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EVALUATION OF THREE ICE-STRUCTURE INTERACTION MODELS

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This work concentrates on the contact between an ice edge and a vertical indenter face during intermittent crushing. The results of a laboratory indentation test series are analyzed and a description of the physical phenomena occurring at the contact is given. The existing dynamic ice-structure interaction models are reviewed. The ability of three models to simulate representative test results is evaluated.

The results of the indentation test series consist of 77 tests of which 23 are analyzed. The tests were carried out in a small basin with a floating sheet of columnar grained freshwater ice. The ice was horizontally loaded with an indenter which was installed in front of a spring-mass system. Special attention was paid to the visual observations in order to find the physical phenomena occurring at the ice deformation process.

The ice failure process was cyclic in the whole analyzed parameter space. The apparent crushing frequency was dependent on the indenter velocity and the lowest natural frequency of the structure. The latter also affected the peak force values: the higher the natural frequency the lower the force.

The ice failed in flaking. The flakes were formed locally by different size macrocracks which separated the flakes from the parent ice. This lead to a chain reaction which caused the deformation of the whole contact face during the load drop. The resulting ice wedge was crushed during the spring-back phase following the load drop.

The results of three representative tests were simulated using three different type dynamic ice-structure interaction models. None of the models performed well when the needed parameter values were obtained straight from the tests or the test conditions. However, when the parameter values were modified, the match improved considerably.
I have worked on this thesis for many years. It all began in 1989 when I had just got my M.Sc. degree and joined the team of the Ship Laboratory of the Helsinki University of Technology. At that time Mr. Antti Joensuu and Prof. Kaj Riska had just carried out the landmark indentation test series where they found the ice contact being a thin wavering line at the watershed of the ice edge. My work continued from there by conducting a laboratory ice indentation test series with thick freshwater ice together with a team of scientists led by Dr. Tuomo Kärnä from the Technical Research Centre of Finland. This test series forms the base of this thesis.

Later on, in the spring of 1990, I had the chance to participate the medium scale ice indentation tests at Hobson’s Choice ice island in the Canadian high Arctic. The scale of the tests was big enough to reveal many details of the ice crushing process. But better than that for a young ice researcher was the spirit of the camp. Just to live in a same hut with Dr. Robert Frederking, Prof. Ian Jordaan, Mr. John McCallum and Dr. Dan Masterson opened up for me a whole new world in ice research. The opportunity for me to participate in these experiments was worked out mainly by Prof. Riska. I have been indebted to him for this ever since.

In spite of all this preliminary work I was not able to delve into my thesis until the beginning of 1994. Even then, sharing the time between studies, family, and job has prolonged the work. I would like to express my sincere gratitude to my supervisors, Prof. Petri Varsta and Prof. Riska for the patience and support they have shown me during my studies. Special thanks belong also to Dr. Kärnä who as a colleague in conducting the laboratory tests in 1989 and later on as an instructor has been for me a model of a true scientist. I thank my colleagues at the Ship Laboratory, Prof. Claude Daley, Dr. Pentti Kujala, Dr. Jukka Tuhkuri, and Dr. Brian Veitch for the enlightening discussions I have had with them over the years.

The simulations done for the thesis were greatly eased by Dr. Devinder Sodhi, Prof. Mauri Määttänen, and Prof. Riska who kindly gave the computerized versions of their models for my
use. I am also grateful to Prof. Määttänen for the guidance he has given in the course of the thesis work.

The funding of the thesis has come from many sources. The laboratory ice indentation tests were conducted under a project funded by the Technology Development Center (TEKES). The one-year scholarship from the Teknillen edistämissäätiö made possible for me to concentrate on the thesis throughout the year 1994. The final touch for this work has been given with the help of the Helsinki University of Technology Library where I have worked since the beginning of 1995. All this funding is gratefully acknowledged.

Finally, this thesis would not have been finished at all without the loving support from my wife, Mrs. Leena Ruskomaa. Taking care of the home and our daughters, Aino and Kaisa, has kept her busy while I have travelled into the depths of my research. My gratitude for this cannot be expressed with words.

Otaniemi, September 1996.

Ari Muhonen
CONTENTS

ABSTRACT 2
PREFACE 3
NOTATIONS 6

1 INTRODUCTION 9
1.1 Scope of work 9
1.2 Evolution of ice-structure interaction modelling 11

2 ICE-STRUCTURE INTERACTION TESTS WITH THICK FRESHWATER ICE 15
2.1 Test arrangement 15
2.2 Test results 18
2.2.1 Quantitative results 18
2.2.2 Qualitative results 24
2.2.3 Pressure measurements 32
2.3 Ice deformation process 35
2.4 Discussion 39

3 THEORETICAL ICE-STRUCTURE INTERACTION MODELS 42
3.1 Interaction calculation methods 42
3.2 Ice force calculation methods 43
3.2.1 Matlock et al.’s model 44
3.2.2 Sodhi’s model 45
3.2.3 Æranti’s model 46
3.2.4 Kärnä’s model 47
3.2.5 Määttänen’s model 49
3.2.6 Risko’s model 50
3.3 Discussion 51

4 COMPARISON OF THREE MODELS TO THE TEST RESULTS 53
4.1 Simulations 53
4.1.1 Sodhi’s model 54
4.1.2 Määttänen’s model 60
4.1.3 Risza’s model 57
4.1.4 Simulation of test No. 17 with modified parameters 74
4.2 Discussion 81

5 SUMMARY AND CONCLUSIONS 84

REFERENCES 86
NOTATIONS

\( a \) half width of structure
\( a_s, a_z, a_z \) measured accelerations
\( A_i \) nominal contact area
\( c \) damping constant, cohesion term (Coulomb failure criterion)
\( c_x, c_y, c_z \) empirical constants
\([C]\) damping matrix
\( d \) diameter of ice fragments
\( D \) width of indenter
\( f \) natural frequency
\( f_n, f_i \) lowest natural frequency
\( f_{app} \) apparent crushing frequency
\( F \) force
\([F]\) force vector
\( F_c \) measured actuator force
\( F_{cr} \) critical zonal ice force
\( F_{exp} \) measured average extrusion force
\( F_i \) generalized force
\( F_{max} \) maximum force
\( F_{min} \) minimum force
\( F_{avg} \) measured average peak force
\( FZ \) ice failure length
\( g \) function
\( h \) ice thickness
\( k \) stiffness
\( k_i, K_i \) penetration constants
\( K_i \) ice stiffness
\( k_s, K_s, K_s^s \) penetration coefficients
\( k_{loc} \) local stiffness
\( k_s \) structural stiffness
\([K]\) stiffness matrix
\( m \) mass
\( m' \) orthonormalized mass
\([M]\) mass matrix
\( N \) number of tooth in contact with ice
\( p \) spacing of teeth, normal pressure
\( p_m \) mean pressure, melting pressure of ice, contact pressure
$p_c$  critical crushing pressure
$p_e$  extrusion pressure
$p_{cr}$  critical effective pressure
$q_i$  generalized coordinate
$(g)$  generalized displacement vector
$t$  time
$T$  ice temperature
$u$  absolute displacement
$u_1, u_2, u_3$  measured displacements
$u_i$  ice sheet rigid body motion
$u_{el}$  ice sheet elastic deformation
$u_p$  failure of ice sheet
$u_s$  global elastic deformation of structure
$u_{el}$  local elastic deformation of structure
$v$  velocity, nominal velocity
$v_o$  velocity of ice sheet
$w_i$  relative displacement of the ice edge and the structure
$W$  height of contact
$x, y, z$  Cartesian coordinates, displacement
$\alpha$  angle of crack
$\alpha_i, \beta_i$  coefficients
$\delta$  deformation of tooth, displacement of structure around its equilibrium position
$\delta_i$  velocity of the indenter at the point of ice action, relative velocity
$\zeta$  damping ratio
$\zeta_i$  generalized damping ratio
$\theta$  integration variable, angle of edge to which crack runs
$\mu$  ice-ice friction factor
$\sigma$  zonal ice pressure
$\sigma_1, \sigma_2, \sigma_3$  crushing strengths
$\sigma_{cr}, \sigma_{cr}, \sigma_{cr}$  ice crushing strength
$\sigma_{pz}$  zonal peak ice pressure
$\sigma_{max}$  maximum zonal ice strength
$\delta_1, \delta_2$  loading rates of ice sheet
$\tau$  shear strength of ice
$\phi$  friction angle (Coulomb failure criterion)
$\omega$  natural frequency
Subscripts

app apparent
c crushing
e ice, index
max maximum
min minimum
pav average peak level
s structure
std standard deviation
sum sum
x, y, z Cartesian coordinates

Abbreviations

ASME American society of mechanical engineers
CGREL Cold Regions Research & Engineering Laboratory
FFT fast Fourier transform
IAHR International Association for Hydraulic Research
IUTAM International Union for Theoretical and Applied Mechanics
MDOF multi degree of freedom
OMAE Offshore Mechanics and Arctic Engineering
OTC Offshore Technology Conference
POAC Port and Ocean Engineering under Arctic Conditions
PVDF polyvinylidene fluoride
SDOF single degree of freedom
1 INTRODUCTION

1.1 Scope of work

When designing a structure, knowledge is needed on the loads, global as well as local, which the structure will encounter during its operations. Dynamic ice loads became of interest some thirty years ago, when stationary platforms were taken in use in arctic oil exploration. Since then, ice crushing phenomena have been studied with numerous ice-structure interaction tests both in laboratories and in full scale. The analysis of the results led in many cases to theoretical interaction models. However, most of them were of limited value due to their semiempirical nature.

In recent years much attention has been paid to finding the physical phenomena occurring at the contact between the ice and the structure during the interaction. The basic idea is to transfer these phenomena into mathematical form and in this way construct a physically sound interaction model. The problem is, however, that ice can fail with many different failure modes and that the situation is affected by several parameters. Therefore only the first steps have been taken in using this approach.

Many test series have revealed details of the ice-structure contact phenomena. The first observers were Timco (1986) and Sodhi (1989). However, Joensuu and Riska (1989) made a fundamental observation that the contact was a thin wavering line on the watershed of the ice edge. Their work was followed by others using similar type laboratory equipment (e.g. Gagnon and Mølgaard, 1991, Fransson et al., 1991, Tuhkuri, 1993). The contact line was also found in medium scale ice indentation tests (Frederking et al., 1990, Meaney et al., 1991) as well as in tests using level ice sheet in laboratory (Muhonen et al., 1992a, 1992b) and in full scale (Fransson, 1994).

One thing common to all these test series was the intensive use of different devices for making visual observations. In particular high speed filming through a transparent indenter face proved to reveal the ice deformation processes well. For example, the the line-like contact was discovered this way.
Figure 1. Flat vertical indenter facing a semi-infinite ice sheet; coordinate axes and some important measures.

