LOCATING ARcing FAULTS USING
RADIO MEASUREMENT METHOD

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Acknowledgments

My humble ambition is always to serve the humanity as a whole. Thus, all the actions that I do, must serve to the maximum number of people. Here is what motivated my choice of studying electricity to be an engineer in the field. According to me, the electricity is the vital hub of our modern society because of its high demand in all sectors of human activities. My dream was then to pursue such a study in a well known university in the world, and the Being of Light comes to the aid of his faithful children, thus the Helsinki University of Technology (TKK- HUT) has opened its doors to me. It was real joy when they announced that I was allowed entrance examination. After five (5) years of hard work, shared my studies, my family and the sport activities since am an professional athlete in the national karate team of Cote d'Ivoire, despite the diversity of my activities, I was able to concentrate on my engineer career with the remarkable support of my lovely wife, Laakso Karoliina Essi (Essi Zoko Ble). To whom I reiterate also my sincere love. We have three (3) beautiful children together. These 5 years of intense studies have been rewarded by the diploma of electrical engineer (M. Sc), which end this current study by the present thesis. I take this opportunity to say thank you to Professor Matti Lehtonen (Head of Department), who gave me the honor to lead this wonderful research study. He really fought for hard in order to be granted with a scholarship throughout the project. He is a very generous and remarkably full of affection. I thank Professor PJ Moore of University of Bath in England who provides me with a very good documentation in close connection with my thesis. I would like to thank also all researchers of the Department of Electrical and Communications Engineering, Power System and High Voltage Engineering for all their support and collaboration regarding this work. Especially researchers like Elkalashy Nagy Ibrahim, Abdel Salam, Jari Hällström and John Mullar, who assisted
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The electrical distribution and transmission are increasingly subject to many faults on electrical lines. This also calls in to question the quality of energy. The most recurring power electric faults are those emitting electromagnetic radiations. The aim of this thesis is to seek for the location of these power arcs. Such faults inject excess current into the network. The problem is that the power arcs are undetectable by the mean of traditional detection devices at beginning since they induce small current which are below the relay setup current for automatic triggering disconnection. The induced current will grow as time increases until the relays trip, by then the network is inevitably damaged. In order to avoid such kind of situation, it is important to develop a new approach to locate these electric arcs at their earlier stage. In this thesis, a laboratory experiment was conducted in order to study all the information regarding the captured signals by the sensors. Several sets of faults were simulated by a spark generator and the electromagnetic radiations have been captured by four (4) radio antennas placed at different locations around the spark source. The statistic analysis and some signal processing methods were explored in order to extract an essential algorithm for the location of this particular type of electric fault. All the methods of analysis have not been exploited, then the initiative has been taken to further our research in future by examining other methods in order to improve the location algorithm accuracy.

Keywords: Antenna, power system, transmission line, distribution network, arcing fault, cross-correlation, wavelet.
Les réseaux de distribution et de transmission sont de plus en plus sujets à de nombreux défauts sur les lignes électriques appelés arcs électriques. Ces arcs émettent des radiations électromagnétiques qui affectent la qualité de l'énergie. L’objectif de ce mémoire est de localiser la position précise de ces arcs électriques car ils injectent un excès de courant électrique dans le réseau qui n’est malheureusement pas détectable par le moyen du dispositif de détection classique, puisque au-dessous de la valeur nominale captée par les relais de détection dans l’ordre de quelques ampères. Ces courants induits vont croître progressivement jusqu’au signalement d’une surtension par les différents relais de contrôle qui vont alors actionner une disconnectin automatiqne. Pour éviter que le réseau électrique soit endommagé avant l’action des relais, il serait mieux de développer une nouvelle approche de détection de ces arcs électriques immédiatement après leur apparition. Dans ce mémoire, une expérience-labo a été menée pour étudier les informations du signal emis par les arcs. Ces signaux sont simulés par un générateur d’étincelles dont les radiations électromagnétiques seront détectées par quatre (4) antennes placées à autour de ce générateur. L’analyse statistique et les méthodes de traitement des signaux ont permis de définir un algorithme de localisation des arcs électriques. Toutes les méthodes d’analyse n’ayant pas été exploitées, alors l’initiative a été prise pour poursuivre nos recherches dans l’avenir par l’examen d’autres algorithmes de localisation des arcs électriques.

Mots-clés: antenne, système électrique, ligne de transmission, réseau de distribution, arcs électriques, la corrélation croisée, et les ondelettes.

Hakusanat: Antenni, sähköjärjestelmä, siirtojohto, jakeluverkko, arcing faults, ristikorrelaatio, wavelet.
List of symbols and abbreviations

- RM ............................................ Radio Measurement
- RF ............................................. Radio Frequency
- PA ................................................ Power Arcing
- SG ................................................ Spark gaps
- XCORR ....................................... Cross Correlation
- VLF ............................................ Very Low Frequency
- UHF ........................................... Ultra High Frequency
- CFO ........................................... Critical Flash-Over
- ARI ............................................ Arc Re-Ignition
- ARF ........................................... Arc Re-strike Frequency
- RA ............................................. Radio Antenna
- TDOA ........................................ Time-Difference Of Arrival
- TOA ........................................... Time Of Arrival
- SFERIC ....................................... atmospheric Signal
- DCC ............................................ Direct Cross Correlation
- H .............................................. Horizontal
- V ............................................... Vertical
- ° ........................................ Degree(angle)
• A .......................................................... Antenna
• d ........................................................... Distance
• m ............................................................. Meter
• s ............................................................. second
• cm ...................................................... centimeter
• τ .......................................................... Time delay
• E .......................................................... Voltage from the source
• U ......................................................... Capacitive Voltage
• R ........................................................... resistor
• C ........................................................... capacitor
• c ................................... Speed of light in empty medium
• DA ............................................. Directional Antenna
• SCADA ...................... Supervisory Control And Data Acquisition
• ID ............................................. Identification number
• LOS .................................................. line-of-sight
• PLo ..................................................... Pathloss
• $E_{rad}$ ........................................... Electromagnetic Field radiation
• WRAF ........................................... Wide Range Air Fuel
• Equation .............................................. Eq
1 Introduction

The Power industry is initiating many projects in order to improve the reliability of the electric distribution system, but many problems like disturbances caused by power electric faults such as power arcing and flash overs of spark gaps are affecting the power quality. These arcs produce electromagnetic waves which can be detected by radio measurement (RM) that utilizes radio frequency (RF) pulses emitted from faulty components as monitoring parameter. Many investigations have been done using RM for detection of partial discharges and lightning. Such detection method using directional finding antenna to locate power arcs is not well known but it seems to be very interesting and useful tool in power electric industry. In this project a measurement circuit is designed in order to locate power arcs using four antennas. The characterization of the signal obtained will be done by conventional analysis using appropriate mathematical tools. The fault types considered are single phase to ground faults involving power arc which is supposed to be a wide spectrum radio transmitter (compare to Marconi’s first radio), the spectrum of which could be measured from distance with an antenna. The goal of this thesis is to find, if it is possible to develop a fault location system based on directional antennas (DA), which are able to define the direction where the fault exists. The direction could be possibly defined by measuring the radio disturbances coming from arcs using four DA and comparing their measurements. If several radio measurement stations are located around the network, the actual fault location can be found in the place where measured directions intersect.

1.1 Outline

- Chapter 2 : Characteristics of power arcing faults In this chapter, the effects of arcing faults on power lines are described and some good
examples are presented.

- Chapter 3: Arcing Fault detection on Overhead Line This chapter explains the techniques developed to detect arcing fault.

- Chapter 4: Causes of Power Arcing fault The conditions such as causes and types of power arcing are discussed here.

- Chapter 5: Ionization theory It is well know fact that arcing fault produces electromagnetic wave therefore we devote this chapter in order to explain the arcing radiation phenomenon.

- Chapter 6: Radio measurements methods This chapter discuss all the theory about radio measurement method.

- Chapter 7: Review of Antenna design Four antennas were build to capture the radiations emitted by the power arcs, the construction of such antennas is described in this chapter.

- Chapter 8: Laboratory test All the steps followed in the laboratory regarding the radio measurement experiment is presented in this chapter, the characterization and the study of the signals captured is done in this chapter, using accurate and useful statistical analysis to locate the exact fault.

- Chapter 9: Signal processing analysis In this chapter we develop the different tools used for the fault location techniques. Four typical algorithms are explored to predict accurately the signal emission source.

2 Characteristics of arcing faults

The Medium voltage distribution networks are subject to environmental stresses, where many obstacles like lightning strikes on lines cause outages[1].
For economical reason, the utility is more concerned by a shortest possible clearing time of the fault, of course if this fault can be located precisely to avoid unexpected hunting on the power lines. Among many types of fault on the distribution lines, for this specific project we are more concerned in Power Arcing (PA) and Spark Gaps (SG) which are the major disturbance factors on power system lines.

