Optical imaging of surface dynamics in microstructures

Lauri Lippiäinen
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Lauri Liptäinen

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Abstract

The research summarized in this thesis covers the design, implementation, and use of optical techniques for characterizing surface movements in microstructures. The main focus of the work has been on developing instrumentation and data analysis methods for investigating surface vibrations in micromechanical components that are based on, e.g., microelectromechanical systems (MEMS), and surface and bulk acoustic waves.

All the scanning single-point and full-field vibration detection setups and methods developed in this work enable phase-sensitive, absolute amplitude measurement of surface vibrations. An unstabilized homodyne interferometry concept is presented for detecting out-of-plane (OP) vibrations with a scanning single-point Michelson laser interferometer. A noninterferometric detection method for measuring in-plane (IP) vibrations is also described that is implemented in this scanning system. The setup enables vibration measurements for frequencies up to 2 GHz, with typical minimum detectable amplitudes of even less than 1 pm and 10 pm for the OP and IP components, respectively. Furthermore, novel methods based on these scanning techniques were implemented to allow for studies of the nonlinear behavior of surface vibrations, which serve to advance the understanding of such effects in microacoustic components. The scanning-based optical imaging methods were applied to two research studies in MEMS resonators that showed unexpected behavior.

The full-field interferometric techniques and analysis methods developed in this thesis work push the performance of the camera-based detection of OP vibrations into new limits. The work advances the stroboscopic white-light interferometric technique to be applicable for characterizing high-frequency devices with vibration amplitudes down to less than 100 pm and with frequencies up to 1 GHz. In addition, a stabilized full-field stroboscopic detection concept was developed and the implemented setup was demonstrated to allow for detecting surface vibrations with minimum detectable amplitudes of less than 30 pm. The stabilized full-field interferometer was also developed further for imaging surface dynamics on microstructures in the time domain with even subnanometer vertical resolution.

The optical imaging methods described in this thesis contribute substantially to the research and development of micromechanical devices as they offer direct information of the underlying device physics. The benefits of these advanced optical characterization methods are clearly highlighted in the two MEMS resonator study cases, in which the optical characterization revealed the physical mechanisms that adversely affect the device performance.

Keywords Laser interferometry, microacoustics, micromechanical devices, surface dynamics

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Tekijä
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Väitöskirjan nimi
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Julkaisija
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Yksikkö
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Tiivistelmä
Väitöskirjatyö käsittlelee optisten menetelmien suunnittelua, toteutusta ja käyttöä pintaliikkeiden karakterisointiin mikrorakenteissa. Työn erityisenä painopisteena on ollut mitataulutietoisten ja datanalyysimenetelmien kehittäminen pintavärähtelyjen tutkimiseksi mikromekaanisissa komponenteissa, jotka perustuvat esimerkiksi mikroelektromekaanisiin systeemihin (MEMS) tai pinta- ja tilavuuskustoihin.


Väitöstyössä kehitetyt kamerapohjaiset interferometriaalitesteet ja -menetelmat parantavat kokokentätatekniikalla toteutetun pystyväärähtelyn mitattamisen suorituskynyn rajoja. Stroboskooppiellä valkoisen valon interferometriaalitekniikalla toteutettu laitteisto edistää kyseisen tekniikan käyttöä: tauluetta 100 MHz:n väärähtelytaajuuuksien asti pienimmän havaittavan väärähtelyamplitudin ollessa alle 100 pm. Tämän lisäksi työssä kehitettiin stabiloiuto Stroboskoopppinen kamerapohjainen interferometriamenetelmä. Tähän tekniikkaan perustuvan laitteiston osoitettiin pystyvän havaitsemaan alle 30 pm:n väärähtelyampludeua. Samasta stabilointi-ideaan kehitettiin edelleen mikrorakenteisten pintojen pystysuuntaisten liikkeiden kuvantamiseen myös aika-alueessa niin, että saavutettiin jopa 1 nm:n resolutio.

Tässä väitöstyössä esitettyjä optiset kuvantamismenetelmät hyödyttävät oleellisesti mikromekaanisen laitteiden tuotekehitystä tarjoten suoran kokeellisen menetelmän selvittää laitteiden toiminnan fysiikaalia perustaa.

Avainsanat
Laseroferometria, mikroakustikka, mikromekaaniset laitteet, pintaliikkeet

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Preface

The research work summarized in this thesis was carried out in the Microacoustics research group at the Department of the Applied Physics at Aalto University School of Science (formerly Helsinki University of Technology) during the years 2008-2016. This research has been financially supported by scholarships from Finnish Cultural Foundation, Helsinki University of Technology, and Finnish Foundation for Technology Promotion. I am most grateful to my supervisor Prof. Matti Kaivola for the unique opportunity to work in such a fruitful and inspiring research group. I also thank him for guiding me through the finish line.

The long-term collaboration with VTT Technical Research Centre of Finland has been an integral part of this research work and has provided a source of motivation for many of the optical imaging concepts described in this thesis. It has been a privilege to work with enthusiastic and wonderful colleagues like Antti Jaakkola and Tuomas Pensala. I want also to thank Antti, Tuomas, and numerous other people from VTT for the interesting and lively discussions on a wide variety of more or less relevant topics. In addition, it was a great pleasure to work together with Sergey Gorelick, Panu Koppila, and many others during several interesting projects that are outside the scope of this thesis work.

The development of the white-light interferometer was a fascinating joint project between the Microacoustics group and the Fiber Optics group of the Department of Micro and Nanosciences, Aalto University School of Science. I want to thank the group leader Hanne Ludvigsen for providing a supportive environment for this successful project. I truly enjoyed working with Igor Shavrin and Steffen Novony when we struggled through the obstacles and finally achieved our research objectives.

I wish to express my gratitude to Olli Holmgren for patiently teaching me all the details of the homodyne interferometer setup. I am forever
indebted to my instructor and long-term friend Kimmo Kokkonen who introduced me to the secrets of interferometry. There are no words to describe the amount of support and contribution that he has provided me throughout my research work. He has also been a gold mine of fresh ideas and creativity from which most of the imaging concepts of this work has been originated from. I will never forget the unique atmosphere of the office, and the hilarious, unconventional humor cultivating our daily research work.

I cannot overstate the importance of my friends, grandparents, parents, sisters and a brother for providing the crucial ingredients of a meaningful and balanced life. Thanks for being there. Finally, I want to thank my wife Piia, my son Roni, and my daughter Sani for their love and patience. It is definitely not an easy task to try to understand the researcher’s mind. In particular, how can such a goofy thesis be so important? I hope I can figure out that one day and give you a good answer.

Munich, Germany, September 27, 2016,

Lauri Lipiäinen
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List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.


Author’s contribution

The author has contributed significantly to the development of the optical methods, analysis, interpretation, and reporting of the results presented in this research work. The author has carried out all the optical measurements of this work. In Publications I-III and VI, VII, the author did most of the planning and implemented the optical detection concepts, developed the data analysis, and performed the optical characterization of the samples. The author performed the sample measurements for the characterization of the optical setup reported in Publication IV, and participated to the design and implementation of the system as well as to the development of the measurement control software. The author was responsible for the experimental part of Publication V and was involved with the data analysis. The author has been primarily responsible for writing the Publications I-III and VI, VII and contributed to the content in Publications IV and V. The author has also presented the results of the work at international conferences, including the IEEE International Ultrasonics Symposium in 2009, 2010, 2012, and 2014.
# List of abbreviations

The following abbreviations are used in the overview:

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>1D</td>
<td>One-dimensional</td>
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<tr>
<td>2D</td>
<td>Two-dimensional</td>
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<tr>
<td>3D</td>
<td>Three-dimensional</td>
</tr>
<tr>
<td>AC</td>
<td>Alternating current</td>
</tr>
<tr>
<td>AIDT</td>
<td>Annular inter-digital transducer</td>
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<tr>
<td>BAW</td>
<td>Bulk acoustic wave</td>
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<tr>
<td>BS</td>
<td>Beam splitter</td>
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<tr>
<td>CO</td>
<td>Collimation optics</td>
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<tr>
<td>DC</td>
<td>Direct current</td>
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<tr>
<td>FDA</td>
<td>Frequency domain analysis</td>
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<tr>
<td>FEM</td>
<td>Finite-element method</td>
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<tr>
<td>FT</td>
<td>Fourier transform</td>
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<td>FWHM</td>
<td>Full width at half maximum</td>
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<tr>
<td>IP</td>
<td>In-plane</td>
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<tr>
<td>LED</td>
<td>Light-emitting diode</td>
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<tr>
<td>MD</td>
<td>Modulation depth</td>
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<tr>
<td>MDR</td>
<td>Modulation-depth-reference</td>
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<tr>
<td>MEMS</td>
<td>Microelectromechanical systems</td>
</tr>
<tr>
<td>MM</td>
<td>Multimode</td>
</tr>
<tr>
<td>MOF</td>
<td>Microstructured optical fiber</td>
</tr>
<tr>
<td>OI</td>
<td>Optical isolator</td>
</tr>
<tr>
<td>OP</td>
<td>Out-of-plane</td>
</tr>
<tr>
<td>PBS</td>
<td>Polarizing beam splitter</td>
</tr>
</tbody>
</table>
PD  Photodetector
PID  Proportional-integrative-derivative
QP  Quadrature point
QWP  Quadrature wave plate
RF  Radio-frequency
SAW  Surface acoustic wave
SC  Supercontinuum
SE  Square-extensional
SPUDT  Single-phase unidirectional transducer
SSFFI  Stabilized stroboscopic full-field interferometry
SWLI  Stroboscopic white-light interferometer
WLI  White-light interferometry
1. Introduction

The progress in the development of micro- and nanosystems with mechanically moving or deforming structures has significantly contributed to the innovative technology that facilitates our everyday life. The intense research on microelectromechanical systems (MEMS) has led to a rapid expansion of attractive solutions for replacing conventional electronics made of discrete components. On the other hand, MEMS technology also enables the design of novel miniaturized components with ever more sophisticated functionalities. The foreseen benefits include compact size, low power consumption, and decreased fabrication costs as well as integrability to other electronics by using manufacturing technologies familiar from semiconductor industry. MEMS-based sensors such as accelerometers, gyroscopes, and pressure sensors, have already achieved a market acceptance in a wide range of applications and are routinely used in, e.g., automobiles, customer electronics, and medical devices.

The wireless transmission and reception of today's mobile communications devices rely on radio-frequency (RF) filters that are realized with electroacoustic components in order to satisfy the demanding specifications [1]. Currently these filters are implemented with both surface acoustic wave (SAW) [2] and bulk acoustic wave (BAW) [3] technologies.

Research efforts have also been devoted to developing reference oscillators with MEMS resonator technology for timing and frequency control applications [4,5], which have for decades been dominated by quartz crystal based components. Such oscillators are found in practically any electronic device that uses a reference timing base for clocking of digital electronics or that needs a reference frequency signal. The first commercial MEMS reference oscillators are already available on the markets.

The ongoing development of ever more sophisticated micromechanical components, together with the demand for better performance, call for
advanced experimental techniques for the characterization of the device operation. Despite of the impressive progress in the development of numerical and analytic methods for modeling these advanced components, the models may not always reveal all the relevant phenomena influencing the device operation in complex microstructures. Direct measurement of the mechanical behavior can give vital information on the underlying device physics. These data serve to validate that the device operates as expected and provide useful feedback for improved device design. Moreover, direct experimental information of the mechanical motion advances the theoretical understanding of the dynamics in the complex micro- and nanostructures of these components. Optical probing techniques, such as interferometry, allow for a direct, non-contact measurement of surface deformations and have proven their strength in the characterization of surface vibrations in SAW, BAW, and MEMS devices [6–10].

The objective of the research work of this thesis was to advance the optical imaging techniques for measuring surface dynamics in microstructures. A particular focus has been on developing instrumentation and data analysis methods for versatile optical characterization of surface vibration fields in SAW, BAW, and MEMS devices, including both scanning single-point and full-field camera-based optical detection setups. In addition to vibration measurements, a full-field detection scheme for measuring surface movements in time domain has also been implemented.

Different optical techniques for the detection of surface vibrations in micromechanical devices are introduced in Ch. 2 together with a brief discussion on important features and performance metrics required for such methods. Next, the principles of the developed detection concepts and analysis methods, the implemented optical setups, and the performance of the setups are described in Ch. 3 and Ch. 4 for the scanning and full-field systems, respectively. This is followed by applying the scanning-based optical imaging methods to two research case studies presented in Ch. 5. In these case studies, optical characterization revealed interesting mechanical behavior in square-plate MEMS resonators, which were operating unexpectedly. Finally, Ch. 6 concludes the main achievements and findings of the work.
2. Optical detection of surface movements in micromechanical devices

2.1 Background

Acoustic waves are propagating deformations in an elastic medium resulting from a combinatorial effect of periodic displacements of individual particles of the medium around their equilibrium position. They can be induced by perturbing the medium, e.g., with external mechanical stimuli. Acoustic waves can have both longitudinal and transversal displacement components, and they propagate with a characteristic velocity in the medium with their energy confined near the surface of the medium (SAWs) or also inside the bulk (BAWs). Typical micromechanical components, whose operation is based on mechanical vibrations, have a resonant structure where acoustic waves are generated within a well-defined volume. The waves are bound to propagate within this active volume only, e.g., by means of reflective structures, and in a favorable geometry are hence superposed to form a standing-wave resonance with a high energy density.

A majority of micromechanical components that make use of vibrating structures are based on the electromechanical principle, in which an electric alternating current (AC) signal is converted to mechanical vibrations. The resulting acoustic wave field is then processed in the (electro)mechanical domain in a desired way and transformed back to an electric signal. The most common ways to convert an electrical signal to an acoustic wave field and back are piezoelectric and electrostatic (capacitive) actuations. The former excitation method requires piezoelectric materials in which an electric field in the material creates a deformation and vice versa. This actuation mechanism is used in RF BAW and SAW components where the acoustic waves are generated in a piezoelectric crystal.
or a thin film. A piezoelectric thin film may also be utilized to excite mechanical resonances to materials that are not piezoelectric, such as silicon. Such mechanism is used in, e.g., piezoelectrically actuated MEMS resonators [5]. In capacitive actuation that is conventionally used in MEMS devices, a periodic electrostatic force between electrodes results in a vibrating movement. This is accomplished by driving AC voltage, typically with a large direct current (DC) voltage bias, to the electrodes that are separated with a narrow gap and attached to structures that can move with respect to each other.

