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Off-design simulation and mathematical modeling of small-scale CHP plants at part loads

Tuula Savola a,*, Ilkka Keppo b,*

a Energy Engineering and Environmental Protection, Helsinki University of Technology, POB 4400, FIN-02015 HUT, Finland
b Energy Economics and Power Plant Engineering, Helsinki University of Technology, POB 4400, FIN-02015 HUT, Finland

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Abstract

In this work a mathematical model of combined heat and power (CHP) plants at part loads was developed on the basis of off-design simulations of four existing small-scale (1–20MW,) CHP plants. The deregulation of the electricity market and the regional availability of biofuels have given new opportunities for small-scale CHP plants. These plants are often operated for long time periods at part load conditions, which makes the knowledge of their part load behaviour very important. The simulation results presented here showed that although the part load power production of a CHP plant can be described quite accurately with a single line, there is a small nonlinear reduction in the power production as the district heat load decreases. Based on the simulation results two and three line regression models describing the electricity production and its nonlinear changes as a function of the district heat load and the outgoing district heating water temperature were developed. According to a comparison with a single and a two line regression model, the three line model describes the simulated CHP processes at part loads most accurately (i.e., has the highest \( R^2 \) value). The developed models can be used, e.g., for more detailed optimisation of a district heating network with a small-scale CHP plant. When evaluating the annual production of the CHP...
plant the accuracy of the heat duration curve may have as high influence on the results as the accuracy of
the power production model.
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**Keywords:** District heating; Efficiency; Part load; Off-design; CHP

1. **Introduction**

This paper presents the development of a simulation based mathematical model describing the part load performance of four small-scale (1–20 MW<sub>e</sub>) combined heat and power (CHP) plants. The data for the regression models is created by simulating existing CHP plants in off-design conditions. The simulation models are based on the state-of-the-art reviews of the small-scale steam Rankine cycle CHP plants in Finland and Sweden [1, 2]. Four of these reviewed plants were selected as example cases and more accurate data from these processes was collected for simulation purposes.

In small-scale CHP plants the part load operation usually covers large periods of the total plant operation time. The load depends on the district or process heat demand with the minimum load for smaller plants often around 40–50% of the maximum heat load. The minimum load of a general biofuel steam turbine CHP system is in previous studies mentioned to be 30% of the full load [3]. Because of the importance of the part load operation the evaluation of the CHP processes or district heating networks requires knowledge on the part load performance of the plants.

The operation of the small-scale steam Rankine cycle CHP plants has previously been studied, e.g., by Orgíro et al. [4], who simulated with Aspen the integration of a district heating production to an existing power plant. Harvey et al. [5] used a simulation code developed by Carcasci and Facchini [6] and created a simulation model on a CHP plant containing a gas turbine and a heat recovery boiler and discussed the optimal part load operation of the process. The simulation program selected for our work was a steady state simulation program Prosim by Endat Oy [7, 8]. The program has been used in power plant simulations both in academics [9] and in industry [10], and has performed well in a comparison between 18 simulation programs presented in the article by Giglmayr et al. [11]. Lately Tveit [12] has used the program also for integrating chemical processes into a larger energy system, as a simulation tool in energy system analysis [13], and in a simulation of a biofueled indirectly fired microturbine [14].

For power plant optimisation purposes the part load operation of the steam turbine has often been modeled with the Willans line [15–17]. The Willans line gives the partial steam flow as a function of the produced power. The Willans line is approximately a straight line between the smallest load, when the turbine is running but not yet producing power, and the load with the maximum efficiency. In the loads above the maximum efficiency the power production increases slower than in steam loads below the maximum efficiency. The Willans line includes the nonlinear change of the turbine isentropic efficiency at partial steam loads, although it itself is a linear function.