The theme of this work is the contact between an ice edge and an indenter face. The work is based on the results of a laboratory indentation test series done with thick freshwater ice. The parameter space is limited to the situation, where a vertical faced indenter meets level ice sheet (Figure 1) and the ice failure mode is crushing with spalling (flaking) as defined by Timco (1986). The relative velocity between the indenter and ice sheet is such that a cyclic force time history with high force level fluctuations is produced. Sodhi (1989) defines this as intermittent crushing.

The tests and their results included in the analysis are presented in Chapter 2. The results are analyzed both quantitatively and qualitatively. The processes occurring at the contact interface are revealed and their effect on the measured time histories examined.

The present theoretical ice-structure interaction models are examined in Chapter 3. The general mathematical formulation of the problem is presented. A closer inspection is done on how the models handle the ice contact force mathematically. The advances and pitfalls of the models are discussed. Three different type models are chosen for detailed examination.

In Chapter 4 the three chosen models are used to simulate some representative test results. This is done in order to test the ability of the models to simulate real time histories.
starting from measured physical quantities (input). The results of the simulation are examined and the way the models should be improved is discussed.

1.2 Evolution of ice-structure interaction modelling

The dynamic ice-structure interaction was first studied by Peyton (1968) and Blenkarn (1970). They both observed independently that a moving ice field can induce severe vibrations to slender offshore structures while conducting intensive full scale tests at Cook Inlet. Even though the observation was the same, the interpretation of the reason was different.

Peyton (1968) noticed that the ice force fluctuated considerably when the ice sheet moved against the structure at low velocity. He defined the phenomenon as "ratcheting". The fluctuations occurred at a fairly constant frequency of 1 Hz (which on the other hand was the lowest natural frequency of the test structure). In a laboratory test series Peyton observed that the natural frequency of the structure did not affect the ice crushing frequency. Based on these observations he concluded that ice has a "characteristic ice frequency".

Blenkarn (1970) reported that the primary mechanism of loading for the structures was direct crushing or shearing failure of ice. Therefore the values of the measured effective contact pressure were compared with laboratory data on the compressive strength of Cook Inlet ice measured by Peyton. This data showed that the ice crushing strength depends on the loading rate as depicted in Figure 2. Since the result of the comparison was reasonable Blenkarn concluded that also the ice force is loading rate dependent. He defined the increase in ice force with an increase in relative velocity as "damping" having the opposite sign than the structural damping. If this velocity derivative of the ice force (dF/dv) exceeds the structural damping the total damping becomes negative (Blenkarn, 1970, App.4). This leads to energy exchange from the ice field to the structure. This phenomenon was then defined as self-excited vibration.
A number of theoretical models have followed the thoughts of Peyton and Blenkarn. These can be divided in two groups: finite crushing depth (or frequency) and self-excited vibration (or negative damping) models. The models of the former group are based on the assumption that the ice fails within a certain distance ahead of the structure when a critical load is reached. The structure then extrudes the crushed ice. A new loading cycle begins when the intact ice is met. The self-excited vibration models assume that the crushing strength of the ice first increases with increasing loading rate and then decreases. This negative slope of the function is used to explain the ice induced vibrations.

Peyton’s idea was first adopted and changed into an ice-structure interaction model by Matlock et al. (1969, revised version of the paper 1971) who studied ice induced vibrations when designing a bridge near Cook Inlet. In this mechanical model the original idea of ice crushing frequency was turned to finite crushing depth. The aim of the model was to validate some ideas which explained the severe vibrations measured by Peyton.

Matlock et al.’s model has later been modified several times to take into account the effect of the relative velocity between ice and the structure (the idea first used by Blenkarn).
Based on their own test results Tsuchiya et al. (1985) proposed that the maximum resistance of ice should be recalculated in each time step taking into account the effect of the relative velocity. Daoud and Lee (1986) suggested that the ice force follows a predefined saw tooth function. The velocity dependence was simply defined with a Heaviside step function having the value 1 when the contact exists (positive relative velocity) and 0 otherwise. Karr et al. (1992) analyzed Matlock’s model and solved the equations in closed form. In addition, they introduced random properties for certain ice parameters.

Sodhi has contributed much to the development of the finite crushing depth modelling. He has carried out several test series where he has studied the physical phenomena occurring at the interaction process (Sodhi and Morris, 1984, Sodhi and Nakazawa, 1988, Sodhi, 1989, Sodhi and Chin, 1992, Sodhi, 1992). Using these data he developed a model where one force cycle is divided into three phases each of which has its own mathematical formulation. The resulting force time histories are simple but they contain the essential features of intermittent crushing.

Eranti presented his ideas of the interaction modelling first in Eranti et al. (1981). In his dissertation (Eranti, 1992) the model is generalized for an arbitrary shaped structure by using the zonal ice force concept introduced by Kry (1978). Eranti assumes that the intender penetrates into each ice zone until a critical ice pressure is reached. This pressure is loading rate dependent and also random. The ice zone fails a finite distance ahead and the indenter is able to move within this zone with only minor resistance of the failed ice.

Kärnä (1992) follows and extends the ideas of Eranti. His model is able to simulate full three dimensional structures of any type. This is ensured with two methods: finite element modelling of the structure, and the zonal approach of the ice force. In addition, the model includes the behavior of the soil-structure interaction. This model has been further developed in (Kärnä, 1994) and (Kärnä and Järvinen, 1994).

Määttänen studied slender steel lighthouses, which had collapsed on the coast of Finland (Määttänen, 1975). He deduced that the reason for the collapse were vibrations which were of self-excited nature. This led Määttänen to extend Blenkarn's
ideas to cover multi-degree-of-freedom (MDOF) structure models. The first version of the model was published in (Määttänen, 1977) and a revised version in (Määttänen, 1978).

Xu and Wang (1986) introduced an "ice force oscillator model" with self-excited vibration properties. Their main idea was that the ice itself has a natural frequency which causes the oscillations in the ice force even if the structure is perfectly rigid. Thus the motion of the ice sheet has to be described with a similar second order differential equation as the structure. The needed parameters for the model should be obtained from laboratory ice indentation tests (Xu and Wang, 1986). A revised version of the model was given in (Xu and Wang, 1988).

Several other models use the idea of self-excited vibrations. Ranta and Räty (1983) and later on Räty (1992) generalized Määttänen’s model for any beam structure of rotational symmetry and of varying radius. In addition, an analytical solution was derived for the equations. Croteau (1983) incorporated the zonal ice force concept of Kry (1978) extending the use of the model to wide offshore structures. Toyma et al. (1983) developed simple formulas for the maximum response of the structure as well as the crushing frequency of the interaction in order to explain the results of a field model test series. Vershinin and Iliadiy (1990) presented an interaction model graphically with a short verbal description. This model was used to design offshore structures for the Sakhalin area.

Riska et al. (1993) present the first model which is based on the new concept of modelling physical phenomena behind the interaction. It uses the ice contact model of Daley (1991). Riska’s model study the interaction between slender structures with vertical walls and a finite level ice floe. It is based on calculating both global and local displacements of the ice and the structure. Five displacements are calculated and united by the kinematic condition and the common force.

A good representation of the existing ice-structure interaction models are given in the state-of-the-art reports of Sodhi (1988) and Määttänen (1988). Riska et al. (1993) contains also a review on the subject.
2 ICE-STRUCTURE INTERACTION TESTS WITH THICK FRESHWATER ICE

2.1 Test arrangement

The Ship Laboratory of the Helsinki University of Technology (formerly the Laboratory of Naval Architecture and Marine Engineering, TKK/LRT) and the Laboratory of Structural Engineering of the Technical Research Centre of Finland (VTT/RAT) conducted together a laboratory ice indentation test series in August 1989 to February 1990 in the small 3x1.5 m² ice tank of VTT/RAT. The aim was to investigate the ice-structure interaction, especially the effect of different parameters to the indentation process and the response of the structure. The test arrangement is described in detail and the main results are given in the two volume measurement report (Muhonen et al., 1992a and 1992b).

The test structure is shown in Figure 3. It consisted of a spring-mass system (referred to here as carriage) in front of which different shape indenters could be attached. The mass and stiffness of the carriage could be changed with lead weights and springs respectively. The system was driven with a servo controlled hydraulic actuator with a maximum force capacity of 400 kW, displacement of 800 mm, and velocity about 80 mm/s.

The measurement system consisted of 50 different transducers. The main variables are shown in Figure 4. The ice force was measured directly behind the indenter in order to minimize the effect of the structural response to the time histories. The local pressures at the indenter face were measured with a matrix of small 2x2 cm² PVDF (polyvinylidene fluoride) elements. The method is described in (Joensuu, 1988) and (Muhonen and Koriseva, 1990). The accelerations and displacements of the carriage, indenter, and the ice sheet were measured using conventional techniques.

A big effort in the test series was put on intensive visual observations and recordings in order to find the physical phenomena occurring at the deformation process. Each test was recorded with a video as well as a 35 mm camera. After the test
Figure 3. Test structure (Muhonen et al., 1992a).

Figure 4. Main measurements (Muhonen et al., 1992a).
the ice edge was inspected and sketched. In 20 tests the contact between the ice and the indenter was filmed with a high speed camera through a transparent window. In many cases samples of the ice edge were taken and thin sections made.

Columnar grained freshwater ice was used in the test series throughout. The nominal thickness of the ice sheet was usually 10 cm. It was relatively thick to ensure good observations of the ice deformation process. The ice sheet was made using spraying technique, and the ice sheet grew in an ambient temperature of about -20°C. In the tests the ice was quite warm. The average temperature was about -0°C to -2°C. The quality of the ice was checked with thin sections, and the strength of each ice sheet was measured with separate ice compression tests. The stiffness of the ice was measured once.

Altogether 77 tests were made during the test series of which 45 with vertical indenters. The parameters varied in the tests are shown in Table 1. The results consisted of the measured time histories and recordings of the visual observations. The time histories, excluding the pressures, were subjected to a 1000 Hz low-pass filter within the amplifiers. The signals were stored with two tape recorders and from them digitized to a computer with 4000 Hz sampling rate. For the analysis the computerized signals were later digitally filtered using 200 Hz low-pass filters. At the same time the apparent sampling rate was dropped to 1000 Hz.

Table 1. Summary of the parameters varied during the laboratory test series.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass m (kg)</td>
<td>2000, and 15000</td>
</tr>
<tr>
<td>Spring stiffness k (kN/mm)</td>
<td>70, 10, and 2.5</td>
</tr>
<tr>
<td>Natural frequency f1 (Hz)</td>
<td>25, 4.1, and 1.9</td>
</tr>
<tr>
<td>Ice thickness h (mm)</td>
<td>50 - 130</td>
</tr>
<tr>
<td>Indenter width D (mm)</td>
<td>100 - 900</td>
</tr>
<tr>
<td>Aspect ratio D/h</td>
<td>1 - 18</td>
</tr>
<tr>
<td>Indenter shape</td>
<td>Flat vertical, inclined, wedge cylindrical, bow shaped</td>
</tr>
<tr>
<td>Nominal velocity v (mm/s)</td>
<td>10, 30, 50, and 80</td>
</tr>
</tbody>
</table>
2.2 Test results

The analysis concentrates on the tests made with vertical faced indenters in the velocity range of 10 mm/s to 80 mm/s. Flat indenter face was chosen to keep the interaction simple. It also allowed high-speed filming through a transparent window and the pressure measurements. This was important for observing the basic phenomena of the ice deformation process. For the same reason the used ice sheets were usually as thick as possible. The velocity range was chosen such that intermittent crushing would occur. The avoidance of creep dictated the lower velocity limit and the capacity of the actuator the higher one.