The first significant achievements in the development of microelectromechanical devices date back to the 1960s. In 1965, White and Voltmer demonstrated a seminal electrode structure, called an interdigital transducer, for efficient transduction of an electric RF signal into SAW and back [11]. The interdigital transducer consists of two interlocking comb-shaped electrode arrays that are deposited on the surface of a piezoelectric substrate to form a periodic structure. This structure enabled industrial use of acoustic waves in filtering and signal processing applications, launching an era of new SAW research and intense development of SAW-based electroacoustic components. Some of the first reviews of the early development of SAW technology in the 1970s can be found in Refs. [12–15]. The concept of a MEMS component was introduced by Nathanson et al. in their pioneering work on a resonant-gate transistor [16, 17] the operation of which was based on resonance of a micromechanical cantilever beam. Another early example of a MEMS device was a resonistor [18].

From then on, microelectromechanical devices with vibrating structures have been intensively developed for a wide range of applications, and experimental characterization of the device performance has been an essential part of the device design from the very beginning. These devices are typically characterized electrically as their electrical output is of primary interest that ultimately determines the relevant performance. Even though advanced electrical measurement techniques have indeed significantly contributed to the research and development of electromechanical components, electrical characterization provides only indirect information of the acoustic behavior and sometimes leaves open questions of the details of the underlying device physics. A more complete picture of the device operation can be obtained if these electrical measurements are combined with direct measurements of the mechanical vibrations using, e.g.,
optical techniques.

Understanding of nonlinear effects in microacoustic components has lately attracted a wide interest. Nonlinearities in microacoustic components offer interesting possibilities for advanced MEMS device designs that challenge the traditional solutions based on linear operation [19–23]. On the other hand, better understanding of the origin of the nonlinearities in, e.g., BAW and SAW filters is needed to further improve the linearity of the components and hence to achieve the stringent performance requirements of the next-generation components [24–27]. To validate and test theories, models, and actual component performance, experimental methods for studying the nonlinearities in these devices are needed. In this research work, optical scanning single-point techniques have been developed for the characterization of nonlinear acoustic wave fields in micromechanical components. These optical methods are discussed in Sections 3.1.2 and 3.2.2.

In addition to vibrating micromechanical components, there are also a multitude of micromechanical components and other functional microstructures whose operation is based on temporal displacements or deformations that are not periodic. Consequently, techniques to detect dynamic movements, such as transients, impulse responses, and repeatable or non-repeatable single events, are needed. Being able to measure time-dependent surface deformations is not only beneficial for industrial research and development purposes, but also of particular interest for physics research in which the movement of a structure is measured as a function of an external physical quantity. For instance, such measurement allows for determining the electromechanical coupling coefficient in a piezoelectrically actuated microstructure by measuring the surface deformation as a function of applied DC voltage. In Sec. 4.2.2, a camera-based interferometer for the full-field time-domain measurement of out-of-plane surface movements of microstructures is presented. Although in this thesis the optical time-domain detection is not treated in such detail as the optical characterization of surface vibrations, it is worth noting that only a few of the optical methods used for the measurement of surface vibrations, discussed in Sec. 2.2.1, are readily applicable for time-domain measurements.
Figure 2.1. (a) Principles of the single-point and full-field detection techniques and definitions of the out-of-plane (OP) and in-plane (IP) vibration components. (b) Three-dimensional view of instantaneous out-of-plane deformations of a square-plate MEMS sample for two different vibration modes (left and middle) and of an acoustic standing wave pattern on the surface of a SAW sample (right). The SAW wavelength in typical SAW devices ranges from \( \sim 1 \mu m \) to a few tens of micrometers whereas in MEMS components the vibration field can be laterally much smoother.

2.2 Characterization of surface vibrations

2.2.1 Overview of optical detection techniques

The invention of the laser in 1960 [28] was a revolutionary milestone for optical metrology, providing a source of coherent, collimated, and monochromatic light that enables measurements of surface features and movements with subwavelength accuracy. It was not long after the development of the first microelectromechanical devices that laser-based techniques were already applied for the detection of acoustic waves in such components. The principles of the earliest methods for measuring the out-of-plane (OP) vibration component of SAWs with micrometer-order lateral resolution were already demonstrated in the late 1960s, including diffraction grating [29–31], knife-edge [32, 33], and interferometric detection techniques [6, 34, 35]. In these single-point detection schemes, a laser beam is focused on the sample surface to a spot with \( \mu m \)-order diameter as illustrated in Fig. 2.1(a). The acoustic wave field over the sample surface area of interest could then be imaged by translating the light beam or the sample laterally, i.e., by raster-scanning the measurement area point by point.

Although all the important SAW parameters can be characterized with the above-mentioned three techniques [36], the diffraction grating and knife-edge techniques require relatively high curvatures for the OP surface deformations as their principles are based on surface relief grating and surface tilt, respectively. In contrast to these two methods, optical interferometry provides a direct measure of the vertical surface dis-
placement. It is hence suitable for measuring laterally smooth OP vibration fields, which are common in many BAW and MEMS devices, see Fig. 2.1(b).

The diffraction grating method was a popular approach in the 1970s for characterizing SAW fields [36–38]. Later on, the two other methods have become more commonly used. The knife-edge detection has been widely applied by many research groups for characterizing different parameters in the SAW fields [39–48]. Several non-scanning or scanning single-point interferometer setups with homodyne, heterodyne, and laser Doppler vibrometric detection schemes have been published for this purpose, see for example Refs. [6, 34, 35, 49–62]. Also laser-based pump-probe techniques with single-point or scanning interferometric OP detection in time domain have been used for the detection of acoustic waves [63–66].

The progress in high-resolution camera technology and computer hardware in the 1990s made it possible to utilize full-field interferometric imaging for the vibration characterization in microstructures. Consequently, since the turn of the millennium, many different full-field interferometer setups with continuous or stroboscopic illumination have been proposed for the measurement of OP vibration fields in microelectromechanical devices. The maturing of the digital technology enabled holographic vibration measurement technique [67–69] to be applied to the characterization of micromechanical devices [70–75]. Full-field interferometric techniques such as speckle interferometry [76, 77] and phase-shifting laser interferometry [78–80] have also been developed for the purpose.

In addition to these laser-based techniques, stroboscopic white-light interferometry and other interference microscopic techniques that use light sources with a shorter coherence length than that of lasers have been implemented for the full-field detection of OP vibrations in microstructures [81–87]. Also a non-interferometric microscopic technique with low-coherence illumination has been reported for this application [81, 88].

The detection of the in-plane (IP) vibration component is particularly important in MEMS research, since many MEMS devices utilize IP vibrations in their operation. In contrast to the wide variety of optical methods available for the OP vibration detection, only few single-point or full-field optical techniques are readily suitable for the one-dimensional or vectorial detection of the IP vibration field in microstructures. Practically all these techniques require a specific surface structure and are hence not
applicable for all types of surfaces.

In the case of rough and scattering surface structures, interferometric approaches such as laser Doppler vibrometry, holographic interferometry, and speckle interferometry have been used for studying IP vibrations with μm-order imaging resolution [75,77,89–91]. A more popular and versatile approach is digital processing of microscopic full-field images using continuous illumination [92] or stroboscopic illumination [81,83,88,93–96]. In these cases, the useful image texture is formed either by the sample surface structure itself or by artificial patterning [97]. Also, a method for detecting IP vibrations on polished metal surfaces has been proposed [98].

Alternatively, IP vibrations can be measured using a knife-edge technique, in which a focused laser beam is positioned at an edge of the moving microstructure, see for example [99]. A similar single-point method has also been presented in a reflection geometry that enables the detection of IP vibrations at any interfaces with varying reflectivity on the surface [100–102].

IP and OP detection methods have been successfully combined to a single system to enable a complete three-dimensional (3D) characterization of the vibration field [81,83,85,88,94,100]. The detection of both vibration components has even been carried out simultaneously with low crosstalk [85,94]. Doppler vibrometry has also been demonstrated to be capable of measuring 3D vibrations in micromechanical devices with scattering surfaces. In this method, the scattered light of a focused laser beam impinging perpendicularly to the sample surface is measured in at least three directions [91].

All the above-mentioned optical imaging techniques are based on diffraction, transmission or reflection of light in the vibrating microstructure. Also variations of other optical quantities caused by the acoustic fields have been utilized for the characterization of surface vibrations. Surface waves can induce, e.g., reflectance changes or reflectance polarization anisotropy [103], or polarization changes of the transmitted beam [104,105].

As optical methods have already been used for the characterization of acoustic waves in micromechanical components over five decades and numerous different optical setups have been published, there exist many excellent review articles [6–8,36,106,107], books [9,108–111], and book chapters [10,112–117] on the topic. Surface vibrations can also be measured with other than optical techniques. Atomic force microscopy enables
excellent lateral imaging resolution and has been attempted to characterize vibrations in micromechanical components [118–120]. It may however damp the vibration amplitude and hence cannot be considered as purely non-contact method [121], although this damping effect can be significantly reduced by dynamic force microscopy [122]. Also scanning electron microscopy has been applied for the vibration characterization of acoustic waves [123–126]. Other more exotic techniques to visualize acoustic fields, some of which are even destructive, include, e.g., X-Ray topography [127, 128], electromigration [129], and utilization of nanoscale smoke particles spread on the sample surface [130].

2.2.2 Considerations of capabilities and performance metrics

It is obvious from Sec. 2.2.1 that there exists a wide variety of optical techniques for the detection of acoustic wave fields in micromechanical components. Consequently, a complete comparison of different techniques with their benefits and drawbacks is a laborious task and left beyond the scope of this thesis. Instead, here the necessary or useful characteristics and performance metrics that an optical technique should have for versatile characterization of surface vibrations in microacoustic components are briefly summarized.

An important requirement is that the measurement should not alter the integrity and the mechanical performance of the device. This is achieved by using a low light power density to avoid heating or even damage of the sample. For quantitative and reliable measurements, the optical method should in general be insensitive to mechanical drifts and disturbances from the environment such as temperature variations, air flows, and acoustic vibrations. In addition, immunity to the variation of surface roughness and surface reflectivity of the sample is required. An ideal optical technique would also provide data of both the IP and OP vibration component and preferably simultaneously. It would also be beneficial to be able to measure vibrations on surfaces featuring large \( \mu \text{m-order} \) height differences. As many low-frequency devices are designed to operate in a low pressure environment, a long working distance is useful to enable in-vacuum measurements through a transparent window of a vacuum chamber.

One of the most important performance attributes is the minimum detectable amplitude that ultimately defines how small signals can be measured, and hence, how weak effects can be studied. The typical maximum
amplitudes of high-frequency devices (>10 MHz) are only few nanometers or even below 1 nm. For studying weak effects of such devices, a minimum detectable amplitude of \( \sim 1 \text{ pm} \) is needed. The capability to measure weak effects buried in strong signals is also a desired property and therefore high dynamic range and maximum detectable amplitude are key performance merits. The lateral imaging resolution should be \( \sim 1 \mu m \) or preferably less to allow for vibration measurements in complex microstructures. Moreover, a large imaging area of more than \( 1 \text{ mm} \times 1 \text{ mm} \) is required to be able to measure the whole vibration field of microacoustic components having a large active area.

In fact, although some optical techniques may be more advantageous and versatile than others, none of them comply with all the above-mentioned requirements. A good compromise is to use a combination of different methods that have many of the desired characteristics and that complement each other.

Scanning single-point detection setups currently offer the best smallest detectable vibration amplitude for both the IP and OP vibration measurements. Using a focused laser beam in the scanning setup, a lateral imaging resolution of even less than \( 1 \mu m \) can be reached. Scanning laser interferometers can provide vibration measurements up to GHz frequency range with even sub-picometer minimum amplitude detection limit for the OP vibration [52–54, 60]. A disadvantage of this OP detection technique is that in a typical implementation, the highest vibration amplitudes that can be accurately measured are limited to the linear operating regime of the interferometer. In practise, the maximum measurable amplitude is less than a few tens of nanometers. Using single-point detection for the IP vibration component, a minimum detectable amplitude of \( \sim 10 \text{ pm} \) has been achieved for vibrations up to GHz range [131].

Some of the full-field detection methods are significantly faster than the point-wise scanning systems for large-area measurements with high lateral imaging resolution. In certain full-field schemes, a single-frequency vibration measurement may require only a few camera frames, enabling almost instant characterization of the vibration fields. The full-field systems, however, cannot typically reach as good a minimum amplitude detection limit as the single-point detection systems, mostly because of the limited dynamic range and inferior noise performance of the camera-based detection when compared to the typical photodetectors, such as photodiodes, used in single-point detection systems.
Figure 2.2. (a) Schematic presentation of a homodyne Michelson interferometer. After propagating through collimation optics (CO), the linearly polarized source beam is divided into orthogonally polarized sample and reference beams by a polarizing beam splitter (PBS). The two beams are reflected back from the sample surface and the reference mirror. The quarter-wave-plates (QWPs) rotate the linear polarization of the back-reflected beams by $\pi/2$ rad such that they are directed to the detection arm at the PBS. In the detection arm, the orthogonally polarized beams are projected to the same linear polarization state, and the resulting interference signal is measured by the photodetector. (b) Interference intensity $I_D$ as a function of the sample displacement $z$ along the beam propagation direction, and illustration of the interferometer response $\Delta I$ to a harmonic sample movement with a peak-to-peak amplitude of $\Delta z$. The modulation depth (MD) is the peak-to-peak magnitude of the periodic $I_D$ signal. The best sensitivity and linearity is obtained when the interferometer is operating at one of the quadrature points marked with black dots, $QP_1$ and $QP_2$ denoting the quadrature points at the positive and negative slope, respectively. The intensities $I_S$ and $I_R$ are defined in Eq. (2.1).

The optical detection setups presented in this thesis for the characterization of surface vibration fields use frequency domain technique, in which the vibrations are excited in the sample with sinusoidal electrical drive signal and the detection is tuned to this drive frequency. In order to completely characterize the OP or IP vibration field, the primary requirement for the frequency domain measurement is to provide absolute amplitude (i.e., the magnitude) and phase data of the mechanical vibrations. For example, without the phase information it would not be possible to separate acoustic waves propagating to different directions. Therefore, for a thorough characterization of acoustic behavior in micromechanical devices, all the optical setups implemented in this work are capable of phase-sensitive absolute amplitude detection of the surface vibrations.

