific mean values, however, the differences are more moderate, the average ratio being 1.59. This value corresponds reasonably well to the value 1.3 used by Valtaoja et al. (1999) in the exponential decomposition of radio flares. The difference between the values can be explained by different flare definitions. In Valtaoja et al. (1999), individual shock components are used while in our approach an activity phase (which may include several shocks) is considered as a flare. On average, the decay times are longer than the rise times of the flares for all sources. However, as the top panel of Figure 5 shows, one source can have flares of very diverse characteristics.

5.2. Time Lags of the Flare Peaks

We made a qualitative analysis of the individual flares of our sample sources, tracking the order in which the flare moved from frequency band to the next and tracing the evolution of the peak flux from its maximum value. We used all available frequencies for each source. In flare 1 of 1308+326 and flare 3 of 3C 446, we took into account only the first component of the flare, although in some frequency bands other components may be stronger. Of the 45 flares included in our sample, 11 (24%) were consistent with the description of a high-peaking flare: the high frequencies peak first with the highest peak fluxes. There were nine (20%) more that were very nearly consistent, for example, with one frequency band peaking "too early." In four (9%) flares, we possibly detected also the plateau stage in the high radio bands after which the flare turns into high peaking. There was only one (2%) flare that was consistent with the low-peaking flare behavior (flare 9 of OJ 287, defined only at three frequency bands), and one that was nearly consistent (flare 4 of BL Lac). Four flares were entirely inconsistent with the shock model, having no sensible order in either the frequency or flux evolution. However, all these four flares occurred in sources with very fast and frequent variability (S5 0716+714, OJ 287, PKS 1749+096, and BL Lac), which means that the different flare components are particularly hard to separate. It is possible that this has affected our analysis. For a third of our flare sample, 15 flares in all, we could make no meaningful analysis of the time lags at all because of the sparse sampling of the flares.

We also calculated the time lags for 27 flares with a sufficiently resolved structure. For some of them, time lags could be determined for only 2 or 3 frequency bands. The mean values of these tentative time delays range from roughly 10 days to 130 days in the observer’s frame at 37 and 4.8 GHz, respectively. The mean and median values of errors in the peak times, in turn, are of the order of 28 and 10 days, respectively, for the whole sample. Thus, the precise time lags cannot be determined, but we chose rather to examine the sequence in which the flare peak reaches each frequency. Each frequency band was assigned a rank number. The first frequency band which displayed the flare was ranked 1, the second was ranked 2, and so on.

Figure 6 shows a bubble plot of the peak time rank number plotted against frequency band. The size of each bubble is proportional to the number of cases having the same values
of rank number and frequency. According to the general shock model of Valtaoja et al. (1992), in high-peaking flares, the higher frequencies peak first with the low frequencies following with increasing time delays. That is, if the flares followed the shock model precisely, we would see a negative correlation in Figure 6, with the highest frequency peaking first (and having the lowest rank number) and other frequencies following in order. In Figure 6, there certainly is a negative trend and the high frequencies have lower rank numbers on average. A significant negative correlation is verified by the Spearman rank correlation test ($\rho = -0.567$ and $P < 0.0005$). In about half of the flares included in this analysis, the highest frequency band was the leading frequency band.

We plotted another bubble plot (Figure 7) describing the dependence between the rank orders of relative peak fluxes ($S_{\text{max}} - S_{\text{min}}$) and the frequency. Of the relative fluxes, the highest was ranked first. According to the shock model, there should be again a negative correlation, which is indeed very strong ($\rho = -0.592$ and $P < 0.0005$ in the Spearman test). In the two highest frequency bands, 90 and 230 GHz, there are some stray data points in the high ranks. There is a strong possibility that the flares have not been detected in these frequencies in their full strength due to sparse sampling. The plot is very similar when the ratio of maximum and minimum fluxes is used as the relative flux.

One should, however, bear in mind that the unambiguous definition of the time lag of the flare peak from one frequency band to another is made difficult by the complex structure of some of the flares. When there are several flare components superposed on each other, it can be tricky to trace the evolution of just one of them. Also, the error range in the peak time can be substantial, depending on the sampling density.

