Backlash compensation in electric vehicle powertrain
Model-based backlash estimation and active compensation in electric vehicle powertrain were studied. The powertrain was modelled as a two-mass system with a flexible shaft. Physical model of the backlash was combined with the two-mass model.

Two discrete-time switched mode backlash estimators from literature were constructed in order to determine the size and position of the backlash in a powertrain. Additionally, a modified discrete-time backlash estimator was designed for practical use. The modification avoids datatype overflowing.

Grey-box modelling techniques were applied in order to fit the constructed powertrain model to a real powertrain of Fiat Doblo -electric vehicle. The backlash gap size estimation was carried out successfully. The backlash position angle estimation turned out to be highly dependent on the powertrain model accuracy.

Furthermore, two active backlash compensation methods were designed, tested and analyzed off-line. The first compensation method was based on a custom control law. This turned out to be a pure PD-controller. The second method was based on a linear quadratic regulator (LQR). Finally it was proven, that the LQR solution leads to exactly the same motion control law as the constructed custom control law.

The controlled system leads to significantly softer landing compared to uncontrolled backlash traverse. However, a trade-off between soft landing and the response time of the system must be done.

**Keywords:** backlash, Kalman-filter, two-mass model, parameter identification, powertrain, electric vehicle


Välkyn koko estimoitiin Fiat Doblo -ajoneuvosta onnistuneesti. Välyskulman estimointi osoittautui suurilta osin riippuvaiseksi voimalinjan mallin tarkkuudesta.


Kompensointi systeemi johti merkittävästi hallitumpaan ja pehmeämpään välken ylitykseen. Hallittu välkyn ylitys johtaa kumminkin kasvaneeseen systeemin vasteaikaan.

**Asiasanat:** välys, Kalman-suodin, kaksimassamalli, parametrien identifiointi, voimalinja, sähköauto

**Kieli:** Englanti
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Espoo, April 18, 2016

Joonas Sainio
## Abbreviations

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<th>Description</th>
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<tr>
<td>ABS</td>
<td>Anti-lock braking system</td>
</tr>
<tr>
<td>A/D</td>
<td>Analog to Digital</td>
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<tr>
<td>CAN</td>
<td>Controller Area Network</td>
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<tr>
<td>DARE</td>
<td>Discrete Algebraic Riccati Equation</td>
</tr>
<tr>
<td>DLQE</td>
<td>Discrete Linear Quadratic Equation</td>
</tr>
<tr>
<td>GND</td>
<td>Ground potential</td>
</tr>
<tr>
<td>HESS80</td>
<td>Inverter made for heavy hybrid vehicles by ABB Oy</td>
</tr>
<tr>
<td>$H_{\infty}$</td>
<td>H-infinity</td>
</tr>
<tr>
<td>LQ</td>
<td>Linear Quadratic</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear Quadratic Regulator</td>
</tr>
<tr>
<td>min</td>
<td>minimum</td>
</tr>
<tr>
<td>max</td>
<td>maximum</td>
</tr>
<tr>
<td>MMSE</td>
<td>Minimum Mean Square Error</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional, Integral, Derivative</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional, Derivative</td>
</tr>
<tr>
<td>PEM</td>
<td>Prediction Error Method</td>
</tr>
<tr>
<td>PRBS</td>
<td>Pseudo random binary signal</td>
</tr>
<tr>
<td>QFT</td>
<td>Quantitative feedback theory</td>
</tr>
<tr>
<td>VCC</td>
<td>Positive supply voltage</td>
</tr>
<tr>
<td>VCU</td>
<td>Vehicle control unit</td>
</tr>
<tr>
<td>ZOH</td>
<td>Zero-order hold</td>
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Symbols

\[ \alpha \] Half of the backlash gap size \([\text{rad}]\)

\[ \alpha_{\text{road}} \] Road grade angle \([\text{rad}]\)

\[ \alpha_{\text{m}} \] Motor angular acceleration \([\text{rad/s}^2]\)

\[ \Phi \] ZOH-equivalent system matrix

\[ \Gamma \] ZOH-equivalent control matrix

\[ \theta_b \] Inner shaft angle of the twisting shaft \([\text{rad}]\)

