Photon-assisted tunneling and charge transport in hybrid circuits

Máté Jenei
Photon-assisted tunneling and charge transport in hybrid circuits

Máté Jenei

The public defense on 5th June 2020 at 13:00 (1 p.m.) will be organized via remote technology.

Link: https://aalto.zoom.us/j/64611149664


A doctoral dissertation completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Science, https://aalto.zoom.us/j/64611149664, on 5 June 2020 at 13:15.

Aalto University
School of Science
Department of Applied Physics
Quantum Computing and Devices
Supervising professor
Prof. Mikko Möttönen

Thesis advisor
Dr. Kuan Yen Tan

Preliminary examiners
Dr. Hans Hübl, Walter-Meißner-Institut, Germany
Dr. Floris Zwanenburg, University of Twente, Netherlands

Opponent
Prof. Thomas Markus Ihn, ETH Zürich, Switzerland

Aalto University publication series
DOCTORAL DISSERTATIONS 81/2020

© 2020 Máté Jenei

ISSN 1799-4934 (printed)
ISSN 1799-4942 (pdf)

Unigrafia Oy
Helsinki 2020

Finland
Abstract

Photonic dissipation and charge sensing are two crucial topics of modern quantum circuit dynamics. Quantum circuits operating at low powers reaching few-photon level require precise control over losses to be a workhorse of quantum information processing. Contrarily, qubit manipulation demands reliable reset protocols. Although charge sensing is considered as a mature diagnostic device in mesoscopic physics, high-frequency charge pumping requires much more sensitive detectors to reveal faster electron dynamics. In all of the topics discussed in this thesis, a multitude of device materials and parameters are explored, consisting of superconductors, normal metals, insulators, field-induced two-dimensional electron gas conductors, and quantum dots. The variety of the mentioned "components" forms the investigated hybrid nanostructures.

In this thesis, the control of the coupling between a superconducting resonator and a dissipative reservoir is investigated. One convenient control method is to employ a normal-metal–insulator–superconductor junction, which functions as a voltage-tunable heat sink that is compatible with superconducting circuit technology. The heat sink is found to modulate the fundamental frequency of a resonator, a signature of a broadband Lamb shift in a microwave circuit. Also, the heat sink can be operated as an accurate thermal microwave source that can be used to calibrate the total gain of an arbitrary amplification chain. Another method, as demonstrated in an experiment, is to couple two resonators capacitively with one of the resonators that was intentionally engineered with a low-quality factor. This highly-dissipative resonator has an integrated magnetic-field-sensitive superconducting quantum interference device that enables the tunability of the resonator natural frequency. Matching the resonances of both resonators then allows to increase the dissipation in the high-quality resonator.

This work also investigates electric current metrology realized through semiconductor field-effect transistor quantum dot pumps. In one aspect, we integrated a high-sensitivity superconducting Josephson-junction-based charge sensor with a silicon quantum dot architecture to examine the noise properties of the system. By studying the noise statistics, we are able to determine the dominant noise mechanism surrounding these pumps thus paving the way to the development of better detectors. In another aspect, the portability of a quantum dot single-electron pump is verified, where we have demonstrated that the relative uncertainty of the quantized current created by the same device is within 1.30 ppm regardless of the location of the experiment.

Keywords Tunnel junction, single-electron transport, microwave photonics, quantum-circuit refrigerator, quantum dot, charge detection
Preface

The work presented in this thesis was carried out in Quantum Computing and Devices group, QCD Labs, Department of Applied Physics, Aalto University from January 2016 to November 2019.

My supervising Prof. Mikko Möttönen accepted my application in 2015 and I joined his group as a research assistant for a few months. These few months started my journey at QCD Labs. I am grateful to Mikko for creating and driving QCD Labs as an entity for which it was worth to wake up and work on every single day. Your guidance and vision kept me on the track! In the beginning, Prof. Matti Kaivola acted as my PhD advisor. I would like to thank him for keeping his office door always open and for his advice on how to pursue the PhD degree.

I express my gratitude to Dr. Kuan Yen Tan for being my thesis advisor and for the countless hours he invested in me. Not just for transferring his theoretical and practical knowledge, but also to show how to be clearheaded when I encounter problems. Initially, Dr. Joonas Govenius and Dr. Russell Lake helped me on settling in low-temperature physics. Besides the great company, they gave lots of hints how to enjoy life in Finland. I would like to thank also to Dr. Tuomo Tanttu for introducing me to the beauty of single-electron pumping and operating a plastic dilution refrigerator. The two people, whom I spent most of my time with in the lab are Dr. Matti Partanen and soon-to-be-PhD Roope Kokkonemi. I am grateful for those unforgettable years.

I am indebted to many current and former QCD Labs members including, but not limited to, Kostya, Tuomas, Sofia, Joni, Shumpei, Jan, Dibyendu, Marci, Akseli, Johannes, Tianyi, Wei, Vasili, Eric, Jinli, Jean-Philippe, Roberto, Timm, Giacomo, Arto, Chengyu, Aarne and Valteri. Thanks for the hard work to the talented summer students whom I worked together: Heikki, Elin, and Heidi. I also had luck to work and spend time together with people from Prof. Jukka Pekola’s group: Dr. Joonas Peltonen, Prof. Matthias Meschke, Dr. Anna Feshchenko, Dr. Dmitry Golubev, Dr. Libin Wang, Dr. Jorden Senior, and Elsa Mannila.
My studies meant a great opportunity to collaborate and learn how to collaborate. From the beginning of my PhD studies, intense scientific discussions with Dr. Elina Potanina and Prof. Slava Kashcheyevs catalysed my work. I am grateful for them being encouraging and supportive. In different experimental projects, I worked together with people whom I want to thank from VTT Technical Research Centre of Finland: Dr. Antti Kempinen, Dr. Antti Manninen, Dr. Janne Lehtinen, Dr. Emma Mykkänen, Dr. Visa Vesterminen, Dr. Alberto Ronzani, Dr. Robab Najafi Jabdaraghi, and Dr. Mika Prunnila. The results presented in this thesis have been achieved together with Dr. Alessandro Rossi, Dr. Stephen Giblin, Dr. Matti Silveri, Dr. Jani Tuorila, and the collaborators from the University of New South Wales: Prof. Andrew Dzurak, Dr. Ruichen Zhao, Yuxin Sun, and Dr. Kok Wai Chan.

The infrastructure where I spent most of my time requires a lot of attention. The provision of Dr. Mika Koskenvuori, Dr. Päivikki Repo, Dr. Ville Vähänässi, and Antti Peltonen directly impacted my work for which I am grateful. Furthermore, I acknowledge the support from EUROMET European Metrology Programme for Innovation and Research (EMPIR) Project No. 15SIB08, e-SI-Amp. I also acknowledge the Doctoral Education Network in Condensed Matter and Materials Physics to support my conference trips.

High-school and undergraduate years shaped my interest undoubtedly and brought me where I am now. I thank to my godparents Ferenc Dargó and Katalin Dargóné to introduce me maths and physics, and to Dr. Tamás Serényi Sch. P. who revealed me how solving physics problems can be joyful. At University of Szeged, I am grateful to Dr. Péter Földi and Dr. Attila Czirják for being my thesis supervisors and showing me the practices in scientific research.

Least but not last I have to say thank you to my family. My wife, Éva Zsófia, supported and loved me in spite of difficulties and taught me how to bring balance in my life. My parents and brothers always encourage me in my plans and work. Without such wonderful people by my side this book would never have come into being.

Helsinki, May 7, 2020,

Máté Jenei
# Contents

<table>
<thead>
<tr>
<th>Preface</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contents</td>
<td>iii</td>
</tr>
<tr>
<td>List of Publications</td>
<td>v</td>
</tr>
<tr>
<td>Author's Contribution</td>
<td>vii</td>
</tr>
</tbody>
</table>

## 1. Introduction  1

## 2. Theoretical background  5

2.1 Quasiparticle density of states  5

2.1.1 Normal conductor  5

2.1.2 Superconductor  7

2.1.3 Charge island and quantum dot  7

2.2 Tunneling through a weak link  8

2.2.1 Tunneling in a single-electron transistor  9

2.2.2 NIS tunneling  11

2.2.3 SIS tunneling  12

2.3 Photon-assisted tunneling in NIS structures  13

2.3.1 Resonators  13

2.3.2 NIS junction coupled to a resonator  14

2.4 Charge detection with superconducting single-electron transistor  16

2.4.1 Charge sensing for error counting  19

2.4.2 Counting statistics and waiting-time distributions  20

2.5 Single-electron pumping in semiconductors  21

## 3. Experimental methods  23

3.1 Sample Fabrication  23

3.2 Measurement procedure  26

3.3 Microwave network analysis  26
This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.


VI M. Jenei∗, E. Potanina∗, R. Zhao, K. Y. Tan, A. Rossi, T. Tanttu,
* These authors contributed equally to this work, December 2019.

Author’s Contribution

Publication I: “Calibration of cryogenic amplification chains using normal-metal–insulator–superconductor junctions”

The author carried out all the measurements, analyzed most of the data, and wrote a major part of the manuscript.

Publication II: “Broadband Lamb shift in an engineered quantum system”

The author contributed significantly to the measurements and to the voltage reflection data analysis by developing the normalization method.

Publication III: “Fast control of dissipation in a superconducting resonator”

The author actively participated in the data analysis, and in the measurements, and commented on the manuscript.

Publication IV: “Flux-tunable heat sink for quantum electric circuits”

The author helped the first author with the numerical simulations, gave extensive technical assistance in device fabrication and measurements, and commented on the manuscript.
Author's Contribution

**Publication V: “Superconducting charge sensor coupled to an electron layer in silicon”**

The author actively participated in the device fabrication, measured and analyzed most of the data, and wrote the first full draft of the manuscript based on the input from coauthors.

**Publication VI: “Waiting-time distributions in a two-level fluctuator coupled to a superconducting charge detector”**

The author actively participated in the device fabrication, measured and analyzed the data, and wrote half of the manuscript based on the input from coauthors.

**Publication VII: “Realisation of a quantum current standard at liquid helium temperature with sub-ppm reproducibility”**

The author carried out all measurements at Aalto University, participated in the data analysis, and commented on the manuscript.
1. Introduction

The explanation of black-body radiation by Max Planck in 1900 gave rise to the concept of photons; the electromagnetic radiation comprises an integer number of energy packets and the energy of a packet is proportional to the frequency of the radiation. A few years later in 1909, the quantization of the electric charge was discovered during the Millikan–Fletcher oil drop experiment [1, 2].

These findings led to the First Quantum Revolution in the beginning of the 20th century rendering a completely new way of describing the natural world. The technical advancements in the 20th century benefited from the laws of quantum mechanics, for instance, the computer industry, laser technology, and nanofabrication processes. These improvements are the prelude of a new era which we are experiencing now, the Second Quantum Revolution where individual quantum systems and phenomena are not just observed, but carefully engineered such that their characteristics are employed directly in various applications [3]. This thesis is inspired and driven by technical problems, in which engineered quantum circuits can serve with potential solutions, such as, how to estimate the gain of an amplification chain, how to deplete photons effectively from a resonator, and how to build self-calibrating high-precision electric current sources.

The size of the engineered circuits discussed in this thesis are so small that their degrees of freedom are restricted. As the temperature drops, the thermal noise and therefore the fluctuations on the degrees of freedom are significantly decreased. If these conditions are fulfilled, three relevant peculiar properties can coexist in a circuit: First, electrons pass through a potential barrier in a sequential manner which is referred to as sequential single-electron tunneling. Here, the total energy of an individual electron is less than the height of the potential barrier. Second, in a portion of a mesoscopic conductor which is tunnel coupled to its environment, electron occupation can be changed only by providing a well-defined minimum amount of energy otherwise the Coulomb repulsion blocks the tunneling. Third, the resistance of certain metals vanishes hence the metal enters into the superconducting phase. In superconductors, two electrons combine
and form a Cooper pair which stores a binding energy, or more specifically energy that is equal to the superconducting gap [4].