2.2.3 Homodyne detection principle

All the interferometers developed in this thesis are based on a homodyne Michelson-type optical setup, which is one of the commonly used optical configurations in microacoustics research. An example of an optical layout of a homodyne Michelson interferometric detection setup is depicted
The interferometer consists of a source arm, a measurement or sample arm, a reference arm, and a detection arm separated by a beam splitter. The reference and sample beams travel a round-trip in the reference and measurement arms, respectively, as they are reflected back from the reference mirror and the sample surface. The two beams are then recombined to interfere on the photodetector (PD). The intensity of the interference signal depends on the optical phase difference between the two beams. Therefore, intensity variations of the interference signal carry information on the changes in the optical pathlength difference of the two beams. In this way, a movement of the sample surface along the beam path is converted to an observable intensity change by the interferometer.

In this Michelson geometry, consider an ideal case where two coherent, collinear, monochromatic beams with the same linear polarization, beam width, and wavefront are superimposed at the detection arm. Assuming noise to be negligible, the resulting intensity of the two-beam interference incident on the PD, averaged over an optical cycle, is

$$I_D = I_S + I_R + 2\sqrt{I_S I_R} \cos(\varphi).$$  \hfill (2.1)

Here \(I_S\) and \(I_R\) are the intensities of the individual beams reflected from the sample and the reference mirror, respectively, after propagating through the polarizer of the detection arm. The third term, in which \(\varphi = \varphi_S - \varphi_R\) is the phase difference of the two beams, defines a constructive or destructive interference of the two beams depending on the sign of the cosine function.

The periodic dependency of \(I_D\) on the optical pathlength difference of the two beams is illustrated in Fig. 2.2(b), where the optical pathlength difference is twice the displacement \(z\) of the sample surface. As seen from the figure, the optical wavelength \(\lambda\) provides a length reference, since one period of the \(I_D\) signal corresponds to a \(z\) displacement of \(\lambda/2\). The closer the match between \(I_S\) and \(I_R\), the higher the modulation depth (MD) of the interference signal. In a perfect balance of \(I_S = I_R\), the interference signal varies between 0 and \(4I_S\).

Assuming a sinusoidal OP surface vibration of \(z(t) = z_{vib} \sin(2\pi f_{vib} t + \Phi_{vib})\), where \(z_{vib}, f_{vib},\) and \(\Phi_{vib}\) are the amplitude, frequency, and phase of the OP vibration, the optical phase difference in Eq. (2.1) varies in time as

$$\varphi(t) = \varphi_0 - \frac{4\pi z_{vib}}{\lambda} \sin(2\pi f_{vib} t + \Phi_{vib}).$$  \hfill (2.2)
The parameter $\varphi_0$ is the quasi-static optical phase difference between the sample and reference beams that, without active control of the path-length, typically varies slowly due to, e.g., variations in the ambient conditions in the sample and reference arms. Inserting Eq. (2.2) into Eq. (2.1), one gets

$$I_D(t) = I_S + I_R + 2\sqrt{I_S I_R} \cos \left[ \varphi_0 - \frac{4\pi z_{\text{vib}}}{\lambda} \sin (2\pi f_{\text{vib}} t + \Phi_{\text{vib}}) \right].$$  \hfill (2.3)

In order to study the frequency content of the interference signal, the time-dependent third term may be expanded into a harmonic series with spectral magnitudes given by Bessel functions of the first kind [131, 132],

$$I_{SR}(t) = 2\sqrt{I_S I_R} \left[ \cos(\varphi_0) J_0 \left( \frac{4\pi z_{\text{vib}}}{\lambda} \right) + 2 \sin(\varphi_0) \sum_{n=1}^{n=\infty} \left\{ J_{2n-1} \left( \frac{4\pi z_{\text{vib}}}{\lambda} \right) \sin \left[ (2n-1)(2\pi f_{\text{vib}} t + \Phi_{\text{vib}}) \right] \right\} \right]$$

$$+ 2 \cos(\varphi_0) \sum_{n=1}^{n=\infty} \left\{ J_{2n} \left( \frac{4\pi z_{\text{vib}}}{\lambda} \right) \cos \left[ 2n (2\pi f_{\text{vib}} t + \Phi_{\text{vib}}) \right] \right\}.$$ \hfill (2.4)

The spectrum of the $I_{SR}$ signal consists of a nearly constant term (varying slowly as $\cos(\varphi_0)$) and different harmonic components. In the limit of small OP vibration amplitude, $z_{\text{vib}} \ll \lambda$, Eq. (2.3) can be further approximated as

$$I_D(t) \approx I_S + I_R$$

$$+ 2\sqrt{I_S I_R} \left\{ \cos(\varphi_0) + \frac{4\pi z_{\text{vib}}}{\lambda} \sin \left[ 2\pi f_{\text{vib}} t + \Phi_{\text{vib}} \right] \sin(\varphi_0) \right\},$$ \hfill (2.5)

since for small $x$, $J_0(x) \approx 1$, $J_1(x) \approx x/2$, and the higher-order terms can be considered to be negligibly small. This linear approximation results in less than 1 % error in the determination of vibration amplitudes of $z_{\text{vib}} \leq 10$ nm, when $\lambda = 632.8$ nm. At larger vibration amplitudes the error starts to accumulate quickly. For example, when $z_{\text{vib}} = 40$ nm, the error is already about 8 %.

The two terms within the parenthesis of Eq. (2.5) depend periodically on the quasi-static pathlength difference $\varphi_0$ of the two beams, the first as $\cos(\varphi_0)$ and the last as $\sin(\varphi_0)$. Assuming now that $\varphi_0$ is fixed to a constant value, the last term oscillates sinusoidally at $f_{\text{vib}}$ and its magnitude is linearly proportional to the amplitude of the OP vibration. The frequency domain detection is synchronized to the excitation of the sample at this vibration frequency, and a signal corresponding to this last term is recorded. Importantly, the largest magnitude of this term is obtained,
when $\sin(\varphi_0) = \pm 1$. If the interferometer is operating at this so-called quadrature point (QP), the highest sensitivity and the best linearity in the small-amplitude detection is achieved, see Fig. 2.2(b).

In practise, the interferometer operation does not typically stay at a QP without active control of the optical pathlength of the reference or sample arm. First of all, the height variations of the sample surface inevitably lead to variations in $\varphi_0$. In addition, the optical paths, particularly in the case of free-space reference and measurement arms, are prone to slow drifts and fluctuations due to, e.g., thermal fluctuations and air flows. These slow fluctuation and drifts in $\varphi_0$ lead to incorrect measurement of the OP vibration amplitude even in the case of a flat sample surface.

Consequently, to enable phase-sensitive absolute amplitude measurements of the OP surface vibration fields, homodyne interferometers are typically stabilized to operate at either $QP_1$ or $QP_2$ by actively controlling the optical phase difference. A calibration measurement is additionally needed to determine the gain of the system in order to obtain absolute amplitude values. A commonly used stabilization technique is based on generating a sinusoidal phase modulation at a frequency $f_M$ to one of the interfering beams and detecting the resulting interference signal both at $f_M$ and $2f_M$ with a PD. The recorded $f_M$ signal provides a reference related to the MD of the measured interference signal (see Sec. 3.1.1) and the $2f_M$ signal is used as a feedback for an active control of the optical path length difference between the two arms of the interferometer, respectively [53, 54, 133]. Another approach is to use quadrature detection [109] that also requires a means to monitor the MD of the interference signal and a calibration procedure.
3. Scanning setups and detection methods

3.1 Scanning homodyne laser interferometer

3.1.1 Unstabilized homodyne detection scheme

In contrast to the stabilized homodyne interferometry described at the end of Sec. 2.2.3, Knuuttila et al. presented an alternative approach to determine the OP vibration amplitude at the QP [52]. Their detection scheme utilized a linear sweep of the optical path length in the reference arm by using a piezo-driven reference mirror. The sweep covered an optical pathlength distance of larger than $\lambda$ and hence several QPs were included in the interference signal as presented in Fig. 3.1. During the cycle when the mirror was moving toward the PBS, a spectrum analyzer recorded the amplified output signal from a PD in a zero-span mode, i.e., tuned to the electrical excitation frequency of the sample. As a result, a signal at $f_{vib}$ was obtained that was periodic as a function of the path-length difference, see Fig. 3.1. From this data, amplitude maxima, located at each QP, were found and averaged.

The determined amplitude value $A_{vib}$ is linearly related to the last term in Eq. (2.5) with $|\sin(\varphi_0)| = 1$ and thereby to the absolute amplitude of vibration in the linear approximation. Unfortunately, this linear factor is unknown and depends on the system gain, which relates the intensity of the interfering beams to the recorded $A_{vib}$. This factor also depends on $\sqrt{I_S I_R}$, hence being affected by changes in the intensity of the reflected sample beam. As a result, the $A_{vib}$ data obtained at different scan points across a sample surface with varying optical reflectivity are incomparable.

The $A_{vib}$ data obtained using the method by Knuuttila et al. cannot hence be used for studying the ratios of the vibration amplitudes on the
Figure 3.1. Illustration of the signals recorded in the unstabilized homodyne detection concept, enabling determination of $A_{\text{vib}}$, that is proportional to the absolute amplitude of the OP surface vibration and phase $\Phi_{\text{vib}}$ of the vibration. The measured amplitude and phase data at $f_{\text{vib}}$ are plotted as a function of the optical path length difference, together with the modulation-depth-reference (MDR) signal that monitors the MD of the interference signal. In stabilized homodyne interferometers, $\phi_0$ is stabilized at a single quadrature point, QP$_1$ or QP$_2$, to obtain $A_{\text{vib}}$ and $\Phi_{\text{vib}}$ unambiguously.

Knuuttila et al.’s detection scheme was developed further in Publication I to allow for phase-sensitive absolute amplitude measurements of OP vibrations. In the modified detection scheme presented in Fig. 3.2, the $\cos(\phi_0)$ interference signal, hereafter called the modulation-depth-reference (MDR) signal, is also recorded during the linear sweep of the optical path length. A vector network analyzer (HP8753E) is used to excite the sample and to enable phase-sensitive detection. The amplitude and phase signals at $f_{\text{vib}}$ and the MDR signal are simultaneously recorded during the < 100-ms sweep that covers an optical pathlength of over $3\lambda/2$ in order to go through at least 3 QPs, see Fig. 3.1. Since the wideband PD used for recording the $f_{\text{vib}}$ signal is internally ac-coupled with a high-pass frequency of 30 kHz, a separate low-speed PD was added to the detector arm to detect the low-frequency ($\sim 10$ Hz) MDR signal. Equally well, a single dc-coupled wideband photodetector could be used to detect both the
TEM\textsubscript{00} -mode HeNe laser $\lambda = 632.8$ nm

Figure 3.2. (a) Optical setup of the unstabilized scanning homodyne laser interferometer with phase-sensitive absolute amplitude detection scheme. The light source is a 10-mW TEM\textsubscript{00} HeNe laser (UniPhase 1135). In this Michelson-type optical configuration, the sample beam is focused on the sample surface with a long-working-distance microscope objective (Nikon 354248 CF N Plan 40 x ELWD) to a 1.8-$\mu$m diameter spot. The sample is mounted on a translational stage consisting of three motor-controlled linear stages (Newport MFN25cc) to control the sample’s $(x,y)$-position during the raster-scan and the beam focus on the sample. In the reference arm, a linear ramp sweep of the optical pathlength is carried out at each scan point by moving the reference mirror along the beam propagation direction with a piezo-stage. In the detection arm, the interfering beam is divided into two beams, which are simultaneously measured with two PDs. On one of the branches, a high-speed photodetector (Newport 818-BB) detects the interference signal at $f_{\text{vib}}$ and the modulation-depth-reference (MDR) signal is acquired with a low-speed PD. In addition, a third PD monitors the light power of the beam reflected back from the sample at each scan point by measuring the stray beam reflected from the QWP, hence providing a light power image of the sample with a point-to-point correspondence with the measured amplitude and phase data.

MDR and $f_{\text{vib}}$ signals.

The QPs are located by finding the positions of steepest slopes of the recorded MDR signal. The amplitude $A_{\text{vib}}$ is calculated as an average over the amplitude values at the QPs identified from the MDR data. The peak-to-peak value $M_R$ of the MDR signal serves as a reference for the determination of the absolute amplitude from $A_{\text{vib}}$ data, see Publication I for a detailed description. This requires that the relative system gain of the MDR and $A_{\text{vib}}$ measurements are known, as in this setup the two signals are detected at separate optical branches with two PDs having different gains of the amplifier electronics. Here the relative gain has been experimentally determined. Since the $M_R$ is obtained from each sweep, it offers an accurate length reference defined by the optical wavelength at each scan point.

As illustrated in Fig. 3.1, there is a $\pi$ rad ambiguity of the vibration phase measurement, depending on the slope of the MDR signal. These $\pi$ rad shifts are therefore corrected in the analysis before the vibration phase value is determined. The vibration phase $\Phi_{\text{vib}}$ is calculated as an
average over these corrected phase values at around each QP.

With the current electronics, the minimum detectable amplitude is typically a few picometers with a 1 kHz intermediate frequency bandwidth of the vector network analyzer, corresponding to a measurement sensitivity of $\sim 0.01 \text{ pm/Hz}$. Decreasing the intermediate bandwidth would result in a better low-amplitude detection limit, at the cost of an increased measurement time. The minimum detectable amplitude is actually limited by the noise level of the vector network analyzer used here. When even better performance for the absolute amplitude detection is required, the sample is excited with a signal generator (HP8648D) and a spectrum analyzer (HP8594E) is used in the detection instead, providing a less than 1 pm minimum detectable amplitude, which is already limited by the noise level of the photodetector electronics itself. The detection bandwidth of the photodetector limits the frequency measurement range of vibrations to 30 kHz–2 GHz. The lateral resolution of the measured vibration fields is approximately 1 $\mu$m, limited by the full width at half maximum (FWHM) spot size of the laser beam focused on the sample. This resolution is sufficient for most of the typical micromechanical components, with the operation frequency ranging from kHz up to several GHz.

