Study of Simulation Parameters and Flow-Controlled Boundary Conditions in a Single-Blade Pump Simulation

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ABSTRACT
A single-blade sewage pump is simulated numerically using a FINFLO flow-solver. Low-Reynolds number approach is utilized with a $k - \epsilon$ turbulence closure. Influence of an internal timestep on a convergence of a time-accurate simulation is studied. The effect of the boundary conditions on a performance prediction, mass balance fluctuation and velocity distributions is studied by using also flow controlled boundary conditions.

MAIN RESULT
Pump head, efficiency and torque are obtained from the simulated flowfield and compared to measured data. The velocity distributions are also compared to the measurements available. The convergence of the pressure is poor which affects significantly pump performance prediction. In spite of that, prediction of velocity distributions is quite good.

KEY WORDS
FINFLO, $k - \epsilon$ model, pump performance, chimera, periodic

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Nomenclature

\( C_p \)  pressure coefficient  
\( E \)  total energy  
\( F, G, H \)  flux vectors in the \( x \)-, \( y \)- and \( z \)-directions  
\( H \)  total head  
\( Pr \)  Prandtl number  
\( Q \)  source term vector  
\( Re \)  Reynolds number  
\( T \)  temperature  
\( U \)  vector of conservative variables  
\( V \)  velocity vector  
\( c_p \)  specific heat at a constant pressure  
\( c_v \)  specific heat at a constant volume  
\( e \)  specific internal energy  
\( k \)  turbulent kinetic energy  
\( \dot{m} \)  mass flow  
\( p \)  pressure  
\( \vec{q} \)  heat flux  
\( t \)  time  
\( u, v, w \)  velocity components in the \( x \)-, \( y \)- and \( z \)-directions  
\( u_\tau \)  friction velocity (\( = \sqrt{\tau_w/\rho} \))  
\( y^+ \)  dimensionless distance from the wall (\( = yu_\tau/\nu \))  
\( \beta \)  artificial compressibility coefficient  
\( \delta_{ij} \)  Kronecker’s delta  
\( \epsilon \)  dissipation of the kinetic energy of the turbulence  
\( \theta \)  temperature difference (\( = T - T_\infty \))  
\( \mu \)  dynamic viscosity  
\( \nu \)  kinematic viscosity  
\( \phi \)  scalar  
\( \rho \)  density  
\( \tau \)  shear stress

Superscripts

\( T \)  transposition  
\( l \)  left side  
\( r \)  right side  
\( w \)  wall value  
\( ' \)  fluctuating component
Subscripts

- $T$: turbulent
- $i, j, k$: $i$-, $j$- and $k$-component
- $t$: tangential component
- $n$: normal component
1 Introduction

In this study a single-blade sewage pump is simulated numerically using FIN-FLO flow solver. A low-Reynolds number $k - \epsilon$ turbulence model is utilized in the simulations. The case has been simulated earlier using the velocity as an inlet and the pressure as an outlet boundary condition [1]. Since in this approach there were severe oscillations in the mass flow, in this study number of internal iterations in time-dependent simulation is increased. This was done in order to get a fully converged result at every time step. Furthermore, boundary conditions are replaced either by placing the pump in a test tank utilizing overlapping grid method or by utilizing periodic boundary conditions. The flow is controlled by changing the area of exhaust pipe cross section, in a similar manner as using a throttle valve.

In the following, the governing equations and turbulence modelling are firstly described. Next, the computational domain and the grid are depicted and, finally, the results of the simulation are presented and compared to measurements as well as previous simulations with the different boundary condition approach.