While the published literature summarized in this work considers rather different types of physics, charge tunneling forms a common foundation for our study. Therefore, the theoretical overview in Chapter 2 starts from the presentation of the materials, continues with the description of the single-electron tunneling, and navigates towards more specific and applicable theories. Photon-assisted tunneling is a special case of the aforementioned sequential single-electron tunneling where the tunneling electron absorbs or emits a photon during the transport.

The meaning of the word “hybrid” in the title of this thesis refers to the different materials that the circuits consist of. In Publications I–III, the key components are normal-metal–insulator–superconductor (NIS) junctions, where a thin layer of copper is the normal metal, aluminium is the superconductor, and these two layers are separated by an aluminium oxide insulator. The superconducting quantum interference device (SQUID) in Publication IV and the superconducting single-electron transistor (SSET) in Publication V and Publication VI are formed by superconductor–insulator–superconductor (SIS) junctions, which are in practice Al–Al$_2$O$_3$–Al junctions. Analogously, metal-oxide-semiconductor field-effect transistors (MOSFETs) comprise an aluminium or a conductive polysilicon conductor layer, an isolating silicon-oxide gate, and a silicon semiconductor substrate employed in Publications V–VII.

On the other hand, these circuits can be considered as hybrids, in which well-established platforms have been unified to exhibit novel properties. In the case of Publications I–III, the NIS junctions are combined with superconducting resonators to cool either the resonator or the normal-metal island forming a quantum-circuit refrigerator (QCR) [5] or they act as a thermal photon source [6] depending on a single control parameter. In Publication V, a two-dimensional electron gas (2DEG) conductor induced by a MOSFET, which is the backbone of contemporary transistor-based classical computing, is combined with a metallic single-electron transistor (SET). The SET was theoretically developed by Averin and Likharev in 1986 [7] and the following year the first report on the experimental realization was presented by Fulton and Dolan [8]. Publications I–IV of this thesis investigate the properties of a superconducting coplanar waveguide resonator, which follows the model of a quantum harmonic oscillator [9] coupled to a tunable dissipative environment. In Publications I–III this tunable environment is a double NIS or SINIS junction regulated by an external voltage and in Publication IV the environment is a dissipative resonator with a tunable resonance frequency.

The typical power in a microwave circuit is at the few-photon level, which requires substantial amplification for the readout [10, 11]. In practice, different types of amplifiers are cascaded and placed at different tempera-
ture stages such that the signal-to-noise ratio is maximal, resulting in a higher quantum efficiency [12]. The experimental estimation of the total gain and noise temperature of such an amplification chain is nontrivial, because the calibration schemes may need multiple cool-downs or require impedance-matched components. In Publication I we present a method using a QCR circuit as a calibration device. The technique requires two consecutive measurement steps which, at the end of the procedure, yield a single coefficient as a fitting parameter to obtain the total gain and noise temperature.

Renormalization of energy levels in atomic physics owing to coupling to the electromagnetic field is known as the Lamb shift [13], which has also been observed in microwave circuits [14] but only in a narrow frequency range. In Publication II we demonstrate the case of a broadband Lamb shift which we directly relate to the change of the coupling between the resonator and the environment introduced by the tunneling electrons thorough a NIS junction.

In Publication III the fast operation of a resonator-reset protocol based on the tunable environment is demonstrated experimentally. Our main motivation to create a dissipation switch is to improve the qubit control by driving the qubit state to ground state in a controllable manner. To this end, we couple a QCR to an intermediate resonator or directly to a superconducting qubit. A similar motivation lies behind the method developed in Publication IV where the reset is carried out using magnetic flux as a control parameter. In this case, the dissipative element is a secondary resonator, which is capacitively connected to ground via a resistive copper nanostructure. The frequency of the resulting dissipative resonator is flux-tunable due to a SQUID which regulates the dissipation experienced by the main resonator. As we show, the quality factor of the main resonator can be tuned by two orders of magnitude.

Publications V–VII concentrate on the electric current metrology with the aim of redefining a new quantum-based current standard, which ties the definition of the electric current to the fundamental constant of elementary charge, $e$ [15]. Single-electron pumps, as potential candidates [16], are periodically-driven nanoscale devices that produce an average electric current $I = n ef$, where $n$ is the transferred number of electrons over a time period $1/f$. Therefore, precision of the current created by the single-electron pump depends on the error in the transferred number of electrons. Nowadays, silicon quantum dots with voltage-tunable potential barriers provide the lowest uncertainty in the pumped current in the sub-nanoampere regime [17].

In Publication VII we show that a transportable silicon quantum dot single-electron pump produces a consistent quantized current in three different laboratories, and both the deviation from $1 ef$ and the uncertainty of the pumped current are at the level of a part per million at
1.05 GHz frequency. Since the current measurement requires room temperature circuitry, due to the added noise, the measured accuracy of the pumped current is bounded by the noise of the measurement system.

On-chip charge sensing helps to overcome this issue. Previous experiments demonstrated how the failed events of a single-electron source can be resolved [18–21], but in all of these cases, the limiting factor in the operation speed was the weakly-coupled charge detector. Based on Publication V, we propose a structure for high-speed error counting and by measuring the charge noise on the superconducting detector and the coupling strength between the detector and a charge reservoir connected to the quantum dot pump, we estimate the potential performance of such self-calibrating current source.

In Publication VI, we use the charge detector to observe charge state transitions of a two-level fluctuator (TLF) that is strongly coupled to the detector. With the help of a waiting-time distribution theory, we reveal the timescales where the harmful effect of the fluctuator can be minimized in terms of charge sensing.

Thesis is organized as follows: Chapter 2 presents the theoretical background for the used materials for charge tunneling in solid-state matter, and for photon-assisted tunneling, as well as a literature overview which puts each Publication in context. Chapter 3 describes the fabrication processes, experimental set-ups, and methods. The key results of this thesis are provided in Chapter 4. Chapter 5 summarizes the thesis and highlights some of the promising future research directions.
2. Theoretical background

In this chapter, we introduce the theoretical concepts pertaining to the experiments used in this thesis. In Sec. 2.1, we present the condensed-matter systems and their electronic properties. Single-charge tunneling through a potential barrier, namely, normal-conductor–insulator–superconductor (NIS) and superconductor–insulator–superconductor (SIS) junctions, and the working principle of a single-electron transistor, are discussed in Sec. 2.2. In Sec. 2.3, the inelastic charge tunneling is presented focusing on the case of a tunnel junction coupled to a resonator. Finally, in Secs. 2.4 and 2.5, the charge sensing and charge pumping are introduced.

2.1 Quasiparticle density of states

Single-charge tunneling is a fundamental phenomenon, in common with all the devices studied in this thesis. To quantitatively describe the charge transport first the employed materials have to be discussed. Charge carriers in the solid state of matter are electrons, holes, or paired electrons in the case of superconductors. Electrons and holes are fermionic-type particles, which means that, at most, a single electron or hole can occupy an available single-particle quantum state \(^{[22]}\). The occupation of these energy levels determines the relevant transport properties: few levels participate in the charge transport, while most of the levels are occupied or empty. The density of states times an infinitesimally small energy step \(\delta E\) equals the number of fermionic states in the energy range \([E, E + \delta E]\).

2.1.1 Normal conductor

Some conductors used in the experiments are in the normal state. The charge carriers of the normal-state electrodes, which are called quasiparticles, can be modeled by the Fermi liquid theory if their momenta are in the vicinity of the Fermi sphere \(^{[22]}\). In these Fermi systems, the behavior of quasiparticles is analogous to free electrons, but they are affected by
Theoretical background

electron-electron interactions, which are encoded in the dispersion relation, i.e., the energy as a function of the momentum. At a finite temperature $T$, the probability of occupying the energy level $E$, with respect to the chemical potential, follows in Fermi–Dirac distribution

$$f(E, T) = \frac{1}{e^{E/(k_B T)} + 1},$$

where $k_B$ is the Boltzmann constant. The Fermi level, or chemical potential, is the energy where the occupation probability is $1/2$. Since the conductor contacts have a large number of electrons, removing or adding one electron to the system does not change the distribution. The normalized density of states $n_N$, in a normal conductor is unity, however, the normalized states available for tunneling depends on the temperature as illustrated in Fig. 2.1(a).

Semiconductor field-effect transistors in charge inversion or in accumulation modes also follow Eq. (2.1). In semiconductors, the conducting domain is confined to a lateral plate in the vicinity of the semiconductor–insulator interface, which is a two-dimensional electron gas (2DEG) in the accumulation mode or a two-dimensional hole gas in the inversion mode.
2.1.2 Superconductor

Certain materials undergo a phase transition below a certain critical temperature and magnetic flux density where the electrical resistance vanishes [4]. Consequently, in a superconducting ring without any external action, a supercurrent can exist longer than the lifetime of the universe [24]. The Bardeen–Cooper–Schrieffer (BCS) theory [25] explains both the vanishing resistance and the expulsion of the magnetic field, known as the Meissner effect, in homogeneous superconductors. According to the BCS theory, in the superconducting phase a weak attractive force appears between the charge carriers, the force originates from electron-phonon interactions, since the movement of an electron shifts the positions of the nearby positively-charged ions towards the moving electron. This lattice deformation changes the initial charge distribution which attracts a consecutive electron. Hence, the two interacting electrons form a Cooper pair, which may break up if an energy of $2\Delta$ is applied, where $\Delta$ is referred to as the superconductor gap parameter.

The quasiparticle density of states in a superconductor resembles the semiconductor density of states, in which a band gap is present [4]. At zero temperature, in the subgap region, no available quasiparticle states exist, but outside the gap, quasiparticle current may be responsible for a finite resistance. In experiments at finite temperatures, however, a non-zero current appears in the subgap regime, which emerges from the interaction of the superconducting structure and its environment yielding a current which is orders of magnitude smaller than the above-gap current [26].

The superconductor is described by the normalized Dynes formula for the density of states which reads as [9, 27]

$$n_S(E) = \frac{N_S(E)}{N_F} = \left| \text{Re} \left( \frac{E/\Delta + i\gamma_D}{\sqrt{(E/\Delta + i\gamma_D)^2 - 1}} \right) \right|,$$  \hspace{1cm} (2.2)

where $E$ is defined as an offset from the Fermi level, $\gamma_D$ is the dimensionless Dynes parameter, and $N_F$ is the normal-state density of states at the Fermi level. As shown in Fig. 2.1(b), the density of states in the vicinity of $|E| = \Delta$ is increased, while a few states within the gap provide quasiparticle transport.

2.1.3 Charge island and quantum dot

At low temperatures, if the physical size of a conductor is small and it has only capacitive and weak tunnel coupling to its environment, the single-electron effect appears. Our focus is on such conductive islands which are located between a source and a drain electrode that are tunnel coupled to the island. In this thesis, both the harnessed metallic structures [28] and semiconductor quantum dots [29, 30] belong to the category of charge
islands. Charge quantization is the main reason why the charge state of a conductor island can be increased in steps dictated by the Coulomb repulsion. The energy cost to load a single electron on a charge-neutral island equals the charging energy, $E_C = e^2/(2C\Sigma)$, where $e$ is the elementary charge and $C\Sigma$ is the total capacitance of the island. The available states of a quantum dot, taking only into account the Coulomb energy, is schematically presented in Fig. 2.1(c).

The spacings between consecutive charge states, or addition energies, are not necessarily equidistant. The spatial confinement of the quantum dot results in atom-like orbitals where each electron state follows a variant of Hund’s rule [31]. The material and the electrostatic confinement creating a quantum dot yield different potential landscapes, which determine the prominence of the energy level spacing compared to the charging energy [32]. In typical experimental setups, due to the finite tunnel coupling between a lead and a quantum dot, the Dirac peaks undergo a broadening effect [9].

Quantum dots have various applications: due to the tunable charging energy they can be used for photon-assisted tunneling electron spectroscopy (PATS) [33–35], quantum dot systems can be employed in thermometry [36–38], as high-accuracy current sources (See Sec. 2.5), and in quantum information processing as both charge [39, 40] and spin qubits [41–45].

