An Evaluation Model for a Ride-Sharing Problem

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Thesis submitted for examination for the degree of Master of Science in Technology.
Espoo 15.3.2017

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**Title:** An Evaluation Model for a Ride-Sharing Problem

**Date:** 15.3.2017  
**Language:** English  
**Number of pages:** 8+62  

**Department of Mathematics and Systems Analysis**

**Professorship:** Systems and Operations Research

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Cars often utilise only a fraction of their capacity. Ride-sharing utilises the excess capacity by connecting the drivers and the passengers. This Thesis develops a model to evaluate the performance of a ride-sharing platform. The objective of the model is to create performance estimates to see if a ride-sharing service is a feasible concept in a given area.

Platforms facilitate the markets between the supply and the demand. The users of a platform benefit from the number of other users. This is called as the network effect. People will not use a service that does not provide them with enough value. To meet this requirement, a service with a network effect must have a sufficient number of users that is called the critical mass. Services are often piloted with a small sample of users. Piloting a platform service that needs a critical mass may be challenging because reliable data cannot be obtained with only a few users.

Agent-based modelling produces comprehensible results about the performance of a modelled platform. The model provides quantitative results that make sense and similar results have been obtained in previous studies. The demand forecast employed in this Thesis do not correspond real data, but they could be easily replaced by results derived from more advanced traffic models.

**Keywords:** Platform, Ride-sharing, Feasibility, Agent-based modelling, Matching
Tekijä: Markus Sallila

Työn nimi: Arviointimenetelmä kimppakyytipalveluille

Päivämäärä: 15.3.2017   Kieli: Englanti   Sivumäärä: 8+62

Matematiikan ja systeemianalyysin laitos

Professuuri: Systeemi- ja operaatiotutkimus

Työn valvoja: Prof. Ahti Salo

Työn ohjaaja: TkT Kimmo Berg


Agenttipohjainen malli tuottaa helposti tulkittavia tuloksia mallinnetun alustan suorituskyvystä. Tulokset ovat tulkinnaisia ja vastaavat aiempia tutkimuksia kimppakyytipalveluista. Käytetyt kysyntäennusteet eivät välitämättä vastaa todellista dataa, mutta ne on helppo korvata kehittyneempien liikennemallien antamilla tuloksilla.

Avainsanat: Alusta, Kimppakyyti, Soveltuvuustutkimus, Agenttipohjainen mallintaminen, Paritus
Preface

The work with the topic was interesting and I learned a great deal, even to the extent that the work of the beginning, put beside the newest parts, looks shamefully bad.

The Systems Analysis Laboratory has an enthusiastic atmosphere toward the work they do and I want to thank everyone there for their supportive help. Especially I want to thank the aid of Doctor Antti Toppila, who helped me get started with the Thesis. And the aid of my advisor Doctor Kimmo Berg, who helped me finish it off.

Everything was enabled by my supervisor Professor Ahti Salo, whom I want to thank for his patient guidance.

Otaniemi, 15.3.2017

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Abbreviations and variables

Abbreviations

DAR  dial-a-ride
DARP  dial-a-ride-problem
DART  dial-a-ride-transit
DRT  demand responsive transportation
FIFO  First-in-First-out
HOV  high occupancy vehicle
MaaS  mobility as a service
PTAL  public transport accessibility level
SOV  single occupancy vehicle

Variables

$\delta_{ij}^*$  distance of the shortest path between $i$ and $j$
$s_k$  speed of a car owned by user $k$
$a(e)$  additional distance of the driver for picking-up and dropping-off a passenger in match $e$
$w(k)$  width of time window for user $k$
$t(k)$  the earliest possible departure time for user $k$
$\bar{t}(k)$  the latest possible departure time for user $k$
$x_p(k)$  travel cost savings as a passenger for user $k$
$x_d(k)$  travel cost savings as a passenger for user $k$
$y_p(k)$  travel cost savings as a passenger for user $k$
$y_d(k)$  travel cost savings as a passenger for user $k$
$h(e)$  the objective value for match $e$
$c_p(e)$  value or surplus for the passenger in match $e$
$c_d(e)$  value or surplus for the driver in match $e$
$c_w(k)$  value for the waiting of user $k$
$c_t(k)$  value of time for user $k$
$c_5$  cost of driving a kilometre
$r_f$  initial charge for a passenger
$r_t$  per kilometre price for a passenger
$o_f$  initial compensation for a driver
$o_t$  per kilometre for a driver
$f_t$  time spent in communication with the system
$t_l$  time which before the information is fed to the system
Chapter 1

Introduction

1.1 Background

Platforms are services that connect different sides of interaction [1]. For instance, a shopping mall is a platform that creates markets inside the mall. Each store can benefit from the customers of the other stores by differentiating their selection. The variety of goods sold in the mall is greater than the selection of any single store. Together they establish a platform ecosystem [1]. What makes digital platforms timely is the advancement of technology [1]. Digitalisation has opened the possibility for new business models.

Major players have been overthrown by smaller platform business; taxi companies have lost their clients to Uber [2][3], Blackberry has fallen behind Apple [4] and retail business has moved to Amazon [5]. The common factor in these developments is that the platform, instead of competing with a product of its own, offers a channel for other businesses to provide their products on the platform.

Many platforms are multi-sided markets, arising from the platform connecting multiple sides of the market within the platform ecosystem. Two-sided markets are the most common, as most businesses have two sides: supply and demand. The platform acts as a broker between supply and demand. For example, the game console connects the game industry and the gamers. Having both sides for the markets creates the chicken-or-egg problem [6]. That is, the platform must have suppliers in order to offer something for consumers, but suppliers are unwilling to join the platform without consumers. The effects in which the quantity of either side has an impact on the value of the product are called network effects.

Network effects in brokerages are commonly cross-side externalities as users benefit mainly from the users of the other side of the market. Same-side externalities
often occur with technology standards [7]. Technology standards have increasing
profitability with the number of the adopters [8]. The markets of technology standards
have a tendency to tip over to a technology that has gained enough adopters compared
to the alternative technologies, making the given technology a dominant design. The
word critical mass may be used to mean the point where the tipping occurs. It can
also refer to the number of adopters, whereafter the network becomes self-sustaining.
Early adopters [9] may try the service, but without enough users, the utility is too low
and using the service will be discontinued. Network effects are present in platform
economics. Seen from this perspective they share similarities in market behaviour
with the technology standards

Demand responsive transportation (DRT) is one of the possible applications for
the platforms. In DRT, the supply is tailored on-demand, in contrast to buses with
timetables and fixed routes. The combined supply makes the service more valuable
to customers. If one were to directly contact the supply of just one taxi driver, there
would be a serious lack of availability, because the taxi driver might already have a
customer. The location of the taxi driver could be inconvenient. The driver could be
on the other side of the town, resulting in a long waiting time for the consumer and
creating costs for the driver. Taxi companies combine the supply provided by many
drivers to one. It is more likely within multiple taxis that at least one to be free
compared to a given taxi. The taxis are dispersed geographically and consequently,
the nearest taxi is likely to be nearer than if only one taxi were contacted. The service
level can thus be significantly improved by increasing the number of taxis. These
effects have been studied [10] and tested with a piloted mass DRT system called
Kutsuplus, which did show the effect of increasing profitability with the number of

The availability of excess resources can give opportunities for business ideas. Such
resources can be almost free because otherwise they would not be exploited. The
ownership of such resources may be dispersed. This gives platform solutions an
advantage because they have the capability to combine the access to the supply of
such resources from multiple suppliers. One such unexploited resource is private cars.
The utilisation rate of cars is low, given that the cars are parked most of the time.
The car has a fixed number of seats, thus empty seats are an excess capacity. The
problem is that of providing access to the excess capacity often requires work, which
the owners are unwilling to provide without compensation.

In ride-sharing, travellers share the same car with other individuals with similar
itineraries and time-schedules [12]. Ride-sharing can be hitch-hiking or some more
organised version, like a carpool, where commuters take turns in driving each other. Ride-sharing is similar to the taxi services, but if the drivers have somewhat regular routes, it resembles public transportation. For the success of public transportation, accessibility is vital. Public transport accessibility level (PTAL) is a method to measure the accessibility of a location by using public transportation [13]. In ride-sharing, the accessibility level of some point depends on the number of drivers. If there are no drivers whose itineraries and time schedules match those of a person who requires transportation, the destination may not be reached with ride-sharing by that person. If the PTAL within the area is too low, by default public transportation will not be used there. If it is not used by default, people will get access to some alternative mode of transportation, such as their own a car. The same will apply to ride-sharing.

Without critical mass, the service is unlikely to be used. The public transport network serves a customer that may interchange between multiple modes to reach nodes of a network that are not directly connected; the lines create complementary value for each other. This is different from ride-sharing because ride-sharing is often considered as a taxi-like service from origin to destination. The network effects are thus even stronger than in the public transport network. The public transport networks are controlled by a management, so that they can have unprofitable lines for the sake of increasing PTAL. Even non-public mass transport can have unprofitable lines if they complement enough the other lines, but in ride-sharing, each driver will expect to get some form of benefit for their participation, which additionally increases the point of a critical mass.