2.1 Power arcs and Spark gaps

An arc occurs when electricity flows through space, and lightning is a good example. Sometime referred as “sferics” [2], arcing occurs when electricity follows the path of damaged insulation, using surrounding dirt, debris and moisture as the conductive medium, or more often the electricity escapes in the high voltage power transmission lines and jumps across a gap in an electrical connection, and generates radio signals. Spark gaps are radio noise transmitters with short range and broad spectrum coverage signals. [3] In the presence of the arcs, high current may be induced at the faulty section that causes many damages in the system because of fault current high temperature. The magnitude of an arc fault current is limited by the resistance of the arc and the impedance of the ground return path. This low level of fault current is often insufficient to immediately trip overcurrent devices installed within the vicinity of the arc fault, resulting in the growth of the arcing fault that increases the system damage. To conclude, it is important to say that an arc fault which is a high power discharge of electricity between two or more conductors is initiated in three ways:

- through spark discharge
- physical contact and
- glow-to-arc transition
2.2 Classification of arcing fault

Taking in account the duration time, the power arcing faults are classified as transient, semi-permanent and permanent. Transient and semi-permanent faults are very fast since they last up to a few seconds with less damage to power network. Usually the traditional methods like circuit breaker automatically clear these fast event faults and then supply restoration by re-closure of the circuit breaker, this process is known as auto-re-closure.[4] The permanent faults last for more than a few seconds and can cause considerable damage to power system plant, after attempted auto-re-closure onto the persisting fault the electricity supply is permanently disconnected until reparation takes place. Figure(1) shows typical examples of electrical power arcs on the transmission and distribution lines. One major characteristic of arcing fault currents is that they are discontinuous, nonlinear, non sinusoidal and are limited by the ground return path impedance and the arc voltage. Two main models developed by researchers are important to be defined, such as Kaufman and Page who employed an instantaneous arc current model: this model stipulates that an arc current begins to flow when the supply voltage provides the necessary re-strike potential. The arc current flows continuously until the stored magnetic energy associated with the systems inductance had been dissipated. The arc current remains zero, until the magnitude of the supply voltage equals the re-strike voltage; at this point, the arc re-strikes with the opposite polarity. Protective devices are usually not very sensitive to parallel arcing fault since their current magnitude are less than that of normal operating current of the system being protected. There are two major approaches to arcing fault model representation: arcing fault model for conventional power systems and arcing fault model for circuit breaker interruption of naval and aerospace systems. But in this thesis, we are more concerned with an electromagnetic radiation model on which the experiment is based on.
2.3 Power systems arcing fault model

Several models of arc fault modeling equations have evolved over time for representing behavior of arcing fault in power system operation. However, the instantaneous arcing fault current model better presents the failure physics of the arc. The instantaneous arc current model, relying on inductive circuit, is based on energy balance and incorporates a flat-topped arc voltage. The arc current in the model is described by Eq(1):

\[ i_{arc} = \int_{t_a}^{t} (V_{max} \cos(wt) - V_{arc}) dt \]  

where

\[ t_a = \frac{1}{w} racsin\left(\frac{V_{strike}}{V_{max}}\right) \]

As a variant of the instantaneous arc current model, a differential equation based arcing fault model can be easily derived in Eq(2):

\[ V_{max} \cos(wt) = i_{arc} R + L \frac{di_{arc}}{dt} + V_{arc} \]  

Then, a variant of the classic differential equation model is given below following series of experiments conducted by the researchers Eq(3).

\[ V_{max} \sin(wt) = i_{arc} R + L \frac{di_{arc}}{dt} + (20 + 534g) i_{arc}^k \]  

where \( g \) is arc conductance, and the constant \( k \) is related to luminance of the arc which is in the range of \( 0.12 \leq k \leq 0.5 \) depending on the arcing medium, the voltage and current levels at which the arc occurs. These equations above are based on free-air arcs generated between both copper and aluminum electrodes.
Figure 1: Arcs [5]
2.4 Arc models for switching system

Different types of arc modeling for circuit breaker interruption have evolved over the years. The primary models are Mayr and Cassie models which relate arc conductance with arc voltage and arc time. The two models are almost identical except that one relates arc conductance with arc voltage over arc power, and the other arc conductance with arc voltage over fixed arc voltage. Both types of arc representation have a common property that the arc current depends largely on the magnitude of the line impedance (X/R ratio) and the arc gap length. The mathematical models of Cassie and Mayr arc shown in Eq(4) and Eq(5): [5]

Mayr arc model:

\[
\frac{1}{g} \frac{dg}{dt} = \frac{d\ln(g)}{dt} = \frac{1}{\tau} (\frac{ui}{p} - 1)
\]  

Cassie arc model:

\[
\frac{1}{g} \frac{dg}{dt} = \frac{d\ln(g)}{dt} = \frac{1}{\tau} (\frac{u^2}{U_c^2} - 1)
\]

where:

- \(g\) is an arc conductance
- \(u\) is the arc voltage
- \(i\) is an arc current
- \(\tau\) is an arc time constant
- \(P\) is a cooling power of arc (arc loss constant)
- \(U_c\) is a constant arc voltage

2.5 Electromagnetic Energy Radiation Model

Basically the two models of Mayr and Cassie describe arcing faults in terms of voltages and currents. In this Thesis the model for detection of arcing
faults relies on the electromagnetic energy radiated from an arcing source, since arcing faults are accompanied by radiation in the form of heat, sound and electromagnetic waves. Rompe and Wiesel conductivity law in physics provides the temporal evolution of the conductivity during the electronic multiplication process. This conductivity law associated with a propagation equation gives the electric field value of each point of the transient arc zone and leads to the space-time evolution of current and potential (voltage). The calculation of the current variation thus leads to the determination of intensity and spectral content of the radiated electric field associated with the transient arc. Mathematically put, the general expression for the electromagnetic field radiation $E_{rad}$ propagated at a distance $R$ by a current component of length $\delta$ oriented along the x-axis is given in Eq(6)[7]

$$E_{rad}(R, t) = \frac{\sin(\theta)}{4\pi\varepsilon_0 R c^2} \int_0^\delta \frac{di}{dt} dx$$

Eq(6)

Where $\theta$ is the angle between current direction and the $R$ vector and $c$ is the velocity of light. The complete variation of current on the element of length $\delta$ can be written in Eq(7) as:

$$\frac{di}{dt} = \frac{\delta i}{\delta t} + \frac{\delta i}{\delta x} \frac{\delta x}{\delta t}$$

Eq(7)

During the formation of the transient arc, the deformation of the initial potential pattern leads to partial variation of $\frac{\delta i}{\delta t}$ in the discharge frames. Under steady state propagation conditions the radiated electric field $E_{rad}$ becomes in Eq(8) and Eq(9):

$$E_{rad}(R, t) = \frac{\sin(\theta)}{4\pi\varepsilon_0 R c^2} \int_0^\delta \frac{\delta i}{\delta x} \frac{\delta x}{\delta t} dx$$

Eq(8)

which reduces to
\[ E_{rad}(\mathbf{R}, t) = \frac{\sin(\theta)}{4 \pi \epsilon_0 R c^2} \nu \Delta i \] (9)

where \( \nu \) is the steady state propagation speed of the discharge and

\[ \Delta i = i_{max} - i_o \] (10)

, where \( i_o \) is the current associated with the primary breakdown phase before the development of the transient arc and \( i_{max} \) is the peak current at the beginning of the propagation. The discharge and the short gap breakdowns at atmospheric pressure shows a current phase characterized by a fast rising front. There are two phases: the transient arc phase which takes place in the breakdown mechanisms and the development of the conductive channel between the electrodes. This strong ionization phase lasting from 5 to 10 ns is the source of the VHF and UHF radiation emitted during the different phases of natural lightning and the conducted arcing experiments. Surveys have shown that the most common type of fault on power systems is the single phase to earth fault. Overhead line faults are 90% predominantly of this kind, and 90% of single phase to ground faults are arcing fault[5].

The air is the main insulation medium of the overhead transmission lines. Therefore the fault path often contains an arc which bridges part of the air insulation gap when breakdown occurs. The arc in the fault path is preceded by an avalanche ionization at arc ignition; this causes non-linear currents to circulate in the fault path. These non-linear currents occur at frequencies in the Very Low Frequency (VLF) band up to the Ultra High Frequency (UHF) band, which cause any matched antenna like structures connected to or forming part of the current path to radiate electromagnetic fields. This mechanism is shown in Figure(2). Since the power system arcing fault induced electromagnetic waves, it is important to introduce a new fault detection and location technology based on the monitoring and data logging.
Figure 2: Arcing wave propagation [7]

of these events. This technique has many advantageous properties for an arcing fault detection and location system because their implementation in the power plant is very cheap and easy to develop. [7] The advantage is that the system does not require a direct connection to the power system for each overhead line monitored; the main advantage of using VHF radiation as the basis for a distribution network overhead line monitoring system is that the relatively low current levels of arcing faults on distribution networks can produce significant radiation in the VHF region of the electromagnetic spectrum. Further advantages are the relatively small reception antenna sizes that will allow portability for experimentation purposes and the development of compact monitoring system equipment.
2.6 Propagation mechanism

The free space path loss with wavelength $\lambda$ at distance $d_o$ in the far-field of the antenna with gains $G_1$ and $G_2$ is given by Eq(11)[7]

$$PL_o(d_o) = -G_1 - G_2 + 20 \cdot \left(\frac{4\pi d_o}{\lambda}\right)$$  \hspace{1cm} (11)

Eq(11) can be used for separate rays, when the first Fresnel zone is free from obstructions. Sometime if the wave is regarded as a plane wave, then the field in point $r$ is given by Eq(12)

$$E(r) = E_o \cdot e^{-jkr}$$  \hspace{1cm} (12)

where $k$ is the wave vector. The distribution of the phase in the radiated field of an antenna is called the phase pattern of the antenna. The phase is relative quantity and it is defined in relation to a phase reference, such as a separate reference signal. A single-frequency field component of the angular frequency $\omega$ can be presented as scalar function of time in Eq(13) as:

$$E(t) = E_o \cos(\omega t + \psi_o) = Re \cdot E_o e^{j\omega t}$$  \hspace{1cm} (13)

Where $E_o$ is the amplitude of the field and $\psi_o$ is the phase at $t = 0$. The phase $\psi$ is the angle of the complex number $E_o e^{j(\omega t + \psi_o)}$. If the field $E$ propagates along $x$-axis with the velocity $\nu = \omega / k$ ($k$ is the wave number) then the field at point $x$ at time $t$ is as shown in Eq(14)

$$E(t) = E_o \cos(\omega t + \psi_o - kx)$$ \hspace{1cm} (14)

At point $x$, $E(t, x)$ is represented by $E(t, 0)$ shifted in the positive direction by $x/\nu$. This is considered as a phase delay as the phase is decreased by $kx$ in the propagation from 0 to $x$. The complex time factor $e^{j\omega t}$ can be written
as $e^{-j\omega t}$. For positive traveling wave the phase factor $e^{-jkr}$ becomes $e^{jkr}$ and the phase is increased by the amount $kx$ in the direction of propagation.

