Abstract—The cores of electrical machines are generally punched and laminated to reduce the eddy current losses. These manufacturing processes such as punching and cutting deform the electrical sheets and deteriorate its magnetic properties. Burrs are formed due to plastic deformation of electrical sheets. Burr formed due to punching on the edges of laminated sheets impairs the insulation of adjacent sheet and make random galvanic contacts during the pressing of stacked sheets. The effect of circulating current occurs if the burrs occur on the opposite edges of the stacks of laminated sheets and incase of bolted or welded sheets, induced current return through it. This induced current causes the additional losses in electrical machine. The existence of surface current on the boundary between two insulated regions causes discontinuity of tangential component of magnetic field. Hence, based on this principle, the boundary layer model was developed to study the additional losses due to galvanic contacts formed by burred edges. The boundary layer model was then coupled with 2-D finite element vector potential formulation and compared with fine mesh layer model. Fine mesh layer model consist of finely space discretized 950028 second order triangular elements. The losses were computed from two models and were obtained similar at 50 Hz. The developed boundary layer model can be further used in electrical machines to study additional losses due galvanic contacts at the edges of stator cores.

Keywords — air gaps, eddy current, finite element method permeability, surface current

I INTRODUCTION

Electrical steel sheets are the indispensable constituent in the construction of cores of electromagnetic devices. Sheets are rolled to their given thickness, and laminated to minimize the eddy current loss. Later, they are cut or punched into desired shape for electromagnetic device. Sheets are laminated or coated before they are cut or punched to ease the punching process and also to prevent the damage of cutting tools and sheet itself [1]. Punching and cutting induces internal mechanical stresses which deforms the sheets and deteriorates their magnetic properties. The behavior of the magnetic properties and iron loss under such stress is studied in [2] where the hysteresis loss was observed due to change in permeability. It was observed in [3] that an annealing process reduces iron losses by 50 % and produces a factor 3 change in permeability of test samples of laminated sheets. However, the cores of the electrical machines and transformers are not perfectly insulated from each other. The laminated sheets of electrical machines are subjected to many foreign particles during assembly and make galvanic conact between inter-laminar sheets and makes thin conducting layer. However, manufacturing process such as punching also introduces burrs at the edges of electrical sheets and makes conducting layer and causes additional losses. The effect of punching has been widely researched in the scientific community. According to Schmidt [4], when cutting by punching, stress region can be from 0.35 mm up to 10 mm [5] from the cut edge and the deformed area can extend for about 0.3 mm due to plastic deformation [6]. It is studied that burr size of commercial material is less than 0.02 mm high in 0.28 mm thick sheet [7]. However, engineering society has agreed upon the average affected cut edge, having a width equal to or larger than the thickness of the lamination [8] and the ISO 13715 standard defines the edge of a work piece as burred if it has an overhang greater than zero [9].

II EFFECTS OF BURRS ON LAMINATED SHEETS

A burr formed during punching of sheets has a strong impact on interlayer short circuits as well as on the cut edge properties. Burr formation occurs due to shearing during the separation of the metal by two blades. The series of the events occur when the moving blade gets in contact to the sheet and rolls over until reaching the fracture shear stress of the sheet [10], [11]. As the load continues to increase it initiates a crack which produces the rapid breakthrough involving a ductile fracture and formation of a burr as shown in Fig. 1 [12]. However, there are many de-burring techniques such as using electrochemical machining, abrasive flow machining or high pressure water jet but no single de-burring operation can accomplish 'burr free' conditions without having side effects [13]. Burrs formed at the edges of laminated sheet impair the insulation of adjacent sheet and make random galvanic contacts during pressing of stacked sheets. The effect of circulating eddy current occurs if burrs occur on opposite edges of the laminations and in case of bolted or welded sheets, induced current returns through these paths. These additional paths increase the loss and it is important to model such phenomena in order to identify the parameters which can minimize these effects. The effect of punching on magnetic properties is studied in [14] and [15] where the increment of hysteresis loss is up to 20-40% compared to guillotine cut and in [16] magneto-mechanical coupled FEM was proposed to model the effect. There are also few studies done regarding the modeling of inter-laminar short circuit losses using artificial burr contacts in [7], [17], [18], [19] and [20] where effect on
permeability due to punching is assumed constant and randomness of burr contacts are not completely addressed. In [7] experimental studies were done to measure the losses due to burr contacts. They drilled the laminated sheets to have the controllable artificial contacts. The contacts were varied by inserting conducting pins. They measured the loss on temperature rise principle. Temperatures were measured by using microprocessor controlled thermistor bridge. They concluded the increase of the loss due to burr contacts was up to 5% of total loss. There are also analytical studies done in [21], [22], [23], [24], [25] and [26] to model thin conducting layers using finite element method. However, the conducting layers formed by burrs within the stacks are uncertain, since they are formed by a stochastic process which depends on a large number of parameters, such as the age of punching die, stacking pressure, short circuit’s geometry, thickness of the insulating layer and the number of sheets [27], [28].

