
© 2008 by authors and © 2008 Springer Science+Business Media

Preprinted with kind permission from Springer Science+Business Media.
1 Designing Interactions for Navigation in 3D Mobile Maps

Antti NURMINEN¹, Antti OULASVIRTA²

¹Helsinki University of Technology, Finland
²Helsinki Institute for Information Technology HIIT, Finland

Abstract. Due to their intuitiveness, 3D mobile maps have recently emerged as an alternative to 2D mobile maps. However, designing interactions for navigation in a 3D environment using a mobile device is non-trivial. Challenges are posed by the severe limitations of the mobile user interface and of the capacities of the mobile user. This chapter analyses the problem of degrees of freedom: how to make navigation quicker and more intuitive by the means of restricting and guiding movement, yet enabling unrestricted access to all reasonable points-of-interests. Insights from empirical studies of mobile map interaction are presented, in the form of a model of interactive search, to draw requirements for interaction design. Then, the design of controls, landmarks, cameras, interest fields, routes, paths etc. are analysed and several higher-level navigation metaphors are discussed. We propose ways to support spatial updating, rapid alignment of physical and virtual spaces, and overcoming the keyhole problem. A working prototype system is used to illustrate different solutions alongside with alternative designs, weighing their pros and cons.

1.1 Introduction

Recent advances in the processing capabilities and interface technologies of mobile devices have brought about a situation where 3D mobile maps are increasingly realistic. During the past five or so years, several developers have presented working prototypes of 3D mobile maps and empirically compared them to 2D maps. Various user benefits have been identified, such as fun, recognisability, efficiency, intuitiveness, and decreased memory load (e.g., Burigat and Chittaro 2005, Laakso 2002, Oulasvirta, Nurminen, and Nivala submitted, Rakkolainen and Vainio 2001, Vainio and Kotala 2002).

Figure 1 presents 2D and 3D maps of the same area. There are important differences between 2D and 3D mobile maps in the way each supports orientation and navigation. Particularly, orientation with 2D maps (electronic or paper) requires identifying possibly abstract cues, symbols, and shapes of a map as well as real world objects, and performing a mental transformation between them. A 3D map can provide a more directly recognisable visualisation of the environment. With a first person view, even the mental transformation would become unnecessary. While a 3D representation seems to provide a very intuitive static view of the environment, (interactive) navigation in such a virtual environment is more complex. Consequently, while 3D
visualisations appear as an appealing option for designers, a major problem lies within the meaningful and efficient control over such visualisations.

Furthermore, there are non-trivial design problems arising from the characteristics of mobile human-computer interaction (HCI). The key challenge in design is the problem of degrees of freedom (DOFs): how to balance freedom of movement with efficiency of navigation. The number of DOFs needed to completely control a 3D view exceeds the amount of input controls in mobile devices, and the problem is accentuated by the fact that mobile devices have small displays, which implies that more motion is needed to gather the same amount of information. Moreover, as mobile users’ capability to invest uninterrupted attention in a mobile device is known to be compromised (Oulasvirta et al. 2005), the interface should allow them to display what ever is currently needed as quickly and easily as possible, without complicated manoeuvring that requires all of the user’s attention.

This book chapter summarises three years of research and development efforts on m-LOMA, a 3D mobile map of an urban environment (the city centre of Helsinki) (Nurminen 2006). Our goal has been to create an efficient interface for navigating in a 3D view with the limited resources available. We first review general goals for designing a 3D navigation interface specifically for mobile devices. A model of user interaction with 3D maps is then presented. Subsequently, we formalise the problem of mapping controls to manoeuvring, and proceed to present mobile input devices along with those of their innate and practical problems that should be considered in the interaction design. We then present and discuss several manoeuvring schemes. The main part of the text concerns real and simulated cases and introduces a set of rules, tools, and metaphors that can ease navigation.

1.2 Definitions

The term 3D map has been used widely in the literature, assuming an intuitive definition. However, this term can be applied to representations whose source data is purely two-dimensional. Correspondingly, a data set which is in fact three-dimensional can be visualised in various fashions resembling a 2D map.

Based on our observations, we have identified two processes involved in producing 3D maps. First, a three-dimensional representation of a physical environment is
formed. This representation can be a data structure, such as a 3D model, where three-dimensional shapes and visual characteristics are stored. The modelling process can simplify the real-world geometry and its visual features, but maintains the spatial relations of objects. Metadata can be included in the representation. Second, this representation is visualised as a two-dimensional image, depicting the three-dimensional aspects of the original environment and conveying the spatial relations of the physical structures of the area. The selected visualisation strategy to yield a final map may depend on context and purpose, emphasising different characteristics of the environment. We define a 3D map to be a two-dimensional visualisation of a three-dimensional representation of a physical environment, emphasising the three-dimensional characteristics of this environment.

3D maps may have a variety of characteristics, depending on their contents, purpose, visualisation and other features. Table 1 presents a common set of such attributes. In addition, there may be other run-time selectable features and modes, such as perspective or orthogonal projection, first-person view, or visualisation styles. For example, thematic visualisation could employ cartographic conventions to colour the buildings based on their type (landmark, office building, shopping mall) instead of applying textures to the façades (Plesa and Cartwright 2007).

By our definition, a photograph is not a 3D map, as it is not created by a two-stage process, even though it is visually indistinguishable from the theoretical ideal map. A 2D road map with a perspective projection would not be a 3D map, as the data structures are not truly three-dimensional. Such a map could be called a “2.5D” map. 3D geometry, portrayed directly from above with orthogonal projection, could be classified as a 2D map, unless the visualisation provides clearly three-dimensional characteristics, such as shadows. Classification of maps based on procedural models would depend on the resulting model geometry and the chosen visualisation method.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>The data set and its visualisation exactly match the real world; a single image is indistinguishable from a photograph.</td>
</tr>
<tr>
<td>Realistic</td>
<td>Map data is visualised in a realistic manner, in an attempt to approach the ideal representation. Typically, this involves use of textures created from the real environment by digitisation.</td>
</tr>
<tr>
<td>Real-time rendered</td>
<td>Visualisation is performed on-the-fly instead of displaying pre-rendered animation sequences or images.</td>
</tr>
<tr>
<td>Navigable</td>
<td>Allows users to control the position and direction of the virtual camera.</td>
</tr>
<tr>
<td>Interactive</td>
<td>A navigable, real-time rendered 3D map, responding to users’ queries and providing navigation-assisting features.</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Contains time dependent elements other than the virtual camera, such as positions of GPS tracked users, public transportation etc.</td>
</tr>
<tr>
<td>Electronic</td>
<td>Emphasises the computerised means in producing the 3D view instead of a drawing or painting.</td>
</tr>
</tbody>
</table>
Urban/outdoor/indoor Description of the represented environment.