The acquisition of the phase and absolute amplitude data of surface vibrations in a piezoelectrically actuated MEMS beam resonator with this unstabilized detection setup is presented in Fig. 3.3 to demonstrate the unstabilized homodyne detection concept. The sample (beam area 40 $\mu$m $\times$ 320 $\mu$m and thickness 28 $\mu$m), designed and manufactured by VTT Research Centre of Finland, is similar to that reported by Jaakkola et al. in [137] except that certain areas of the metallization of the top surface are patterned with a grid of etched holes of 3-$\mu$m diameter. The hole pattern results in up to 90 % variations in the optical reflectivity of the resonator surface. This sample is therefore well suited for studying the performance of the absolute amplitude detection, since the absolute amplitude data should be insensitive to the surface reflectivity. The effects caused by reflectivity variations are clearly present in $A_{vib}$ data shown in Fig. 3.3(a). The variations in the optical reflectivity of the sample surface due to the hole pattern are clearly observable in the $M_R$ data in Fig. 3.3(b), but the absolute amplitude data presented in Fig. 3.3(c) are practically immune to such effects as they are compensated for with the help of the MDR signal. The ambiguity in phase detection is demonstrated by the data shown in Fig. 3.3(d), where the phase value is obtained from the first
Figure 3.3. Demonstration of obtaining the phase and absolute amplitude data of the OP vibration field of a 10.1-MHz resonance mode of a beam resonator using the non-stabilized homodyne detection concept. (a) The measured $A_{vib}$ data and (b) peak-to-peak amplitude data $M_R$ of the MDR signal are used to calculate the (c) absolute amplitude map (logarithmic z scale) of the vibration field. (d) The measured raw phase data of the $f_{vib}$ signal acquired from the first QP that occurs in the optical pathlength sweep of the reference beam, without taking into account the slope of the interference signal at the QP. This data clearly show the π ambiguity in phase detection due to the drifts in the interferometer operation point. (e) The correctly determined phase data of the surface vibration. (f) The instantaneous, three-dimensional shape of the beam obtained by combining the amplitude and phase data of the 10.1 MHz mode at maximum deflection of the resonator edges.

QP in the optical pathlength sweep at each scan point, pointing out the need for identifying the slope at the QP for the correct determination of the phase of vibration. In Fig. 3.3(e), the phase of the vibration is properly determined by finding the phase value from the QP located at the positive slope. The instantaneous deformation of the beam due to the flexural vibration mode is visualized in Fig. 3.3(f).

3.1.2 Measuring nonlinear out-of-plane vibrations

One of the most important attributes in the characterization of nonlinear vibration fields in micromechanical and electroacoustic devices is the capability of measuring vibrations over a wide dynamic range with at least $\sim 1$ pm minimum detectable amplitude. In particular, the nonlinearly excited acoustic fields in SAW and BAW filters due to harmonic and in-
termodulation generation can have several orders of magnitude smaller vibration amplitudes than that of the primary, linearly excited field at the operation frequency. For instance, the maximum amplitude of the acoustic field at the operational frequency may be $\sim 1$ nm, whereas the nonlinear fields may have magnitudes less than 10 pm. Although the nonlinearities are weak, the sensitivity of the receiver can still be significantly impaired [27,138] due to the nonlinear response. Optical characterization of such weak nonlinear effects, therefore, requires state-of-the-art performance in low-amplitude detection. Scanning laser interferometry provides an attractive approach for studying nonlinearities in OP surface vibrations of micromechanical devices as it offers a wide dynamic range for the amplitude measurement combined with a capability of detecting even sub-picometer amplitudes of OP vibrations with frequencies up to GHz range.

In a frequency domain measurement, a straightforward approach to study acoustic nonlinearities in micromechanical devices is to measure the vibration amplitude as a function of the electrical excitation power $P_{in}$, with the detection being tuned to the same excitation frequency. Any deviation from the linear dependence between the measured vibration amplitude and excitation power in this power sweep indicates a nonlinear behavior. A more advanced approach, often readily available in frequency domain detection, is to carry out a frequency sweep for the sample excitation at various power levels. During the frequency sweep, the detection is again tuned to the excitation frequency. By sweeping the frequency both into increasing and decreasing frequency, this method allows for, e.g., studies of bifurcation of a resonance mode. Although these measurements may often yield useful insight into the nature of the nonlinearity observed, they do not however provide any information of the spectral content of the detected surface vibrations outside the excitation frequency. Nonlinear excitation of vibrations at other frequencies than the excitation frequency is a common phenomenon in electroacoustic and micromechanical components and has currently evoked increasing research interest. Synchronized excitation and detection at different frequencies would therefore be a useful approach to obtain more complete information of the nonlinear behavior of the surface vibrations.

The unstabilized homodyne laser interferometer setup was further developed to enable the use of different excitation and detection frequencies for studies of nonlinearities in OP vibration fields. The main challenge of
Figure 3.4. Example of optical mixing artifacts in an OP measurement due to the nonlinear nature of the interferometric detection. The resonator sample is a square-plate resonator with a plate side length of $L = 160 \mu m$. (a) Two signal generators ($G_A$ and $G_B$) drive the sample at two different frequencies to excite two vibration modes in the sample simultaneously. Both modes are excited well within their linear operation regime. The measured OP amplitude data of (b) the first mode at $f_A = 6.874$ MHz and (c) the second mode at $f_B = 26.397$ MHz, which are simultaneously excited into the linearly operating resonator sample. (d) Schematic representation of the frequency spectrum of the measurement signal, including signal artifacts caused by the nonlinearity of the interferometric detection. The artificial signals solely due to optical mixing at the (e) difference frequency ($f = 19.523$ MHz) and at the (f) sum frequency ($f = 33.271$ MHz). The logarithmic amplitude scales are normalized to the maximum amplitude (0 dBm) of each data set.

This frequency-domain measurement technique arises from the inherent nonlinearity of the interferometric detection, see Eq. (2.1). Due to the nonlinear nature of the two-beam interference, all the mechanical vibration fields which have a sufficiently strong OP component and which simultaneously exist in the sample at different frequencies are mixed in the detection, resulting in artificial signals. This mixing behavior is illustrated in Fig. 3.4 by simultaneously exciting two resonance modes in a linearly operating MEMS resonator. In addition to the OP vibration fields measured at the excitation frequencies of the two modes shown in Figs. 3.4(b) and 3.4(c), artificial signals presented in Figs. 3.4(e) and 3.4(f) also appear in the measured data due to the optical mixing, e.g., at the sum and difference frequencies of the two modes. Without any knowledge of the two true mechanical modes, these mixing artifacts could be falsely interpreted as true vibration fields.

To get a further insight into the nonlinear mixing experimentally demon-
strated in Fig. 3.4, it is instructive to consider the interference term in Eq. (2.3) in a case of two sinusoidal surface vibrations existing simultaneously in the sample at different frequencies. The interference term in this case is

$$I_{SR, \text{mix}} \propto \cos [\varphi_0 - \beta_1 \sin(2\pi f_1 t + \Phi_1) - \beta_2 \sin(2\pi f_2 t + \Phi_2)], \quad (3.1)$$

where the parameter \( \beta_i = 4\pi z_i/\lambda \) has been defined for the sinusoidal vibration \( i (i = 1, 2) \) with vibration frequency \( f_i \), amplitude \( z_i \), and phase \( \Phi_i \). Using Bessel functions of the first kind, Eq. (3.1) can be expanded as

$$I_{SR, \text{mix}} \propto \sum_{n=\infty}^{n=-\infty} \sum_{m=\infty}^{m=-\infty} \left\{ J_n(\beta_1)J_m(\beta_1) \cos [\varphi_0 - n(2\pi f_1 t + \Phi_1) - m(2\pi f_2 t + \Phi_2)] \right\} \propto J_0(\beta_1)J_0(\beta_2) \cos(\varphi_0)$$

$$+ 2J_1(\beta_1)J_0(\beta_2) \sin(\varphi_0) \sin(2\pi f_1 t + \Phi_1)$$

$$+ 2J_0(\beta_1)J_1(\beta_2) \sin(\varphi_0) \sin(2\pi f_2 t + \Phi_2)$$

$$+ 2J_1(\beta_1)J_1(\beta_2) \cos(\varphi_0) \left\{ \cos [2\pi(f_1 + f_2)t + \Phi_1 + \Phi_2] \right\} - \cos [2\pi(f_1 - f_2)t + \Phi_1 - \Phi_2] \}$$

$$+ \ldots \quad (3.2)$$

Here only the first terms that include the zeroth and first order Bessel functions are explicitly shown. As in Eq. (2.4), the term \( J_0(\beta_1)J_0(\beta_2) \cos(\varphi_0) \) depends periodically on the static optical pathlength difference of the two interfering beams, hence providing information on the MD of the interference. The next two terms correspond to the interference signal at the vibration frequencies \( f_1 \) and \( f_2 \). The last term with the \( J_1(\beta_1)J_1(\beta_2) \) factor gives rise to the mixing at frequencies \( f_1 + f_2 \) and \( f_1 - f_2 \) that were experimentally observed in Fig. 3.4. Due to the mixing effect in the two-beam interference, artificial signals are also detected at higher-order harmonic frequencies of the vibrations as well as at the linear combinations of their sum and difference frequencies.

From Eq. (3.2) it is also observed that the measured amplitude data at one of the two vibration frequencies depend on the \( J_0 \) factor and thus on the amplitude of the surface vibration at the other frequency. If the surface vibration, e.g., at \( f_1 \), has a large amplitude, it affects the amplitude term measured for the vibration at \( f_2 \). As a consequence, quantitative analysis can be particularly tedious. For instance, obtaining absolute amplitude data of vibration fields that simultaneously exist at different
frequencies in the sample requires a rigorous analysis, if any of the vibration fields is so strong that it affects to the measured amplitude data of the vibrations existing at other frequencies. Such strong nonlinearly excited fields are possible, e.g., in vibrating MEMS and NEMS devices with nonlinear behavior associated with parametric coupling [23, 139]. The existence of a strong field rules out the validity of the small-vibration approximation for $J_0$ and $J_1$ terms used in Eq. (2.5). Instead, the Bessel coefficients need to be included in the analysis.

Optical characterization of higher-order harmonic generation is severely complicated by the fact that the amplitude spectrum of the interference signal always contains peaks at the higher harmonics of the vibration frequency due to the nonlinear nature of the interference, as seen in Eq. (2.4). This occurs even though the fundamental vibration field is highly linear and no higher-harmonic vibration fields exist. Therefore, if true vibration fields are nonlinearly excited to the sample via harmonic generation, the measured data at these harmonics are a combination of the true mechanical field and the artificial signal, the latter originating from the nonlinear interferometric detection of the fundamental field. If the spatial shape of the higher-harmonic vibration field differs from that of the fundamental field, it is in principle possible to discern the amplitude data of the true vibration field from the measured amplitude data by using the fact that the artificial signal has the same shape as the fundamental vibration field.

In conclusion, characterization of nonlinear OP vibration fields with interferometric techniques requires a thorough understanding of the nonlinear nature of the detection method. The applicable measurement and analysis methods in general depend essentially on the nonlinear behavior of the surface vibrations and there is no universal approach that is feasible for every case. A rigorous use of interferometry can however reveal crucial insight into the nature of nonlinearities observed in electromagnetic devices and, hence, significantly advance the understanding of the device physics.
3.2 Scanning laser technique for in-plane vibrations

3.2.1 Detection principle

A non-interferometric technique for detecting a signal that is proportional to the absolute amplitude of IP vibrations has earlier been developed to the scanning homodyne interferometer [100,131]. In this thesis work, this IP detection concept has been further developed to obtain phase-sensitive absolute amplitude data of lateral vibrations. The IP detection method is basically a generalization of the knife-edge detection method in a 2-dimensional (2D) case and in reflection geometry [100–102, 131]. It is based on measuring the modulation of the reflected beam intensity, when a focused laser beam is positioned on an optical reflectivity gradient on a sample surface that vibrates laterally [100, 102, 131]. A sharp, step-like change in the optical reflectivity on the sample surface (see Fig. 3.5) is preferred for the best sensitivity. For example, interfaces of two materials with different optical reflectivities on the sample surface, vertical edges of the sample, and holes can be utilized.

As depicted in Fig. 3.5(a), the detection is sensitive to the vibration component along the steepest gradient of the optical reflectivity. Consequently, to enable measurement of 2D IP vibration fields with relatively high spatial resolution, a suitable grid of optical contrast features on the sample surface is needed. In the measurements of MEMS resonators presented in this thesis, the vectorial IP detection is obtained by measuring the intensity modulation of a Gaussian laser beam reflected from the edges of circular etch holes in the top-electrode metal layer that covers, e.g., the whole surface of the resonator plate.

Intriguingly, the IP detection can be straightforwardly implemented to the homodyne scanning laser interferometer shown in Fig. 3.2 by blocking the reference beam [100]. In this way, only the beam reflected from the laterally vibrating sample surface is detected. The intensity modulation at the frequency of the surface vibration and average power $P_{DC}$ of the beam are recorded by the high-speed and low-speed PDs at the detection arm, respectively.

The mean reflected light power profile $P_r$ across interfaces with different reflectivities on the sample surface can be determined from the average light power data $P_{DC}$. The inset on the left in Figure 3.5(b) presents the recorded $P_{DC}$ data in a square-plate Si MEMS resonator. The top surface
Figure 3.5. (a) IP detection principle and the model geometries of the detected optical power reflected from the sample surface when a Gaussian beam with total incident power of $P_0$ crosses a 1D interface contrast (“1D edge” model shown on top-left) and a circular contrast (“hole” model, top-right) with a step change in optical reflectivity from $R_1$ to $R_2$. The simulation result with the hole model is marked with blue crosses, using a hole radius of $r_h = 1.2w$. The green line illustrates the 1D edge model curve with $R_1 = 0.10$ and $R_2 = 0.95$, and the black line the same model scaled to the minimum and maximum values of the simulation result of the hole model. (b) The measured optical power across a single hole etched on a metal surface of the resonator plate of a square-extensional mode MEMS resonator. The sample structure (plate size $257 \mu m \times 257 \mu m \times 28 \mu m$) is discussed in more detail in Ch. 5. The light power image of the sample is shown on the left together with a zoomed inset where the dashed red line indicates the scan line of the Gaussian beam across a single hole.

of the Si plate is covered with piezoelectric AlN thin film. The top Mo electrode deposited on the AlN film is patterned with a grid of circular holes, see the detailed structure of the sample in Fig. 5.1 of Ch. 5. The
measured optical power profile $P_r$ across a single hole is plotted on the right inset of Fig. 3.5(b). The best sensitivity and linearity is obtained at the position of the steepest slope of the curve.

In our setup, the transverse intensity profile of the laser beam is Gaussian. When a focused Gaussian beam is translated across an infinite 1D interface with a step change in reflectivity as shown in Fig. 3.5(a), the total power of the back-reflected beam $P_{r,1D}$ follows the form of a cumulative distribution function of a normal distribution with a standard deviation of $w/2$, where $w$ is the $1/e^2$ beam spot radius on the surface [102]. The scaling and offset of this function are defined by the reflectivities $R_1$ and $R_2$ across the edge [101], see Fig. 3.5(a).