In carpooling, the lost flexibility is the main reason for not participating [14]. Despite the efforts to increase the popularity by government subsidized high occupancy vehicle (HOV) lanes that are allowed to be used only by cars that have more passengers than a predefined limit, the share of carpoolers has been decreasing [15]. Ride-sharing is about the driver selling his flexibility. The driver loses the possibility to alternative time usage options. For many, the loss of flexibility is greater than the benefits of carpooling has to offer.

Casual carpooling, which is also known as slugging, is one of the most successful ride-sharing modes. In slugging, the passengers flock in given locations, often near big highways [16]. In the United States, there are three cities where slugging is active. Washington has over 30-years of history with slugging, giving slugging creditability as a transportation mode. The spontaneous nature is preferred to prearranged carpooling, as the flexibility is compromised less. The slugging spots have a transit
possibility in proximity, removing uncertainty whether destination can be reached if slugging were to fail. The passengers gain free trips and both drivers and passengers save time with HOV lines. Even then slugging is not popular.

Dynamic ride-sharing has the flexibility benefit of slugging while being a taxi-like solution, where passengers are picked up wherever instead at a fixed set of locations. Many ride-sharing programs have failed, while only a few have survived [17]. In a survey study on the feasibility of dynamic ride-sharing in Berkeley California, 20% of respondents reported that they would be willing to use dynamic ride-sharing for commuting [17]. About 12,000 passengers commute daily by single occupancy vehicle (SOV) at Berkeley, of which 7,000 persons live in areas considered to be dense enough to be feasible. The feasibility was defined to be such that the match rate would be at least 60%. Free parking was found to be an effective mean to subsidise. A guaranteed free taxi ride back home was considered important. People also wanted that the service could be ordered earlier, which was also one of the desired characteristics of Kutsuplus [11]. Mobility as a service (MaaS) refers to the concept in which access to logistic resources is layered behind interface making a set of actual transportation modes an abstraction of one single mode. One idea for making ride-sharing feasible has been to integrate it as part of MaaS solution, it would provide cost savings, while not having the problem of accessibility to some locations.

1.2 Research objectives

This Thesis develops and tests a method that could be used to evaluate, whether a platform is likely to work. For example, a service business may be required to generate sufficient profit.

The feasibility is subjective and depends on the stakeholder’s preferences. The feasibility studies may use technical, economic, legal, operational and scheduling metrics as in the TELOS framework for the feasibility study [18]. In the context of project management, an approach where a case of an application is developed from a concept in phases is called the phase-gate model [19]. Each gate between phases is a feasibility study to evaluate whether the development should be continued. This often saves resources, as evidence about applications infeasibility may be found during the project.

For a platform, the operational performance is non-trivial. The platform may not function successfully without a critical mass, thus it cannot be reasonably piloted with only a few users. This is especially true if the platform relies on the supply of
resources owned by its users. To evaluate the performance, an agent-based model is built to simulate the platform. A customer-centric approach can be vital for a platform. Users will not use the platform if the platform is not feasible for them from their own point of view. This means that the market of the platform is mutually beneficial for all sides of the market. This can make managing service quality and incentives important in the design of the platform.

Mechanism design is a game-theoretic framework for designing an optimal outcome for a governance, making it relevant for the platforms. Optimality is decided by the designer of the mechanism and is thus subjective. In contrast to the traditional mechanism design, the platform is a service with similar quality requirements as any other service. Different mechanisms may have different interfaces. The users may prefer open access to information and options to choose, instead of a governance that decides the outcome and enforces it, but with a preferred outcome.

The design of the ride-sharing platform has many possible architectures and features. This Thesis designs one platform that maximises the profit of the platform. The platform is simulated for a demand scenario. Parameters and models for the scenario are optimistic. The simulation accounts for the fact that the users will not use a service that does not provide enough value for them. Appraising quality would need much information on the preferences of the users. Such information is easy to integrate for the developed method.
Chapter 2

Literature review

Already in 1991, it was found that automated matching has a better match rate than manual work in ride-sharing [20]. As the technology advances, new possibilities for automatisation have been introduced. Dial-a-ride-problems (DARP) are closely related to DRT, as DRT is sometimes also called dial-a-ride-transit (DART). The word dial-a-ride (DAR) is often used in pre-scheduled cases like transportation services for handicapped and elderly people, while DRT has come to mean more dynamic taxi-like services of small buses. In DARP, a fleet of vehicles with given capacity pick up people from different locations [21]. Many DARP-algorithms have been developed [22]. The objectives of a service may also include quality, which has been studied for DAR services [23]. Some algorithms consider quality as a desirable attribute and handle it in the objective function while some treat it as a constraint, ensuring sufficient quality. The optimisation has been also studied specifically for dynamic-ride-sharing [24]. Optimisation is only a one of the activities of a ride-sharing service.

2.1 Classification framework for ride-sharing services

The following activities identify the user-side of a ride-sharing: planning, pricing, and payment [12]. Rides can be shared in four different patterns introduced in Figure 2.1. A driver can serve multiple passengers or the number of passengers may be constrained to only one.

In ride-sharing the route always begins from the driver’s origin and ends at the driver’s destination. A passenger can be picked-up via the route the driver would
Figure 2.1: Positional elements of ride-sharing. Blue and white are used to distinguish between two passengers. In detour ride-sharing, there are two possible routes. The passenger picked up first could be dropped off before picking another up [12].

Figure 2.2: Classes of ride-sharing along axis of two main taxonomic criteria [12].
normally travel, this is shown as a straight arrow from the driver’s origin, to the
driver’s destination in Figure 2.1. In such cases, there are multiple ways of locating
the passenger. What makes inclusive and partial ridesharing different is that in
partial the passengers may travel to a location for the pick-up and continue from the
drop-off, while in inclusive ride-sharing the origin and destination of the passenger
are required to be on the drivers’ route.

There are taxonomic criteria which classify services and emphasise how different
types of service are distinct, see Figure 2.2.

Primary search criteria describe how the users are matched. Routing and time is
the dynamic ride-sharing, which requires information about the routes. On-Demand
pair and time refer to pairing systems, where the routing is aggregated to areas.
Keywords are on a list, from which users may search possibly interesting offers.
On-Demand pair and first-come first-serve is basically the same as casual carpooling
or slugging.

The target market describes the demand the service is designed to meet. On-
Demand is considered similar to conventional taxi services, whereas Commute is
for commuting. Long-distance is for the longer distance travels. Event-based trips
like festivals and conferences are considered as long-distance. Along these axes, the
classes of service can be identified. The names are clear or have been introduced
except for the One-shot ride-match which means the type of services that do not
provide automated pricing.

Decisions about planning of the trips, pricing and how the payment is conducted
depend on the class of the ride-sharing service. Planning is relevant in Routing and
time, and On-Demand pair and time. The other two leave the task of matching to
users. Planning decisions are often algorithmic. The service types for a ride-sharing
platform are introduced in Figure 2.3.

Pricing can be a catalog price, rule-based or negotiation-based. Catalog prices are defined by the users, rule-based pricing has some formula that may include parameters like distances and in the negotiation-based pricing the price is decided between the driver and passengers. Pricing for dynamic real-time ridesharing introduces additional problems for the pricing mechanism.

The mechanism should induce truthful information from the users for serving them as well as possible. It is possible that the users manipulate the system for their own benefit. The participants may feel that the mechanism is unfair if only drivers or passengers benefit. Another problem arises when there are differences in the outcome created by the submission time and the time window that can be used in matching. Behaviour in which the people play a game with the submissions timing has been noticed in online auctions [25]. For robustness, the pricing may need to adapt to different situations, like the change of schedules and no-shows.

Payments can be done directly between the driver and the passenger or via a third party, for example via bank. It is only the latter that may be used in a commercial application. This is because if the cash does not go through the platform, it is inconvenient for the platform to get its share. There is also a difference on how tailored or fixed the timetable and the route are. All ride-sharing businesses are considered to be motivated by the cost-sharing between participants. Taxis are considered to seek to maximise their revenue, whereas DAR has a neutral motivation between cost-sharing and generating revenue.

Users may feel insecure when travelling with strangers. In many services, trust to strangers is tried to achieve with profiles of users that are visible to other users. Profiles include feedback from previous matches which proves the user’s trustworthiness. An escrow mechanism that does not proceed the payment to a driver that does not exceed a threshold quality has been proposed [12]. Safety is just a part of the quality features.

### 2.2 Service quality

SERVQUAL is the best known framework in marketing discipline for the quality of the services [23]. The framework is also known as RATER which is an abbreviation of its determinants: reliability, assurance, tangibles, empathy and responsiveness. In SERVQUAL framework, the gap model, shown in Figure 2.4, describes quality with gaps of expectations and perceptions [26]. The gaps explain the possible sources
Figure 2.4: The gap model of the SERVQUAL framework. The black vertical line separates the sides of customers and the provider of the service.

Figure 2.5: Expectation confirmation theory as a model.
that lead to dissatisfaction of the customers. The provider of the service may have
difficulty in managing the different levels of an organisation to meet the objectives of
the management in the operations. The communication can raise expectations, but
also has a direct effect to the perceived service. It is also that the management may
have an erroneous idea about what the customers would expect.