2.2 Tunneling through a weak link

In this section, we present a quantitative description of quasiparticle tunneling through a potential barrier. If two mesoscopic conductor leads, left and right, are separated by a thin potential barrier, through which the electronic wave function can penetrate such that it does not vanish on the opposite side of the barrier, quantum mechanics allows the transport of quasiparticles to the other electrode. This effect is referred to as quantum tunneling. Experimentally, such a potential barrier can be realized in tunnel junctions or in a potential landscape shaped by electrostatic field in semiconductors. A description of quasiparticle tunneling in small junctions can be found in Ref. [7], and in Ref. [46] which we follow in this section and use their equations.

The Hamiltonian of the system in second-quantized form can be written as

$$H = H_L + H_R + H_T,$$

(2.3)

where $H_L$ and $H_R$ are the Hamiltonians of the left and right electrodes, respectively, and $H_T$ is the tunneling Hamiltonian. If the tunneling resistance is larger than the von Klitzing constant $R_K = h/e^2$, where $h$ is the Planck constant, and the tunneling of a single particle does not es-
Theoretical background

essentially change the thermal equilibrium of the leads, the tunneling can be treated as a weak perturbation. The first-order transition rate of the quasiparticles from a microscopic state $|i\rangle$ with energy $E_i$ to a final state $|f\rangle$ corresponding to $E_f$ can be calculated using Fermi’s golden rule as

$$
\Gamma_{i\to f} = \frac{2\pi}{\hbar} |\langle f | H_T | i \rangle|^2 \delta(E_i - E_f),
$$

where $\delta$ is the Dirac delta function. The tunneling rate for all possible $|i\rangle$ and $|f\rangle$ states can be expressed using the fact that the occupation probability of the quasiparticles follows the Fermi–Dirac distribution. Furthermore, we presume that the tunneling matrix element $|\langle f | H_T | i \rangle|$ assumes an average value that is embedded in the tunneling resistance $R_T$, together with a constant density of states in the left and right electrodes. The energy level difference $(\delta E_{L\to R})$ between the two electrodes is controlled with an external voltage source connected to one of the leads, shifting the electrochemical potential by $eV$ with respect to the opposite electrode.

The tunneling rate for all the initial and final states is written in an integral form [47]

$$
\Gamma_{L\to R}(\delta E_{L\to R}) = \frac{1}{e^2 R_T} \int_{-\infty}^{\infty} n^L(E)n^R(E + \delta E_{L\to R})
\times f_L(E, T_L) [1 - f_R(E + \delta E_{L\to R}, T_R)] dE,
$$

which has a straightforward physical meaning: the tunneling rate from the left electrode to the right electrode depends on the number of occupied states in the left electrode and the number of empty states on the other side. The tunneling is driven by the external voltage source which is connected to the right electrode. For example, in normal-metal–insulator–normal-metal junctions the tunneling rate is calculated by combining Eq. (2.5), Eq. (2.1), and using $n^L_N(E) = n^R_N(E) = 1$.

### 2.2.1 Tunneling in a single-electron transistor

A single-electron transistor (SET) is a charge island which is in tunnel contact with two leads and capacitively coupled to a gate electrode [8]. An SET is schematically illustrated in Fig. 2.2(a). Surprisingly, such single-electron transistor is an effective model for charge transport in many mesoscopic devices. However, as will be shown later, in superconducting SETs additional features appear in the measurements which is beyond the scope of the presented theory.

The operation of an SET requires at least two voltage sources, namely, a voltage source connected to the two leads and a voltage source connected to the gate. The charge island has a charge state $n$, a charging energy $E_C > \max(k_B T_L, k_B T_R)$, and a total capacitance $C_\Sigma = \frac{e^2}{2E_C}$. The tunnel junctions are assumed to be symmetric. The flow of quasiparticles can
create a net current if an external bias voltage \( V_{SD} = V_L/2 = -V_R/2 \) is applied to the electrodes and an available island state is in between the Fermi levels of two electrodes as illustrated in the energy diagram in Fig. 2.2(b). Using the gate voltage \( V_g \), the energy levels of the island can be shifted to achieve the aforementioned Coulomb resonance condition for the charge tunneling.

The tunneling rate between a lead and the island can be calculated by inserting Eq. (2.1) and \( \delta E_{L \to I}(n) = -eV_{SD} + E_{ch}(n) - E_{ch}(n+1) \) into Eq. (2.5), where

\[
E_{ch}(n) = \frac{e^2 (n + n_g)^2}{2C_\Sigma} = E_C(n - n_g)^2, \tag{2.6}
\]

is the electrostatic energy which depends on the charge state of the island.

If only elastic sequential tunneling is taken into account, a master equation can be constructed for \( P(n, t) \), which describes the probability of the system occupying charge state \( n \) at time \( t \). The details of the master equation and the analytical solution for the stationary state are elaborated in Refs. [47, 49]. A numerical solution for a time-independent transport is exemplified in Fig. 2.2(c) using the first three conduction electron states \( n = 0, 1, 2 \). The domains, where the current is suppressed by the Coulomb blockade, correspond to a well-defined island charge state, which are separated by the degeneracy points at \( n_g = n + 1/2 \). Single-electron transistors serve in practical applications as Coulomb blockade primary thermometers [50–52], sensitive charge detectors [53–55], readout circuits for qubits [56–59], and signal amplifiers [60–62].
2.2.2 NIS tunneling

Tunnel junctions, which are formed by the composition of a normal conductor and a superconductor, are the main components in Publications I–III. Because the quasiparticle distribution in the normal conductor is more sensitive to the temperature change compared to the superconductor which is biased near the gap voltage, NIS junctions are employed as microcoolers for semiconductor 2DEGs [63, 64] and in normal-metal structures [52, 65, 66].

The tunneling rate through an NIS can be calculated by combining Eqs. (2.1), (2.2), (2.5), and \[ \delta E_{L \rightarrow R} = eV_{dc} \], where \( V_{dc} \) is the external voltage applied to the junction terminals. The tunneling rate from a normal conductor to the superconductor yields [67]

\[
\Gamma_{NIS}(V_{dc}) = \frac{1}{e^2 R T} \int_{-\infty}^{\infty} n^R_S(E + eV_{dc}) f_L(E, T_N) \left[ 1 - f(E + eV_{dc}, T_S) \right] dE.
\]

(2.7)

The average net current through a junction is the sum of two tunneling directions; quasiparticle transport from the normal-metal to the superconductor [Eq. (2.7)] and vice versa. The total current therefore can be obtained by [67]

\[
I_{NIS}(V_{dc}) = \frac{1}{e R T} \int_{-\infty}^{\infty} n^S_S(E) \left[ f(E, T_N) - f(E + eV_{dc}, T_N) \right] dE,
\]

(2.8)

where the current depends only on the temperature of the normal-conductor and not the superconducting lead if the temperature-dependence of the superconducting gap is negligible [68]. The heat transport through the junction reads as [67]

\[
P_{NIS}(V_{dc}) = \frac{1}{e^2 R T} \int_{-\infty}^{\infty} dE \left( E + eV_{dc} \right) n^S_S(E) \left[ f(E, T_S) - f(E + eV_{dc}, T_N) \right]
\]

\[
= \frac{1}{2e^2 R T} \int_{-\infty}^{\infty} dE \left( E + eV_{dc} \right) \left\{ eV_{dc} \left[ f(E - eV_{dc}, T_N) - f(E + eV_{dc}, T_N) \right] + E \left[ 2f(E, T_S) - f(E + eV_{dc}, T_N) - f(E - eV_{dc}, T_N) \right] \right\},
\]

(2.9)

where the first term expresses Joule heating through the junction and the second term corresponds to the cooling effect of the junction. In contrast to the NIS current, the heat transport depends directly on the temperature of the superconductor. Furthermore, at a vanishing bias voltage the current flow is suppressed, but heat transfer between the two electrodes is possible in the presence of a finite temperature gradient.
The elastic tunneling assumption explains the cooling effect, but in experiments the tunneling quasiparticles interact with the electrostatic environment, which enriches the physics of tunneling [69]. Photon-assisted tunneling in NIS junctions is detailed in Sec. 2.3.

2.2.3 SIS tunneling

The combination of two superconducting electrodes separated by an insulator, through which particles can tunnel, is called a Josephson junction [70]. The quasiparticle tunneling at low temperature $T_C > T$, where $T_C$ is the critical temperature of the superconductor, similar to the previous sections, can be written by taking the appropriate density of states Eq. (2.2) and inserting it into Eq. (2.5). As a results, the current is described by [9]

$$I_{\text{SIS}}(V_{\text{dc}}) = \frac{1}{eR} \int_{-\infty}^{\infty} n_S(E) n_S^R(E + eV_{\text{dc}}) \left[ f(E, T_L) - f(E + eV_{\text{dc}}, T_R) \right] dE.$$  

(2.10)

According to Eq. (2.10), to measure a quasiparticle current outside the gap, one has to apply a bias voltage $e|V_{\text{dc}}| > 2\Delta$.

The supercurrent, i.e., an electric current which appears across the junction at vanishing bias voltage and which is associated with Cooper pair transport, has been observed in superconductor-insulator-superconductor (SIS) junctions since 1960 [49, 71, 72]. Based on the Ginzburg–Landau theory, which is capable of explaining Cooper pair tunneling [73], the superconducting condensate is described by its phase difference $\Delta \phi$ across the junction. The current through the junction is described by the dc Josephson relation [4]

$$I(t) = I_c \sin (\Delta \phi),$$  

(2.11)

where $I_c$ denotes the critical current which sets a boundary for maximum supercurrent across the junction. The phase evolution depends on the potential difference between the leads, $V$, which follows the ac Josephson relation [4]

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar} = 2\pi V/\Phi_0,$$  

(2.12)

which implies a linear phase build-up with time in the case of a constant bias voltage. Here, $\Phi_0$ denotes the magnetic flux quantum. The ac Josephson effect, i.e., alternating current at constant voltage bias, can be obtained if one substitutes Eq. (2.12) into Eq. (2.11).

If the two junctions are in parallel, we obtain a superconducting quantum interference device (SQUID), which is a magnetic-flux-tunable element em-
Theoretical background

ployed in Publication IV. The SQUID inductance depends on the magnetic flux through the loop $\Phi$:

$$L(\phi) = \frac{\Phi_0}{2\pi I_c |\cos (\pi \phi / \Phi_0)|}.$$  \hfill (2.13)

Two Josephson junctions in series can form a superconducting single-electron transistor, which is employed in Publications V and VI are discussed in more detail in Sec. 2.4.

2.3 Photon-assisted tunneling in NIS structures

If the tunneling electron interacts with an electromagnetic environment, the electron may absorb or emit energy to the coupled electromagnetic mode. Since the energy of the electron is not conserved during transport process, it is referred to as inelastic tunneling. The environment is modeled with the $P(E)$ function, which expresses the probability of a tunneling electron emitting a photon with an energy $E$. In this case, the tunneling rate follows a more general form \[46, 69\]:

$$I(V_{dc}) = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} dE' \{ f(E, T_L) \left[ 1 - f(E', T_R) \right] P(E + eV_{dc} - E') 
- \left[ 1 - f(E, T_L) \right] f(E', T_R) P(E' - E - eV_{dc}) \}.$$ \hfill (2.14)

2.3.1 Resonators

In Publications I-III a photonic environment capacitively couples to the tunneling electrons. The environment is a single-mode superconducting transmission-line resonator \[5\]. This system is theoretically studied in Ref. \[74\]. The resonator works as a filter circuit that transmits electromagnetic radiation in the vicinity of the resonance frequency, while reflecting any other frequency components. The transmission-line-type resonators are terminated from each side with small capacitors which induce a standing wave to determine the fundamental frequency of the mode \[75\]. The resonator, in our case, is a coplanar waveguide resonator. The resonance frequencies at a certain length $l$ and speed of light $v_{ph}$ can be calculated using \[75\]

$$f_n = (n + 1) \frac{v_{ph}}{2l},$$ \hfill (2.15)

where $n$ is the mode index. If not stated otherwise, the fundamental mode ($n = 0$) is addressed, which constrains the energy of the photons that
The theoretical background can be absorbed or emitted by the tunneling electrons to \( E = \hbar \omega_0 \), where \( \omega_0 = f_0/(2\pi) \). The transmission line is characterized by a lumped-element inductance \( L \) and capacitance \( C \), which form an \( LC \) oscillator with a characteristic impedance \( Z_r = \sqrt{L/C} \). The average inverse lifetime of a photon in the resonator equals the cavity damping rate \( \gamma \) which depends on the losses and leakages in the resonator described by the quality factor \( Q = \omega_0/\gamma \).

