Power Balance in the Finite Element Analysis of Electrical Machines

Bishal Silwal
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A doctoral dissertation completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Electrical Engineering, at a public examination held at the lecture hall AS1 of the TUAS building on 16 June 2017 at 12 noon.

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Abstract

This dissertation deals with the study of the power balance in the numerical simulations of electrical machines. In the numerical analysis of electrical machines using the finite element method, several methods and techniques to compute torque and force exist, but the existing methods are not free from accuracy issues. In this dissertation, approaches based on the power balance of the machine to obtain torque and force are presented, and the possibility to use the power balance approach to validate the existing methods is shown. Both healthy machines and machines with an eccentric rotor have been considered. The applicability of the power balance approach to study the electromagnetic damping of the mechanical vibrations during eccentricity is also assessed.

This dissertation carefully compares a conventional torque and force computation method and the power balance approach, based on the finite element mesh used in the air gap of the machine during simulations. Variations in the air gap mesh is brought by changing the number of layers of elements and the layer used for the rotation of the rotor and for the torque and force computation. Results show that the conventional method suffers from an accuracy issue mainly related to the finite element discretization of the air gap of the machine. Variations in the air gap mesh do not affect torque from the power balance. However, force computation from the power balance is not yet robust.

The influence of the rotor eccentricity on the torque of the machine was also studied. Results show that an eccentric machine does not exhibit the same torque as a healthy machine. The harmonic components around the principal slot harmonic are most affected. In this thesis, a measurement set-up to measure the torque harmonics of a machine was designed. The measured results were compared with the simulated results.

Keywords  eccentricity, electromagnetic torque, finite element method, forces, harmonics, induction machine, mesh, power balance, rotor dynamics
Tekijä
Bishal Silwal

Väitöskirjan nimi
Tehotasapaino sähkökoneiden elementtimenetelmän analyyssissä

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Tiivistelmä
Väitöskirja tutkii tehotasapainon käyttöä sähkökoneiden numeerisessa simuloinnissa. Voinen ja väätömomentin laskemiseen on kehitetty useita elementtimenetelmään perustuvia numeerisia malleja, mutta niihin kaikki sisältyy epätarkkuuksia. Tämä väitöskirja esittää tehotasapainoon perustuvia menetelmiä voimien ja väätömomentin laskemiseen, ja lisäksi näyttää että tehotasapainoa voi käyttää olemassa olevien mallien validointiin. Työssä tutkitaan sekä ehjä sähkökoneita, että koneita joiden roottorit on epäkesken. Lisäksi työ arvioi tehotasapainon soveltuvuutta mikaanisten virhätelyjen sähkömagneettisen vaikutumisen tutkimukseen.


Avainsanat
elementtimenetelmä, epäkeskisyys, epätähtikone, roottoridynamikka, sähkömagneettinen väätömomentti, tehotasapaino, verkko, voimat, yliailut

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This thesis was pre-examined by Prof. Nelson Sadowski and Dr. Mircea Popescu. I thank them for taking time to review my work.

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I sincerely thank Ari Haavisto for managing all the technical resources needed during my PhD and also for helping me to build the measurement set-up. I would also like to acknowledge Dr. Erkki Lanto for helping me during the measurement. My humble thanks to Pia Metso and Heidi Koponen for taking care of the administrative and travel related matters.

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Bishal Silwal

2nd May 2017, Espoo
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References

Publications
This doctoral dissertation consists of a summary and of the following publications, which are referred to in the text by their Roman numerals.


**Author’s Contribution**

**Publication I: “Computation of Torque of an Electrical Machine with Different Types of Finite Element Mesh in the Air Gap”**

In this paper, the effect of different variations of a finite element mesh used in the air gap of a machine on the numerical calculation of the torque of a cage induction machine is studied. A method of torque calculation based on the power balance of the machine is presented and is used to assess the torque computed from the Coulomb’s virtual work method.

Bishal Silwal performed the numerical simulations and wrote the paper. Paavo Rasilo, Lauri Perkkiö, Antti Hannukainen, Timo Eirola and Antero Arkkio contributed through discussions and commenting on the paper.

**Publication II: “Numerical Analysis of the Power Balance of an Electrical Machine with Rotor Eccentricity”**

This paper introduces the concept of whirling power in the analysis of an electrical machine under rotor eccentricity. The error in the numerically computed power balance of the machine is studied by introducing the whirling power into the power balance. The possible sources of errors in the power balance are discussed.

Bishal Silwal performed the numerical simulations and study and wrote the paper. Paavo Rasilo and Antero Arkkio helped analyse the results and also contributed through discussions and commenting on the paper. Lauri Perkkiö, Antti Hannukainen and Timo Eirola contributed by commenting on the paper.

In this paper, the influence of dynamic rotor eccentricity on the torque of a cage induction machine is studied. The main focus of the study is the behaviour of the torque harmonics with increasing eccentricity.

Bishal Silwal did the numerical simulations and wrote the paper. Paavo Rasilo and Anouar Belahcen helped through their suggestions, discussions and comments. Antero Arkkio contributed by commenting on the paper.

Publication IV: “Prospects and Limitations of Power Balance Approach for Studying Forces and Electromagnetic Damping in Electrical Machines”

This paper mainly deals with the study of electromagnetic forces due to dynamic rotor eccentricity and the effect of different mesh variations on the computed forces. The use of the power balance of the machine to calculate the forces and to study the damping of mechanical vibrations is presented and explained. The limitations associated with the approach are discussed.

Bishal Silwal performed the numerical simulations, wrote the paper and analysed the result. Paavo Rasilo and Antero Arkkio contributed through discussions and commenting on the paper.

Publication V: “Measurement of Torque Harmonics of a Cage Induction Machine under Rotor Eccentricity”

In this paper, a new measurement rig to measure torque and its harmonic components is presented. The measurement set-up is very flexible in the sense that it can be used to measure with a wide range of load and under a wide range of rotor eccentricity. Some initial test results have been presented. The measured results are also compared with the simulations.

Paavo Rasilo and Anouar Belahcen gave the idea of the measurement. Bishal Silwal designed and built the measurement set-up, performed the measurements and simulations, analysed the results and wrote the paper. Ari Haavisto helped build the set-up. Antero Arkkio contributed through suggestions for the measurement and commenting on the paper.
List of Symbols and Abbreviations

\( \beta \) Index for the time-integration method
\( \gamma \) Smoothing function used in the Eggshell method
\( \varepsilon \) Permittivity of the material
\( \mu \) Permeability of the material
\( \mu_0 \) Permeability of free space
\( \nu \) Reluctivity of the ferromagnetic material
\( \rho \) Electric charge density
\( \sigma \) Conductivity of the material
\( \tau \) Maxwell stress tensor
\( \phi \) Reduced electrical scalar potential
\( \psi \) Magnetic scaler potential
\( \psi_{ki}, \psi_{kf} \) Flux linkages of phase \( k \) at the beginning and the end of a certain time interval
\( \omega_m \) Mechanical angular speed of the rotor
\( \omega_n \) Natural frequency of the rotor
\( \omega_w \) Whirling frequency of the rotor

\( A \) Magnetic vector potential
\( a \) Unbalance vector
\( B \) Magnetic flux density vector
\( B_x \) Component of magnetic flux density along \( x \)-direction
\( B_y \) Component of magnetic flux density along \( y \)-direction
\( B_z \) Component of magnetic flux density along \( z \)-direction
\( B_r \) Component of the magnetic flux density along radial direction
List of Symbols and Abbreviations

\( B_t \) \hspace{1cm} 
Component of the magnetic flux density along tangential direction

\( D \) \hspace{1cm} 
Electric flux density

\( D_e \) \hspace{1cm} 
Damping provided by the electromagnetic system

\( D_m \) \hspace{1cm} 
Viscous damping from the surrounding fluid

\( E \) \hspace{1cm} 
Electric field strength

\( e_z \) \hspace{1cm} 
Unit vector parallel to \( z \)-axis

\( \text{err} \) \hspace{1cm} 
Relative error in the power balance

\( F \) \hspace{1cm} 
Force vector

\( F_x \) \hspace{1cm} 
Component of force along \( x \)-direction

\( F_y \) \hspace{1cm} 
Component of force along \( y \)-direction

\( F_r \) \hspace{1cm} 
Component of the eccentricity force along radial direction

\( F_t \) \hspace{1cm} 
Component of the eccentricity force along tangential direction

\( f_s \) \hspace{1cm} 
Supply frequency

\( f_w \) \hspace{1cm} 
Whirling frequency of the rotor in Hz

\( G \) \hspace{1cm} 
Jacobian matrix of the iso-parametric mapping

\( H \) \hspace{1cm} 
Magnetic field strength vector

\( i \) \hspace{1cm} 
Current in the conductor

\( J \) \hspace{1cm} 
Electric current density

\( K \) \hspace{1cm} 
Stiffness of the shaft

\( l \) \hspace{1cm} 
Length of the machine

\( m \) \hspace{1cm} 
Number of phases of the machine

\( M \) \hspace{1cm} 
Mass of the rotor

\( n \) \hspace{1cm} 
Unit vector normal to the surface

\( n \) \hspace{1cm} 
Order of harmonics

\( N_{\text{aug}} \) \hspace{1cm} 
Number of elements in the integration region, Coulomb’s method

\( P_{\text{in}} \) \hspace{1cm} 
Power input to the machine

\( P_{\text{loss}} \) \hspace{1cm} 
Electromagnetic losses

\( P_{\text{whirl}} \) \hspace{1cm} 
Whirling power

\( p \) \hspace{1cm} 
Number of pole pairs in the machine

\( p_c \) \hspace{1cm} 
Centre point of the rotor

\( Q_r \) \hspace{1cm} 
Number of rotor slots

\( R \) \hspace{1cm} 
DC resistance of the conductor

\( r \) \hspace{1cm} 
Radius of integration used for torque and force computation
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s, r_i$</td>
<td>Radii of concentric boundaries of the integration region in Arkkio’s method</td>
</tr>
<tr>
<td>$S_{ng}$</td>
<td>Cross-sectional area of the integration region in Arkkio’s method</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque of the machine</td>
</tr>
<tr>
<td>$u$</td>
<td>Voltage</td>
</tr>
<tr>
<td>$V$</td>
<td>Integration volume</td>
</tr>
<tr>
<td>$v$</td>
<td>Velocity vector of the whirling motion</td>
</tr>
<tr>
<td>$W_c$</td>
<td>Co-energy functional for the finite element domain</td>
</tr>
<tr>
<td>$W_f$</td>
<td>Energy of the electromagnetic field</td>
</tr>
<tr>
<td>$W_{in}$</td>
<td>Energy input to the system</td>
</tr>
<tr>
<td>$W_{loss}$</td>
<td>Energy loss in the rotor cage</td>
</tr>
<tr>
<td>$W'$</td>
<td>Co-energy of the magnetic field</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D</td>
<td>Two-Dimensional</td>
</tr>
<tr>
<td>3-D</td>
<td>Three-Dimensional</td>
</tr>
<tr>
<td>AMB</td>
<td>Active Magnetic Bearing</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>IPM</td>
<td>Interior Permanent Magnet</td>
</tr>
<tr>
<td>PSH</td>
<td>Principal Slot Harmonic</td>
</tr>
<tr>
<td>RSH</td>
<td>Rotor Slot Harmonics</td>
</tr>
<tr>
<td>SMPM</td>
<td>Surface Mounted Permanent Magnet</td>
</tr>
<tr>
<td>UMP</td>
<td>Unbalanced Magnetic Pull</td>
</tr>
</tbody>
</table>
1. Introduction

1.1 Background

Electrical motors are the biggest consumer of electricity and account for about 45% of the global electric energy consumption (Waide & Brunner 2011). Moreover, with the growing trend of global industrialization, the demand of electrical motors is increasing more and more every year. On the other hand, electrical generators are the largest source of electrical energy generation all over the world. Therefore, engineers are working to make rotating electrical machines, both motors and generators, better and efficient, considering all the related aspects such as design, manufacturing and operation.