1: Roll over
2: Sheared zone
3: Ductile fracture
4: Burr

Fig. 1. Burr formation.

The conducting layer formed by the burred edges can be modeled with finite element method with a very fine mesh layer and usually adaptive mesh is used but the fine mesh layer may consist of degenerated element or very high number of elements. The degenerated elements may lead to the system of ill conditioned matrix and hence the alternative method of modeling the thin conducting layer is required.

### III Mathematical Modeling and Methods

#### A. Thin Boundary Layer Formulation

Burr formed at the edges of electrical sheets deteriorates the insulation and make galvanic contacts. The surface current on the contact edges of a laminated sheet causes the discontinuity of the tangential component of the magnetic field [29]. It can be written as

\[ \mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \int_0^1 \mathbf{J} \, dx. \]  

(1)

It is assumed that the laminated sheets are parallel to the xy plane and the current density \( \mathbf{J} \) is perpendicular to the plane and assumed constant in thin conducting layer. Under quasi-static approximation, the current density of a slab shown in Fig. 2 is given by \( \mathbf{J} = \mathbf{E}. \) The current density is integrated along the conducting layer and surface current in terms of vector potential \( \mathbf{A} = A_z \mathbf{k} \) is given by (1) where \( \nu_1, \nu_2 \) are the reluctivities of iron and air respectively.

![Thin conductive layer](image)

**Fig. 2.** Thin slab.

\[ \mathbf{n} \times (\mathbf{v}_1 \nabla \times \mathbf{A} - \mathbf{v}_2 \nabla \times \mathbf{A}) = -\sigma h \frac{\partial \mathbf{A}}{\partial t}. \]  

(2)

The magnetic field can be expressed in terms of magnetic density using the material equation as

\[ \mathbf{n} \times (\nu_1 \nabla \times \mathbf{A} - \nu_2 \nabla \times \mathbf{A}) = -\sigma h \frac{\partial \mathbf{A}}{\partial t}. \]  

(3)

The magnetic vector potential \( \mathbf{A} \) in 2D is in \( z \) direction and its gradient is written as in (7). Normal component can be decomposed in the two dimensional plane by writing as \( \mathbf{n} = n_i \mathbf{i} + n_j \mathbf{j}. \) Cross product of the vectors in (3) can be written as

\[
\begin{bmatrix}
  i & j & k \\
n_i & n_j & 0 \\
-\nu_1 \frac{\partial A_z}{\partial y} & 0 & -\nu_2 \frac{\partial A_z}{\partial x}
\end{bmatrix} = -\sigma h \frac{\partial A_z}{\partial t} \mathbf{k}.
\]

(4)

The surface current in terms of magnetic vector potential can be written as

\[ \nu_1 \nabla A_z \cdot \mathbf{n} - \nu_2 \nabla A_z \cdot \mathbf{n} = \sigma h \frac{\partial A_z}{\partial t}. \]  

(5)

The magnetic flux density is the curl of magnetic vector potential and in two dimensional study, it can be expressed as

\[ \mathbf{B} = \begin{bmatrix}
  \frac{\partial A_z}{\partial y} \\
  -\frac{\partial A_z}{\partial x}
\end{bmatrix}. \]  

(6)

The gradient of \( A_z \) can be expressed as

\[ \nabla A_z = \begin{bmatrix}
  \frac{\partial A_z}{\partial x} \\
  \frac{\partial A_z}{\partial y}
\end{bmatrix}. \]  

(7)

The magnetic flux density can be expressed in terms of gradient of magnetic vector potential with the introduction of matrix term. It can be written as
\[ \mathbf{B} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{\nabla} A_z. \]  

(8)

The material equation \( \mathbf{B} = \mu \mathbf{H} \) is used. Equation (8) is substituted in material equation. The introduced matrix is inversed and \( \mathbf{v} \mathbf{\nabla} A_z \mathbf{n} \) is expressed in terms of \( \mathbf{H} \) as in (9).

The expression \( \mathbf{v} \mathbf{\nabla} A_z \mathbf{n} \) can be graphically represented as the tangential component of magnetic field in Fig. 3.

\[ \mathbf{v} \mathbf{\nabla} A_z = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{H} \]  

(9)

The expression \( \mathbf{v} \mathbf{\nabla} A_z \mathbf{n} \) can be graphically represented as the tangential component of magnetic field in Fig. 3.