Due to the fact that the internal losses and the rate of emitting/loading photons to/from a transmission line for the resonator are different, we distinguish two quality factors: the internal quality factor \( Q_{\text{int}} = \omega_0/\gamma_{\text{int}} \) which is associated with the resonator losses due to internal dissipation [76] and the external quality factor \( Q_{\text{ext}} = \omega_0/\gamma_{\text{tr}} \), which indicates how strongly the resonator is coupled to a transmission line that is used for probing the resonator. During the measurements, information is obtained about the loaded or total quality factor, which has the following form in the case of a symmetric coupling

\[
\frac{1}{Q_L} = \frac{1}{Q_{\text{int}}} + \frac{1}{Q_{\text{ext}}}. \tag{2.16}
\]

Superconducting resonators have the benefits of vanishing ohmic losses [77] and a weakly coupled coplanar waveguide provides low radiative losses [78].

### 2.3.2 NIS junction coupled to a resonator

In our experimental realization, we employ a double junction where a mesoscopic normal-metal island with negligible charging energy is in tunnel contact with two superconducting leads. The resulting superconductor–insulator–normal-metal–insulator–superconductor (SINIS) structure is called quantum-circuit refrigerator (QCR). This section presents the working principles of the QCR following Ref. [74]. The lumped-element diagram of the system discussed is depicted in Fig. [2.3]. In the formalism below, one tunnel junction with tunneling resistance \( R_T \) and a capacitance \( C_j \), interacts with a resonator while the other junction is modeled as a capacitor \( C_m \). The total capacitance of the normal-metal island is \( C_N \). The capacitive coupling between the normal-metal island and the \( LC \) resonator is \( C_c \). The other end of the resonator couples to a transmission line with a characteristic impedance \( Z_{\text{tr}} \) through \( C_g \). The external bias voltage applied to the symmetric double junction yields a voltage drop on a single junction \( V = V_b/2 \).

The normalized forward tunneling transition originating from the quasiparticle transport in an SINIS junction is given by [74]

\[
\vec{F}(E) = \int d\varepsilon\ n_S(\varepsilon)\frac{1}{\hbar}\frac{f(\varepsilon - E) - f(\varepsilon)}{[1 - e^{-E/(k_B T_N)}]}, \tag{2.17}
\]

where \( T_N \) is the temperature of the normal metal which is assumed to be
in thermal equilibrium with the superconducting leads. The resonator recognizes the SINIS as a high-impedance load. The fundamental mode of the resonator has an occupation number \( m \), which the QCR can change by absorbing or emitting a single photon at a time. The multiphoton process, i.e., a tunneling electron interacts with more than a single photon during a tunneling event, is neglected.

The environment that the tunneling electrons induce on the resonator can be treated as a thermal reservoir, which changes the resonator occupation according to the rates \[74\]

\[
\Gamma^{T}_{m,m-1} = \gamma_{T}(N_{T} + 1)m, \quad (2.18a)
\]
\[
\Gamma^{T}_{m,m+1} = \gamma_{T}N_{T}(m + 1), \quad (2.18b)
\]

where \( N_{T} = 1/\{\exp(h\omega_{0}/(k_{B}T_{T})) - 1\} \) is the mean thermal occupation of the reservoir that has an effective temperature \( T_{T} \), and \( \gamma_{T} \) is the damping rate owing to the SINIS junction. The damping rate and the effective temperature of the thermal reservoir can be calculated using Eq. \(2.17\) and written as \[74\]

\[
\gamma_{T} = \bar{\gamma}_{T} \frac{\pi}{\omega_{0}} \sum_{\ell,\tau = \pm 1} \ell \tilde{F}(\tau eV + \ell h\omega_{0}), \quad (2.19)
\]
\[
T_{T} = \frac{h\omega_{0}}{k_{B}} \left[ \ln \left( \frac{\sum_{\tau = \pm 1} \tilde{F}(\tau eV + h\omega_{0})}{\sum_{\tau = \pm 1} \tilde{F}(\tau eV - h\omega_{0})} \right) \right]^{-1}, \quad (2.20)
\]

where the \( \bar{\gamma}_{T} \) a is shorthand notation for the asymptotic damping rate \( \bar{\gamma} = 2C_{v}^{2}Z_{c}\omega_{0}/(C_{v}^{2}R_{T}) \). The average power which radiates from the SINIS junction to the resonator depends on the transition rates in Eqs. \(2.18a\)
Theoretical background

and (2.18b), and their sum over \( m \) weighted by the probability of \( m \) photons in the resonator [74]

\[
P_T = \hbar \omega_0 \left( \langle \Gamma_{m,m+1}^T \rangle - \langle \Gamma_{m,m-1}^T \rangle \right) = \hbar \omega_0 \gamma_T (N_T - \langle m \rangle), \tag{2.21}
\]

where \( \langle m \rangle \) is the mean photon number in the resonator. The first term of Eq. (2.21) describes the heating of the resonator, and the second is the cooling term. Depending on the bias voltage, the power can be positive or negative in the case of heating or cooling the resonator, respectively. A schematic energy diagram that shows the working principle of the QCR is depicted in Fig. 2.4.

The coupling strength between the resonator and the transmission line is given by

\[
\gamma_{tr} = \frac{Z_r}{Z_{tr}} \frac{\omega_0^3}{\omega_0^2 + (Z_{tr} C_g)^{-2}}, \tag{2.22}
\]

which is used to estimate the radiated power from the transmission line to the resonator:

\[
P_{tr} = \hbar \omega_0 \gamma_{tr} (N_{tr} - \langle m \rangle). \tag{2.23}
\]

The thermal photon number in the transmission line, \( N_{tr} \), depends on the effective temperature of the transmission line and the direction of the radiation is determined by the sign of \( P_{tr} \).

2.4 Charge detection with superconducting single-electron transistor

As presented in Sec. 2.2.1, the SET is a device that can store a well-defined number of electrons on its island under certain conditions. In contrast, the all-superconducting single-electron transistor (SSET) has a superconducting gap in the density of states of the electrodes, as well as on the island. As a result, the features that are observed on the \( V_{dc}-n_g \) scan, are not only the result of the quasiparticle-related conductance, but also the contribution of the Cooper pair transport.

From the Coulomb stability diagram of a superconducting SET, which is depicted in Fig. 2.5(a), different families of resonances can be identified [79]. The threshold normal-state SET behavior discussed in Sec. 2.2.1 matches with the high-bias regime, \( 4\Delta < eV_{dc} < 4\Delta + 2E_c \). Similarly to the SINIS junction, \( 4\Delta \) of energy is required to have available states for only quasiparticle-mediated currents. In SSETs, the threshold voltage, which separates the subgap and the quasiparticle current, depends on
Figure 2.4. Schematic energy diagram of the photon-assisted tunneling in a SINIS junction coupled to a resonator at increasing bias voltage, $V$, from (a) to (d). Only a single side of the QCR is presented for clarity, whose essential component is a normal-metal–insulator–superconductor junction. The wavy arrows indicate the photon absorption (blue) or emission (red) by the tunneling electron. (a) At vanishing bias voltage no tunneling is possible due to the lack of available states. (b) High-energy electrons tunnel to the superconductor and decreasing the temperature of the normal metal. The blue arrows indicate the possibility to absorb a photon from the resonator and tunneling electron gains an energy quantum $\hbar \omega_0$, which is possible in this biasing regime. (c) Voltage configuration, where the maximum photon-absorption rate can be achieved. (d) In the high-bias regime, both photon absorption and emission (red) are possible and the temperature of the normal-metal island increases.
Theoretical background

Figure 2.5. (a) Differential conductance, $G_{\text{diff}} = dI_{\text{dc}}/dV_{\text{dc}}$, of a superconducting SET charge detector as a function of normalized bias voltage and gate charge presented in Publication VI. Above the threshold for sequential quasiparticle tunneling (yellow dashed line) the detector follows the characteristics of a normal-state single-electron transistor. The Josephson-quasiparticle process on each sides of the junctions (blue dashed lines) intersects at the double Josephson-quasiparticle cycle (red circle). The parameters of the device are $E_c = 160 \, \mu\text{eV}, \Delta = 195 \, \mu\text{eV}$, and $R_T = 180 \, \text{k}\Omega$. (b) Schematic representation of the double Josephson-quasiparticle cycle.

The charge state. The charging energy directly relates to the threshold voltage. In the subgap regime, resonant lines appear where Cooper pairs can tunnel into the island from one of the junctions, and two quasiparticles leave at the other junction [80]. This feature corresponds to the Josephson-quasiparticle cycle (JQP). At bias voltage, $eV_{\text{dc}} = 2E_c$, the two JQPs become resonant and the corresponding voltage appoints the double Josephson-quasiparticle (DJQP) cycle [80, 81]. Besides the presented features, additional charge transport mechanisms are also observed in the subgap regime, such as the Andreev process [79] and $2e$-periodic currents [82]. Resonators coupled to SSETs revealed that the quasiparticle processes are interacting with the environment [83, 84].

In Publications VI and V the charge detector is biased at the voltage corresponding to the DJQP cycle that is schematically presented in Fig. 2.5(b). Initially, the island is occupied by $n$ number of extra charges. If the island becomes resonant with the Fermi level of the first electrode, a Cooper pair can tunnel into the island, followed by a quasiparticle tunneling out to the second electrode. Owing to the shift in the Fermi level of the island, the Cooper pair resonance occurs with the other electrode. As a Cooper pair leaves the island, one quasiparticle tunnels from the other side and the island arrives back at the original charge state $n$. 
2.4.1 Charge sensing for error counting

The aim of charge sensing is to monitor the charge state of the desired system, such as a charge reservoir coupled to a quantum dot [18–20] or a quantum dot directly [85, 86]. As discussed in Sec. 2.2.1, single-electron transistors can be used for high-sensitivity charge detection. In this section, the charge sensing in a direct-current detector is detailed. A fixed detector current $I_{dc}$ flows through the voltage-biased SSET until the local electrostatic field changes the charge state of the island, yielding a variation in $I_{dc}$. Due to the fixed distance of the detector Coulomb oscillation peaks [see Fig. 2.5(a)], the current noise can be expressed as a corresponding charge noise using [87]

$$\delta n_g = \left( \frac{\partial I_{dc}}{\partial n_g} \right)^{-1} \delta I_{dc},$$

(2.24)

where $\partial I_{dc}/\partial n_g$ is the detector sensitivity. To maintain the detector sensitivity constant, an electrostatic feedback is applied to the detector using an auxiliary gate [21]. This feedback has a negligible hindering effect on the detection signal. The capacitive coupling between the monitored charge island and the detector, $C_c$, determines the signal amplitude on the detector if the charge state of the monitored island changes by a single electron [88] and is expressed as $\delta q_e = eC_c/C_{\Sigma CI}$, where $C_{\Sigma CI}$ is the total capacitance of the reservoir island, if the counting island charging energy is $E_{\Sigma CI} \ll E_c$.

Measuring the charge noise can help to identify the types of noise. In the case of this thesis, the noise has a linear dependence on temperature, which is a signature of the white-noise regime. On the other hand, in a more sophisticated setup where a qubit is capacitively coupled to an SET that is biased at the voltage corresponding to the DJQP, the detection approaches the quantum limit of efficiency which is described by [84, 89]

$$\chi = \sqrt{\frac{6E_c^2 S_{sq}}{I_{dc} R^2}} \geq 1,$$

(2.25)

where $S_{sq}$ is the intrinsic charge noise of the detector. As Eq. (2.25) indicates, approaching the quantum limit of the detector depends only on the detector characteristics and not on the coupled charge island. In the case of $\chi = 1$, the weak-coupling SET reaches its ideal detection conditions, where the rate of the qubit readout and the qubit decoherence rate due to readout are equal [90]. The lowest charge noise figures are achieved in radio-frequency (rf) reflectrometry [57, 91, 92] using a fully superconducting SET. Note that using the supercurrent for charge detection has also been proposed [93].