6. THE CORRESPONDENCE BETWEEN TIME SCALES AND FLARE PARAMETERS

6.1. Notes on Individual Sources

In the following, we present a brief description of the flaring sources and their behavior at all available frequency bands individually (flux curves of the sources are available in the electronic edition of the Journal). We also compare their flux curves to the timescales discussed in Section 4.

**S2 0109+22.** Three distinct flares can be discerned at 4.8, 8, 14.5, 22, and 37 GHz; the higher frequencies are too sparsely sampled. The first two flares, peaking in 1993 and 1998, lasted for about three years, while the last one, in 2000, lasted for up to six years in the low frequencies. All three were relatively weak; the 1998 flare had the highest peak flux of 3.13 Jy at 37 GHz. The third flare reached its peak in approximately one year, depending on the frequency, but took up to 5.6 years to decrease back to the base level flux. The flares consist of multiple components, which makes it difficult to find the peak especially in the low-frequency flux curves. In the first flare the 8 GHz flux peaks first, higher frequencies follow within a month. The 4.8 GHz flux peaks almost six months later. In the second one, 37 GHz leads, with the other frequencies peaking almost a year later, and 4.8 GHz being the last one again with a time lag of over a year.

The $T_{\text{SF}}$ values for S2 0109+22 range from 0.86 (22 GHz) to 4.817 (8 GHz) years. In 22 GHz the flares are much stronger, and the SF is affected by the relatively well sampled and fast rises and falls of the flares. In 8 GHz, the flux curve is less dramatic, and this can be seen as a longer $T_{\text{SF}}$. In 8, 14.5, and 22 GHz, the DCF seems to pick up the interval between flares 1 and 3, giving timescales between 5.82 and 7.73 years. At 4.8 GHz, where the peaks of the flares are barely discernible, $T_{\text{DCF}} = 1.85$ years. At 37 GHz, the three flares are clearly above the base level flux, and more evenly separated. This affects also the DCF, which now gives a timescale of 2.94 years, a little more than half of the interval between flares 1 and 2, and almost exactly the interval between flares 2 and 3.

**AO 0235+164.** Here we have four distinguishable multifrequency flares in 1987, 1990, 1992, and 1998. The maximum peak fluxes are 4.44 Jy at 14.5 GHz, 4.20 Jy at 90 GHz, 6.88 Jy at 37 GHz, and 5.56 Jy at 37 GHz, respectively. The flares have some substructure, but the peaks are still quite clearly defined. Although relatively intense, the flares last for three years or less in all frequencies. We were able to determine time lags for two flares. In both, the flare commences at 230 GHz. The other frequencies follow some tens of days later in a rather random order. The longest time delay, 104 days, is at 8 GHz in the 1987 flare, where 230 GHz was the leading frequency band.

The high level of activity in the total flux density flux curve for AO 0235+164 is reflected in its SF timescales, which are mostly below one year. Only at 37 GHz, we find the most significant
timescale to be $T_{SF} = 2.71$ years, but another timescale of $T_{SF} = 0.87$ years is also seen in the SF. The approximate intervals between flares 1 and 4 are four, two, and six years. We find roughly the same numbers listed in Table 2 as $T_{DCF}$ and $T_F$. In most cases, the most significant timescale is around 5.5 years. In 4.8 GHz, the periodogram picks up an 11.5-year timescale, which is twice the $T_F$ of the other frequencies (see also Section 4).

**PKS 0422+004.** The flux curves are quite poorly sampled, but two events can be discerned. One flare took place in 1994 with a peak flux of 1.37 Jy and another in late 2001 with a peak flux of 2.40 Jy. These low flux levels make every small variation stand out, hindering the clear definition of the flare. In the first the duration ranges from 2.3 to 6 years, depending on the frequency band, while the second one lasted for about two years. In the second flare, there are no long time lags, all available frequencies peak within 37 days.

The $T_{SF}$ obtained for PKS 0422+004 are mostly lower limits of approximately ten years. For 14.5 and 37 GHz, we got $T_{SF} = 3.83$ and $T_{SF} = 2.71$, respectively. The DCF did not produce any timescales at 22 and 37 GHz, and there were no timescales in the periodogram analyses of any of the frequency bands. In the lower frequencies, $T_{DCF} = 6–7$ years, which corresponds well to the interval of flares 1 and 2, which is about seven years.