Electrical machines are electromechanical converters, and the non-linearity of the materials used, and also the fact that they include a rotating system, makes them more complex systems, at least from the design and analysis point of view. Therefore, it requires a special tool to accurately study these devices. The numerical modelling technique using the Finite Element Method (FEM) is one of them. The development of the computing resources has made FEM one of the most widely and commonly used tool for the design and analysis of electrical machines. Usually, a three-dimensional model is required to understand the exact electromagnetic phenomena; however, a two-dimensional approximation gives results with enough accuracy to understand and analyse the machine.

Electrical machines work on the principle of energy conversion through the electromagnetic field, which makes their analysis to be based on the solution of the field or its approximation. The accuracy of the solution obtained from FEM is related to the finite element discretization or the mesh used in the simulations. Some electrical machines, for instance, induction machines, have a very small air gap length. As in an electrical machine, the electromagnetic field transfers the power through the air gap, an accurate computation of the field is desired particularly in the air gap region, therefore, a proper mesh in that region is desired. A coarse mesh will lead to an inaccurate solution, which will in turn affect the accuracy of different machine quantities derived from the
electromagnetic field solution, for example, the magnetic flux density, which will further lead to an inaccurate calculation of quantities such as torque, forces and losses.

The electromagnetic torque of a motor is very important, as it is used to transfer the input electrical power fed to the machine into mechanical power output from the shaft. Generally, in the finite element analysis of electrical machines using a magnetic vector potential solution, torque is calculated from the magnetic flux density components, which are obtained from the curl of the vector potential. The spatial differentiation is the main source of error, which leads to inaccurate torque and force computation. Such accuracy issues have been reported in the literature. Although, some methods were developed in the course of time by addressing the accuracy issues in the conventional torque and force calculation methods, a reliable method that can be used as a reference for the assessment of the conventional methods is missing.

Rotating electrical machines are designed and manufactured such that the net radial force acting on the rotor should be zero; and a tangential force component that produces torque exists. To let this happen, the rotor should be concentric with the stator, which is an ideal condition. However, in practice, the rotor may be eccentric. Two types of eccentricity are very common – static eccentricity and dynamic eccentricity. In static eccentricity, the rotor is shifted from the centre of the stator bore with a certain radius, while in dynamic eccentricity, the rotor is additionally whirling around the centre point of the stator. An eccentric rotor produces an imbalance force, which tends to pull the rotor in radial direction. This phenomenon is often termed as the unbalanced magnetic pull (UMP). The radial forces and the magnetic pull have been researched for a long time. However, very little attention has been given to the tangential component of the force and its importance. When the machine is under dynamic eccentricity, the forces are circulatory. In that case, the tangential component of the eccentricity force is parallel to the whirling motion of the rotor. Therefore, they combine to produce power. This power, however very small compared with the rated power of the machine, can play an important role in the electromechanical interactions within the machine. This brings a necessity to study and calculate this power and its effect on the operation of the machine.

The non-uniform air gap in an electrical machine caused by rotor eccentricity creates asymmetrical flux density distribution in the air gap. Besides producing electromagnetic forces, this might also affect other operational quantities of the machine such as the losses, torque etc. The additional losses due to an eccentric
rotor have already been studied. A comprehensive study on the behaviour of torque and torque harmonics in an eccentric machine is however still lacking.

1.2 Aim of the Work

The main objective of this research is to identify and implement an alternative approach for the calculation of the electromagnetic torque and forces in electrical machines, such that this method can be used as a reference method to validate or verify existing methods. The new method, together with the conventional methods, will be evaluated based on the variations in the finite element mesh. In other words, the effect of the air gap mesh on the torque and force computation will be studied. The same method will be applied to study the rotordynamics and the electromechanic interaction in electrical machines with an eccentric rotor.

1.3 Scientific Contributions

The scientific contributions of this dissertation can be summarized as follows:

- The energy-conserving property of two different time-integration methods commonly used in the numerical simulation of electrical machines are compared, and based on the results, the trapezoidal rule is proposed as a method that conserves energy in both linear and non-linear problems.

- A method for the computation of torque, based on the power balance of the machine, is presented. This method is seen to be insensitive to the type of finite element mesh used in the air gap. Therefore, accurate torque calculation even with a sparse mesh in ensured.

- A detailed investigation of the influence of the finite element mesh in the computation of torque and electromagnetic forces by conventional methods is presented. It has been found that the density of the mesh and the choice of the band of elements for torque or force computation and for the rotation of the rotor affect the output torque and force.
• The power balance aspect of the rotordynamics of an electrical machine is presented. The concept of the *whirling power* is introduced, and the method to calculate it is proposed.

• An approach to calculate the eccentricity force and to study the electromagnetic damping of mechanical rotor vibrations, from the power balance of the machine, is proposed.

• Generally, in the numerical computation of torque, the radius of integration is chosen by considering the geometrical centre point of the stator as the centre, which in case of an eccentric rotor is not true. A correct way of calculating torque of a machine with dynamic rotor eccentricity is proposed.

• The torque of an eccentric machine is thoroughly investigated. It has been shown how the eccentricity increases the amplitude of some of the harmonic components around the frequency close to the frequency of the principal slot harmonics (*PSH*). The results could be useful for diagnostics purposes.

• A novel measurement rig for the measurement of the torque waveform and harmonics of a cage induction machine is designed and tested.

### 1.4 Structure of the Dissertation

The research work carried out for this dissertation is divided into and written as five different chapters. The current chapter is Chapter 1, which gives a brief introduction and background of the research topic, discusses the aim of the research and presents the scientific contributions. Chapter 2 presents a review of the literature relevant to the research. In Chapter 3, an overview of different methods used to carry out the research work is presented. The results and the findings are presented and discussed in Chapter 4. Chapter 5 presents a brief conclusion based on the results obtained in this dissertation.
2. Review of Relevant Literature

2.1 Force and Torque Calculation using FEM

The finite element analysis of electrical machines was a subject of extensive research during the 80's. However, it was already applied for the electromagnetic field problems in early 70's. Chari & Silvester (1971) and Chari (1971) used the finite element method to study two-dimensional stationary magnetic field in a turboalternator and a dc machine. The technique was further developed in the 80's (Tandon et al. 1983; Ito et al. 1981; Bouillault & Razek 1983; Brunelli et al. 1983; Strangas & Theis 1985; Strangas 1985; Shen et al. 1985; Arkkio 1987), and the 2-D time-stepping finite element method, simultaneously solving the field and circuit equations, was made possible.

Force and torque computations are key issues in the numerical analysis of an electrical machine. Many methods have been put forward in the course of time to compute the electromagnetic force and torque. In general, the force computation can be divided into two categories – computation of force as a global quantity, that is, acting on a body, and second – the computation of local forces, for instance, force density on the surface of a material. Bastos & Sadowski (2003) also classified different methods and techniques to calculate the electromagnetic forces and torques into two categories. First, direct methods that are based on the magnetic vector potential solution, and second, the methods based on the force density over the magnetic material surface. The commonly used direct methods are described here first.

The Maxwell Stress Tensor Method

The Maxwell stress tensor method is one of the oldest yet one of the most commonly used methods to compute forces and torque acting on a body placed
in a magnetic field (Reichert et al. 1976; Wignall et al. 1988). This method requires the knowledge of the magnetic field on the whole surface enclosing the body. Therefore, it is generally used in conjunction with the computation of the magnetic field. The force acting on the body is calculated by integrating the Maxwell stress tensor along a surface placed in the air and enclosing the body.

The stress tensor expressed in terms of the magnetic flux density components is given by

\[
\boldsymbol{\tau} = \frac{1}{\mu_0} \begin{bmatrix}
B_x^2 - \frac{1}{2}B_z^2 & B_xB_y & B_xB_z \\
B_yB_x & B_y^2 - \frac{1}{2}B_z^2 & B_yB_z \\
B_zB_x & B_zB_y & B_z^2 - \frac{1}{2}B_z^2
\end{bmatrix}
\]

where, \(B_x\), \(B_y\) and \(B_z\) are components of the magnetic field density in x-, y- and z-directions, respectively.

In electrical machines, the force exerted on the rotor is calculated by integrating the stress tensor along a surface in the air gap of the machine.

\[
F = \oint_S \boldsymbol{\tau} \cdot d\mathbf{S}
\]

Equation (2) can also be written as

\[
F = \oint_S \left\{ \frac{1}{\mu_0} (\mathbf{B} \cdot \mathbf{n}) \mathbf{B} - \frac{1}{2\mu_0} \mathbf{B}^2 \mathbf{n} \right\} d\mathbf{S}
\]

where, \(\mathbf{n}\) is the unit normal vector to the boundary \(S\).

In the two-dimensional analysis of electrical machines, the surface integral in (3) can be reduced to a line integral in the air gap. Moreover, in radial-flux electrical machines, the normal component of the flux density is often assumed to be significantly larger than the tangential component. Therefore, the tangential component is usually ignored. If a circle of radius \(r\), concentric with the rotor, is taken as the integration path, the force is given by

\[
F = \frac{l}{2 \mu_0} \int_0^{2\pi} r B_z^2 d\theta
\]
where, $l$ is the length of the machine, $\mu_0$ is the permeability of the free space, $B_r$ and $B_\theta$ are the radial and tangential components for the flux density, respectively. The torque of the machine can be similarly calculated as

$$T = \frac{l}{\mu_0} \int_0^{2\pi} r^2 B_r B_\theta d\theta.$$  \hfill (5)

**Method Proposed by Arkkio**

The force and torque resulting from (4) and (5) should be independent of the radius of integration $r$ in the air gap, but in practice, the results can vary substantially (Arkkio 1987). More reliable results are obtained, as proposed by Arkkio, if the line integral in (4) and (5) is transformed into a surface integral as

$$F = \frac{l}{2\mu_0 (r_s - r_i)} \int S_{ag} B_r^2 dS$$ \hfill (6)

$$T = \frac{l}{\mu_0 (r_s - r_i)} \int S_{ag} r B_r B_\theta dS$$ \hfill (7)

where, $r_s$ and $r_i$ are the radii of two concentric boundaries of the integration region in the air gap, and $S_{ag}$ is the cross-sectional area of the integration region.

**Coulomb’s Virtual Work Method**

This method, put forward by Coulomb (1983), is based on the virtual work principle and is used to determine the force and torque of a body. In this technique, the elements in the air surrounding the body are virtually deformed, and the energy function of the elements is differentiated with respect to the virtual displacement. The energy functional of the finite element domain is given by

$$W = \int_V \left( \int_0^H \mathbf{H} \cdot d\mathbf{B} \right) dV.$$ \hfill (8)
The virtual work principle is applied at the constant magnetic scalar potential. For this, the nodal values of the magnetic vector potential are kept constant while calculating the derivative of the energy. The force is calculated as the derivative of the energy functional with respect to the virtual displacement, for example, the force in $x$- and $y$-direction is given by

$$ F_{x,y} = \frac{dW}{dx, y} . \quad (9) $$

Substituting (8) into (9) and using local coordinates instead of the global ones for the subset of the virtually displaced finite elements gives

$$ F_{x,y} = \frac{1}{\mu_0} \sum_{e=1}^{N_{\text{eg}}} \left[ -B^T G^{-1} \frac{\partial G}{\partial x, y} B + \frac{1}{2} B^2 |G|^{-1} \frac{\partial |G|}{\partial x, y} \right] d\Omega \quad (10) $$

where, $N_{\text{eg}}$ is the number of elements in the integration band, and $G$ is the Jacobian matrix of the isoparametric mapping between the local coordinates and the global coordinates. In electrical machines, the summation is performed over a band of elements in the air gap of the machine.