Mobile Electronic, running on a mobile, battery-operated device, such as a PDA or smart phone. Also implies navigability, interactivity and real-time rendering.

Immersive A stereo projection system, where a separate view is produced for each eye to achieve an illusion of being immersed in the environment.

**Table 1.** Common 3D map attributes.

### 1.3 General requirements for mobile navigation interfaces

The design of a mobile map suited for users’ needs is a challenging task, and may require several trade-offs (Meng and Reichenbacher 2005). With mobile 3D maps, the situation is further accentuated. This section summarises user requirements for the design of mobile navigation interfaces. A real-life example of cumbersome interaction is presented.

#### 1.3.1 Support for use in multitasking situations

When a user operating an application is completely concentrating on the task, unaware of the existence of the interface, the interface is said to be transparent to the user (Norman 1988). This should be achieved also for 3D navigation. Mobile users have short attention spans, in extreme mobile situations in the order of just few seconds (Oulasvirta et al. 2005). Therefore, the mobile application should provide prompt aid. Moreover, users’ operative modalities are often limited; for example, they may need to operate with one finger only, or may not hear anything due to environmental noise. In addition, users may have limited time or motivation to learn complex controls. To summarise, we assign three goals for mobile controls in pursuit of transparency:

- minimise cognitive load (as defined by working memory load, amount or duration of cognitive task processing, or complexity of mental computations),
- minimise motor effort and procedural complexity,
- minimise use of time.

#### 1.3.2 Support for navigation

While experiencing a city, the user may be performing one of the many tasks related to navigation. She may be exploring the environment, observing interesting features, searching for something, or conducting a naïve search, that is, extensively and at times exhaustively searching the environment (Darken and Sibert 1996). Or, she may already know the approximate position of a target, proceeding for a closer look, on a
primed search (Darken and Sibert 1996). Perhaps a route has been provided, and the user is attempting to maintain orientation while manoeuvring towards the target, or is spotting the next turn point. While the user moves and observes the environment, she simultaneously develops a cognitive map (Downs and Stea 1977). At all times, the user attempts to maintain a sense of orientation and avoid any situations that might lead to disorientation.

To support navigation, we assign the following goals for the 3D view:

- maximise information that helps orientation,
- maximise information that helps performing the current task,
- minimise information that leads to disorientation,
- maximise information that helps forming an accurate cognitive map.

Fulfilling these objectives ensures that the user

- does not get lost,
- is able to find and visit all places of interest,
- is able to re-visit places,
- feels familiar with the space.

1.3.3 Support for embodied interaction

In general, what makes mobile maps distinct and different from typical virtual environments (VEs)—such as virtual reality and desktop-based navigation systems—is that the user is physically embedded in the world that the virtual model represents. The dual presence is not symmetric: the roles of eye, head and body movements for acquiring information are emphasised in physical environments (PEs), whereas VEs are typically associated with decreased field of view and low fidelity of landmarks and non-visual cues. There are four implications of this.

First, what makes mobile maps different from VEs and map artefacts is the strong influence of the keyhole property (Woods and Watts 1997). The keyhole property means that “the number of potential views is much greater than the physical size of the available viewports” (p. 619). Since direct recognition is often not possible because of this property unless the target happens to be on the display, users have to move within the space or spaces to achieve a position in which the target can be found. In contrast to VEs, mobile maps assume search within two spaces instead of just one.

Second, because of the keyhole property, the alignment of the representation with the represented space is often difficult. Alignment is important for orientation, because human spatial knowledge is known to be viewpoint-dependent. This also concerns knowledge of dynamic scenes (Garsoffky, Schwan, and Hesse 2002). Hence, when objects in a map do not correspond to stored representations, the user has to transform or rotate the representation, which entails mental or physical effort (Levine 1982). Mou and McNamara (2002) elaborate this view by arguing that spatial memories are defined with respect to intrinsic frames of reference, which are selected on the basis of egocentric experience and environmental cues. The availability of cues is exactly where 3D maps differ from other representations. It is also worth mentioning that viewpoint-dependence of spatial knowledge may be exaggerated in the case of
mobile maps where the size of display is small. Presson, DeLange, and Hazelrigg (1989) claim that small displays do not "afford", as does movement in PE, large spatial arrays to be coded by the perceptual system during movement. Moreover, they do not support as much perceptual exploration and scanning (cf. Roskos-Ewoldsen et al. 1998).

Third, because the use of mobile maps is supposed to be possible when the user is moving, the map’s support for spatial updating—the mechanisms involved in locating positions in space relative to oneself after a given spatial transformation—is emphasised (Wraga 2003). Wang and Brockmole (2003) have examined what happens when the environment is divided into nested structures (e.g., city consisting of district consisting of blocks). They noted that spatial updating is not carried out in all structures simultaneously nor with the same accuracy. When switching to a new environment, one often loses track of one's position relative to old environments. Providing alternative views in 3D might support the user in updating and re-constructing such structures when needed.

Fourth, the small scale of transformations (e.g. Golledge 1999) in mobile maps may lower the informativeness and recognisability of objects. Only recognisable objects can be utilised as landmarks that help the tasks of mapping and orientation. Therefore, 2D is different from 3D by trading off informativeness of an object to informativeness of an area. Witmer, Sadowski, and Finkelstein (2002) showed that adding aerial views enhances users’ ability to navigate through VE. Furthermore, in order to help in understanding the relationship between the virtual and the physical, landmarks in the virtual model also have to be distinctive (stand out), which depends on the visual as well as the structural qualities of the view (Sorrows and Hirtle 1999).

In Section 1.5, several solutions to these problems are presented and analysed.