For district heating network and CHP plant optimisation purposes different approaches for modeling the power production at partial district heating loads have been developed. Sundberg et al. [18, 19] used a power-to-heat factor for defining the linear dependence between the power production and the district heat load. The power-to-heat factor was given different values accord-
ing to the selected time periods and the expected district heating loads during them. Also Benonys-
son et al. [20] described the power production of a CHP plant as a linear function of the district
heating load with the power-to-heat factor. They used the electrical efficiency for calculating the
power-to-heat factor and took into account the temperature of the hot district heating water when
defining the electrical efficiency. Zhao et al. [21] presented the production of a CHP plant by
describing the fuel consumption as a linear function of the power and district heating production
and the hot district heating water temperature.

The purpose of this work is to simulate the electricity production as the district heat load de-
creases in small-scale CHP plants, and to create a linear mathematical model of the power pro-
duction as a function of the district heat load. The model should be suitable for a district
heating network optimisation and should include the nonlinear reduction of electricity production
at part loads. The developed model can be used for evaluating the electricity production over a
longer operation period of a district heating network with different sizes small-scale CHP plants.
Especially, the effect of the district heating network part load operation on the feasibility of the
small-scale CHP plant investment can be estimated.

2. The simulation models

Four existing CHP plants producing 14, 11, 6.1 and 1.8 MW electricity were selected to present
the state-of-the-art steam Rankine processes of the CHP production from biomass. Some basic
data of these processes in full load design conditions is presented in Table 1. All of the selected
plants were quite new and three of them had started their operation in the year 2002, when also
the process data of these CHP plants was collected.

A Prosim process model consists of modules and streams. The modules can be chosen from a
module library, which contains most of the typical power plant equipment (e.g., burners, boilers,
turbines, and heat exchangers). Stream analyses (e.g., fuel analysis) are available from an analysis
library. After the process model is constructed and the relevant parameters for the modeling case are

<table>
<thead>
<tr>
<th>Process</th>
<th>MW&lt;sub&gt;e&lt;/sub&gt;</th>
<th>1.8</th>
<th>6.1</th>
<th>11</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boiler type</td>
<td>Grate</td>
<td>BFB</td>
<td>BFB</td>
<td>BFB</td>
<td>BFB</td>
</tr>
<tr>
<td>Wood fuel input</td>
<td>MW</td>
<td>11.5</td>
<td>26</td>
<td>42</td>
<td>48</td>
</tr>
<tr>
<td>Temperature after boiler/in bed</td>
<td>°C</td>
<td>650</td>
<td>850</td>
<td>870</td>
<td>850</td>
</tr>
<tr>
<td>Steam values</td>
<td>°C/bar</td>
<td>355/16.5</td>
<td>510/60</td>
<td>510/92</td>
<td>515/93</td>
</tr>
<tr>
<td>Condenser pressure</td>
<td>kg/s</td>
<td>3.8</td>
<td>7.9</td>
<td>13.8</td>
<td>16.1</td>
</tr>
<tr>
<td>District heat in/out temperature</td>
<td>°C</td>
<td>55/85</td>
<td>55/85</td>
<td>55/85</td>
<td>55/70/85</td>
</tr>
<tr>
<td>Feed water tank temperature</td>
<td>°C</td>
<td>105</td>
<td>120</td>
<td>160</td>
<td>158</td>
</tr>
<tr>
<td>Flue gas exit temperature</td>
<td>°C</td>
<td>174</td>
<td>176</td>
<td>172</td>
<td>174</td>
</tr>
<tr>
<td>Net electricity production</td>
<td>MW&lt;sub&gt;e&lt;/sub&gt;</td>
<td>1.8</td>
<td>6.2</td>
<td>11</td>
<td>13.6</td>
</tr>
<tr>
<td>District heat production</td>
<td>MW</td>
<td>8.3</td>
<td>16.5</td>
<td>25.8</td>
<td>28.4</td>
</tr>
<tr>
<td>Electrical efficiency</td>
<td></td>
<td>0.16</td>
<td>0.24</td>
<td>0.26</td>
<td>0.28</td>
</tr>
<tr>
<td>Power-to-heat ratio</td>
<td></td>
<td>0.22</td>
<td>0.38</td>
<td>0.43</td>
<td>0.48</td>
</tr>
<tr>
<td>Total efficiency</td>
<td></td>
<td>0.88</td>
<td>0.87</td>
<td>0.88</td>
<td>0.88</td>
</tr>
</tbody>
</table>
inserted in the modules, the design simulation of the process model can be run. In a design simulation the energy and mass balances of the process modules are calculated. Also, the physical properties of the plant equipment modules are calculated according to the given data. For each module Prosim uses an iterative Newton–Raphson method for the balance calculations, which are performed in the order of the user defined numbering of the modules. After the design model of the process has been created the properties of the equipment modules can be fixed for off-design calculations.