2 Flow Equations

A low-Reynolds number approach is used in FINFLO. The Reynolds-averaged Navier-Stokes equations, and the equations for the kinetic energy ($k$) and dissipation ($\epsilon$) of turbulence can be written in the following form

$$\frac{\partial U}{\partial t} + \frac{\partial(F - F_v)}{\partial x} + \frac{\partial(G - G_v)}{\partial y} + \frac{\partial(H - H_v)}{\partial z} = Q$$

(1)

where the unknowns are $U = (\rho, \rho u, \rho v, \rho w, E, \rho k, \rho \epsilon)^T$. The inviscid fluxes are

$$F = \begin{pmatrix} \rho u \\ \rho u^2 + p + \frac{2}{3} \rho k \\ \rho u v \\ \rho u w \\ (E + p + \frac{2}{3} \rho k) u \\ \rho u \epsilon \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho v^2 + p + \frac{2}{3} \rho k \\ \rho v w \\ \rho v k \\ (E + p + \frac{2}{3} \rho k) v \\ \rho v \epsilon \end{pmatrix}, \quad H = \begin{pmatrix} \rho w \\ \rho w v \\ \rho w w \\ \rho w k \\ (E + p + \frac{2}{3} \rho k) w \\ \rho w \epsilon \end{pmatrix}$$

(2)

where $\rho$ is the density, the velocity vector by using Cartesian components is $\vec{V} = u \vec{i} + v \vec{j} + w \vec{k}$, $p$ is the pressure, $k$ is the turbulent kinetic energy and $\epsilon$ its dissipation, and the total energy $E$ is defined as

$$E = \rho e + \frac{\rho \vec{V} \cdot \vec{V}}{2} + \rho k$$

(3)
where $e$ is the specific internal energy. The viscous fluxes are

$$F_v = \begin{pmatrix}
0 \\
\tau_{xx} \\
\tau_{xy} \\
\tau_{xz} \\
u\tau_{xx} + v\tau_{xy} + w\tau_{xz} - q_x \\
\mu_k(\partial k/\partial x) \\
\mu_{\epsilon}(\partial \epsilon/\partial x)
\end{pmatrix}, \quad G_v = \begin{pmatrix}
0 \\
\tau_{xy} \\
\tau_{yy} \\
\tau_{yz} \\
u\tau_{xy} + v\tau_{yy} + w\tau_{yz} - q_y \\
\mu_k(\partial k/\partial y) \\
\mu_{\epsilon}(\partial \epsilon/\partial y)
\end{pmatrix}$$

$$H_v = \begin{pmatrix}
0 \\
\tau_{xx} \\
\tau_{yy} \\
\tau_{zz} \\
u\tau_{xx} + v\tau_{yy} + w\tau_{zz} - q_z \\
\mu_k(\partial k/\partial z) \\
\mu_{\epsilon}(\partial \epsilon/\partial z)
\end{pmatrix}$$

Here the stress tensor, $\tau_{ij}$, includes laminar and turbulent components. The fluid is assumed to be Newtonian and, therefore, the laminar stresses are modelled by using Stokes hypothesis. The Reynolds stresses $\rho u'_i u'_j$ are included in the stress tensor $\tau_{ij}$.

$$\tau_{ij} = \mu \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{2}{3}(\nabla \cdot \vec{V})\delta_{ij} \right] - \rho u'_i u'_j + \frac{2}{3}\rho k\delta_{ij}$$

For the Reynolds stresses, Boussinesq’s approximation

$$-\rho u'_i u'_j = \mu_T \left[ \frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} - \frac{2}{3}(\nabla \cdot \vec{V})\delta_{ij} \right] - \frac{2}{3}\rho k\delta_{ij}$$

is utilized. Here $\mu_T$ is a turbulent viscosity coefficient, which is calculated by using a turbulence model, and $\delta_{ij}$ is the Kronecker’s delta. In the momentum and energy equations, the kinetic energy contribution $2/3\rho k\delta_{ij}$ has been connected with pressure and appears in the convective fluxes, whereas the diffusive part is connected with the viscous fluxes. The viscous stresses contains a laminar and a turbulent parts. The heat flux can be written as

$$\vec{q} = -(\lambda + \lambda_T) \nabla T = -\left( \mu \frac{c_p}{Pr} + \mu_T \frac{c_p}{Pr_T} \right) \nabla T$$