In our analysis, the $P_r(x)$ data are assumed to follow this one-dimensional (1D) edge model. This approximation is intuitively justified when the hole radius $r_h \gg w$. When $r_h \approx w$, the back-reflected optical power needs to be calculated numerically using the hole model presented in Fig. 3.5(a). In this numerical analysis, the hole edge is modeled as a 2D circular step change in the optical reflectivity and the overlap between the Gaussian beam and the hole is calculated at each beam position.

The simulation of the detected optical power profile $P_{r,h}$ across a hole with $r_h = 1.2w$ is presented in Fig. 3.5(a). As observed from the figure, the $P_{r,h}$ curve is shifted towards the center of the hole compared to the $P_{r,1D}$ curve whose steepest slope is at the edge of the hole. The simulations however verify that when the 1D edge model function is scaled to the minimum and maximum value of the $P_{r,h}$ curve and shifted in $x$ such that the location of its highest gradient is aligned with that of the $P_{r,h}$ curve, the 1D edge model is well matched with the simulation result in the vicinity of the steepest slope for $r_h \geq 1.0w$, see the zoomed part in Fig. 3.5(a). Moreover, the 1D edge model is well in agreement with the experimental data presented in Fig. 3.5(b), in which the hole radius $r_h \approx 1.5 \mu m$ is also close to the 0.9-$\mu m$ waist size of the Gaussian beam.

A second important observation from the simulations (using $w = 1.0 \mu m$, $R_1 = 0.95$, and $R_2 = 0.10$) and experiments is that, for $r_h \geq 1.0w$, a linear approximation of the $P_{r,1D}$ function at the location of the steepest slope results in a less than 2% error for vibration amplitudes up to 200 nm. Consequently, to obtain a measurement accuracy of about 2% for the measurements of small vibration amplitudes of $x \leq 10$ nm, at least one of the measurement points in the raster-scan should be located within 200 nm distance from the position of the steepest slope at around each hole edge.
A scan step of as high as $\sim 400$ nm is sufficient to achieve such accuracy.

The gradient at the steepest slope of the $P_{r,1D}$ function can be calculated as

$$k = \sqrt{\frac{2}{\pi}} \frac{P_0 R_1}{w} \left( 1 - \frac{R_2}{R_1} \right),$$

(3.3)

where $P_0$ is the total power of the Gaussian beam incident onto the sample surface. The values proportional to $P_0 R_1$ and $P_0 R_2$, i.e., to the detected optical powers of the sample beam when reflected solely from the metal surface and from the surface at the bottom of the hole, can be readily found from the $P_{DC}$ data. For example, these values can be determined for each hole by finding the maximum value on the metal surface surrounding the hole and the minimum value inside the hole. With known $w$ and relative gain between the two PDs recording the $P_{DC}$ signal and signal at $f_{vib}$, the absolute amplitude of vibration can then be obtained without the need of function fitting or reconstruction of model curves, thus greatly simplifying the analysis.

A close-up of measured IP data from a square-plate MEMS resonator is presented in Fig. 3.6. The metal surface of the sample is patterned with holes as seen in the light power image $P_{DC}$ data in 3.6(a). These $P_{DC}$ data are utilized in the IP detection to determine the relative optical reflectivities of the metal surface around each hole and of the bottom surface of each hole. The magnitude $A_{IP}$ and phase $\Phi_{IP}$ of the detected signal at $f_{vib}$ are presented in Figs. 3.6(b) and 3.6(c). Note that for convenience the $A_{IP}$ data are here processed further to correspond to the absolute vibration amplitude values. The amplitude of the IP vibration is determined from these data as an average of the maxima of the two amplitude lobes located at the opposite sides of each hole as seen in Fig. 3.6(b). The direction of the IP vibration is along the line which goes through these two amplitude maxima and the center of the hole. The red arrows in the phase data in Fig. 3.6(c) indicate the direction of the vibration. As is seen from the phase data, the two amplitude maxima have opposite phases due to the different signs of the reflectivity gradients at opposite edges. By taking into account the direction of the reflectivity gradient, the phase of the vibration is unambiguously determined.

With a 1 kHz detection bandwidth of the HP8753E vector network analyzer, the typical minimum detectable IP amplitude is a few tens of picometers. As in the case of OP detection, the combination of the HP8648D signal generator and the HP8594E spectrum analyzer provides a better low-amplitude detection limit, offering less than 10 pm minimum detec-
Figure 3.6. IP vibration data measured from a similar square-plate resonator as that shown in Fig. 5.1. (a) The normalized $P_{DC}$ data, showing the light power image of a zoomed area on the resonator plate. (b) Amplitude data of the signal recorded at $f_{vib}$ with logarithmic scale, normalized to correspond to the amplitude values of the lateral vibration. The maxima of the two amplitude lobes at the edge of each hole are used to define the absolute amplitude and direction of the IP vibration. (c) Phase data with red arrows superposed on the data to indicate the direction of IP vibration. The phase has binary values at each hole, the nodal lines being aligned perpendicular to the direction of the vibration.

3.2.2 Measuring nonlinear in-plane vibrations

Often, the interferometric OP measurements of nonlinear surface vibrations alone provide only a limited view of the nonlinearities observed in the micromechanical components. Many of these devices may also vibrate laterally, and furthermore, in certain components, pure IP fields may exist with no detectable OP component. In general, a more complete understanding of the nonlinearities could therefore be achieved if information of the IP component of the nonlinear vibration fields were also available. Since the capability of the scanning interferometer to use different excitation and detection frequencies in the OP measurements is readily applicable to the IP detection mode, studies of nonlinear IP vibrations are possible as well.

It is however obvious from Sec. 3.2 that the detected intensity profile across interfaces with different reflectivities is nonlinear. Therefore, the IP detection technique is spatially nonlinear, resulting in that at certain measurement points the nonlinear terms are more prominent than at other locations. As in the case of OP detection, optical mixing effects complicate the measurements of nonlinear IP fields, if at least one IP vibration field is simultaneously present in the sample at a different frequency than the fundamentally driven field. Fortunately, in a case where several IP fields simultaneously exist in the sample, but their vibration directions
Figure 3.7. Example of optical mixing artifacts in measured IP data. (a) The absolute IP amplitude data of the first mode at $f_A = 16.7$ MHz, and (b) the second mode at $f_B = 0.725$ MHz that are simultaneously excited in a MEMS square-plate resonator in the study described in Sec. 5.2. (c) The mixing of the two signals in the IP detection results in an artificial signal observed at $f_A - f_B = 15.9$ MHz. An edge of a single hole is depicted with a dashed circle to visualize the fact that the amplitude maxima of the artificial signal are not located at the edges of the holes. Such amplitude data are unexpected for a true IP mode.

along the surface of the sample are different, the artificial signals due to optical mixing typically have (almost) unphysical IP field shapes. As an example, the two simultaneously excited IP modes with the measured IP amplitude fields shown in Figs. 3.7(a) and (b) result in a mixing artefact at their difference frequency that is presented in Fig. 3.7(c).

Artificial signals also exist at higher harmonics of the excited field. Due to the spatial sensitivity profile of the detection, these higher harmonic artifacts also result in different amplitude and phase data than true IP vibration fields. The artificial signal at the second harmonic frequency, for instance, shows two maxima at each side of an edge with a step-like reflectivity change. This fact can be made use of if a true harmonic IP surface vibration field is simultaneously excited with the fundamental mode. The amplitude maxima in the measurement data due to the artificial signal are at different locations than those of the true vibration field and hence these mixing artifacts can be separated from the data.

In Publication III, these nonlinear IP and OP detection methods were successfully applied to study the nonlinear behavior of a square-plate MEMS resonator. Even though the nonlinearities of the devices were found to be related mostly to the nonlinear behavior of the IP vibration modes, both IP and OP measurements were needed to understand the nature of the nonlinear phenomena. These studies clearly indicate that the research of nonlinear mechanical behavior in micromechanical components greatly benefits from the combined use of nonlinear IP and OP
optical imaging methods, particularly if the origin and mechanism of the nonlinearity are unknown beforehand.
4. Full-field imaging setups and detection concepts

4.1 Stroboscopic white-light interferometer

4.1.1 Detection principle

White-light interferometry (WLI), also called as low-coherence interferometry, is a well-established method for non-contact profiling of surface height features over a wide range extending from a centimeter down to even a nanometer scale. In contrast to laser interferometry, WLI readily enables unambiguous mapping of the surface topography with height differences that extend over a single period of the interference signal ($>\lambda/2$), i.e., a single interference fringe. WLI is also attractive for full-field imaging of the surface profile, since the low-coherence light source enables speckle-free illumination. Furthermore, with a suitable spectrum of the light source, a resolution comparable to that of full-field laser interferometry is achievable.

In WLI, the limited coherence length of the light source results in a localized interference pattern with the maximum of its envelope located at the zero pathlength difference of the interfering beams in a dispersion-compensated interferometer, as illustrated in Fig. 4.1. The shape of the envelope depends on the spectrum of the light source, and the broader the spectrum the narrower the envelope of the interference pattern. In a particular case of a Gaussian source spectrum, the envelope is also Gaussian [140].

When the sample is translated along the propagation direction $z$ of the sample beam with small, well-defined steps, the optical pathlength difference is varied in a controllable way between the two arms of the interferometer. By recording the interference data at each step of this $z$ scan,
Figure 4.1. Visualization of the white-light interferometer principle for vibration measurements, using a simplified optical setup of a Michelson-type interferometer. (a) Due to the low-coherent light source, the detected interferometric signal has an envelope with limited length along the beam path. As an example, a measured spectrum of a green LED is presented together with the resulting interference signal. The topography $h(x, y)$ of the sample is obtained by recording the interferogram at each measurement point $(x, y)$ on the sample surface when either the sample or the reference mirror is translated along the direction of the beam propagation. (b) The stroboscopic illumination with short light pulses synchronized to the vibration frequency of the surface motion effectively enables a recording of the instantaneous surface deformation at a specific phase of the vibration, as illustrated in the figure in cases of 0 degree and 160 degrees phase delay between the sample motion and stroboscopic illumination. By tuning the phase delay $\theta$ between the sample excitation and stroboscopic illumination with equal phase steps through the whole vibration cycle, the phase and absolute amplitude of the vibration can be determined with the use of static optical profilometry methods.

the height map $h(x, y)$ over the sample surface is obtained provided that the scanning range covers the most prominent, major part of the interference fringe envelope at each lateral measurement point, see Fig. 4.1. Unambiguous height information can be obtained by locating the position of the fringe envelope at each lateral measurement point on the sample surface. Several methods have been developed for the determination of the fringe envelope position, including, e.g., Fourier or Hilbert transforms [141,142], center of gravity analysis [143], and frequency domain analysis (FDA) [144,145]. Although ideally the resolution of these envelope local-
ization methods is improved with a narrower fringe envelope and hence with a broader spectrum of the light source, the presence of noise limits the resolution obtainable from these analyses in practice [146]. At best, these envelope localization analysis methods provide a resolution of several tens of nanometers.

The height resolution of WLI can be further improved by combining the analysis of the fringe envelope position with the analysis of the interferometric data itself. In this enhanced WLI technique, the former analysis enables the unambiguous height determination whereas the latter provides a higher $z$ resolution for the topographic measurement but is ambiguous due to the periodicity of the signal. Consequently, the enhanced WLI technique makes use of the advantages of both methods, and the surface topography can be unambiguously measured with sub-nanometer resolution over a wide height range [145, 147–149].

In our WLI-based instrumentation, the FDA method described in detail in [144, 145] has been utilized as it simultaneously provides the envelope position information and uses the interferometric data for improved height resolution. In FDA, a Fourier transform of the interferometric data along the $z$-coordinate is calculated first at each measurement point, i.e., at each pixel in the case of camera-based full-field detection, before further analysis. The benefit of this approach is that all measured data points of the fringe pattern contribute to the final result. Furthermore, in contrast to methods for locating the fringe envelope which assume a particular functional form of the envelope, the FDA is not sensitive to the shape of the envelope, and hence to the spectrum of the light source [144].

The use of stroboscopic illumination synchronized to the sample excitation enables measurements of periodic OP surface vibrations with static surface profiling methods such as WLI. Stroboscopic illumination with short enough light pulses effectively "freezes" the vibrational motion as the sample is illuminated at a specific vibration phase only. Periodic vibrational motion can then be characterized by repeating the measurement for different phase delays between the sample excitation and the light pulses, see Fig. 4.1(b).

In the vibration analysis of our stroboscopic white-light interferometer (SWLI) described in Publication IV, the phase delay $\theta$ between the stroboscopic illumination and the sample excitation is varied in equal phase steps over the vibration cycle and interference images are recorded at each $\theta$ value and sample scan position $z$. This data corresponds to WLI
measurement of instantaneous surface deformations at each $\theta$ which are determined by applying the FDA method. To obtain the absolute amplitude and phase data of vibration, a Fourier transform is then calculated for this set of instantaneous deformations at each measurement point on the sample surface. When the phase delay values span exactly one period of the surface vibration, the vibration amplitude and phase are obtained from the second bin of the discrete Fourier transform while the surface topography can be obtained from the first bin.

### 4.1.2 Light sources

Even though WLI is a well-established technique, many of the published works on stroboscopic WLI of electromechanical devices have been limited to the measurement of low-frequency (up to a few MHz), high-amplitude (even several $\mu$m) vibrations only with a detection limit ranging from few nanometers to $\sim 100$ nm [79,82,86,87]. Our objective was to achieve a sub-nanometer, state-of-the-art detection limit for SWLI that is also comparable to that of existing full-field interferometric technologies. Provided that the duration of the light pulses were short enough and sub-nanometer amplitude detection were achievable, SWLI would also be applicable for measuring high frequency micromechanical components, for which the typical vibration amplitudes are only a few nanometers or even less. For versatile use of the instrument, we aimed to extend the capabilities of SWLI for measuring vibrations up to GHz range by developing a customized light source.

Achieving these objectives sets specific requirements for the light source that are challenging to fulfill. Fleischer et al. [146] have studied the theoretical resolution limit obtainable from unambiguous height measurement using WLI with combined fringe envelope location and interference data analysis. They showed that in the presence of noise there is an optimum source spectrum for the best resolution. In addition, the spectrum of the light source should also be temporally stable and spatially uniform to enable full-field imaging. To avoid depth and imaging resolution impairment due to speckles, the spatial coherence of the light source needs to be sufficiently low. In addition to the suitable spectral properties needed for enhanced WLI analysis, the vibration studies with SWLI technique require that the source emits short-enough pulses with stable, high-enough intensity and low noise. Furthermore, the optical pulse repetition rate should be freely adjustable such that it can be synchronized to the exci-
tation frequency of the vibration with low pulse jitter. For camera-based
detection of vibrations with frequencies greater than the camera frame
rate, it is also necessary that the pulse repetition rate is high compared
to the camera frame rate to ensure a sufficient number of light pulses per
camera frame such that picking up or losing a single pulse does not cause
a significant error.