The part load operation of the modeled CHP plants was simulated by changing the fuel mass flow into the fluidized bed or grate boiler in the off-design mode. The minimum heat load in part load simulations depended on the plant size and was 35% for the larger 11 and 14 MWₑ plants and 45% for the smaller 1.8 and 6.1 MWₑ plants. The used fuel was wood, the lower heating value of which was 6.24 MJ/kg. The excess combustion air factor \( \lambda \) was 1.1 and the temperature of the combustion air after the air preheater was adjusted in design case to be between 200 and 300 °C. In the processes with bubbling fluidized bed the evaporator and the superheater were considered to be placed parallel to each other in the fluidized bed. This means that the bed temperature (850–870 °C) indicates the flue gas exit temperature from both the evaporator and the superheater. Therefore the bed temperature defines the steam mass flow in the fluidized bed design cases, if the fuel input and thus the flue gas flow remain constant. In the grate boiler case a radiant furnace model with an evaporator was used and the flue gas exit temperature from the radiant furnace with an evaporator defined the steam flow in design case with the constant fuel flow.

A steam turbine was modeled as several turbine modules each corresponding to one turbine stage (i.e., the expansion between two steam extractions). A similar decomposition principle of steam turbines for modeling purposes has been presented by Chou and Shih [22]. The temperature of the outgoing district heating water defines the back pressures in the steam turbine both at full and at part loads. The pressures of the other turbine stages are defined by the turbine constant, which can be derived from the cone rule presented, e.g., in [23], and is calculated from the data given in the design point. To keep the pressure of the superheated steam constant also at the partial steam loads, the first turbine stage is a regulating stage, which adjusts the steam flow so that the required constant pressure is obtained. The friction losses in blading and the relative efficiency changes of the regulation valves adjusting the steam flow decrease the efficiency of the regulation stage at part loads. The efficiency of the regulation stage is typically designed to be at its maximum at partial steam load (90% load).

Here the part load performance of the regulation stage is defined in the turbine module as user defined second order polynomial functions, which are calculated for each CHP plant case separately. The functions are based on the estimation that the maximum efficiency of the regulation stage, 0.80, is gained at around 90% steam load. At full load the efficiency is 0.75 and as the steam load decreases towards 10% the efficiency goes to zero. This estimation, where the maximum efficiency of the steam turbine is gained at part load, corresponds the usual conditions in a CHP plant. With these estimations the turbine efficiency starts to decrease rapidly, when the steam load is less than 80–70%.

The efficiencies, \( \eta \), of the working turbine stages are calculated by Prosim using Eq. (1), which is based on the turbine design specifications from the late 1990s.

\[
\eta = 0.023521 \cdot \ln(v) + 0.749538
\]  
(1)
The average volume flow, \( v \), is calculated as

\[
v = \frac{m \cdot dh_s}{p_{in} - p_{out}}
\]

(2)

where \( m \) is the mass flow, \( dh_s \) is the isentropic enthalpy change and \( p \) is the pressure.

When the mass flow of steam decreases during the part load operation, then also the inlet pressure of the turbine stage decreases accordingly. Thus the average volume flow and therefore the isentropic efficiency of the turbine stage are not affected by the load changes. Overall, the part load operation affects the efficiency of the whole turbine system by changing the efficiency of the regulation stage and the exhaust losses at the turbine exit. The exhaust losses of the last turbine stage were calculated according to the reference data of exhaust loss versus relative volumetric flow provided in [8].