where $\lambda$ is a molecular and $\lambda_T$ a turbulent thermal conductivity coefficient and $Pr$ is a laminar and $Pr_T$ a turbulent Prandtl number, and $c_p$ is a specific heat at constant pressure. The diffusion of turbulence variables is modelled as

$$\mu_k \nabla k = \left( \mu + \frac{\mu_T}{\sigma_k} \right) \nabla k$$
\[ \mu_\epsilon \nabla \epsilon = \left( \mu + \frac{\mu_T}{\sigma_\epsilon} \right) \nabla \epsilon \]  

(9)

where \( \sigma_k \) and \( \sigma_\epsilon \) are turbulent Schmidt’s numbers of \( k \) and \( \epsilon \), respectively. Density is obtained from an equation of state \( p = p(\rho, T) \) Since this case is essentially incompressible, pressure differences \( p - p_0 \) are solved instead of pressure. The components of the source term \( Q \) are non-zero in possible buoyancy terms and in turbulence model equations. In this study the buoyancy terms are insignificant and not applied.

In the present study an artificial compressibility approach is used to determine the pressure. The flux calculation is a simplified version of the approximate Riemann-solver utilized for compressible flows [2]. It should be noted that in this approach the artificial sound speed affects the solution, but the effect is of the second-order and it should not be visible as the grid is refined. The effect is similar to the Rhie and Chow interpolation method applied in commercial codes. The solution method applied is described in [3]. In time-accurate simulations the artificial compressibility approach is used inside a physical time step [4]. Each time step is treated as a steady-state case and iterations are made inside the time step. The time derivative term is treated as a source term. The method is fully implicit and a three-level approximation is used for the time derivative.

### 3 Turbulence Modelling

As mentioned, turbulent stresses resulting from the Reynolds averaging of the momentum equation are modelled by using Boussinesq’s approximation (6). The turbulent viscosity coefficient \( \mu_T \) is determined by using Chien’s [5] low-Reynolds number \( k - \epsilon \) model from the formula

\[ \mu_T = c_\mu \frac{k^2}{\epsilon} \]  

(10)

where \( c_\mu \) is a empirical coefficient. The source term of Chien’s model is

\[ Q = \left( \begin{array}{c} P - \rho \epsilon - 2 \mu \frac{k}{y_n^2} \\ c_1 \frac{\epsilon}{k} P - c_2 \frac{\rho \epsilon^2}{k} - 2 \mu \frac{\epsilon}{y_n^2} e^{-y^+ / 2} \end{array} \right) \]  

(11)

where \( y_n \) is the normal distance from the wall, and the dimensionless distance \( y^+ \) is defined by

\[ y^+ = y_n \frac{\rho u_\tau}{\mu} = y_n \sqrt{\frac{\rho \tau_w}{\mu}} \approx y_n \left[ \frac{\rho |\nabla \times \vec{V}|}{\mu} \right] w^{1/2} \]  

(12)

Here \( u_\tau \) is friction velocity and \( \tau_w \) is friction on the wall, and the connection between them is \( u_\tau = \sqrt{\tau_w / \rho} \). The unknown production of the turbulent
kinetic energy is modelled using Boussinesq’s approximation (6)

\[ P = -\rho u_i u_j \frac{\partial u_i}{\partial x_j} \]
\[ = \left[ \mu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right) - \frac{2}{3} \delta_{ij} \rho k \right] \frac{\partial u_i}{\partial x_j} \]  \hspace{1cm} (13)

The turbulence model presented above contains empirical coefficients. Those are given by [2]

\[
c_1 = 1.44 \quad \sigma_k = 1.0 \\
c_2 = 1.92(1 - 0.22 e^{-Re_T^2/36}) \quad \sigma_\epsilon = 1.3 \\
c_\mu = 0.09(1 - e^{-0.0115y^+}) \]  \hspace{1cm} (14)

where the turbulence Reynolds number is defined as

\[ Re_T = \frac{\rho k^2}{\mu \epsilon} \]  \hspace{1cm} (15)

Chien’s model is very robust, but it has several shortcomings. It usually overestimates the turbulence level and is not performing well in a case of an increasing pressure gradient. However, in the present study the main emphasis is to study the behaviour of the numerical method, where turbulence plays a minor role.