Expectation confirmation theory illustrated in Figure 2.5, seeks to explain hierar-
chically different aspects behind customer satisfaction [27]. It has been found that
performance explains quality better than expectation, and that customer satisfaction
explains future purchase intentions better than service quality [28]. SERVPERF was
created from SERVQUAL to handle with performance related parameters better [28].
The hierarchy between determinants and attributes can lead to estimation errors.
Not having a hierarchy improves estimation accuracy. Also, disconfirmation of beliefs
is often almost meaningless. Dropping them reduces the number of attributes by
half [28].

The quality measurements of DAR services have been explained with the case
specific quality dimensions instead of RATER determinants [23]. The determinants
are replaced by these dimensions. Abstract dimensions have observable attributes. An
example of such model has been made [29], which is illustrated in Figure 2.6. Service
quality of DAR services has also been researched in the context of transportation
models [23]. The attributes can be measured directly in simulations and optimisations.
The most often used quality specifications in operations research literature are: difference between actual and desired delivery time, maximum ride time and excess ride time over direct time. The time window, during which the destination is to be reached, is often taken as a constant for all of the users. In the operations research models of DAR, quality is in the objective function or ensuring that the quality attributes achieve the predefined threshold levels [30] as constraints.

2.3 Routing and matching algorithms

DAR services can be identified to be something between static and dynamic [22]. In static services, all transportations are known, and in dynamic services, the routing is done as vehicles begin operating. Often it is something between, as a subset is known. Also in dynamic ride-sharing the routes are often considered to be submitted before departure [24]. This creates lead-time, which makes the optimisation more static. What makes most of the DARP different from ride-sharing is that in DARP, the fleet is assumed to be permanent. In ride-sharing, the drivers have their own destinations that need to be met within their own time window. Ride-sharing is thus DARP with an additional constraint.

Ride-sharing has some specific features to DARP which are sometimes taken into account in optimisation [24]. It is dynamic because the rides can be ordered within a short time. The fleet is not owned, and the drivers are not employed by a central organisation. The trip-related costs are divided between the passenger and the driver. The service brokers only non-recurring trips. The participants have prearranged the ride contrary to hitch-hiking or slugging. The service provides some automation that helps matching and facilitates communication between the users.

The matching algorithm has been studied in the literature [24]. The algorithms may optimise the outcome, but sometimes they use heuristics or auctions. The objective is often to minimise system-wide vehicle-miles and travel times or maximising the number of participants. This is unlike in DARP, where the objective considers often to minimise fleet size or driven journey, though some algorithms try to take quality into optimisation [22]. Still, the motivator behind DART is usually to reduce social welfare costs of transporting passengers that cannot use many of the other modes of transportation and the services are often provided by a government [23].

In dynamic ride-sharing constraints are the time windows of the users. This is similar to DAR, but matching the time windows between passengers and driver makes time the dominant constraint rather than the capacity of a vehicle. For this reason,
many of the ride-sharing algorithms consider only detour ride-sharing with a single passenger. What makes many DARP algorithms unusable for dynamic ride-sharing is that in dynamic ride-sharing it must be accepted that not all of the customers may have a match. Some of the DARP algorithms may turn a customer down. If the time windows cannot be matched with a fixed fleet size the algorithm tries to perform as well as possible.

What makes dynamic ride-sharing fundamentally different from DARP is that the service providers are independent. They have their own objectives and thus they are not willing to sacrifice their own benefit to reach the system-wide social optimum. This has motivated agent-based models, because the actions are done for self-interest, which can lead to a difference from the social optima called the price of anarchy [24]. One could argue that auction-based optimisation heuristic takes this into account, because every user bids in auctions according to their willingness to pay [31], although the users’ willingness to pay in auction algorithms is a heuristic to maximise the objective function, not an actual benefit. The auction type of optimisation is part of the decentralised agent-based optimisation method family. They are often faster than the centralised system optimisation that produces better system-wide results. One may benchmark an algorithm against performance of a static binary integer program which solves the system-wide optimal solution for matching the users [31].

Matching independent agents opens up problems which can be addressed with game theory [12]. Designing a mechanism for selfish agents in a game-theoretic environment is a mechanism design problem [32]. Two-sided matching is a widely studied problem in a mechanism design. It has been applied in allocating medical interns to hospitals [33].

2.4 Mechanism design

Social choice theory is a framework for aggregating agents’ preferences for deciding about outcomes affecting the agents [32]. Mechanism design is a social choice theory with strategic agents. The agents tell the preferences that they expect to maximise their benefits. This leads to a Bayesian game in which the preference or payoff for an agent is not known. The problem in mechanism design is to find a mechanism that optimises the outcome even if the agent would not necessarily tell their true preferences. If the optimal strategy for the agents is to tell true preferences; to be truthful, the mechanism can be called truthful, strategy proof, incentive compatible or direct [25][32].
The mechanism designer may have some objectives for the outcome that the mechanism must fulfil. If the sum of the agents’ values for the outcome is maximised, the mechanism implements a social choice function. In this special case, the revelation principle says that if such mechanism exists, there exists a strategy-proof mechanism. If such a mechanism is not possible or if some other objectives conflict, the mechanism may only minimise the inefficiency from the strategic behaviour, in which case it is called the price of anarchy minimisation [32].

The mechanism can exchange cash between agents. In such cases, the designer may want to maximise the revenue. The mechanism may seek zero profit. This could be the case if for example the mechanism would regulate the use of public goods. It could be that some nations are creating an organisation to share rights for pollution or fishing. The nations would not like to have an organisation that enforcing the mechanism makes profit. It would create a problem on how to share the profits, which may lead to a conflict. Then the mechanism could seek a budget balance by sharing all of the cash, minimising the maximum revenue or at least not to make losses.

The mechanism may need to incentivise the agents to participate in the mechanism. If the agents value fairness, the mechanism may seek maxmin fairness where it is maximised the smallest value of any agent. The outcome may need to provide value for every agent making agents ex post individual rational. The constraint can be relaxed to hold only as an expected value. Then the agents are said to be ex interim individual rational [32].
Chapter 3

Model of ride-sharing

3.1 The problem

Reasons for travelling are often tied to specific locations and times. This generates a spatial demand for transportation.

There are many transportation modes to satisfy the demand. In order to serve the purpose of travel, the offered transportation must go from the origin to the destination at the given time. People have some demand flexibility, to the extent that they are prepared to walk and wait, the origin and the destination of the transportation mode do not have to be exactly the origin, nor the destination of the person. The public transportation has fixed schedules and locations where one can get on and off the vehicle and the trip can consist of multiple modes.

The preferences include among others, how willing the persons are to wait or walk. Different people have different preferences; elderly people would not like to walk, but have plenty of time to wait and commuters that could walk need to reach their workplaces before their work shifts begin. People’s preferences influence the price they are willing to pay. Payments make it possible to change value between the flexibilities and alternative goods and services.

Many people have a car to satisfy their need for transportation. A car gives its owner access to possibly the most flexible transportation mode. The car is often located close to the owner’s origin and can be used at any time. A car can reach most destinations, and parking lots are built in the vicinity of many possible destinations. This differs from public transportation which is designed to meet the aggregated demand.

The public transportation may fail to satisfy all demand. A taxi is often used in such cases. The taxi needs to be ordered before the transportation, but it serves the
need as does one’s own car without the need for a parking lot. The taxi company and the public transportation provider have their costs from operating their fleet. Owning a car entails costs. Even if the government were to subsidise trips, there can be some limits for the costs. The price can be higher than the willingness to pay. The need for transportation can be left unsatisfied. Most cars have multiple seats, which leaves a lot of the capacity unexploited.

3.2 Model of a service

In ride-sharing, the service can be provided through a platform. The two-sided market is between drivers and passengers. The platform organises the market by making matches. A match is a deal that a driver will drive the passenger from the origin to the destination, at the appointed time. From now on, the word user will be used to refer a person using the platform, if it is not specified whether the person is a driver or a passenger or if it does not matter which of them that person is.

The platform considered in this Thesis is classified as a detour ridesharing, dynamic real-time ridesharing service for a single passenger [12]. The platform would work the best when the users from dispersed settlement areas travel to the cities. The service type is an integrated service with a rule-based pricing and the payment is done via a third party. Otherwise, users would need to negotiate the price between each other. With rule-based pricing, the platform offers a clear payment scheme which is accepted before the service is used.

The deals are proposed by the platform. The platform has no prior information about possible routes of its users or it does not utilise predictions in a matching algorithm, albeit the algorithm can forecast with a reasonable confidence that the users will commute the same route, provided that they have commuted that route before. The schedules are often regular. Getting a match is not certain. For this reason, dynamic ride-sharing may call for an alternative mode of transportation as a complement, which is flexible enough to meet the demand within a short time. This gives users time flexibility which they can utilise by giving it for the platform to make matches. Often the most flexible such mode is having a car. Thus, the users in the simulation have cars and they all will act as potential drivers and passengers.

The users provide following information when they ask for a ride:

- Earliest possible departure (time of day, for example 08:00)
- Latest possible arrival (time of day)
- Origin (address, for example Otakaari 1 in Espoo)
- Destination (address).
- The roles they are willing to take (either yes or no for both driver and passenger roles).

This information is the minimum for matching users while meeting constraints set by their schedules. The users can decline an offered match and the matches can be changed afterwards, until the driver departures, with a condition that both the driver and the passenger find another match. Changing matches would be too inconvenient if the users would lose the matches they already had.

