Deflection of atoms by a pulsed standing wave: effects of laser field coherence

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Abstract. Deflection of two-level atoms by a pulsed standing wave with a pulse duration of a few nanoseconds is studied by using the density matrix formalism. The effect of the limited coherence time of the pulsed laser field on the momentum distribution of the deflected atoms is investigated. In particular, we determine the coherence time at which the deflection by a pulsed standing wave differs significantly from the zero-relaxation case.

1. Introduction

Deflection of an atomic beam by a resonant monochromatic standing wave has been subject to extensive theoretical and experimental studies during recent years [1–12]. The goal in these studies has been to determine the momentum distribution of deflected atoms after interaction with the standing wave. It has been shown for the case of zero external relaxation that atoms will be deflected into discrete angles forming a peaked deflection profile [2–4]. This kind of phenomenon can be useful, for example, in splitting the atomic wavefunction for applications in atom optics. If the external relaxation cannot be neglected, the coherence of the deflection will be reduced and the resulting momentum distribution will contain a diffusive element. This has been shown both theoretically and experimentally for relaxation by spontaneous emission [4, 6, 7, 9, 11, 13].

In this work we study the deflection of two-level atoms by a pulsed standing wave with a pulse duration of a few nanoseconds. With the high intensity available in a pulsed laser, many successive stimulated emission–absorption processes can occur during the interaction time. In this case one would expect the momentum distribution of the deflected atoms to consist of a doubly peaked structure with relatively large separation between the maximum deviations. However, this applies only to the case of zero relaxation. In practice, the phase of a pulsed laser field fluctuates rapidly causing reduction of the coherence of the atom–field interaction. With increasing fluctuations the peaked structure of the deflection profile should therefore start to smear out and in the limit of fast relaxation it should vanish completely. This effect resembles the reduction of coherence by spontaneous emission. The final momentum distribution for our case will, however, be different because of the lack of randomly emitted photons.

To determine the effects of phase fluctuations we calculate the momentum distribution of the deflected atoms as a function of the coherence time of the laser field. If this time is much shorter than the Rabi-flipping period, the deflection can be described by the random-walk model [3]. The final momentum distribution will in this case have a Gaussian shape.
The coherence time of a pulsed laser can, however, be of the order of the pulse duration. The deflection profile should then retain some characteristics of coherent deflection and has to be determined using quantum mechanical calculations.

In this work, the temporal evolution of atoms in the laser field is modelled in a similar way as was done in [4]. Because of the short pulse duration the effects of spontaneous emission will be neglected. Phase fluctuations of the laser field will be included in the calculations by introducing relaxation terms for the nondiagonal density matrix components. The resulting equations for the density matrix are solved in the Raman–Nath regime for the atomic Wigner function. The final momentum distributions are then calculated by numerical integration of the Fourier expansion coefficients of the propagator for the Wigner function.

We calculate the final momentum distributions at laser intensities typical of CW lasers and pulsed lasers. The effects of phase fluctuations are determined by comparing the momentum distributions at different relaxation rates, which are chosen such that the different types of deflection can clearly be seen. The implications of the results for deflection experiments will be discussed.

2. Model

We consider a case where a monoenergetic beam of two-level atoms, initially in their ground state, is deflected by a pulsed standing wave as shown in figure 1. The atoms are assumed to travel parallel to the $z$-axis with velocity $v_0$ and the standing wave is assumed to be along the $x$-axis and tuned into resonance with the atomic transition. The time dependence of the pulsed standing wave is chosen to be square shaped with a pulse duration $T$. Within the pulse duration the electric field of the standing wave is

$$E(x, t) = 2E_0 \cos(kx) \cos(\omega t) \quad (1)$$

where $E_0$ is the amplitude of the field, $k$ is the wavenumber and $\omega$ is the frequency. In equation (1) and in the following calculations the electric field is assumed to be linearly polarized. Also, each atom is assumed to experience a constant intensity during the interaction time regardless of its initial $z$-position.

In order to simplify the calculations we restrict the atomic motion to the Raman–Nath regime. In that case the atomic motion in the transverse direction can be neglected during

![Figure 1. The standing wave geometry used for the calculations of the final momentum distributions of the deflected atoms.](image)
the interaction time and the resulting density matrix equations will be local in \( x \). Effects of the phase fluctuations of the laser field on the atomic motion are taken into account by introducing a relaxation constant \( \gamma \) to the nondiagonal terms of the atomic density matrix.