### 2.6.1 Refraction

The energy can propagate behind obstacles through both refraction and diffraction. These two phenomena can be evaluated with Snell’s law that refraction behind $90^\circ$ corners requires $\epsilon_r < 2$.

### 2.6.2 Reflection

The dielectric medium has a relative dielectric permittivity $\epsilon_r = \epsilon'_r - j\epsilon''_r$. Fresnel reflection coefficient $r_\perp$ and $r_{II}$ for perpendicular and parallel polarizations, respectively.

### 2.6.3 Transmission

For layer with thickness ($d$) of homogeneous dielectric medium the field transmission coefficient is is shown in Eq(15)

$$T = \frac{(1 - r^2) \cdot e^{-j\delta}}{1 - r^2 e^{-j2\delta}}$$

(15)

Where $r$ can be either parallel or perpendicular field reflection coefficient and,

$$\delta = \frac{2\pi d(\sqrt{\epsilon_r})}{\lambda \cdot \cos(\phi_2)}$$

(16)

Where $\phi_2$ is the direction of propagation inside the dielectric layer with respect of the normal of the boundary. The real part of $\delta$ is the electrical length of the path of the wave inside of the dielectric medium. The field reflection coefficient inside a layer, see Eq(17)
\[ R = \frac{r(1 - e^{2j\delta})}{1 - r^2e^{-j2\delta}} \] (17)

### 2.6.4 Diffraction

Diffraction is analyzed according to Huygens principle, which states that propagation of a wave can be calculated from equivalent sources. The Fresnel-Kirchhoff theory gives the scalar value of the field behind an ideally conducting knife-edge.[7]

### 3 Arcing Fault detection on Overhead Line

In the proposed monitoring system the existence of breakdowns in overhead line insulation due to arcing faults is exposed by detecting and locating the origin of the arc induced sferic radiation; this system is principally aimed to provide fault information for overhead distribution networks see Figure(3). The most important feature of any potential solution is its functionality, the equipment must be able to accurately detect and locate arcing faults. The factors of the equipment functionality are:

- Good reliability of the system
- Compatibility with other information systems already in use
- Low equipment cost
- Simple, quick, low cost installation
- Low maintenance requirements

The functionality specification of the potential application of an arcing fault location system for overhead line distribution networks are:
• **Location Tolerance**: A useful fault location tolerance for operational purposes would be to locate the fault within an acceptable distance square of the actual fault location.

• **Fault Location Information Availability**: The system should be able to calculate the fault location to the given tolerance within a few minutes of the fault event. This will allow rapid dispatch of repair teams and engineers to the fault location.

• **Network Coverage**: The system should provide coverage primarily for the 11 kV networks. Coverage of other networks would be useful but is not as critical.

• **Sensitivity**: A high percentage of arcing faults must be detected even if sensitive detection equipment reduces discrimination from other events because the events can be correlated with protection operation information provided by existing SCADA systems to confirm their validity.

• **Fault Type Discrimination**: The discrimination of the fault type, i.e., how many phases are involved and the existence of an earth path is not a priority but would be useful for identifying faults in the field if it is not expensive to implement. This specification forms the basis for the development of the equipment for power system arcing fault location based on VHF radio wave propagation.[7]

4 Causes of arcing faults

A power system arc occurs when the insulation medium between two electrodes of unequal potential breaks down, the arc constitutes an ionized cur-
rent path between the electrodes. The power system arcs can be classified as either arcs that occur during switching operations or arcs that occur due to a fault on the electricity supply network.[7]

### 4.1 switching system induces arcing

A switching arc occurs when the operation of switchgear causes two contacts to either separate or combine enabling the disconnection or connection of the power supply. These arcs are necessary for power systems operation to reconfigure a network or to clear a fault; this means that switching arcs are intentional and can therefore be partially controlled.[8, 9]

### 4.2 Arcing fault on overhead line

A fault arc is an unwanted current path between an item of energized plant and an object at a different potential. These events occur due to breakdown of system insulation in the following circumstances:[4, 11]
1. **Overvoltages** on the system caused by: Lightning strikes (direct strike or strike near to line inducing over-voltages) and Switching operations

2. **Insulation failure**: Insulation electrical strength deterioration (due to imperfections), pollution of insulator surface from salt and chemical and ice deposits

3. **Mechanical** compromise of the insulation: Conductor clashing due to high velocity winds, tree growth into lines, wind driven objects falling onto line, mechanical failure of line support structure (due to high winds, weathering), mechanical failure of line conductor (due to ice build up, weathering) and vandalism

4. **Human error** by authorized personnel working on network (rare):

   Failure to remove equipment from line and incorrect operational procedure.

5. **Ionization theory**

   When the Critical Flash-Over (CFO) voltage has been surpassed the potential gradient in the air gap accelerates any available free electrons, increasing their kinetic energy to a high value, freeing more electrons in the process. There is now an increased population of both positively charged gas molecules and negatively charged electrons, these ions are accelerated in the electric field toward the cathode and anode respectively. The increased population of accelerated free electrons in the gas cloud cause further ionization and free more electrons; this cycle leads to an avalanche effect that eventually enables the ionized gas cloud to act as a conductor with a high resistance value. This process is known as avalanche ionization breakdown.\[11, 12\] For a typical power system fault arc a stable conducting current path does not form immediately after the first avalanche ionization breakdown, instead a number
of momentary breakdowns occur, known as sparks, before a conducting arc is ignited.

5.1 Critical Flash-Over (CFO)

The magnitude of the CFO voltage has been extensively researched; the most simplified definition is given in terms of the gap length $d$ and electrode shape configuration coefficient $k$ in Eq(18). This relationship is based on results for transmission system arcs. It should be noted that this gives the CFO voltages for a constant set of standard atmospheric, weather conditions and electrode materials.[12, 13]

$$V_{cfo} = 500 \times k \times d^{1.6} (kV)$$  \hspace{1cm} (18)

The Arc Re-Ignition (ARI) voltage is affected by the same processes as the first arc ignition, however the after effects of the previous arcing make re-ignition more favorable. These after effects increase electron population in the insulation medium available for avalanche ionization breakdown; the ARI voltage is therefore lower than the CFO voltage.[14]

5.2 Arcing fault ignition evaluation

The CFO voltage is dependent on the atmospheric conditions of air temperature, pressure and humidity, these affect the population of ionized particles available for avalanche ionization breakdown[12]. The condition of the electrode is affected by the build-up of surface contaminants such as water or pollution that significantly reduce the CFO voltage, increasing the chances of arc formation.[15] Certain weather conditions can affect the arc length and path. The most common example is wind where the ionized arc products can be dispersed between arc ignitions creating a winding arc path, in strong
5.3 Characteristics of power arcing voltage and current

The arcing fault can be classified by their electrical properties, to aid this approach the representation of a fault arc can be simplified to a conducting condition and a non-conducting condition; this method, when applied to a single phase to earth fault with the line and earth fault path impedance included, is shown in Figure(4).[16]

Figure (4 A) shows the normal system condition with no arc present, there is no current flowing in the fault circuit, in this case the system voltage appears across the insulating gap. When the arc is conducting in Figure (4 B) it is characterized by the following attributes:[4, 13, 18]

- Near square wave voltage (due to constant positive and negative half cycle arc voltage $e_{arc}$)
• Near sinusoidal arcing current (with a current discontinuity between each half cycle arc)

• Dynamic arcing resistance, conductance

These characteristics of the arc are all dependent on the change in the conduction properties of the insulation medium due to the arc ignition. Plots for the per-unit (Pu) voltages and current waveforms for the model developed in Figure(4) are shown in Figure(5)[15]. Figure(5) shows that the time taken for the voltage across the air gap to reach VCFO, the Critical Flash-Over voltage at time \( t_1 \) is directly dependent on the system voltage. At \( t_1 \) the arc ignites and the voltage across the air gap now falls to \( e_{arc} \) for the period of the current conduction through the arc column. The current rises until the system voltage system falls below the arc voltage level, this occurs at \( t_2 \). The current flowing through the arc column now reduces, conduction is still maintained after the voltage zero crossing since the circuit is dominated by the inductive line impedance \( Z_{line} \), the current flow finally ceases after the current zero that occurs at time \( t_3 \). The duration and magnitude of current flow are directly proportional to \( e_{arc} \) since energy is stored in the inductance while \( e_{system} > e_{arc} \), and is represented by area \( A \), this maintains the current flow after the voltage zero until all of the energy is discharged with the equal area criterion of \( A = B \) applying. The cycle then begins again but for a negative half cycle.[12] During this sequence of events the arc exhibits a switching characteristic, extinguishing and re-striking at every current zero at the Arc Re-strike Frequency (ARF). If the fault circuit contains a higher resistance then the less current will flow and less energy will be stored in the circuit inductance.
6 Radio measurements methods

As mentioned earlier, RF measurement consist of using Radio Antenna (RA) to detect any particular signal that has to be analyzed in order to find the precise location of the signal emission source. In this method the flight time difference will be needed to evaluate the actual location of the arcing fault, using four directional antennas. The calculation of the time-difference of the arrival (TDOA) between antenna signals will be explored. Beside the propagation path phenomenon, one of the major problems in the detection estimation is to define the exact start-time of the RF signal from each measuring antenna. It is very important to determine accurately that start-time since it plays a critical role within any location algorithm used. This measurement method needs at least four antennas with the relative coordinates of each antenna known, in order to produce an accurate result.[14] See Figure(6a) for the position of the antennas around the arcing source.
6.1 Advantage of radio measurement

In conventional fault location systems, the major drawback is the need for a direct connection to every line being monitored; since the cost of installing these systems on the entire network would be very high, these systems have not been widely implemented. Since arcing induced sferic, fault detection and location requires no direct connection to overhead line plant, it can monitor a geographic area and is therefore comparatively inexpensive. This lack of direct connection also allows the plant to be retrofitted without the need to disconnect any electricity network supply. The development of the sferic radiation monitoring system has many of the positive attributes of a modern digital measurement system. These system attributes include:

- Accuracy, speed and flexibility of digital sampling and processing
- Modular design with separate outstations allowing for system expansion
- Modular design of monitoring station equipment allowing for easy maintenance and upgrade
- Stand alone capability of outstations (no reliance on operation of a single master station)
- Distributed processing at each condition monitoring station to reduce communications.[11]

6.2 Experimental system

The components in Figure(6a) were used for the experiment:

- Power cable
- Directional finding antenna
The experiment consists of simulation of a spark gap fault which will propagate spherically RF wave signal. Four directional finding antennas situated on different coordinates scale around the source in order to detect the emitted waves by the faulty point. Logically the crossed section point of these four direction antennas’ paths will establish the actual location of the fault position. These antennas are bi-conical and their covering frequency range is set between 20 $MHz$ to 300 $MHz$ since most of the RF pulses in the distribution system are detected by the broadband characteristic of the antennas. The lower limit of 20 $MHz$ will eliminate the surrounding interferences, which are often low frequency signals.
6.3 Physical application of antennas

The RF direction finding antenna method will be more suitable for the distribution network, see Figure(6b). The objective is to design antenna that can be capable to detect any arcing or spark gap fault at a distance of 10 Km. The antenna will be positioned on the pole just a bit under the power cables. Any hazard caused by power arcs is immediately detected by the antenna which on other hand send the information to the control room. Each detector will have a specific identification number (ID). by calling to the control board we will exactly know the area of the faulty section. This way the fault hunting time will be reduced to $\frac{1}{10}$ of its normal value.