**S5 0716+714.** This source also has two multifrequency flares, in late 1998 and 2003. Both flares were monitored in radio frequencies up to 37 GHz, unfortunately high-frequency data are missing. The first one peaked at 2.54 Jy; while the second one was considerably stronger at 6.28 Jy. Both peak fluxes occurred at 37 GHz. The latter flare was extensively monitored in a WEBT multifrequency campaign, including INTEGRAL (Ostero et al. 2006). It was very fast, lasting approximately for 2.5 years. It is noteworthy that the absolute peak flux of the 2003 flare is strongly dependent on frequency. At 4.8 GHz it is only 1.9 Jy, and the flare barely stand out from the base level flux. From there on, it steadily rises at each frequency to exceed 6 Jy at 37 GHz. The same effect can be seen in the 1998 flare, albeit to a lesser extent.

The SF timescales of S5 0716+714 seem to be either very fast ($T_{SF} \leq 1$ for 4.8, 14.5, and 37 GHz) or very long ($T_{SF} \geq 6$ for 8, 22, and 90 GHz). This discrepancy is most likely due to the sparser sampling of the prominent 2003 flare in 8, 22, and 90 GHz. Especially at 37 GHz, it is very frequently sampled, and thus dominates the flux curve and timescales. The DCF and periodogram timescales are close to five years in 4.8 and 8 GHz, and roughly two years in the higher frequencies. The interval between flares 1 and 2 is of the order of five years. Thus, the two flares seem to define the timescales in the low frequencies, where they are very low in amplitude, but fail to do so from 14.5 GHz upward, where they are very strong. This is probably because of the increasing dominance of the 2003 flare, while at the same time the sampling in the beginning of the flux curve gets poorer in the higher frequencies.

**PKS 0735+178.** The flux curve is dominated by a single, double-peaked flare in all available frequencies. Frequent sampling and reasonably low short-term variability make its definition simple. The first component of the double peak was the stronger one in all frequencies except 4.8 GHz. The peak occurred in most frequencies in 1989, and the peak flux was 5.30 Jy at 14.5 GHz. The duration of the flare was notably long, ranging from 10 to 12 years. The flux decline in particular was slow, lasting approximately eight years.

The SF timescales, determined for frequencies up to 90 GHz, are diverse, ranging between 2.15 and 6.07 years. They are quite long, and the reason is evident in the flux curves: PKS 0735+178 has minimal short-term flux variation. $T_F$ is available for only one frequency, 8 GHz, being 14.11 years. This clearly reflects the modest pace of the fluctuations of the flux curve. As the source only has one flare, it is impossible to comment on the compatibility of the flare intervals and timescales. The interval between the components of the double peak of flare 1 is of the order of 1.5 years at all frequencies, which is distinctly less than any timescale obtained in this analysis.

**PKS 0754+100.** This source exhibits two modest flares that have multifrequency data: one peaking in late 1996 and one in 2003. The peaks can be discerned quite effortlessly, although the flux levels are not very high. The first flare peaks at 2.94 Jy at 37 GHz and the latter at 2.57 Jy at 14.5 GHz. There was a more intense flare in the mid-1980s, peaking over 3 Jy at 8 and 14.5 GHz, but it was not monitored in Metsähovi and thus is not included in our analysis.

The $T_{SF}$, $T_{DCF}$, and $T_F$ values could be determined for all frequencies up to 37 GHz. They are not very consistent: $T_{SF}$ is approximately 0.5–4.3 years; both $T_{DCF}$ and $T_F$ are approximately 2.8–10.8 years. There is also a clear discrepancy between the high and low frequencies, the latter having much shorter timescales. Much of the inconsistencies are due to the 1980s flare which is included in the 4.8–14.5 GHz data, but missing from 22 and 37 GHz data. The flare was strong and broad, and thus has lengthened the low-frequency timescales. The interval between the two flares accounted for in this analysis is approximately 6.5 years at all frequencies, best reflected by the 22 GHz timescales. The longest timescale, $T_{DCF} = 10.88$ years is found at 14.5 GHz. In that frequency band, the peak of flare 1 is particularly well sampled and strong, probably strengthening the timescale corresponding to the interval between the 1980s flare and flare 1, which is roughly 12 years.