In a coordinate frame moving with the atoms at velocity \( v_0 \) the atomic motion can then be described by the equations:

\[
\begin{align*}
\frac{\partial}{\partial t} \sigma_{gg} &= -i \left[ \kappa^* (x + \frac{1}{2}u) \sigma_{eg} - \kappa (x - \frac{1}{2}u) \sigma_{ge} \right] \\
\frac{\partial}{\partial t} \sigma_{ee} &= -i \left[ \kappa (x + \frac{1}{2}u) \sigma_{ge} - \kappa^* (x - \frac{1}{2}u) \sigma_{eg} \right] \\
\frac{\partial}{\partial t} \sigma_{ge} &= -\gamma \sigma_{ge} - i \left[ \kappa^* (x + \frac{1}{2}u) \sigma_{ee} - \kappa (x - \frac{1}{2}u) \sigma_{gg} \right] \\
\frac{\partial}{\partial t} \sigma_{eg} &= -\gamma \sigma_{eg} - i \left[ \kappa (x + \frac{1}{2}u) \sigma_{gg} - \kappa^* (x - \frac{1}{2}u) \sigma_{ee} \right]
\end{align*}
\]

where \( \sigma_{ab} \) are the density matrix components for the two-level atoms, \( \gamma \) is the relaxation rate and \( \kappa(x) = \Omega_0 \cos kx/2 \) is the atom–field coupling constant. The resonant Rabi frequency is defined as \( \Omega_0 = \mu_{eg} E_0 / \hbar \), where \( \mu_{eg} \) is the dipole matrix element for the transition.

Equations (2) are simplified versions of the generalized Bloch equations and include both the internal and the external dynamics of the atoms.

Since equations (2) are local in space and the electric field has a constant amplitude, the general solution can be written in the form:

\[
\sigma_{ab}(x, u) = I_{ab}(x, u, T) \sigma_{ab}^i(x, u)
\]

where \( \sigma_{ab}^i(x, u) \) and \( \sigma_{ab}(x, u) \) are the initial and final distributions and \( I_{ab}(x, u, T) \) is the impulse response of the system. The impulse response can be determined by using Laplace-transform methods. In this work we are interested in finding the distribution of the atoms in momentum space. For this we solve the diagonal terms of the density matrix using equation (3). By taking the Fourier transform of the sum of the diagonal elements we can determine the atomic Wigner function \( w(x, p) \) [4, 14]

\[
w_i(x, p) = \int dq \ G(x, q, T) \ w_i(x, p - q)
\]

where \( G(x, q, T) \) is the Fourier transform of the impulse response with respect to \( u \) and the indices \( i \) and \( o \) refer to input and output, respectively. The distribution of atoms both in coordinate and momentum space can now be determined by using the properties of the Wigner function. To calculate the final distribution we need the initial state of the atoms. In this work we assume the initial state to be localized in momentum space such that \( \delta p_i \ll \hbar k \).

For the momentum distribution this gives [4, 14]

\[
\mathcal{P}_o(p) = \int dx \ G(x, p, T) \ \mathcal{P}_i(x)
\]

where \( \mathcal{P}_i(x) \) is the initial distribution in coordinate space which for the chosen initial state is spread over many wavelengths.

Next we consider the case of deflection of an atomic beam by a pulsed standing wave. The impulse response for the sum of the diagonal elements can be determined by solving equations (2). By defining a new atom–field coupling constant \( K(x, u) = \kappa(x + \frac{1}{2}u) - \kappa(x - \frac{1}{2}u) = -\Omega_0 \sin(kx) \sin \left( \frac{1}{2}ku \right) \) the impulse response can be written in the form:

\[
I(x, u, T) = e^{-\frac{1}{2} \gamma T} \left[ \cos(\omega_1 T) + \frac{\gamma}{2\omega_1} \sin(\omega_1 T) \right]
\]
where \( \omega_1 = \sqrt{K^2 - (\gamma/2)^2} \). From equation (6) it can be seen that the phase fluctuations of the laser field cause the impulse response to decay with a rate of \( \gamma/2 \). In the limit of \( \gamma = 0 \) the impulse response reduces to that of coherent deflection and gives a diffraction-type momentum distribution [2–4, 8]. For fast relaxation, i.e. \( 4K^2/\gamma^2 \ll 1 \), the phase of the laser field will change many times during a Rabi period. In this case the impulse response is given by the rate-equation approximation [1]. For fast relaxation the deflection profile can also be calculated by using the random-walk model and will result in a diffusive deflection profile [3]. In the intermediate region the deflection profile has both a diffractive and a diffusive component and will have to be calculated using the impulse response given in equation (6).