The duration of the light pulses must in general be much shorter than
the time scale of the process being observed. For example, in vibration
measurements the fringe visibility is decreased when the duty ratio of
the pulse duration to the vibration period is too high, typically resulting
in an underestimated vibration amplitude as described in [150,151]. From
these theoretical and experimental studies, it can be concluded that for vi-
bration amplitudes up to $\lambda/2$, a duty ratio of less than 10% is still feasible
for determining the absolute amplitude within 2% accuracy. Even as high
a duty ratio as 30% results in an absolute amplitude error of less than
15% in the same vibration amplitude range. Furthermore, if the duty
ratio and the temporal shape of the optical pulse are known, the error in
the absolute amplitude detection at high duty ratios can be compensated
for to improve the accuracy [150, 151].

A light-emitting diode (LED) with a suitable spectrum offers a low-cost
light source that readily fulfills most of the aforementioned requirements.
Achieving shorter than nanosecond pulses with high enough power from
a LED source is however technically challenging, and thus measuring vi-
bration with frequencies of over 10 MHz are currently impractical with
LED illumination. LED is therefore an attractive and low-cost choice
for vibration measurements up to MHz range. Shorter pulses with high-
enough power can be obtained with lasers, but their spectral properties
are often inappropriate or difficult to adjust for the purpose. Among laser-
based solutions, a custom-designed supercontinuum-based illumination
source holds a promise to fulfill all the desired properties for the needs of
high-resolution SWLI as its spectral properties can be tailored.

Designing the supercontinuum (SC) source according to these require-
ments provides many technologically challenges. In particular, the spec-
tra of the SC pulses should ideally remain unchanged at different pulse
repetition rates to enable vibration measurements at freely selectable fre-
cuencies. This means in practise that the peak powers of the amplified
pulses from the pump source should be independent of the repetition rate.
Furthermore, the peak powers of the pump source need to be balanced to
be sufficiently high for efficient SC generation, but still low enough to minimize possible heat-induced drifts in the SC-generation. If the spectral shape of the SC pulses remains relatively stable at any repetition rate of interest, the proper spectral properties for high-resolution SWLI can then be obtained by spectral filtering of the output pulses after SC generation. Such SC source was developed in [152] for our setup to provide both optimized spectral properties in the visible and a freely adjustable repetition rate of the short optical pulses, thus enabling SWLI applications over a wide frequency range.

4.1.3 Measurement setup and performance

The optical setup of the full-field SWLI illustrated in Fig. 4.2(a) is based on a typical Michelson-type configuration already discussed at the beginning of Sec. 2.2.3. A non-polarizing beam splitter divides the optical pulses emitted from the light source into a sample and reference arm. The light pulses reflected back from the sample and reference surfaces are recombined in the beam splitter and the resulting interference pattern is recorded with a monochrome camera (Point Grey BFLY-PGE-09S2M) equipped with imaging optics (Volpi AS 11/50). The sample z-scan is accomplished by a piezoelectric translator stage. The scan typically covers a z range of at least 7 μm with 20 nm steps. The light pulses are triggered by a function generator \(G_1\) (Agilent 33250A) operating in a pulsed mode, whereas a similar function generator \(G_2\) drives the sample. The two synchronized generators enable an accurate and precise control of the phase delay \(\theta\) between the vibration and illumination pulses required for the measurement.

A LED-based stroboscopic illumination was first implemented in the SWLI system to optimize the minimum detectable vibration amplitude obtainable with the FDA method. In Publication IV it was shown with a demonstrative measurement of a MEMS resonator that a close-to-optimum performance was achieved with a green LED having a center wavelength of 510 nm and spectral width of 35 nm (FWHM), enabling detection of vibration amplitudes of less than 100 pm in typical micromechanical components. In fact, the effective detection spectrum depends both on the source spectrum and the spectral sensitivity of the camera and hence the optimal source spectrum is always interrelated to the camera used. In the LED-illuminated system, the custom-built LED driver enabled optical pulse durations down to 8 ns, thus allowing for measurement of OP
Figure 4.2. (a) Schematic presentation of the Michelson-type SWLI setup. The camera frames are recorded for each excitation-phase delay value $\theta$ at each $z$ step. (b) Schematic diagram of the supercontinuum source and the measured spectrum of its illumination pulses, for more details, see [152]. BPF$_1$ – bandpass filter with 1064 nm center wavelength; OI – optical isolator; MOF – microstructured optical fiber; BPF$_2$ – bandpass filter with 500 nm center wavelength; MM – multimode.

Surface vibrations up to $\sim$ 10 MHz with $\sim$ 10 % duty ratio. The LED-based SWLI setup was demonstrated in Publication IV to be capable of measuring an OP vibration field in a square-plate MEMS resonator at 14 MHz with a maximum amplitude of 300 pm only. In the measurement, a noise of less than 100 pm was achieved, mainly limited by the camera pixel noise. This combination of minimum amplitude detection limit and frequency range already advances the performance of SWLI to new limits. The minimum detectable amplitude is already comparable to that of full-field laser interferometry [71].

After demonstrating that a sub-100 pm detection limit is achievable with LED illumination, a custom-made SC source was implemented to the SWLI setup in order to extend its capabilities to higher-frequency vibration measurements. The SC source design in our Michelson-type SWLI setup is presented in Fig. 4.2(b) and described in detail in [152].

The temporal width of the resulting light pulses were measured to be shorter than 310 ps [152], hence allowing for OP vibration measurements even up to GHz range. To optimize the spectral characteristics of the SC source for high-resolution SWLI, the output pulses from the microstructured optical fiber (MOF) are first filtered with a 40-nm (FWHM) bandpass filter (BPF$_2$) centered at 500 nm, see the optical spectrum in Fig. 4.2(b). The spectrally filtered light pulses are then guided through a 2-m long multi-mode optical fiber, which serves to reduce the spatial coherence of
the SC illumination and hence to diminish speckle formation in the full-field interference image. As an additional benefit, the multimode fiber also modifies the spectrum to be more uniform across the transversal extent of the light pulses. The pulse repetition rate of the source is controllable with an external trigger and can be freely adjusted from a single shot up to 50 MHz with a pulse timing jitter specified to be less than 6 ps. Vibration frequencies higher than the 50-MHz maximum pulse repetition rate can be measured by synchronizing the SC pulsing to the subharmonic frequency of the vibration.

The minimum detectable amplitude limit of the SC-illuminated SWLI setup was evaluated in [152] by characterizing the amplitude and phase fields of a vibration mode at about 3 MHz in a square-plate MEMS resonator. This resulted in an amplitude detection limit comparable to that obtained with LED illumination from the same mode in the same sample. It was thus confirmed that the use of the SC source enables high-resolution SWLI without performance degradation with respect to the stroboscopic LED illumination. In Publication V, to demonstrate the capabilities of the SC-illuminated SWLI setup in characterizing surface vibrations at high frequencies previously unattainable with the SWLI technique, amplitude and phase of surface acoustic wave fields were measured in an annular interdigital transducer (AIDT) structure operating at 74 MHz. The AIDT structure was designed to focus the SAWs on the piezoelectric material to a single, diffraction-limited spot that shows a large concentration of acoustic energy [153]. As in the case of the lower frequency MEMS sample, a minimum detectable amplitude of less than 100 pm was reached again. Such performance provided a sufficient dynamic range for the measurement even though the maximum amplitude at the focal spot of the wave field was less than 3 nm.

A SAW field in a 167 MHz three-track single-phase unidirectional transducer (SPUDT) shown in Fig. 4.3(a) was also measured to demonstrate the feasibility of the SWLI for vibration studies above 100 MHz. The measurement area is at the center of the middle track illustrated with a small red-shaded rectangle in Fig. 4.3(a).

The amplitude and phase data of the OP wave field are presented in Figs. 4.3(b) and 4.3(c) from the measurement area. For better visualization, the measurement data are illustrated severely stretched along the wave propagation direction, since the wave field has a small wavelength compared to the width of the track. The data show a standing
wave pattern confined in the waveguide, with a maximum amplitude of about 600 pm occurring at the center in the transverse direction of the track. Speckle effects are however clearly visible as a graininess of the measured data due to remnants of spatial coherence of the source pulses, indicating a need of even better elimination of these effects. A rotating diffuser, for example, could be used instead of, or, in addition to the multimode fiber for more efficient reduction of the spatial coherence.

These results confirm the fact that the SC-illuminated SWLI setup combined with FDA pushes the performance of SWLI to new limits in the characterization of OP vibrations in micromechanical components. Now the SWLI capabilities allow for unambiguous determination of vibration amplitudes with sub-100 pm minimum detection limit for vibration frequencies from nearly DC up to even GHz range.
4.2 Stabilized full-field interferometer

4.2.1 Detection of vertical surface vibrations

In contrast to the traditional method for stabilizing the interferometer operation point described in Sec. 2.2.3, we have developed an alternative stabilization scheme for camera-based interferometric detection of OP surface vibrations. Intriguingly, in our full-field concept reported in Publication VI, no phase modulation of the interfering beams is required and no additional optical components such as a PD or other hardware are needed for monitoring the optical path length difference. Instead, the reference signal is obtained directly from a freely selectable reference region of the interference data recorded by the camera, thus allowing for a compact interferometer design. A software-based proportional-integrative-derivative (PID) controller uses this feedback signal for stabilizing the interferometer to a pre-determined operation point, such as the QP.

The only essential prerequisite for this stabilized stroboscopic full-field interferometry (SSFFI) concept is that a static, flat, non-vibrating region suitable for the acquisition of the reference signal exists in the vicinity of the vibrating area, such that both regions fit within the imaging area. Such structures are common for many types of micromechanical components. For instance, the vibrating structure of a MEMS resonators is typically surrounded by a flat Si substrate.

While the interferometer is stabilized, the phase delay $\theta$ between the illumination and excitation of the sample are varied in equal phase steps throughout the vibration cycle and the interference pattern is recorded at each $\theta$, in a similar way as was done in the SWLI system for every $z$ step. This allows for a rapid characterization of the whole OP vibration field. For example, using 18 phase steps ($\pi/9$ phase step) for the vibration cycle and a camera frame rate of 30 Hz results in a measurement time of only $t = \frac{18}{0.033} = 0.6$ s for a single vibration frequency.

The relatively fast measurement with this stabilized stroboscopic full-field detection enables a straightforward approach to further improve its minimum detectable amplitude limit. Since one of the main factors limiting the minimum detectable amplitude in the full-field system is the camera noise, averaging over multiple frames at each illumination-excitation phase delay can be used to reduce the effect of this random noise and hence to improve the signal-to-noise ratio of the measurement. Neverthe-
less, the minimum detectable amplitude cannot be infinitely improved via further averaging as the bit depth of the camera sensor ultimately limits the height resolution of the measurement. As long as the random noise dominates the detection, averaging of $N$ frames improves the signal-to-noise ratio by $\sqrt{N}$. As a drawback, the measurement time is linearly increased with $N$. Still, the duration of a vibration measurement with 36 phase steps ($\pi/18$ phase step) and averaging of 100 frames at 30-Hz camera frame rate is only 2 minutes.

The SSFFI setup schematically presented in Fig. 4.4 is built onto an optical table with active vibration isolation (Newport i-2000) to eliminate mechanical perturbations from the environment at frequencies above several Hz to a negligible level for the measurement. The remaining noise contributions to the interferometer operation arise from the slow drifts and low-frequency fluctuations in the optical path lengths of the interfering beams that are compensated for by the active stabilization.

In contrast to SWLI, the SSFFI method allows for the use of a wide variety of light sources such as LEDs and lasers provided that sufficient coherence length for the interferometric detection is achieved and the duration of the optical pulses are short enough for the stroboscopic detection. In laser based illumination the often highly spatially coherent light illumination results in a speckled light field that significantly impairs the performance in our approach, and therefore elimination of the spatial coherence may be required when using such sources. In the setup, a red LED with a center wavelength of 630 nm and spectral FWHM width of 15 nm is used for stroboscopic illumination.
For stabilization, the optical path length difference between the two arms of the interferometer is controlled with a software-based feedback loop such that the interference signal in a non-moving reference region on the sample surface stays at a constant intensity. This is accomplished by calculating the mean intensity $I_{\text{ref}}$ over the pixels in the reference area for each interference image recorded by the camera. The $I_{\text{ref}}$ is then compared against a predetermined set point value $I_{\text{set}}$, and the difference is used as an error signal in a software-based PID controller, which adjusts the interferometer to the chosen operation point, defined by $I_{\text{set}}$.

The set point $I_{\text{set}}$ is defined at the beginning of the measurement by recording $I_{\text{ref}}$ as a function of the $z$-position of the sample during a linear $z$-sweep driven by the piezoelectric translation stage, see the inset in Fig. 4.4. In the case of stabilizing the interferometer to a QP, the set point is found as a midpoint between the maximum and minimum value of the interference fringe having the highest modulation depth $M_R$. In general, the value of the set point can be freely selected between the maximum and minimum value of this best fringe, or even of a different fringe, to optimize the measurement also in cases where the reference area is at a different height than the surface under study.

After the determination of the set point and stabilizing the interferometer to it, the vibration data is measured at all frequencies of interest while the stabilization loop is simultaneously updated at the camera frame rate. Calculating the $I_{\text{ref}}$ value over a large number of camera pixels is advantageous as it improves the precision and accuracy of the stabilization by diminishing the pixel noise. For example, the Si substrate of a typical MEMS resonator often allows for the use of a large reference area.

In addition to obtaining the set point $I_{\text{set}}$, the $z$-sweep initiating the measurement also provides reference data at each pixel in the vibration area, a sort of look-up table, for the determination of the absolute amplitude of vibration. The reference data are necessary since the reference and vibrating areas may have a static $z$-offset due to small height variations or slight tilt of the sample surface, which in practice is rarely perfectly flat and smooth in the nanometer scale. Furthermore, the modulation depth of the interferogram at each pixel can vary due to effects such as variation of the reflectivity of the sample surface and pixel-by-pixel differences in camera sensitivity. When the instantaneous $z$ positions at each phase delay $\theta$ are determined for each pixel, the absolute amplitude and phase data are obtained using a Fourier transform as in the case of SWLI vibra-
A detailed description of the data analysis is given in Publication VI under the assumption of unambiguous detection as follows: The instantaneous z position at each pixel within the vibrating area and reference area at each excitation-illumination phase delay $\theta$ should be located on the same slope on the interferogram as the operation point on which the interferometer is stabilized to. This means that the vibration amplitude and the static height differences over the sample surface are small compared to the half period of the interferogram, $\lambda/4$, not exceeding the extrema of a single fringe. This assumption greatly simplifies the data analysis but, in general, the method can be extended to cases when this condition is not fulfilled.