The use case of using the platform is shown in Figure 3.1. In order to feed the information to the platform’s system, the user fills a form similar to the left part of Figure 3.2. The users give the information some minutes before they leave at earliest. This is justified because the probability of being matched depends on the width of the time window. Wider time windows increase the number of possible matches. The users give the information as soon as they are certain of their schedule because it increases their probability of a match without the inconvenience of waiting. Users are delayed until their latest possible time for departure. The later matches are preferred by the algorithm. These increase the time that the user can be used for matching as a passenger or as a driver. This increases the number of other users with whom the user can be matched.

Matches that do not benefit the driver or the passenger are not driven. It could be thought that the user declines a match. Waiting is assumed not to be a sunk effort; the users will consider it in the benefit of the match. It actually is sunk, but the population may have users that for the service is not suitable, which needs to be accounted for. Due to pricing, the users know whether they would benefit from the service as a passenger, but for the driver, the benefit from a match is not known. Users can live in an area with too low accessibility for them to actually use the service. The accessibility is not known before the simulation results.

### 3.3 Generation of demand

The origins of users and the destination depend on the case specific parameters. For example, the commuters typically leave from their homes. It would mean that users are located in residential zones. Their destinations would be the workplaces, in industrial, commercial, etc. zones. Vice versa, the people travel back to their
Figure 3.1: Use case of the platform. Dashed line means answering.

Figure 3.2: Illustration of the systems UI. On the left is the form that a user fills. On the right the user is prompted to accept the proposed match.
homes. This generates a demand for transportation between residential zones and other mentioned zones. The parameters for zones would be the quantities of residents and some parameters that are related to the reasons of travel. From them, it can be predicted how the traffic is formed. In either case, there is a generation-attraction-matrix that tells how the trips are oriented. Each element has the value of how many people are going from a given location to another given location.

This type of approach has been used to forecast traffic in Helsinki region commuting area [34]. Similarly, the time aspect when the trips are executed is determined by the reason to travel.

### 3.4 Time flexibility

The users have some flexibility when the destination is to be reached. When people are aiming to meet their destinations approximately at the same time, traffic congestions can arise. If they were to depart later or earlier, they would not need to wait in a congestion. Even then, they are not willing to change their departure time. Congestion implies low flexibility. In carpooling, lost flexibility is the main reason for not participating [14].

Time flexibility is the possibility to move an action later or also earlier in time. The range of time when the action can be executed is the time window. The actual time window can be wide. The user may not need transportation for several days, for example, shopping groceries is not critical if the user has stocked food in the fridge. This Thesis will refer to the time window as the time between earliest and latest possible departure times set by the user. The user has some time he would like to travel but is willing to be flexible.

The user has options on how to use the time window. In ride-sharing, the lost options include the actions that would make the user incapable of being at the promised location, when the trip is meant to start. The user’s willingness to give up flexibility is highly incorporated with the benefit they expect the service to provide them. The width of a time window is likely to increase the user’s probability of getting a match. The user will give such time window to be such that it maximises the expected benefit. It depends on the rewarding mechanism of the platform and on the case what the options for time would be and thus what is their benefit.
3.5 Comparison to alternative transportation modes

The ride-sharing platform competes with other transportation modes. Thus, it makes sense to compare it to those modes; handling the benefits with alternative costs. They are comprehensive, unlike the benefit of possibility to change a location.

Driving involves work in that it calls for an action. Waiting has alternative options on how to spend time, but the work does not. The Department for Transportation in the UK uses the value of time in its transport analysis guidance that is a transport appraisal framework [35]. Time of value is the opportunity cost for time spent travelling. The value of time is the upper limit for the price of time flexibility. The valuation of waiting time is complex, but there are methods for estimating the willingness to buy. It may depend on multiple case specific factors [36]. Time is not liquid nor substitutable [37]. Lost time is more valuable than saved and waiting for longer increases the value of time. In this Thesis, we assume the waiting time has the same value as the additional driving time. Without the platform, a passenger would have driven by himself, so the value of the trip is the price of that trip done by a car.
Chapter 4

Mathematical model

4.1 Transportation network

The locations and routes are aggregated to a transportation network consisting of locations as nodes $i, j \in V$ and major roads as edges $(i, j) \in Q \subseteq V \times V$. They form an adjacency matrix $\delta$, in which the length of all arcs is $\delta_{ij} > 0, \forall i \neq j$. If the locations $i$ and $j$ are not directly connected by a major road then $\delta_{ij} = \infty$. The distances between the nodes are big enough for not to be walked.

A path between $i, j \in V, i \neq j$ is a sequence of nodes $p = (i_0, i_1, \ldots, i_n)$ such that $i_0 = i, i_n = j, (i_g, i_{g+1}) \in Q, g = 0, \ldots, n - 1$, where $n$ is the number of vertices the path visits. The set of all paths connecting $i$ to $j$ is $P_{ij}$. The distance of the path is $\delta_p = \sum_{g=0}^{n-1} \delta_{i_gi_{g+1}}$. The shortest path between $i$ and $j$ is $\delta_{ij}^*$, for which holds:

$$\delta_{ij}^* = \min_{p \in P_{ij}} \delta_p.$$

This distance matrix $\delta^*$ is used primarily as the distance between points.

The network consists of roads that have the same speed limit. All the cars are assumed to drive that same speed; the time it takes to travel an edge is linearly dependent on the distance, thus the relative difference between edges is the same in the time it takes to travel, as in the distance. The drivers choose the quickest path. The network is known to everyone and the users prefer to minimise the time they spend travelling. The optimal route could be given by the platform because the platform uses the optimal routes in its rule-based pricing.

Car $k$ has the speed $s_k$. The time is discretised so that $\delta_{ij}/(s_k\Delta t) \in \mathbb{Z}^+, \forall i, j \in V$. This means that the distance from any node to another will be covered after some multiple of $\Delta t$’s.
4.2 Ride-sharing problem

There are $m$ users in a finite set $U$ of all users. Thus, we may refer to a user with a unique integer $k \in U = \{1, \ldots, m\}$. The ones offering to drive also have a need for transportation. Drivers may be matched as passengers if they do not specify their willingness to participate only as drivers. Some might not be willing to travel without their own car, if they need the car at work, or are not guaranteed to get a ride back home.

Each user $k$ has a corresponding service request $r_k \in R$. A request consists of the need for transportation from location $i$ to $j$, having to depart at the earliest $\underline{t}$ and the latest time for departure $\overline{t}$, which is by the platform processed automatically from the latest possible arrival to destination. This can be done by approximating the duration that driving the route $(i, j)$ would take by $\delta_{ij}^*/s_k$. User $k$’s request is $r_k = (i, j, \underline{t}, \overline{t}) \in R$.

For a passenger’s request, there can be multiple drivers that could pick him up. A driver can have multiple passengers that he could pick-up. A potential match can be made between the passenger and the driver if their service requests share similar itineraries and time-schedule. The match is not made if it cannot be driven within time determined by $(\underline{t}, \overline{t})$ of the driver and the passenger. The driver and the passenger can have only one potential match because the platform decides the matches. It has a decision rule for the time when the match will start. The decision rule used in the case is to choose the latest possible moment.

Every passenger has a set of possible drivers that could pick him up $E_k$. For example, if a user referred as 1 could be driven by users 2, 3 and 5, then $E_1 = \{2, 3, 5\}$. The set of all possible matches $E = \bigcup_{k \in U} k \times E_k$. Thus, a match $e \in E$ is an ordered pair $(p, d)$ where $p$ and $d$ are the indices of the passenger and the driver respectively. The network for matching has users $U$ as nodes and matches $E$ as edges.

The platform makes the decision about the matches by a matching algorithm. Matching is a special packing problem in a graph, to find an optimal pairing between users that are connected by possible matches. The optimality of a matching is determined by the values $h(e)$ associated with each edge $e \in E$. If the drivers would never act as passengers, the problem would degenerate to matching bipartite graphs. The matching problem is illustrated in Figure 4.1. Each node can have at most one edge after the matching. In the Figure, the user $u_3$ is left without a match, while other users share an edge with the user they are matched to. The routes and time intervals are specified by the users. In dynamic ride-sharing, the platform matches
the users with incomplete information, because future service requests are not known. The system-wide optimisation is run periodically. The ones that were matched on previous periods, must be matched on later periods.

The platform has a price $x_p(e)$ it charges from the passenger $p$ and a compensation $x_d(e)$ for the driver $d$ for the match $e$. The platform can profit from the difference between charges and compensations. The platform can have objectives other than maximising profits $h(e) = x_p(e) - x_d(e)$, like maximising matches by defining $h(e) = 1$ or reducing traffic congestion. The algorithm can be used to guide the assignment to desired outcomes, by objective function or additional constraints.

The passenger $k$ is willing to pay for the trip according to its benefit $y_p(k)$, which is the alternative cost for that trip. Making a detour has a cost, so the driver is willing to accept matches for a compensation $y_d(e)$. For the platform to create value, it should be that the passengers have in general greater willingness to pay than the drivers’ costs for making detours.