1.3.4 3D navigation with direct controls: example from a field study

To exemplify an inefficient search strategy, let us present a case of manoeuvring in an urban canyon with direct mapping of controls to the navigation state (See 1.5.1.). The case is taken from our field experiment (Oulasvirta, Nurminen, and Nivala submitted), at a time when the subject has completed about half of the tasks. In that experiment, the subjects were shown a target building in the virtual space and asked to move to the corresponding object in the physical space.
A test subject located at A attempts to manoeuvre a 3D map view from A to B (Figure 2A), to spot a landmark known to her in advance. The 3D map is running on a PDA with initial orientation of Figure 2B. The map view is controlled by the PDA's joypad, which switches the viewpoint forward or backward, and rotates it left or right, as long as the control is held down. Manipulating the joypad moves the viewpoint at a constant speed and rotates it at a constant angular velocity. Additional controls affect elevation and pitch, but roll is fixed to a constant up direction. Thus, the subject has four degrees of freedom available for controlling the viewpoint. In the beginning of this task, the subject orients to the environment, spotting a recognisable façade (Figure 2D in m-LOMA and 2C in the real world). The subject then determines a direction of movement and starts to manoeuvre along a road. After two blocks, the subject finds the target landmark and stops. Figure 2E presents the path and points of orienta-

---

1 The subject had trained the controls 15 mins during a training session, and about 30 mins during similar tasks at field before this one, without prior experiences of 3D games or 3D maps.
tion resulting from performing the task. Figure 2F presents the related joypad state as a function of time. To perform this simple task, the user performed a total of 20 rotations and 10 forward movements, using only two degrees of freedom from the available four. The subject did not move backward. The task took 135 seconds to complete, and the controls were used in two sequences: 5 seconds during initial orientation and 29 seconds during manoeuvring towards the target. The remainder of the time, the subject observed the environment and waited for a car and a tram to pass (the tram remained in view for almost a minute). All actions on controls were performed sequentially.

The presented case contains three of the four stages of navigation presented by Downs and Stea (1977): 1) the initial orientation at start, 2) manoeuvring towards a target and 3) recognising the target. The final stage, 4) maintaining the orientation to the target, is not relevant, as the distance to travel after the target has come into view is short and the route straightforward. The task appears simple, but the actual manoeuvring by the subject is unnecessarily complex. At several points, the viewpoint approaches a wall, and correcting the orientation provides only a temporary solution, as yet another correction must soon take place. Even when the subject simply rotates around one axis, a number of button presses is required to find a suitable view.

This real episode taken from our data clearly illustrates the fact that four degrees of freedom are too much for the majority of users. In Section 1.5 we will go through several interface solutions that address this problem. We shall make a distinction between manoeuvring—actions towards a subgoal such as the orientation or moving one block, and micro-manoeuvring—corrective or adjusting actions that happen within a manoeuvre (such as rotating back and forth several times to find a suitable view or direction of travel).

1.4 A model of interactive search on mobile maps

Before turning to concrete design solutions to the DOF problem, we need to explain some aspects of how users interact with mobile maps in real life situations. The model of Oulasvirta, Nurminen, and Nivala (submitted) is rephrased here.

In their influential paper, Kirsh and Maglio (1994) distinguished between two kinds of action: pragmatic and epistemic. The former refers to action that transforms the physical problem space, for example moving a disk from one pole to another in the Tower of Hanoi task. The latter refers to action that does not directly contribute to the solving of the problem; rather, its objective is to have an effect on the cognitive state of the agent itself. Epistemic action can have three functions:

1. to reduce time complexity (how long something takes),
2. to reduce space complexity (how much cognitive processing is required),
3. to reduce uncertainty (how certain is an outcome) in the problem.

For example, Tetris players often quickly rotate a zoid around one or several times after it appears in an attempt to see how the zoid fits the landscape beneath it. This in effect changes the task from mental rotation (slow) to recognition (fast).
1.4.1 Pragmatic search action

Figure 3 presents the model of pragmatic search (cf. Jul and Furnas 1997). In the core of the model are 1) a match process taking care of the comparison between the target description (kept in mind) and the perceived space and 2) a motor action process that transforms the space according to the cue-search strategy. The search-action loop involves acting based on a selected strategy for searching for a cue (e.g., searching for a certain rooftop shape by scanning horizontally while keeping view toward rooftops). Carrying out the strategy produces a change in the state of the world. This new state is perceived and a new match is attempted. A failed match process leads back into the search-action loop. Importantly, search action can take place both in the virtual and the physical world, unlike in VEIs where the search always takes place in the virtual space. When the current strategy does not produce satisfactory matches, it has to be changed. This triggers the hypothesis-restructuring loop, which involves acquiring a new target description by choosing a new cue in the source environment (e.g., noticing that the target building is the lowest one). This does not always happen as an internal process, but is often manifested in the subject physically returning to a position where a new description of the target can be extracted or the earlier one elaborated.
1.4.2 Epistemic search action

Below, we report a total of seven behaviours, observed by Oulasvirta, Nurminen, and Nivala (submitted), that can be interpreted as epistemic action rather than pragmatic action (for visualisations of related search paths, see Figure 4):

1. Scanning the immediate context of the target, in order to elaborate the description of the target held in working memory (e.g., by looking at colours of neighbouring buildings).
2. Scanning for landmarks such as statues, in order to elaborate the description of the target or to use prior semantic and spatial information.
3. Egocentric positioning (locating oneself in the map), in order to utilize the (stronger) representations of the PE in action for the VE.
4. Circling (walking around a full circle in VE) and viewing the buildings, thus creating initial mental representations to support search later on.
5. Exploring the proximal area (e.g., moving around in an area of few blocks from the starting position) in the first experimental trial to familiarise oneself with the model, thus reducing search costs in the subsequent trials.
6. Peeking around a corner to identify action alternatives and familiarise oneself with the surroundings, thus elaborating a representation of the vicinity and reducing uncertainty in the upcoming trial.
7. Rotating: the view is quickly rotated horizontally in the beginning of the trial to see the surrounding buildings. May serve egocentric positioning, landmark search etc. (Similar to tactics 4, 5, and 6.)

But what is actually achieved by such epistemic actions from the perspective of the task of finding a given target? We posit that there are three main functions of epistemic actions in interacting with a mobile map:

1. **Improving cue descriptions.** In some of the epistemic actions, the user scans the target and its immediate neighbourhood. This can result in elaboration of the mental representation of the target, which in turn can facilitate the subsequent match process (i.e., reduction of uncertainty).
2. **Improving match process.** Some of the epistemic strategies can be explained as attempts to shift the processing burden from the limited and effortful short-term working memory system by switching from mental computation and maintenance to recognition. This strategy reduces uncertainty and decreases the time complexity of the task.
3. **Improving search strategy and search action.** Naturally, richer representations of the environment can participate in strategy selection and implementation of search action. For example, having a hypothesis of the target’s position in relation to a landmark helps decisions on where to look. This can make exhaustive scanning unnecessary. Such epistemic actions as exploring and peeking in effect enhance the user’s possibility to know where to search for the target by narrowing down the number of alternatives.