Similarly, torque is given as

$$ T = \frac{1}{\mu_0} \sum_{e=1}^{N_{\text{eg}}} \left[ -B^T G^{-1} \frac{\partial G}{\partial \varphi} B + \frac{1}{2} B^2 |G|^{-1} \frac{\partial |G|}{\partial \varphi} \right] d\Omega . \quad (11) $$

**Method of Co-energy Variation**

In this method, the force and torque are calculated as the derivative of the magnetic co-energy with respect to the virtual displacement of the body, when the current is constant. The co-energy of the magnetic field is given by

$$ W^* = \int_{V} \left[ \int_{L} B dH \right] dV \quad (12) $$

where, $V$ is the integration volume (Bastos & Sadowski 2003). The force acting on the body along the direction of the virtual displacement, for instance, along $x$-direction, is calculated by
The derivative of (13) is approximated by the difference between two successive solutions. The solutions must be obtained with a constant current, and the rotor displaced by a distance equal to \( dx \) between the two calculations, which makes it tedious to calculate the force when the machine is supplied by power converters (Burakov 2007).

The second category, as described by Bastos & Sadowski (2003), consists of several methods based on the application of equivalent currents, equivalent magnetic charges and the combination of magnetic charges and currents. All these methods are associated with the concept of superficial force density. The main idea is to replace a magnetic material with a non-magnetic material, having a superficial distribution of the force density. The force is then integrated from the force density. Compared with the direct methods described above, the force density methods require more computational effort.

**Air Gap Element Method**

Over time, many other methods were put forward for the numerical computation of the force and torques in electrical machines. The Air-Gap Element method presented by Abdel-Razek et al. (1981) is one of them. This method is based on the idea that the uniform part of the air gap of the machine is discretized with a single element, often referred to as a ‘macro element’ or an ‘air-gap element’ (Abdel-Razek et al. 1982). An analytical solution of the field in the air gap is obtained, and the nodal values of the magnetic vector potential in the stator and the rotor surfaces are used as the boundary conditions. It may be referred to as a coupled analytical-FEM method. Once the field solution is obtained, the force and torque can be calculated by, for instance, the Maxwell stress tensor method.

**The Eggshell Approach**

In the past decade, a more general approach to calculate the force has been commonly used, called the Eggshell method (Henrotte et al. 2004). This method is based on introducing a thin layer or shell surrounding the object and a
smoothing function \( \gamma \) such that the value of \( \gamma \) is 1 on the internal boundary of the shell and 0 on the outer boundary. The force is then calculated by the integral

\[
F = \int_{V} \tau \cdot \nabla \gamma dV
\]  

(14)

where, \( \tau \) is the stress tensor, and \( V \) is the volume of the shell. It could be argued that this method is equivalent to the Maxwell stress tensor method and the Coulomb’s method. However, it is advantageous in regards to its very general concept (Giet & Franck 2010).

**Other Methods**

Several other methods for force and torque computation were developed, used and discussed in the literature, for example, the method based on the stored energy presented by Marinescu & Marinescu (1988) and Dorrell et al. (2006). A semi-analytical method is presented by Hameyer et al. (1998), in which the magnetic field in the air gap is solved analytically, and the numerically computed magnetic vector potential in the rotor and stator boundary is used as a boundary condition. Similar method has also been presented by Popescu (2006). Popescu et al. (2005) presented two other methods, one of which is similar to the semi-analytical method described above, while the other is based on the virtual work principle and segregates the average and pulsating torque components. These methods are tested for a brushless PM motor. Comparison between some numerical and analytical torque computation methods can be found in the following literature: (Zarko et al. 2009; Xun & Ariu 2010; Lee & Tolbert 2009). Henneberger et al. (1992) presented a method based on the equivalent magnetizing currents for the computation of forces and concluded that the method yielded similar results to the Maxwell stress tensor for total force, but the results differed in case of local force computation.

The computation of local force distribution is needed when one is interested in the vibrations and noise of electrical machines. Bossavit (1992) introduced a method to compute the local force, using an edge-element formulation. Ren & Razek (1992) used the same method together with the virtual work principle to obtain local force in the elemental level. In this method, the magnetic energy or coenergy of one element is differentiated with respect to the virtual displacement of the nodes of that element to obtain the force. Therefore, the
forces obtained are called nodal forces. A method to calculate the nodal forces based on the Maxwell stress tensor has also been presented by Kameari (1993), which is applicable to both node-based and edge-based formulations (Nishiguchi et al. 1999). These methods are commonly used even in recent years for nodal and local force computation in electrical machines (Xiuke et al. 2003; Sathyan et al. 2016).

**Accuracy Issues of the Common Methods**

In the finite element analysis of electrical machines, the Maxwell stress tensor method or its variant, Arkkio’s method, and Coulomb’s method are the most commonly used methods for the computation of forces and torque. Coulomb’s method is closely related to the Maxwell stress tensor method, therefore, it can be used where the former applies (Reichert et al. 1976). A recently published work shows that these methods yield almost the same results (Silwal et al. 2013). The accuracy of the Maxwell stress tensor depends on the choice of the radius of integration. This method is also sensitive to the finite element discretization around the object, where the force has to be calculated, and a sharp change in the magnetic field in the iron-air interface could result in large errors in the calculated force. The dependence on the radius of integration is addressed by Arkkio’s method. However, Arkkio’s method is also not very accurate in the case of an eccentric rotor, as the air gap is non-uniform (Tenhunen 2003). On the other hand, for the same kind of finite element discretization used, Coulomb’s method yields more accurate results than the Maxwell stress tensor method (Fu & Ho 2009). Nevertheless, several other accuracy issues have been reported for these methods (Sadowski et al. 1992; Arkkio & Hannukainen 2012). Hameyer et al. (1998) and Popescu et al. (2005) describe the possible numerical errors in the field computations that result in the inaccuracy in torque calculated with the Maxwell stress tensor and Coulomb’s method and also propose different ways to enhance their accuracies.

The dependence of torque and force computation on the finite element mesh have also been reported. Burow et al. (1995) studied three types of mesh structures in the air gap of the machine to study the effect on torque and concluded that torque is mainly dependent on the remeshing algorithm used and the amount of distortion in the element brought by the movement of the rotor. Similarly, Allegre et al. (1996) also presented the influence of the mesh deformation due to a moving body on the force calculation. The results show
that a less-deformed mesh gives smaller oscillations on the force computed. The effect of meshing on the accuracy of torque has also been discussed by Miwa et al. (2004), and an adapting meshing technique to increase the accuracy has been presented. Mizia et al. (1988) also compared the common force computation methods and showed that the discretization of the air gap affects the force calculation. The force and torque calculations in these studies were based on the Maxwell stress tensor and Coulomb’s method.

The accuracy of the Air Gap Element method is also known to be dependent on the quality of the meshes in the stator and the rotor (Abdel-Razek et al. 1981). One other disadvantage of this method is the huge computational time required. In recent years, a new formulation based on this method has been presented and is claimed to be computationally efficient and accurate (Wang & Kamper 2010; Li & Aliprantis 2013).

2.2 Methods Based on Energy and Power

In the finite element analysis of electrical machines, the concept of calculating force and torque from the field solution, considering the stored or dissipated energy, has existed for a long time (Coulomb 1983; Marinescu & Marinescu 1988). A method to predict the torque from the energy converted over one electrical cycle has been presented in (Staton et al. 1996) and (Miller et al. 2008). The approach in both the studies was based on the energy loop obtained as the flux-MMF diagram such that the average torque is proportional to the area of the loop. Bianchi et al. (2007) extended the study by including the flux linkage harmonics in the analysis, which was not taken into account previously. The study focused on interior and surface-mounted permanent magnet motors (IPM and SMPM). Lately, the method has been used in the design and optimization of the PM machines, owing to its efficient computational advantage (Sizov 2013; Sizov et al. 2013).

Moreover, some recent interest in the numerical power and energy balance studies can also be found in the literature. The power balance of the machine has been used to verify the iron-loss models for 2-D and 3-D analysis of electrical machines (Dlala et al. 2010; Lin et al. 2010; Pippuri 2010; Rasilo et al. 2012). A time-stepping method for the transient analysis of machines has been presented by Ho et al. (2012) and is claimed to be power-balanced. In the same paper, the authors study the energy conservation properties of the most commonly used backward-difference method and trapezoidal rule for time-integrations in the
Simulations. However, they only focused on the average reactive power over one excitation cycle in a steady state. Fu & Ho (2013) discussed the instantaneous power balance, considering the calculation of active and reactive powers from a coupled field and circuit computation for an inductor.

Dorrell et al. (2006) considered using power balance to compute the torque of a rotating electrical machine. In recent years, Arkkio & Hannukainen (2012), Silwal et al. (2013) and Niu et al. (2013) have also proposed approaches to compute the electromagnetic torque based on the power balance of the machine. The former two works, the focus was on the average torque of the machine, while the latter one deals with instantaneous torque. The time-integration used in the simulation is very important for the accuracy of the power balance approach. The energy-balanced properties of the trapezoidal rule are discussed by Hairer et al. (2006). Niu et al. (2013) have used the backward difference method in their work, which is itself known to be energy-consuming, even in the linear case (Ho et al. 2012). Recently, Rasilo et al. (2014) studied instantaneous power balance in the numerical simulation of electrical machines. The authors propose an approach called the collocation approach that allows studying the power balance in a time step continuously.

### 2.3 Eccentricity and Forces

Reviews of the literature on the subject of rotor eccentricity and the unbalanced magnetic pull created by it presented by Dorrell (1993), Tenhunen (2003) and Burakov (2007) suggest that the discussion on this subject dates back to Fisher-Hinnen (1899). Early works on this topic were published by Rosenberg (1918), in which the UMP was evaluated by the difference in the magnetic flux densities over the opposite poles of the machine. Robinson (1943) tried to analytically describe the UMP in synchronous and induction machines, taking into account static rotor eccentricity.

Summers (1955) began to develop the theory of electromagnetic forces and UMP based on the rotating magnetic field components. He studied a two-pole induction machine. Robinson (1963) later used the same theory to study vibrations in a.c. machines and presented the vibrating characteristics of the rotor at line frequency and twice the line frequency. Based on the theory of the rotating fields, Freise & Jordan (1962) and Jordan et al. (1967) presented the UMP as a result of the interaction between the fundamental component of the magnetic field and the field harmonics having a lower and higher number of
poles. Freise & Jordan (1962) also presented the damping effect of the equalizing currents on the force and also the change of the direction of the force from the direction of shortest air gap. The theory of rotating fields has been widely used in studies related to the calculation of UMP and noise and vibrations in electrical machines (Ellison & Yang 1971; Heller & Hamata 1977; Dorrell 1993; Timar 1989).

Fruchtenicht et al. (1982) presented an analytical method based on the permeance harmonic analysis to study the electromagnetic force in induction machines. He studied the forces as a function of varying whirling frequencies of the rotor. Similar analysis was used by Berman (1993) to study the damping effect of the equalizing currents on the forces in induction machines produced due to static and dynamic eccentricity. His calculations were validated by experiments. The permeance harmonic theory was also used by Stavrou & Penman (2001), who presented a general model of an induction machine with dynamic eccentricity.

The effects of parallel stator windings on the forces and UMP are reported to have been studied in the past by Robinson (1943) and Krondl (1956). Later, Dorrell & Smith (1994) presented a general analytical method, which uses the conformal transformation technique coupled to the winding impedance approach, to study the UMP in an induction machine. The study also deals with the influence of the parallel stator windings to reduce the magnitude and the direction of the UMP. An analytical approach based on the winding impedance approach has been presented by Smith & Dorrell (1996) to study cage induction motors with static eccentricity and for any series/parallel stator winding connections. The method could also be extended to include dynamic eccentricity. The accuracy of the results are assessed by experimental investigations (Dorrell & Smith 1996).