When the users specify a time window for the platform to use in matching, they lose their flexibility, because they are required to fulfil their role. The lost time flexibility is the greatest reason not to share rides [14]. As drivers, they need to drive the passengers to their destinations, or as passengers be ready to be picked-up when the drivers come to pick them up. Whether a user will use the platform depends on their alternative modes of transportation. The users choose a mode such that they maximise their value, which is the difference between the benefit and the price [38]. To compromise the lower benefit from waiting, the price must be lower. The pricing of the platform should thus create savings for the users. In order to know how much savings it should create, this Thesis uses agent-based modelling to determine passenger’s willingness to pay and driver’s costs.
The cost of commitment time $c_w(k)$ is the value for the time the user is prepared to wait. It is the compensation for which a user would be willing to wait if the service would not give a match. The value of $c_w(k)$ is known before matching takes place.

The users might not get a match every time, and the drivers do not know the distances of the detours before a match is offered. For these reasons, the users can make decisions only with the expected value determined by expected savings. The value created by a match $e = (p, d)$ for the passenger $p$ is $v_p$ and for the driver $d$ is $v_d$.

$$v_p(p) = -c_w(p) + [y_p(p) - x_p(e)] \quad (4.1)$$
$$v_d(e) = -c_w(d) + [x_d(e) - y_d(e)]. \quad (4.2)$$

The term $[y_p(p) - x_p(e)]$ will be called as the value of the deal for the passenger and $[x_d(e) - y_d(e)]$ for the driver.

For a feasible service, the value of the deals is positive in general. Then drivers’ costs from detours are lower than the savings the ride-sharing creates for passengers $y_p(k) - y_d(e) \geq 0$. Also, the value of the deals needs to be greater than $c_w(k)$ for the users to be willing to use it.

Every user has an own point of view for the expected value. The platform is not necessarily a suitable option for everyone. The platform’s feasibility is subjective depending on the platform’s vision of the scale of market penetration and profitability.

The time flexibility is not fixed. The users may wake up earlier or postpone their reason to travel. Thus, the users can decide the width of their time windows $w(k) = \tilde{t}_k - \tilde{t}_k$. The users have means to control the $c_w(k)$ by controlling $w(k)$. The wider time window they give, the higher probability of finding a match.

The probability of a match depends on the number of users. And the number of users that are at the moment available for matching depends on the width of their given time windows. These both depend on the value the users expect to gain by using the platform. The platform controls the value by pricing. Thus, the pricing may give an incentive to participation and users to give wider time windows.

### 4.3 Pricing

In a match $e$ in Figure 4.2, where the route of driver $d$ is $(i, j)$ and the route of passenger $p$ is $(i', j')$, the driver incurs cost $y_d(e)$ from the additional distance $a(e) = \delta_{i'i'} + \delta_{i',j'} + \delta_{j,j'} - \delta_{i,j}$ of picking the passenger up and dropping him off
\[ y_d(e) = \frac{a(e)}{s_d} c_t(d) + a(e)c_5, \]

where \( c_t(d) \) is the value of time and \( c_5 \) is the cost of transportation per kilometre.

![Diagram](image)

Figure 4.2: The route of driver \( i \rightarrow j \) when picking-up on \( i' \) and dropping-off on \( j' \) a passenger. The route that the driver would otherwise drive is marked with a dashed arrow.

The compensation \( x_d(e) \) needs to be greater or equal than the cost with the value of time spent waiting, for the driver to accept the match

\[ x_d(e) \geq y_d(e) + c_w(d). \]  \hfill (4.3)

The benefit of travel \( y_p(p) \) for the passenger \( p \) is approximated by the cost of the alternative mean of transportation. Driving alone route \((i', j')\) with one’s own car has a cost

\[ y_p(p) = \delta_{i'j'} c_5. \]

The price \( x_p(e) \) the passenger \( p \) is willing to pay to accept the matching needs to be less or equal than the benefit of travelling without the value of waiting.

\[ x_p(e) \leq y_p(p) - c_w(p). \]  \hfill (4.4)

Passengers pay the initial \((r_f)\) and per kilometre \((r_i)\) charges. This is also applied to the drivers’ compensation, but with different values for initial \((o_f)\) and per kilometre \((o_i)\) prices.

The price \( x_p(e) \) for passenger \( p \) with route \((i', j')\) depends only on the route. The passenger does not pay for the detour of the driver.

\[ x_p(p) = \delta_{i'j'} r_i + r_f. \]
And the compensation $x_d(e)$ for the driver

$$x_d(e) = a(e) o_t + o_f.$$ 

From these, the profit $h$ from a match $e$ can be found to be

$$h(e) = \delta^*_e r_t + r_f - a(e) o_t - o_f.$$ 

If the platform wants to subsidise the service, it could be that the profits are negative.

### 4.4 Determining the time flexibility

The users are assumed to be ready to depart during the time-window they give. This is inconvenient for the users because it restricts the activities they can do while waiting. The compensation that the driver gets depends on the detour they take, which is not known before the driver is matched. The expected additional distance depends on the number of users, like in the case of taxis, the geographically dispersed taxis are likely to drive a shorter distance to pick-up location of their customer. The savings of the passenger are known, from the pricing and the route they have. The value from the deal for a user $k$ as a passenger is

$$[y_p(k) - x_p(k)].$$

The platform may not match the users every time. The expected savings should take this into account, but this would create a dependency on the results. This Thesis has a policy not to have such dependencies.

Expected savings and the price required to wait create the value the user expects to gain for their participation. This Thesis assumes that the users are willing to share the value equally with the platform as a time window. Users give a time window $w(k) = \bar{t}_k - \underline{t}_k$ by scaling the value the users are willing to provide for the platform by the value of time

$$\frac{[y_p(k) - x_p(k)] - c_w(k)}{2c_t(k)} = w(k),$$

where $w(k)$ is the width of the time window for user $k$ and $c_t(k)$ the value of time. The time window should grow with expected savings from the deal. Also, the ratio $w(k)/c_w(k)$ should converge as $w(k)$ increases. If time became almost free or infinitely
expensive, the function would be implausible. The ratio \( c_w(k)/w(k) \) should decrease as \( w(k) \) increases; short waiting times are experienced as less valuable per time [37]. Function that would fit the specifications is

\[
c_w(k) = w(k)c_t(k).
\]

With this function the time window becomes

\[
\frac{[y_p(k) - x_p(k)] - w(k)c_t(k)}{2c_t(k)} = w(k) \Rightarrow w(k) = \frac{[y_p(k) - x_p(k)]}{3c_t(k)}.
\]

(4.5)

4.5 Algorithm for maximising profit

To make a match \( e \) for a driver with passenger as in Figure 4.2, the time windows need to be feasible.

\[
\bar{t}_p \geq t_d + \delta_{\omega}/s_d \quad \text{(passenger} \ p \ \text{can be reached in time by the driver} \ d) \\
\bar{t}_d - a(e)/s_d \geq \bar{t}_p.
\]

The driver can attain own destination in time.

Besides these physical constraints, for the users to have positive values from matches there is constraints

\[
x_d(e) \geq y_d(e) + c_w(d) \quad (4.3), \quad \text{for the driver}
\]

\[
x_p(e) \leq y_p(p) - c_w(p) \quad (4.4), \quad \text{for the passenger}.
\]

The drivers leave as late as possible to increase the time they are on the platform. Thus, the departure time of the driver in all matches is minimum of the latest possible times of both users. The two different cases are illustrated in Figure 4.3. The driver must departure either so that he has time for the detour and arrive at the latest possible time for arrival, or so that

\[
\min(\bar{t}_p - \delta_{\omega}/s_d, \bar{t}_d - a(e)/s_d)
\]

The matching system illustrated, in Figure 4.4 takes the service requests in \( t_i \) before \( \bar{t} \). For each increase of discretised time, the system makes matches and fulfils the matches scheduled with the given time by notifying them that the match is locked. The system also removes unmatched users that for the given time equals \( \bar{t} \).
Figure 4.3: The two different cases that determine the latest possible time. Blue boxes are the time windows. Red boxes are the time that is constraining. Black line shows the last possible time for the driver to departure in a match.

Figure 4.4: The steps of a matching system as a flow.
For matching, the following optimisation algorithm is used.

For each user, the system searches feasible matches from requests of other users. In a binary decision vector variable $z$, each element corresponds to a feasible match $e \in E$

$$z_e = \begin{cases} 1, & \text{if match } e \text{ is executed} \\ 0, & \text{otherwise.} \end{cases}$$

Each user is matched at most once (the user may only either serve one request as a driver or have own request served as a passenger). This will be the only constraint for the optimisation as $z$ already is by definition such that the other constraints are met. In binary constraint matrix $A$, there is for every user a row where each column tells whether the user is either the passenger or the driver in the corresponding element of $z$.

$$A_{ke} = \begin{cases} 1, & \text{if user } k \text{ is a participant in a match } e \\ 0, & \text{otherwise} \end{cases}$$

The matches can be changed, if alternative matching yields higher profits. Person given a match earlier must have a match. The corresponding rows of matched users in matrix $A$ are moved to a matrix $B$ to force this rule. This adds to the matching problem of Figure 4.1 an additional problem illustrated in Figure 4.5. The system runs periodically optimisation on the matching, which works as in Figure 4.6.