Implications of this descriptive user model to design are discussed in Section 1.5.
Fig. 4. Visualisations of users’ epistemic actions when searching a target. A) Learning the model by exploring it from a top-down view; B) Walking around in a city square and peeking around corners; C) Diving to street-level; D) Scanning, diving and scanning at multiple levels, and walking around at the street level. From the movement log data of the experiment of Oulasvirta, Nurminen, and Nivala (submitted).

1.5 Designing controls

An integral part of interaction design of navigation is the assignment of interface controls to movement and action in the mobile map environment. Typical mobile devices capable of running 3D graphics include personal digital assistants (PDAs) and smart phones (see Figure 5). The controls offered by PDAs commonly include a touch screen, a few buttons, and a joystick. The touch screen essentially provides a direct pointing mechanism on a 2D plane, and a two-dimensional analog input. The touch screen is operated by a stylus (a pointing stick). The buttons and the joystick are simple discrete inputs. A direct text input method may be missing due to the lack of buttons,
but in that case, it is compensated by a virtual keyboard, operable by the stylus. A smart phone commonly provides only discrete buttons. Usually a few of the buttons are assigned to serve menu selections, and the rest are used either in typing text or dialling. Sometimes a joypad is provided for easier menu navigation. Exceptions to these prototypical examples exist, such as Nokia's Communicators, which include a full keyboard. Some even contain both a keyboard and a touch screen, such as the Palm Treo 650. Sometimes, a small keyboard can be attached to a PDA.

When a control state is propagated to, and received by an application, it is called an input event. Mobile operating systems commonly prevent input events to be generated in parallel, in a forced attempt to reduce possible control DOFs to 1; if one button is down, the next possible event is the up event from that button. Platform-dependent software development kits provide information to override this restriction.

![Fig. 5. Mobile devices differ in the availability and layout of interfaces.](image)

### 1.5.1 Mapping controls to navigation

The use of controls can be described as a mapping from control space $G$ to navigation space $N$. The navigation space can be described by a navigation state, which contains all the variables related to navigation, including the camera position and orientation, but also other variables such as speed, field of view (FOV) etc. The mapping can also be called the function $g$ of the input $i$, $g: G \rightarrow N$. We will call the number of relevant user actions the control DOF and the navigation DOF the number of variables available for movement. The mapping provides movement on a guide manifold, a constrained subspace of the navigation space (Hanson and Wernert 1997). Discrete control inputs can be used for providing either a single event or multiple independent discrete events. The mapping can depend on time, possibly in a nonlinear manner. A 2D input can be used similarly for a single 2D event, a discrete sequence of such events $(x, y)_i$, or a time dependent input $(x, y, t)$. The motion derivatives $(x', y')$ can also be used as an input. If a control is given cyclic behaviour, the mappings depend on the current cycle $c$ of the inputs. $g: G_c \rightarrow N$. In this case, one button can control two or more functions, each button release advancing the function to next on the cycle.
1.5.2 Control delays

Physical controls provide a multitude of error sources. A single binary discrete event may simply not occur when a button malfunctions. Time-dependent events are prone to errors in timing. This can be caused either by the user or by the device. The user simply may not be able to estimate timings accurately, but the same may be true for the device. For example, with the Symbian operating system, the accuracy of the system clock currently available to applications is less than 20ms (1/64s). There might be constant minimum delays between events, caused by an operating system or a device controller, or the controller output may be sampled at discrete intervals, causing seemingly random output timings if the sampling rate is too low. Event propagation from the OS to the application may also depend on available CPU, which may be sparse during 3D rendering. For example, when a user presses a rotate button, rendering may start immediately, but when the button is released, it can take a while before the rendering engine receives the event. If the user was attempting to aim at a certain location, the possible crosshair would have passed the target. Combining these problems may yield a temporal, context-dependent inaccuracy of worse than 100ms. While a coherent delay can be anticipated by a user, incoherent temporal errors are difficult to adapt to. Moreover, all such delays contribute to degradation of user experience and, consequently, of usability. Therefore, identifying least-latency control mechanisms is among the very first tasks of 3D interaction design for mobile devices.

1.6 Designing for navigation

Navigation is a process involving both mental and physical action, both way-finding and movement (Darken and Sibert 1996). All movement requires manoeuvring, performing a series of operations to achieve subgoals. Whereas manoeuvring in a 2D view can be mapped to a few input controls in a relatively straightforward manner, movement in a 3D world cannot. In addition to 3D position, one needs to specify 3D orientation as well. Direct control over such a view would require simultaneous specification of at least six degrees of freedom. Generally, producing decent motion requires even more variables, but a mobile device only has a few controls, of which the user might want to use only one at a time. We assess that developing interaction for a mobile 3D map depends heavily on solving this problem.

Therefore, when designing mobile maps, designers have to implement both cartographic (amount of information presented, symbolisation, generalisation, simplification) and interaction solutions, and this latter part is often overlooked. The model of pragmatic action presented above suggests that the key way to minimise the possibility of choosing a failing strategy is to guide and direct the user to use those cues that are known to be efficient in that particular model (e.g., for m-LOMA, façades of buildings are more effective than street geometry). Similarly, designers can deliberately make it more difficult to use those cues that are inefficient. Here, this design strategy is called guidance.

Second, the model of epistemic action suggests supporting 1) the elaboration of the target description, 2) the construction of a representation of the area in the PE, 3) the
use of prior knowledge, and 4) transforming the locus of task processing from working memory to perceptual modules. However, if guidance was optimally effective, one could argue that users would not need to relapse to epistemic action and other “corrective” behaviours. This, we believe, is not the case. Because of substantial individual differences in representing the environment and in the use of cues and landmarks (e.g., Waller 1999), and because information needs vary between situations, the best solutions are those that support flexible switches between efficient strategies.