Charge sensing is a key component for error counting experiments [94]. Error counting is a rigorous estimation of the accuracy of a single-electron
source which is reviewed in Sec. 2.5. Instead of using a room temperature
circuitry to evaluate a single-electron source, an in-situ charge sensor
is employed, which yields shorter integration times and self-referenced
current source realization [19, 21, 95, 96]. Because the failed events due
to an error are relatively rare compared to the total number of transferred
charges, typically 1 out of million events, the charge detector should resolve
only these transitions. The low error rate in the electron source justifies
why direct-current detectors are feasible for error counting experiments.

2.4.2 Counting statistics and waiting-time distributions

If a parasitic two-level fluctuator (TLF) is capacitively coupled to a readout
circuit, its transitions from the microscopic state \( |0 \rangle \) to \( |1 \rangle \) and vice versa
may be resolvable [97–99]. In most of the cases, the presence of TLFs are
unfavorable [76, 100]. However, counting switching events between the
microscopic states potentially reveals the timescales, on which the system
is dominated by TLF transitions [101, 102].

The full counting statistics for single-electron events which are not corre-
lated [103] follow a Poisson probability distribution, which has the analytic
form

\[
P(n, t) = \frac{(\Gamma t)^n}{n!} \exp(-\Gamma t),
\]

(2.26)

where the expectation value of \( n \) is \( \langle n \rangle = \Gamma t \) and the transition rate \( \Gamma \) is
fixed. Here, \( P(n, t) \) denotes the probability of having \( n \) tunneling events
within a time interval of duration \( t \). The probability of an idle time \( \tau \)
is defined by \( \Pi(\tau) = P(n = 0, \tau) \) which is connected to the waiting-time
distribution, i.e., the probability distribution of \( \tau \) delay between two con-
secutive tunneling events, for stationary transport [104, 105] through the
expression

\[
W(\tau) = \langle \tau \rangle \partial^2_\tau \Pi(\tau).
\]

(2.27)

Using the full counting statistics from Eq. (2.26), one can construct the
moment generating function

\[
\mathcal{M}(\chi, t) = \sum_{n=0}^{\infty} P(n, t)e^{in\chi},
\]

(2.28)

where \( \chi \) is a counting field. The moment generating function predicts
all the moments corresponding to the statistics for \( n \) by employing the
formula

\[
\langle n^m \rangle(t) = \partial^m_{\chi} \mathcal{M}(\chi, t)|_{\chi \to 0}.
\]

For instance, the expectation value is con-
structed by taking the first derivative, \( m = 1 \), the second moment is the
second derivative, \( m = 2 \), and so on. In a more general approach, where
multiple tunneling rates are involved in the stationary transport, the time
evolution of the probability distribution is described by a master equation

\[
\frac{d}{dt} p(t) = \mathbf{L} p(t),
\]

(2.29)
where \( p(t) = [p_0(t), p_1(t)]^T \) is a probability vector that gives the occupation of the \( |0\rangle \) and \( |1\rangle \) respectively, and \( L \) is the tunneling rate matrix. Following the partition of the rate matrix presented in Ref. [106], the solution of the master equation, \( p(\chi, t) \) that depends on the counting field

\[
p(\chi, t) = e^{L(\chi)t} p(\chi, 0),
\]

where the rate matrix \( L(\chi) \) reads as

\[
L(\chi) = \begin{pmatrix}
-\Gamma^+ & \Gamma^- \\
e^{i\chi} \Gamma^+ & -\Gamma^-
\end{pmatrix}.
\]

As is shown in Sec. 4.6, the transition rate matrix elements, \( \Gamma^+ \) and \( \Gamma^- \), are related to the waiting-time distribution and the idle-time distribution respectively [107], which takes into account the finite bandwidth of the detector [102, 108].

### 2.5 Single-electron pumping in semiconductors

Transferring electrons one by one to create a high-precision electric-current source has been proposed more than three decades ago [7]. Over the years many different realizations have been demonstrated, initially metallic structures [109, 110] operating in the few-MHz regime and in semiconductor quantum dots [111]. The mean electric current in these devices \( I = e f \), where \( f \) is the pumping frequency, the relative uncertainty was on the level of a few percents. The piezoelectric effect in gallium arsenide opened a way to use surface acoustic waves that drives electrons through a potential barrier, which can achieve \( f = 2.7 \) GHz with a relative uncertainty of \( 10^{-4} \) [112]. Ten years later, the uncertainty of the pumped current decreased drastically with the voltage-tunable barriers [17, 113, 114] in addition to the novel families of single-electron sources such as graphene pumps [115], SINIS turnstiles [116], and quantum phase slip devices [117].

Single-electron sources can be categorized into two groups: turnstiles which need an external source-drain bias voltage for their operation, and pumps which operate with a vanishing potential difference between the source and drain leads. This work focuses on semiconductor quantum dot pumps which employ electrostatic confinement using finger gates [118]. The quantum dot, which is in tunnel contact with the source and drain electrodes, follows the characteristics of a single-electron transistor. To create a quantized electric current, a radio-frequency modulation is needed. The periodically driven pump transfers charges in cycles, each of which consists of three phases as depicted in Fig. 2.6. First, in the loading phase electrons, tunnel into the quantum dot. Second, the entrance barrier
Theoretical background

**Figure 2.6.** Potential energy diagrams for charge pumping in the case of one-parameter driving. In phase I, electrons are loaded from the source (S) electrode. In phase II, the back-tunneling rate for the extra electrons in the quantum dot $\Gamma^{(S)}$ is higher than the modulation speed of the entrance barrier so that only a single electron is eventually localized in the quantum dot. Finally, in phase III the trapped electron is emitted to the drain (D) with tunneling rate $\Gamma^{(D)}$ while the back-tunneling is negligible.

Potential ramps up to trap electrons, which are emitted in the final phase by tunneling out through the exit barrier.

Depending on the modulation strategy, different theoretical models are used to describe charge transport [119]. In the case of low-frequency pumping $f \ll \Gamma$, where $\Gamma$ is the in-tunneling rate for electrons in the quantum dot, two driving signals are required for having a quantized current. On the other hand, faster pumping is achieved in a single-parameter ratchet-mode operation, where the rf drive is applied to a tunneling barrier. This process employs a non-adiabatic transport that vanishes below a cut-off frequency if the drive amplitude is constant [120]. For example, all electrons may escape from the quantum dot before the trapping phase. If both barriers are driven at the same frequency, the direction of the pumping is controllable by the phase offset between the two driving signals [121].

A unified theory for non-adiabatic pumping, which describes the measurement universally, does not yet exist because different non-adiabatic mechanisms contribute to the transport. However, two important models have been found: the decay cascade model [122] and the thermal model for electron capture [123]. The crossover between the two models has been investigated [20], as well as the breakdown frequency [124]. Excited quantum dot states [114, 125] and transfer mediated by parasitic states [126] yield additional feature in the pumping data.
3. Experimental methods

In this chapter, we present the sample fabrication methods used for the measured devices and briefly discuss the measurement methods.

3.1 Sample Fabrication

The samples studied in the experiments are fabricated using standard microfabrication and nanofabrication processes. Although the samples are manufactured in different cleanroom facilities, the fabrication procedures are similar. The devices used in Publications I–III are entirely fabricated in the Micronova Nanofabrication Center, Espoo, Finland. The devices reported in Publications VI and V are fabricated mostly in the Australian National Fabrication Facility, University of New South Wales, Australia and Micronova Nanofabrication Center, Espoo, Finland. The device in Publication VII originates from NTT, Tokyo, Japan. A common feature in all of the samples is the substrate material which is highly resistive silicon.

Quantum-circuit refrigerator

The fabrication starts with the preparation of the four-inch wafer. The QCR samples require a 300-nm-thick silicon oxide layer which is grown by wet oxidation. In the following step, a niobium layer, which is the material of the CPW resonators, is sputtered. The resonator patterning employs an optical lithography mask: First, the wafers are primed with polydimethylsiloxane in a vacuum oven that removes the moisture from the surface and increases the adhesion of the consecutive spin-coated photoresist layer. Those sections of the wafer, which are not covered by the metallized photomask, are exposed to ultraviolet (UV) light. After the development, the patterned photoresist goes through a hard baking process to prepare the surface for reactive ion etching that anisotropically removes the 200-nm-thick niobium layer. To create a dielectric separation
between the central conductor of the CPW resonator and the subsequent SINIS junction, Al$_2$O$_3$ is deposited using atomic-layer deposition (ALD) at 200 °C. Trenches are cut on the backside of the wafer using a dicing saw whereby the wafer can break into smaller pieces.

The patterning of the junctions starts with spin-coating a layer of copolymer Poly(methyl metacrylate-co-methacrylic acid) [P(MMA-MMA)]. After baking the first layer of resist, another layer of polymethyl-metacrylate (PMMA) solution in anisole is spun and baked similarly to the first layer. The nanoscale pattern is written with a Vistec EPBG5000 electron beam lithography system with 100 kV acceleration voltage. After the exposure, the mask is developed by immersing the chip in methyl isobuthyl ketone (MIBK) solution, after which the chip is soaked in isopropyl alcohol and dried with a N$_2$ gun.

The opened windows on the resist mask define the metallization area that are evaporated using physical vapor deposition (PVD). The metal is situated in a graphite crucible and heated by a focused electron beam. The bilayer resist provides an undercut for the multi-angle evaporation technique during the SINIS junction deposition. The first evaporation defines the 20-nm-thick aluminium layer that is oxidized in situ by injecting 1-5 mbar of O$_2$ gas into the deposition chamber for a controlled amount of time. The normal-metal domains are evaporated under a second angle, which also yields a 20-nm-thick metal layer. The metallization finishes with the removal of the excess metal, which is on the top of resist mask, by soaking the chip in acetone. Since the resist is acetone-solvable, the metal layer lifts off except for the area where the features adhere to the wafer. The lift-off can be enhanced using elevated temperature, in the case of acetone maximum 52 °C, or ultrasonic agitation if needed. The lift-off procedure finishes with an isopropyl alcohol soaking and N$_2$ drying.

**Flux-tunable coupled resonators**

The methodology for fabricating the flux-tunable resonators is quite similar to that used in the QCR sample described in the previous section. The first difference is the lack of oxide on top of the silicon substrate in order to provide a higher quality factor. Here, the direct contact between the silicon and the sputtered niobium is critical, which is achieved by employing a thorough ion beam etching before the sputtering. In addition, the Al$_2$O$_3$ layer is not used for these types of samples.

The nanofabrication requires only a single lithography step. The metal deposition, however, is carried out in two phases: First, the Cu shunt resistor is evaporated, while the window for the SQUID is covered by a metallic mask. Second, the metallic mask is moved to cover the Cu resistor, and the Al SQUID is deposited using double-angle evaporation, where the two layers of Al are separated by in-situ oxidation.
Experimental methods

Silicon quantum dot pumps with metallic gates

The MOS structures require more fabrication steps compared to the previously discussed circuits. The fabrication procedure is discussed in detail in Ref. [127]. The microfabrication begins with defining the boundaries of the $n$-type channel using a $p$-type region that is referred to as the channel stopper. Employing a lithography mask, the field oxide is etched selectively, which reveals the silicon substrate on the area where the $p$-type doping is desired. To achieve $p$-type doping, in our case boron is the acceptor ion which diffuses into the silicon substrate in a furnace. Afterwards, the boron-contaminated field oxide is removed (deglassing), and a fresh layer of $\text{SiO}_2$ is grown. The fabrication of the actual $n$-type channel involves a patterned phosphorus doping of the wafers to provide a galvanic contact throughout the thick-field oxide region. The diffusion doping is carried out in a furnace to achieve high dopant density in the range of $10^{19} - 10^{20}$ cm$^{-3}$, which yields finite conductance at cryogenic temperatures. The next lithography layer patterns the 5-nm gate oxide, which is located in the center of each device under the nanostructure. Next, metallic pads are deposited which form ohmic contacts with the $n$-type channel. In the final microfabrication step, the wafer goes through a forming gas annealing to improve the metal-semiconductor connection.