A wide-spread study of the subject related to eccentricity and the UMP in a cage induction machine can be found in other publications of D. G. Dorrell. For instance, (Dorrell 1995; Dorrell 1996) presented the effects of a rotor cage skewing on the magnitude of the UMP. The results show an increase in the UMP in a skewed rotor. The reduced magnitude of the UMP resulting from the saturated iron core has been presented in (Dorrell 1999). (Dorrell 2000) investigates the non-uniform rotor eccentricity (rotor and stator axes are not parallel to each other) in a three-phase cage induction machine. His recent works on a similar subject are presented in (Dorrell & Hsieh 2010; Dorrell 2011;
Dorrell et al. 2011; Dorrell et al. 2013) and mainly use numerical methods for the study.

Some of the initial studies on electromagnetic forces due to eccentricity based on numerical simulations can be found in the works of Salon et al. (1992), DeBortoli et al. (1993) and Arkkio & Lindgren (1994). The former two publications considered both static and dynamic eccentricity and were mainly focused on the effect of the parallel windings on the forces. In the latter, the authors used a 2-D time-stepping FEM based on the magnetic vector potential formulation to study the force acting on the rotor of a high-speed machine. The forces were computed by using Coulomb’s method. A numerical analysis of the forces produced by static and dynamic eccentricity in a conventional cage induction machine can also be found in a study by Arkkio (1996). The effect of parallel windings, saturation of the iron core and the loading of the machine were the focus of the study.

Neves et al. (1998) studied the vibration characteristics of a switched reluctance machine by using numerical analysis. An analytical investigation of the radial force characteristics in a similar machine was performed by Garrigan et al. (1999). Electromagnetic forces of an IPM and SMPM with rotor eccentricity have been studied by Kim et al. (2001). The study showed that the UMP is higher in IPM motors compared to the SMPM motors due to substantially stronger magnetic field asymmetry. Stability problems in a large synchronous hydrogenerator due to the forces produced by the eccentric rotor have been discussed by Lundström et al. (2007). The damper winding in the synchronous machine reduces the force and also affects the stability and eigen frequencies of the rotor. Static eccentricity in a large salient-pole synchronous machine and the effect of saturation on the UMP have been studied by Perers et al. (2007). The results showed a negative effect on the eccentricity harmonics and the UMP.

A broad study of the forces produced by a whirling cage rotor has been presented by Arkkio et al. (2000). The author calculated the forces over a wide range of whirling frequency. The rotor is forced to move along a circular path around the geometrical centreline of the stator with a certain radius and whirling frequency. The author also presents a parametric model for force calculation, where the unknown parameters are calculated from the results of numerical simulations. The results are seen to have good agreement with the measurement.
The forced whirling method presented by Arkkio et al. (2000) takes a huge computation time because in order to obtain the result over a wide range of whirling frequencies, many time-stepping FEA simulations are required. To overcome this problem of the computational burden, Tenhunen et al. (2003) presented a method called the Impulse method. This method is applied in the finite element analysis by moving the rotor from its central position for a short period of time. This disturbs the magnetic field, which in turn produces forces between the rotor and stator. Thus, the frequency response function of the force is calculated by using spectral analysis techniques. The force calculated by this method showed very good agreement with that computed from the conventional method, even requiring less than 5% of computation time. The author used this method further to study the effect of equalizing currents in the rotor cage and parallel stator winding on the UMP, the effect of saturation on UMP and conical eccentricity and mixed eccentricity problems (Tenhunen 2003).

According to the literature review presented above, it is evident that rigorous research has been done to study the eccentricity, forces and the unbalanced magnetic pull created by the forces. However, very few publications regarding the study of the behaviour of torque in the case of an eccentric machine was found. Schlensok & Henneberger (2004) studied an induction machine and compared a healthy and an eccentric rotor. The results showed a slight reduction in the average torque in an eccentric rotor. The torque in an eccentric switched reluctance machine was studied by Dorrell et al. (2005). The results showed an increase in the torque of the machine with increasing eccentricity. (Sheth & Rajagopal 2004) also comes to a similar conclusion about a similar type of machine. The influence of the rotor eccentricity on the torque ripple of a surface-mounted brushless DC motor was studied by Salon et al. (2001). The authors reported that the torque ripple of such machine is not much affected by the eccentricity. It is because of the fact that the magnets increase the effective air gap of the machine, and an eccentric rotor position makes negligible change in the effective air gap of the machine. A brief discussion about the difference in torque behaviour of a cage induction machine in a healthy and eccentric case can be found in a study by Belahcen & Arkkio (2010). An analytical study on the torque of an eccentric cage induction machine was done by Dorrell (1994). The author concluded that the eccentricity has negative effect on the torque of the machine. Some studies have also been done focusing on the induction machine under mixed eccentricity, which is a case when the machine has both static and dynamic eccentricity simultaneously. For instance, (Faiz et al. 2008) employs the
stator current spectrum for the diagnosis of the mixed rotor eccentricity and also presents the effect of mixed eccentricity on the time variation of torque. Similarly, (Polat et al. 2015) briefly compares the peak value of the instantaneous torque of an induction motor for a healthy case and cases with static, dynamic and mixed eccentricity, but the authors do not elaborate on the effect of eccentricity on the average output torque of the machine or torque harmonics.

Another aspect of rotor eccentricity in electrical machines is the mechanical vibrations. The electromagnetic forces were studied from the mechanical point of view by Kovacs (1977). The author stated that the UMP acts as a spring force. Fruchtenicht et al. (1982) showed the importance of the tangential component of the force on the electromechanical interaction. The tangential force acts as a damping force that either enhances or reduces the vibration, depending on the direction of the force. Belmans et al. (1985) studied the electromechanical vibration in a flexible-shaft machine by including the electromagnetic spring constant and electromagnetic damping constant. The result showed that the electromagnetic damping constant is important to accurately study the stability of the machine. Garvey (1999) showed that the tangential force can sometime be more significant than the radial force, especially in large machines.

A coupled electromagnetic and structural FEM to study the mechanical vibrations caused by dynamic eccentricity has been presented by Ha & Hong (2001). The paper presents the dynamic response of a switched reluctance motor rotor. Laiho et al. (2009) presented control methods to suppress the rotor vibration in a two-pole cage induction machine. More recent works on the electromechanical interactions can be found, for example, in (Werner & Binder 2006; Sinervo et al. 2011; Werner 2016; Sobra et al. 2016; Yang et al. 2016).
3. Method of Analysis

3.1 Test machines

The study presented in this thesis is mainly performed in two cage induction machines of different sizes. In Publication I, the torque of a 4-pole 37 kW cage induction machine, the main parameters of which are shown in Table 1, is investigated. The two-dimensional cross-section of the machine is shown in Figure 1. The upper half of the figure shows the typical second order triangular mesh used in the simulations and the lower half shows the magnetic field solution obtained from a time-stepping simulation. The equipotential lines are also plotted. In Publication II, the effect of inaccurate torque computation due to meshing on the numerical power balance of the machine is studied for a 15 kW cage induction machine. The 4-pole machine also has the parameters listed in Table 1 and a cross-sectional geometry, as shown in Figure 2. The eccentric rotor position and the concentration of the flux density near the shorter air gap region (on the left side) can be seen from the figure.

In Publication III, the study is performed on the 15 kW machine, while Publication IV compares the force computation in both the 37 kW machine and the 15 kW machine. However, the 37 kW machine used in Publication IV has a series connected stator winding unlike the one used in the Publication I, which is the original motor with two parallel paths in the stator winding. The measurement set-up designed in Publication V consists of the same 15 kW machine as a test machine.
Figure 1. 2-D cross section of the 4-pole 37 kW cage induction machine used in Publication I. The upper half of the figure shows a typical second order triangular mesh used in the simulations. The lower half shows the magnetic flux density distribution obtained from a time-stepping simulation along with the equipotential lines.

Table 1. Parameters of the test machines used in the study

<table>
<thead>
<tr>
<th>Parameter</th>
<th>37 kW Machine</th>
<th>15 kW Machine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of poles</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Connection</td>
<td>Star</td>
<td>Delta</td>
</tr>
<tr>
<td>Rated voltage [V]</td>
<td>400</td>
<td>380</td>
</tr>
<tr>
<td>Supply frequency [Hz]</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Rated current [A]</td>
<td>70</td>
<td>31</td>
</tr>
<tr>
<td>Rated torque [Nm]</td>
<td>240</td>
<td>106</td>
</tr>
<tr>
<td>Rated power [kW]</td>
<td>37</td>
<td>15</td>
</tr>
<tr>
<td>Number of stator slots</td>
<td>48</td>
<td>36</td>
</tr>
<tr>
<td>Number of rotor slots</td>
<td>40</td>
<td>34</td>
</tr>
<tr>
<td>Outer diameter of stator [mm]</td>
<td>310</td>
<td>235</td>
</tr>
<tr>
<td>Inner diameter of stator [mm]</td>
<td>200</td>
<td>145</td>
</tr>
<tr>
<td>Outer diameter of rotor [mm]</td>
<td>198.4</td>
<td>144.1</td>
</tr>
<tr>
<td>Inner diameter of rotor [mm]</td>
<td>70</td>
<td>53</td>
</tr>
<tr>
<td>Radial air gap length [mm]</td>
<td>0.8</td>
<td>0.45</td>
</tr>
<tr>
<td>Axial length of machine [m]</td>
<td>0.249</td>
<td>0.195</td>
</tr>
<tr>
<td>Moment of inertia [kg m²]</td>
<td>0.256</td>
<td>0.063</td>
</tr>
<tr>
<td>Mass of the rotor [kg]</td>
<td>63</td>
<td>30</td>
</tr>
</tbody>
</table>
3.2 Numerical Method

3.2.1 Electromagnetic Field Computation

The magnetic field in the cross section of the machine is solved by using a two-dimensional finite element analysis. The time-stepping method based on the $A - \phi$ formulation, where $A$ is the magnetic vector potential and $\phi$ is the reduced electric scalar potential, is used. An electrical machine can be assumed to have a quasi-static magnetic field. In such a case, the Maxwell equations in the differential form are written as

\begin{align}
\nabla \cdot \mathbf{D} &= \rho \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} &= \mathbf{J}
\end{align}
and the material equations that relate to the quantities in the above equations with each other are given by

\begin{align}
D &= \varepsilon E \\
B &= \mu H \\
J &= \sigma E
\end{align}  \tag{19} \tag{20} \tag{21}

where, $D$ is the electric flux density
$B$ is the magnetic flux density
$E$ is the electric field strength
$H$ is the magnetic field strength
$J$ is the electric current density
$\rho$ is the electric charge density
$\varepsilon$ is the permittivity of the medium
$\mu$ is the permeability of the medium and
$\sigma$ is the conductivity of the medium.

The magnetic vector potential can be defined as

$$B = \nabla \times A$$  \tag{22}

which satisfies (16). Using (22) together with the material equation (20) and the Maxwell equation (18), we obtain

$$\nabla \times (\nu\nabla \times A) = J$$  \tag{23}

where $\nu$ is the magnetic reluctivity. Now, defining $\phi$ as the electric scalar potential and using (17), (21) and (22), we obtain

$$J = -\sigma \frac{\partial A}{\partial t} - \sigma \nabla \phi.$$  \tag{24}

Combining (23) and (24), we obtain

$$\nabla \times (\nu\nabla \times A) + \sigma \frac{\partial A}{\partial t} + \sigma \nabla \phi = 0.$$  \tag{25}
Also, the current density satisfies the divergence $\nabla \cdot J = 0$, which gives

$$\nabla \cdot \left( \sigma \frac{\partial A}{\partial t} \right) + \nabla \cdot (\sigma \nabla \phi) = 0. \quad (26)$$

Equations (25) and (26) are the governing equations of the finite element method used in this study.