4.2.2 Measuring vertical movements in time domain

With suitable modifications, the active stabilization concept described in Sec. 4.2.1 offers an elegant full-field technique for measuring vertical surface movements in time domain. As in SSFFI, a non-moving area within the imaged sample surface is needed. From this non-moving surface area, a reference region is selected from which the monitoring signal for the feedback loop controlling the interferometer operation point is acquired. The implemented stabilized full-field interferometer setup for the detection of surface deformations in time domain was introduced in Publication VII. In this setup, the stabilization of the interferometer is accomplished in a similar way to the SSFFI setup, shown in Fig. 4.4. In the time-domain setup, no stroboscopic illumination is needed. Instead, continuous-intensity illumination is used to record the temporal data of the surface deformation. The temporal data is obtained at the camera frame rate while simultaneously updating the stabilization loop at the same rate.

The basic idea of the data analysis is to track the time-dependent intensity trace along the interferogram at each pixel. This analysis is well suited for measuring small time-dependent movements on flat surfaces where interference fringe peaks are not crossed. In such measurements, the setup enables a z-resolution of about 1 nm for each pixel at a sampling rate limited only by the maximum camera frame rate. For larger deformations and height differences of the sample surface, the analysis in general depends on the nature of the surface movements. Furthermore, the intensity noise, always present in the measurement, makes an unambiguous
determination of the surface movement challenging at around the fringe extrema, particularly if the surface moves into both vertical directions and the direction of the movement can change during the measurement at any time instant.

It is, however, still possible to unambiguously measure the time-dependent surface deformation fields for a wide variety of surface movements even if the height range extends over multiple periods of the periodic interference signal. In such measurements, the rate of change of the surface deformation needs to be small enough with respect to the camera frame rate such that the time-dependent interference signal can be recorded without ambiguity. Furthermore, a priori knowledge of the characteristics of the surface dynamics is typically required. The data analysis procedures to measure large deformations are discussed in more detail in Publication VII.

The method has several advantageous features for use in versatile studies of vertical surface movements in time domain. For example, the temporal resolution, the vertical resolution of the surface movement, and the lateral imaging resolution are ultimately limited by the camera refresh rate, camera pixel noise, and the imaging optics, respectively. These three performance attributes are therefore defined by the fundamental limits of the full-field technique, and the data analysis itself does not result in additional constrains.

4.2.3 Sample measurements and obtained performance

The performance of the stroboscopic stabilized full-field interferometer was studied in Publication VI by characterizing the OP vibration field of the resonance mode at 12 MHz in a capacitively excited, square-plate MEMS resonator presented in Fig. 4.5(a). The MEMS resonator developed for frequency reference applications by VTT is designed to operate at a lateral square-extensional mode at 13 MHz [154]. In addition to the main mode it also has several other modes such as the 12-MHz vibration mode with a strong OP component studied here.

The feedback signal for the stabilization has been selected from a rectangular $59 \times 61$-pixels reference area on the substrate outside the vibrating area shown in Fig. 4.5(a). In the vibration measurement, 36 phase steps ($\pi/18$ phase step) over the vibration cycle have been used to record the instantaneous deformations and 400 frames have been averaged for each $\theta$ value to reduce the pixel noise.
Figure 4.5. (a) Microscopic image of a capacitively driven 320 μm × 320 μm square-plate resonator. The resonator plate is released from the silicon substrate using deep reactive ion etching on a silicon-on-insulator wafer. For the sacrificial oxide release etch, a grid of holes (diameter 2 μm) has been fabricated on the resonator plate as shown in the zoomed inset. (b) Absolute amplitude and (c) phase data of the 12-MHz OP vibration field correspond to a standing wave pattern characteristic to the eigenmode of the resonance. (d) The amplitude histogram extracted from the reference area indicated with the red rectangle in the micrograph. The mean amplitude value of 27 pm over the pixels in the reference area is marked on the histogram with a red dashed line.

The absolute amplitude and phase data presented in Figs. 4.5(b) and 4.5(c) show the shape of the complex standing wave OP field. The measured vibration field has several amplitude maxima higher than 2 nm surrounded by sharp nodal lines, and the neighboring maxima vibrate at opposite phase. These results agree well with those previously obtained with a scanning laser interferometer [100, 155].

Without averaging, in this measurement the current performance of our stabilized full-field system would allow to achieve a minimum detectable amplitude of less than 600 pm within 1.2 s measurement time by using 36 phase steps and 30-Hz camera frame rate. The averaging of 400 frames, however, reduces the pixel noise by a factor of about \( \sqrt{400} = 20 \), resulting in an amplitude detection limit of less than 30 pm, as can be observed in the amplitude histogram from the reference area plotted in Fig. 4.5(d). The amplitude values at the nodal lines in the vibration field are also consistent with this result.

The obtained minimum detectable amplitude is already better than that obtained with our SWLI setup using similar components, thus clearly demonstrating the potential of this SSFFI concept in low-amplitude detection. Furthermore, despite the relatively low-cost interferometer setup, with an inexpensive LED as a light source and a low-cost camera in detection, the 30-pm minimum amplitude detection limit achieved in these
first results is already comparable to or even slightly better than that of other full-field interferometers [71, 86, 94].

In Publication VII, the time-domain stabilized interferometer is used to measure the deformation of an AlN membrane as a function of time-dependent pressure difference applied between the top and bottom surface of the membrane. The measurement sample shown in the inset in Fig. 4.6(a) is a part of a transducer considered for differential pressure sensing applications. The structure consists of a 300-nm AlN thin film with Al top and bottom electrodes that forms a freely standing membrane on a Si substrate. To create a pressure difference across the membrane resulting in a mechanical deformation, a round 700-μm-diameter hole has been drilled in the center of the Si substrate beneath the AlN membrane to expose the bottom surface of the membrane to a vacuum chamber.

The detailed description of the experiment and the data analysis are described in Publication VII. The results of the surface movement are presented in Fig. 4.6 when a monotonically increasing pressure difference from 0 Pa to roughly 1.2 kPa is applied across the AlN membrane such that the pressure $p_b$ in the vacuum chamber below the membrane slowly changes from ambient pressure $p_0$ to roughly $p_b = p_0 - 1.2 \text{ kPa}$. The data do not show the true topography of the membrane but only the difference...
from the initial state. The temporal behavior of the surface is sampled at 6 Hz camera refresh rate, and the pressure difference is initiated at the 100th frame, corresponding to about 17 s after the start time of the measurement.

The temporal course of the displacement observed from a single pixel at the center of the membrane is presented in Fig. 4.6(a). In the new equilibrium state reached at about \( t_i = 500 \), the center of the membrane is deflected about 330 nm from its initial position. The membrane area on top of the hole deflects towards the lower pressure. The deformation is circularly symmetric on the hole area and the maximum deflection occurs at the center of the hole as seen from the surface deformation data of the AlN membrane shown in the inset in Fig. 4.6(b) and from the cross-sectional profile plotted in the same figure.

To estimate the resolution of the measurement, the noise level of the analyzed data was determined at the final equilibrium state \( t_i = 500 \) from the rectangular region in the membrane surface shown in the inset in Fig. 4.6. This region is outside the hole area and hence expected to stay stationary at its initial position. The data values in this area were within \( \pm 3 \) nm from the initial position, and the standard deviation over all pixels in this region was 0.84 nm. The noise analysis clearly indicated that a resolution of about 1 nm is obtainable for this measurement.
5. Characterization of square-plate MEMS resonators

VTT has aimed at developing a piezoelectrically excited silicon MEMS resonator that satisfies the specifications of temperature-compensated crystal oscillators. Such oscillators are crucial components in demanding timing control applications used, e.g., to provide the frequency reference signal in wireless communication. Until now these components have been realized with an AT cut quartz crystal, but the continuous strive for lower cost components with smaller size and lower power consumption has raised the need to replace quartz with other solutions.

One of the VTT’s designs was a square-extensional (SE) mode MEMS resonator shown in Fig. 5.1. In SE-mode operation, the resonator plate expands and contracts, while preserving its shape. To validate and optimize the device design, electrical impedance measurements were performed to empirically characterize the performance of the SE-mode MEMS resonators. In the electrical characterization, two unexpected phenomena were observed in the operational mode that adversely affected the device performance. To get physical insight into the nature of the observed behavior, the vibration fields of the components were characterized using the laser scanning setup and the detection concepts and analysis methods discussed in Sections 3.1 and 3.2.

5.1 Splitting of the main mode branch

The SE-mode resonator design aimed at providing a simple and cost-effective means to fabricate devices with a wide range of different operational frequencies, that would allow using the same structure for multiple applications. A finite element model (FEM) predicted that the operation frequency of the SE mode should be inversely proportional to the side length of the resonator plate. The contribution of the anchors and the top
Figure 5.1. (a) Schematic view of the square-plate MEMS resonator. The 28-μm-thick, single-crystalline resonator plate is attached to the substrate at its corners with 4-μm-wide, meander anchors. The Mo layer provides the top-electrode connection to the piezoelectric AlN thin film whereas the Si resonator plate itself acts as a bottom electrode. (b) A light power image of the resonator sample with a plate side length of \( L = 209 \ \mu m \) and the rectangular scan areas I–III for the OP and IP measurements. The top Mo electrode is patterned with a 5-μm-period grid of circular, 2.5-μm-diameter holes to enable the vectorial IP detection over the whole resonator plate.

metal electrode were assumed to be negligible and hence excluded from this FEM model.

The fabrication and design of the square-plate resonator with a resonator plate side of 160 μm × 160 μm × 20 μm was previously reported in [156]. In the study, the resonator was found to operate in the SE mode at 26 MHz as expected. After obtaining successful performance, the scalability of the SE mode frequency was studied over a wide range of operational frequencies ranging from 13 MHz to 30 MHz by fabricating and electrically characterizing a set of resonators with different plate sizes [157]. For comprehensive characterization, a total of 1200 devices were fabricated in two wafers with 64 different plate sizes, the length of the plate side varying from \( L = 131 \ \mu m \) to \( L = 320 \ \mu m \) in steps of 3 \( \mu m \). Each size variation was replicated about 10 times in both wafers, and the replicas were distributed over the whole wafer. The design and the dimensions of the anchors were kept the same for all resonators.

In the electrical impedance measurement results presented in Fig. 5.2, a clear deviation from the \( 1/L \) scaling law was unexpectedly obtained. Especially, at intermediate plate side lengths of 160 μm < \( L < 260 \ \mu m \), instead of a single SE resonance, two separate resonances were observed in the vicinity of the predicted SE resonance frequency, one above and the other below it. In these intermediate plate sizes, the SE mode branch seems to be split into two distinct frequency branches whereas at other plate dimensions the measured resonance frequency agrees well with the original FEM model. An analytic 1D spring-mass equivalent model shown in the inset A in Fig. 5.2 is fitted to the electrical data (gray curve) to qualita-
**Figure 5.2.** Electrically measured resonance frequencies of the 1200 devices as a function of the plate side length (black ‘+’) and a selected set of optical characterization results. Only the electrically characterized resonance frequencies closest to the SE mode frequency curve (gray line) predicted by the spring-mass 1D model illustrated in the inset A are presented. The red curves represent a least squares fit of the coupled resonator model shown in the inset B to the electrical data. The black circles indicate the resonances that were optically measured from eight samples of different plate sizes. An example of the increased anchor activity within the 20-22 MHz range is presented in the inset C, in which IP amplitude data measured from a single point on the top-right anchor of a selected resonance, with a high vibration amplitude at the anchors, are plotted as a function of the drive frequency. In the data insets illustrated with jet colormap, the IP (black arrows) and OP vibration fields (colormap) are shown for the SE mode of the $L = 305$-$\mu$m resonator and for the two resonances with different frequencies in the splitting region of the $L = 209$-$\mu$m resonator. Since the IP and OP vibration fields of the $L = 305$-$\mu$m resonator show almost ideal characteristic shape of the SE mode, it can be observed that the two resonances of the $L = 209$-$\mu$m resonator have the typical IP field shape of the SE mode whereas the OP data deviate from the ideal SE mode shape. Insets D and E: IP phase data of the two resonances from the top-right anchor of the $L = 209$-$\mu$m resonator. The red and yellow arrows depict the instantaneous movement directions of the anchor and the resonator plate, respectively.

To understand the mechanism leading to this adverse behavior, IP and OP vibration fields were measured from eight selected samples using the homodyne scanning Michelson laser interferometer setup. The selected set consisted of devices of different plate sizes with their $L$ ranging from 140 to 320 $\mu$m.
137 μm to 305 μm in steps of 24 μm.

The rectangular scan areas for the OP and IP measurements are depicted in the light power image of the $L = 209$-μm sample presented in Fig. 5.1(b). OP data over the resonator plate, anchors and a part of the substrate were first measured from the scan area I to get an overview of the acoustic behavior. Then the IP vibration fields from the whole resonator plate were obtained using the scan area II. Finally, IP and OP data from the top-right anchor area and the part of the resonator plate were measured from area III for more detailed analysis, based on the observations from scans of areas I and II.

As detailed in Publication II, the combined use of the phase-sensitive IP and OP measurements were crucial to reveal the origin of the observed splitting phenomenon of the SE mode branch. The most relevant results for explaining the mechanism of the splitting are illustrated in Fig. 5.2. An important finding in the OP measurement (scan area I) was an increased anchor activity within 20–22 MHz at all samples studied, indicating the possibility of anchor resonances existing at these frequencies that could couple to the SE mode, see the inset C. A second key finding was that the IP vibration fields of the resonances in the two frequency branches were in most cases still characteristic to the SE mode, see the IP data for the $L = 209$-μm presented in Fig. 5.2. The measured IP phase data from the anchor scan III shown in insets D and E further supported that the splitting effect is caused by a coupling of the SE mode to an anchor mode. From this phase data it is observed that on the higher-frequency branch the resonator plate and the anchor are moving in opposite phase whereas on the lower-frequency branch they vibrate in phase. Such behavior and modal splitting is characteristic for a strongly coupled IP oscillation between the SE mode and the anchor.