7 Review of Antenna design

Based on the characteristics of the arcing faults source of electromagnetic wave propagation, it is important to design a proper antenna capable of detecting radio frequencies generated by these faults. From a single piece of wire, we made a one-turn loop with two ends sticking out from the plane of loop, each is a $\frac{1}{4}$ wavelength long. One diode will be used to rectify the microwave developed across the loop. The loop will consist of providing mechanical stability for the antenna and also it will act as a one-turn inductor in order to prevent low frequency interferences.[11]

7.1 Building the antenna

These are the steps followed for the construction of our directional antenna:

1. Take the insulation off on a piece of a wire

2. Wrap it around cylindrical tube to form a loop of about 11 : 5 mm in diameter in the middle of the wire.
3. Bend the ends of the wire so that the plane of the loop is perpendicular to the line form by the two ends of the wire, see Figure(7)

4. Make the bend as closer as possible to the loop.

5. Trim the length of each half of the antenna to 31 \text{mm} as measured from the plane of the loop

6. The overall length is about 75 \text{mm}

7. Solder the connector to the antenna, placing a small resistance plate in between

8. Solder the antenna to one end of the Bat 86 diode to the and the other end to the connector

9. Solder the other end of the diode to the remaining wire from the loop

10. cover on the side the antenna by adjusting a half plane plate reflector vertically.

7.2 Antenna theory

- Every structure carrying RF current generates an electromagnetic field and can radiate RF power to some extent.

- A transmitting antenna transforms the Radio Frequency (RF) energy produced by a radio transmitter into an electromagnetic field that is radiated through space.

- A receiving antenna transforms the electromagnetic field into RF energy that is delivered to a radio receiver.
(a) Antenna components

(b) Completed antenna design

Figure 7: Antenna design.
Most practical antennas are divided in two basic classifications:

1. Marconi Antennas (quarter-wave, which is the Monopole and derivate).
2. Hertz Antennas (half-wave, which is the Dipole and derivate). [11, 16]

7.2.1 Radiated Fields

When RF power is delivered to an antenna, two fields evolve. One is an induction field or near-field, which is associated with the stored energy; the other is the radiated field, see Figure(8a) At the antenna, the intensities of these fields are large and are proportional to the amount of RF power delivered to the antenna. The radiated field is divided in three distinctive regions:

1. Reactive Near-Field is the region close to the antenna where the electric-E and magnetic-H fields are not orthogonal and anything within this region which couple with the antenna will distort the radiated pattern. Antenna Gain is not a meaningful parameter here.

2. Radiating near-field (transition region or Fresnel region) is the region between near and far field. In this region the antenna pattern is taking shape but is not fully formed. Antenna gain will vary with distance.

3. Far-field (Fraunhofer region) region is defined as that region of the field where the angular field distribution is essentially independent of the distance from the antenna. The antenna gain is constant with distance.

Where

\[ R_1 = 0.62 \sqrt{\frac{D^2}{\lambda}} \] and

\[ R_2 = \frac{2D^2}{\lambda} \]

7.2.2 Polarization

Figure(8b) presents some examples of polarization.
A radiated wave polarization is determined by the direction of the lines of making up the electric-E field.

If the lines of electric-E field are at the right angles to the earth’s surface, the wave is vertically polarized.

If the lines of electric-E field are parallel to the earth’s surface, the wave is horizontally polarized.

A vertical antenna receives vertically polarized waves, and a horizontal antenna receives horizontally polarized waves.

If the field rotates as the waves travel through space, both horizontal and vertical components of the field exist, and the wave is elliptically polarized.

Circular polarization describes a wave whose plane of polarization rotates through $360^\circ$ as it progresses forward. The rotation can be clockwise or counter-clockwise. The circular polarization occurs when equal magnitudes of vertically and horizontally polarized waves are combined with a phase difference of $90^\circ$. Rotation in one direction or the other depends on the phase relationship.

7.3 Antenna types

7.3.1 Monopole Antennas

A monopole antenna is a $\frac{\lambda}{4}$ whip placed over a ground-plane.

- The ground-plane of this antenna can be the metal case of a radio, the body of a vehicle, the metallic roof of a house, or $\frac{\lambda}{4}$ radials.
7.3.2 Dipole Antennas

- A half-wave dipole antenna is a wire or conducting element whose length is half the transmitting wavelength and is fed at the center.
- In free-space a thin dipole at resonance presents an input impedance of approximately 73 Ω.
- This impedance is not difficult to match to 50 Ω transmission lines, and a number of convenient matching circuits have been designed to make the transition from various coaxial and other transmission lines.

7.3.3 Loop Antennas

The Loop Antenna refers to a radiating element made of a coil of one or more turns.
The dissipative resistance in the loop, ignoring dielectric loss, is depended by the loop perimeter, the conductor width/thickness, the magnetic permeability $\mu$, the conductivity $\sigma$, and by the frequency.

The loop’s inductance is determined by the circumference, the enclosed area, the conductor width/thickness, and the magnetic permeability. The loop antennas can be divided in three groups:

1. Full-wave Loop antenna
2. Half-wave Loop antenna
3. Series-loaded, Small-loop antenna

7.3.4 Patch Antennas

- A rectangular patch antenna is defined by its length $L$ and width $W$.

- For a simple micro-strip line, the width is much smaller than the wavelength. For the patch antenna, the width is comparable to the wavelength to enhance the radiation from the edges.

- The length $L$ should be slightly less than $\frac{\lambda}{2}$, where $\lambda$ is the wavelength in the dielectric medium. Here, $\lambda$ is equal to $\frac{\lambda_0}{\sqrt{E_{eff}}}$, where $\lambda_0$ is the free-space wavelength and $E_{eff}$ is the effective dielectric constant of the patch. The value of $E_{eff}$ is slightly less than $E_r$, because the fringing fields around the periphery of the patch are not confined in the dielectric substrate but are also spread in the air.

- The electric field is zero at the center of the patch, maximum (positive) at one side, and minimum (negative) on the opposite side.

- The far-field radiation pattern is orientated orthogonal to the surface conductor.
• The surface conductor does not form the radiating element as it does in a dipole. Instead, radiation occurs from along edges \( L \) and \( W \), and which edge depends upon the electromagnetic mode of radiation the antenna is operating in. confined in the dielectric substrate but are also spread in the air.

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• The far-field radiation pattern is orientated orthogonal to the surface conductor.

• The surface conductor does not form the radiating element as it does in a dipole. Instead, radiation occurs from along edges \( L \) and \( W \), and which edge depends upon the electromagnetic mode of radiation the antenna is operating in.

7.3.5 Slot Antennas

The basic slot antenna is a 2-wave slot cut in a conducting sheet of metal. The feed point is across the center of the slot and it is balanced. The feed impedance is high, typically several hundred ohms. Even if mechanically the slot antenna is the opposite of a dipole because is a nonconducting slot in a sheet of metal (compared to a wire in a free space), the slot antenna has a lot of similarities to a dipole. However, it does exhibit some differences as follows:

• The feed point is across the center instead of in series, so the feed point impedance is high instead of low.

• E and H fields are switched so that the polarity is opposite.
7.3.6 Helical Antenna

A conducting wire wound in the form of a screw thread can form a helix antenna. Usually the helix uses a ground plane with different forms. The diameter of the ground plane should be greater than $\frac{3\lambda}{4}$. In general the helix is connected to the center conductor of a coaxial transmission line and the outer conductor of the line is attached to the ground plane. The parameters which characterize a helix antenna are:

- $N$ ............................................... the number of turns
- $D$ ............................................. the diameter of the Helix
- $S$ .............................................. the spacing between each turn
- $L$ .............................................. total Length of the antenna
- $\alpha$ .............................................. the Pitch angle,

which is the angle formed by the line tangent to the helix wire and a plane perpendicular to the helix axis. The Figure(9) presents all the different types of antennas.

7.4 Tracking types

There are several tracking technologies in use, although the most common are magnetic, acoustic, and optical technologies. Let us examine an overview of the different technologies and their advantages and disadvantages.[18, 19, 20]

Mechanical

- Description : Measure change in position by physically connecting the remote object to a point of reference with jointed linkages
Figure 9: Types of antennas
• Strengths: Accurate, Low lag no line of sight (LOS) or magnetic interference problems good for tracking small volumes accurately

• Weaknesses: Intrusive, due to tethering subject to mechanical part wear-out

**Magnetic**

• Description: Use sets of coils (in a transmitter) that are pulsed to produce magnetic fields. Magnetic sensors (in a receiver) determine the strength and angles of the fields. Pulsed magnetic field may be AC or DC.