Because of the periodicity of the standing wave the coupling constant \( K(x,u) \) and thus the impulse response are periodic in \( u \) with a period of \( \Delta u = \lambda \). This allows us to expand the impulse response in a Fourier series in \( u \) with the resulting expression

\[
I(x,u,T) = \sum_{n=-\infty}^{\infty} C_n(x,T) e^{i n ku} \tag{7}
\]

where the Fourier coefficients \( C_n(x,T) \) are given by

\[
C_n(x,T) = \frac{1}{\lambda} \int_{-\lambda/2}^{\lambda/2} du \, I(x,u,T) e^{-i n ku}. \tag{8}
\]

Since the \( u \) dependence in equation (7) is only in the exponential, the Fourier transform of the impulse response can be easily calculated. By using the properties of the Dirac \( \delta \)-function the propagator for the Wigner function can be written as

\[
G(x,p,T) = \sum_{n=-\infty}^{\infty} C_n(x,T) \delta(p - n\hbar k). \tag{9}
\]

The distribution of the deflected atoms can now be determined with the aid of equations (5), (8) and (9).

Equation (5) for the final momentum distribution can be somewhat simplified by using the fact that the initial state is spread over many wavelengths and by taking into account the periodicity of the impulse response. With this knowledge the infinite integral can be replaced by integration over one period and the final momentum distribution becomes [4]

\[
P_o(p,T) = \sum_{n=-\infty}^{\infty} P_n(T) \delta(p - n\hbar k) \tag{10}
\]

where

\[
P_n(T) = \frac{1}{\lambda^2} \int_{-\lambda/2}^{\lambda/2} dx \int_{-\lambda/2}^{\lambda/2} du \, I(x,u,T) e^{-i n ku}. \tag{11}
\]

The coefficients \( P_n(T) \) can be interpreted as probabilities to end up in a final momentum state \( n\hbar k \). To determine the deflection profile one thus has to calculate the coefficients \( P_n(T) \) as a function of \( n \). In this work we calculate the final distribution by numerically integrating equation (11) for values of \( n \) which give a significant contribution. We do the integrations for different relaxation rates in order to determine the effects of phase fluctuations of the laser field.

In the case of a pulsed standing wave with a high field intensity, the calculation of the momentum distributions could also be done by using the quasiclassical approximation
3. Results

In this section we present calculated results for a beam of two-level atoms deflected by a pulsed standing wave. The deflection profiles are calculated for several relaxation rates at two different field intensities. The relaxation is assumed to be caused by the finite coherence time of the laser field. In this case the rate constant \( \gamma \) can be taken as the inverse of the coherence time. First we calculate the deflection profiles for a low-intensity standing wave corresponding to the case of a focused CW laser beam. The low-intensity case is studied in order to determine the effects of the finite coherence time of the laser field on experiments done with CW lasers. In the second case the field intensity is chosen to be of the order of the available intensities in commercial pulsed dye lasers. The frequency of the standing wave is chosen to be on resonance with the \( D_1 \)-transition (589 nm) of sodium. The relation between the Rabi frequency and the laser intensity in this case is

\[ \Omega_0 = 17.6 \times 10^6 \sqrt{I \text{[mW cm}^{-2}]} \text{ s}^{-1} \]

Figure 2 shows the distribution of atoms in momentum space after deflection from a standing wave with Rabi frequency \( \Omega_0 = 700 \times 10^6 \text{ s}^{-1} \) and interaction time \( T = 20 \text{ ns} \). These values correspond roughly to those in deflection experiments performed with focused CW lasers [8, 9]. The final momentum distributions were calculated by using the impulse response given by equation (6). The different distributions correspond to different values of the bandwidth \( \delta \nu = \gamma / \pi \) of the laser field, which were selected to show the transition from diffractive to diffusive deflection. The probability for observing an atom in the momentum state \( n\hbar \) is indicated by the bars for each bandwidth. To illustrate the deviations from the coherent deflection, the momentum distribution at zero relaxation has been included in each figure with crosses. From the calculated momentum distributions it can be seen that the relaxation caused by the finite coherence time can be neglected for bandwidths up to \( \delta \nu \approx 1.6 \text{ MHz} \). At 16 MHz bandwidth the effects of relaxation can already be clearly seen. At this value the exponential decay factor of the impulse response, equation (6), has reduced the sinusoidal terms by 10\%. For bandwidths larger than 16 MHz the momentum distributions tend to be peaked at zero momentum and they quickly assume a diffusive shape. An almost Gaussian momentum distribution can already be seen at bandwidths around 320 MHz even if this value is below the conventional validity region of the rate-equation approximation which in this case would be \( \delta \nu \gg 450 \text{ MHz} \).