Altogether 26 tests met the selection criteria for the analysis. However, two tests (Nos. 37 and 38) had to be excluded due to their non-comparable test conditions and one (No. 4) due to unsuccessful force measurement. The selected 23 tests are listed in Table 2 and shown in Figure 5 arranged with the aspect ratio (D/h) and apparent strain rate (indenter velocity over two times indenter width, v/2D).

The test results are analyzed both quantitatively and qualitatively. The peak force level of the time histories are examined as a function of the velocity of the indenter, v, the aspect ratio, D/h, and the lowest natural frequency of the structure, f,. The spectrum of three representative tests are evaluated. In the qualitative analysis the form of the time histories are analyzed in the light of the visual observations.

The natural frequency of the carriage was varied changing both its mass and stiffness. Three different combinations of these were used in the tests as was shown in Table 1. In the following these combinations are referred to as the stiff, medium stiffness, and flexible structure.

2.2.1 Quantitative results

An example of a typical force time history is given in Figure 6a. It consists of repetitive cycles. One cycle can be divided in two phases, loading and spring-back, as defined by Kärnä et al. (1993). These are marked in Figure 6b.
### Table 2. Tests included in the analysis.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$k$ kN/mm²</th>
<th>$m$ kg</th>
<th>$D$ mm</th>
<th>$v$ mm/s</th>
<th>$h$ mm</th>
<th>$f$, Hz</th>
<th>$D/h$</th>
<th>$f_{o6}$ Hz</th>
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<td>2000</td>
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<td>2000</td>
<td>300</td>
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<td>10</td>
<td>15000</td>
<td>300</td>
<td>80</td>
<td>110</td>
<td>4.1</td>
<td>2.7</td>
<td>1.5</td>
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<tr>
<td>56</td>
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<td>15000</td>
<td>100</td>
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<td>3.0</td>
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<td>4.1</td>
<td>1.0</td>
<td>2.1</td>
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<tr>
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<td>10</td>
<td>15000</td>
<td>100</td>
<td>10</td>
<td>65</td>
<td>2.0</td>
<td>1.5</td>
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<td>2.0</td>
<td>1.0</td>
<td>0.5</td>
</tr>
<tr>
<td>69</td>
<td>2.4</td>
<td>15000</td>
<td>100</td>
<td>80</td>
<td>100</td>
<td>2.0</td>
<td>1.0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

1) Apparent crushing frequency, see page 23.

![Figure 5. Tests included in the analysis.](image-url)
Figure 6a. An example of the measured load time histories (test 15).

Figure 5b. Definitions of the phases of a force cycle.

The force level of the time history can be analysed by examining the peak force values. They are defined as the highest force value of each cycle. This is depicted in Figure 6a. The level of the force peaks fluctuates considerably. This is seen from Table 3 where the mean of the force peaks, $F_{net}$, as well as the standard deviation, $F_{std}$, maximum, $F_{max}$, and minimum, $F_{min}$, are given. The results of only 11 tests are shown here. In the other 12 tests the magnitude of the force signal was erroneous due to malfunctioning of the force measurement system and therefore they had to be omitted from the peak force analysis.
Table 3. Variation of the peak forces.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$F_{pav}$ kN</th>
<th>$F_{10s}$ kN</th>
<th>$F_{max}$ kN</th>
<th>$F_{5x}$ kN</th>
</tr>
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<tbody>
<tr>
<td>6</td>
<td>115</td>
<td>26</td>
<td>162</td>
<td>51</td>
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<td>78</td>
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<td>151</td>
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<td>16</td>
<td>118</td>
<td>44</td>
<td>204</td>
<td>57</td>
</tr>
<tr>
<td>17</td>
<td>102</td>
<td>26</td>
<td>173</td>
<td>34</td>
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<td>18</td>
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<td>17</td>
<td>118</td>
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<tr>
<td>67</td>
<td>35</td>
<td>11</td>
<td>56</td>
<td>19</td>
</tr>
</tbody>
</table>

a) effect of velocity  
b) effect of natural frequency  

Figure 7. Measured effects of parameters to the average peak force level.

The small number of the data points makes it difficult to analyse thoroughly the effect of the parameters to the force level. However, Figures 7a-b show some trends. The effect of velocity is contradictory. In the tests done with stiff structure the average peak force level decreases with increasing velocity. However, the two points obtained with the medium stiffness structure show the opposite trend. The effect of the natural frequency of the structure is clear: the higher the
natural frequency the lower the force. The aspect ratio does not have any clear effect to the force level.

Figures 8a-c show the frequency spectra of the force time histories of three representative tests. They were obtained using fast fourier transform (FFT) analysis. The macro "Spectrum" of the used DaDiSP program "returns the magnitude of the first half of the FFT of a signal normalized by the length of the signal" (DaDiSP worksheet manual, p. 365).

![Spectrum graphs]

a) test No. 17  
b) test No. 50  
c) test No. 67

Figure 8. Force spectra of three representative tests.
The highest frequency peak in each spectrum is rather narrow (especially for the test No. 5c). It correlates well with the calculated apparent crushing frequency, \( f_{\text{app}} \), marked in the figures. This is defined as the number of force peaks within a certain time interval. Hence, the apparent crushing frequency is a good approximation to the dominant frequency of the spectra. As it is easy to obtain it is used in the analysis throughout instead of the dominant frequency.

\[ \begin{array}{c}
\bullet D/h = 1.4 \quad \bullet D/h = 2.7 \quad \Delta D/h = 1.0 \quad \times D/h = 1.0
\end{array} \]

![Graph](image)

**a) effect of velocity**

\[ v = 80 \text{ mm/s} \quad \bullet v = 50 \text{ mm/s} \]

\[ v = 80 \text{ mm/s} \quad \Delta v = 30 \text{ mm/s} \]

![Graph](image)

**b) effect of aspect ratio**

![Graph](image)

**c) effect of natural frequency**

**Figure 9. Measured effects of parameters to the apparent crushing frequency.**
Figures 9a-c show the effect of the parameters to the apparent crushing frequency. Both increasing velocity and increasing natural frequency of the structure increase the apparent crushing frequency. Especially the latter trend is very clear. The big scatter hides the effect of the aspect ratio.

2.2.2 Qualitative results

The form of the time histories gives information about the physical phenomena occurring during the interaction. When these are compared with the visual observations an idea of the ice deformation process can be obtained. No synchronization was used to couple the time histories and the video recordings during the test series. However, the beginning of a test can be located on the video within a few frames and calculating the frames from there on gives an adequate accuracy for the comparison.

The trends given in the previous Chapter show that the natural frequency of the structure affected strongly on the force level and on the spectrum of the force time history. Therefore the time histories of three different tests (Nos. 17, 50, and 67) are examined here separately. These tests are representative showing typical features found in the time histories of other tests.

Tests done with stiff structure

Example time histories (force, displacement, and acceleration) of the test No. 17 done with stiff structure are shown in Figure 10.

The loading phase of each force cycle is quite linear. Only the very top rounds a little before the load drop. The structure stays practically still as can be seen in the displacement time history. The movement is so small that it cannot be seen in the video. Instead, when the ice debris is not obstructing the view, radial cracks and cleavage cracks can often be seen to emanate during this phase. The high speed filming does not reveal any movement of the ice at the contact face, either.
Figure 10. Time histories of the test No. 17 (Muhonen et al., 1992b).
When the peak is reached the force drops abruptly, but within a finite time. The force levels for a moment before another small drop. However, it always remains positive in most tests; only in the test No. 16 (v=10 m/s) it goes to zero. The structure accelerates and starts to move. The video shows how ice debris flows away from the contact area. It piles up in front of the indenter and soon blocks the view to the contact area.

The high speed filming shows how ice debris is moving up and down. The area at the watershed of the ice face is hard to make any observations on. In some cases there is a clear thin line which resembles the one reported by Joensuu and Riska (1989). However, mostly it seems that the debris flows both up and down irregularly from the area. The size of ice debris varies. This is depicted in Table 4 which shows the results of sieving done after four tests.

Table 4. Results of the ice debris sieving. The numbers give the cumulative mass (in grams) of fragments smaller than d. d is the diameter of the fragments in millimeters.

<table>
<thead>
<tr>
<th>d</th>
<th>Test No. 4</th>
<th>Test No. 6</th>
<th>Test No. 15</th>
<th>Test No. 23</th>
</tr>
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<tbody>
<tr>
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<td>2</td>
<td>4</td>
<td>3</td>
<td>6</td>
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</tr>
<tr>
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<td>520</td>
<td>332</td>
<td>317</td>
</tr>
<tr>
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<td>1146</td>
<td>990</td>
<td>392</td>
<td>739</td>
</tr>
<tr>
<td>32</td>
<td>1807</td>
<td>1153</td>
<td>509</td>
<td>1031</td>
</tr>
<tr>
<td>Total</td>
<td>2010</td>
<td>1277</td>
<td>634</td>
<td>1221</td>
</tr>
</tbody>
</table>

Tests done with medium stiffness structure

Example time histories of the test No. 50 done with medium stiffness structure are shown in Figure 11.

The loading part of the force looks quite similar to the one obtained with stiff structure, the loading rate is just slower. However, the spring-back phase is different. The force
Test 50, indenter P100, v=30 mm/s, h=112 mm, m=15000 kg, k=19 kN/mm

Figure 11. Time histories of the test No. 50 (Muhonen et al., 1992a).
stays a long time at a low level and then drops to zero. The structure moves a long distance during the low force and then reverses. The contact is lost at that moment.

During the low force the structure vibrates. This leads in some cases to banging against the ice face. This produces sharp force peaks before a new cycle. The vibration can also be seen in the loading phase of the following cycle.

The visual observations of the tests done with medium natural frequency of the structure succeeded well. Test No. 50 was especially well documented with a video camera. This was because pressurized air was used to remove the ice debris from the contact area. Also the video camera was placed on the side of the carriage thus giving a close view to the contact. The main observations are commented in the following.

The test starts when the indenter comes into contact with the ice edge. The indenter stops and during most of the loading phase it cannot be seen to move. One radial crack emerges some time during the loading, but it cannot be seen when exactly this happens. As the load gets higher the ice in front of the indenter becomes whitish.

When the load is high the ice deforms and the indenter starts to move. A semicircular crack forms the edge of a big spall which separates a big ice piece from the ice sheet. The moving indenter squeezes out the ice piece which flies some distance ahead. After this the indenter surges ahead with high velocity. Different size ice particles can be seen to fly out from the cavity left in the ice sheet. The cavity diminishes as the indenter moves forward, but it is not eaten totally away.