• Strengths: Inexpensive, Accurate, no LOS problems, good noise immunity, map whole body motion, large ranges - size of small room.

• Weaknesses: Ferromagnetic and metal conductive surfaces cause field distortion. Electromagnetic interference from radios. Accuracy diminishes with distance. High latencies due to filtering

**Sourceless, non-inertial**

• Description: Use passive magnetic sensors, referenced to the earth’s magnetic field, to provide measurement of roll, pitch, and yaw, and as a derivative, angular acceleration and velocity.

• Strengths: Inexpensive. Transmitter not necessary portable.

• Weaknesses: Only 3 DOF difficult to mark movement between magnetic hemispheres

**Optical**
• Description: Use a variety of detectors, from ordinary video cameras to LEDs, to detect either ambient light or light emitted under control of the position tracker. Infrared light is often used to prevent interference with other activities.

• Strengths: High availability. Can work over a large area. Fast. No magnetic interference problems. High accuracy

• Weaknesses: LOS necessary limited by intensity and coherence of light sources, weight, and expensive

**Internal**

• Description: Use accelerometers and gyroscopes. Orientation of the object is computed by jointly integrating the outputs of the rate gyros whose outputs are proportional to angular velocity about each axis. Changes in position can be computed by double integrating the outputs of the accelerometers using their known orientations.


• Weaknesses: Only 3 DOF. Drift. Not accurate for slow position changes

**Acoustic (ultrasonic)**

• Description: Use three microphones and three emitters to compute the distance between a source and receiver via triangulation. Use ultrasonic frequencies (above 20 MHz) so that the emitters will not be heard

• Strengths: Inexpensive. No magnetic interference problems. Lightweight
- Weaknesses: Ultrasonic noise interference. Low accuracy since speed of sound in air varies with environmental conditions. Echoes cause reception of “ghost” pulses. LOS necessary

**Radio** (GSM location)

- Description: Use three or more transmission radio emitters to compute the distance between a source and receiver via triangulation. Can use different modalities to obtain the location: measuring signal attenuation, angle of arrival, time difference of arrival

- Strengths: Inexpensive because can use the existing GSM infrastructure. Lightweight.

8 Laboratory test

In total eight measurements have been done, base on the position of the antenna such as horizontal and vertical directions. Regarding the orientation two angles were considered (\( \alpha = 0^\circ \) and \( \beta = 45^\circ \)). The aim is to find how the position and orientation angles can affect the antenna output. Tables(3), in appendix (A) shows the antenna coordinates around the the spark gaps in a specific direction and position. The spark is placed on the origin of the xy-coordinate system. All the 4 antenna have the same height \( h_a = 26.5cm \), almost identical to the height of the spark \( h_s = 26.7 cm \). In this experiment the spark is produced by a spark voltage generator. An AC high voltage of 2kV is applied to the spark gap, using air as medium in the small gap of 0.25 \( mm \). The general setup of the experiment is shown in the Figure(10a) and Figure(10b) describes the approximate simplified electrical circuit. A series resistance of \( 50 \times 10^6 \Omega \) coupled with a capacitor of \( 5 \times 10^{-9} F \) placed across the gap. The resistor has two different functions. When the system is switched on, the resistor acts as charging resistance with the charging current \( i_c(t) \) flowing through it. When the capacitor is fully charged, the reverse situation takes place, meaning that the capacitor start discharging through the gaps with a capacitive voltage of \( U_c(t) \) causing the sparks between the gaps that induces the electromagnetic propagation waves which are captured by the antennas connected to a digitizer which records the output signal data. During the discharging period the resistor exhibits a circuit protection function. The charging time \( (\tau) \) is equal to:

\[
\tau = R \times C \tag{19}
\]

When the capacitive voltage is:

\[
\tau = 50 \times 10^6 \times 5 \times 10^{-9} = 0 \times 25s
\]
\[ U_c(t) = E - (E - U_{co})e^{-\frac{t}{RC}} \]  \hspace{1cm} (20)

Assuming that at \( t = 0 \) and \( U_{co} = 0 \). Therefore

\[ U_c(t) = E(1 - e^{-\frac{t}{RC}}) \]  \hspace{1cm} (21)

Since

\[ i_c(t) = C \frac{dU_c(t)}{dt} \]  \hspace{1cm} (22)

Then

\[ i_c(t) = \frac{E}{R} e^{-\frac{t}{RC}} \]  \hspace{1cm} (23)

RF signal is emitted from the spark gap discharges as a point source at position \( P(x, y, z) \) with \( (x = 0, y = 0, z = 2.67 \text{ Cm}) \). It is assumed in this experiment that the wave propagation speed is same as the velocity of light in empty space, that is \( C = 3 \times 10^8 \text{ m/s} \). Four antennas have been setup as \( A_1 \), \( A_2 \), \( A_3 \) and \( A_4 \) to capture the emitted signals from the source. The respective traveling time from the arcing source to each antenna is \( T_1, T_2, T_3 \) and \( T_4 \); and the difference time between antenna \( A_1 \) and \( A_2 \) is \( T_{12} \) similarly between \( A_1 \) and \( A_3 \) is \( T_{13} \) and between \( A_1 \) and \( A_4 \) is \( T_{14} \). The corresponding distance \( (d) \) can be evaluated by this standard definition of the distance see Eq(24):

\[ d = v \times t = c \cdot t \]  \hspace{1cm} (24)

Where \( v = \text{velocity} \) and \( t = \text{time} \).

The shortest distance from the hall to the experiment system is \( 1 \text{ m} \). Therefore the reflection occurs almost at:

\[ t = \frac{2m}{3.10^8 \text{m/s}} = 6.67 \times 10^{-9} \text{s} \]
Figure 10: Experiment setup and its Circuit
\[ t = 6.67 \text{ ns} \]

In order to avoid the interferences from the reflected signals, it is important to consider the first signals which are obviously not contaminated. The output signals recorded by the digitizer are presented in the Figure(11). Matlab simulation has been used to produce signals output from each antenna for analysis purpose, see Figures(12a), (12b), (12c), (12d). The experiment measurement were done as follow:

8.1 Measurement 1a

Antennas are placed at the same distance from the source (about 42 cm). All the four antennas have an orientation angle of \( \beta = 45^\circ \) Measurements were done in two positions (Horizontal and vertical). Since all the four antennas are at the same distance from the spark gap source, it is clear that the delay time between those antennas are equal to zero.

8.2 Measurement 2b

Here the antennas are at different distances from the source, therefore the time delay of the arrival signals between these antennas should be taken into account. The orientation angle is \( \beta = 45^\circ \) and measurements are done in horizontal and vertical positions.

8.3 Measurement 5e

The antennas are placed in line such that they have the same x-coordinate with an orientation angle \( \alpha = 0^\circ \). The delay time is considered except antenna \( A_1 \) and \( A_2 \) since they are at the same distance from the source.
Figure 11: Digitizer output
Figure 12: Antennas output signals
8.4 Measurement 7g

The antennas are aligned such that they have same $y$-coordinate. Their orientation angle $\alpha = 0^o$, since they are at different distances from the source the time of arrival (TOA) signal needs to be considered. In order to find TOA, the xcorr method in matlab is used. The results are presented in Table(4b) in Appendix. Comparison between the theoretical values and the measured values shows that both data are almost correlated even though at first sight the percentage error seems to be significant. This abnormality is due to three main reasons:

- The covering detection range of the antennas
- The short distance between the antennas Since the speed of the wave propagation is very high, with the distance in the range of few centimeters ($cm$). It is clear that the short distance has a great effects on the output signals.
- The short sampling time Dealing with a wide band spectrum, it is important to increase the sampling time in order to get a very accurate result. but still the results obtain are clearly mean full and show the capability to detect the arcing faults with the aid of DA.

Base on the antennae different coordinates the following equations can be estimated.

\[(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = (v \times t)^2 \quad (25)\]

\[(x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2 = (v \times (t + t_{12}))^2 \quad (26)\]

\[(x - x_3)^2 + (y - y_3)^2 + (z - z_3)^2 = (v \times (t + t_{13}))^2 \quad (27)\]
\[(x - x_4)^2 + (y - y_4)^2 + (z - z_4)^2 = (v \times (t + t_{14}))^2 \quad (28)\]

From the Eq(25), Eq(26), Eq(27) and Eq(28) an accurate estimation of the arcing source can be determined using of course a number of possible iteration algorithms if the time difference \((T_{12}, T_{13}, \text{and} T_{14})\) are measured. The \((x, y, z)\) coordinates of the four antennas arranged around the arcing fault source and measured are described in the Table(3a), (3b), (3c) in appendix. The distance \(d_i\) with \(i = 1, 2, 3, 4\) from each antenna to the source wave radiation, referred to the coordinates system can be express in Eq (29):

\[d_i = \sqrt{(x_o - x)^2 + (y_o - y)^2 + (z_o - z)^2} \quad (29)\]

\((x_o, y_o, z_o)\) are the coordinates of the electromagnetic radiation source and \((x, y, z)\) each antenna coordinates. Using the Eq(25), Eq(26), Eq(27), Eq(28) and Eq(29) and the coordinates presented in Table(3a), (3b), (3c), the theoretical propagation times between antennas are calculated and presented in Table(15b), (4a), (4b) in appendix (C).

9 Signal processing analysis.

The RF signals captured from the antennas are always contaminated by impulse noises which need to be filtered before examination of these signals. Any elimination process will produce a very important and precise analysis with correct results. Depending on the type of signal obtained we will choose a perfect signal processing method to be applied. The signal processing is the analysis, or interpretation and manipulation of any captured signal. Here the signal of our interest will be the one detected by the four directional finding antennas. Processing of such signal includes storage and reconstruction, separation of information from noise.
9.1 Statistical analysis

The statistical analysis is performed in order to extract better features of different RF signals emitted by the arcing fault. In this analysis the signals will be presented in terms of a set of properties or parameters. The most common measurements in statistics are:

1. arithmetic mean
2. standard deviation
3. auto-correlation coefficient
4. variance

All these parameters actually compute the value about which the data are concentrated. All measures of central tendency are used to estimate the mean.