Fig. 3. Graphical representation of \( \mathbf{v} \mathbf{\nabla} A_z \mathbf{n} \) as tangential component of \( \mathbf{H} \).

\[ \mathbf{1}_{2} = \mathbf{0}. \]  

(10)

Equation (10) was space discretized by replacing weight function \( \mathbf{w} \) with shape functions of active nodes. The coupling of boundary layer model to 2D finite element method results to system of equations

\[ \mathbf{S} \mathbf{A} + \mathbf{T} \dot{\mathbf{A}} = \mathbf{0} \]  

(11)

where,

\[ \mathbf{S}_{ij} = \int_{\Omega} \mathbf{v}_i \mathbf{\nabla} N_j \cdot \mathbf{\nabla} N_j \, d\Omega, \]

\[ \dot{\mathbf{A}} = \frac{\mathbf{\varepsilon}_a}{\delta t}, \]

\[ \mathbf{T}_{ij} = \int_{\Gamma} \sigma h N_i N_j \, ds, \]

\[ i, j = 1, \ldots, 3. \]

The system of equations is solved as \( \mathbf{v} = \mathbf{v}_2 \) (constant for air) and \( \mathbf{v} = \mathbf{v}_1 \) (linear insulated iron region). \( \mathbf{S} \) is the stiffness matrix and \( \mathbf{T} \) is the damping matrix which accounts for time dependent terms. The coupling of the boundary layer model in existing system of equations results in an additional term in time dependent matrices. It is important to know that the additional term in \( \mathbf{T}_{ij} \) is the line integration along the material boundaries and shape functions \( N_i \) and \( N_j \) corresponds to only nodes that belong to the material boundaries.

IV RESULTS

The derived mathematical boundary layer model is given by (1). It is compared with fine mesh model in electrical UI sheets in finite element software COMSOL. In fine mesh layer model, thin conducting region is finely space discretized. It consisted of 950028 quadratic triangular elements as shown in Fig. 5.

Fig. 4. Insulated iron and air region.

B. Coupling of Boundary Layer Model into 2D Model

Maxwell equations in terms of magnetic vector potential is solved in two non conducting regions using Green’s theorem and weighted residual method. The weight function vanishes along the Dirichlet boundaries \( \Gamma_{\text{dir}} \) and \( \Gamma_{\text{air}} \). The presence of surface current along the boundaries \( \Gamma_{\text{ai}} \) and \( \Gamma_{\text{al}} \) causes the discontinuity of tangential magnetic field. Hence, the coupling of the boundary layer model into 2D finite element of two insulated iron and air region as shown in Fig. 4 is given by

\[ R = \int_{\Omega_{\text{ir}}} \mathbf{v}_i \mathbf{\nabla} A_z \cdot \mathbf{\nabla} w d\Omega + \int_{\Omega_{\text{air}}} \mathbf{v}_i \mathbf{\nabla} A_z \cdot \mathbf{\nabla} w d\Omega \]

\[ -\int_{\Gamma_{\text{air}}} \mathbf{w} \sigma h \frac{\partial A_z}{\delta t} = 0. \]  

(10)

Equation (10) was space discretized by replacing weight function \( \mathbf{w} \) with shape functions of active nodes. The coupling
The two models are compared with same mesh in the frequency domain, changing frequency from 50 Hz to 150 Hz and parametrizing thickness of conducting layer from 0.05 mm to 0.2 mm. The air gap flux density of two different model, obtained from COMSOL was compared. The difference in the air gap flux density between the U and I sheet in different frequencies and conducting width can be seen in Fig. 6 and Fig. 7.

The galvanic contacts along the edges of UI sheets can be modeled by assigning constant conductivity of iron at the edges. The losses due to galvanic contacts were computed from both the model. The loss obtained from two models behave very closely at 50 Hz and 100 Hz.

The losses increased as frequency increased and this behaviour can be seen in Fig. 8. However, at high frequency and at conducting width of 0.15 mm, the losses start to decrease, probably because of shielding effect. In fine layer model, at high frequency the flux cannot penetrate near the skin depth and hence the losses start to decrease. However, boundary layer model is less affected by skin depth. It can be seen from Fig. 8 that the two models behave closely in loss computation.

In conclusion, the boundary layer model can compute the loss similar to fine mesh layer model at 50 Hz and at burr width less than 0.2 mm. However, boundary layer model predicts more losses than fine mesh model at 150 Hz frequency and 0.2 mm burr width. The maximum difference between the losses computed from these two model was 19%. The boundary layer model provides the mesh free solution at the thin conducting region. This model can be used to study the additional losses due to interlaminar galvanic contacts in 37 kW induction machine considering the random conductivity. This application will be presented in the future paper.

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**REFERENCES**


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