4 Computational Grids and Boundary Conditions

All computational grids are based on grid that is described in detail in reference [1]. Only the second grid level (every other gridpoint) is used in this simulation, since the main purpose is to study the effect of boundary conditions and grid converged results are not important.

4.1 Fixed Velocity

The massflow of this case is 33 kg/s. A quasi-steady simulation is used as a starting guess for the time-accurate simulation. Velocity and turbulence quantities are used as a pump inlet boundary condition and the pressure is given as the pump outlet boundary condition.

4.2 Test Tank

In test tank case the pump grid is placed inside retangular tank. The tank grid is quite coarse, but is clustered around pumps inlet and outlet area. Clustering is necessary for the Chimera interpolation method, where the Chimera and
background grids should have about same density. In this simulation the total number of grid cells is 403,072. There are 162,944 grid cells in the impeller part of the grid, 111,104 in the volute part and 129,024 in the tank part. The surface grid is depicted in Fig. 1.

Fig. 1: Surface grid of tank and the pump.
Also in this case a quasi-steady simulation is used as a starting guess for the time-accurate simulation. The quasi-steady simulation is started using velocity and turbulence quantities as a pump inlet boundary condition and the pressure is given as the pump outlet boundary condition. A value of 800 000 Pa is given for the outlet pressure in order to prevent negative pressures during the iteration. The pressure level is insignificant in a present incompressible case. As the flow through the pump is established, the inlet and outlet boundary conditions are changed to overlapping boundaries. Since the flow is not regulated by the boundary condition, the area of the outlet surface has to be adjusted in order to get a correct massflow. The diameter of the outlet is reduced from from 100 mm to 80 mm, thus the area is reduced by 38.5 %. In the quasi steady simulation the massflow was 29 kg/s, but during the time accurate simulation massflow changed to 32 kg/s, which was considered to be close enough the massflow used in other simulations.

4.3 Periodic Loop

This grid is same as pump part in test tank simulation, except the outlet pipe is extended and, because of the implementation of periodic boundary condition in FINFLO, it is bended down. Also there is a non-matching surface after the bend, since the periodic boundary condition needs to have a same grid distribution in both ends. The total number of grid cells is 295 808, there are 21 760 cells in the added duct. The surface grid is depicted in Fig. 2.

![Fig. 2: Surface grid of a periodic loop pump.](image_url)
5 Results

A quasi-steady simulation was used as an initial guess for the time-accurate simulations. With the inlet/outlet boundary conditions, eight impeller revolutions were simulated with 200 internal iterations, after that four revolutions with as much as 2000 iterations.

With the tank case five impeller revolutions and with the pipe loop case seven revolutions were simulated. In the both of these simulations, 200 internal iterations are used for two first impeller rounds and 400 iterations after that. The Courant number of the internal iterations is equal to one in all simulations.

A multigrid acceleration could not be used to accelerate convergence in the time-accurate simulations due to unknown instability, in the quasi-steady computations two multi-grid layers were used.

Unfortunately the pipe loop case results suffer from the fact that non-matching boundary is reducing the time-dependent nature of the flow. In Fig. 3 massflows at upstream and downstream sides of non-matching connection and at periodic inlet and outlet are shown. It can be seen that there is also a small difference in massflows between periodic inlet and outlet but amplitude is in same phase and magnitude. Since massflow remains constant over non-matching connection in quasi-steady simulation, the problem is extremely difficult and could not be solved within this study.