9.1.1 arithmetic mean

The arithmetic mean of the sample of signal collected from each antenna is calculated using the Eq(30) and the corresponding results are shown in Table(1a).

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]  

(30)

Regarding the mean value, we can clearly notice that the measurement with the antenna in horizontal position is more accentuated than the one obtained with antenna in vertical position. This shows that the antenna are more effective in horizontal position. Antenna A1 and A4 respectively in measurement 5e and 2b present highest mean value compared to antenna A2 and A3 which are almost similar. The objective of signal features extraction
Table 1: Average and Standard deviation

is to represent the signal in term of a set of properties or parameters, such as arithmetic mean. It computes the value about which the data are centered. In this case measuring of the central tendency estimate the average of the considered sample, see Eq(30). The comparison of the result obtained is shown in Figure(13a). The mean values of each antenna have been grouped together for each experiment $2b - 5e - 7g$. The variations observed are due to the antenna corresponding position. the more closer the antenna is to the source, the more efficiently it captures the signal. This result demonstrates clearly that antenna should be near the hazardous region in order to produce accurate solution.
(a) Auto-correlation coefficients table

<table>
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<tr>
<th></th>
<th>$A_1$</th>
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<th>$A_3$</th>
<th>$A_4$</th>
</tr>
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<tbody>
<tr>
<td>2b</td>
<td>1.0000</td>
<td>-0.0134</td>
<td>-0.0585</td>
<td>-0.2065</td>
</tr>
<tr>
<td>5e</td>
<td>1.0000</td>
<td>0.3591</td>
<td>0.3550</td>
<td>0.1498</td>
</tr>
<tr>
<td>7g</td>
<td>1.0000</td>
<td>0.1917</td>
<td>0.0669</td>
<td>0.1059</td>
</tr>
<tr>
<td>V</td>
<td>$A_1$</td>
<td>$A_2$</td>
<td>$A_3$</td>
<td>$A_4$</td>
</tr>
<tr>
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<td>-0.2923</td>
<td>0.0171</td>
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<tr>
<td>5e</td>
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<td>0.1891</td>
<td>0.3003</td>
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<td>7g</td>
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<td>-0.1486</td>
<td>-0.1706</td>
<td>-0.1441</td>
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</table>

(b) Variance table

<table>
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<td>0.1853</td>
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<td>5e</td>
<td>0.2940</td>
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<td>0.5807</td>
<td>0.0951</td>
<td>0.0707</td>
<td>0.0326</td>
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<tr>
<td>V</td>
<td>$A_1$</td>
<td>$A_2$</td>
<td>$A_3$</td>
<td>$A_4$</td>
</tr>
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<td>5e</td>
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<td>0.0889</td>
<td>0.1448</td>
<td>0.0926</td>
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<td>7g</td>
<td>0.2239</td>
<td>0.0676</td>
<td>0.1259</td>
<td>0.0316</td>
</tr>
</tbody>
</table>

Table 2: Autocorrelation and variance
9.1.2 Standard deviation

The standard deviation measures the dispersion of a set of sample. It is most often measured by the deviation of the samples from their average. The sum of these deviations is zero. The sum of the squares of the deviation is positive. The standard deviation of the samples is formulated by the Eq(31) and the corresponding calculated results are presented in Table(1b)

\[
s = \sqrt{\frac{\sum f_i(x_i)^2 - (\sum f_i x_i)^2}{n - 1}}
\]  

(31)

Similarly like in the case of the arithmetic mean, comparison of the the value of standard deviation for each antenna has been made and presented in Figure(13b). Again the antenna position affects considerably the signal detection process. This result confirms our hypothesis stated above. But the results obtained in the case of average value are more accurate compared to those presented here, since for a single sample the standard deviation drops by \(\sqrt{N}\) times when \(N\) samples are considered. Therefore in this case convergence is not guaranteed.

9.1.3 Auto-correlation coefficient

The auto-correlation coefficient measures the correlation between samples at different distances apart. These coefficients often provide insight into the probability model that generates the data. The concept of auto-correlation is similar to ordinary correlation coefficient, namely that given \(N\) pairs of samples on two variables \(x\) and \(y\), the correlation coefficient is given by the Eq(32) and the results are shown in Table(2a).
\[ r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} \] (32)

The Figure(14a) shows the comparison of the resulting auto-correlation coefficient obtained from each measurement. The value presented in Table(2a) shows that the antenna have similar characteristics and they are working properly.

### 9.1.4 variance

The variance is the average of the squared standard deviations, that is shown in Eq(33) and the corresponding calculated results are presented in Table(2b).

\[ s^2 = \frac{\sum(x_i - \bar{x}_i)^2}{n - 1} \] (33)

Similarly, comparison of the variance values are presented in Figure(14b). Since the variance is the average of the squared standard deviation, the results obtained are same as the one presented in the case of the standard deviation.

### 9.2 Cross correlation techniques

The arrival time difference between the antennas needs to be taken in account. From a cross-correlation method, we will be able to establish the delay time. This technique finds the similarities between two signals, by interpolation, a good resolution of the time-delay is obtained from fractions of sample interval. Having two signals \( x_i \) and \( x_j \) as one is displayed through
Figure 13: Average and Standard deviation graphics
Figure 14: Autocorrelation and Variances graphics
time $\tau$ relative to other, a signal obtained at two separated antennas can be mathematically expressed in Eq(34) and Eq(35) as:

$$x_i = S(t) + n_i(t)$$

(34)

$$x_j = \alpha S(t - t_{ij}) + n_j(t)$$

(35)

Where $S(t)$ is the source signal, and $n_i(t)$ and $n_j(t)$ are uncorrelated noise sources, therefore they have zero mean values. $\alpha$ is the attenuation factor, and $t_{ij}$ is the delay time between antennas $A_i$ and $A_j$ received signals. The Direct Cross Correlation (DCC) can be used to measure similarities of these two signals. The cross correlation ($r_{xi,xj}$) is expressed in Eq(36) as:

$$r_{xi,xj}(\tau) = E[x_i(t)x_j(t + \tau)]$$

(36)

with $i = 1, 2$ and $j = 1, 2$ where $\tau$ is called the time-lag or time-delay. It shows the shift between the signal pair $x_i$ and $x_j$. From this clear definition, it is very important to keep in mind that this Cross Correlation (xcorr) method is implemented to estimate the time delay by detecting the time that maximizes it’s function. Using matlab simulation code, the xcorr is done by applying this code in the programming interface to simulate the empirical data captured from each antenna. For the corresponding graphical output, see Figure(15a), are presented in appendix of figures see Figures(21a), (21b). The xcorr method has been applied each time between $(A_1$ and $A_2)$, $(A_1$ and $A_3)$, $(A_1$ and $A_4)$. Here for simplicity: $H$ stands for horizontal and $V$ for vertical and $1a$ for the antenna in $\alpha = 0^o$ and $3c$ for $\beta = 45^o$. The corresponding time($t$) of the maximum amplitude of the signal gives the time-delay between each pair of the antennas. The results are presented in Table(15b). The antenna taken in pairs and subjected to cross-correlation method was able to achieve the time delay between the antennas. Tables(15b), (4a) and (4b)
present all the results for each type of measurement. In the tables, $t$ is the real
time obtained using Eq (36). $t_h$ and $t_v$ are the times when the antennas are
in horizontal and vertical positions as indicate indexes h and v respectively.
The error margin $E_1$ is for the antennas in a horizontal position while $E_2$ is
for the vertical position. The Eq(37) and Eq(38) below show the calculation
of the margin of error:

$$E_1 = t - t_h$$  \hspace{1cm} (37)$$

$$E_2 = t - t_v$$  \hspace{1cm} (38)$$

9.3 Location algorithm

The arcing fault is found by using simultaneous solution of a set of nonlinear
equations, presented in Eq(40), Eq(41), Eq(42). Considering the rectangu-
lar coordinates $(x, y, z)$ with subscript ”s” denoting the spark source, and
subscript 1 to 4 (more generally ”q”) indicating the antenna position. The
propagation is shown in this general Eq(39) :

$$C^2(t_q - \tau)^2 = (x_s - x_q)^2 + (y_s - y_q)^2 + (z_s - z_q)^2$$  \hspace{1cm} (39)$$

since $\tau$ is unknown, and the TOA are only known as differences, the propa-
gation equation can be more usefully presented as follow :

$$Ct_{12} = d_1 - d_2$$  \hspace{1cm} (40)$$

$$Ct_{13} = d_1 - d_3$$  \hspace{1cm} (41)$$
Figure 15: xcorr analysis results

<table>
<thead>
<tr>
<th>$b$</th>
<th>$t(ns)$</th>
<th>$t_\alpha(ns)$</th>
<th>$E_1$</th>
<th>$t_\beta(ns)$</th>
<th>$E_2$</th>
</tr>
</thead>
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<td>$A_1A_2$</td>
<td>0.60</td>
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<td>0.96</td>
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</tr>
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<td>-0.19</td>
<td>0.13</td>
<td>0.28</td>
</tr>
<tr>
<td>$A_1A_4$</td>
<td>0.95</td>
<td>0.34</td>
<td>0.61</td>
<td>0.11</td>
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<tr>
<td>$A_2A_3$</td>
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<tr>
<td>$A_2A_4$</td>
<td>1.34</td>
<td>0.64</td>
<td>0.70</td>
<td>0.11</td>
<td>1.23</td>
</tr>
<tr>
<td>$A_3A_4$</td>
<td>0.59</td>
<td>0.91</td>
<td>-0.32</td>
<td>0.10</td>
<td>0.49</td>
</tr>
</tbody>
</table>

(b) b-type time delay estimation
\[ Ct_{14} = d_1 - d_3 \]  \hfill (42)

Where
\[ d_q = \sqrt{(x_s - x_q)^2 + (y_s - y_q)^2 + (z_s - z_q)^2} \]  \hfill (43)

Eq(40), Eq(41), Eq(42) contain three unknowns \((x_s, y_s, z_s)\) and can be solved by iteration. Newton-Raphson solution approach used to present satisfactory solution. Execution of this algorithm can result in three possible outcomes:

1. The algorithm converges on the source location

2. The algorithm cannot converge, but enters into a limit cycle where the position of subsequent iteration lie on line passing through the antenna array and the source.