**OJ 287.** Being one of the most-studied sources in the Tuorla-Metsähovi observing project, OJ 287 has ample data. Its flux curves in all frequencies are characterized by vigorous variability superposed on a long-term fluctuation of the base level flux. The yearly mean values of the flux density at 37 GHz range between almost 8 Jy and less than 2 Jy. The determination of single flares is very difficult due to the sheer number of them. We count as many as nine multifrequency outbursts during the 25 years of our monitoring. They are quite brief in duration, typically lasting for less than two years, or, in many cases, less than a year. The highest peak flux, 9.63 Jy, occurred in 1983 at 22 GHz. The undertakings of OJ 287 are of special interest because of its claimed optical periodicity. This almost 12-year periodicity is thought to be possibly the result of a binary black hole interaction (Sillanpää et al. 1988; Lehto & Valtonen 1996; Valtonen et al. 2008).

The rapid and frequent outbursts result in very short SF timescales, between 0.2 and 0.5 years at all frequencies. The DCF timescales, however, determined for 8 and 37–230 GHz, are considerably longer. They range approximately between 4 and 6.5 years. The DCF probably picks up also the base fluctuation, which can have a timescale as long as 20 years. The sole $T_F$ value that could be calculated was 1.03 years at 90 GHz. It describes the source well: the calculation of the flare intervals gives mostly values below two years. The intervals
between flares 1 and 2 as well as 6 and 7 are of the order of five years. There are, however, minor flares evident in the flux curves during those intervals as well, but they are not included in this analysis.

1308+326. This source had a flare that lasted for the entire 1990s. It peaked in 1992 at 4.37 Jy at 22 GHz. In the lower frequencies it was clearly double peaked, and had several components also in 22 and 37 GHz. After a slow decline, the flux levels soared again in 2003, reaching a peak flux of 3.5 Jy at 14.5 GHz. An optical period for the source has been reported (Fan et al. 2002), but no periodicity in the radio data has been detected to our knowledge.

The interval between the two flares of 1308+326 is eight years at 8 GHz and roughly 11 years at 14.5–37 GHz. This is evident in the $T_{\text{DCF}}$ values, which are between 10.34 and 11.70 years. Pyatunina et al. (2007) derived an activity cycle of $\geq 14$ years for this source, which also reflects its long timescales of variability. At 90 GHz, $T_{\text{DCF}} = 3.77$ years, but this value has exceptionally large errors and is based on a poorly sampled flux curve where the flares cannot be discerned properly. The periodogram gives mostly similar results as DCF. At 8 GHz $T_{\text{p}} = 14.59$ years, which is quite high compared to the $T_{\text{DCF}}$ values. The SF timescales range as $T_{\text{SF}} = 2.71–4.29$, telling that 1308+326 has little short-term variability.

PKS 1413+135. The flux curves are very erratic, making it difficult to find well-defined outbursts. Two multifrequency events can be discerned. The first flare peaks at 4.55 Jy at 37 GHz in the end of the year 1990. The second peaks at the same frequency at 2.46 Jy seven years later, in 1997. The peak of the latter flare in particular is ill-defined. There is a clear flux boost lasting for more than six years, during which there are several minor maxima in all frequencies. There is also a pronounced flare visible in the low-frequency data in the early 1980s, but the high-frequency data are missing.

Also noteworthy is the strong evolution of the flux curve with frequency. At 4.8 GHz, the flux curve is flat, but gradually it gets more eventful toward higher frequencies. This is reflected by the timescales. Both $T_{\text{SF}}$ and $T_{\text{DCF}}$ are clearly longer in the low frequencies. $T_{\text{SF}}$ is 2.71 years at 8 GHz and 1.51 years at 37 GHz; $T_{\text{DCF}}$ is 9.38 years at 4.8 and 8 GHz, declining slowly to 2.26 years at 90 GHz. Curiously enough, $T_{\text{p}}$ values seem to remain unaffected by the evolution of the flux curve with frequency, ranging between roughly 7 and 9 years. In addition, the 14.5 GHz timescales are affected by the 1980s flare which is not included in the high-frequency data. The intervals of the flares are hard to determine because of the ambiguous definition of the peak of the second flare. At 8 GHz it is roughly four years, at 14.5 GHz two years, and at 22 and 37 GHz roughly seven years. Thus, it is approximately comparable to $T_{\text{p}}$, except at 14.5 GHz.