The platform in charge of matching maximises its own profit. Let vector $h$ be the profits of the matches, each corresponding to an element of $z$. Preference for later matches is achieved by adding to each element of $h$, $\epsilon$ times the value of discretised time the match would begin.

$$\max h^T z \quad \text{maximize the profit}$$

$$s.t.$$

$$Az \leq 1 \quad (The \ person \ can \ only \ take \ part \ in \ one \ fulfilled \ match)$$

$$Bz = 1 \quad (The \ person \ that \ had \ a \ match \ previously, \ must \ have \ a \ match)$$
Figure 4.5: Matching problem case of Figure 4.1, after increase of time. Blue balls mean previously matched users. Users $u_1$ and $u_2$ have left the system. Users $u_8$ and $u_9$ have arrived. Because $u_6$ had previously a match marked in dashed line, $u_9$ is connected to him instead of $u_3$, who would have been closer.

Figure 4.6: The timeline for the optimisation system. Trapezoids are the points where the platform runs the system-wide optimisation.
Algorithm 1 Matching system
1: initialise requests from data and calculate time windows and benefits
2: matched ← empty set of matched users
3: left ← empty boolean set of users left
4: for time t to reach ending time do
5: user pool ← requests not left and t ≥ ℓ - ℓ
6: matches ← empty set
7: for all passenger ∈ user pool do
8:   for all driver ∈ user pool \ passenger do
9:     if driver can be matched with the passenger then
10:        if the driver and the passenger have a positive benefit then
11:           to matches is added a match (passenger, driver, value)
12:         end if
13:     end if
14:   end for
15: end for
16: A ← users of matches not matched
17: B ← users of matches matched
18: g ← values of matches
19: matching ← arg max \( h^T z \mid Az \leq 1, Bz = 1, z = binary \)
20: left ← left \( \cup \) matching marked to leave at \( t \) or \( t = \bar{t} \)
21: matched ← matched \( \cup \) matching
22: end for

\[
\begin{array}{c|c|c|c|c}
\text{(match)} & H_0 = 0 & H_1 = -5 & H_2 = -5 & H_3 = 5 \\
\hline
\text{profit} & (1,2) & (1,2) & (1,5) & (2,6) \\
& +5 & +5 & +10 & -5 \\
& (3,4) & & & \\
& -5 & & & \\
\end{array}
\]

Figure 4.7: The matching with cumulative profit.

4.6 Maximising the number of matches

The optimisation of the number of matches can be done by changing the objective function into \( \bar{1} z \), where \( \bar{1} \) is a vector of ones.
Additionally, there is a new constraint

\[ h^T z - H_t \geq 0, \]

where \( H_t \) is the cumulative profit from the people left, up-to time \( t \) with initial value \( H_0 = 0 \). The cumulative profit is updated after optimisation \( H_{t+1} = H_t + h^T z \). This ensures that after optimisation the platform has a positive profit. \( H_t \) can take negative values, but it is always ensured that the matches and the cumulative profit sum to positive profit, which is illustrated in Figure 4.7. \( H_t \) is \(-5\) because the match (3, 4) leaves, making \(-5\) profit. This is possible because the match (1, 2) makes the profit to compensate that. In later matchings, the match (1, 2) can be changed, but the match (1, 2) is an option.

### 4.7 First-in-first-out (FIFO)

The matching can be done in a similar manner to the first-in-first-out (FIFO) principle. This will nevertheless be addressed as FIFO, due to the resemblance. The algorithm for the matching is fast and easy to implement as it does not optimise anything. The results of FIFO are for comparing the results from optimisation, into results that are not optimised. This will tell whether it is worth the effort to implement any optimisation.

There is a queue of the requests in the order in which they were given. A new request will be compared to the requests in the queue. If the request can be matched with the request it is compared to, the requests are removed from the system as they are matched. If the request cannot be matched with any request in the queue, then the request is added to the queue. The unmatched requests leave the queue when their latest possible time for departure comes.

The matches have the same constraints as in the optimisation. Additionally, the platform only makes matches that generate profit; the driver’s compensation is not higher than the price of the passenger. The same occurs implicitly when the platform optimises the profit, but for the FIFO it needs to be explicitly added.
Chapter 5

Case South-Western Finland

5.1 Case environment

Users are the travellers in south-western Finland at peak hours. Greatest cities in the area are Tampere, Lahti, Turku and Helsinki. The users that have a car may act as potential drivers and passengers, whereas users without a car can only act as passengers. This Thesis uses transportation network data from Andelmin (2014) [39]. In Figure 5.1 is the road network of south-western Finland. The shape of a

![Road network diagram](image)

Figure 5.1: The road network of south-western Finland used in simulations.
peak is taken from a morning commuters [40]. At mornings the peak starts around city centres, when commuting begins. The commuters that have a longer distance from home to their workplace often start their trips earlier.

The process of generating dummy data using topology of the network is explained in Appendix A. The process creates the generation attraction matrix and the distribution of earliest departure times. The generation attraction matrix is \((\text{origin}, \text{destination})\) pairs as the distribution of users as shares for it to be scalable with the number of users. It takes into account that the users are going from the less populated areas to cities. The distribution of earliest departure visually resembles the data about the number of cars that pass a measuring point [40].

Figure 5.2 shows the trend of the users with a longer distance from home to city start their trips earlier; the mean and variation increase with closeness to city centres. In Figure 5.3 is a pooled distribution of earliest departure times of a generated population.

All of the cars will drive the speed limit and thus have the same speed. For the speed \(s_k\), it can be used 100 km/h as that is somewhat the aggregate speed of the road network. The discretisation of the time and the network allows handling of distances as times or vice versa. The discretisation is 5 minutes, which is small enough compared to travel times to not have much effect. 5 minutes is also reasonably good for a system-wide optimisation rounds.
Figure 5.3: Earliest departure times from the whole population. The peak hours are discretised to 50 periods. The number of generated users is 1,000,000 for a visualisation purpose.
The users input the information to the system $t_l = 15 \text{ min}$ before their earliest possible time for departure $t_l$. This is justified as the probability of being matched is highly affected by the width of the time window for the user being usable for matches. The users give the information as soon as they are certain of their schedule because it increases their probability of a match without the inconvenience of waiting. Because the service is dynamic ride-sharing it can be considered that the routes are given 15 minutes before.

5.2 Prices

In the appraisal of transportation and road projects, the value of travel time savings is called the value of time. The time spent by drivers on their detour is approximated to be worth same as the value of time. It is assumed that each user is an individual, not a group. The value of time $c_t(k)$ for an individual commuter can be approximated as 10.68 €/hour (Table 5.1).

Table 5.1: Value of travel time savings for a car; used by liikennevirasto [41].

<table>
<thead>
<tr>
<th>Purpose of travel</th>
<th>persons/car</th>
<th>€/person/hour</th>
<th>€/car/hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work(6.8%)</td>
<td>1.15</td>
<td>23.68</td>
<td>27.23</td>
</tr>
<tr>
<td>Commute(20.6%)</td>
<td>1.10</td>
<td>10.68</td>
<td>11.75</td>
</tr>
<tr>
<td>Leisure(72.6%)</td>
<td>1.60</td>
<td>6.79</td>
<td>10.86</td>
</tr>
<tr>
<td>Average</td>
<td>1.46</td>
<td>8.33</td>
<td>12.16</td>
</tr>
</tbody>
</table>

The price of gasoline and car wear down per kilometre is $c_g$. The Finnish Transport Agency uses 0.1457 €/km in appraisal [41]. The same value is used in this Thesis. It does not take capital expenditure for a car into account in the same way as does kilometre allowance from Finnish tax authority which would be 0.43 €/km for year 2016 [42]. The users have a car, so they pay the capital expenditure for it.
Chapter 6

Computational results

The simulation could be run with different sets of features to evaluate, which set of features optimises the objective of the platform. It makes no sense to analyse results of parameters that are not taken into account in the platform’s design.

The most important metrics are profit and the number of matches because they are the objectives for which the features of the platform were designed. Also, the value for users defined in Equations (4.1) and (4.2) will be analysed. This could be also called customer surplus. The rest of the possible parameters are not considered to be important or sensible.

The simulation is run 5 times, with initial charges $o_\ell, r_f \in \{0, 2, \ldots, 30 \text{ €}\}$, compensation from detour $o_\ell \in \{0, 0.08, \ldots, 0.56 \text{ €}\}$ and the price for distance $r_f \in \{0, 0.02, \ldots, 0.14 \text{ €}\}$.

The plotting format of Figure 6.1 can be read in a way that the smaller plots are on top of each other horizontally and vertically. This property makes the four

![Diagram](image)

Figure 6.1: The basic format of plotting contours. Each plot is averaged over results of simulation runs. When a plot is a constant, it is left white.

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Figure 6.2: The plot for the profit of the platform using the plotting format of Figure 6.1. The number of users is 2000.

dimensional results more comprehensible. The objective of these plots is to create some intuition into how pricing affects the outcome and what is the relation between the different metrics. The euro signs (€) will be omitted in the plots because it would take space and there are no other units of value used, so there is no risk of confusion.

Figure 6.2 shows that the results are likely to be almost everywhere continuous. It can be seen that the plots close to each other are similar.