After the surge the indenter backs a little and the contact is lost. When a new cycle begins there is still a considerable size cavity left in the ice in front of the indenter. This means that the contact is far from the nominal area (indenter width, D, times ice thickness, h). It means that the previous spalling and extrusion activity defines the contact area in the next loading.
Figure 12. Time histories of the test No. 67 (Muhonen et al., 1992a).
Tests done with flexible structure

Example time histories of the test No. 67 done with flexible structure are shown in Figure 12.

The flexible structure shows one new feature in the interaction process: during the spring-back phase the structure moves ahead a long way. The corresponding part of the force time history contains many small peaks. The video shows that the ice is flaked several times as sketched in Figure 13. The situation was analyzed in (Kärnä et al., 1993). It was concluded that at least some of the force peaks are caused by the same type of process which produce the high peaks.

![Diagram](image)

Figure 13. Flake formation during the third loading cycle in the test No. 69 (redrawn from Kärnä et al., 1993).

Effect of velocity and aspect ratio

The effect of velocity to the force time histories can be seen in the results of the tests done with stiff structure. The force signal of the test No. 17 (Figure 10) is a good example of the general trend. However, the result of the test No. 16 (v=10 mm/s) differs from that as can be seen in Figure 14a. The force there drops to zero at the end of each cycle. The form of the time history resembles the ones obtained with medium stiffness structure. The effect of velocity is therefore of the same type as the effect of the natural frequency.
Figure 14a. Force time history of the test No. 16 (Muhonen et al., 1992b).

Figure 14b. Force time history of the test No. 23 (Muhonen et al., 1992b).

The aspect ratio does not affect the form of the force time history as can be seen in Figure 14b. Both phases of the force cycle can be seen in this test done with 900 mm wide indenter. The form of the time history is very similar to the one of the test No. 17.
2.2.3 Pressure measurements

When examining the ice deformation process it is important to know the pressure distribution along the indenter face. This was examined in the test series with a matrix of PVDF pressure elements. A sketch of the element panels is shown in Figure 15. Unfortunately, the PVDF technique was not very reliable in this test series and it worked only for the first few tests. However, the results of test No. 4 can be analyzed.

Figure 15. Sketch of the PVDF pressure panels (Muhonen et al., 1992a).

Figures 16a-c show examples of the pressure distribution during different loading conditions. Three bottom rows of the pressure transducers cover the central part of the ice sheet. The broken elements are shaded in the figure.

The pressure distribution fluctuates in time and space in a complex way. In general, the pressures in the watershed area of the ice face are high when the load is high (Figure 16a). However, the area of the high pressures changes from one force peak to another. This indicates that the contact area varies from peak to peak. The pressures diminish when the load drops (Figure 16b) but they are still notable at the centered row of the transducers. The pressures go up again a little during the spring-back phase (Figure 16c).
The pressures above and below the ice sheet center line were measured with the transducers No. 23 and No. 32 (see Figure 15). A six second window of their pressure time histories is given in Figure 17. They show that there is very seldom high pressure simultaneously on both transducers. This can also be seen from the crosscorrelation plot shown in Figure 18a. The situation is different at the corner of the indenter. The crosscorrelation of the pressure transducers No. 25 and 30
Figure 17. An example of pressure time histories.

a) Crosscorrelation of the elements Nos. 23 and 32, test No. 4.

b) Crosscorrelation of the elements Nos. 25 and 30, test No. 4.

Figure 18. Crosscorrelation of two pressure elements.
situated right above each other is rather good as depicted in Figure 18b. Thus it can be concluded that the pressure distribution is not uniform along the whole contact area. It should also be noted that the pressure is also zero at times as can be seen in the time histories.

2.3 Ice deformation process

The visual observations give support to the observation of uneven contact. The video from the test No. 50 showed clearly that there were cavities in the ice in front of the indenter during loading. The videos of the wide structure tests (for example test No. 24) show, on the other hand, how ice debris is flown out from the contact area apparently simultaneously from several places along the width of the indenter during the spring-back phase of each cycle.

The thin sections made after the test from the contact area (Figure 19) show how the parent ice is separated from the deformed ice by macrocracks. These macrocracks are curvy reaching from the watershed area of the ice face to the top and bottom surfaces of the ice. The parent ice forms a wedge where the tip is in contact with the structure. The layer of the pulverized ice is the widest at the surfaces.

Based on the observations listed above (especially the uneven contact) an idea of the ice deformation process can be formulated. During the loading phase the stresses in the ice increase. The uneven contact leads to uneven pressure and stress distributions with local peaks. In spite of that the total force increases rather linearly and the ice shows elastic behaviour. Also some permanent deformation occurs as reported by (Zarná, 1994) and (Eranti, 1992).

The force increase ends when at least one of the local stress peaks reaches a critical value. The ice deforms locally resulting in the change of the contact geometry and redistribution of the stresses. This leads to a chain reaction and load drop during which the whole ice contacting the indenter fails. The resulting contact geometry is a wedge created by
macrocracks. The structure starts to move releasing the energy stored in the springs.

An interesting part from the pressure measurements supporting the assumption of many consecutive flakes is shown in Figure 20. It shows the time histories of three pressure elements during one force cycle. The pressure at elements No. 24 and No. 30 starts to drop before the total force drop whereas at the elements No. 26+27 (measured together) the pressure drops a few milliseconds later.

During the spring-back phase the structure crushes the remaining wedge. The contact with the parent ice is first narrow but increases as the wedge is crushed. At the same time the energy stored in the springs of the carriage is consumed. When the equilibrium position is passed the springs start to decelerate the structure. Finally the structure stops and it may even reverse (if the stiffness of the structure is low enough) before a new cycle.

It is important to notice that the process contains a geometric coupling. The forming macrocracks separate a certain
volume of ice from the parent ice. The ice between the crack and the indenter face is deformed and possibly extruded. However, the structure does not necessarily move ahead as far as the macrocrack reach on the surface of the ice. Thus a cavity is left in the ice face, which is either void (as in the test No. 50) or filled with ice debris. These cavities affect the contact geometry. It means that the spall of the former cycle gives dimensions to the ice wedge contact area to be loaded and that way also the peak force level in the next cycle.

Radial cracks were observed during the loading phase of the cycles. However, there are no signs of these in the force time histories. Thus they do not affect the process globally. Instead they have a local effect: the radial cracks can be thought to act as stress relievers.
Radial cracks emerge from the corners of the indenter. This is in accordance with the classical theory (Timoshenko and Goodier, 1951) which shows that there are the highest stresses (Figure 21). This has also been measured by Fransson (1994) and there are traces of that also in this test series as shown in the pressure distribution in Figure 16a. When the local stress becomes too high at the corner, a radial crack is formed and the stress drops. This even out the global stress distribution along the whole indenter face. This helps the global load to increase further and higher total load is achieved.

![Figure 21. Theoretical load distribution (Timoshenko and Goodier, 1951, adopted here from Fransson, 1994).](image)

The time histories and the events seen on the videos of all the analysed tests look the same. Therefore it can be concluded that the basic ice crushing process is valid on the whole parameter space examined in this Chapter. However, the velocity of the structure as well as the structural stiffness do have their effect on the details.

The results of the test No. 16 (done with slow velocity) and No. 50 as well as a few other tests done with medium stiffness structure look very much alike regarding the profiles of the force time histories and the occurrences seen from the video. In spite of differing parameters the loading rate is quite the same in their force cycles. This would suggest that the essential parameter is neither velocity nor stiffness but the loading rate (N/s or MPa/s) of the ice.
2.4 Discussion

Figures 22a-b show the peak pressure as a function of aspect ratio (D/h) and apparent strain rate (v/2D) respectively collected from different test series. The quantitative results given in Chapter 2.2.1 are added in the figures. They compare well with the other measurements. It is hard to find any general trend due to big scatter as was the case in Figures 7 in Chapter 2.2.1.

Figure 22a. Peak pressure vs. aspect ratio.

Figure 22b. Peak pressure vs. nominal strain rate.
Tuhkuri (1995) has reported an ice indentation test series which concentrated on examining the ice crushing process. The tests were done with cold ice blocks having brittle behaviour. An example time history of his tests is given in Figure 23. It shows the triangular shape of the force cycles. Each event "consisted of a 'bang' sound and movement of crushed ice. Hence each event was reshaping the ice surface" (Tuhkuri, 1995, p. 270).

![Figure 23. An example force time record from (Tuhkuri, 1995).](image)

The ice deformation process in Tuhkuri's tests was crushing with flaking. Each flaking event was observed to create a macrocrack which removed a flake from the parent ice. When there was no confinement near the ice surface a line type contact was formed. When there was confinement the contact area consisted of "compacted crushed white ice" with no direct contact of the parent ice to the indenter.

The ice deformation process reported by Tuhkuri has many similarities with the one explained in chapter 2.3. In both cases the ice deformation mode was crushing with flaking and the flakes were formed by macrocracks. An area of compacted crushed
ice was a very typical feature in the tests reported in this chapter, and a few times the line type contact was clearly observed.

The biggest difference between the processes is in the force drop. Tuhkuri reports that each event removed one flake whereas in Chapter 2.3 it was suggested that the force drops due to a chain reaction caused by consecutive macrocracks. Three reasons can be thought for the difference. In Tuhkuri's tests the ice was cold and behaved in truly brittle manner. The structure was stiff and its transient vibration small. And finally, ice blocks were used for testing having different end conditions than an ice sheet.

The similarity of the ice deformation process described in Chapter 2.3 with Daley's contact model is interesting. In both of these the force is affected by the contact geometry. Daley simplifies the process to two dimensional as shown in Figure 24. However, the figure shows how the flakes are three dimensional in reality and of different size. This concept is very similar to the one described in this work.

Figure 24. Two dimensional idealization of the flaking process (Daley, 1991).
3 THEORETICAL ICE-STRUCTURE INTERACTION MODELS

3.1 Interaction calculation methods

The time dependent behaviour of a structure can be modelled with second order differential equations. The ice loading is taken into account as a forcing function. The difficulty in solving the equations is that the ice force is usually highly nonlinear. It also contains back coupling since the displacement and/or the velocity of the structure affects the magnitude of the ice force. Different ways of modelling the structure is presented in the following. Formulas 1 to 4 are taken from (Thomson, 1988).

The simplest way to model a structure is to assume it to have only single degree of freedom (SDOF). The equation of motion is then

\[ m \ddot{x} + c \dot{x} + kx = F \quad (1) \]

where \( m \), \( c \), \( k \), and \( x \) are mass, damping coefficient, stiffness, and displacement of the structure respectively. \( F \) is the ice contact force which can be a function of time, relative or absolute displacement of the ice and the structure, relative or absolute velocity of the ice and the structure and the initial ice velocity (Määtänen, 1989). This simple form of modelling the structure is common in the early interaction models.