\[ \theta_b \] Backlash position angle \([\text{rad}]\)

\[ \hat{\theta}_b \] Backlash position angle estimate \([\text{rad}]\)

\[ \dot{\theta}_b \] Backlash angular speed \([\text{rad/s}]\)

\[ \theta_d \] Total shaft displacement \([\text{rad}]\)

\[ \hat{\theta}_d \] Total shaft displacement estimate \([\text{rad}]\)

\[ \dot{\theta}_d \] Total shaft displacement speed \([\text{rad/s}]\)

\[ \theta_l \] Load position \([\text{rad}]\)

\[ \dot{\theta}_l \] Angular speed of the load inertia \([\text{rad/s}]\)

\[ \ddot{\theta}_l \] Angular acceleration of the load inertia \([\text{rad/s}^2]\)

\[ \theta_{\text{m}}, \theta_1 \] Motor position \([\text{rad}]\)

\[ \dot{\theta}_m \] Angular speed of the motor inertia \([\text{rad/s}]\)

\[ \ddot{\theta}_m \] Angular acceleration of the motor inertia \([\text{rad/s}^2]\)

\[ \theta_{\text{mref}} \] Reference motor position \([\text{rad}]\)

\[ \theta_{1,\theta_3,\theta_{\text{load}}} \] Load position \([\text{rad}]\)

\[ \theta_s \] Shaft twist \([\text{rad}]\)

\[ \dot{\theta}_s \] Shaft twist speed \([\text{rad/s}]\)

\[ \theta_t \] Transmission position \([\text{rad}]\)

\[ \theta_w \] Wheel position \([\text{rad}]\)

\[ \omega_e \] Engine angular speed \([\text{rad/s}]\)

\[ \omega_{\text{m}} \] Motor angular speed \([\text{rad/s}]\)

\[ \omega_{\text{mref}} \] Reference motor speed \([\text{rad/s}]\)

\[ \dot{\omega}_m \] Motor angular speed estimate \([\text{rad/s}]\)

\[ \omega_w \] Wheel speed \([\text{rad/s}]\)

\[ \dot{\omega}_l, \dot{\omega}_{\text{load}} \] Load angular speed \([\text{rad/s}]\)
\( \dot{\omega}_l \) Load angular speed estimate \([rad/s]\)

\( \rho \) Air density \([kg/m^3]\)

\( \tau \) Time delay \([s]\)

\( \theta_{0+} \) Positive offset parameter \([rad]\)

\( \theta_{0-} \) Negative offset parameter \([rad]\)

\( a \) Longitudinal acceleration \([m/s^2]\)

\( a_{vehicle} \) Vehicle acceleration \([m/s^2]\)

\( A, A_{aug}, A_{co}, A_{ble} \) System matrix

\( A_f \) Vehicle frontal area \([m^2]\)

\( b_e \) Engine friction coefficient \([Nms/rad]\)

\( b_l \) Load friction coefficient \([Nms/rad]\)

\( b_m \) Motor friction coefficient \([Nms/rad]\)

\( b_v \) Equivalent vehicle friction coefficient \([Nms/rad]\)

\( B, B_{aug} \) Control matrix

\( c, c_1, c_2 \) Shaft damping \([Nms/rad]\)

\( cr1 \) Non-speed dependent rolling resistance coefficient

\( cr2 \) Speed dependent rolling resistance coefficient

\( c_w \) Wheel damping \([Nms/rad]\)

\( C, C_{aug}, C_{pos}, C_{neg}, C_e \) Measurement matrix

\( C_D \) Drag coefficient of a vehicle

\( d \) Diameter \([m]\)

\( D \) Feed-through matrix

\( f_r \) Rolling resistance coefficient

\( F_{air} \) Air resistance force \([N]\)

\( F_r, F_{roll} \) Rolling resistance force \([N]\)

\( F_{climb} \) Climbing resistance force \([N]\)

\( F_{inertia} \) Inertial resistance force \([N]\)

\( g \) Gravitational acceleration \([m/s^2]\)

\( i_f \) Final gear ratio

\( i_g, i \) Gear ratio

\( i_{g1} \) Gear ratio in first gear

\( i_t \) Transmission gear ratio

\( J_d \) Driveshaft inertia \([kgm^2]\)

