Advances on Portfolio Choices

Nasim Dehghan Hardouroudi
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**Abstract**

This doctoral dissertation focuses on models, methods, and their applications in portfolio optimization and risk management. The major contribution is developing and analyzing stochastic models on these type of problems. The dissertation consists of five articles, all of which together contribute to the subject by providing novel results and circumventing some shortcomings presented in the literature.

Firstly, we analyze the problem of finding optimal mean-variance portfolios under a cardinality constraint. This problem is a mixed integer quadratic programming (MIQP) and is hard to solve. We propose a novel approach to reformulate the problem as a mixed integer linear programming (MILP) problem, which can be solved using any available relevant solver. Computational results show that our MILP approach is significantly more robust and faster than MIQP. Moreover, our MILP approach compares favorably with equivalent problems with coherent risk measures conditional value-at-risk (CVaR) and mean absolute semi-deviation.

Secondly, we study the problem of portfolio selection using stochastic dominance (SD) constraints based on second order stochastic dominance (SSD). In particular, we introduce a self-contained theory with some new results. We develop simple arguments to formulate SSD constraints for mathematical programming. We then propose some methods of finding non-dominated optimal solutions and analyze mean-risk models as objectives involving absolute semi-deviation as well as CVaR as risk measures. Moreover, we introduce a relaxation of SSD, named directional SSD (DSSD), and show how it is operationalized for DSSD constrained portfolio optimization. We also presents a thorough comparison of computational performance of alternative approaches to SD constrained optimization. In addition, we consider the out-of-sample study of SSD and DSSD constrained portfolio optimization problems using stock market data of the US. The results suggest that DSSD based approach do well compared to SSD based, given that risk aversion exhibited by the objective function is relatively mild.

Lastly, we consider second, third, fourth and fifth order SD (SSD, TSD, FOSD and FISD respectively) as well as decreasing absolute risk aversion (DARA) stochastic dominance (DSD) and stochastic dominance based on exponential utility (ESD). We derive necessary and sufficient efficiency tests under the six types of SD. For well-known SSD and TSD efficiency tests, we provide simple arguments, which are then used to develop new FOSD, FISD, DSD and ESD efficiency tests. Our FOSD and DSD tests circumvent shortcomings in some recent literature. We provide numerical demonstration for each test using stock market data of the US. The results indicate that the market portfolio is inefficient and dominated under those types of SD, for various investment horizons. Interestingly, the results of DSD and ESD are almost identical.

**Keywords** portfolio optimization, second order stochastic dominance, DARA stochastic dominance, Nth order stochastic dominance, portfolio efficiency
To my parents
Nadali and Shahnaz

And to my husband
Abolfazl

For all their love
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List of Articles

This doctoral dissertation consists of an introduction and of the following articles which are referred to in the text by their numerals


PART I: Overview of the dissertation
The importance of investment in economy development of a country, growing individuals’ wealth, fighting inflation, saving for individuals’ retirement and etc., is known for many people. Portfolio optimization and risk management are the most significant topics in investment science and finance. Developing effective models and methods in portfolio choice would not only benefit individuals with managing their own portfolios but also have a crucial impact for banks to stay away as much as possible from financial crisis.

The milestone of the modern era of finance theory was reached by Harry Markowitz (1952). He introduced a fundamental mathematical model in portfolio optimization, called mean-variance (MV) model. By means of MV optimization procedure, given a set of assets, one is able to minimize the overall portfolio risk for a given expected return, or equivalently maximize the expected return for a given amount of portfolio risk by allocating her wealth efficiently to the different assets. However, There have been some criticisms towards the structure of Markowitz optimization framework. Specifically, the optimization scheme in his model is computationally unstable due to the sensitivity to estimation errors of the inputs, namely the expected return and the covariance matrix (see Bordie, Daubechies, De Mol, Giannone and Loris, 2009). Another difficulty of Markowitz original model is choosing variance as a measure of risk. This is justified if stock returns are normally distributed or if utility functions of agents are quadratic (Markowitz 1959, 2014). However, neither condition holds in practice: stock returns are not normally distributed as they usually relate to fat tails (see e.g., Mandelbrot 1993) and quadratic utility functions violate decreasing absolute risk aversion and non-satiation. Furthermore, variance is not a coherent measure of risk (Artzner et al. 1999) and it penalizes similarly both favorable and unfavorable deviations of return from mean return thereby violating preference for skewness. Therefore, some alternative risk measures, specifically those penalizing the unfavorable deviation (downside part) from mean return, were studied and proposed for mean-risk portfolio optimization problems. The downside risk measures are such as lower partial moments and Conditional Value at Risk (CVaR) (Rockafellar and Uryasev, 2000, 2002). However, since the information about the risk attitude of decision makers are not known, the ambiguity about choosing the most appropriate risk measure for mean-risk models exist. All things considered, MV rule is still the most cited model for ranking portfolios and many practitioners appreciate MV based portfolio selection methods including some refinements which makes MV model more realistic.
An attractive refinement is imposing a cardinality constraint which limits the number of assets in the portfolio to the MV problem. Using cardinality constraint, portfolio managers control transaction costs and other managerial concerns. However, incorporating a cardinality constraint to the Markowitz model yields a mixed integer quadratic programming problem (MIQP) which is hard to solve. In the first article (Dehghan Hardoroudi, Keshvari, Kallio and Korhonen, 2017), we address this issue by proposing a novel exact approach to reformulate this problem as a mixed integer linear programming (MILP) problem, for which computer codes are readily available. For numerical tests, we find cardinality constrained minimum variance portfolios of stocks in S&P500. A significant gain in robustness and computational effort by our MILP approach relative to MIQP is reported. Similarly, our MILP approach also competes favorably against cardinality constrained portfolio optimization with coherent risk measures CVaR and mean absolute semi-deviation (MASD). Finally, our approach may be of theoretical interest, as it seems to reduce computational burden relative to the equivalent MIQP approach. Therefore, the ideas can perhaps be extended to other problems as well.