The SDOF equation can also be written in another form

\[ \ddot{x} + 2 \zeta \omega_n \dot{x} + \omega_n^2 x = \frac{F}{m} \quad (2) \]

where \( \zeta \) and \( \omega_n \) are the relative damping coefficient and (circular) natural frequency of the structure respectively. The closed form solution of Sodhi's model is obtained starting from this equation.
In the advanced models the structure is described with beam elements. This way its vibration modes can be defined and more realistic behaviour can be deduced. The solution of these multi degree of freedom (MDOF) models is based on mode superposition techniques. One equation of motion is needed for each mode taken into account. This is usually presented using generalized coordinates $q$. Then the equation for the $i$th natural mode is

$$q_i'' + 2\zeta_i\omega_i q_i' + \omega_i^2 q_i = F_i$$ (3)

where $\zeta_i$ is the generalized damping, and $F_i$ is the generalized force. The equation group can also be written in the matrix form

$$[M]\ddot{q} + [C]q' + [K]q = [F]$$ (4)

where $[M]$ is the mass matrix, $[C]$ is the damping matrix, $[K]$ is the stiffness matrix $[q]$ is the generalized displacement vector, and $[F]$ is the ice force vector. Eq. 2 can also be used to describe the behaviour of each natural mode.

Due to the difficult nature of the ice force the only possibility in solving the interaction equations is to use numerical integration. Several different methods have been used in the models. Only in two simple cases (Karr et al., 1992, Sodhi, 1994) the equations have been solved in closed form and thus they readily give the response of the structure as a function of time.

3.2 Ice force calculation methods

In order to calculate the ice force the behaviour of ice under loading has to be idealized. Each interaction model has its own manner of doing this. Some important models are presented in the following. It should be noted that the original notation has been used in the formulae. This may cause some difficulties in comparing the different models.
3.2.1 Matlock et al.'s model

In the mechanical model of Matlock et al. (1969) ice is assumed to be an ideal elastic-brittle material. It is represented by a carriage transporting a series of uniformly spaced elastic-brittle teeth as shown in Figure 25. The carriage moves with a constant velocity \( u \) where \( u \) is the absolute displacement. The deformation of the tooth in contact with the mass can be determined from

\[
\delta = (z - x) - (N - 1) \times p
\]

where \((z - x)\) is the relative displacement, \( N \) is the number of the tooth in contact or next to be contacted, and \( p \) is the spacing of the teeth as defined in Figure 25.

When a tooth is in contact with the mass, the force is assumed to be linearly dependent on the deformation of the tooth. At the maximum deformation \( F_{\text{max}} \) is reached, the tooth fractures completely, and is discarded. This occurs instantaneously permitting the force drop to zero. The force remains zero until a new tooth comes into contact with the mass.

Figure 25. Mechanical model (Matlock et al., 1969).
1.2.2 Sodhi's model

Sodhi's model (Sodhi, 1994) is based on imitating the force time history of intermittent crushing mode. One cycle of loading is divided in three phases (loading, extrusion and separation) as shown in Figure 26. Each phase is modelled with its own formula.

![Figure 26. Force time history of intermittent crushing (Sodhi, 1994).](image)

In the loading phase the resisting force is simply the stiffness of the ice, $k_i$, multiplied by the elastic displacement of the ice edge:

$$F(t) = k_i((vt-z)\cdot x)$$  \hspace{1cm} (6)

where $(vt-z)$ represents the forward movement of the ice sheet within one cycle and $x$ is the deformation of the structure as defined in Figure 27. Thus it gives the relative displacement of the ice sheet and the indenter. The loading phase ends when a critical contact force is reached. It is defined as

$$F_{cr} = P_{cr}Dh$$  \hspace{1cm} (7)
where \( p_c \) is the critical effective pressure (8 MPa to 13 MPa according to Sodhi, 1989), \( D \) is the width of the structure, and \( h \) is the thickness of the ice sheet.

During the extrusion phase the structure moves with high velocity extruding the crushed ice. The extrusion force is assumed to be constant:

\[
F(t) = p_e Dh
\]  

(4)

where \( p_e \) is the extrusion pressure (2 MPa to 4 MPa according to Sodhi, 1989). The separation phase starts and the force drops to zero, when the forward velocity of the structure changes its sign. If this does not occur at all, the interaction mode is changed to continuous crushing. Then the structure continues to move forward with a constant velocity and constant force to the end of the simulation.

3.2.3 Eranti’s model

In Eranti’s model (Eranti, 1992) one loading cycle is divided in two phases, penetration and failure. It should be noted that these are not the same as Sodhi’s loading and extrusion phases.

In the penetration phase the ice is assumed to penetrate into the structure. As this is physically not possible the
penetration is "fictitious" (Eranti, 1992, p. 23). It represents the elastic and permanent deformation of the ice edge. The corresponding force is calculated with the formula

\[ F = k_i c_i h \sigma \]  

(9)

where \( k_i \) is "the ordinary penetration constant" (Eranti, 1992, p.24), \( c_i \) an empirical constant, \( h \) is the thickness of the ice, and \( \sigma \) is the zonal ice pressure. If the indenter reverses during the penetration phase, this unloading is handled with another penetration constant, \( k_\sigma \). It takes into account the permanent deformation of the ice.

The penetration phase ends when the critical zonal ice force, \( F_{cr} \), is reached. This is a function of the scaled penetration rate. The ice is assumed to fail a finite length

\[ F_{cr} = c_3 \sigma_{cr} h - c_4 h \]  

(10)

where \( c_3 \) and \( c_4 \) are constants, \( \sigma_{cr} \) is the zonal peak ice pressure corresponding to \( F_{cr} \), and \( h \) is the ice thickness.

When the critical ice force is reached the force is assumed to drop suddenly to a low value which is kept until the contact is lost or a new cycle begins.

The maximum zonal ice strength, \( \sigma_{cr} \), and five other constants (including \( c_3 \)) to \( c_4 \) mentioned above) are obtained from measured time histories. In addition, a random number generator is used to give the maximum zonal ice strength a random value with a lognormal distribution.

### 3.2.4 Kärnä's model

In Kärnä's model (Kärnä, 1992) the ice sheet is divided into a far field and near field component. The former takes into account the rigid body motion of the ice sheet. The boundary between the far field and the near field works as a buffer. This ensures that the static displacements caused by the interaction
will not go to infinity as would be the case for a semi-infinite ice sheet. Both fields are then divided further into zones following the procedure of Eranti's model.

The calculation of the ice force is done in the near field area. The procedure is based on dividing the force time history in two parts (loading and unloading) and constructing separate equations for both of them. The contact force is a function of the relative displacement, \( w_i \), of the ice edge and the structure, and its first time derivative. In the loading phase the relative displacement gives the compression of the ice in the near field area.

During the loading phase the ice force is calculated using a simple linear correlation between the displacement and the force

\[
F_i = K_i w_i \quad (11)
\]

where \( K_i \) is the penetration coefficient. It is obtained from test results. As in Eranti's model the elastic unloading is handled with another penetration coefficient, \( K^\prime \).

The loading phase ends when a critical force level is reached and the ice is deformed a finite distance ahead. The critical force level is calculated with the help of the critical crushing pressure, \( p_m \), and the nominal contact area, \( A \). The critical crushing pressure is loading rate dependent and random with lognormal distribution. The length of the failed ice zone added with the elastic rebound of the ice sheet is called as "the finite failure depth". It is an empirical parameter and it is obtained from measured force-displacement time histories.

The unloading phase consists of two parts. The force drop occurs at a finite time. This is caused by the formation the spalls and extrusion of the big ice pieces. After this the ice force remains at a low level, about 10% to 20% of the critical force (Kärnä, 1974b). When the solid ice edge is reached again, the cycle begins anew.
3.2.5 Määttänen’s model

The contact force is calculated in Määttänen’s model (Määttänen, 1978) using a simple assumption: the ice is in a constant crushing state. Therefore Peyton’s graph can be used to determine the relation between ice loading rate and crushing strength at each time step. Thus it is enough to calculate the loading rate of the ice sheet. The ice crushing strength is obtained from the graph and the force can be calculated using appropriate contact area.

The loading rate of the ice sheet is calculated with the equation originally deduced by Blenkarn:

$$\sigma = (v_s - \dot{\delta}_i) \frac{4\sigma_c}{\pi a} \cos \theta$$  \hspace{1cm} (12)

where $v_s$ is the velocity of the ice field, $\dot{\delta}_i$ is the velocity of the indenter at the point of ice action, $\sigma_c$ is the ice crushing strength and $a$ is the radius of the structure. The cosine term takes into account the circular form of the loading. The ice crushing strength can now be obtained from the simplified graph (see Figure 28) using the formula

$$\sigma_{ci} = \alpha_i + \beta_i \sigma$$  \hspace{1cm} (13)

where $\alpha_i$ and $\beta_i$ are the slopes of the increasing and decreasing lines of the modified Peyton’s graph respectively. The total force can now be calculated by integration.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{simplified_peyton_graph.png}
\caption{Simplified Peyton’s graph (redrawn from Määttänen, 1978).}
\end{figure}
3.2.6 Riska’s model

The ice force in Riska’s model is calculated using Daley’s contact model. It assumes that the indenter moves with a constant velocity towards an ice block as shown in Figure 29. When the contact is reached, the load builds monotonically, because the ice is assumed to be a rigid-brittle material. The contact pressure, $p_c$, in the loading is determined by pressure melting.

![Diagram](image)

Figure 29. Mechanics of Daley’s contact model (Daley, 1991).

The ice fails in flaking. Coulomb macroscopic failure criterion is used to determine the critical shear stress level. After the flake occurs the contact width and also the force is immediately divided in half. The flaking process is such that the flakes are formed sequentially. They also occur in different levels.

The model is stochastic since each flake results in the change of the edge geometry. This affects the failure conditions for the next flake. The flakes are assumed to fly freely away so that extrusion of pulverized ice is not considered.

The ice force is calculated in an iterative process where the result of one time step is used to calculate the force of the following time step. The formulas are quite complicated and are therefore not presented here. They can be found in (Daley, 1991) or (Daley, 1992).
Risk'a's interaction model consists of five subsystems as depicted in Figure 30 each one having its own equation of motion. These equations are used to calculate the corresponding displacements: failure of the ice sheet, \( u_s \), global elastic deformation of the structure, \( u_g \), local elastic deformation of the structure, \( u_{el} \), ice sheet rigid body motion, \( u_r \), and ice sheet elastic deformation, \( u_{se} \). The ice force is treated as a sixth variable. The calculated displacements and the ice force are coupled together with the kinematic condition

\[
g(t) = u_s(t) + u_{se}(t) + u_g(t) + u_r(t) + u_{el}(t)
\]

where the signs correspond to the directions shown in Figure 30.

3.3 Discussion

As mentioned in Chapter 1.2 most of the models are based either on the idea of the finite crushing depth or the self-excited vibration. Both of these ideas have been criticised.

Finite crushing depth is a measure which has vague physical support (see for example Riska et al., 1993, p. 5). Its values...
have to be taken from indentation tests and therefore the models are semiempirical. The models are based on imitating force or displacement time histories. This is done by dividing the time histories in cycles and phases. The more details are modelled the more phases (and parameters) are needed. This makes the models complicated and difficult to understand. It also limits their validity. Therefore these models cannot be generalized and their usage is limited to a narrow (though important) parameter space.