The district heating production was modeled as a condenser, which incoming district heating water temperature was 55°C and the outgoing water temperature changed between 75 and 105°C. By combining the off-design simulation cases with different outgoing district heating water temperatures it was possible to model the part load conditions corresponding the real operation of a CHP plant with varying heat demand curves.

3. Sensitivity analysis of the simulation models

The verification of the simulation models can be done by acquiring data from measurements of the operation system of the simulated processes. To get process data that is reliable enough is often very difficult and in this case such accurate data was not available. Here the sensitivity analysis was used to evaluate the effect of the model input data uncertainty on the simulation results. The analysis shows, which are the most sensitive result values for input parameter variations and which input parameters have the greatest effect on the results. This way the sensitivity analysis is also indicating the most important process parameters for the data collection in the future. In this case the fuel flow, the flow and temperature of the combustion air, the flue gas temperature reduction in the evaporator, the temperature of the outgoing district heating water, and the temperature of the feed water were expected to be important parameters defining the power production of the processes, and they were thus selected to be the changing parameters in the sensitivity analysis. The parameters were varied \( \pm 5\% \) and the changes in the electricity and district heat production were recorded. The results of the sensitivity analysis for the four CHP models are presented in Table 2.

The important parameters that were used to define the flue gas and the steam mass flow in the processes were fuel flow and the flue gas temperature reduction in the boiler, respectively. Thus the change of these values resulted to an almost similar change of the electricity and district heat production. The important factor affecting the flue gas flow is the ratio of the excess combustion air, which defines the total combustion air flow. The change in the ratio of excess air caused also a high impact on the production of the CHP plant. The change of the outgoing district heating temperature had an important effect on the electricity production, but only minor effects on the district heating production.
According to the sensitivity analysis the simulation models are most sensitive to the changes in the fuel flow and in the flue gas temperature reduction in the boiler. The fuel flows were known quite accurately from the full load design data of the CHP plants. The flue gas temperature decrease in the boiler was possible to adjust by comparing the simulated steam mass flow, which is defined by the temperature difference, to the design value of full load operation. In this case the input values, which had the highest effect on the results according to the sensitivity analysis, were also the ones that were most accurately known from the collected plant data. The analysis shows that also in the future modeling especially the accurate values of the fuel flow and the flue gas temperature after the boiler should be of high priority in data collection.

4. Simulation results

The net power production \( P_{net} \) at part loads versus the district heating load \( Q \) for the four simulated models with different outgoing district heating water temperatures are presented in Figs. 1–4. Although the dependence between the district heat load and the produced net power seems to be quite linear, there is a small nonlinear reduction in the \( P_{net}(Q) \) function at part loads. The reason for this nonlinear decrease is the behaviour of the steam turbine isentropic efficiency.
The isentropic efficiency decreases rapidly at steam loads below 80–70% as is defined for the regulation stage of the turbine in the design model.

5. The mathematical model for power production in part loads

The power production of a CHP plant can be described using the district heat production ($Q$) and the temperature levels of the outgoing hot and incoming cold district heating water ($T_h$ and $T_c$, respectively) as variables. The general linear model for power production is

$$P(Q, T_h, T_c) = a \cdot Q + b \cdot T_h + c \cdot T_c + d$$

(3)
This formulation allows to exclude the more complicated description of the steam turbine and makes it possible to describe the whole district heating process without expanding the problem too much. Similar formulation can also be used to describe the consumption of fuel, $Q_{\text{fuel}}$.

If the steam turbine has a two stage district heating, $T_c$ can refer either to the cold incoming district heating water temperature or to the temperature between the heat exchangers (e.g., the outgoing water temperature from the first district heat exchanger), since these two temperatures depend on each other. In the one stage district heating case the temperature of the cold incoming district heating water does not affect the power production. As the temperature of the cold incoming district heating water was not changed in the simulations, the $T_c$ factor was not taken into account in the regression model coefficient calculations. If it had been included in the simulations,
it would have affected only the coefficients of the 14 MW<sub>c</sub> process with the two stage district heating.