Another well-established model in portfolio choice is the expected utility maximization which has been first introduced to decision theory by Von Neumann and Morgenstern (1962). In expected utility theory, random returns are compared by their values of expected utilities. However, the expected utility values depend on the utility function whose specification is a subjective matter. Consequently, this approach is less applicable in practice.

A theoretically sounder approach for portfolio choice is using stochastic dominance (SD) relation in which the above difficulties using the mean-risk and expected utility maximization are overcome. Unlike the mean-risk criteria which takes into account only the two attributes of return distributions (mean and risk), SD allows for the whole distribution of random returns. Furthermore, SD is closely related to expected utility theory. However, in SD approach one is not required to specify utility function, rather the random returns are ranked partially based on assumptions about general characteristics of utility functions. In portfolio selection, second order stochastic dominance (SSD) is of practical interest as the underlying set of utility functions accounts for non-satiable and risk-averse preferences which together indicate the observed economic behavior. Empirical evidence, however, motivates some other types of stochastic dominance as well: third order SD (TSD) which in addition to non-satiation and risk aversion accounts for prudence (preference for higher skewness); forth order SD (FOSD) that assumes also temperance (preference for lower kurtosis); decreasing absolute risk aversion (DARA) SD (DSD) which accounts for non-increasing absolute risk aversion, in addition to non-satiation, risk aversion and prudence; and ESD based on concave CARA utility functions which accounts for non-satiation and constant non-negative absolute risk aversion. Although SD rules are theoretically attractive, they are difficult to apply in practice. This motivates us to focus on incorporating the different types of SD rules mentioned above in portfolio selection problems. Therefore, the main contribution of this
doctoral dissertation is working on both theoretical and practical aspects of portfolio optimization and risk management problems with SD relations.

In the second article (Kallio and Dehghan Hardoroudi, 2018), we consider the problem of portfolio selection using SD constraint, including constraints based on SSD and some of its relaxations. We contribute by introducing a self-contained theory with some simple arguments and novel proofs for some known results. In particular, we develop new and simple arguments to formulate SSD constraints. We discuss some methods of finding non-dominated optimal solutions and analyze mean-risk models as objectives involving absolute semi-deviation as well as CVaR as risk measures. We introduce directional SSD (DSSD) as a relaxation of SSD and show how it is operationalized for DSSD constrained portfolio optimization. We also provide a thorough comparison of the computational efficiency of seven approaches for SD constrained portfolio optimization employing a variety of objective functions, benchmarks and the size of the scenario tree.

In the third paper (Dehghan Hardoroudi, 2019), we compare the out-of-sample performance of SSD and DSSD constrained portfolio optimization problems, using stock market data of the US. When the market index is taken as benchmark, out-of-sample tests show that both SSD and DSSD constrained optimization with a variety of objective functions outperform the benchmark in terms of average return, and DSSD based approach compares favorably against SSD based, given that risk aversion exhibited by the objective function is relatively mild.

As decision makers would like to know if their portfolios are the best ones or there exists a better portfolio conforming their preferences, the need for stochastic dominance efficiency tests arises. Several recent studies developed methods to identify portfolio efficiency. For instance, Post (2003) and Kuosmanen (2004), proposed tests for SSD efficiency; Bawa et al. (1985), and Post and Kopa (2013) developed a test for TSD efficiency; Post et al. (2015) developed an approximate test for DSD efficiency. However, there have been some shortcomings in some of these literatures which motivated us to circumvent them in the next two papers.

In the fourth paper (Kallio and Dehghan Hardoroudi, 2019a), we derive efficiency tests for SSD, TSD, DSD and ESD. We provide extreme simple derivation for SSD and TSD efficiency tests. We present novel necessary and sufficient efficiency tests for DSD and ESD. The efficiency tests for DSD and ESD are based on convex optimization and a sequence of linear programming problems, respectively. First, Post, Fang and Kopa (2015) developed an efficiency test for DSD; however, it was only a necessary test, as they missed the convexity property. Their DSD efficiency test is a relaxation of ours. To illustrate our approaches under these four types of SD as well as fourth order SD (FOSD) as decision criteria, we use stock market data of the US and test the efficiency of the market portfolio with respect to convex combinations of 10 stock portfolios and the one-month US treasury bill. The results indicate that the market portfolio is
inefficient and dominated under the five types of SD, for all the monthly, quarterly, semiannual and annual investment horizons. Interestingly, the results of DSD and ESD are almost identical.

In the fifth article (Kallio and Dehghan Hardoroudi, 2019b), we present new results for FOSD and fifth order SD (FISD) circumventing shortcomings in Post and Kopa (2013) and Fang and Post (2017). We derive necessary and sufficient efficiency tests for FOSD and FISD efficiency. An FOSD efficiency test was first attempted in Post et al. (2013); however, due to an error in Theorem 1, they end up with a linear programming (LP) formulation in contrast to our convex nonlinear programming (NLP) test problem. The FOSD test of Post et al. (2013) is a restriction of our test, and is only a sufficient one. Although, Fang and Post (2017) aimed to overcome the problems of Post and Kopa (2013), the issues remain unsolved, as the proof of the main result Lemma 2 is incomplete.
References


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