Figure 6.3 has one of the plots with $o_f = 8$ €, $o_t = 0.16$ € from 6.2. There are no matches when the passengers have low fixed charge and low per kilometre price or when they are both high. For the drivers, it is the same, but not as radical because there are some matches with high fixed and per kilometre compensation. These effects are created because the passengers are not willing to pay more than the benefit of the trip. If the passengers do not pay enough, the drivers cannot be compensated. The drivers have no similar threshold from the benefit because it is possible that the passengers’ origin and the destination are via the route the driver would be driving anyway; picking up a passenger without driving a detour is free for the driver.
Figure 6.3: The contour plot of profit for the pricing $o_f = 8 \, \text{€}$, $o_i = 0.16 \, \text{€}$. The number of users is 2000.

Figure 6.4: The saved driving as a share of total demand plot using the plotting format of Figure 6.1. The number of users is 2000.
The optimal pricing to maximise matches is different to the optimal pricing to maximise the profit in Figure 6.2. The results of matches are slightly miss-aligned with the profits, while being located closely within the same area. This means that the match rate correlates with the profit.

Figure 6.4 has the result for the share of driving that is saved. Profit, match rate and the saved driving correlate with each other, but the different locations for optimality mean that as objectives they conflict.

Figures 6.5 and 6.6 show that the passengers and the drivers gain similar expected value by using the service. The shapes of the results are similar; the pricing affects them equally. For this reason, the passengers and the drivers aggregated values can be handled as the customer surplus. The optimal customer surplus clearly conflicts with the profit and the match rate. The match rate is one of the most important metrics of quality in ride-sharing. It would likely have an effect to the user’s willingness to pay and thus the customer surplus.

The result for profit when the system uses FIFO matching changes the shape of results less than changing the number of users. This would imply that the shape is highly determined by the demand, rather than the algorithm. The optimisation manages better with the profits on the border case pricing where there are not many matches, but the optimal pricing is the same.

The results show that the optimal price when the profit is optimised is \( o_f = 8 \text{ €}, r_f = 18 \text{ €}, o_t = 0.16 \text{ €}, r_t = 0.02 \text{ €}. \) For FIFO, the optimal pricing is the same. The number of matches is maximised when the number of matches maximised with the pricing \( o_f = 10 \text{ €}, r_f = 12 \text{ €}, o_t = 0.16 \text{ €}, r_t = 0.02 \text{ €}. \) For these prices, the simulations were run 1000 times for user populations of sizes \( \{100, 200, \ldots, 5000\} \). When the number of matches was optimised, simulations were run only up to the population size of 1600 because the optimisation slowed radically.

The profit distribution in Figure 6.7 shows the variation between simulations and that it increases slowly with the number of users. Results of FIFO do not differ significantly, which implies that the optimisation does not affect the variation. The outliers are mostly lower to the mean.

Figure 6.8 shows that the consumer surplus is higher to FIFO when the profit is optimised, which results from the saved driving as shown in Figure 6.9. Optimising the number of matches increases consumer surplus and saved driving a lot, but the profit ends up close to zero.

In Figure 6.10 FIFO has only slightly lower match rate. This means that the value per match is a more significant contributor to profit than the match rate. Optimising
Figure 6.5: The drivers’ aggregated value using the plotting format of Figure 6.1. The number of users is 2000.

Figure 6.6: The passengers’ aggregated value using the plotting format of Figure 6.1. The number of users is 2000.
Figure 6.7: The distribution of profit. The solid line shows the mean and the dashed lines are in the distance of three standard deviations, which gives the confidence level of 99.73%. Balls mark the outliers.

Figure 6.8: The average profit and consumer surplus.
Figure 6.9: The saved driving as a share of total demand. The solid blue line is the result of optimised profit, the red dash line is the result of FIFO and green points are the result of optimised number of matches.

Figure 6.10: The average match rates. The solid blue line is the result of optimised profit, the red dash line is the result of FIFO and green points are the result of optimised number of matches.
Figure 6.11: The average profit per match. The solid blue line is the result of optimised profit, the red dash line is the result of FIFO.

The number of matches increases radically the number of matches.

The profit per match improves for optimised profit while decreases for FIFO with the number of users in Figure 6.11. The customer surplus per match does not improve as significantly with FIFO as it does with optimised in Figure 6.12. The surplus by the number of matches’ maximisation is significantly higher than profit maximisation and FIFO. Profit maximisation and FIFO has almost same average distances for the matched passengers in Figure 6.13, but the optimisation decreases while FIFO slightly increases the detours of drivers. Optimising the number of matches decreases average distances for the matched passengers, and slightly increases the detours compared to FIFO.

The distribution of the lengths of passengers’ trips are in Figure 6.14 and the detours in Figure 6.15. It can be seen that both optimisations make profit with long matches, but that to increase the match rate, the match rate optimisation loses by making matches that have a shorter trip for the passenger and the longer detours for the drivers.
Figure 6.12: The average customer surplus per match. The solid blue line is the result of optimised profit, the red dash line is the result of FIFO and green points are the result of optimised number of matches.

Figure 6.13: The average distance of a passengers trip and the detour for a driver per match.
Figure 6.14: Distribution of the lengths of the passengers' trips. The solid blue line denotes the case where profit was optimised. The red dash line indicates the profitable matches when the number of matches is optimised, and the solid green line shows the distribution of unprofitable ones. The number of users is 1000.

Figure 6.15: Distribution of the lengths of the detours. The solid blue line denotes the case where profit was optimised. The red dash line indicates the profitable matches when the number of matches is optimised, and the solid green line shows the distribution of unprofitable ones. The number of users is 1000.
Chapter 7

Discussion

7.1 Agents

In ride-sharing case, it is reasonable to assume that the users do not decide about or try to affect the platform’s strategy nor the implementation. They are considered myopic enough and either accept or decline the offers from the platform. The agents are said to be active, which means that the agents react to changes in the environment with simple pre-determined actions. Their process of thought and analysis on how they end up to the decisions is not modelled, so they are not cognitive.

The market dynamics of users are considered to be in free competition between alternative modes of transportation. The users are modelled as price-takers. The agents will accept offers that benefit them, but they will not plan strategies to change the dynamics of the markets for their benefit. The users are assumed not to collaborate or to take collective action; there is no cooperation.

For implementation, the decisions about trips need to be reasonably simple. The deal brokered by the platform has a restricted format. The users could be provided meta-data-templates for conditional route definitions or something else that makes it possible to customise the deals. For a simpler matching algorithm, it may be reasonable to limit the deals to individual trips, with a given set of conditions. It would also make the assumption about myopic users that do not plan strategies invalid because the users would need a motivation behind their customised deals and the motivation would most likely be strategic and non-myopic. The platform’s business model is only dynamic ride-sharing, not brokerage of transportation contracts between individuals for carpooling.
7.2 Pricing

The platform’s pricing is rule-based. It is assumed that the driver gets paid and the passenger does pay. For the platform to enforce rule-based pricing, the payments are done via a third party, not directly between the driver and the passenger. This also allows the platform to subsidise the trips; the passengers may pay less than the driver gets. The threshold quality for the pricing requires transparency and ease of use; they may want to understand the mechanism and not worry about bargaining. For transparency, the platform is not surge pricing by changing the rules between users. The users can predict their savings and make the judgements based on the pricing mechanism they know. As the platform is assumed to not have prior information about routines of its users, the pricing rule can only employ information that the users give and what can be deduced, like properties of possible matches.

The users are paid or charged only for the realised matches. It would be unreasonable to charge users that used their time flexibility for matching; they would pay for a negative benefit. Pricing does not reward users for giving a wider time window \( w(k) \). The incentives for wider time windows are in the value of the matches, affected by the pricing mechanism. Absurdly wide time-windows could occur if the users would benefit from the width of the time window. The pricing mechanism should work primarily to maximise profits and incentivise the users to participate. Other mechanisms could work with a matching algorithm to meet other objectives like match rate or social welfare.

The pricing rule is similar to the pricing of taxis. The prices need to be transparent for the passengers to accept them. If the users could not predict their costs, then they could potentially lose money for their lost time flexibility. Alternatively, the users can tell their maximum willingness to pay as passengers or the willingness to accept as drivers. Users would then have two decision variables: the price and the width of the time window. They would depend on each other, making decisions hard. Drivers can get with different matches, varying additional distances. It could be that the drivers would require a per kilometre fee, because otherwise they would need to charge for the worst case scenario, or would not be incentivised to give wide time windows. This would make it hard for the drivers because they would need to price their own time and costs. For the modelling choices, it would not make sense because this type of pricing requires spatial games and strategies for testing their pricing. This type of problems are sometimes solved with reinforcement learning [43]. These methods do not always converge, which can be part of the dynamics of
the market. For corporate entities that would be acceptable, but for price taking consumers probably not. Also, it would make it fairly similar to the pricing of taxis, but less transparent.