3. The algorithm diverges asymptotically.

The outcomes 1 and 2 correspond to the sparks source being located close to or far from the antenna array. The outcome 3 is a function of the ratio of the WRAF signal to background noise. In the presence of the noise, the TOA of the WRAF signal can be distorted since the background noise varies spatially, it is possible for a set of TOA to be calculated that do not correspond to a physical source position. In this case the algorithm can fail to converge. This is a particular problem where the antenna spacing is small. It is possible to recover this situation by making small adjustments to the time delays to find the nearest position of the convergence.

### 9.4 Fourier analysis

In this technique we will transform the captured signal by the Fourier transform process following a predefined mathematical formula named as fre-
quency function, see Eq(44). Fourier analysis is a powerful method to describe a signal based on frequency in a periodic event that occurs.

\[ X(\omega) = \int_{-\infty}^{+\infty} x(t) \exp(-jwt)dt \]  

(44)

Here the Fourier analysis method is used to determine the frequency of the propagation of the signal of the arcs. It is clear that such frequency is function of the distance between the source and the receivers. Three cases have been explored to implement the impact of the distance upon the frequency. All these cases are shown in series of figures from Figure(16b), (23a), (23b), obtained with matlab simulation. The corresponding results calculated using Eq(44), are shown in Table(16a)
<table>
<thead>
<tr>
<th>$H(\text{Hz})$</th>
<th>$A_1(\times 10^8)$</th>
<th>$A_2(\times 10^8)$</th>
<th>$A_3(\times 10^8)$</th>
<th>$A_4(\times 10^8)$</th>
</tr>
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<tbody>
<tr>
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<td>3.46</td>
<td>3.46</td>
<td>3.46</td>
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<tr>
<td>$5e$</td>
<td>4.95</td>
<td>3.96</td>
<td>4.46</td>
<td>3.96</td>
</tr>
<tr>
<td>$7g$</td>
<td>3.96</td>
<td>2.97</td>
<td>2.97</td>
<td>3.96</td>
</tr>
<tr>
<td>$V(\text{Hz})$</td>
<td>$A_1(\times 10^8)$</td>
<td>$A_2(\times 10^8)$</td>
<td>$A_3(\times 10^8)$</td>
<td>$A_4(\times 10^8)$</td>
</tr>
<tr>
<td>$2b$</td>
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<td>3.46</td>
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<tr>
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<td>$7g$</td>
<td>4.46</td>
<td>3.96</td>
<td>3.46</td>
<td>4.95</td>
</tr>
</tbody>
</table>

(a) Energy level

(b) Fourier of 2b-H

Figure 16: Fourier analysis results
10 Brief description of other methods

10.1 Wavelet analysis

The Wavelet analysis is like a kind of localized Fourier transform in both time and frequency domain. The wavelet transform is done by dilating and translating the analysis wavelet and by convolution with the mother function of the captured signal. Applying this technique is helpful in order to analyze the high frequency section of the signal by the short analysis wavelet and the low frequency part of the signal is analyzed with the long analysis wavelet. The wavelet coefficient is shown in Eq(45)

\[
W(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \Psi\left(\frac{t - b}{a}\right) dt
\]

(45)

where the signal \( f(t) \) is a function of a continuous variable, \( a \) is the scale parameter, \( b \) the translation parameter, and \( \Psi(t) \) is the mother wavelet in time domain by \( a \) and shifted by \( b \). The value of \( W(a, b) \) will indicate the similarity between the mother wavelet \( \Psi(t) \) and the original signal \( f(t) \). The application of cross-correlation techniques on the wavelet will increase the accuracy of the result. With the combined method the distance between the source fault and the receiver will be determined in order to indicate the exact location of the arcing fault. Using \( f(t) = xcorr(bHa_1, bHa_2) \), we obtain a wavelet coefficient decomposition. This method scales the signal simultaneously into time and frequency domain. It gives important details of the signal compare to Fourier analysis. The information extracted confirms the results already obtained from a cross-correlation method. To make a meaningful analysis with wavelet method when the signals detected by the antennas. It would be very interesting to compare the frequency energy level of each signal captured by 4 antennas individually as presented in Figures(17a), (17b) and (17c) below. Then we explore the energy level of signals given by the pair after techniques
cross-correlation and observations are made, by comparing these frequencies with those obtained using wavelet. In our case we considered the pair of antennas \((bHa_1, bHa_2)\) it should be noted that this concerns only the pair \(b\)-type antenna measurement \(bHa_1\) exhibits a frequency \(f_{bHa_1} = 0.04455\)Hz for a given level of energy of \(E_{bHa_1} = 692.3\)Ws when the antenna \(bHa_2\) presents a \(f_{bHa_2} = 0.03465\)Hz frequency corresponding to a level of energy \(E_{bHa_2} = 1390\)Ws so that their pairs trained in cross-correlation gives the following result, \(f_{xcorr} = 0.03465\)Hz and \(E_{xcorr} = 0.8157 \times 10^9\)Ws.

Figure 17: Energy level
10.2 Phase difference method

The phase $\beta$ of the signal with carrier frequency $f_c$, modulation frequency $f_b$ and peak phase deviation $\Delta\phi$ can be presented in Eq(46) as

$$\beta = 2\pi f_c t + \Delta\phi \sin 2\pi f_b t$$  \hspace{1cm} (46)

When the modulation signal is noise, the $rms$ phase deviation is equal to the standard deviation assuming Gaussian distributed phase noise amplitude. So the phase noise standard deviation $\sigma_{\phi_{DSB}}$ is shown in Eq(47)

$$\sigma_{\phi_{DSB}}(f_b) = \frac{\Delta\phi}{\sqrt{2}}$$  \hspace{1cm} (47)

The Eq(48) the frequency of the signal can be written as the derivative of phase:
\[ f = \frac{1}{2\pi} \cdot \frac{\partial \beta}{\partial t} f_c + \Delta \phi f_b \cos(2\pi f_b t) \]  

(48)

It can be shown by examining a Bessel-function series expansion of Eq(48) that for the small values of modulation index associated with random phase noise only frequency \( f_c, f_c - f_b \) and \( f_c + f_b \) are significantly high in energy. Therefore the relation between the single sideband phase variance \( \sigma_{SSB}^2(\text{rad}^2) \) related to the single sideband (SSB) phase noise level can be written :

\[ \sigma_{\phi SSB}^2(f_b) = \frac{P_{SSB}(f_b)}{P_c} \]  

(49)

The noise components of different frequencies sum quadratic. The total phase variance of SSB integrated over period \( T \) as function of SSB spectral density \( \xi(f_b) \) is then estimated in Eq(50) as :

\[ \sigma_{\phi SSB}^2(f_b) = \int_{-\frac{T}{2}}^{\frac{T}{2}} \xi(f_b) df \]  

(50)

The total phase variance is the sum of the two SSB and in the bi static measurement system the transmitter and the receiver noise sum quadratically.

### 10.2.1 Time difference of arrival (TDOA)

When radio waves propagate, it reaches the receiving antennas at different time. The signal will take \( t_1 \) time to reach the antenna 1. In the same way it would reach the antenna 2 in time \( t_2 \), antenna 3 in time \( t_3 \) and antenna 4 in time \( t_4 \). It is shown that if the Antenna receives a signal at time \( t_1 \), one can find the distance of spark gap to the antenna by using \( d_1 = ct_1 \), where \( c \) is the velocity of radio waves. By taking these four antennas as the center and drawing four arcs (with calculated distance as radius) would intersect at one point. This point is the location of the arcing fault. Time difference of arrival has been the preferred technology of choice for high
accuracy location systems since the advent of radar. The global positioning system is a TDOA based system as are most of the systems proposed for the location and monitoring service (LMS). When a signal is transmitted from the spark gaps, the signal propagates to all of the antenna sites where the signal reception is time stamped. The differences in time stamps are then combined to produce intersecting hyperbolic lines from which the location is estimated. TDOA systems are subject to many multi path problems as angle of arrival systems. In this case, multi path distorts the shape of the signal and the group delay, causing the TDOA system difficulty in accurately determining the point in the signal to be measured by all receivers. For this reason, most TDOA systems have historically been wide band (> 1MHz).

10.2.2 Measuring the angle of arrival (AA)

This second technique for estimating location is known as angle of arrival or direction finding. Angle of arrival was first developed for military organizations and was later applied to cellular signals. The most common version of this technique is known as small aperture direction finding, which requires a complex antenna array at each of several cell site locations. The antenna arrays have 4 to 12 antennas at a spacing of less than 1 wavelength, and can in principle work together to determine the angle from signal source. When several antennas determine their respective angles of arrival, the signal source location can be estimated from the point of intersection of projected lines drawn out from the source at the angle from which the signal originated. The measured direction of arrival (DOA) is defined in Eq(51) as:

\[ r_i = \arctan \frac{y - y_i}{x - x_i} \]  

(51)
11 Conclusion

A precise comparison of the results from the methods enumerated in the signal analysis section is done to determine the exact location of the fault. From the investigation a very good analysis technique can be established for feature research in the electrical engineering development. The location accuracy depends on the configuration of four antennas array and distance to the source. It is important to understand that radio frequency directional finding can improve power distribution network fault location. Intelligent methods should be used to cope with the wide receiver bandwidth and environmental interferences. Different fault conditions simulated in laboratory create and emit different radio frequency signals, which are captured by the antenna. Although some of these RF signals have distinctive features compared to others, there are several faulty conditions which the radio frequency signals are very similar in terms of voltage level, time period and signal pattern in time domain. As the number of fault conditions increases, the problem will
be greater. Therefore a good analysis method must be determined in order to reveal more unique feature of the particular signal. Among the different methods discussed above the cross correlation has showed the least distinctive feature compare to the statistical analysis, Fourier transform. Furthermore, the result of cross correlation will be greatly influenced from the surrounding area.