PKS 1749+096. This object exhibits violent variability. There are five multifrequency flares peaking in 1993, 1995, 1998, 2001, and 2002. The 1993 flare had the highest peak recorded in our data, 12.07 Jy at 90 GHz. Given the relative brevity of the flares and their intensity, PKS 1749+096 also exhibits some of the fastest grand-scale rises and declines of the flux. For example, in the 1993 flare it reached the peak flux in approximately 80 days, rising over 9 Jy. While normally a flat spectrum source, as BLOs in general, this object has been reported to have an inverted spectrum during outbursts (Torniainen et al. 2005).

PKS 1749+096 is another example of a flux curve evolving with increasing frequency. The flares, barely discernible at 4.8 GHz, get more intense at higher frequencies. As in the case of PKS 1413+135, the evolution is reflected in the timescales, albeit less clearly. Mostly it can be seen in the $T_{\text{DCF}}$ values. They decline from 4.45 years at 4.8 GHz to 1.30 years at 90 GHz. At 8 GHz, $T_{\text{DCF}}$ is the longest, 6.78 years. The intervals between the flares 1 and 4 are roughly 2–3 years. The interval between flares 4 and 5 is approximately one year. Also, as with PKS 1413+135, the $T_{\text{p}}$ seem to be less affected by the changing behavior of the flux curve with growing frequency. $T_{\text{p}}$ has its highest value at 37 GHz, where $T_{\text{p}} = 9.81$. The SF timescales are quite short, below two years, except for 8 GHz ($T_{\text{SF}} = 2.15$) and 90 GHz ($T_{\text{SF}} = 3.04$). The beginning of the flux curve is particularly well sampled and relatively stable at 8 GHz, which probably lengthens $T_{\text{SF}}$ compared to the high frequencies, which have less data. On the whole, the timescales of 14.5 and 22 GHz seem to correspond best to the flares included in the analysis.

S5 2007+77. This BLO has one outburst, which was reasonably well monitored also at the Metsähovi frequencies. It occurred in 1991–1992 and reached a peak flux of 3.69 Jy at 14.5 GHz. The low-frequency flux curves reveal that the source was variable also prior to that time, but since the early 1990s it has been in a quiescent state with only modest variability. Unfortunately, the flux curves are not very well sampled and data from 8 GHz and the very highest frequencies, 90 and 230 GHz, are missing completely from the flare analysis.

The SF timescales are mostly the typical 0.5–2 years, only at 22 GHz it is as long as 3.83 years. The 22 GHz flux curve, however, is undersampled and the timescales are tentative at best. In 37 GHz, the $T_{\text{SF}}$ did not show a plateau and no timescale could be determined. The rise and decay times of flare 1 are between one and two years, so $T_{\text{SF}}$ at 4.8 and 14.5 GHz describe them accurately. $T_{\text{DCF}}$ produced significant timescales only at 4.8, 8, and 37 GHz, and $T_{\text{p}}$ only at 37 GHz. All values are approximately 2–3 years, roughly compatible with the 1980s flaring evident in the low-frequency flux curves.

BL Lac. The archetype of all BL Lacertae objects certainly has a very variable flux curve. Unfortunately, the major flare of early 1980s is not included in our analysis due to poor sampling in 22 and 37 GHz. Since then, however, we count nine multifrequency outbursts in 1987–1988, 1989, 1992, 1993, 1996, 1997–1998, 2000, 2002, and 2003. All are similar in shape in the higher frequencies, peaking at 3–6 Jy. At 4.8 and 8 GHz, the 1987–1988 and 1996 flares (flares 1 and 5) are more prominent with fast rises, and outbursts following them seem to be partially superposed on their slow decline. With growing frequency, flares 1 and 5 seem to lose their dominance and blend into the other flares.