The coefficients for the single linear regression models for fuel input and power production are calculated as least square estimators for a multiple linear regression model using the LINEST function in Microsoft Excel. The results are presented in Table 3. The coefficients refer to the original plants and therefore they have to be scaled, if the plant size should be changed.

The \( R^2 \) is the coefficient of determination, which describes the accuracy of the approximation. It can be calculated according to

\[
R^2 = 1 - \frac{\sum(Y_j - \hat{Y}_j)^2}{\left(\sum Y_j^2 \right) - \left(\sum Y_j \right)^2 /n}
\]

where \( Y_j \) is the simulated value and \( \hat{Y}_j \) is the value from the approximation.

If there is no difference between the simulations and the estimated value, the value of \( R^2 \) equals one. If \( R^2 \) equals zero, there is no correlation between the estimation and the simulations. \( R^2 \) is the proportion of the variability in the response explained by the regression model.

The values of the \( R^2 \) calculated for the single multiple linear regression model are good enough for the accuracy required from a regression coefficients used in an optimisation model. However, this model would not take into account the more steep changes of the power production at part load operation. This would especially harm the accuracy, when the load is very low, since the non-linearities become more apparent then. This can be seen, e.g., from the derivatives of the \( P_{\text{net}}(Q) \)

<table>
<thead>
<tr>
<th>Plant size, MW&lt;sub&gt;c&lt;/sub&gt;</th>
<th>Single line model of fuel input</th>
<th>Single line model of power production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
</tr>
<tr>
<td>1.8</td>
<td>1.49</td>
<td>-0.0191</td>
</tr>
<tr>
<td>6.1</td>
<td>1.70</td>
<td>-0.0426</td>
</tr>
<tr>
<td>11</td>
<td>1.73</td>
<td>-0.0614</td>
</tr>
<tr>
<td>14</td>
<td>1.82</td>
<td>-0.0601</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Two line model of power production</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
<td>( d )</td>
</tr>
<tr>
<td>1.8</td>
<td>0.180</td>
<td>-0.015</td>
<td>1.55</td>
</tr>
<tr>
<td>6.1</td>
<td>0.373</td>
<td>-0.035</td>
<td>3.00</td>
</tr>
<tr>
<td>11</td>
<td>0.421</td>
<td>-0.050</td>
<td>4.37</td>
</tr>
<tr>
<td>14</td>
<td>0.493</td>
<td>-0.049</td>
<td>3.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Three line model of power production</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a )</td>
<td>( b )</td>
<td>( d )</td>
<td>( L_1 )</td>
</tr>
<tr>
<td>1.8</td>
<td>0.136</td>
<td>-0.0152</td>
<td>1.92</td>
<td>0.90</td>
</tr>
<tr>
<td>6.1</td>
<td>0.358</td>
<td>-0.0349</td>
<td>3.23</td>
<td>0.75</td>
</tr>
<tr>
<td>11</td>
<td>0.378</td>
<td>-0.0502</td>
<td>5.41</td>
<td>0.85</td>
</tr>
<tr>
<td>14</td>
<td>0.463</td>
<td>-0.0491</td>
<td>4.49</td>
<td>0.80</td>
</tr>
</tbody>
</table>
lines, which increase at part loads as presented in Fig. 5. To take these changes into account, the power production was described also with two and three linear representations instead of only one. The function then takes the form

\[ P(Q, T_h, T_c) = \frac{a}{C_1}Q + \frac{b}{C_1}T_c + \frac{c}{C_1}T_c + d \quad \text{for } L_1 \cdot Q_{\text{inv}} \leq Q \leq L_2 \cdot Q_{\text{inv}} \]

where

\[ w_1 = (L_1 \cdot Q_{\text{inv}} - Q) \cdot r_1 \]

\[ w_2 = (L_2 \cdot Q_{\text{inv}} - Q) \cdot r_2 \]

are positive coefficients that describe how much the power production is affected by the part load operation. For a two line model only the two upper functions from Eq. (5) are used.