![Massflows at upstream and downstream side of the non-matching connection (left) and at periodic inlet and outlet (right).](image)

A head is calculated from

\[ H = \frac{p_2 - p_1}{\rho g} \]  

(16)

where \( p_1 \) is the inlet pressure, \( p_2 \) the outlet pressure; \( \rho \) the density, and \( g \) the acceleration due to gravity. The efficiency is obtained from

\[ \eta = \frac{\Delta E}{T \omega} \]

(17)

where \( \Delta E \) is the difference between the mechanical energy flux at the inlet and the outlet and \( T \omega \) is the required axial power. The axial power should
be smaller than the measured one, since several components of the loss, for example the friction on the outside of the impeller and bearing losses, are not taken into account. The head, efficiency and axial power needed are presented in Table 1. The test tank and pipe loop cases are compared to the standard fixed velocity inlet and fixed pressure outlet case, which are simulated using 200 and 2 000 internal iterations.

Table. 1: Pump performance.

<table>
<thead>
<tr>
<th>Case</th>
<th>Total Head</th>
<th>Efficiency</th>
<th>Shaft Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet/outlet 200</td>
<td>12.95 m</td>
<td>82.4%</td>
<td>5289 W</td>
</tr>
<tr>
<td>Inlet/outlet 2000</td>
<td>13.2 m</td>
<td>80.3%</td>
<td>5857 W</td>
</tr>
<tr>
<td>Test tank</td>
<td>12.53 m</td>
<td>62.1%</td>
<td>5593 W</td>
</tr>
<tr>
<td>Pipe loop</td>
<td>9.1 m</td>
<td>64.6%</td>
<td>5457 W</td>
</tr>
<tr>
<td>Experimental</td>
<td>12.4 m</td>
<td>71.1 %</td>
<td>5675 W</td>
</tr>
</tbody>
</table>

The head, mass balance, efficiency and axial power over a one impeller cycle are shown in Figs. 4 – 5. It can be seen, that fixed velocity with 200 internal iterations cause heavy oscillation in head value, as well as in outlet massflow and efficiency. Increasing number of internal iterations reduces maximum amplitude but in turn it produces high-frequency oscillations instead of the original sinusoidal head curve. Utilizing the test tank, the maximum amplitude resembles the case of the fixed velocity with 2000 iterations, but without small oscillations.

Fig. 4: Head as a function of the impeller angle in the time-accurate simulations with the test tank and the pipe loop geometries as well as the fixed velocity cases.
Fig. 5: Massflow in (top) and massflow out (bottom) as a function of the impeller angle in the time-accurate simulations with the test tank and the pipe loop geometries as well as the fixed velocity cases.
Fig. 6: The efficiency as a function of the impeller angle in the time-accurate simulations with the test tank and the pipe loop geometries as well as the fixed velocity cases.

Fig. 7: The shaft power as a function of the impeller angle in the time-accurate simulations with the test tank and the pipe loop geometries as well as the fixed velocity cases.
The problem of fluctuating pressure is related to convergence problems. When a number of internal iterations is increased, pressure is changing linearly after first 50 iterations. And it continues changing, which was tested even with 30,000 iterations. But, opposite to one might expect, the amplitude of pressure fluctuations is smaller when iteration number is increased. On the other hand the number of pressure fluctuations per impeller revolution is increased. Convergence histories of average pressure and maximum pressure change within a timestep (2,000 internal iterations) are shown in Fig. 8.

![Graph showing convergence history of average pressure and maximum change of pressure](image)

**Fig. 8**: Convergence history of average pressure (left) and maximum change of pressure (right) within a timestep.

Pressure distributions on the surface of the impeller and on a half of the volute, as well as velocity vectors and velocity distributions at the measurement stations, given in Fig. 9, are shown in Figs. A-1 – A-8 of Appendix A.

In velocity distributions there are no significant differences between the cases. Pressure distributions are quite different between the cases, and that is partly because secondary oscillations of the pressure are affecting the distribution quickly especially in case of 2,000 internal iterations.