$T_{\text{SF}}$ values are mostly below one year, except for 4.8 GHz ($T_{\text{SF}} = 3.83$) and 22 GHz ($T_{\text{SF}} = 2.41$). In the former case, flares 1 and 5 dominate the small-scale variations, leading to a long SF timescale. Especially the decay time of flare 1 is long. In 22 GHz, the reason for a long SF timescale is less clear. In the case of $T_{\text{DCF}}$ and $T_{\text{p}}$, long timescales clearly dominate. The typical values are $T_{\text{DCF}} = 7.5$ years and $T_{\text{p}} = 8.5$ years. This contrasts with the average flare interval of approximately two years. The reason for such long timescales is probably the strong flare on early 1980s, which is unequalled in flux density. This conclusion can also be drawn from the 8 GHz timescales. They are considerably shorter, $T_{\text{DCF}} = 2.81$ years and $T_{\text{p}} = 3.80$ years, while 8 GHz is also the only frequency that has data from the beginning of 1970s when BL Lac was very bright, flux levels being comparable to those of the 1980s flare.
3C 446. The flux curve is marked by three flares, two of them quite broad and strong. The first peaked approximately in 1990. In 230 GHz, however, the peak occurred as early as 1988, when the flux levels reached 11.71 Jy. The other two peaked in 1996 and 2000, at 6.30 Jy and 9.29 Jy, respectively, at 22 GHz. All outbursts have multiple components and the frequency evolution is apparent.

3C 446 has relatively few short-scale flux variations and rise and decay times of 3–4 years in flares 1 and 3. This can be seen in the SF timescales which are quite long, roughly \( T_{\text{SF}} = 3 \) years, except for 8 GHz, for which \( T_{\text{SF}} = 1.5 \) years. This could be due to either the randomly sampled early flux curve at 8 GHz, which is missing in other frequencies, or just minor variations. The DCF and periodogram timescales were significant only at the low frequencies, and \( T_{\text{DCF}} \) also at 90 GHz. All \( T_{\text{DCF}} \) and \( T_{\text{P}} \) are long, mostly ten years or above, reflecting the rather sedate behavior of the source. However, their behavior is not very consistent: at 14.5 GHz, \( T_{\text{P}} = 5.71 \) years, which is considerably shorter than at 4.8 and 8 GHz, but \( T_{\text{DCF}} = 11.16 \) years, which is clearly longer than at the lower frequencies. It is of the same order as the 12-year activity cycle for 3C 446 obtained by Pyatunina et al. (2001). At 90 GHz, \( T_{\text{DCF}} = 5.96 \) years, which roughly corresponds to the 6–7-year interval between flares 1 and 2. The average interval between flares 2 and 3 is 3.5–5 years.

### 6.2. Correlations

In Section 6.1, we described in detail how, for each source, the timescales and the observed flux curve related to each other. To illustrate the correspondence between the timescales and the temporal parameters of the outbursts more quantitatively we plotted (1) SF timescales of the source against both the rise times, \( \Delta t_R \), and the decay times, \( \Delta t_D \), of the flare, averaged for each source and frequency (Figure 8), (2) DCF and periodogram timescales against the peak-to-peak intervals of consecutive fluxes, \( \Delta t_{\text{PP}} \), averaged for each source and frequency (Figure 9). In all the plots, the dashed line represents an ideal one-to-one correspondence. All parameters are observational and have not been corrected for redshift nor for Doppler boosting.

From Figure 8, we see that the plot is very similar for both the rise and decay times. There is considerable scatter at low \( \Delta t_R \) and \( \Delta t_D \) on both sides of the one-to-one line, and at high values the SF timescales seem to be significantly shorter than \( \Delta t_D \). The lower limits of \( T_{\text{SF}} \) were not included in the plot. According to the Spearman rank correlation test, there is a significant positive correlation between \( T_{\text{SF}} \) and both \( \Delta t_R \) and \( \Delta t_D \). For rise times, \( \rho = 0.600 \) and \( P < 0.0005 \), and for decay times \( \rho = 0.607 \) and \( P < 0.0005 \).

The distribution of the \( T_{\text{DCF}} \) and \( T_{\text{P}} \) values plotted against \( \Delta t_{\text{PP}} \) (Figure 9) are also scattered, but seems to roughly follow the one-to-one line. According to the Spearman test, the positive correlation is significant for both the DCF (\( \rho = 0.366 \) and \( P = 0.005 \)) and periodogram (\( \rho = 0.420 \) and \( P = 0.008 \)).