The self-excited vibration models are quite straightforward. Their basic assumptions are simple. They do not need any experimental parameters apart from Peyton’s graph. However, the method works only if the graph is valid. This has been disputed ever since it was published.

The idea of loading rate dependency of the ice crushing strength is widely accepted. It has also been measured many times (see for example Michel and Toussaint, 1977). However, there is no unified view of the behaviour of ice at high loading rates. This is because it is very difficult to produce reliably such high loading rates which are needed at the end of the graph. Another problem of the graph is that it has been measured in a laboratory using Alaskan ice. It is doubtful that no ice properties would affect the ice crushing strength. So in order to use the graph it should be produced for other ice types as well.

Daley’s contact model has many promising features. It takes the needed parameters from measured ice properties or they are calculated using simple formulae. The main drawback of the model is its use of naive assumptions. However, their real effect to the usability of the model have not yet been really tested. The model is also based on the results of one laboratory indentation test series. It should be evaluated using other data as well.
4 COMPARISON OF THREE MODELS TO THE TEST RESULTS

4.1 Simulations

Three of the models reviewed in Chapter 3 are chosen for a closer evaluation, namely Sodhi’s, Määtänen’s and Riska’s. They represent different approaches to the interaction modelling. The models of Eranti and Kärnä cannot be evaluated here, because they have already used the test results presented in Chapter 2 to obtain numerical values for the empirical parameters needed in their models.

The evaluation is done by simulating the results of three representative tests (Nos. 17, 50, and 67). The needed parameter values are taken straight from the tests or the test conditions. This examines how well the models perform in a situation where the help of existing time histories is not available.

The values for the parameters common to all three models are given in Table 5. The used indentation velocities are obtained from the displacement time histories with linear regression. They differ slightly from the nominal ones which were shown in Table 2 (p. 19).

Table 5. Values for the parameters common to all three models.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Structure m kg</th>
<th>k kN/mm</th>
<th>ζ</th>
<th>D mm</th>
<th>Ice h mm</th>
<th>V_{true} mm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>2 000</td>
<td>65</td>
<td>0.1</td>
<td>300</td>
<td>115</td>
<td>45</td>
</tr>
<tr>
<td>50</td>
<td>15 000</td>
<td>10</td>
<td>0.1</td>
<td>100</td>
<td>112</td>
<td>30</td>
</tr>
<tr>
<td>67</td>
<td>15 000</td>
<td>2.4</td>
<td>0.1</td>
<td>100</td>
<td>65</td>
<td>20</td>
</tr>
</tbody>
</table>
4.1.1 Sodhi’s model

Sodhi’s model is based on a few simple principles. The structure has only one degree of freedom. The ice is assumed as an elastic material having a stiffness, \( k_i \), until a critical contact pressure, \( p_c \), is reached. This can be calculated with the formula

\[
P_t = \frac{F_{\text{avr}}}{Dh}
\]

\( \text{(15)} \)

where \( F_{\text{avr}} \) is the measured average peak force, \( D \) is the width of the indenter, and \( h \) is the thickness of the ice sheet. The average peak force was defined in Chapter 2.2.1.

The force drops to a low constant level for the extrusion phase. The corresponding extrusion pressure can be calculated from

\[
P_e = \frac{F_{\text{avr}}}{Dh}
\]

\( \text{(16)} \)

where \( F_{\text{avr}} \) is the measured average level of the low force during the spring-back phase.

The parameters needed to use Sodhi’s model are shown in Table 6 and the values for them in Tables 5 and 7. The ice stiffness was measured once during the test series (Muhonen et al., 1992a, p. 61). The value used here is a mean of four separate values.

The program used in the simulation is the original version of the model obtained straight from Dr. Sodhi. It is based on

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Ice parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width, ( D )</td>
<td>Stiffness, ( k_i )</td>
</tr>
<tr>
<td>Mass, ( m )</td>
<td>Thickness, ( h )</td>
</tr>
<tr>
<td>Stiffness, ( k_a )</td>
<td>Velocity, ( v )</td>
</tr>
<tr>
<td>Damping, ( \zeta )</td>
<td>Failure pressure, ( p_r )</td>
</tr>
<tr>
<td></td>
<td>Extrusion pressure, ( p_e )</td>
</tr>
</tbody>
</table>
Table 7. Values of the input parameters used in Sohli’s model simulations.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$K_i$</th>
<th>$P_i$</th>
<th>$P_e/P_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>55</td>
<td>3.0</td>
<td>0.28</td>
</tr>
<tr>
<td>50</td>
<td>55</td>
<td>7.1</td>
<td>0.19</td>
</tr>
<tr>
<td>67</td>
<td>55</td>
<td>5.2</td>
<td>0.27</td>
</tr>
</tbody>
</table>

the closed form solution formula given in (Sodhi, 1994). The output consists of the ice force and the response of the structure (displacement, velocity, and acceleration). The simulation is done with a constant 1 ms time step, and the outputs are 5 s long time histories. It would have been good to have control over these two parameters. Now the simulation time is too short for slow velocity tests like the test No. 67.

The simulated time histories (force, indenter displacement, and acceleration) are given in Figures 31-33. The signal on the background is the measured time history between the time instants 2.582 s to 3.551 s. This represents the total signal better than its beginning with a very high initial force peak and a few subsequent very low force peaks.

The simulated force time history consists of equal, extremely simple cycles. Even though this general trend is correct the result differs from the measured signal in important details. The peak force level does not change in the simulation result (because it was given as input) whereas the peaks of the measured signal vary considerably. The loading rate is too high and this leads to overly high apparent crushing frequency.

The most fundamental difference is in the unloading phase of the cycles. The extrusion and separation phases are vital parts of the simulated force time history, but they do not exist in the measured signal. The corresponding part there consists only of a small decay in the force drop before the next cycle.

The response of the structure is modelled quite well except for the fact that the apparent crushing frequency is too high. The form of the simulated acceleration signal is good and the
Figure 31. Sodhi's model simulation result of test No. 17.
Figure 32. Sodhi’s model simulation result of test No. 50.
Figure 33. Sodhi's model simulation result of test No. 67.
peak level close to the measured one. However, Sodhi assumes the force to drop abruptly both from the peak level to the extrusion level and from that to zero. These jumps cause abrupt changes in the acceleration. The problem could have easily been avoided had the force been allowed to drop within a short, yet finite time. This would have been in accordance with the measured signal. This problem does not exist in the displacement time history.

The force time history of the test No. 50 has all the features of intermittent crushing. This is easily noted when comparing Figures 26 (p. 45) and 32. In spite of that the simulation fails again. The constant peak force level and the too high loading rate are present as they were in the simulation of the test No. 17. In addition, the transient vibration of the structure is too big. This causes an enormously exaggerated transient peak after the first loading cycle. That again affects the second loading cycle which becomes very short in duration.

Once again a fundamental difference occurs. The indentation mode changes into continuous crushing after three cycles in the simulated signal. This contradicts with the measurements. The change of mode is extremely sensitive to even small variation of parameters. The crushing mode returns cyclic if the structural damping is changed from 0.1 to 0.105 or down to 0.05, or if the velocity is changed from 30 mm/s to 29 mm/s. This kind of performance suggests that the assumption of the mode change is problematic.

The simulation result of the test No. 67 is shown in Figure 33. Sodhi’s model does not allow flaking and force peaks during the extrusion phase. Therefore this part of the time history is not modelled correctly.

The spectra of the simulated force time histories are given in Figures 34. The spectrum of the test No. 17 consists of one peak and its multiples. Thus it is too simple to represent any real signal. The spectra of the simulated and measured signals of the two other tests are not comparable. The mode change in the simulated test No. 50 changes also the form of the spectrum. In the test No. 67 the length of the simulation is too short for the process to stabilize and therefore the spectrum does not represent the stabilized situation.
Figure 34. Spectra of the simulated force time histories of Sodhi’s model.

4.1.2 Määttänen’s model

In Määttänen’s model the structure is described with beam elements. The response of an MDOF structure to ice loading is solved with mode superposition method. The ice properties are defined with the simplified Peyton’s graph which was presented
Table 8. The numerical values of the parameters which define the simplified Peyton's graph.

<table>
<thead>
<tr>
<th>Crushing strength N/mm²</th>
<th>Loading rate N/mm² /s</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ₀</td>
<td>1.67</td>
</tr>
<tr>
<td>σₙ</td>
<td>2.56</td>
</tr>
<tr>
<td>σᵣ</td>
<td>0.83</td>
</tr>
<tr>
<td>φ₁</td>
<td>0.20</td>
</tr>
<tr>
<td>φ₂</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 9. Input parameters of Määttänen's model.

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Ice parameters</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, m</td>
<td>Thickness, b</td>
<td></td>
</tr>
<tr>
<td>Stiffness, kₐ</td>
<td>Velocity, v</td>
<td></td>
</tr>
<tr>
<td>Damping, c</td>
<td>Coefficients defining Peyton's graph</td>
<td>Duration of simulation</td>
</tr>
<tr>
<td>Width, D</td>
<td></td>
<td>Time step</td>
</tr>
<tr>
<td>Initial displacement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial velocity</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

in Figure 28 (p. 49). The needed crushing strength and loading rate values given by Määttänen (1978) are shown in Table 8.

The original model was too complicated for simulating a simple spring-mass system. Therefore the program code given by Prof. Määttänen was altered to suit SDOF structure modelling. The input parameters needed for the simplified model are summarized in Table 9.

The original model simulates circular piles, so for this work the calculation procedure had to be changed to suit flat indenters. This was done using two assumptions. The pressure distribution was assumed uniform along the whole width of the indenter and the loading rate of the flat indenter the same as the maximum loading rate of the circular structure at its centerline. This way the formula given by Määttänen (Eq. 12, p. 49) could be used for calculating the maximum pressure.

The simulated time histories are given in Figures 35-37. The simulation time histories of the test No. 17 resemble the ones obtained with Sodhi's model. The signal is too repetitive the force peak level being constant, the apparent crushing
Figure 35. Määttänen's model simulation result of test No. 17.
Figure 36. Määttänen’s model simulation result of test No. 50.
Figure 37. Määttänen's model simulation result of test No. 67.
frequency is about two times too high, and the force drops to zero after the extrusion phase forming a gap between the ice and the structure.

The response results have been manipulated somewhat for representation purposes. The original model gives the displacement of the structure around its equilibrium position. This has been changed to the advancement, $x$, of the indenter with the formula

$$x = vt - \delta$$  \hspace{1cm} (17)

where $v$ is the velocity of the ice, $t$ is time, and $\delta$ is the displacement of the structure around its equilibrium position. The acceleration result was smoothed by taking a ±5 point moving average to remove the numerical noise.

The general form of the simulated displacement resembles the measured one. However, the structure does not move during the simulated loading phase opposing the trend in the measured signal. The discontinuities of the simulated force signal in the beginning and at the end of the contact separation cause abrupt changes in the acceleration. The level of the acceleration is about 90% of the measured one.