\( J_e \) Engine inertia \([kgm^2]\)

\( J_f, J_{fg} \) Final gear inertia \([kgm^2]\)

\( J_{g1}, J_{prim} \) Gear box primary side inertia \([kgm^2]\)

\( J_{g2}, J_{sec} \) Gear box secondary side inertia \([kgm^2]\)

\( J_h \) Hub inertia \([kgm^2]\)

\( J_m, J_{motor} \) Motor inertia \([kgm^2]\)

\( J_l \) Load inertia \([kgm^2]\)

\( J_t \) Transmission inertia \([kgm^2]\)
\( J_t \)  
Thread inertia \([kgm^2]\)  
\( J_{v, \text{vehicle}} \)  
Vehicle equivalent inertia \([kgm^2]\)  
\( J_w, J_{\text{wheel}} \)  
Wheel inertia \([kgm^2]\)  
\( k, k_1, k_2, k_{\text{shaft}} \)  
Shaft stiffness \([Nm/rad]\)  
\( k_1, k_2 \)  
PD-controller tuning parameters \(^1\)  
\( K, K_{\text{aug}}, K_{\text{pos}}, K_{\text{Neg}}, K_{\text{Wait}} \)  
Kalman gain  
\( K_{\infty} \)  
Steady-state optimal feedback gain  
\( l \)  
Length \([m]\)  
\( m \)  
Mass \([kg]\)  
\( M_{\text{Obsv}} \)  
Observability matrix  
\( P_k \)  
Covariance of state estimate  
\( Q \)  
State weighting matrix  
\( r \)  
Radius \([m]\)  
\( r_{\text{eff}} \)  
Effective tire radius \([m]\)  
\( r_{\text{stat}} \)  
Static tire radius \([m]\)  
\( r_{w, \text{wheel}} \)  
Wheel radius \([m]\)  
\( R \)  
Control input weighting matrix  
\( S_k \)  
Innovation covariance  
\( S_{\infty} \)  
Steady-state solution of DARE  
\( T_{\text{const}} \)  
Time constant \([s]\)  
\( T_{\text{eng}} \)  
Engine torque \([Nm]\)  
\( T_{\text{load}, T_l} \)  
Load torque \([Nm]\)  
\( T_m \)  
Motor torque \([Nm]\)  
\( \dot{T}_m \)  
Derivative of motor torque \([Nm/s]\)  
\( T_{\text{req}} \)  
Motor torque request \([Nm]\)  
\( T_{\text{roll}} \)  
Rolling resistance torque \([Nm]\)  
\( T_s, T_{\text{shaft}} \)  
Shaft torque \([Nm]\)  
\( T_s \)  
Sample time \([s]\)  
\( u \)  
Control signal \([Nm]\)  
\( v, V \)  
Longitudinal vehicle speed \([m/s]\)  
\( \psi_k \)  
Measurement residual  
\( V_W \)  
Wind speed \([m/s]\)  
\( \dot{x}_k \)  
State estimate  
\( x_{\text{ref}} \)  
Reference state  
\( \dot{z}_k \)  
Measurement prediction

\(^1\) Not shaft stiffness in this context!
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Chapter 1

Introduction

The existence of the backlash in a vehicle powertrain is a well-known problem in the field of powertrain control. The primary problem is that the backlash excites oscillations in the powertrain. The ignorance of the backlash complicates significantly the already difficult powertrain control tasks such as oscillation damping. Secondary problem is that the mechanical parts wear out due to impulse caused by the uncontrolled backlash traversal. This impact can generate also audible noise. One obvious solution against the backlash is to produce more precise mechanical parts such that backlash would not exist. However, this is expensive and mechanical parts such as cogwheels in a gearbox wear out during their use which causes backlash in long term anyway.

The revolutionary increased computation power in microprocessors has enabled the implementation of more advanced control techniques in the powertrain control applications. The state-of-the-art techniques of control engineering could be thus utilized also when compensating the backlash in a powertrain. Furthermore in electric vehicle powertrains the electric drive provides much faster and precise torque response than conventional combustion engine. These two facts certainly give more flexibility to the compensation of the backlash in the powertrain when the backlash can be driven closed in a fast but controlled manner.