### 7. DISCUSSION

Throughout this paper, it is important to remember that this sample represents only a small fraction of the BLO population. Most BLOs are too faint in the radio frequencies, or even if their flux density is above the detection limit, they simply lack the long-term data needed for this kind of analysis. Also, the high-energy BLOs (HBLs) are sorely under-represented: only two of them are included in the timescale analysis, and none at all in the flare analysis. It is also noteworthy that only 13 of the 24 sources included in the timescale analysis had significant, well sampled flares to analyze during the observing period. Some of the remaining 11 sources simply do not have a very variable flux curve, which indicates that even some radio-bright BLOs are surprisingly steady emitters. For example, Mark 421, B2 1147+24, and Mark 501 have remarkably uneventful radio flux curves. This is also confirmed by other authors (e.g., Venturi et al. 2001; Blażejowski et al. 2005; Lichti et al. 2008).

While the shock-in-jet scenario gives the general guidelines of AGN variability and its causes, there are many additional factors, such as relativistic boosting, properties of the ambient medium, turbulence and bending of the jet (Marscher 1996), affecting the flux behavior we observe. These effects together with the shock mechanism generate the diverse flux curves observed also in our sample, ranging from the rapid spikes of OJ 287 to the modest pace of 1308+326 and PKS 0735+178.

As stated in Section 5.2, there are two things that complicate the analysis of the time lags of the BLO flares: the very complex structure of most of the flares and the regrettably sparse sampling. The first affects especially the small flares, where it can be impossible to separate the components from each other and thus their evolution cannot be traced. The latter creates errors in both the peak time and peak flux of the flare. In many cases, errors in the peak time in different frequency bands do...
Raiteri (1999) developed a model to explain the spectral variations of large variations in the flux curves of many BLOs. Villata & Raiteri (1999) developed a model to explain the spectral variations of Mark 501 with a helical jet produced by a binary black hole system. The model was able to describe the peculiar X-ray part of the spectral energy distribution (SED) very well but the low frequency optical to radio part remained fairly constant. They concluded that the low-frequency variations could be due to inhomogeneities in the rotating jet or intrinsic brightness variations. Ostorero et al. (2004) applied the model to the SED and the radio and optical flux curves of AO 0235+164. The model was based on the 5.7-year quasi-periodicity suggested for the source by Raiteri et al. (2001). The periodic flares are explained by the rotation of the helix. Observed signatures are similar to the shock model so that first the high-frequency portion of the jet approaches the line of sight and Doppler boosting increases causing the flux density to rise. As the helix rotates, different frequency portions approach the line of sight and this way the time delays between the frequency bands can be explained. In their model, the nonperiodic flares were explained with intrinsic brightness variations (e.g., shocks). As the model was based on the observed periodicity of 5.7 years, it should be modified now that the period did not repeat after the year 2000 (Raiteri et al. 2006). It should be noted that the sources in our sample are not strictly periodic in the radio regime and usually the observed quasi-periodicities last only a short time in the flux curve (Hovatta et al. 2008a).

The model by Villata & Raiteri (1999) has also been used to explain the variations in BLOs S4 0954+65 (Raiteri et al. 1999), ON 231 (Sobrito et al. 2001), and S5 0716+714 (Ostorero et al. 2001, using data prior to the extreme flare in 2003). In addition, VLBI polarization observations have revealed helical magnetic fields in many BLOs (e.g., Gabuzda et al. 2004; Mahmoud & Gabuzda 2008). Some of those are also in our sample, but they are mostly sources for which we have not performed a detailed flare analysis because they do not have distinct flares in the radio frequencies. It is indeed possible that in these sources the variations are caused by changes in the Doppler beaming due to curved jets rather than intrinsic phenomena such as shocks. However, testing this scenario would require detailed studies of simultaneous SEDs, which is beyond the scope of this paper.

We showed in Section 6 that the computational timescales correlate fairly well with the observed temporal parameters. To our knowledge, such a straightforward, but revealing comparison has not been done before. The statistically significant correlation in Figures 8 and 9 confirms that the SF, DCF, and L-S periodogram timescales are indeed directly linked to the source behavior we observe. Unfortunately, the scatter is substantial. For example, in our data a source with a DCF timescale close to eight years, can have a real peak-to-peak flare interval of 2–10 years. The average absolute deviation of the computed timescale from the one-to-one correspondence is 0.98 and 1.24 years for $T_{\text{SF}}$ against $\Delta t_{PP}$ and $T_{\text{DCF}}$ against $\Delta t_{PP}$, respectively, and 2.24 and 3.23 years for $T_{\text{DCF}}$ and $T_{\text{TP}}$ against $\Delta t_{PP}$, respectively. In representing the peak-to-peak intervals of the flares, $T_{\text{DCF}}$ and $T_{\text{TP}}$ are near equivalent. However, at least in the cases of PKS 1413+135 and PKS 1749+096, the periodogram results are less affected by the frequency evolution of the flux curve between the frequency bands.