If this formulation is to be used in an optimisation problem, it is in its general form

\[ P(Q, T_h, T_c) = a \cdot Q + b \cdot T_h + c \cdot T_c + d - \sum_{i=1}^{n} w_i \]

with the restrictions

\[ w_i \geq (L_i \cdot Q_{\text{inv}} - Q) \cdot r_i \]

\[ w_i \geq 0 \]
Since the change is always negative and the power production is therefore decreased, the optimisation will try to keep $w_i$ as low as possible, as long as the price of electricity is positive. However, when the district heat production is lower than $L_i \cdot Q_{\text{inv}}$, it has to give these variables positive values according to Eq. (9).

In the case, where three linear descriptions are used $n = 2$ and $i \in [1, 2]$ in Eqs. (8) and (9). Therefore, if the district heat produced is lower than the limit $L_1$, but higher than $L_2$, the equation for power production is

$$P = (a + r_1) \cdot Q + b \cdot T_h + c \cdot T_c + d - L_1 \cdot r_1 \cdot Q_{\text{inv}} \quad (11)$$

If the heat produced is lower than $L_2$, the equation becomes

$$P = (a + r_1 + r_2) \cdot Q + b \cdot T_h + c \cdot T_c + d - (L_1 \cdot r_1 + L_2 \cdot r_2) \cdot Q_{\text{inv}} \quad (12)$$

The coefficients $a, b, c, d$ and $r_i$ for the two and three line models are found using the Microsoft Excel solver to maximise the $R^2$ and using the coefficients as variables. The values for $L_1$ and $L_2$ are found by giving them different values and then choosing the pair that gives the highest $R^2$ value. The values for $L_1$ and $L_2$ are therefore not globally optimal, but they do give an accurate enough description for the power production. The coefficients and limits are presented in Table 3.

In addition to having higher $R^2$ values, using the two and three line models takes into account the nonlinear changes that happen during the part load operation. The difference between these approximations and the approximation using the single line is presented in Fig. 6, where the squares of errors, $e^2$, are compared to the simulation results. The example case is the 11 MW_e process with the outgoing district heating water temperature of 85°C.

The significance of the two and three line models to the electricity production and the income from the electricity production was estimated by calculating the annual production according to the simulations and the different mathematical models. The annual electricity production was calculated by scaling down an estimation of the district heating load duration curve presented in [5], so that for each district heating network case around 65% of the district heat peak load was produced with the CHP plant. The lowest loads for the CHP plant cases were between 35% and 45%.

![Fig. 6. Difference between the square of errors, $e^2$, with single line, two line, and three line models. 11 MW_e process with outgoing district heating water temperature of 85°C as an example.](image-url)
The electricity price was changed so that at full load it was 50 €/MWh and decreased to 5 €/MWh when the load was reduced to 45%. The annual power production and the income from the produced power estimated with different models are presented in Table 4.

When compared to the simulated results, the three line model gave the closest estimations of the annual power production and electricity income in the 14 MWe case. The estimations of the two and three line models were almost equally good in the 6.1 MWe case. Despite that the three line model corresponded better the simulation results in Table 3, when all loads were expected to be equally significant, the two line model gave better estimations in the 1.8 and 11 MWe cases, when the duration of the loads during the annual operation time was considered. This highlights the importance of the duration curve coefficients to the results of the annual production. The advantage of the three line model is that it describes well also the loads with a very short annual duration. The two line model describes very well the loads in the linear parts of the model but the accuracy decreases at the loads outside the two linear equations, e.g., at very small loads with short durations. Thus the two line model may give better results in the annual production calculations than the three line model if the duration coefficients (i.e., load duration) for the loads outside the two linear equations are very small and the coefficients for the loads in the linear parts of the model are very large.

According to the comparisons, the single line, the two line, and three line models all describe the electricity production of these CHP plants fairly well. The three line model describes the simulated results more accurately, but using the single line or the two line model should be considered case by case according to the needed accuracy of the estimation. When evaluating the annual production of a CHP plant a careful selection of as an accurate duration curve as possible for the CHP plant may be as important as the selection of the power production model.