In general, one requires a full three-dimensional model to completely describe the actual electromagnetic phenomena. But the complex geometry of the machine, the non-linearity of the material and the movement of the rotor makes the three-dimensional problem complex. Therefore, the problem is reduced to a two-dimensional model by assuming that the magnetic field does not depend on the $z$-coordinate. The magnetic field is solved in the $x-y$ plane of the machine cross section, $z$-axis being parallel to the axis of the shaft. Thus, the magnetic vector potential and the current density are given by

$$A = A(x, y, t) e_z \quad (27)$$

$$J = J(x, y, t) e_z \quad (28)$$

where, $e_z$ is the unit vector parallel to the $z$-axis.

If the current density is integrated over the cross section of the conductors, a relation between the voltage $u$, current through the conductors $i$ and its dc-resistance $R$ can be obtained as

$$u = Ri + R \int_S \sigma \frac{\partial A}{\partial t} dS \quad (29)$$

The windings and the rotor cage circuit equations are formed by using Kirchoff’s law and (29) and then simultaneously solved with the magnetic field equations. The end winding fields are modelled approximately by adding the end winding impedances to the circuit equations. In reality, the ferromagnetic materials used in the electrical machine construction are highly non-linear. In this dissertation, both linear and non-linear materials have been considered. For a linear problem, the core of the machine is considered to have a constant relative permeability, which is set to 1000. For a non-linear problem, the non-linearity of the material is modelled by using a single valued magnetization curve.
The rotation of the rotor is modelled by the moving band technique. In this method, the mesh in the stator and the rotor does not change, and the field and circuit equations are written in their own frames of reference. The solutions are then forced to match in the air gap. At each time-step, the rotor is rotated by an angle corresponding to its angular speed. The air gap mesh is remeshed at every time step.

Publications II, III and IV deal with the study of the cage induction machine under rotor eccentricity. The static eccentricity is modelled by shifting the stator mesh with a fixed displacement equal to the relative eccentricity expressed as a percentage of the radial air gap length. The whirling motion of the rotor in case of dynamic eccentricity is modelled by forcing the centre position of the rotor to move along a circular path at a constant speed. The radius of the circular path, also called the whirling radius, is equal to the relative eccentricity expressed as a percentage of the radial air gap length. For instance, if the eccentricity is set to 11%, the whirling radius equals 11% of radial air gap length.

3.2.2 Air Gap Mesh

The finite element mesh used in all the simulations in this dissertation consist of second order triangular elements. The number of elements and the nodes in the mesh depend on the number of symmetric sectors of the machine cross-section under study. For instance, if a normal healthy machine is studied, it is sufficient to simulate only one symmetric sector of the machine. In such a case, the mesh used in the simulation of the 37 kW machine has around 3500-4500 elements and around 7000-9000 nodes, depending on the type of mesh used in the air gap in Publication I. However, if the machine under study has a whirling rotor, the full cross section of the machine should be simulated. In such a case, the mesh used in the simulation of the same 37 kW machine under dynamic eccentricity has around 14000-18000 elements and around 28000-36000 nodes.

One primary objective of this dissertation is to investigate the accuracy of typical torque and force computation methods for different types of mesh used in the simulations. For that purpose, different variations of the mesh in the air gap of the machine are produced and used in the simulations. Therefore, the exact number of the elements and nodes in the whole mesh depends on the type of mesh used in the air gap of the machine. Usually, the air gap can be divided into one or more layers of elements. Figure 3 shows four different arrangements of a finite element mesh in the air gap consisting of single, double and triple layers with two different shapes of elements in the middle band.
In addition to the number of layers, the band used for movement of the rotor and for torque or force computation can be changed to bring more variations. For instance, in a single-layer mesh, both the rotation of the rotor and torque or force computation is done by using the same band of elements. But for a double- and triple-layer mesh, the band of elements used for the rotation of the rotor and for torque or force computation may be the same or different. Altogether 14 different combinations of the number of layers and the bands used for the torque or force computation and the rotor movement, as shown in Table 2, were made.

![Finite element mesh in the air gap](image)

**Figure 3.** Finite element mesh in the air gap (a) one-layered (b) two-layered (c) three-layered with right-angled triangular elements in the middle band; (d) three-layered with equilateral triangular elements in the middle band.
Table 2. Different combinations of layers of elements and bands used for rotor movement and torque or force computation

<table>
<thead>
<tr>
<th>Combination</th>
<th>Number of Bands</th>
<th>Movement</th>
<th>Torque/Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
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<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
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<td>1</td>
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</tr>
<tr>
<td>7</td>
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<td>1</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
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<td>2</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
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<td>2</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

### 3.3 Power Balance Method

This method is based on the principle of energy conservation in electrical machines. An expression to compute the average torque of the machine is derived from the power balance of the machine. The power balance of an electrical machine in motoring mode is given by

\[
P_{in} = P_{loss} + \frac{dW_f}{dt} + T\omega_m
\]  

(30)

where \(P_{in}\) is the input power, \(P_{loss}\) is the electromagnetic losses, \(W_f\) is the energy of the electromagnetic field and \(T\omega_m\) is the power transmitted by the torque.

If the angular speed \(\omega_m\) is assumed to be constant, the power balance expression given by (30) can be integrated for a certain period of time \(\Delta t\) to obtain an expression for the average torque of the machine.

\[
\int_{t_0}^{t_0+\Delta t} T\,dt = \frac{1}{\omega_m} \int_{t_0}^{t_0+\Delta t} \left( P_{in} - P_{loss} - \frac{dW_f}{dt} \right) dt
\]  

(31)

\[
T^a = \frac{P_{in}^a - P_{loss}^a - \frac{\Delta W_f}{\Delta t}}{\omega_m^a}
\]  

(32)
Here, the superscript \(^{\text{a}}\) denotes the average value. Since torque is obtained by integrating the power, this method of calculating torque is also called the energy balance method, for example, in Publication I.

The stator winding has been modelled as a filamentary winding without eddy currents. In this case, the resistive stator losses can be excluded from the power balance and the input power to the machine is calculated using the currents and the flux linkages of the stator phases

\[
P_{\text{in}}^{\text{a}} = \frac{\Delta W_{\text{in}}}{\Delta t} = \frac{1}{\Delta t} \sum_{k=1}^{m} t_k \, d \psi_k
\]  

(33)

where \(i_k\) is the current of phase \(k\), \(\psi_{ki}\) and \(\psi_{kf}\) are the flux linkages of phase \(k\) at the beginning and end of the period \(\Delta t\). \(m\) is the number of phases.

\(P_{\text{loss}}\) is obtained as

\[
P_{\text{loss}}^m = \frac{\Delta W_{\text{loss}}}{\Delta t} = \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \left( -E \frac{\partial A}{\partial t} \right) dtdV
\]  

(34)

\[
= \frac{1}{\Delta t} \int_{t_0}^{t_0+\Delta t} \sigma \left( \frac{\partial A}{\partial t} - \frac{u}{l} e_z \right) \frac{\partial A}{\partial t} dtdV
\]  

(35)

where \(V\) is the volume of the solution region, \(E\) is the electric field strength, \(A\) is the magnetic vector potential, \(\sigma\) is the conductivity, \(u\) is the electric scalar potential and \(l\) is the length of the machine. In the simulations, the core material has been treated as a non-conducting material without losses.

The change in the magnetic field energy \(W_t\) over the period of time is calculated from the magnetization curves of the materials and flux densities \(B_i\) and \(B_f\) at the beginning and the end of the period.

\[
\Delta W_t = \int_{B_i}^{B_f} \mathbf{H} \cdot d\mathbf{B} dV
\]  

(36)

This method gives average torque in a time interval. The time variation of the torque can be calculated if \(\Delta t\) in (32) is equal to the length of one time step used in the simulation. For an average torque in the steady state of the machine, the
time interval $\Delta t$ should be chosen to be at least one fundamental period of the machine. The time derivatives are approximated by first order difference ratios and the time integrals in \((31)-(35)\) are summed up time-step by time-step. Over one time step, the input energy and the energy consumed by the resistive loss in the rotor cage are, respectively,

$$\Delta W_{\text{in}} = \sum_{n=1}^{m} \left[ \beta i_{n,k+1} + (1 - \beta) i_{n,k} \right] (\psi_{n,k+1} - \psi_{n,k})$$  \(37\)

$$\Delta W_{\text{loss}} = \int \sigma \cdot \frac{\Delta t}{\nu} \left[ -\beta u_{k+1} + (1 - \beta) u_k \right] e_z \cdot \left( A_{k+1} - A_k \right) \cdot dV$$ \(38\)

where $\beta$ is the variable that defines the time integration method used in the simulation. For example, $\beta$ is equal to 0.5 for the trapezoidal rule and 1 for the implicit Euler method.

### 3.4 Energy Conserving Time-Integration Method

For the method described in the previous section to yield accurate results, the time integration method used in the simulation should be energy-conserving. To identify the suitable time-integration method, the error in the power balance of the machine in a locked rotor case has been studied. A time-stepping finite element analysis is performed on the 37 kW machine. One period of the fundamental supply frequency is divided into 600 time steps. Since the shaft power is zero in the locked rotor condition, one can see if the input power is in good agreement with the resistive losses and the magnetic field energy. The relative error is calculated as

$$\text{err} = \frac{(P_{\text{in}} - P_{\text{loss}} - \Delta W_{\text{f}})^a}{P_{\text{in}}^a}.$$ \(39\)

The trapezoidal rule and the implicit Euler method are both tested for power balance. Both linear and nonlinear magnetic properties of the core materials are used for each of the time integration methods. The absolute values of the relative
error when the trapezoidal rule is used are shown in Figure 4 and Figure 5 shows when the implicit Euler method is used. The relative error is shown with respect to the number of time steps per period of the supply frequency.

![Figure 4](image_url)

**Figure 4.** Relative error in the power balance as a function of the number of time-steps, when the trapezoidal rule was used for time-integrations in the simulation.

![Figure 5](image_url)

**Figure 5.** Relative error in the power balance as a function of the number of time-steps, when the implicit Euler was used for time-integrations in the simulation.

The trapezoidal rule shows a relatively small error. In the linear case, the error is very small and reducing the time-step size does not change the error significantly, however, it has a decreasing trend in the nonlinear case. With a higher number of time steps per period, the error in the nonlinear case reduces significantly. In the case when implicit Euler is used, the order of the error for both linear and nonlinear case does not have much difference. However, these
are very high compared to that obtained for the trapezoidal rule. In the time-stepping finite element simulations of electrical machines, one period of supply frequency is typically divided into around 400-800 time steps. Results show that for this number of time steps, with trapezoidal rule used, the power balance is well-fulfilled to allow torque computation based on it, whereas the implicit Euler does not conserve the energy well. The two time-integration methods are further compared in Publication I. In other publications, only the trapezoidal rule is used.

3.5 **Whirling Power and Force Computation**

The power balance of (30) is only valid for a normal healthy machine. When the machine is under eccentricity, the non-uniform air gap creates asymmetrical flux density distribution in the air gap, which produces forces, as shown in Figure 6.

![Figure 6. Forces in an eccentric machine.](image)

The force produced by static eccentricity is mainly directed towards the shortest air gap. When the machine is under dynamic eccentricity, the forces are circulatory by nature. As can be observed from Figure 6, the tangential component of the force and the whirling motion are parallel. In that case, the tangential component of the force and the velocity vector of whirling combine to produce power. This power is termed as ‘whirling power’.