Variability of BLOs in the lower radio frequencies of 4.8, 8, and 14.5 GHz was also studied by Aller et al. (1999). They studied the variability behavior of a complete flux-limited sample of 41 BLOs using e.g., the SF. Only two of the BLOs in our sample (OJ 425 and 4C 56.27) are not included in the sample of Aller et al. (1999). They used UMRAO data from 1980 to 1996, while we have used the same database updated until...

![Figure 9. $T_{\text{DCF}}$ (top panel) and $T_{\text{TP}}$ (bottom panel) plotted against the peak-to-peak intervals, $\Delta t_{PP}$ of the flares, averaged for each source and frequency. The dashed line represents a one-to-one correspondence.](image-url)
2005 April. We have compared our results to see if the source behavior has changed during the past ten years. On average, the timescales have remained quite similar, the average longest timescale from SF in Aller et al. (1999) is 2.9 years compared to our average SF timescale for BLOs which is 3.7 years (median is 2.7 years; Hovatta et al. 2007). When individual sources are studied, there are some differences and we believe it is mainly due to the longer data set used in our analysis. Similar results were obtained in Hovatta et al. (2007) when the SF timescales for the whole AGN sample at 22 and 37 GHz were compared to analyses made ten years earlier.

In many of the BLOs in our sample, short intraday variations are seen in optical and radio frequencies (e.g., Wagner & Witzel 1995). However, the sampling density of our monitoring programs is not frequent enough to detect such rapid variations, unless a special campaign is arranged. The only example for which intraday variations have been observed at 37 GHz using Metsähovi data is S5 0716+714 (Ostorero et al. 2006). During which intraday variations have been observed at 37 GHz using Witzel1995). However, the sampling density of our monitoring are seen in optical and radio frequencies (e.g., Wagner & Witzel1995). However, the sampling density of our monitoring.

The 45 BLO flares in our analysis confirm the generalized shock model of Valtaoja et al. (1992) with no clear, undisputed exceptions. However, very frequent sampling on several radio frequencies is needed for the accurate, observational determination of the flare components and time lags. Based on the evolution of the relative peak flux and the time lags from one frequency band to the next, we find that the BLO flares are mostly high peaking. Probably they reach their maximum development in the millimeter to submillimeter wavelengths.

4. The computational timescales, $T_{SF}$, $T_{P_{DCF}}$, and $T_p$, have a statistically significant correlation with the temporal flare parameters obtained directly from the flux curves. However, scatter is considerable, and the average deviation from one-to-one correspondence is of the order of 1–3 years, depending on the parameter and timescale in question.

We gratefully acknowledge the funding from the Academy of Finland (project numbers 205793, 210338, and 212656). UMRAO is supported in part by a series of grants from the NSF, most recently AST 0607523, and by funds from the Department of Astronomy, University of Michigan.

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8. CONCLUSIONS

We have studied the long-term radio variability and the flare morphology of radio-bright BL Lacertae objects. The main conclusions are as follows:

1. Radio-bright BLOs exhibit a range of flaring behavior, with few common features. Especially the quasar-like objects PKS 0735+178, 1308+326, and 3C 446 have distinctively long outbursts with modest short-term variability, contrasting with the more erratic behavior of other sample sources. The long flare of 1308+326 clearly stands out from the rest of the sample even after correcting for the relativistic boosting effects. Our findings confirm the quasar-like nature of 1308+326 and 3C446 and indicate that PKS 0735+178 also has radio behavior different from typical BLOs.

2. The median duration of a flare in a radio-bright BLO is of the order of 2.5 years, and the peak flux density typically reaches about 5 Jy at 37 GHz. On average, the decay time of the flare is 1.6 times longer than the rise time. When the Doppler boosting effect is taken into account, the peak flux of the flare does not depend on the duration of the flare, indicating that the energy release in a flare does not depend on its duration.

3. The 45 BLO flares in our analysis confirm the generalized shock model of Valtaoja et al. (1992) with no clear, undisputed exceptions. However, very frequent sampling...