An analytic model of two coupled spring-masses depicted in Fig. 5.2 was used to find out if the observed splitting could be explained with a modal coupling of an anchor and the SE mode. In the model, the resonator plate is described with an effective mass $m_1$ and a spring constant $k_1$ whereas another spring-mass system $(m_2, k_2)$ represents the contribution of all the four corner anchors. This analytic model already resulted in a good fit with the electrically measured data as presented with the red curves in Fig. 5.2, thus supporting the hypothesis of an anchor-plate coupling phenomenon.

Consequently, the FEM model was refined to also take into account the
anchors. This more accurate model successfully reproduced the observed splitting phenomenon [158].

5.2 Nonlinear behavior

Electrical impedance measurements of the same set of SE-mode MEMS resonators as in Sec. 5.1 also revealed an unexpected nonlinear phenomenon. In some of the devices, a compression of the amplitude was observed above a certain input drive power and hence the power handling capacity of these resonators was degraded. Two devices were selected for optical characterization, that both showed the compression behavior but otherwise the nonlinear behavior was not similar. In particular, the frequency dependency of the electrically measured resonant responses of the two resonators were different in the nonlinear regime of operation. It was hence assumed that the nonlinearities observed in the two devices may arise from different mechanisms.

The first resonator R1 has a plate side length of \( L_1 = 305 \, \mu m \) and the second (R2) \( L_2 = 257 \, \mu m \). The SE resonance frequencies of the two resonators are \( f_1 = 14.14 \, MHz \) and \( f_2 = 16.67 \, MHz \), the motional resistance \( R_m = 80 \, \Omega \) is the same for both resonators and the quality factors are only slightly different, \( Q_1 = 17000 \) and \( Q_2 = 18000 \). Interestingly, these electrically characterized parameters of the two resonators are very similar, although the observed nonlinearities have distinct characteristics.

In Publication III and [159], the nonlinear acoustic behavior in the two resonators was studied with the scanning laser interferometer, using the nonlinear techniques described in Secs. 3.1.2 and 3.2.2. The nature of the acoustic nonlinearities was first thoroughly studied with single-point measurements at the upper right corner of the resonator plate. It turned out that the capability of using different excitation and detection frequency combinations was essential to discover the nonlinear mechanism. Based on the findings in the single-point experiments, areal measurements were carried out on the resonator plate at selected excitation and detection frequency combinations in which the nonlinear acoustic behavior was observed. OP vibration fields were measured from the whole resonator plate whereas, due to symmetry, the IP vibration data were obtained only from the upper right quadrature of the plate.

In the single-point measurements, it was found out that the amplitude of the SE mode was indeed compressed in both resonators above a cer-
tain threshold drive power $P_{th}$. This amplitude compression can be observed in Fig. 5.3(a), where the IP vibration amplitudes of the SE modes in both resonators are presented as a function of the input drive power $P_{in}$. More importantly, the single-point measurements revealed that when the amplitude compressed, another mode at a subharmonic frequency of the SE mode was simultaneously excited. In the two resonators, the nonlinearly excited modes have, however, very different subharmonic frequencies. In R1, the nonlinear mode is excited at half the SE mode frequency $f_{SE/2} = 7.07 \text{ MHz}$, whereas in R2 the resonance frequency of the nonlinearly excited mode $f_R = 0.725 \text{ MHz}$ corresponds to the 23rd subharmonic frequency of the SE mode.

The IP amplitudes of the nonlinearly excited modes are also plotted in Fig. 5.3(a) as a function of the input drive power. From these data it is evident that the subharmonic modes are not excited at all below the threshold drive power but they abruptly emerge at $P_{th, 1} = -7 \text{ dBm}$, and $P_{th, 2} = -10 \text{ dBm}$ in R1 and R2, respectively. Above $P_{th}$, the IP amplitudes of the SE modes eventually saturate at a constant level when the input power is increased, whereas the amplitudes of the nonlinearly excited modes increase, first more than 1 dB per 1 dB increase of input power after which the slopes asymptotically approach unity. Therefore, in the asymptotic behavior, all the added power appears to linearly increase the amplitude of the nonlinearly excited modes.

Furthermore, an interesting nonlinear effect was observed in R2 that was not found in R1. The IP vibration amplitudes of the SE and nonlinearly excited modes of R2 are illustrated in Fig. 5.3(b) as a function of the drive frequency, using three different drive power levels in the nonlinear regime ($P_{in} > P_{th}$) and one in the linear regime. The amplitude cut-off effect is clearly present in the SE mode data in the nonlinear regime. In addition, the amplitude behavior is different when the drive frequency is swept into increasing or decreasing frequency, showing amplitude-frequency hysteresis that is characteristic to spring hardening. The same effect is also observed for the nonlinearly excited mode at $f_R$.

In R1, the areal measurements showed that the nonlinearly excited mode at $f_{SE/2} = 7.07 \text{ MHz}$ has both IP and OP vibration components. The measured OP amplitude data of this nonlinearly excited mode are presented in Fig. 5.3(c), when the resonator is driven in the nonlinear regime at $f_{SE}$ with input power $P_{in} > P_{th}$. In the case of R2, a purely rotational mode is nonlinearly excited at the 23rd subharmonic frequency of the SE
Figure 5.3. (a) IP amplitude data (logarithmic scale with arbitrary normalization) of the SE ('+') and nonlinearily excited mode ('o') in R1 (black) and R2 (green) with respect to the input drive power $P_{in}$. The threshold powers $P_{th,1} = -7$ dBm and $P_{th,2} = -10$ dBm for the onset of the subharmonic parasitic modes are indicated with dashed lines. (b) IP data of the SE mode (top) and the nonlinear mode (bottom) of R2 when the input frequency $f_{in}$ is swept across the SE resonance, using four different $P_{in}$ levels indicated in the figure. In the nonlinear regime, $P_{in} > -10$ dBm, the frequency up-sweeps (blue lines) and down-sweeps (red lines) result in different amplitude-frequency dependencies with a Duffing-like spring hardening behavior. (c) OP amplitude data of the nonlinearly excited mode in R1 detected at $f_{SE}/2$, when the SE mode is excited at $f_{in} = 14.1$ MHz in the nonlinear regime with $P_{in} = -6$ dBm. (d) IP vibration amplitude and the vector fields of the SE mode and (e) $f_R$ mode of R2 when the sample is driven with $P_{in} = -6$ dBm above $P_{th,2}$.

The symmetric IP amplitude data of the SE mode and the rotational mode are shown in Figs. 5.3(d) and 5.3(e) from the top-right quadrature of the plate. The IP data of the $f_R$ mode correspond to a rotation of the whole resonator plate around its center point in the lateral plane, with no deformations of the plate.

It was confirmed with optical measurements that the $f_{SE}/2$ mode of R1 can also be directly excited by driving the resonator at $f_{in} \approx f_{SE}/2$. On the contrary, in R2 the $f_R$ mode cannot be piezoelectrically actuated directly at its resonance frequency in this electrode geometry. Accordingly,
the $f_R$ mode cannot be electrically detected at all. Consequently, for resonator R2 it is evident that electrical characterization alone does not provide essential experimental data about the mechanism of the nonlinearities observed. The optical measurements readily revealed the nonlinearly excited modes, hence clearly demonstrating the strengths of the optical characterization methods for studying nonlinear vibrations in electromechanical devices.

The observed nonlinearities severely degrade the power handling capacity in both resonators well below the expected limit at which the mechanical nonlinearities of the vibrating AlN-Si structure and the electromechanical nonlinearity of the AlN thin film become significant. For example, using the theoretical model reported by Kaajakari et al. [160], the critical vibration amplitude at which mechanical nonlinearities are prominent is estimated to be well above 300 nm for the SE mode in these resonators. The saturated IP amplitude data of the SE mode is, however, more than an order of magnitude smaller, see Fig. 5.3(d).

The obtained results in the optical measurements support that the nonlinear mechanism in the resonators is related to autoparametric coupling of two mechanical modes. The dynamic instability of the resonator results in the SE mode acting as a parametric driver for the subharmonic unwanted mode. A study of autoparametric resonances in capacitively excited MEMS resonators was previously published by Avoort et al. [161], where similar amplitude saturation behavior of the operational mode was noticed. The observed spring hardening characteristics in R2 may originate from the mechanical nonlinearity of the $f_R$ mode, which is induced to the SE mode by an interplay of the two modes.

The optical results in the two resonators indicate that the autoparametric coupling mechanism in this resonator structure can be complex. Since the SE mode was coupled to very different type of parasitic modes in the two resonators studied here, it is likely that there are in general a number of parametric resonances that could couple to the SE mode provided that their resonance frequency occur close to subharmonic or superharmonic frequencies of the main mode. Furthermore, even with a fixed plate size, the electrical characterization results showed that only some of the devices featured nonlinear behavior at such low input drive powers, suggesting that small variations in the manufacturing process may contribute significantly to the behavior of the autoparametric coupling.
6. Conclusion

The research work of this thesis covers the design and implementation of state-of-the-art optical measurement setups as well as the development of detection methods and data analysis techniques for characterization of surface dynamics in microstructures. All these vibration detection methods provide both the phase and absolute amplitude of either the vertical or lateral component of the mechanical vibration field. Such data enables a complete reconstruction of the corresponding vibration field component. In addition to the vibration measurements, a full-field detection concept for measuring vertical surface movements in time domain was demonstrated. These optical techniques offer an attractive combination to address the need for advanced experimental methods in the research and development of novel electromechanical components with improved performance and increased functionality.

In Publication II, the unstabilized homodyne scanning interferometry concept described in Publication I and the in-plane detection method discussed in Sec. 3.2 were applied to investigate an unexpected splitting phenomenon of the square-extensional mode in square-plate MEMS resonators. In the study, phase-sensitive in-plane and out-of-plane measurements were used to find out that the observed splitting effect of the square-extensional mode branch was due to coupling of a parasitic anchor resonance to the square-extensional mode. These experimental results brought valuable information of the underlying device physics, resulting in an improved model of the square-extensional mode resonator.

In the thesis work, novel methods were implemented to the scanning interferometer to allow for measurement of nonlinear in-plane and out-of-plane surface vibrations. The inherent nonlinearity of these detection methods constitutes a challenge for the nonlinear vibration studies, resulting in optical mixing artifacts in the measured data. The use of these
concepts therefore requires a thorough understanding of how the measurement signal should be interpreted and analyzed to be able to correctly characterize the true mechanical behavior of the device.

The nonlinear measurement techniques were utilized in Publication III to study the adverse nonlinear behavior of the square-extensional resonance found in two square-plate MEMS resonators. Both the nonlinear in-plane and out-of-plane detection techniques were needed in order to completely characterize the nonlinear mechanical behavior. The measurements revealed an autoparametric excitation mechanism with simultaneous excitation of a subharmonic mode together with the square-extensional resonance above a certain threshold drive power in both resonators. This study clearly highlights the benefits of direct optical measurement of the mechanical nonlinear behavior. In particular, the in-plane rotational mode excited at the 23rd subharmonic frequency of the square-extensional mode in the other resonator is not detectable with indirect, electrical measurements in this electrode geometry.

Publications IV–VI describe advanced full-field interferometric techniques for characterizing surface vibrations in micromechanical devices. The progress in instrumentation and data analysis reported in Publications IV and V pushes the performance of stroboscopic white-light interferometry in vibration measurements into new limits. In Publication IV, a LED-illuminated stroboscopic white-light interferometer setup with 8-ns optical pulses was implemented and enhanced frequency domain data analysis was used to achieve a minimum detectable amplitude of less than 100 pm in a MEMS resonator sample. The obtained minimum amplitude detection limit outperforms the current state-of-the-art stroboscopic white-light interferometry and is already comparable to that of full-field laser interferometry. In addition, this approach allows for unambiguous determination of high-amplitude vibrations that are larger than the period of the interference signal.

To enable higher-frequency measurements with the stroboscopic white-light interferometric technique, a supercontinuum source emitting 300-ps pulses was developed and implemented to the stroboscopic white-light interferometer setup [152]. The spectral properties of the supercontinuum source were tailored suitable for the use of the frequency domain analysis to achieve high performance in low-amplitude detection. Again, as in the case of a LED-based setup, a minimum detectable amplitude limit of less than 100 pm was obtained with the same sample. In Publication V,
the supercontinuum-based stroboscopic white-light interferometer setup was used to measure the focusing of surface acoustic waves at 74 MHz by an annular interdigital transducer structure. These results confirm that stroboscopic white-light interferometry is no longer limited to low-frequency measurements of MEMS devices that typically have high amplitudes, but is also applicable to the characterization of high-frequency devices with vibration amplitudes down to even sub-nanometer scale and vibrations up to GHz range as well.

The stabilization concept introduced in Publication VI allows for a simple and elegant homodyne full-field interferometer design for the characterization of low-amplitude vibrations. The applicability of the stabilized stroboscopic full-field interferometry concept was demonstrated by a MEMS sample measurement, in which a minimum detectable amplitude of less than 30 pm was achieved, a performance which is already slightly better than that of existing full-field interferometers. The method and the obtained results in these first measurements therefore hold a promise for reaching a minimum detectable amplitude of even $\sim 1$ pm with full-field interferometry.

Publication VII reports further development of the stabilized full-field interferometry concept to enable measurements of out-of-plane movements in time domain. The ability to unambiguously measure surface deformations with even subnanometer vertical resolution and down to $\sim 1$ $\mu$m lateral resolution makes the method versatile for studying surface dynamics of a wide range of different magnitudes in microstructures. The implemented setup is well suited for measuring single-event dynamic surface phenomena and for studying deformations as a function of an external physical quantity such as voltage or pressure. Such a time-domain technique benefits the development of advanced microelectromechanical devices and the experimental research of, e.g., novel functional materials with deforming surfaces and the characterization of material properties in piezoelectric crystals or thin films.

The detection concepts and data analysis methods described in this thesis advance the optical imaging techniques for the characterization of surface movements in micron scale structures. The benefits of the use of the advanced optical measurement techniques in the research and development of micromechanical devices are clearly highlighted in the two MEMS resonator study cases in which the optical characterization contributed substantially to the understanding of the mechanical behavior of
the devices.

The demand for components with improved performance and ever more sophisticated functionalities has directed the microacoustics and micromechanics research towards more complex device structures that are ever more challenging to design and simulate. Direct characterization of surface movements may therefore become a more integral part of the device design in the near future, which sets interesting prospects for the further development and research of the advanced optical imaging methods.
References