Therefore, the power balance of an eccentric machine should also include the whirling power in the power balance expression. If \( \mathbf{F} \) is the force vector and \( \mathbf{v} \) is the velocity vector of the whirling motion, the whirling power can be calculated by

\[
P_{\text{whirl}} = \mathbf{F} \cdot \mathbf{v}.
\]  

(40)
Thus, the power balance of the eccentric machine is given by

\[ P_{in} = P_{loss} + \frac{dW}{dt} + T\omega_{in} + P_{whirl}. \]  

(41)

The power balance of (41) should always hold for an eccentric machine. In that case, it can be used to validate the force (tangential component, because the radial force component is perpendicular to the whirling motion, therefore, does not contribute to the whirling power) computed by other existing force computation methods. For this, the whirling power computed from (40) is validated against the whirling power computed from (41). Alternatively, (41) can also be used to compute the tangential component of the force itself. If the other terms in (41) are known, the whirling power can be calculated, and thus, the tangential force can be deduced from the whirling power. However, this approach is not yet robust enough for force computation. This has been explained in Chapter 4 of this dissertation. More analysis can also be found in Publications II and IV.

### 3.6 Analysis of Electromagnetic Damping

The behaviour of the eccentricity forces as a function of the whirling frequency can be studied by performing the numerical impulse test (Tenhunen et al. 2003). Such test was performed on the 15 kW machine, and the forces as a function of the whirling frequency are shown in Figure 7.

The eccentricity harmonics dampen the forces in all other frequencies, except some frequencies near the vicinity of the mechanical angular frequency of the rotor. Therefore, both forces have sharp maxima near those frequencies. The tangential force not only has a sharp maximum, but also changes its direction within this frequency range. The whirling power described in the earlier section is mainly contributed by the tangential force, since the radial force is orthogonal to the velocity vector of the whirling. Thus, the whirling power also changes its direction with the frequency range, which means that the power is either added to or lost from the system, depending on the frequency of the whirling motion.
Method of Analysis

Figure 7. Eccentricity forces as a function of whirling frequency computed using the impulse method for the 15 kW machine. An impulse amplitude equal to 20% of the air gap length was applied to the rotor position in a horizontal direction, and the forces were calculated from the frequency response function. The result shows that the forces have a sharp maximum near the mechanical frequency of the rotor (24.2 Hz). The tangential force changes direction within some range of frequency.

The whirling power acts as the stabilizing or destabilizing power depending on the direction of the force. Therefore, it is associated with the damping of the vibrations. The damping provided by the electromagnetic system to the mechanical vibration can be estimated from the whirling power. Thus, the power balance approach has been used to study electromechanical interaction by calculating the electromagnetically induced damping (EID) from the whirling power and solving the mechanical behaviour of the system. Here, the mechanical behaviour of the system is modelled by the Jeffcott rotor model, as shown in Figure 8.

Figure 8. Jeffcott rotor model. It consists of a rigid disc in the middle of a massless flexible shaft. The disc is assumed to move only in x-y plane.
The rotor is modelled as a rigid disc of mass \( M \) in the middle of a uniform, massless and flexible shaft with stiffness \( K \). The shaft is supported by rigid bearings at both ends, and the disc is assumed to move only in \( x - y \) plane. If \( \vec{p}_c \) defines the centre point of the rotor as \( x(t) + jy(t) \), \( D_m \), \( a \), and \( \omega_m \) are the viscous damping from the surrounding fluid, the amplitude of the unbalance and the rotor speed, respectively, the equation for the rotor motion can be given as

\[
M\ddot{\vec{p}}_c + (D_m - D_e)\dot{\vec{p}}_c + K\vec{p}_c = Maa_m^2e^{j\omega_m t}
\]

where, \( D_e \) is the electromagnetically induced damping calculated from the whirling power as

\[
D_e = \frac{2P_{\text{whirl}}}{(a\omega_m)^2}.
\]

### 3.7 Design of a measurement set up

A measurement system was designed, and some preliminary results were presented in Publication V. The measurement rig is shown in Figure 9. It consists of the test machine suspended with four horizontal arms connected to the frame of the machine in both the drive and the non-drive ends. The machine was equipped with active magnetic bearings (AMB) used to create eccentricity and the whirling motion of the rotor and also measure the resulting forces. Only radial bearings were used. The electrical motor acts as an axial bearing. The radial bearings used were eight-pole heteropolar bearings with bias-current linearization. The operating principle and the control parameters are described by Lantto (1999). Although the AMB can be used to create eccentricity, as well as measure the eccentricity forces, it was only used to create eccentricity, as the force measurement procedure is not fast enough for the intended use of harmonic analysis.
Four piezoelectric force sensors were placed vertically beneath the horizontal arms, which measured the vertical forces. These force sensors only measured the dynamic force components. The output of the sensor was a charge signal that was converted to a voltage signal, using an industrial charge amplifier. The range of the amplifier can be adjusted to get good resolution of the measured signal. The force was obtained by using the amplifier gain and its output voltage. The sampling frequency used was 100 kHz. The schematic diagram of the axial top view of the measurement rig for clear illustration of the position of the sensors can be seen in Publication V. If \( F_{z1}, F_{z2}, F_{z3}, \) and \( F_{z4} \) were the vertical forces measured by sensors 1, 2, 3 and 4, respectively, the net force in the vertical direction is given by

\[
F_z = F_{z1} + F_{z2} + F_{z3} + F_{z4} .
\]  

Torque can be extracted from the measured force. Since the sensor only measures dynamic force, we only get the dynamic component of torque. The harmonic components of torque were obtained by performing a fast Fourier transform (FFT) of the torque signal. The complete measurement rig also consisted of the load, which was coupled to the shaft of the test machine through a flexible coupling. A torque transducer was connected to the shaft, next to the coupling, to measure the dc-component or the static component of torque. Thus, the complete torque waveform could be obtained by the superposition of the static and the dynamic components of torque. A 37 kW induction machine as connected as the load machine.
4. Results and Discussion

4.1 Influence of the Finite Element Mesh in Torque Computation

The torque computed from the power balance approach is compared with that computed from Coulomb’s method for different types of mesh variations shown in Table 2. For the linear case, the result is shown in Figure 10 and Figure 11 for the nonlinear case. Here, the simulation is performed on the 37 kW induction machine, and the torque is averaged over two periods of supply frequency.

![Figure 10. Average torques for different mesh combinations of Table 2. The results shown are for a 37 kW induction machine simulated at rated load. The core material has linear magnetic properties.](image)

Results show that the torque computed from the Coulomb’s method is sensitive to the type of mesh used in the air gap, while the torque computed from the power balance approach seems to be independent from such variations in the mesh. Both the linear case and the nonlinear case have a similar trend, although the amplitudes of torque vary. It is also worth noticing that when the same band of elements is used for both rotation and torque computation (mesh combinations 1, 2, 5, 6, 10 and 14), torque from both methods are very close.
Results and Discussion

Figure 11. Average torques for different mesh combinations of Table 2. The results shown are for a 37 kW induction machine simulated at rated load. The core material has non-linear magnetic properties.

The above results are obtained from a healthy machine. Figure 12 shows the result of a similar study on a 15 kW machine under dynamic eccentricity. The influence of the mesh seems severe in cases of a whirling rotor, considering the amplitude of the variation in the torque.

Figure 12. Average torques calculated for different mesh combinations of Table 2 for 15 kW cage induction machine, when the machine is under rotor eccentricity (whirling rotor). The whirling motion adds more error in the torque computed from Coulomb’s method.

Next, the effect of the shape and size of the elements in torque computation is studied. For this purpose, a three-layer mesh is used in the air gap of the machine, and the middle band is used to compute torque (mesh combination 13). The element in torque computation band is set to be an equilateral triangle (Figure 3(d)). The elements’ sides in each element can be shifted, so that the
element gradually becomes a right-angled triangle (Figure 3(c)). Then, the elements were gradually shifted back to the equilateral ones. Figure 13 shows the effect of the change in shape of the elements on average torque, for a linear case. Figure 14 shows the same for the nonlinear case. The horizontal axis in the figures represents the shift of the element sides scaled by the length of the element side. When the shift is 0, the shape of the element corresponds to a right-angled triangle, and when the shift is ±0.5, the shape of the elements corresponds to an equilateral triangle.

**Figure 13.** Effect of the shape of elements in average torque calculation, 37 kW cage induction machine; core material has linear magnetic properties.

**Figure 14.** Effect of the shape of elements in average torque calculation, 37 kW cage induction machine; core material has non-linear magnetic properties.

The torque from the power balance method seems to be independent of the shape of the elements. The typical behaviour of the torque computed by Coulomb’s method when the element shape is changed is interesting. The torque from both methods is almost same when the elements are equilateral.
triangles, but the differences increase as the shape changes to a right-angle. However, the difference between maximum and minimum torque is not significantly large.

Furthermore, total harmonic distortion in the torque was studied for different mesh variations. For this, two cases were considered. In the first case, the same band of elements was used for both rotation of the rotor and for torque computation. In the second case, different bands were used. In addition, each case was studied for a two-layer mesh and a three-layer mesh. So, altogether four different mesh combinations were studied. Figure 15 shows the distortion in the torque as a function of the number of time-steps per period used in the simulation when torque is calculated with Coulomb’s method. The numbers in the legend correspond to the mesh combinations as shown in Table 2, for example, 2-2-1 means that the mesh used is a two-layer mesh with the second band used for rotation and the first band used for torque computation. The result shows relatively higher distortion when the same band of element is used for both rotation and torque computation, for both a two-layer and three-layer mesh. It should be noted here that the total harmonic distortion in the torque computed by Coulomb’s method is also dependent on the mesh in the air gap.

The power balance method, however, requires a smaller step size to accurately model the harmonics in the torque, as shown in Figure 16. The result shows that the choice of the band for rotation and torque computation did not affect the distortion. However, slight differences are observed when the number of the layers of elements is different.
Results and Discussion

4.2 Influence of Finite Element Mesh on Force Computation

The influence of the mesh on the computation of the force was studied for both 37 kW and 15 kW machines, when the machines were under dynamic rotor eccentricity. Same mesh variations as in Table 2 were used. The eccentricity was set to 33%, and the machines were simulated at the rated voltages and rated slips. The forces were calculated using the Coulomb’s virtual work method (Equation (10)) and are shown as an average force over an interval.

Figure 17 shows the radial and the tangential component of the forces for all 14 mesh combinations for the 37 kW machine as well as how the variations in the air gap mesh result into varying forces. Although the variations are visible, the difference between the maximum and the minimum values of force is not significantly high. For radial force, the difference is about 6 N and for the tangential force, it is about 0.7 N. The variations above are negligible, if one takes into account the mean force. But the situation can get worse if, for example, the quality of the mesh is reduced. The 15 kW machine has the radial air gap about half the length of the 37 kW machine. In addition, the number of nodes in the stator and rotor surfaces are fewer compared with the 37 kW machine. This affects the shape and the regularity of the elements in the air gap. In that case, for the same type of mesh used, the elements in the 37 kW machine are more uniform, regular and of better quality than in the 15 kW machine. This can be seen in Publication IV, from the comparison of meshes of the two machines.
The force computed for the 15 kW machine is shown in Figure 18. Large variations in the force can be seen in this case. The shape of the elements used also shows considerable effect on the forces. Figure 19 shows the forces when the shape of the elements in the force computation band was changed from an equilateral triangle to a right-angled triangle and back to the equilateral in the 37 kW machine. Figure 20 shows the same for the 15 kW machine. The variations are again significant in the case of the 15 kW machine.

The difference between the maximum and the minimum value of force when the mesh type and the shape of elements are changed is shown in Table 3 for both machines as a percentage, with respect to the mean force among all combinations in each cases.

**Figure 17.** Forces computed for different mesh combinations shown in Table 2, 37 kW machine.

The force is seen to vary with different mesh combinations, but the variations are small.