The gate structure in the gate oxide regime consists of three different metallic layers. Each layer requires a single-layer PMMA spinning, nanopatterning, and thermal evaporation of Al. The layers are separated by native $\text{Al}_x\text{O}_y$ grown on a hotplate at $150^\circ$C. After finishing the gate structure fabrication, the charge detectors are patterned and the devices are shipped to Finland, where the superconducting SETs are deposited.

Nanowire pumps

The single-electron pump in [Publication VII] follows a recipe discussed in Ref. [96]. The quantum dot pump is defined on a semiconductor-on-insulator wafer, which is a silicon substrate that is covered with a 400-nm-thick buried oxide. On top of the buried oxide, a layer of silicon is etched to form a nanowire, which is covered by silicon oxide created by thermal oxidation. The polycrystalline silicon finger gates are deposited by pyrolyzing, where silane gas in a low-pressure furnace thermally decomposes to a doped silicon to increase its conductivity. The gate structure is covered by an interlayer silicon oxide and a final layer of polysilicon top gate is deposited. The ohmic contacts are $n$-type ion-implanted regions masked by the topmost gate layer.
3.2 Measurement procedure

The packaging of the samples is carried out by a wedge-bonder machine that connects the pins of a sample holder to the pads of the chip using Al wires. All of the investigated phenomena in this thesis call for low temperatures, typically 1 kelvin or $-272^\circ C$. The boiling temperature of liquid helium, 4.2 K, is enough for single-electron pumping presented in Publication VII. Here, the sample holder is attached to a probe stick and immersed in an isolated dewar. On the other hand, in the rest of the works subkelvin temperature is achieved using $^3$He - $^4$He dilution refrigerators.

The signal from the sample to the room temperature is channeled through various cables. Two types of cablings are used in the experiments: the direct-current lines and the radio-frequency transmission lines. As the cryostat has many different temperature stages, at each stage the lines have to be heat sunked. Different types of rf lines are cascaded, which have different attenuation and heat conductance. Near the base temperature, superconducting niobium coaxial cables provide low attenuation and low heat conductance, while above 1 K, beryllium-copper coaxial cables have higher attenuation. Two types of dc lines are employed in the experiments: cryoloom twisted pair lines (phosphor-bronze or beryllium-copper) and resistive coaxial cable (Thermocoax). At room temperature, the measurement instruments are connected to the lines leading to the sample. Floating voltage sources provide a low-noise voltage signal that serves as dc biases, and dc current is amplified by high-gain transimpedance amplifiers and measured with digital multimeters and phase-sensitive lock-in amplifiers. Arbitrary-waveform generators and continuous-wave generators are used to create rf signals for the microwave measurements. The rf readout lines are equipped with cryogenic broadband amplifiers and room temperature amplifiers. A spectrum network analyser, a vector network analyser, and a PXI-based analog-to-digital converter are used to measure the rf signal response.

3.3 Microwave network analysis

The methods to analyse the microwave circuits discussed in this thesis are presented in this section. Quantum systems are accessible through analog signals consisting of continuous variables [12]. In our case, analog signals are mediated by microwave resonators and transmission lines, and measured by room temperature instruments. The interpretation of these analog signals provide insight on the quantum behavior of the investigated components. Classical network analysis is therefore central technical part of the experiments and simulations.

Microwave circuits can be regarded as two-port devices, on which one
Experimental methods

describes the transmission and reflection properties of the microwave
circuit by calculating the currents and voltages instead of solving Maxwell's
equations. The scattering matrix establishes a connection between the
incident voltage amplitudes $V_1^+$ and $V_2^+$ and the outgoing voltages $V_1^-$ and
$V_2^-$, as shown in Fig. 3.1(a) as \[128\]

$$
\begin{pmatrix}
V_1^- \\
V_2^-
\end{pmatrix} =
\begin{pmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{pmatrix}
\begin{pmatrix}
V_1^+ \\
V_2^+
\end{pmatrix}
$$

(3.1)

The transmission coefficient from Port 1 to Port 2 can be calculated using
$S_{21} = V_2^- / V_1^+$, if $V_2^+ = 0$. Similarly, the reflection coefficient on Port 1 can
be described with $\Gamma = S_{11} = V_1^- / V_1^+$, if $V_2^+ = 0$. These are the quantities
that are frequency dependent and can be measured with a vector network
analyzer.

Figure 3.1. Schematic representation of a two-port network. (a) Incident and outgoing
voltages that are connected via the scattering matrix. (b) Voltages and cur-
rents which are related through the $ABCD$ matrix.

Another description which relates the voltage and current on Port 1 to
the voltage and current on Port 2 is the transmission or $ABCD$ matrix
method \[128\]

$$
\begin{pmatrix}
V_1 \\
I_1
\end{pmatrix} =
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix}
\begin{pmatrix}
V_2 \\
I_2
\end{pmatrix}
$$

(3.2)
as shown in Fig. 3.1(b). The sign convention is such that current $I_1$ flows
in from port 1 and $I_2$ flows out through port 2. The $ABCD$ matrix method
can be used efficiently when multiple components are cascaded. The
total $ABCD$ matrix of the cascaded network is the matrix product of the
$ABCD$ matrices of each component. The relation between the transmission
coefficient $S_{21}$ and the matrix elements of the $ABCD$ matrix is given by
\[128\]

$$
S_{21} = \frac{2}{A + B/Z_L + CZ_L + D},
$$

(3.3)

where $Z_L$ is the characteristic impedance of the transmission line that is
connected between the two-port network and the measurement apparatus.
4. Results

This chapter highlights the main results of this dissertation. Section 4.1 details the results of the gain calibration protocol, where a cascaded amplification chain is characterized. Next, we experimentally demonstrate a broadband Lamb shift, which is followed by the measurement results of the fast reset of a resonator. Subsequently, the flux-tunability of the coupled resonators is presented. In Sec. 4.5 the noise properties of a superconducting charge detector is presented and a similar hardware in Sec. 4.6 is used in practice to characterize a two-level fluctuator. Finally, in Sec. 4.7 the consistency of the pump current from a single-electron pump is verified at different laboratories.

4.1 Gain calibration of an amplification chain using tunnel junctions

The results from Publication 1 are discussed in this section. Here, we develop a primary gain calibration method, schematically presented in Fig. 4.1 which is insensitive to impedance mismatch and experimentally conveniently implementable. In low-temperature physics, the typical energy scales are significantly smaller than the energy scales in the optical regime (1 eV) and the thermal radiation at room temperature (10 meV). As a result, gaining information from low-temperature systems often requires substantial signal amplification. The characteristics of cascaded amplifiers, such as the total gain and the noise temperature, impose conditions on the measurement time of signal given its level at the input of the amplifier chain. The novelty of the presented method incorporates a two-step measurement which can be carried out within a single cooldown and requires only one external control parameter to implement the gain calibration. In Sec. 2.3 the theoretical background of the system is introduced, and here we exploit the incoherent photon generation in the high-bias-voltage regime of the SINIS junction as was first demonstrated in Ref. [6].
In the high-bias regime, the second-order approximation of Eqs. (2.19) and (2.20) can be simplified using Sommerfeld expansion. The power that the resonator emits to the transmission line after the approximation has the form of

$$P_{tr} \approx \frac{\gamma_{tr} \tilde{\gamma}_T}{\gamma_{tr} + \tilde{\gamma}_T + \gamma_x} \left\{ \frac{eV}{2} + \hbar \omega_r \left[ \frac{\gamma_x (N_x - N_{tr})}{\tilde{\gamma}_T} - N_{tr} - \frac{1}{2} \right] - \frac{1}{4} \Delta^2 \left( 1 + \frac{\tilde{\gamma}_T}{\tilde{\gamma}_T + \gamma_{tr} + \gamma_x} \right) \right\}, \quad (4.1)$$

where we assume that the resonator is coupled to another dissipative reservoir representing excess loss with a coupling strength $\gamma_x$, and an average photon number $N_x$. If the voltage-dependent calibration signal goes through a multi-stage amplification chain the measured power at the end of the chain is $P_{out} = GP_{tr} + P_{noise}$, where $G$ is the total gain of the chain and $P_{noise}$ is the total noise generated by the amplifiers. The voltage dependence of Eq. (4.1) after the amplification reads as

$$P_{out}(V) = aV + b + c/V, \quad (4.2)$$

where \{a, b, c\} are fitting parameters, hence the total gain can be expressed as

$$G = \frac{2a \tilde{\gamma}_T + \gamma_{tr} + \gamma_x}{e \tilde{\gamma}_T \gamma_{tr}}. \quad (4.3)$$

The coupling strengths $\tilde{\gamma}_T$, $\gamma_{tr}$, and $\gamma_x$ have to be determined based on reflectometry by measuring the reflection coefficient $\Gamma$. Experimentally, we
Results

drive the resonator through a transmission line connected to a cryogenic circulator which directs the driving signal to the resonator and the reflected signal to the amplification chain. The voltage-dependent reflection coefficient is given by

\[ \Gamma(V) = \frac{(2 - r)\gamma_{tr} - r(\gamma_T(V) + \gamma_x) + 2ir[\omega_p - \omega_T]}{\gamma_T(V) + \gamma_{tr} + \gamma_x - 2i(\omega_p - \omega_T)}, \]  

(4.4)

where \( r \) is a Fano resonance factor owing to a capacitive crosstalk between reservoirs.

The measured reflection coefficient, Eq. (4.1), yields the characteristic coupling strengths. Since we encounter a Lamb shift of the resonator resonance frequency in the measurements (detailed in Sec.4.2), the measured signal corresponding to a finite \( V \) is divided by the zero-bias trace and the ratio of two instances of Eq. (4.4) yields the normalized reflection coefficient \( \Gamma_N(V_b) = \Gamma(V_b)/\Gamma(0) \), where the \( \omega_r \) and \( \gamma_T \) depend on the bias voltage. An example fit is presented in Fig. 4.2(a). In the next step, the power spectral density (PSD) is measured as a function of the bias voltage. Since the central frequency of the resonator \( f_r = \omega_r/(2\pi) = 4.67 \) GHz determines mean photon energy of the calibration signal, the PSD is recorded only in the vicinity of \( f_r \), which is depicted in Fig. 4.2(b). Combining the characteristic damping rates and the parameter \( a \), which is fitted based on the measured PSD, the total gain of the amplification chain is estimated to be \( G = 51.48 \pm 0.10 \) dB, where the 1\( \sigma \) uncertainty is valid in the vicinity of \( f_r \) and includes all the uncertainty of the damping rate fitting, the uncertainty of the power fitting, and the uncertainty arising from the second-order approximation.

4.2 Lamb shift in microwave circuits

In this section, we discuss [Publication II], which investigates the low-bias behavior of a device and employs a setup similar to those in Fig. 4.1 where the absorption of the input power is controlled via the bias voltage applied to the SINIS junction. In addition to the microwave radiation, we observe a voltage dependence on the resonance frequency of the resonator. This effect is associated with the Lamb shift as discussed below.

The reflection coefficient is recorded as a function of the bias voltage and frequency detuning \( \delta = \omega_p - \omega_T \), where \( \omega_T \) is the resonance frequency corresponding to zero bias voltage. The measured data which is recorded by a vector network analyzer is depicted in Fig. 4.3. In the \([-1, 1] \times eV/\Delta \) voltage range, the signal goes through a \( 2\pi \) phase winding over the measured detuning range, while in the high-bias regime the phase variation is more moderate. Interestingly, the magnitude vanishes only around \( eV/\Delta = \pm 1 \), which exhibits the case where the coupling strength of the resonator to the transmission line matches the total internal losses of the
Results

Figure 4.2. (a) Real and imaginary part of the measured normalized voltage reflection coefficient $\Gamma_N$ (blue cross) and a fit (red dashed line) according to the ratio of the two instances of Eq. (4.4). (b) Measured output power $P_{out}$ as a function of the bias voltage (blue dots). Each point corresponds to the integral of a power spectral density (PSD) measurement depicted in the inset which shows the difference between the finite-voltage and zero-bias PSDs. Each PSD is averaged over 100 frequency sweeps. The total gain $G$ is fitted using Eq. (4.2) (red line) in the voltage range $eV_b/(2\Delta) \in [1.07, 8.83]$. The theoretical prediction for the power is represented by the dotted black line. The magenta circle illustrates the system noise at zero bias. The figure is adapted from Publication I.