Momentum distributions after deflection by a high-intensity standing wave are shown in figure 3. The Rabi frequency and the pulse duration in this case are \( \Omega_0 = 1 \times 10^{12} \text{ s}^{-1} \) and \( T = 7 \text{ ns} \), which are typical values for pulsed dye lasers. The first profile shows the momentum distribution in the case of zero relaxation. This is the typical doubly peaked structure of the Bessel function distribution for coherent deflection [2–4]. Because of the high field intensity the splitting between the side maxima is quite large. For the chosen parameters the maximum transverse momentum is about \( 3500\hbar \) which for sodium
Figure 2. Deflection of atoms by a standing wave of low intensity for various values of the laser bandwidth. The interaction time and the Rabi frequency are in this case $T = 20$ ns and $\Omega_0 = 700 \times 10^6$ s$^{-1}$, respectively. The probabilities for observing atoms in momentum states $\hat{n}\hat{h}\hat{k}$ are represented by the bars. The crosses correspond to the momentum distribution at zero relaxation.

corresponds to a velocity of approximately $100$ m s$^{-1}$. The next two profiles in figure 3 correspond to the narrowband and the broadband operation of a typical pulsed dye laser. For the narrowband operation the linewidth of the laser is chosen to be $\delta \nu = 0.15$ GHz and for the broadband operation a linewidth of 3 GHz is used. Since the Rabi period in both cases is much shorter than the coherence time of the field, some coherent effects are expected to be seen. For the case of narrowband operation the coherence time is long enough so that only a few phase jumps will occur on average during the interaction time.
Deflection of atoms by a pulsed standing wave of high intensity for various values of the laser bandwidth. The interaction time and the Rabi frequency are in this case $T = 7$ ns and $\Omega_0 = 1 \times 10^{12}$ s$^{-1}$, respectively. The probabilities for observing atoms in momentum states $\mathbf{n} \hbar \mathbf{k}$ are represented by the bars.

The effect of these phase jumps can be seen in the calculated momentum distribution as a transfer of atoms mostly to the low momentum states. If the coherence time is further decreased, the number of atoms in the two side maxima will be significantly reduced. In the case of broadband operation the momentum distribution already resembles a Gaussian distribution with long tails.

4. Conclusions

Deflection of two-level atoms by a pulsed standing wave was studied by applying the density matrix formalism in the Raman–Nath regime [4]. The relaxation due to the
phase fluctuations of the laser field was described with a single decay parameter. The final momentum distributions were solved for a square-shaped pulse profile by numerical integration of the Fourier expansion coefficients of the propagator for the Wigner function. Although a square-shaped pulse profile with a duration of a few nanoseconds is very hard to realize experimentally it was adopted in this work in order to simplify the calculations.

The calculated results for the case of a low-intensity standing wave show that the effects of the phase fluctuations will become significant as soon as the coherence time of the laser field is reduced to the order of the interaction time. For still shorter relaxation times the coherence of the deflection will be quickly lost and the final momentum distribution will take a diffusive shape. The transition from the diffractive to the diffusive deflection will occur well below the rate-equation limit which assumes no coherence between the processes induced by the two travelling wave components of the standing wave. The calculated results indicate that even for a short interaction time a relatively narrowband laser should be used in order to achieve good fringe visibility in the deflection experiments.

Deflection of atoms by a standing wave of high field intensity was studied for relaxation rates corresponding to the bandwidths of typical narrowband and broadband dye lasers. In the single-mode operation of a pulsed laser the bandwidth can be reduced close to the Fourier limit, which for a 7 ns laser pulse is \( \sim 130 \text{ MHz} \). For pulsed multimode dye lasers the bandwidth is typically of the order of a few GHz. In both cases the calculations suggest that the effects caused by the relaxation of the coherence should be clearly observed as a transfer of atoms in momentum space to the region around zero momentum rather than to high momentum states. For the typical bandwidth values of multimode dye lasers the deflection appears to be mainly diffusive but for the narrow bandwidth the two side maxima of coherent deflection should be observable. This indicates that a pulsed standing wave could be useful for deflecting atoms when using single-mode dye lasers. Experiments on the deflection of a sodium beam using broad and intermediate bandwidth standing-wave pulses have been reported in recent years [17, 18].

The spread in the momentum distributions calculated for realistic experimental parameters is of the order of \( 10^4 \hbar k \). For light atoms, such as sodium, the transverse displacement of the atoms during the interaction time might thus have some effect on their final momentum distribution, and the Raman–Nath assumption might not be valid. This effect was, however, assumed to be negligible, which is justified at least at lower intensities and for heavier atoms. At high intensities the use of a single-mode pulsed laser could be interesting for an experimental study of deflection beyond the Raman–Nath regime. Moreover, since the deflection is due to stimulated processes, we believe that a pulsed standing wave could also be useful for the manipulation of molecular beams.

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References

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