The simulated force time history of the test No. 50 has many features which can also be found from the measured signal: during the loading phase the force fluctuates a little before leveling to a linear rise; the force drop occurs within a finite time; the extrusion phase ends with a small peak; and the second cycle begins with a small force peak. The two abrupt force level changes in the beginning and at the end of the contact separation are the features of the model alone. The damping is too small as was the case with Sodhi’s model. This causes too violent vibrations of the structure during the spring-back phase and in the beginning of the following cycle.

The response of the structure in the simulation of the test No. 50 resembles that of test No. 17. The peculiarities caused by the abrupt force level changes can clearly be seen in the acceleration signal. However, they do not exist in the displacement signals.
The simulation result of the test No. 67 does not add much to the results achieved so far. The form of the time history is quite good except that the crushing frequency is again too high. The load level is approximately 15% smaller than the measured peaks. The extrusion force is constant and no sign of the fast velocity flakes can be seen.

The spectra of the simulated force time histories are given in Figures 38. They all consist of one high peak and its multiples. As with the result of the Sodhi's model they are too simple to represent the measured spectra.

a) test No. 17
b) test No. 59
c) test No. 67

Figure 38. Spectra of the simulated force time histories of Määttänen's model.
4.1.3 Riska’s model

Riska’s model is very versatile. It can be used to simulate slender, vertical and flat faced MDOF structures interacting with either an ice sheet (having a constant velocity) or a floe. In the latter case the rigid body motions of the floe are taken into account.

The parameters needed for the simulation of an SDOF structure are listed in Table 10. The orthonormalized mass can in this case be obtained with the formula

\[ m' = \frac{1}{\sqrt{m}} \]  

(18)

where m is the mass of the structure. The local stiffness of the structure was calculated in (Riska et al., 1993) using finite element modelling. The result 680 MN/m is used throughout.

Table 10. Input parameters of Riska’s model needed in this work.

<table>
<thead>
<tr>
<th>Structural parameters</th>
<th>Ice parameters</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local stiffness, (k_{\infty})</td>
<td>Velocity, (v)</td>
<td>Duration of simulation</td>
</tr>
<tr>
<td>Width, D</td>
<td>Crushing pressure, (p_c)</td>
<td>Time step</td>
</tr>
<tr>
<td>Natural frequency, (f)</td>
<td>Cohesion, (c)</td>
<td></td>
</tr>
<tr>
<td>Damping, (\zeta)</td>
<td>Friction angle, (\phi)</td>
<td></td>
</tr>
<tr>
<td>Orthonormalized mass, (m')</td>
<td>Number of flaking levels</td>
<td></td>
</tr>
<tr>
<td>Thickness, (h)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The needed ice parameters can be calculated with the following formulae given in (Daley, 1991). The formation of the ice flakes is calculated with the Coulomb failure criterion which is formulated as

\[ \tau = c + p \tan \phi \]  

(19)

where \(\tau\) is the ice shear strength, \(p\) is the normal pressure, and \(c\) and \(\phi\) are the cohesion and friction angle of ice respectively.
These material parameters can be obtained with the following formulae:

\[ c = 1.0625 \sigma_{\text{cr}} \sqrt{\frac{1 - \mu}{\mu + 3.25}} \]  \hspace{1cm} (20)

and

\[ \sin(\phi) = 0.47055\mu + 0.52941 \]  \hspace{1cm} (21)

where \( \sigma_{\text{cr}} \) is the stress to cause crack propagation in compression, and \( \mu \) the ice-ice friction factor. The friction factor as well as the pressure melting value of ice (corresponding to the crushing pressure \( p_c \)), depend on the temperature of the ice. The equation for the former one has been obtained empirically:

\[ \mu = \frac{0.39 T}{3.18 - 0.39 T} \]  \hspace{1cm} (22)

The latter one can be calculated with

\[ p_c [\text{MPa}] = 10 \times T \]  \hspace{1cm} (23)

where \( T \) is the ice temperature in °C.

The parameters used in the simulation are given in Table 11. The formulae given above were used to calculate the failure criterion parameters. The needed ice crushing pressure, \( \sigma_{\text{cr}} \), was taken from the test results and modified to the correct temperature with the graph shown in Figure 39. This procedure was used also in (Riska et al., 1993). The number of flaking levels is 2. The same number was used by Daley (1991), and Riska et al. (1993).

The program for the simulations was given by Prof. Riska. It is a straight computerization of the formulae presented in Riska et al. (1993). The generality of the program is not fully utilized in this work. The modelled structure has only one
Table 11. Values of the input parameters used in Riska’s model simulations.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>P_c</th>
<th>C</th>
<th>(\phi)</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>21</td>
<td>1.15</td>
<td>39</td>
<td>-2.1</td>
</tr>
<tr>
<td>50</td>
<td>31</td>
<td>1.50</td>
<td>36</td>
<td>-1.1</td>
</tr>
<tr>
<td>67</td>
<td>21</td>
<td>1.06</td>
<td>39</td>
<td>-2.4</td>
</tr>
</tbody>
</table>

Figure 39. Ice compressive strength as a function of ice temperature (Mellor, 1979).

principal mode and the ice sheet is supposed to have a constant nominal velocity.

The simulated time histories are given in Figures 40–42. They begin with non-zero values. This is because in the model the ice edge is assumed wedge-shaped whereas in the measurements it was flat. The difference in the signals disappears after the first loading and therefore it is omitted from the results.

The simulated force signal of the test No. 17 resembles somewhat the measured one. The general form of the cycles is of the same type, and the peak force level fluctuates. This did not occur in the results of the two previous models. However, in this model the loading rate of the force is too small opposing
Figure 40. Riska’s model simulation result of test No. 17.
Figure 41. Riska's model simulation result of test No. 50.
Figure 42. Riska’s model simulation result of test No. 67.
the trend obtained with Godhi's and Määttänen's models. When the ice fails the force drops abruptly. The new cycle begins right after the force drop; there is no levelling as in the measured signal.

The simulated displacement and acceleration signals of the two first peaks are both in form and magnitude quite like the measured ones. This is said omitting the fact that the apparent crushing frequency is not correct. The drop of the simulated force level in the two following cycles lessens the response as well and therefore the similarity with the measured signals diminishes.

The simulation result of the test No. 50 is not as good as the one of the test No. 17. The loading rate of the force is much too small, and the basic features of the intermittent crushing (extrusion, gap) are missing. The simulated displacement is too smooth, the nearly total stop of the loading phase and rapid advancement of the unloading phase are both missing. The magnitudes of the acceleration peaks are about half of the measured values.

The simulation result of the test No. 67 contained a lot of numerical noise. This was removed by taking a +5 point moving average through the whole signal. The filtered signal shows the general trends of the force fluctuation. The similarity between the simulated and measured signal is not very good. However, the force drop from peaks is not abrupt. There are also more force peaks than surges in the displacement curve. This means that the force has peaked also during the extrusion phase. This is in accordance with the observations in the tests.

The spectra of the simulated force time histories are given in Figures 43. They all consist of many harmonic components which is in accordance with the measurements. This is because the level of the force peaks does vary and the load cycles are not equal in form and magnitude. On the other hand, there are too many components involved in the spectrum of the test No. 17, because the action does not concentrate on a narrow frequency band as in the spectrum of the measured signal. The same is true for the test No. 50, the spectrum of the simulated force lacks the sharp frequency peak of the measured signal. The simulated and measured spectra of the test No. 67 look quite similar.
Figure 43. Spectra of the simulated force time histories of Riska's model.

4.1.4 Simulation of test No. 17 with modified parameters

None of the three ice-structure interaction models performed well in the simulations with the parameter values taken straight from the test conditions. It is of interest, though, to check how the values should be changed in order to
get an optimal simulation result. Test No. 17 is used as a test case. The force time histories of the optimized simulations are given in Figures 44.

In Sodai's model the peak and extrusion force levels are defined a priori. Therefore the main parameter which affects the form of the force time history is the ice stiffness, $k_i$. It can be used to modify the loading rate of the force.

The apparently good simulation result is obtained by approximately halving the measured ice stiffness to the value $k_i = 27$ MN/m. This is not far from the lowest measured single value (33 MN/m). One should also bear in mind the effect of the permanent deformations in ice during the loading phase which lower the effective ice stiffness.

The peak force level of the measured signal does not change much at this part of the time history, therefore the predefined peak level of the simulation looks reasonable. However, it should be remembered that this is not very typical as was shown in Chapter 2.2.1. The loading cycles overlap also well, but the basic problems of the simulation (gap, extrusion) already mentioned in Chapter 4.1.1 remain.

Määttänen's model used Peyton's graph which is a measured result defining a certain ice property. Therefore it cannot be changed as such. But if the graph is interpreted as a mere correlation between loading rate and ice failing pressure the alteration is possible. In this way also the parameters defining the simplified form of the graph can be given physical meanings.

The turning points of the simplified Peyton's graph have a big effect to the form of the simulated force time history. The maximum crushing pressure and therefore also the maximum force is defined by $\sigma_i$, and the extrusion force by $\sigma_e$. This can be deduced from the form of the graph. The loading rates affect the forms of the different parts of the loading cycle. $\dot{\sigma}_i$ defines the loading rate of the maximum force and $\dot{\sigma}_e$ the loading rate where the extrusion phase begins. Their difference $\dot{\sigma}_e - \dot{\sigma}_i$ is also important. The bigger the difference the longer time it takes for the force to drop from the maximum to the extrusion level. When $\dot{\sigma}_i$ is very small even small loading rates start the extrusion phase. The system may then achieve equilibrium and the extrusion may continue until the end of the simulation. This
Figure 44. Simulation results of the test No. 17 using modified parameters with Sodhi’s model (top), Määttänen’s model (middle), and Riska’s model (bottom).
corresponds with Sodhi’s definition for continuous crushing mode.

The modified Peyton parameters used in the optimized simulation are given in Table 12. The crushing strength values \( \sigma_s \) and \( \sigma_a \) have been lowered in order to get the peak force level right without affecting the slope of the rising part of the graph. The apparent crushing frequency has been corrected by dropping the ice field velocity about 60% to 19 mm/s. This takes into account the stiffness of the ice and the permanent deformations occurring during the ice loading.

Table 12. Modified values of the parameters which define the simplified Peyton’s graph.

<table>
<thead>
<tr>
<th>Crushing strength (N/mm²)</th>
<th>Loading rate (N/mm²/μs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_s )</td>
<td>1.00</td>
</tr>
<tr>
<td>( \sigma_i )</td>
<td>1.95</td>
</tr>
<tr>
<td>( \sigma_o )</td>
<td>0.83</td>
</tr>
<tr>
<td>( \dot{\sigma}_s )</td>
<td>0.20</td>
</tr>
<tr>
<td>( \dot{\sigma}_i )</td>
<td>0.80</td>
</tr>
</tbody>
</table>

The simulated force time history is almost identical with the measured one for the first second of the simulation as shown in Figure 44b. It contains all the important features: the force curves a little during the loading phase, the curve near the peak force is round, the force drop occurs within a finite time, the force levels for a moment before a new cycle, and the force stays always above zero. Only the force peak at the end of the extrusion phase and the constant peak force level differ from the form of the measured signal. This was seen also in Chapter 4.1.2.