**Manoeuvring** in a VE can be realised with various levels of control over movement. Table 2 presents a set of manoeuvring classes, in decreasing order of navigation freedom. Beyond simply mapping controls to explicit manoeuvring, one can apply metaphors in order to create higher-level interaction schemes. Research on virtual environments has provided several metaphors (see Stuart 1996). Many but not all of them are applicable to mobile 3D maps, partly due to restrictions of the input methods and partly due to the limited capacities of the user. Several methods exist for assisting or constraining manoeuvring, for guiding the user's attention, or for offloading unnecessary micro-manoeuvring. For certain situations, pre-animated navigation sequences can be launched via shortcuts. With external navigation technologies, manoeuvring can be completely automatic. It is essential that the special circumstances and potential error sources typical to mobile maps are taken into consideration in navigation design. Selecting a navigation scheme or metaphor may also involve striking a balance between support for direct search for the target (pragmatic action) on the one hand and updating cognitive maps of the area (epistemic action) on the other. In what follows, several designs are presented, analysed, and elaborated in the framework of navigation stages (Downs and Stea 1977) from the user's perspective.

<table>
<thead>
<tr>
<th>Manoeuvring class</th>
<th>Freedom of control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explicit</td>
<td>The user controls motion with a mapping depending on the current navigation metaphor.</td>
</tr>
<tr>
<td>Assisted</td>
<td>The navigation system provides automatic supporting movement and orientation triggered by features of the environment, current navigation mode, and context.</td>
</tr>
<tr>
<td>Constrained</td>
<td>The navigation space is restricted and cannot span the entire 3D space of the virtual environment.</td>
</tr>
<tr>
<td>Scripted</td>
<td>Animated view transition is triggered by user interaction, depending on environment, current navigation mode, and context.</td>
</tr>
<tr>
<td>Automatic</td>
<td>Movement is driven by external inputs, such as a GPS device or electronic compass.</td>
</tr>
</tbody>
</table>

*Table 2. Manoeuvring classes in decreasing order of navigation freedom.*
1.6.1 Orientation and landmarks

The first stage of any navigation task is initial orientation. At this stage, the user does not necessarily possess any prior information of the environment, and her current position becomes the first anchor in her cognitive map. To match this physical position with a 3D map view, external information may be necessary. If a GPS device is available, the viewpoint can be commanded to move to this position. If the map program contains a set of common start points potentially known to the user, such as railway stations or major bus stops, a selection can be made from a menu. With a street database, the user can walk to the nearest intersection and enter the corresponding street names. When the exact position is known, the viewpoint can be set to the current position, perhaps at street level for a first-person view. After resolving the initial position, we further encourage assigning a visual marker, for example an arrow, to point towards the start point. If the user's attempts at localisation fail, she can still perform an exhaustive search in the 3D map to find cues that match her current view in physical world.

For orientation purposes, landmarks are essential in establishing key locations in an environment (Evans 1980, Lynch 1960, Vinson 1999). Landmarks are usually considered to be objects that have distinguishable features and a high contrast against other objects in the environment. They are often visible from long distances, sometimes allowing maintenance of orientation throughout entire navigation episodes. These properties make them useful for epistemic actions like those described in Section 1.4. To facilitate a simple perceptual match process, a 3D map should reproduce landmarks in a directly recognisable manner. In addition, a 3D engine should be able to render them from very far distances to allow visual searches over entire cities and to anchor large scale spatial relations.

Given a situation where the start point has been discovered, or the user has located landmarks in the 3D map that are visible to her in PE, the user still needs to match the two worlds to each other. With two or more landmarks visible, or a landmark and local cues, the user can perform a mental transformation between the map and the environment, and triangulate her position (Levine, Marchon and Hanley 1984). Locating landmarks on a 3D map may require excessive micro-manoeuvring, even if they are visible from the physical viewpoint. As resolving the initial orientation is of such importance, we suggest assigning a direct functionality to it. The landmark view would automatically orient the view towards landmarks or cues as an animated view transition, with one triggering control (a virtual or real button, or a menu entry). If the current position is known, for example with GPS, the landmark view should present both the landmark and the position. Without knowledge of the current position, the same control would successively move the camera to a position where the next landmark is visible. Implementation of such functionality would require annotating the 3D model with landmark information.

Sometimes, no major landmarks are visible or in the vicinity. In this case, other cues must be used for matching the virtual and real environments, such as edges or areas, street names, topological properties, building façades, etc. Local cues can be unique and clearly distinguishable, such as statues. Some local cues, such as restaurant logos, are easy to spot in the environment even though they are not unique. We suggest populating the 3D environment with local cues, minor landmarks, and provid-
ing the system with related annotation information. Again, a single control would trigger camera animation to view the local cues. As this functionality draws the attention of the user to local cues, it requires knowledge of the user's approximate position to be effective.

As landmarks are often large objects, we suggest assigning landmark annotation to entire entities, not only to single points. An efficient 3D engine with visibility information available can enhance the landmark view functionality by prioritising those landmarks that are at least partially visible to the user in PE.

1.6.2 Manoeuvring and exploring

After initial orientation is obtained, the user can proceed with any navigational task, such as a primed search (Darken and Sibert 1996). In a primed search, the target's approximate position is resolved in advance: a point of interest could be selected from a menu, the user could know the address and make a query for coordinates, a content database could be searched for keywords, or the user could have a general idea of the location or direction based on her cognitive map. A primed search consists of the second and the last of navigational stages, that is, manoeuvring close to the target and recognising the target during a local browse. We suggest assigning another marker arrow to the target.

The simplest form of navigation would be immediately teleporting the viewpoint to the destination. Unfortunately, instant travel is known to cause disorientation (Bowman et al. 1997). The commonly suggested way of travelling to long distances in generally straightforward direction is the steering metaphor, where the camera moves at constant speed, or is controlled by accelerations. By controlling the acceleration, the user can define a suitable speed, but doesn't need to use the controls to maintain it, relieving motor resources for orientation. Orientation could indeed be more directly controlled while steering, in order to observe the environment. In an urban environment, moving forward in a straight line would involve positioning the viewpoint above rooftops in order to avoid entering buildings.