**Figure 18.** Forces computed for different mesh combinations shown in Table 2, 15 kW machine.

Large variations in forces can be seen.
Results and Discussion

**Figure 19.** Force as a function of the shape of the elements used for computation, 37 kW machine. The radial force is seen to have an increasing trend when the element shape is changed from equilateral to right-angled. For the tangential force, the difference between the forces is seen to be increasing when changing the shape of the element from equilateral to right-angled.

**Figure 20.** Force as a function of the shape of the elements used for computation, 15 kW machine.

**Table 3.** Difference between the maximum and the minimum value of the force shown in percentage with respect to the mean force among all combinations in each case.

<table>
<thead>
<tr>
<th></th>
<th>37 kW Machine</th>
<th>15 kW Machine</th>
</tr>
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<tbody>
<tr>
<td><strong>Mesh combination changed</strong></td>
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<td></td>
</tr>
<tr>
<td>Difference in radial force</td>
<td>0.8 %</td>
<td>11 %</td>
</tr>
<tr>
<td>Difference in tangential force</td>
<td>0.3 %</td>
<td>7 %</td>
</tr>
<tr>
<td><strong>Shape of elements changed</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Difference in radial force</td>
<td>0.08 %</td>
<td>9 %</td>
</tr>
<tr>
<td>Difference in tangential force</td>
<td>0.11 %</td>
<td>11 %</td>
</tr>
</tbody>
</table>
4.3 Whirling Power and Force from Power Balance

The origin and the calculation method of the whirling power was explained in Chapter 3. Studies have shown that in a machine under eccentricity, forces increase linearly with eccentricity. Therefore, the whirling power also increases with increased eccentricity. Figure 21 shows the whirling power computed using (40) as a function of relative eccentricity. It is noticeable that the whirling power is very small compared to the rated power of the machine. Therefore, to be able to calculate the whirling power from the power balance (41), the power balance should be very accurate. The power balance was seen to be reasonably accurate for torque computation in a healthy machine. Here, the accuracy of the power balance is checked by substituting the whirling power computed by using (40) in (41). In this case, the error in the power balance is shown in Figure 22. All the simulations are performed at the rated load.

![Figure 21](image1.png)

**Figure 21.** Whirling power calculated by using (40) as a function of the whirling radius.

![Figure 22](image2.png)

**Figure 22.** The error in the power balance of (41) of the machine as a function of eccentricity.

The whirling power is calculated by using (40) and substituted in (41). The torque is calculated by using (11).
The results suggest that in a healthy machine, the power balance is satisfied well, but when the whirling power is added, the error increases. As the whirling power increases with eccentricity, an increasing trend is seen in the error as well. One source of error is the third term in the right-hand side of the power balance expression (41). It has already been shown that an accurate computation of torque is very much dependent on the type of finite element discretization used in the air gap of the machine. The shape and the uniformity of the elements also affects the computation, and in case of an eccentric machine, where the air gap is non-uniform, a non-uniform mesh is likely. Figure 23 shows the average torque and the corresponding error in power balance calculated for five different types of mesh in the air gap of the machine, with 33% eccentricity. The results describe the sensitivity of torque well, and thus, the error in power balance with respect to the type of mesh used. Even though the change in torque values seems much smaller, it has a significant effect relative to the magnitude of the whirling power. For instance, the difference between the maximum and the minimum torque in the five cases studied is 0.121 Nm. The corresponding power related to this difference is 18.4 W, which is significant in this case.

![Graph showing average torque and error in power balance](image)

**Figure 23.** The average torque calculated with (Coulomb’s method) 5 different types of mesh in the air gap and the corresponding error in the power balance. It is clear that the error is sensitive to the type of mesh used. The combinations used are 1-1-1, 2-2-1, 2-2-2, 3-2-1 and 3-3-2.

Additional error is brought by the remeshing done for the rotation of the rotor. This can be explained by comparing the two cases. In the first case, the rotor is rotated at its rated speed, and in the second case, the rotor is locked. The errors in the power balance in such cases are shown in Figure 24. The results clearly
show that the error in the locked rotor case does not change much with an increasing eccentricity, while it increases when rotation is modelled. In the locked rotor case, the term $T\omega_m$ is equal to zero. This suggests that the error in the power balance mainly comes from the type of mesh used in the air gap and remeshing, which actually affects the term $T\omega_m$, thus affecting the power balance. More error is introduced in the case of dynamic eccentricity because, in this case, in addition to the remeshing, the rotor mesh is also moved in a circular path around the geometrical centreline of the stator. This adds non-uniformity in the mesh in the air gap, which contributes to more error. This can be understood from a comparison between the dynamic eccentricity and static eccentricity cases, the result of which is shown in Figure 25.

Figure 24. The error in the power balance (41) in the case of dynamic eccentricity (whirling rotor) under rated load and locked rotor. In locked rotor cases, when the rotor is not rotating, the error does not change, but when the rotor is rotating at the rated speed, the error increases.

Figure 25. Power balance error in the case of both dynamic and static eccentricity, simulated at rated speed. In the static eccentricity case, there is no whirling and the power related to it.
The results shown in Figure 21-25 are obtained from the study performed on the 15 kW induction machine. As explained in the previous section, the air gap mesh of the 37 kW machine has better quality elements. The results shown in Table 3 also add confidence to this claim. This suggests that the power balance error should be minimal in this case. For 33 % eccentricity, the errors in the power balance of (41) for the 37 kW machine are shown in Figure 26. The errors are shown in watts. Results show good power balance except for the mesh combinations 3, 8, 9, 11 and 12. The reason for large errors in those mesh combinations is due to inaccurate torque. It has been shown in Publication IV that the inaccuracy in torque describes the corresponding error in the power balance shown in Figure 26.

![Figure 26. Error in the power balance of (41) for different mesh combinations of Table 2, for the 37 kW cage induction machine. The power balance is good except for some mesh combinations.](image)

So, for the 37 kW machine, power balance can be alternatively used to calculate the whirling power. The force can then be deduced from the whirling power. However, the limitation associated with this approach is that both torque and the whirling power have to be considered at the same time. Small inaccuracies in torque will lead to significant differences in the magnitude of the whirling power, which ultimately leads to inaccuracies in the computed force, which is evident from Figure 27. The tangential component of the force computed from the power balance approach for the 37 kW machine is shown in Figure 27. The result is inaccurate in cases when the error is large, but in other cases, the results show reasonable agreement.
4.4 Electromagnetically Induced Damping

The whirling power computed using (40) for the 37 kW cage induction machine under 33% eccentricity at rated slip is shown as a function of the whirling frequency in Figure 28. In this case, the rotor is forced to move around a circular path at the given whirling frequency. When the whirling power is positive, for instance, at 20 Hz and 22 Hz whirling frequency, it means that the tangential component of the force is in the same direction as the whirling motion. Therefore, it can be accounted as negative damping from the mechanical standpoint. On the other hand, when the whirling power is negative, it means that the tangential force acts in the direction opposite to the whirling motion. This is positive damping from the mechanical standpoint.

Figure 29 shows the response of the mechanical system, when the electromagnetically induced damping coefficient is calculated from the whirling power, which is obtained from the power balance of (41). The damping coefficient is calculated by using (43). In this case, the study is performed on the 37 kW machine simulated at rated slip with 33% dynamic eccentricity. For the mechanical model, the mass of the rotor is 63 kg and the stiffness is 0.15 GN/m. The viscous damping from the surrounding fluid is assumed to be 2000 Ns/m. At rated speed, the whirling power is negative, which means that the damping is positive. This can also be seen in Figure 29. The amplitude of the displacement at the natural frequency is damped. For the critically damped case, the damping coefficient is $2\sqrt{K \times M}$. 

![Figure 27](image-url) 

*Figure 27.* Force computed from the power balance approach for the 37 kW machine, computed together with Coulomb’s method.
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Figure 28. Whirling power as a function of whirling frequency computed from a 37 kW cage induction machine. The whirling radius is 33% of the air gap.

Figure 29. Response of the Jeffcott rotor model, when the unbalance corresponds to 33% of the air gap. The electromagnetically induced damping coefficient is obtained from the whirling power calculated from the power balance of the machine.

4.5 Influence of Rotor Eccentricity on Torque

When the torque is integrated over the elements in the air gap of the machine, the radius of integration is important. Generally, in FEM, the radius of integration is taken by considering the geometrical centre point of the stator as the centre, which in case of an eccentric machine is not true. Such computation will lead to torque \( T' \) which is the sum of two components. The first component \( T_e \) being the torque, corresponding to the whirling radius, and the second component \( T_r \), which is the real torque acting in the rotor. Therefore,
for an accurate calculation of torque, the effect of the whirling radius should be eliminated. The figurative explanation of this idea is shown in Figure 30.

![Figure 30](image)

**Figure 30.** Torque components, corresponding to the radii of integration. The actual torque acting in the rotor is calculated such that \( T_r = T' - T_e \).

To obtain the results shown in Figure 22-23, the torque is calculated using the above technique. Although, it gives the idea of choosing the correct radius of integration, the above technique does not make Coulomb’s method free from the dependencies on the air gap mesh. Therefore, to study the influence of the eccentricity on the torque of the machine, the power balance method is used to calculate the torque.

An eccentric machine does not exhibit the same torque as a normal healthy machine. The FEM simulations were carried out for the 15 kW machine and the average torque was calculated. Figure 31 shows the average torque of the machine as a function of the whirling radius. The average torque of the machine is seen to increase with increasing eccentricity. In the figure, 0% relative eccentricity refers to a concentric healthy machine.

![Figure 31](image)

**Figure 31.** Average torque as a function of the whirling radius calculated for the 15 kW cage induction machine at rated load.
The influence of eccentricity is mainly seen in torque harmonics. Using fast Fourier transformation, the harmonic components of torque are calculated. Figure 32 shows the frequency spectrum of torque for a healthy machine, and Figure 33 shows the same for a machine with dynamic eccentricity. The figures show the frequency range of 200 – 2000 Hz because the harmonic components having considerably larger amplitude other than the fundamental component are seen to occur with this frequency range. For a healthy machine, some of the largest torque components are found to be at 300 Hz and 1700 Hz.

In case of dynamic eccentricity, a drastic increase in the amplitude of the 870 Hz component is seen. The 600 Hz component also shows similar behaviour with an increase in eccentricity. Among other larger harmonics, the 300 Hz component also shows a slight increase in the amplitude, but it is very small compared to the increase in the 870 Hz component. The higher order harmonic around 1700 Hz has a slightly reduced amplitude with increasing eccentricity.

The origin of the harmonics in the torque waveform can be found in the harmonics of the stator current. Therefore, to investigate further, the harmonic components in the stator current are calculated next. The principal slot harmonics (PSH) for the type of machine under study should occur at around 872.8 Hz, but for the PSH to be seen in the current spectrum, the pole pair number \( Q_r \pm n p \) (where \( Q_r \) is the number of rotor slots, \( n \) is the harmonic order and \( p \) is the fundamental pole pair) should be equal to the pole pair number of the space harmonics produced by the stator winding. As the number of rotor slots for the machine under study is 34, it makes the above condition not true. Therefore, the PSH is not seen in the current waveform. However, the rotor slot harmonic (RSH) exists and is seen at around 1695.6 Hz.

![Figure 32](image-url)
Results and Discussion

Figure 33. Frequency spectrum of the torque of the 15 kW cage induction machine under dynamic rotor eccentricity. A drastic increase in the 870 Hz component is seen.