The simulation result could not be this good unless Peyton’s graph would have some truth in it. The graph should not be taken as a measured ice property (which it originally is) but a correlation of two parameters reflecting the physical processes occurring in the ice during indentation. With this new interpretation its disputed origin does not count anymore, nor Määttänen’s incorrect assumption of the continuous crushing state of ice.
Optimizing the simulation result with Riaka's model is difficult, because several parameters affect the form of the signal. The basic parameters describing the ice flaking process are the ice crushing pressure, $p_c$, ice shear strength, $r$, and the cohesion angle, $\phi$. The angle of the flake, $\alpha$, is needed when examining the loading rate. It is calculated with the formula

$$\alpha = \frac{\theta + \phi}{2} \quad (24)$$

where $\theta$ is the angle of the edge to which the crack runs (90° for level ice sheet). The different angles are also shown in Figure 45.

![Figure 45. Definitions for the angle of the flake, $\alpha$, and the angle of the edge to which the crack runs, $\theta$. (Daley, 1991).](image)

The loading rate was found too low in the simulation done with the original parameters. The problem can be examined with the basic formula describing the contact force:

$$F = p_c D W \quad (25)$$

where $p_c$ is the ice crushing strength, $D$ is the width of the structure and $W$ is the height of the contact. As the two first parameters are constant, the loading rate depends only on the
rate of change of the contact height. This again is a function of the velocity of the ice sheet, $v$, and the flaking angle, $\alpha$. As formula 25 shows the flaking angle cannot be less than 45° for level ice sheet. However, according to the test results it should be less than that (see for example Figure 19, p. 36). Therefore the only way of correcting the loading rate is the change of the velocity.

Tuhkuri (1996) assumed in his interaction model that the loading rate is dependent only of the elastic response of the ice edge and got good results. While this holds for truly brittle ice it is not the case when the ice is warm. A simple calculation using the measured ice stiffness and the results of the test No. 17 shows that the assumption of the elastic ice edge response produces loading rates which are about a thousand times higher than the measured ones.

Daley (1991) has shown that the main parameter affecting the peak force level is the cohesion, $c$. Therefore it is used to obtain the correct peak force level. The number of the flaking levels has to be kept in mind for maintaining the variance of the force peaks. However, the second level flakes must not occur during the loading phase. This can be prevented by choosing an appropriate value for the ice crushing pressure, $p_c$. The linearity of the loading phase can be broken by lowering the stiffness of the structure. This gives more reality to the force signal. The best fit was obtained with the parameter values given in Table 13.

The modification of the parameter values has improved considerably the simulation result when compared with the one done with the original values (Figure 40). Both the loading rate and the peak force level are close to the measured ones. Second

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$v$ m/s</th>
<th>$\omega$ 1/s</th>
<th>$p_c$ MPa</th>
<th>$c$ MPa</th>
<th>$\phi$ deg.</th>
<th>$T$ °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>115</td>
<td>14</td>
<td>1.5</td>
<td>1.00</td>
<td>39</td>
<td>-2.1</td>
</tr>
</tbody>
</table>
level flakes do vary the peak forces but they do not exist during the loading phase. The lowered natural frequency, ω, allows greater displacements and this adds reality to the simulated force signal. The lack of the extrusion phase is the biggest drawback. The force starts to rise right after it has dropped. This shortens the duration of the simulated loading cycle and increases the apparent crushing frequency.

The spectra of the simulated force time histories of different models are given in Figures 46. They have not changed.

**Figure 46.** Spectra of the simulation results of the test No. 17 with modified parameters.
much when compared with the ones shown in Figures 4.3. The frequency peaks coincide with the measured ones. However, the spectra of the force time histories of Sodhi's and Määtänen's model are still too simple and the one of Riska's model contains too many harmonic components. These were seen already in Chapter 4.1.1 - 4.1.3.

4.2 Discussion

None of the models performed well in the simulations done with the parameter values taken straight from the tests or test conditions. In spite of differing theoretical background Sodhi's and Määtänen's models gave similar type results when considering the form and phases of the force cycles. Riska's model produced its own type of time histories.

Four major problems emerged in the results obtained with Sodhi's and Määtänen's model: too high loading rate, constant force peak level and extrusion force, and the existence of the separation phase regardless the change of the parameter values used. The common reason for all these is the oversimplification of the ice behaviour. Each problem is commented upon in the following.

Both models underestimated the effects of ice elasticity and permanent deformations. This led to too high loading rate and thus too high apparent crushing frequency. This was improved in the modified simulations by lowering the ice stiffness (Sodhi's model) or ice velocity (Määtänen's model).

The few needed ice parameters were constants in both models. This led to equal force cycles with constant peak and extrusion force. This was seen in the spectra of the force time histories as high peaks at the apparent crushing frequency and its multiples. In the measurements the force peak level varied due to the changing contact area. This was caused by the geometric coupling of the successive loading cycles. The measured load increase in the extrusion phase was also caused by the change of the ice geometry: when the ice was crushed the contact area and thus the load increased. Therefore the change of the contact area should be included in the models.
Damping was too low in the simulated calculations, which led to two phenomena: the transient vibration during the loading phase did not die down as fast as in the measured signal, and the structure swung too far in the spring-back phase the counter reaction taking it too much backwards separating the contact. The needed extra damping must be caused by ice since the damping of the structure is known from the calibrations. One explanation is given by Määttänen (1974, p.24). According to him the energy required to crush and grind the ice and push the ice pieces away as well as friction between ice and the structure cause positive damping. Kärnä (1992), on the other hand, warns about the radiation damping effects in the case of semi-infinite ice sheet.

Riskka's model is based on ice flaking mechanism. This represents a realistic ice behaviour. The change of the contact area leads to varying force which is in accordance with the measurements. The ice sheet elastic deformation is also included in the model. However, the restrictive and naive assumptions made in the ice contact model oversimplify the situation. This leads to non-realistic looking force time histories.

The flaking angle, $\alpha$, was found to be the reason for the too low loading rate in the simulations. The assumption of linear macrocrack and halved contact height after the flake formation leads to another problem. In the laboratory tests the macrocracks were found to be curvy and the contact height small. This way the wedge which is formed by the macrocracks may be smaller than the one created by Daley's model. Therefore Daley's model overestimates the resistance of the force in the spring-back phase of the force cycle. This prevents the indenter to surge forward far enough. This again leads to too small penetration into the ice. Therefore the indenter does not pack and no gap is formed. Kujala (1994) has studied this problem and suggested spiral form for the modelled macrocracks.

Taking the needed parameter values straight from the tests or test conditions did not produce good simulation results. The reason for this does not necessarily lie in the models, but in a problem of measuring ice properties. Ice stiffness was measured only once during the whole test series. Ice crushing pressure was not measured in situ but afterwards in a separate
test. Daley’s contact model needed two parameters which were calculated using semiempirical formulae. Big scatter in the ice property measurements adds to the difficulty. Therefore it is impossible to say what are the true values of the parameters in a single test and simulating the test even with a good interaction model is more or less an educated guess.

The conditions of the test series presented in Chapter 2 are far from the complexity of the real world with wide MDOF structures (leading to non-simultaneous ice deformation) vibrating with all natural frequencies and varying ice conditions. Therefore the validity of the test series can be argued. However, the test conditions were chosen to be as simple as possible in order to reveal the basic processes occurring during the interaction. Therefore the use of the SDOF structure is justified. This also led to a simple test structure, and the test results were simpler to collect, analyze and interpret.

The simple conditions of the test series created a problem in the simulations. Even though both Määttänen’s and Riska’s models are meant for MDOF structures they were used to simulate the behaviour of an SDOF structure. Many of the properties of the models were not used at all; quite to the contrary they were discarded. In spite of that both models showed their flexibility performing well in the optimized simulation.
Present day ice-structure interaction research focuses on finding physical phenomena occurring at the contact. Many test series have been conducted which have utilized intensive visual observation techniques and revealed important details of the interaction. This has led to new ways of modelling the process. However, due to the complexity of the ice only first steps have been taken in using this approach.

This work concentrates on examining the contact between the edge of an ice sheet and a vertical indenter face during intermittent crushing process. The results of a laboratory indentation test series are analyzed and a description of the physical phenomena occurring at the contact is given. The existing dynamic ice-structure interaction models are reviewed. The ability of three models to simulate representative test results is evaluated.

The peak force level in the tests was affected by the velocity and the stiffness of the structure. The latter one also changed the form of the loading cycles, especially the spring-back phase. With the stiff structure it hardly existed, and the force stayed above zero at all times. When the stiffness was intermediate, the energy stored in the structure made the structure to surge leading to a low extrusion force and a subsequent loss of contact. The spring-back phase dominated the whole cycle when the structure was very flexible. The structure advanced a long way with a high velocity and the ice flaked several times during the surge.

The ice contact process was studied in the tests with pressure measurements and visual observations. The pressure distribution was found uneven varying in space and time in a complex way. The highest pressures were measured at the centerline of the ice face. This supports the visual observation that the contact was line-like.

The ice failed in flaking. The flakes were formed by different size macrocracks which separated the flakes from the parent ice. The pressure measurements showed that this occurred at different times in different places. In addition, the force peaks were round. These observations suggest that the load drop
was not simultaneous across the whole contact face. However, the whole ice face was crushed during the load drop. Thus the situation can be called apparently simultaneous.

The ice crushing process contains a geometric coupling. This means that the spall created by the former force cycle gives dimensions to the ice wedge contact area to be loaded. This affects the peak force level of the following cycle. The changing contact area is also important during the extrusion phase. The surging indenter crushes the ice wedge created by the macrocracks. The contact area increases as does the contact force.

The results of three representative tests were simulated using three different type dynamic ice-structure interaction models, namely Sodhi's, Määtänen's, and Riska's. The needed parameter values were obtained straight from the tests or test conditions. None of the models performed well. This was mainly due to oversimplification of ice behaviour. However, when the parameter values were modified, the match improved considerably.

Sodhi's model was found too simple and valid in too narrow parameter space for any practical use. Määtänen's model performed quite well in spite of the fact that Peyton's graph which was used to define the ice properties is 30 years old and obtained with Alaskan, not Baltic ice. When Peyton's graph was modified the simulated signal became almost identical with the measured one. This shows the fundamental nature of the ice loading rate to the interaction process. Riska's model is based on ice flaking mechanism, which was found to represent realistic ice behaviour. However, it suffered from the naive assumptions made in Daley's contact model. Especially the macrocrack angle limited the loading rate too much and this led to non-realistic looking force time histories.

Further development of the models should be based on the found physical processes of the ice-structure interaction. The ice crushing pressure dependence of the loading rate seems an interesting field of research. The macrocracks limiting the peak force level open another problem area worth studying. If these two questions could be united it would open new views to the whole ice-structure interaction research.
REFERENCES