If the user is not yet willing to travel to a destination, she could start exploring the environment as epistemic action, to familiarise herself with it. Again, controls could be assigned according to the steering metaphor. For a better overall view of the environment, the user should be allowed to elevate the virtual camera to a top-down view, requiring an additional control to turn the view towards the ground. This view would allow her to observe the spatial relationships of the environment in a metrically accurate manner. If the user wishes to become acquainted with the target area without unnecessary manoeuvring, the click-and-fly paradigm can be applied, where the user selects a target, and an animated view transition takes her there. Animated view transitions should also be possible when start and end points are defined, for instance by selecting them from a list of known destinations or by having direct shortcuts assigned to them.
1.6.3. Maintaining orientation

When a user is navigating in an environment, during exploration or on a primed search towards a target, she should constantly observe the environment to enrich her cognitive map. Frequent observations are necessary for maintaining orientation, and learning the environment decreases the user's dependency of artificial navigational aids. Where major landmarks provide a frame of reference, local (minor) landmarks help making route decisions (Steck and Mallot 1998).

Following the work of Hanson and Wernert (1997), we suggest using interest fields as a subtle approach to drawing the user's attention to cues in the environment. When the user manoeuvres in an environment, an assisted camera scheme points the camera towards landmarks or local cues such as statues or restaurants with noticeable logos. The attentive camera metaphor (Hughes and Lewis 2000) suits this automatic orientation well. It orients the view towards interesting cues, but lets the movement continue in the original direction. When the angular distance between movement vector and view vector becomes large, the view returns to pointing forward. In addition, the assisted camera could support orientation (Buchholz, Bohnet, and Döllner 2005, Kiss and Nijholt 2003). When the camera is elevated, this scheme automatically orients the camera slightly downwards, in order to avoid filling the view with sky. The user can intervene in the suggested assistance and prevent it with a single click on a control opposite the orientation direction.

In cases where distinguishable local cues are missing, the local position and orientation can be verified directly with features that have been included in the 3D model, such as building façades. Individually textured façades provide a simple way of matching PE and VE almost anywhere. Unfortunately, not all façades provide distinguishable features (or are otherwise memorable), to which end the guidance provided by the system should prioritise other cues, if present.

During the initial orientation, the user was provided with a button that triggers a scripted action for viewing the closest landmark. When she is manoeuvring, the interest fields will mainly be guiding her attention to new local cues, or she can verify her position from other features such as building façades. However, such local information will not necessarily develop her cognitive map, and neglecting to frequently observe known anchor positions can lead to disorientation. Therefore, it is advisable to reorient the view to known landmarks from time to time. The user can achieve this using the same landmark view operation that was used initially, showing one or more landmarks, and then returning to normal navigation mode. Or, the system can suggest this action automatically, as an assisting feature.

An example of the assisted camera scheme is provided in Figures 6A-6D. When the user first approaches a landmark, the system provides the view presented in Figure 6A (at the user’s discretion). The user’s current position is marked with a red dot. Fig. 6B presents the user’s path, depicted with a long arrow. As the user approaches a corner, the view is automatically oriented towards the landmark (6C), and returned to normal view as the user proceeds forward. After a while, the system suggests looking backward (Figure 6D). In Figure 6A, note the two other landmarks in the horizon. Figure 6D includes two local cues, a statue and a bar’s logo. Automatic orientation in such a manner requires optimisation of the view's orientation value based not only on elevation, but the presence of visible cues and landmarks.
**Fig. 6.** An assisted camera scheme. When approaching a landmark (the tower), a quick overall view (A) is suggested. As the landmark comes into view, an automatic glimpse is provided (B and C). When the landmark has been passed, an overall view is suggested again (D).
1.6.4 Constrained manoeuvring

Constrained manoeuvring above rooftops appears to provide a simple, unconstrained 3D navigation space. However, one of the strengths of a 3D map is the possibility of providing a first person view at street level. Unfortunately, manoeuvring at that level will immediately lead to the problem of entering buildings through their façades, which is known to cause disorientation. The solution is a collision avoidance scheme that keeps the viewpoint outside objects. The simplest form of collision avoidance merely prevents movement when a potential collision is detected, which causes micromanoeuvring as the user must correct her position and orientation before continuing. A better solution would be to allow movement along a colliding surface, but even then the view would be filled by the façade, again causing disorientation (Smith and Marsh 2004).

We suggest applying street topology in order to limit the navigation space. Given a street vector database that contains street centrelines, and matching the coordinate system with the 3D model, the view is forced to remain along the street vectors, staying at a distance from building façades. We will call this manoeuvring scheme the tracks mode. Manoeuvring in this mode consists of moving along tracks and selecting from available tracks at crossings.

The usual assisted camera scheme keeps the camera pointed towards local cues. In addition, when the user orients towards façades, the assisted camera maximises the information value by moving the camera away from that surface, inside the building behind if necessary (Figure 7). The 3D engine should allow such motion, and avoid rendering the inner façade of the penetrated wall. Alternatively, the field-of-view can be widened, but that may lead to unwanted perspective distortions, depending on the situation.

1.6.5 Reaching a destination

At the end of a primed search, the user needs to pinpoint the exact goal of the search. This may require naïve search within the vicinity of the target. It may be sufficient to perform this search in the PE, but the user might also conduct it as epistemic action in the 3D map before arriving at the location. The search can be performed using the above-mentioned manoeuvring methods, perhaps at street level. Alternatively, the user can select a pivot point, around which the search is performed in a target-oriented manner. In this case, the navigation subspace is cylindrical and the view centred on a pivot point. An explicit manoeuvring scheme in a cylindrical navigation space would require 3 DOFs, namely radius, rotation, and elevation. A similar spherical control mapping would involve radius and angular location on the sphere surface.
1.6.6 Complementary views

The previous sections provide cases where viewpoint is sometimes set at street level, sometimes at rooftop level, and sometimes in the sky looking down. These viewpoints are informationally complementary, each associated with different interaction modes designed particularly for finding those cues that are informative in that view. We suggest two alternatives: as already mentioned, the explicit manoeuvring scheme would include controls for elevation and pitch, which would be aided by the assistance scheme that maximises the orientation value of the view, orienting the view downwards as the elevation increases. As a second alternative, we suggest assigning a control that triggers an animated view transition between a street level (small scale: first-person view), rooftop level (medium scale: local cues visible) and top-down view (large scale: spatial relations). Assigned to a single control, this would be a cyclic action. With two controls, the direction of animation can be selected. Figure 8 presents a rooftop view and a top-down view. In addition, separate 2D map views would be useful, for example to better convey the street topology. Rakkolainen and Vainio (2001) even suggest simultaneous use of 2D and 3D maps.