(a) | (b)
---|---
![Figure 4.2](image)

Figure 4.3. Measured (a) amplitude of the reflection coefficient as a function of bias voltage and frequency detuning with respect to the zero-bias resonance frequency $\omega_r$ and (b) phase. The figure is adapted from Publication II.

(a) | (b)
---|---
![Figure 4.3](image)

(resonator, i.e., those from the tunneling environment and from the excess losses. In the aforementioned critical coupling region, the minimum of the reflection coefficient shifts with respect to the zero-bias value. The interaction between the resonator and the tunable reservoir arising from the tunneling electrons causes this effect, which is known as the Lamb shift. The Lamb shift appeared first in experimental atomic physics [13].

First the damping rates are extracted using Eq. (4.4) assuming constant Fano factor and voltage-dependent $\omega_r$. The damping rate owing to the SINIS junction as a function of bias voltage is depicted in Fig. 4.4(a), which shows that the dissipation due to the photo-assisted tunneling varies between at least two orders of magnitude. The resonance frequency (shown in Fig. 4.4(b)), exhibits a negative Lamb shift below the critical coupling
and asymptotically settles at a positive value. As discussed in [Publication II] the Lamb shift arises from two contributions that both depend on the coupling strength of the dissipative environment $\gamma_T$. Firstly, there is a frequency-dependent dynamic part. Secondly, and then there is a static part that is independent of frequency and not observed in Fig. 4.4(b)).

![Figure 4.4](image)

**Figure 4.4.** (a) Extracted coupling strength (dots) $\gamma_T$ as a function of the bias voltage, the corresponding $1\sigma$ uncertainty (blue shading) and the theoretical fit (blue line) based on Eq.(2.19). The uncertainty corresponding to the damping rate originating from the excess losses (green dotted line) and from the external coupling (purple dashed line) are comparable with the line widths. (b) Measured Lamb shift as a function of the bias voltage (dots), the $1\sigma$ uncertainty of the frequency shift (red shading), and the predicted dynamic contributions (dashed lines). Because of the uncertainty of the measurement, the static and dynamic shifts cannot be separated. The figure is adapted from [Publication II].

### 4.3 Pulsed quantum-circuit refrigeration

As mentioned above, the QCR presented in Fig. 4.1 is a potential realization of a controllable circuit refrigerator, which is potentially applicable in quantum computing for initializing quantum bits to their ground states [5]. As discussed in Sec. 2.3, the photon absorption of a QCR depends on the voltage applied to the junction terminals. If the photon absorption in
the QCR reaches a high rate, the photon occupation number of a resonator can be efficiently lowered or a qubit can be driven to its ground state. After the reset, if voltage applied to the QCR sets to zero, the resonator or qubit reaches again the low-dissipation regime and photons have long lifetimes. The major novelty of using QCR for resetting the state of a microwave circuit lies in the bias-voltage tunability and the weak interference to other components, e.g., magnetic-flux sensitive SQUIDs or superconducting resonators, and yet the QCR can be controlled on the nanosecond scale. In [Publication III] we drive the resonator with a pulsed excitation, and measure the decay of the resonator excitation while we pulse the bias voltage of the SINIS junction, as shown in Fig. 4.5(a).

The decay of the resonator signal, originating from the change of the coupling strength of the QCR, is extracted in the following way: The initial pump tone excites the resonator with a $-110\text{dBm}$-amplitude. After the pump stops, the resonator signal decays exponentially, and the decay is stimulated by the QCR pulse. As a result of the cooling pulse, the photon-assisted tunneling absorbs the photons from the resonator, and therefore the coupling strength takes a higher value. This in turn expedites the decay. Once the cooling pulse is over, the decay rate assumes the same value as before the cooling pulse. The QCR-enhanced decay rate is measured from the change of signal amplitude during the cooling pulse. Figure 4.5(b) shows the amplitude change as a function of the pulse length. Pulses shorter than 8 ns cause constant decay, because of the engineered on-chip $RC$ low-pass filtering of the QCR lines. Temporal lengths longer than 8 ns enhance the decay and the amplitude ratio depends exponentially on $\tau$, where the decay constant yields $\gamma_{QCR}$ as described in Fig. 4.5(b).

Following the procedure explained in the previous paragraph, we measure the $\gamma_{QCR}$ for different bias voltages, the results of which are shown in 4.5(c). The QCR damping rate cannot be resolved if the $\gamma_{QCR}$ is lower than the coupling strength between the resonator and transmission line, $\gamma_{t}$, because the decay is in this case dominated by the external coupling. However, the damping rates at bias higher than $0.6 eV/(2\Delta)$ are measurable and follow the theory-predicted bias-voltage dependence in Eq. (2.19). As a result, the dissipation using a pulsed operation of the QCR achieves comparable cooling properties, i.e., damping rates, as the steady-state operation.

4.4 Flux-tunable microwave environment

In [Publication IV] we present another microwave circuit providing tunable photon dissipation. In contrast to the QCR-based devices, the dissipation is tuned with magnetic field controlled by an external coil installed in the vicinity of the chip. The sample, shown in Fig. 4.6, consists of a
main resonator (Resonator 1) with a high-quality factor, $10^5$, which is used to extract the scattering parameters of the system and a secondary capacitively-coupled resonator (Resonator 2). The secondary resonator incorporates a SQUID and a resistor. The Josephson inductance of the SQUID depends on the magnetic field, and hence it is possible to control the fundamental resonator frequency with a current in a coil magnet. The resistor contributes as a dissipative element which decreases the average photon lifetime and therefore decreases the quality factor of the resonator.

The $ABCD$ matrix, defined in Eq. (3.2), of the total system can be written in the following form:
Results

Figure 4.6. Device architecture (a) Top-view photograph of the measured sample. (b) Capacitive coupler \( (C_T) \) between Resonators 1 and 2 in false-color SEM image. (c) Micrograph of one of the capacitors, \( C_c \), that are situated on each end of Resonator 1. (d) Micrograph of the SQUID loop that is capacitively coupled through \( C_L \) each side to the center conductor of the CPW and a zoom-in view of the junctions that yield the Josephson inductance \( L \). (e) Micrograph of Resonator 2 termination via \( C_{R1}, R \), and \( C_{R2} \). The magnified view shows the capacitive coupling \( C_{R1} \). (f) Circuit diagram of the measured sample. The transmission coefficient is measured through a transmission line, characteristic impedance of which is \( Z_L \), which differs from the impedance of the CPW resonator \( Z_0 \). The resonator lengths are defined using \( x_1 \) and \( x_2 \). The figure is adapted from Publication IV and Ref. [129].

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = M_1 M_2 M_3 M_2 M_1 = \begin{pmatrix} 1 & \frac{1}{\omega C C} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cosh (\gamma x_1) & Z_0 \sinh (\gamma x_1) \\ \frac{1}{Z_0 \sinh (\gamma x_1)} & \cosh (\gamma x_1) \end{pmatrix} \begin{pmatrix} 1 & \frac{1}{\omega C C} \\ 0 & 1 \end{pmatrix},
\]

(4.5)
where \( M_1 \) and \( M_2 \) denote the \( ABCD \) matrix corresponding to capacitor \( C_C \) and the Resonator 2, respectively. The matrix \( M_3 \) represents the capacitively coupled flux-tunable resonator, with input impedance

\[
Z_{r2} = \frac{1}{i\omega C_T} + \frac{Z_0}{Z_0 + \tanh(\gamma x_2)} \left\{ \frac{Z_S + Z_0 \tanh(\gamma x_2)}{Z_0 + \tanh(\gamma x_2)} \right\}.
\] (4.6)

The rest of the symbols are defined in Fig. 4.6. The flux-dependent term is in \( Z_S = i\omega L + 2/(i\omega C_L) \), where \( L \) is the Josephson inductance from Eq. (2.13) and \( Z_{\text{term}} = R + 1/(i\omega C_{R1}) + 1/(i\omega C_{R2}) \) is the impedance of the termination formed by the two capacitors and the resistor which terminate the center conductor of the CPW to ground. The parameter \( \gamma \) is the wave propagation coefficient and it depends on the internal \( Q \) factor of Resonator 1 and the phase velocity in the CPW resonator.
Figure 4.7. Amplitude of the measured (a) and simulated (b) transmission coefficient of Resonator 1 as a function of the frequency detuning and magnetic flux for the first four modes. The resonator modes are defined Eq. (2.15). In the fundamental mode and in the third mode, denoted here as $f_1$ and $f_3$ respectively, the $S_{21}$ parameter is independent of the magnetic flux, because at the coupler $C_T$, the standing wave has a voltage node. On the other hand, the even modes depend on the magnetic flux and the dependence shows periodicity, which agrees well with the simulation using Eq. (3.3). The quality factor of the second and the fourth modes depends on the magnetic flux as summarized in Fig. 4.8(a). The quality factor can be reduced by a factor of 30 in the forth mode using the flux control. In
Results

comparison, modulation range in the second mode is drastically narrower. The significant difference between the two modes can be explained with the behavior of Resonator 2, depicted in Fig. 4.8(b). Resonator 2 has a flux-dependent resonance around 10 GHz, which can be tuned to be in resonance with the fourth mode of Resonator 1. However, with the lower resonator mode in Resonator 2, this coincidence is not present, hence the quality factor modulation is relatively weak.

Figure 4.8. (a) Experimentally extracted loaded quality factors of the second mode (blue) and fourth mode (red) and their theoretically predicted values (dashed and dashed dotted line, respectively) as a function of magnetic flux. (b) Simulated transmission coefficient amplitude of Resonator 2. The figure is adapted from Publication IV.
4.5 Charge sensing with a superconducting charge detector

The charge noise properties of a superconducting single-electron transistor are discussed in this section summarizing the work in Publication V. The voltage-biased SSET, which is biased at the voltage corresponding to the DJQP, is capacitively coupled to a field-induced charge island. The noise of the detector at constant sensitivity has been recorded at different bath temperatures and different operation modes.

The top-view and cross-sectional structure of the device are presented in Fig. 4.9. Each detector has a pair of source and drain electrodes denoted by DS and DD, respectively, and a plunger gate (DG). The metallic island is located close to the top gate lead L, which can induce a 2DEG island that can be decoupled from a drain ohmic contact D using a switching barrier SB. On the other side, a quantum dot single-electron pump is deposited, the finger gates of which are kept at ground potential throughout the experiments. The bias voltage $V_{dc}$ is used to set the detector to the feature matching the DJQP cycle and the common voltage $V_c$ tunes the detector coupling to the charge island by inducing a 2DEG region under the detector structure. The two operation modes are denoted as standard coupling mode ($V_c = 0\, V$) and enhanced coupling mode ($V_c = 1\, V$) visualized in Fig. 4.9(b).

In this section, only from the standard coupling mode results are discussed. A proportional-integral-derivative (PID) controller maintains the charge sensitivity of the detector and compensates for $1/f$ noise.

![Figure 4.9](image-url)

First, the charge noise of the detector is measured as a function of the bath temperature presented in Fig. 4.10(a). The charge noise is obtained by converting the measured current noise using Eq. (2.24) to charge noise.
and calculating the standard deviation of a 80-s-long time segment. The temperature is considered constant in each segment. If an equal voltage is applied to the SB and L gates, the charge noise figures increase linearly with respect to $T_T$. This is an indication that the detector operates at white-noise regime far from the quantum limit considered in Eq. 2.25. The difference in the linear fits on the charge noise data at different gate-voltage configurations suggests that the charge noise is sensitive to the dc voltage configuration on the device.