Passengers pay the initial \( (r_f) \) and per kilometre \( (r_t) \) charges. This is applied also to the driver’s compensation, but with different values for initial \( (a_f) \) and per kilometre \( (a_t) \) prices. \( c_w(k) \) could be an inhibitor for routes that have the passenger’s route within if drivers were not compensated in case of \( a(e) = 0 \). The initial charge works on the passenger side as a deterrent for short trips that generate less revenue than longer ones. The average cost per kilometre converges to marginal costs with more distance. The platform could alternatively impose a minimum distance rule so that the users would not give too short routes, which could increase the inconvenience felt by the passengers. Drivers have no preference to drive alone, nor with a passenger. Drivers are not compensated by the distance that they drive their passenger because it decreases revenue and as an incentive does not play any role in overcoming the inconvenience and costs of the driver. It does increase the willingness of drivers to take longer distance passengers increasing consumer surplus.

### 7.3 The price of inconvenience and time flexibility

This Thesis uses the value of time as the value for waiting. This may overestimate the value of waiting. The value of waiting varies between occasions. People have other things in their lives, which affect the value of waiting. It is hard to model peoples’ lives and preferences in complete detail and thus some alternative approach is required. As the service in the thesis does not tailor the pricing and the matching for individuals, the resolution required in modelling is lower; good approximations could be gained by using estimates of the real aggregated distributions for the preferences. Using the value of time as the value for waiting is a distribution where every person is similar.

The most critical part is the user model. The agents in the model are willing to share equally their value with the platform in order to gain value. In reality, it can be likely that the more value the users would gain, the greedier they would become; the agents want a greater share of the value if there is more value. The effect has been studied in economics [44].

The users set their time windows depending on the value they expect to get. Before the platform is simulated, the value is not known. For an expected value, the probability distribution for the properties of the match is not known. Platform
Figure 7.1: Simulation system that improves the initial guess by using outcome of the simulation as a feedback.

Figure 7.2: Simulation system with an initial guess.
could be simulated with an initial guess and the results then used in a next iteration round, as shown in Figure 7.1. As the population consists of agents, this would be reinforcement learning.

If there is complexity in the system, as is the case in our simulation, the effects of changing actions are uncertain. The agents’ decisions affect the outcomes. The agents learn from the same outcomes. For this reason, the agents need to be realistic for the simulation to have a realistic outcome. If the agents learn to play a game in unrealistic dynamics created by the agents themselves, they would create unrealistic dynamics for them to learn more about. Even if the simulation would be terminated after few rounds, instead of waiting for convergence, the users would have learned to behave unrealistically. The reinforcement learning may not converge [43]. There can also be path dependency, meaning that different initial states could converge to different points.

One possible model is that the users would give a time window that maximises the expected value. This would require the information how the time windows effect the probability of matching. Because every user would do this, finding time windows would become a game between users. It is unlikely that the users could forecast the system dynamics with a good precision. The users could learn from the outcomes of the simulation. Because the users may not forecast the outcomes, it could be better to have adoption model for the users to make a decision whether they will use the platform. In the adoption model, some users would try the service and if they would perceive that using the service benefits them, then they would keep using the service. There can be a lag as expected value is determined by using the service [27].

With a feedback, the error from the structure of the model would be iterated multiple times, possibly amplifying the effect from an error. With a feedback, the system would be much more complex and there is no data to verify whether the model is reasonable. To decrease the effect of the model it was decided not to have the feedback for the model in this Thesis. Thus the model is as illustrated in Figure 7.2.

7.4 Matching algorithm and simulation

The algorithm chosen for matching is binary integer linear programming. The other faster algorithms have been developed, for actual matching use, whereby binary integer linear programming is used as a benchmark [31]. Heuristics are designed for an application case. The development of algorithms may take a long time, and their
feasibility is about speed and the capability to implement. These are not taken into account in this Thesis. Binary integer linear programming gives an upper bound for the matching algorithms the services would use.

The rolling horizon simulates the dynamic nature of partial information when matches are done. The users may have some patterns that emerge from regular schedules. They could be taken into account for a more optimistic evaluation by using the full information for matching.
Chapter 8

Conclusions

In simulations, the platform manages to produce value for the customers as seen in Figure 6.12 for the customer surplus. Whether the value is enough, considering that the model is optimistic, is uncertain. Using optimisation algorithms gave a clear advantage over FIFO in profit of the platform and the number of matches. Thus, the optimisation algorithm can be of importance for the feasibility of the service. The ride-sharing platform might have a possibility to be a feasible service. The properties of the platform cannot be decided without making a decision about the objectives of the platform. Different platforms have different performance as seen for example between optimisations and FIFO.

The model for time flexibility could have been removed and replaced by a constant width for the time windows. The generated dummy data captures some real phenomena, but as the effects of real phenomena are not thoroughly researched in this Thesis, they could have been replaced by uniform distributions. The rolling horizon property for the matching algorithm is also controversial. The platform can utilise some forecasting. The assumption that the platform does not utilise it is not optimistic and leads to inferior performance. Matching the users and later changing the users they were matched with might be reasonable to change into such that the users are only notified to leave as the ride they are assigned to begin. This also produces inferior performance, while the existence of such a feature is not necessarily part of threshold quality [30], meaning that the users would not require such a feature.

The result of match rate has a similar shape as a function of the number of users as in the literature [10][45]. The existence of a price region where the pricing produces matches and where it does not is trivial. The results are largely continuous in the model parameters and make sense. Even though the model for the time flexibility
is questionable, the agent-based modelling approach would seem to work well for a feasibility study.

The system is modular; agents and the platform are not the same but only interact, so it is easy to make changes to the platform for the development of implementations, or to improve the analysis by integrating more refined models of traffic and customer behaviour.

A similar method for a feasibility study could be applied in any case where time constrained resources are distributed on two-sided markets. One such example would be that of renting construction machines. The machines are stored on the construction sites, but used only for a fraction of the time. Increasing the utilisation rate of them would decrease the capital intensiveness of the industry.
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Appendix A

Generating data

Simulating traffic requires data about the times, origins and destinations of travel. This Thesis generates dummy data to be used in the simulation. The generated data tries to approximate people mostly around the cities travelling to their closest city.

To separate the nodes of the network that are more likely places for a user to start their route, network centralities are used. These metrics try to measure the centrality of a node, with the topology of a network the node is in [46]. The network centrality metric $a : V \rightarrow \mathbb{R}$ for a node in the network $V$, is the average degree of neighbours [46]. The $N$ nodes are indexed in a decreasing order of centrality. The node 1 is the node with the lowest centrality, $N$ the highest. The nodes within the cities often have more edges. The nodes around cities are connected with nodes within cities. This makes the nodes connected to the cities have higher centrality than the nodes connected to them, but not cities.

The normalized network centrality of a node $i$ in the network $V$ of $N$ nodes is

$$
\hat{a}(i) = \frac{a(i)}{\sum_{h=1}^{N} a(h)}.
$$

Because the network centrality of the node corresponds to the motivation for the travel to cities, the relative values are maintained in the probability of the node $i$ being the origin $p_O$ for a user by reversing

$$
p_O(i) = \hat{a}(N - i). \quad (A.1)
$$

The destination depends on the origin by distance. The distance from $i$ to $j$ is normalised

$$
\hat{\delta}_{i}(j) = \frac{\delta_{ij}^*}{\sum_{h=1}^{N} \delta_{ih}^*}.
$$
The shorter the distance from origin, the more probable the node is to be the destination. The distances from each node \( i \) have a mapping from the ordering to the index of the node \( \tilde{\delta}_i : [1, 2, \ldots, N - 1] \to V \). \( \tilde{\delta}_i(1) \) is the farthest node from \( i \), \( \tilde{\delta}_i(N - 1) \) is the closest one.

Specifically, \( p_{D_i}(\tilde{\delta}(n)) \) is the probability for \( n \):th closest node from \( i \) \( \tilde{\delta}_i(n) \) to be the destination for an origin \( i \). The probability depends on the centrality of \( \tilde{\delta}_i(n) \) and its distance from \( i \)

\[
p_{D_i}(\tilde{\delta}(n)) = \frac{\tilde{\delta}(\tilde{\delta}_i(N - n)) \cdot \hat{a}(\tilde{\delta}_i(n))}{\sum_{h=1}^{N-1} \tilde{\delta}(\tilde{\delta}_i(N - h)) \cdot \hat{a}(\tilde{\delta}_i(h))}, \quad \forall n \in [1, 2, \ldots, N - 1] \quad (A.2)
\]

where the denominator normalises the distribution by summing over every possible destination.

The earliest possible departure time \( t \) is modelled to look similar to the data about the number of cars that pass a measuring point \([40]\). A visual inspection of the data suggests that the shapes resemble beta-distributions and that more central nodes have the peak later than the nodes with lower centrality. The distribution is

\[
t \sim Beta(3, 2a(i)),
\]

where the parameters are estimated to produce visually similar shape to the data.

Now that the user has the origin and the destination, the width of the time window \( w \) can be computed by Equation (4.5). The latest possible departure time \( \bar{t} \) of the user is \( t + w \). All of the service requests can then be made with algorithm 2.

**Algorithm 2** User initialisation

1: for all user \( \in \) user pool do
2: the origin \( i \) of user \( \sim p_O \) (A.1)
3: the destination \( j \) of user \( \sim p_{D_i} \) (A.2)
4: the earliest possible departure time \( t \) of user \( \sim Beta(3, 2a(i)) \)
5: the width of timewindow \( w \) of user calculated from \( \delta_{ij}^* \) (4.5)
6: the latest possible departure time for user \( \bar{t} = t + w \)
7: end for