1.6.7 Routing

Given a topological street database, routing functionality can be implemented for example using the A* search algorithm (Hart et al. 1968). When start and end points are set, a route along the streets can be calculated and visualised. Figure 8 presents a route with start and end points marked by arrows and the route visualised as a semitransparent wall.

Routing offloads parts of the way-finding process of the user, letting her concentrate on the local cues necessary for following the pre-calculated path. While the user still could navigate freely, following a route naturally suits our constrained manoeuvring scheme. Given a route, the path is now essentially one-dimensional, and re-
quires very little interaction from the user. With a GPS device, movement along the route would be automatic. An assisted camera scheme would constantly provide glimpses at local cues, minimising the need to orient the view. At each crossing, the assisted camera scheme would orient the view towards the correct direction.

As support for epistemic action, a separate control could be assigned to launch a walkthrough of the route, in order for the user to familiarise herself with local cues related to important decision points such as crossings.

During navigation, the user would mostly be involved in simple recognition processes, observing cues of the local environment. Our primary suggestion is to offer a street-level view, minimising the need for spatial transformations. Secondarily, route navigation could be target-oriented, the viewpoint orbiting at rooftop level around a pivot point. In this case, controls would affect the movement of the pivot point and the supposed current location. A GPS could control the position of the pivot point automatically. To maintain orientation, the user should be encouraged to keep observing large scale features such as landmarks as well, as suggested in the previous section.

![Fig. 8. Route guiding mode. Route visualisation in bird’s eye and top-down views.](image)

### 1.6.8 Visual aids

The examples above have presented a few artificial visual aids for navigation in addition to a realistic 3D model: marker arrows, a GPS position point, and route visualisation. The markers could also display the distance and the name or logo of the target. We also suggest further visual cues: for example, the arrows in our system are solid when the assigned point is visible and outlined when it is not (Fig. 8). In addition to the assisted camera scheme, temporary markers could be assigned to cues that lie too far away from the orientation of the view provided by the attentive camera, with transparency depicting the angular distance. When users encounter subjectively sali-
ent cues, they should be allowed to mark them as landmarks, and assign a marker as a spatial bookmark.

As overlay information, the current manoeuvring metaphor, camera assistance status, or street address could be rendered on the display. A graphical compass could also help in orientation. Figure 8 presents markers with distance, a compass and current navigation mode (the most recent setting). In addition, location-based content could be integrated into the system, represented for example by billboards. If these billboards were to present graphical company logos in easily recognisable manner, they could be used as local cues for the assisted camera scheme.

1.7 Input mechanisms

In the previous section we implicitly assumed that all interaction except for animated view transitions would involve time-dependent, explicit manoeuvring. As long as a button is being pressed, it will affect the related navigation variables. We now present two alternate mechanisms to complete the interaction palette, and proceed to design an integrated navigation solution.

1.7.1 Discrete manoeuvring

With explicit, continuous manoeuvring, the user is constantly involved with the controls. The requirement to navigate both in the PE and the VE at the same time may be excessively straining, especially with an unrestricted, unassisted navigation scheme as described in Section 1.3. Especially at street level, each intersection poses a challenge, as the user must stop at the correct position and orient herself accurately towards the next road before proceeding. The tracks mode helps by constraining the navigation space, but the user still needs to constantly manage the controls in order to manoeuvre the camera. In the case of route following, the essentially one-dimensional route may suffice, as the user mainly just proceeds forward.

As an alternative to continuous manoeuvring, discrete navigation can provide short animated transitions between positions, requiring user attention only at certain intervals. Step sizes can be configured. At crossings, angular discretisation can depend on the directions of the streets. A simple angular discretisation scheme is presented in Figure 9, where rotation of the view will continue until it is aligned with one of the preset directions. The need for accuracy is reduced as the system is pre-configured. The user may be able to foresee what actions will soon be required, for example when approaching a crossing. Therefore, the system should cache the user's commands and execute them in order.

The downside of discrete manoeuvring is the lack of freedom to explicitly define position and orientation, which may reduce the possibility to observe cues in the environment. Thus, the importance of an assisted camera scheme is emphasised, as without automatic orientation towards cues, the user might not notice them.
1.7.2 Impulse drive

A compromise between explicit, continuous manoeuvring and explicit, discrete manoeuvring would be floating, similar to steering, where controls would give the virtual camera impulses. Each impulse would increase the first derivative of a navigation variable, such as speed of movement or rotation. Continuous thrust would provide a constant second derivative, such as acceleration. Both the impulse and thrust should be configurable by the user. By setting the thrust to zero, acceleration would still be possible with a series of impulses. In all cases, a single impulse opposite the direction of motion would stop the movement. In addition, friction would act as a small negative second derivative (deceleration) to all navigation variables, preventing infinite movement.

1.7.3 2D controls

Several mobile devices include a touch screen, operated by a stylus. As an input device, a touch screen produces 2D position events. A single event can be used to operate software UI components, or as a direct pointing paradigm. A series of events could be produced by pressing and moving the stylus on the display. Such a control could drive navigation variables in a seemingly analogous manner, given that the events are consistent and sufficiently frequent (see Section 1.5.2).

1.8. Navigation interface

Navigation in a 3D space with limited controls is a challenging optimisation task for the interface designer. The previous sections have introduced a set of navigation tasks and cases, with several supporting navigation designs and mechanisms. A real appli-
cation must strike a balance between these solutions to yield a complete, integrated navigation interface.

1.8.1 Combined navigation functions

Table 3 presents a collection of the discussed functions and provides a selection method for each function. Shortcuts are offered only to functions that are needed relatively often. Certain functions should be allowed to affect each other. For example, if a route is defined and tracks are turned on, movement is limited to the route. Also, we turn off collision detection in orbiting mode. Available combinations are also affected by current modes. If the viewpoint is tied to the GPS, steering or floating are not available, but orbiting and selection of the level of view (street level view, bird’s eye view or top-down view) are possible.