Velocity vector distributions at the planes through measurement point 1 and an enlarged part of the plane are shown in Fig. A-9 and A-10 of Appendix A. When 2,000 internal iterations are made, the flow is more complex with a secondary vortex whereas the flow is quite similar in cases simulated with 200 or 400 internal iterations.

Distributions for the velocity and turbulent kinetic energy, taken on a plane at a distance of 40 mm from the bottom of the impeller, are shown in Figs. A-11 - A-14 of Appendix A. The velocity in the figures is in an inertial coordinate system. In the velocity distributions there are only small differences between the fixed velocity cases. In the test tank case the flow field is smoother inside the impeller near the wake of the blade. Also the turbulent kinetic energy
distribution of the test tank case is different from the others especially in pressure side of the blade near the hub.

6 Discussion

In this study a single-blade sewage pump is simulated numerically using a low-Reynolds number $k - \epsilon$ turbulence closure. The purpose of the study was to investigate possible reasons for the numerical anomalies found with the simulations using the FINFLO code. The goal of the study was also to examine, how much a better convergence during the time step affects the error in the mass balance and also reflections from the outlet, which occur as unphysical pressure fluctuations. A test tank case without pressure fluctuations by nature was also simulated. A pipe loop case using similar outlet pipe area reduction was also tried. Unfortunately the pipe loop case results suffer from the fact that the non-matching boundary is reducing time-dependent nature of the flow.

With a highly oscillating flow in single blade pump simulation, boundary conditions play a significant role in pressure oscillations, but quantitative values as well as the velocity distributions are not affected so much. Velocity distributions were predicted with a good accuracy. Use of the test tank eliminated oscillations, but there are some problems conserving the massflow.
References


Fig. A-1: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 1. From top to bottom: The fixed velocity boundary conditions with 200 internal iterations, with 2 000 internal iterations and the test tank case. The impeller angle is 0°.
Fig. A-2: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 1. The orientation as in Fig. A-1. The impeller angle is 90°.
Fig. A-3: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 1. The orientation as in Fig. A-1. The impeller angle is 180°.
Fig. A-4: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 1. The orientation as in Fig. A-1. The impeller angle is $270^\circ$. 
Fig. A-5: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 2. The orientation as in Fig. A-1. The impeller angle is 0°.
Fig. A-6: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 2. The orientation as in Fig. A-1. The impeller angle is $90^\circ$. 
Fig. A-7: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 2. The orientation as in Fig. A-1. The impeller angle is 180°.
Fig. A-8: Pressure distribution on the surface of the impeller and volute, velocity vectors and velocity distributions at measurement point 2. The orientation as in Fig. A-1. The impeller angle is $270^\circ$. 
Fig. A-9: Projected velocity vectors in a plane from center of the impeller to measurement point 1. Fixed velocity case with 200 internal iterations (top), 2 000 internal iterations (middle) and the test tank case (bottom).
Fig. A-10: Projected velocity vectors in a plane from center of the impeller to measurement point 1 zoomed in to upper side of the volute. Fixed velocity case with 200 internal iterations (top), 2 000 internal iterations (middle) and the test tank case (bottom).
Fig. A-11: Distribution of velocity (left) and turbulent kinetic energy (right) at plane 40 mm from the bottom of the impeller. From top to bottom the fixed velocity boundary conditions with 200 internal iterations, with 2 000 internal iterations and the test tank case. The impeller angle is 0°.
Fig. A-12: Distribution of velocity (left) and turbulent kinetic energy (right) at plane 40 mm from the bottom of the impeller. The orientation as in Fig. A-11. The impeller angle is 90°.
Fig. A-13: Distribution of velocity (left) and turbulent kinetic energy (right) at plane 40 mm from the bottom of the impeller. The orientation as in Fig. A-11. The impeller angle is $180^\circ$. 
Fig. A-14: Distribution of velocity (left) and turbulent kinetic energy (right) at plane 40 mm from the bottom of the impeller. The orientation as in Fig. A-11. The impeller angle is 270°.