**Table 4**

Annual power production and income from the power production calculated with different models and their differences to the simulation results

<table>
<thead>
<tr>
<th>Model</th>
<th>Power, MWh/year</th>
<th>Income, k€/year</th>
<th>Power, MWh/year</th>
<th>Income, k€/year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.8 MW&lt;sub&gt;e&lt;/sub&gt;</td>
<td>6.1 MW&lt;sub&gt;e&lt;/sub&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>6845</td>
<td>185.9</td>
<td>24,684</td>
<td>668.0</td>
</tr>
<tr>
<td>Single line model</td>
<td>6858</td>
<td>187.4</td>
<td>24,724</td>
<td>673.2</td>
</tr>
<tr>
<td></td>
<td>(0.19%)</td>
<td>(0.79%)</td>
<td>(0.16%)</td>
<td>(0.78%)</td>
</tr>
<tr>
<td>Two line model</td>
<td>6838</td>
<td>185.5</td>
<td>24,688</td>
<td>668.4</td>
</tr>
<tr>
<td></td>
<td>(−0.10%)</td>
<td>(−0.24%)</td>
<td>(0.02%)</td>
<td>(0.06%)</td>
</tr>
<tr>
<td>Three line model</td>
<td>6836</td>
<td>185.3</td>
<td>24,689</td>
<td>667.9</td>
</tr>
<tr>
<td></td>
<td>(−0.14%)</td>
<td>(−0.34%)</td>
<td>(0.02%)</td>
<td>(−0.02%)</td>
</tr>
<tr>
<td></td>
<td>11 MW&lt;sub&gt;e&lt;/sub&gt;</td>
<td></td>
<td>14 MW&lt;sub&gt;e&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>Simulation</td>
<td>46,055</td>
<td>1195.1</td>
<td>57,046</td>
<td>1481.8</td>
</tr>
<tr>
<td>Single line model</td>
<td>46,130</td>
<td>1203.3</td>
<td>57,139</td>
<td>1492.0</td>
</tr>
<tr>
<td></td>
<td>(0.16%)</td>
<td>(0.68%)</td>
<td>(0.16%)</td>
<td>(0.69%)</td>
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<tr>
<td>Two line model</td>
<td>46,062</td>
<td>1195.8</td>
<td>57,066</td>
<td>1483.4</td>
</tr>
<tr>
<td></td>
<td>(0.02%)</td>
<td>(0.06%)</td>
<td>(0.04%)</td>
<td>(0.10%)</td>
</tr>
<tr>
<td>Three line model</td>
<td>46,033</td>
<td>1193.6</td>
<td>57,039</td>
<td>1481.2</td>
</tr>
<tr>
<td></td>
<td>(−0.05%)</td>
<td>(−0.13%)</td>
<td>(−0.01%)</td>
<td>(−0.05%)</td>
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</table>
6. Conclusions

The off-design simulations based on the data from existing small-scale CHP plants showed that there is a nonlinear reduction in the net power production at the partial district heat loads. This is caused by the decreasing isentropic efficiency of the steam turbine at partial steam loads.

On the basis of the simulation results a linear regression model of the power production of the CHP plant was developed for the district heating network optimisation purposes. The input variables for the model are the district heating load and the temperature of the outgoing district heating water. Special for this model is that the nonlinear change of the turbine isentropic efficiency and thus the nonlinear reduction of the electricity generation at part loads were taken into account during the model development.

The accuracy of a single line model was improved in this paper by dividing the power production function in two and three parts according to the district heat load. With these two and three line regression models the nonlinear behaviour of the electricity production at part loads could be taken into account without transforming the corresponding optimisation problem into a nonlinear one. The two and three line model approaches are useful, e.g., when optimising a district heating network with different CHP plant alternatives. When evaluating the annual production of the CHP plant also the selected heat demand curve of the district heating network has a high influence on the results. In the future simulation and modeling work, also the temperature of the incoming district heating water could be taken into the consideration especially, if there are more than one district heat exchanger in the modeled CHP plant cases.

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References