The capacitive coupling between the detector and the charge island $C_c$ determines the readout bandwidth. To estimate the capacitive coupling, we measure an ac signal as a function of a voltage applied to the gate SB utilizing a lock-in amplifier presented in Fig. 4.9(b). This method yields a coupling $C_c = 45 \text{ aF}$. When a single electron is loaded to the 2DEG charge island, the detection bandwidth of the charge state variation is $\Gamma_{\text{err}} = 5.87 \times 10^3 \text{ 1/s at 300 mK bath temperature}$, which is a typical temperature in pumping experiments [87]. If a single-electron pump, at $f = 1 \text{ GHz}$ driving frequency, periodically modulates the occupation number of the island, the worse pump performance that still can be resolved by the charge sensor presented in this work is $5.87 \text{ ppm}$, which is an improvement by a factor of 90 compared to a similar silicon SET [21].

![Figure 4.10.](image.png)

### Figure 4.10.

(a) Charge noise as a function of bath temperature at different gate voltages and their linear fits (dashed). (b) The measured capacitive coupling (circle) as a function of switch barrier voltage. The detector-to-counting island coupling is estimated by the difference in the measured capacitance from the SB open and closed. The gray color is a guide for the eye. The figure is adapted from Publication V.

#### 4.6 Charge transition statistics of a strongly coupled two-level system

Defects are detrimental to charge sensing and ultimately result in $1/f$ noise in many different experiments with silicon [130, 133]. In this section, the characterization of a two-level fluctuator is presented using waiting-time distribution and full-counting statistics. An overview of the results in
Publication VI is presented here.

For this experiment, we use a similar SSET structure as the one presented in Sec. 4.5. Instead of a charge island, here a two-level system is capacitively coupled to the detector as shown in Fig. 4.11(a). The two-level fluctuator is assumed to always occupy one out of the two available states. The two-level fluctuator is in tunnel-contact with a charge reservoir with in- and out-tunneling rates to the fluctuator $\Gamma^-$ and $\Gamma^+$, respectively. The detector that records the transitions corresponding to the switching between the two states yields a signal exemplified in Fig. 4.11(b) indicating the waiting time ($\tau$) and residence time ($\tau^*$) of the subsequent tunneling events. The measured waiting-time distribution and residence-time distribution are constructed using the statistics of a 16-h-long time trace.

The connection between the tunneling rates $\Gamma^-$ and $\Gamma^+$ and the distributions of waiting times can be established through Eqs. (2.30), (2.27), and $\Pi(\tau) = \sum_j p_j(i\infty, t)$, where $p_j(i\infty, t)$ is an $j$th component of the vector $p(\chi, t)|_{i\chi \to i\infty}$, yielding

$$W(\tau) = \Gamma^+ \Gamma^- e^{-\Gamma^- \tau} - e^{-\Gamma^+ \tau} \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ - \Gamma^-}. \tag{4.7}$$

Figure 4.11(c) shows the measured and fitted waiting-time distributions, where the tunneling rates are $\Gamma^+ = 15.8 \times 10^{-3}$ s$^{-1}$ and $\Gamma^- = 473.2 \times 10^{-3}$ s$^{-1}$. The asymmetric tunneling rates can originate from the intrinsic potential landscape of the fluctuator and also partially from the electrostatic feedback.

The residence-time distribution requires a modification on the rate matrix presented in Eq. (2.31) by taking into account a detection rate $\Gamma^D$, since the temporal resolution of the detector is comparable to $\langle \tau^* \rangle$, where $\langle \rangle$ denotes an ensemble average. Thus, the rate equations include the dark-count events, where no transition occurs but the detector indicates an event. The missed events take place, when the detector cannot resolve the charge transition. As a result, the residence-time distribution reads as

$$W_r(\tau^*) = \frac{2\Gamma^D \Gamma^-}{\Lambda} e^{-\tau^* \sum_\alpha \Gamma^\alpha / 2} \sinh(\tau^* \Lambda / 2), \tag{4.8}$$

where $\Lambda = \sqrt{(\sum_\alpha \Gamma^\alpha)^2 - 4\Gamma^D \Gamma^-}$ and $\alpha = +, -, D$. In an ideal detector $\Gamma^D \to \infty$ the residence-time distribution follows an exponential decay $W_\infty(\tau^*) = \Gamma^- e^{-\tau^* \Gamma^-}$. The fitted residence-time distribution is indicated in Fig. 4.11(c), where $\Gamma^+$ and $\Gamma^-$ are obtained from the waiting-time distribution and the detector bandwidth is $\Gamma^D/(2\pi) = 10.4$ Hz, which is in good agreement with the experimentally set value $\Gamma^D/(2\pi) = 11.25$ Hz.
4.7 Consistency of high-accuracy single-electron pumping in silicon

In this section, we present the results from Publication VII that discusses a portable single-electron source. The source has been measured in three different laboratories and reached $1ef$ electric current within at least 1.5 ppm relative uncertainty at $f = 1.05$ GHz. The other novelty of this work is that the measurements are carried out in liquid helium ($T = 4.2$ K), which is a less stringent condition compared to previous high-precision pumping experiments [16]. The device was first measured at NPL, United Kingdom, then transported to Finland. The measurements continued at VTT MIKES, Espoo, and finished at Aalto University.

The device measured in this experiment has been previously utilized in Ref. [134]. The quantum dot is located between an entrance and an exit gate which are on top of a silicon nanowire. A large top gate induces the 2DEG and the quantum dot used for the pumping. For the ratchet-mode
pumping, the rf signal generator is connected to the entrance gate. If the voltages on the entrance and exit gates $V_{\text{ENT}}$ and $V_{\text{EXT}}$ respectively, are scanned when the ac drive is on, the so-called pump maps can be recorded. Example maps are shown in Fig. 4.12(a) measured at the three different institutions. The pump maps in all the three sets of measurements are in good agreement. However, the amplitudes of the rf drive are different for the different institutions. The location of the plateaus yield the $N_{\text{ef}}$ electric currents. Here, we focus on the first plateau, $N = 1$. The tuning of the pump, i.e., to find the ideal operation point to produce the lowest uncertainty, is discussed in detail in Refs. [16, 135].

In all experiments, the total uncertainty does not only depend on the standard deviation of the pumped current, but also depends on the calibration uncertainty of each of the instruments. Namely, the uncertainty of the transimpedance amplifier gain, of the digital voltmeter reading, and of their possible drifts. The high-precision data is acquired by measuring a segment of 250 s when the rf driving is on, then a segment of 250 s when the rf driving is off. These on-off cycles help to tackle with possible drifts in the instruments, because on the segment timescale, both on and off currents should suffer the same drift. Leakage current, when the potential barrier is opaque, can be a potential error source, which has been measured and corrected, where it was possible.

The results of the high-precision measurement including the total uncertainty are shown in Fig. 4.12(b). The deviation of the mean pumped current from $1_{\text{ef}}$ in all of the cases is within the 1-ppm range, and the maximum total uncertainty is as low as 1.30 ppm for the highest-uncertainty data. The uncertainty in the Aalto data is dominated by the calibration of the digital voltmeter that had a longer calibration chain compared to that at NPL and MIKES: the instrument was calibrated against a Zener voltage standard, which was transported from VTT MIKES to Aalto after it was calibrated against a Josephson voltage standard.
Figure 4.12. (a) Derivative of the pumped electric current along $V_{\text{EXIT}}$ as a function of $V_{\text{ENTRY}}$ and $V_{\text{EXIT}}$ measured at three different institutions as indicated. The dBm equals the rf driving power, and the crossing points of the dashed pink lines indicate the operation point for the precision measurements in panel (b). (b) Results of the high-precision pumping at fixed gate values. All quantities are expressed as deviations from current $I_{ef}$. The figure is adapted from Publication VII.
The two objectives of this thesis were to investigate tunable dissipation in superconducting microwave circuits and to study silicon metal–oxide–semiconductor architectures in the aspect electric current metrology. We introduced and demonstrated lossy tunable environments. Moreover, the noise properties of a superconductor–semiconductor hybrid platform related to error counting experiments were presented. We also studied the portability of a silicon single-electron pump quantum current standard.

We developed a novel gain calibration method for amplification chains in Publication I, where we utilized an NIS junction in the high-bias regime as a calibration source that is coupled to a coplanar waveguide resonator. We proposed and carried out a two-step calibration protocol in a single cooldown cycle. First, we extracted by reflectometry the characteristic damping rates of the circuits. Then the power spectral density of the generated signal is measured, leaving a single coefficient as a fitting parameter. Consequently, we obtained the uncertainty of the total gain to be within 0.10 dB.

A broadband Lamb shift was investigated in Publication II owing to a voltage-tunable reservoir. The reservoir arises from the electrons tunneling through an NIS junction, simultaneously absorbing photons from a superconducting resonator. Consequently, the observed shift of the central frequency of the fundamental resonator mode is in excellent agreement with the theoretical first-principle model.

Fast control of dissipation was the focus of Publication III. We examined the dynamics of the resonator population as a voltage pulse on an NIS junction activates the absorption. The employed device shows enhanced dissipation driving a pulse of at least 8-ns-long. At chosen parameters, the amplitude-dependence of the cooling pulse resembles the characteristics acquired in the steady-state, which suggests that the QCR is a reliable hardware component for future quantum circuits.

In Publication IV, we created a system that consists of a high-quality resonator capacitively coupled to a dissipative resonator incorporated with a SQUID. The resonance frequency of the lossy resonator can be tuned
by an external magnetic field which thus controls the dissipation in the high-quality resonator. As a result, the quality factor in this engineered system can be tuned by two orders of magnitude.

In Publication V, we investigated a system of a superconducting charge detector integrated next to a 2DEG island in silicon. The detector noise in standard and enhanced coupling modes were measured systematically at different voltages applied to the MOSFET gates. Furthermore, the mutual capacitance was extracted between a charge island and the detector. Under less stringent conditions, namely, elevated bath temperature, the estimated charge sensitivity of the system increased significantly compared with a silicon-based detector.

Two-level fluctuator strongly coupled to a charge detector was studied in Publication VI. We characterized a strongly coupled parasitic two-level system using a superconducting charge detector and employed the theory of full-counting statistics on the switching events to extract the transition rates. Since the acquired tunneling rates were highly asymmetric, we included the finite detection bandwidth in the theory which we consequently explained our measurements.

We verified the universality of a tunable-barrier quantum dot pump in Publication VII, where the same device was measured in three different laboratories demonstrating the spatial and temporal invariance of the quantum current standard at a generated current of $\sim 168 \text{ pA}$ within a total relative uncertainty of 1.30 ppm. Moreover, the parameter space of the operation, including the bias voltages applied to the barriers and the ac driving amplitude, showed similarity throughout the experiments indicating the robustness of the pump immersed in liquid helium.

This thesis investigated different novel aspects of circuits operating at cryogenic temperatures driven by practical interest. All of the discussed properties require hybrid nanostructures and the incorporation of different materials. In the near future, the focus will be on the behavior of the QCR in a more complex microwave circuit, and how to integrate NIS junctions with superconducting qubits and resonators. Specifically, if existing superconducting resonators which have quality factors of $10^6$ and state-of-the-art superconducting qubits are combined with a QCR to create a monolithic quantum processor, how well the original specifications of the resonators and superconducting qubits can be maintained. In Publication I, the bandwidth of the QCR-based gain calibration is limited to the central frequency of the resonator, therefore the calibration is valid for a narrow bandwidth compared to other gain calibration techniques. One solution would be to integrate a SQUID loop into the resonator to obtain flux tunability.

The urge for high-speed error counting still exists. The hardware, namely, a high-accuracy robust charge pump and a sensitive direct current charge sensor, have been demonstrated, but to date no satisfactory error count-
ing experiment has been realized. Importantly, self-referenced quantum current standards operating at low cost constitute a realistic and yet an extremely important goal for electric current metrology, from which the entire society would greatly benefit. The initial high-speed error counting experiments are going to determine the direction of development of the charge sensors. Since the detection bandwidths of the rf SSET charge detectors are significantly broader [57], they are also a potential candidate for the future error counting experiments.


References


References