**Appendix A : Antenna coordinates**

**Antenna coordinates**

**View of each antenna in the coordinate plan**
Table 3: Type-of-measurements

<table>
<thead>
<tr>
<th></th>
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<th>$A_4$</th>
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<td>26.7</td>
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(a) b-type of measurement

<table>
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<tr>
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<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
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(b) e-type measurement

<table>
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<th>$A_3$</th>
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<tr>
<td>$z_s(cm)$</td>
<td>26.7</td>
<td>26.7</td>
<td>26.7</td>
<td>26.7</td>
<td></td>
</tr>
<tr>
<td>$d_0(m)$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.42</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>$t_0(ns)$</td>
<td>1.05</td>
<td>1.05</td>
<td>1.41</td>
<td>1.41</td>
<td></td>
</tr>
</tbody>
</table>

(c) g-type measurement
Figure 20: Antennas in a Coordinates plan
Appendix B : Cross-correlation examples

The concept of auto-correlation and cross correlation play an important role in signal analysis, particularly in electrical engineering, for the investigation of signal detection and time-delay estimations. The auto-correlation function of a random signal describes the general dependence of the values of the samples at one time on the values of the samples at another time. Consider a random process $x(t)$ (i.e. continuous-time), its auto-correlation function is written in Eq(52) as:

$$R_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)x(t + \tau)dt$$  \hspace{1cm} (52)

Where T is the period of observation. $R_{xx}(\tau)$ is always real-value and an even function with a maximum value at $\tau = 0$. For sampled signal , the auto-correlation is defined as either biased or unbiased defined as follows:

- biased :
  $$R_{xx}(m) = \frac{1}{N - |m|} \sum_{n=1}^{N-m+1} x(n)x(n + m - 1)$$  \hspace{1cm} (53)

- unbiased :
  $$R_{xx}(m) = \frac{1}{N} \sum_{n=1}^{N-m+1} x(n)x(n + m - 1)$$  \hspace{1cm} (54)

with $m = 1, 2, ..., M+1$ where M is the number of lags. The cross correlation function however measures the dependence of the values of one signal on another signal. For two WSS (Wide Sense Stationary) processes $x(t)$ and $y(t)$ it is described by Eq(55) and Eq(56) :

$$R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t + \tau)dt$$  \hspace{1cm} (55)

$$R_{yx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t)x(t + \tau)dt$$  \hspace{1cm} (56)
where $T$ is the observation time. For sampled signals, it is defined as shown in Eq(57):

$$R_{qq}(m) = \frac{1}{N} \sum_{n=1}^{N-m+1} y(n)x(n + m - 1)$$  \hspace{1cm} (57)

with $m = 1, 2, ..., N + 1$, where $N$ is the record length (i.e. number of samples). Figure 2 presents the cross-correlation between antennas 1 and 2. (ie. $bHa_1$ and $bHa_2$).

### Appendix C : Time delay estimation

Many location algorithms based on delay time of arrival (TDOA) estimation have been explored. A pulse $x(t)$ is transmitted, the reflected signal from an object is returned to the receiver. The returned signal $r(t)$ is delayed (i.e. $D$ seconds), noisy and attenuated. The objective is to measure (estimate) the time delay between the transmitted and the returned signal.

#### Delay estimation in time domain

Let the transmitted signal be $x(t)$, then the returned signal $r(t)$ may be modeled in Eq(58) as:

$$r(t) = \alpha x(t - D) + w(t)$$  \hspace{1cm} (58)

where $w(t)$ is assumed to be the additive noise during the transmission. $\alpha$ is the attenuation factor ($\alpha < 1$) and $D$ is the delay which is the time taken for the signal to travel from the transmitter to the target and back to the receiver. A common method of estimating the time delay $D$ is to compute the
Figure 21: Cross-correlation of 5e and 7g
cross-correlation function of the received signal with the transmitted signal \( x(t) \). See Eq(59), Eq(60) and Eq(61)

\[
R_{rx}(\tau) = E\{r(t)x(t + \tau)\} \tag{59}
\]

\[
R_{rx}(\tau) = E\{\alpha x(t - D)x(t + \tau) + w(t)x(t + \tau)\} \tag{60}
\]

\[
R_{rx}(\tau) = \alpha R_{xx}(\tau - D) + R_{wx}(\tau) \tag{61}
\]

Note: \( E \) is the expectation operator. Therefore, the cross-correlation \( R_{rx}(\tau) \) is equal to the sum of the scaled auto-correlation function of the transmitted signal (i.e. \( \alpha R_{xx}(\tau) \)) and the cross correlation function between \( x(t) \) and the contaminated noise signal \( w(t) \). Assuming that the noise signal \( w(t) \) and the transmitted signal \( x(t) \) are uncorrelated then, \( R_{wx}(\tau) = 0 \). Hence the cross-correlation function between the transmitted signal and the received signal may be written in Eq(62) as:

\[
R_{rx}(\tau) = \alpha R_{xx}(\tau - D) \tag{62}
\]

If we plot \( R_{rx}(\tau) \), it will only have one peak value that will occur at \( \tau = D \). A typical plot of \( R_{rx}(\tau) \) is shown in Figure(22) and Table(15b) shows the time delay estimation obtained from cross-correlation method. There is no big difference between real time and those obtained by theoretical simulation. The margin of error is due to the very short distance between the antennas and the spark source on one hand as well as the distance between the antennas themselves on other hand.
Figure 22: Time delay estimation

### Table 4: Time delay estimation

#### (a) e-type time delay estimation

<table>
<thead>
<tr>
<th></th>
<th>$t$ (ns)</th>
<th>$t_h$ (ns)</th>
<th>$E_1$</th>
<th>$t_v$ (ns)</th>
<th>$E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1A_2$</td>
<td>0.67</td>
<td>0.52</td>
<td>0.15</td>
<td>0.70</td>
<td>-0.33</td>
</tr>
<tr>
<td>$A_1A_3$</td>
<td>0.67</td>
<td>0.68</td>
<td>-0.01</td>
<td>0.64</td>
<td>0.03</td>
</tr>
<tr>
<td>$A_1A_4$</td>
<td>2.00</td>
<td>0.53</td>
<td>1.47</td>
<td>0.73</td>
<td>1.27</td>
</tr>
<tr>
<td>$A_2A_3$</td>
<td>0.67</td>
<td>0.68</td>
<td>-0.01</td>
<td>0.68</td>
<td>-0.01</td>
</tr>
<tr>
<td>$A_2A_4$</td>
<td>0.13</td>
<td>0.45</td>
<td>-0.32</td>
<td>0.68</td>
<td>-0.55</td>
</tr>
<tr>
<td>$A_3A_4$</td>
<td>2.00</td>
<td>0.52</td>
<td>1.48</td>
<td>0.52</td>
<td>1.48</td>
</tr>
</tbody>
</table>

#### (b) g-type time delay estimation

<table>
<thead>
<tr>
<th></th>
<th>$t$ (ns)</th>
<th>$t_h$ (ns)</th>
<th>$E_1$</th>
<th>$t_v$ (ns)</th>
<th>$E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1A_2$</td>
<td>0.67</td>
<td>0.65</td>
<td>0.02</td>
<td>0.57</td>
<td>0.10</td>
</tr>
<tr>
<td>$A_1A_3$</td>
<td>0.33</td>
<td>0.64</td>
<td>-0.31</td>
<td>0.39</td>
<td>-0.06</td>
</tr>
<tr>
<td>$A_1A_4$</td>
<td>0.67</td>
<td>0.62</td>
<td>0.05</td>
<td>0.47</td>
<td>0.20</td>
</tr>
<tr>
<td>$A_2A_3$</td>
<td>0.33</td>
<td>0.66</td>
<td>-0.33</td>
<td>0.46</td>
<td>-0.13</td>
</tr>
<tr>
<td>$A_2A_4$</td>
<td>1.00</td>
<td>0.63</td>
<td>0.37</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>$A_3A_4$</td>
<td>0.67</td>
<td>0.65</td>
<td>0.02</td>
<td>0.72</td>
<td>-0.05</td>
</tr>
</tbody>
</table>
Delay estimation in frequency domain

The delay can be estimated well as in frequency domain, using the following method:

Consider the returned signal once again:

\[ r(t) = \alpha x(t - D) + w(t) \]  \hspace{1cm} (63)

Taking the Fourier transform of both sides of equation 8 yields:

\[ P_r(\omega) = \alpha X(\omega)e^{-j\omega D} + W(\omega) \]  \hspace{1cm} (64)

or taking the Fourier Transform of the cross correlation of \( r(t) \) and \( x(t) \) gives:

\[ P_{rx}(\omega) = \alpha P_{xx}(\omega)e^{-j\omega D} + P_{wx}(\omega) \]  \hspace{1cm} (65)

Assuming that the transmitted signal and the contaminated noise are uncorrelated, we get:

\[ P_{xr}(\omega) = \alpha P_{xx}(\omega)e^{-j\omega D} \]  \hspace{1cm} (66)

Therefore by having an estimate of the cross spectral function of the transmitted and the received signal, we can estimate the time delay from its phase.

\[ \text{Phase} = \omega D \]  \hspace{1cm} (67)

Substituting \( \omega \) by \( 2\pi \), this yields

\[ \text{Phase} = 2\pi fD \]  \hspace{1cm} (68)
, hence by taking the slope of the phase

\[
\frac{d}{df}[\text{Phase}]
\]

we have the slope of the phase which may be written as

\[
\text{Slope} = 2\pi D
\]

and finally we can compute \( D \) as:

\[
D = \frac{\text{Slope}}{2\pi}
\]

Appendix D : Results of Fourier analysis
Figure 23: Fourier of 5e-H and 7g-H
Figure 24: Fourier for 2b-V, 5e-V and 7g-V
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