When the machine is under dynamic eccentricity, the harmonics corresponding to $nf_c \pm 2f_w$ appear in the spectrum, where $n$ is the harmonic order in the supply, and $f_w$ is the whirling frequency. Although, the PSH does not appear in the healthy machine, $PSH \pm 2f_w$ components exist in the eccentric case, and it can be seen from Figure 34, that these components show a significant increase in their amplitude with the increase in the eccentricity. This explains the drastic increase in the torque component at around 870 Hz. However, the eccentricity harmonics, corresponding to the rotor slot harmonics, do not show much change in their amplitudes, the result of which is also seen in the spectrum of the torque shown in Figure 33. The slight increase in the 600 Hz component of the torque can also be justified from this result.

To further study the effect of eccentricity on torque, the machine is simulated at a quarter load, half load and full load, and the difference between the torque of a healthy machine and an eccentric machine is plotted as a function of the relative eccentricity. The result is shown in Figure 35.
Figure 34. Frequency spectrum of the line current of the 15 kW cage induction machine under dynamic rotor eccentricity. An increase in the $PSH \pm 2 f_\omega$ component is seen, which explains the increase in some of the torque harmonic components.

Figure 35. Difference between the torque of an eccentric motor and a non-eccentric motor, with different cases of loading under dynamic eccentricity, of the 15 kW induction machine.

4.6 Measurements

The measurement rig presented in this paper is quite flexible and can perform measurements for a wide range of load and eccentricity. However, the results shown in this dissertation are limited to those obtained from initial measurements of a locked rotor condition and under static eccentricity only. A
sinusoidal supply of 25 V was used for both the measurements and the numerical simulations. The reason for choosing such low voltage was because at higher voltage, the machine produced vibration, which would have added to the signals detected by the piezoelectric sensors, thus producing erroneous results. The vibrations could have been the result of improper tuning of AMB controller to work under the locked rotor condition. The measured results are compared with the simulations.

First, a healthy machine was measured. The torque waveforms of the healthy machine, both measured and simulated, are shown in Figure 36. Here, the average torque component has been removed, and only the dynamic component is shown. The agreement between the measured and simulated waveforms shown in Figure 36 can be considered reasonable if one takes into account the dynamics of the support. The measured torque contains some additional harmonics, which are otherwise not seen in the simulation. The harmonic components were obtained by performing a fast Fourier transform (FFT) of both the measured and the simulated torques and are shown in Figure 37.

![Figure 36](image)

**Figure 36.** Torque waveform of a healthy 15 kW cage induction machine measured and simulated with locked rotor at 25 V supply. It should be noted that the average torque component is removed and only the dynamic component is shown.

Next, the machine was subjected to eccentricity. The measurement was done at 22% and 33% eccentricity. The spectrums of torque for both cases were obtained. Figure 38 compares the measured spectrum of torque with those obtained from simulations, for 22% and 33% eccentricity. The effect of eccentricity is clearly seen in the measurements. The measured spectrum in this case, when compared with that of a healthy machine, suggests that some additional harmonics are introduced in the torque waveform due to the
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eccentricity. However, the simulation does not yield similar results. It could possibly be because at 25 V, the machine is still linear in the simulations. To check, additional simulations were performed by increasing the supply to 50 V and 100 V. The rotor was locked and the eccentricity was set to 33%, static. Figure 39 shows the spectrum of the torque obtained from simulations at different voltages. It can be seen that for 50 V and 100 V supply, additional harmonics appear that were otherwise not visible for 25 V supply.

Figure 37. The frequency spectrums of the torque obtained by performing FFT on a healthy 15 kW cage induction machine, measured and simulated with a locked rotor at 25 V supply.

Figure 38. Measured and simulated spectrums of the torque with 22% and 33% static eccentricity of the 15 kW cage induction machine under a locked rotor condition. Measurements show the additional harmonics due to eccentricity, but the simulated result (overlap) is not similar.
Figure 39. Spectrums of the torque of 33% static eccentricity obtained from simulations at different voltage supplies, from the 15 kW cage induction machine. The figure shows that the machine behaves linearly when simulated at lower voltage, thus not delivering accurate results.
5. Conclusion

5.1 Summary of the Work

In the finite element analysis of electrical machines, electromagnetic torque and forces are generally calculated by using conventional direct methods. This dissertation carefully presents the influence of the air gap mesh used in the simulations on the results obtained from such methods. When studying an induction machine, generally, a mesh with two layers of elements is used in the air gap. This reduces the computation time, since the air gap of induction machines is narrow and discretizing it with three or more layers will increase computational burden. However, the use of three layers of elements ensures accuracy, and therefore, is recommended whenever resources allow for it. The results of the study in this dissertation show that the numerical computation of torque and force from the direct method is dependent on the type of mesh used in the air gap, more specifically, on the number of the layers of elements used in the air gap, the band used for rotation of the rotor and for torque or force computation and also on the shape of the elements used in the computation. The study was performed on cage induction machines, and Coulomb’s virtual work method was used for torque and force computation.

The use of the same band of elements for torque computation and the movement of the rotor gave better results, especially when the average torque was considered, as Figures 10-12 show. This is due to the averaging of all the shapes of elements caused by the remeshing done to model the rotation of the rotor. But at the same time, torque may have slightly higher ripples. This is because when the mesh is changed at every time step, the corresponding element matrix is not continuous, which may cause jumps in the torque. This can, however, be minimized using finer elements. Figures 13-14 reveal that torque is less sensitive to the change in the element shape in case of a three-layered mesh than the two-layered mesh. Equilateral elements gave the best
results. The above is true for both linear and non-linear cases. The choice of air gap mesh affects not only the average torque, but also the total harmonic distortion of torque.

The influence of the mesh on the calculation of the radial and the tangential forces produced due to eccentricity were also studied. The variation in the air gap mesh resulted in varying forces. Although nothing about the ‘best’ mesh can be concluded from the results obtained for the forces because of the lack of a reference, an improved result is obtained for a machine with a relatively larger air gap length. The choice of the number of layers of elements, the band of elements used for force computation and the change in the shape of elements showed higher variations in the forces computed for the 15 kW machine than the 37 kW machine, which has the radial air gap length almost twice greater than the former. The reason behind this is that when the non-uniform air gap is meshed, relatively more irregular and non-uniform mesh is formed when the air gap is small, and more non-uniformity is added by the whirling of the rotor. The situation is slightly improved when the air gap length is relatively greater.

An approach to compute the average torque by integrating the power balance of the machine was presented. Different finite element discretization of the air gap did not affect the torque computed from this method. The choice of the torque computation band did not affect the torque computed from the power balance method. The torque from this method seems insensitive to the type of mesh used in the air gap of the machine. This means that a relatively sparse finite element mesh can also result in an accurate torque computation when the power balance method is used. This is very advantageous in terms of the computing resources and time needed.

When direct methods are used for the computation of torque, the spatial differentiation of the magnetic vector potential to obtain the magnetic flux density is one of the probable sources of errors in torque computations. In the power balance method, the spatial differentiation of the potential is needed only to calculate the magnetic energy. Furthermore, when calculating the average torque over a period of fundamental frequency in a steady state, the change in the magnetic energy is zero. Therefore, term, defining the change in the magnetic energy, vanishes or is practically negligible, which makes this method less susceptible to such errors.

The power balance approach provides a reliable method for computing torque in the finite element analysis of electrical machines, but it can only be applied if the torque transmits a significant part of the input power. It is not valid in the
locked rotor condition, as the speed of the rotor is zero. This method can be used to validate the accuracy of other torque computation methods.

The power balance of an eccentric machine was also presented as an approach to verify the force computation or to alternately calculate the eccentricity forces. For this, the power balance of a machine with dynamic rotor eccentricity was defined by including the power produced by the interaction of the force and the velocity vector of the whirling motion. The computation of the whirling power from the power balance serves both the purposes of verifying existing force computation methods and computing force. However, the results shown in Figures 21-27 explain the limitations associated with this approach. This approach is not yet robust because both torque and the whirling power have to be computed at the same time and torque is affected by the type of mesh used in the air gap and the remeshing done to model the rotation of the rotor. In addition, the rotor mesh is moved in a circular path to model the whirling motion, which was also seen to bring about additional error.

The knowledge of the whirling power and its accurate computation from the power balance could also provide the power balance of the machine as an approach to study electromechanical interaction and the effect of electromagnetically induced damping in the mechanical vibration of electrical machines.

The validity of the power balance was checked for a locked rotor case. The accuracy of the power balance approach is also dependent on the time-integration method used in the simulations. A careful comparison between the trapezoidal rule and the implicit Euler method for time-integration in the simulation of a locked rotor case identified the trapezoidal rule as a time-integration method that conserves energy, even in the non-linear case.

In addition, the influence of the dynamic rotor eccentricity on the torque of the machine was studied. The torque was calculated by the power balance method. It was found that an eccentric machine exhibits higher torque than a healthy normal machine. An investigation about the effect of eccentricity on the torque harmonic components showed that the harmonics with frequency close to the $PSH$ of the machine had significant increment with increasing eccentricity. Other larger harmonics were also slightly increased. The reason for the increased amplitude of the torque harmonics was found in the spectrum of the line current in the stator winding. The additional torque might play an important role to induce torsional vibrations in cases when two machines are coupled, and one of them becomes eccentric due to some practical problem, for instance, bearing breakdown or misalignment. The knowledge of the behaviour
of the harmonic components can also be applied for the detection of faults related to the eccentricity.

In this dissertation, a novel measurement rig for the measurement of torque, its waveform and its harmonic components has been presented. The measurement rig can be used to measure the torque waveform of both a healthy machine and an eccentric machine. It was tested by measuring the torque of a 15 kW cage induction machine with up to 33% static eccentricity, and the results were compared with those obtained from finite element simulations. Most of the measurements results were acceptable and had a reasonable match with the simulation results. The measurement rig presented can be extended to measure a wide range of load as well as a wide range of eccentricity. It can also be used to measure forces and vibrations related to broken bars and other rotor faults.

5.2 Suggestions for Future Work

In this work, the prospects of the power balance approach in the finite element methods of electrical machines have been studied. The approach is mainly focused on torque and force calculations. The limitations associated with the approach have been described. Some aspects that can further extend this work in the future are presented here.

The core materials used in the simulations in this work were considered to be non-conducting materials without losses. Only the resistive losses were considered in the power balance expression. In the future, the prospects of this approach can be studied by taking the core losses into account and by considering core materials with losses.

The choice of the time-integration method is important for the accuracy of the power balance method. In this work, the trapezoidal rule has mostly been used, which is a second-order method. It was seen that this method is energy conserving, as Publication I has shown. Besides, the implicit Euler method was also compared, which did not display good energy conserving properties. Other higher order time-integration methods could be tested for power balance.

The effect of the mesh in the air gap of the machine in torque and force calculations has been broadly studied in this dissertation. For this, the air gap was discretized with one, two or three layers, and the choice of the element band for the rotation of the rotor and for the torque and force computation was varied. This was implemented in the in-house code of the Research Group of Electromechanics. Nowadays, several commercial and open-source mesh generators and FEM solvers are available. Additional variations in the mesh can
be produced and the effect of the mesh on torque and force calculations can be studied in various software.

In this dissertation, the power balance approach has been studied for cage induction machines. The same approach can be tested for other machine types, for example, synchronous machines, reluctance machines etc. Also, the air gap of the induction machines is narrow in respect to the other machine types. On the other hand, the saliency in some synchronous or reluctance machines may have an effect on the uniformity in the shape of elements in the air gap mesh, especially near the rotor surface. Therefore, studying different types of machines will add more confidence to the results shown in this dissertation.
References


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Since the energy conversion in an electrical machine is based on the electromagnetic torque, its computation is a key issue in the numerical analysis of electrical machines. Accuracy issues related to the methods used for the torque computation have been reported. In this thesis, torque computation based on the power balance of the machine is presented. A careful comparison between the power balance approach and the conventional method has been done to study the influence of finite element discretization on the computation of torque and the eccentricity forces in the machine.