1.8.2 Control mappings

Mapping manoeuvring methods to controls depends on the available inputs. Figures 10 A through C present sample mappings for common PDA hardware buttons for direct movement, steering, and orbiting. Bindings and shortcuts for a touch screen are presented in Figure 10D. We reserve the lower part of the screen for a menu and shortcuts. The icons from the left present shortcuts to help, landmark view, routing widget, direct/orbit mode, fly to GPS, view transition, tracks mode and 2D map. Touch screen margins are mapped to pitch (left), elevation (right), pan (low) and zoom (up) in direct manoeuvring mode. Stylus movement in the centre of the screen in direct mode moves the viewpoint forward, backward or rotates it. Movement or rotation continues if the stylus reaches any of the margin areas. As a touch screen allows direct pointing, we have also implemented context-sensitive menus (Figure 11). Using the fly to functionality, the user can perform a point-and-fly scripted action. The menus allow, among other things, insertion of start and end points for routing and triggering the scripted action fly along route (the epistemic action of an assisted walkthrough). Currently, PDA hardware buttons are assigned to discrete movement, as the touch screen provides an analog interface.

Mappings for a smart phone are presented in Figure 12. Currently, all controls are assigned to explicit manoeuvring. Other functions are only available via a menu, launched by a hardware button. In smart phones, movement is currently set to be continuous for explicit manoeuvring, and discrete for the tracks mode.

The presented mappings are provided as an example from our implementation of a 3D map. It is advisable to let users configure the bindings to their liking, for example via a configuration file.
<table>
<thead>
<tr>
<th>Navigation type</th>
<th>Function/mode</th>
<th>Selection method</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explicit</td>
<td>Direct/steering/orbiting</td>
<td>Shortcut/menu</td>
<td>If following route, orbit around route points.</td>
</tr>
<tr>
<td>Explicit</td>
<td>Discrete</td>
<td>Menu</td>
<td>N/A for floating; configure impulse and thrust.</td>
</tr>
<tr>
<td>Assisted</td>
<td>Assisted camera</td>
<td>Menu</td>
<td>Assistance intervention possible via an action against assisted motion.</td>
</tr>
<tr>
<td>Constrained</td>
<td>Tracks</td>
<td>Shortcut/menu</td>
<td>Triggers animated transition to nearest road, or to route, if defined.</td>
</tr>
<tr>
<td>Constrained</td>
<td>Route definition</td>
<td>Widget/Point-and-define</td>
<td>When start and end points defined, always generate a route.</td>
</tr>
<tr>
<td>Constrained</td>
<td>Collision detection</td>
<td>Menu</td>
<td>Assisted camera may temporarily turn off. Off in orbiting mode</td>
</tr>
<tr>
<td>Scripted</td>
<td>Landmark view</td>
<td>Shortcut/menu</td>
<td>Street/bird/top-down view</td>
</tr>
<tr>
<td>Scripted</td>
<td>View mode up</td>
<td>Shortcut/menu</td>
<td>Street/bird/top-down view</td>
</tr>
<tr>
<td>Scripted</td>
<td>View mode down</td>
<td>Shortcut/menu</td>
<td>If start point defined</td>
</tr>
<tr>
<td>Scripted</td>
<td>Fly to start</td>
<td>Shortcut/menu/point-and-fly</td>
<td>If end point defined</td>
</tr>
<tr>
<td>Scripted</td>
<td>Fly to end</td>
<td>Shortcut/menu/point-and-fly</td>
<td>If route defined If GPS enabled Ties viewpoint to GPS (street/bird/top-down and orbiting applicable).</td>
</tr>
<tr>
<td>Scripted</td>
<td>Route walkthrough</td>
<td>Widget/shortcut/menu</td>
<td>If route defined If GPS enabled Ties viewpoint to GPS (street/bird/top-down and orbiting applicable).</td>
</tr>
<tr>
<td>Scripted</td>
<td>Fly to GPS</td>
<td>Shortcut/menu</td>
<td>Ties viewpoint to GPS (street/bird/top-down and orbiting applicable).</td>
</tr>
<tr>
<td>Automatic</td>
<td>GPS</td>
<td>Menu: enable GPS</td>
<td></td>
</tr>
</tbody>
</table>

*Table 3. Navigation functions.*
Fig. 10. Current controls in the PDA version of m-loma for A) direct movement, B) steering movement, and C) target-oriented movement, and D) active areas for stylus input.

Fig. 11. Context menus for A) a building and B) for a route marker arrow.
1.9 Implementation notes

Several of the presented techniques require efficient implementation in order to be affordable. For example, straightforward implementation of collision avoidance may require substantial computational resources not available in mobile devices. In addition, certain functionalities depend on content management along with support from the 3D map engine. For example, landmark positions and possibly even their geometry may need to be known to the system. In order to function according to expectations, the assisted camera scheme requires visibility information, which may not be available without implementing highly sophisticated solutions. Real-time rendering of large, richly textured 3D models on mobile devices is itself a substantial technical challenge. Nurminen (2006) provides technical details on the m-LOMA system implementation.

Summary

3D maps provide several potential improvements over their 2D counterparts. Orientation can be performed visually by direct comparison between the map and the environment. During navigation, focus can be shifted from labels (street names) to direct visual cues. The success of this shift depends on the design of the cues and the user interface. Nevertheless, three-dimensionality in itself does not necessarily prove easier navigation, unless the visualisation and user interface suit the navigation tasks.

We have asserted goals and problems for navigation with mobile 3D maps, concentrating on manoeuvring in urban environments. The problems have been identified and a model has been presented as a solution framework. Interaction guidelines have been provided for 3D navigation. Using common navigation tasks as cases, we have applied these guidelines to yield a collection of interaction designs. 3D navigation is a
complex problem and design solutions can be contradictory. Navigation efficiency is also highly context sensitive. An optimal 3D user interface is always a compromise, but we believe that the designs presented here lead to a positive user experience. Our future work concerns testing these solutions in the field.

It may seem that many of the challenges can be solved by technological advances. For example, urban positioning may be based on WLAN technologies or artificial GPS signal generators. 3D hardware will speed up rendering, and may release resources for better I/O management. However, GPS positioning may not be accurate or reliable in urban canyons, software-based rendering speed even with an optimised 3D engine may not suffice, and interface technologies such as mobile touch screens may not function perfectly. In any case, we are heading toward better solutions which will eventually enable creating host of new applications for urban pedestrians.

Acknowledgements

The m-LOMA 3D map project has been supported by EU Interreg IIIA. The Academy of Finland supported the work of the second author. We thank the lead programmers Ville Helin and Nikolaj Tatti. We also thank Sara Estlander who helped in the proofreading of this manuscript.

References