The backlash makes the powertrain control highly nonlinear. However, due to the structure of the backlash dynamics the overall system can be seen as a piecewise linear system. For this reason this thesis focuses on the compensation of the backlash by means of linear model-based control and estimation. Model-based systems give a possibility to predict the dynamics of the system beforehand. This information can then be utilized by the controller. Model-based estimation techniques allow to construct state estimators that can extract data from the internal states of the system which
are not otherwise measurable. This kind of state is for example the backlash angle that gives information whether the backlash gap is closed or not. In this thesis three discrete-time state estimators are designed. The first one estimates the backlash gap size during normal driving conditions. The second estimator is designed to track the backlash position angle. The third estimator is a modified version of the second estimator. It is designed to get rid of absolute position measurements due to limited numerical ranges and accuracy confronted in embedded systems. However, the last two estimators share the same dynamics. The modified version is only a better choice for real implementation. Due to piecewise linear system characteristics all the estimators were additionally designed to be switched type estimators. That is, their dynamics change when the powertrain operates in contact- or backlash region.

In this thesis the backlash compensation is carried out by two compensation methods. The first solution is a custom control law that can be interpreted as a proportional-derivative (PD) controller from the field of classical control. The second solution is a model-based Linear Quadratic Regulator (LQR) from the field of optimal control theory. The LQR -technique makes the tuning of the controller more intuitive compared to the custom control law where the designer adjusts non-intuitive controller gains in order to adjust the performance of the control system. Finally the thesis shows how the two presented controllers lead to the same control configuration in the backlash control - only the tuning procedure will be different.

Before testing the constructed estimators and controllers the models had to be parametrized according to the target powertrain. This thesis explains how the parametrization of the models was carried out with a real electric vehicle. Based on measurement data the unknown powertrain parameters were estimated by utilizing grey-box modelling methods provided by System Identification Toolbox in Matlab -software.

Finally, the estimators were tested by taking measurements from a real electric vehicle and then running the estimators against the measured data off-line. The on-line tests could not be carried out because the inverter unit did not support two position measurements at the same time as required by the constructed estimators. The off-line results seem to be promising and thus the algorithms should be working fine in on-line use. The on-line tests were forwarded to the future. The ideas of the switched mode techniques in this thesis are largely based on research published in [1].
1.1 Objectives of the thesis

The objectives of this thesis were set by ABB Drives -unit. The objectives are listed below:

- Electric drive should compensate the backlash even though there would be multiple inertias in the powertrain.
- Backlash compensation could be carried out when springback factors exist.
- Tuning of the system could be done manually / automatically.

In this thesis the aim is to construct a control system that tries to fulfill the requirements. In order to utilize the modern control system theory the model-based approach was chosen to tackle the problem. The tuning of the estimators and controllers would be mostly done by parametrizing the models according to the target powertrain.

1.2 Structure of the thesis

Chapter 2 introduces the problem of having backlash in the powertrain, more carefully. Some backlash compensation techniques are reviewed from the literature. In order to continue towards model-based techniques, the chapter introduces different dynamic models that could be used for powertrain modelling. Two well-known backlash model candidates are also introduced. In Chapter 3, the chosen powertrain model and backlash model are combined and built in Simulink environment for simulation purposes. The system behaviour is verified against the literature. Chapter 4 introduces the way to parametrize the constructed powertrain model based on grey-box identification techniques. The constructed powertrain model is parametrized based on the measurements of a real electric vehicle powertrain. In Chapter 5, three different backlash estimators based on Kalman filtering theory are constructed. The first estimator estimates the backlash gap size and the second estimator keeps track of the backlash position angle. The third estimator is a modified version of the previously mentioned backlash position angle estimator. It also shares the same dynamics but it is more suitable for real world implementation. In Chapter 6, two control algorithms for backlash compensation are designed. First technique is based on basic PD-control
and second on the optimal pole-placement technique called linear quadratic regulator (LQR). Chapter 7 tests the designed estimators based on real vehicle measurement data and analyses the results. Chapter 8 gives a conclusion for the work and discusses the improvements based on the results. Chapter 9 suggests future work related to the on-line implementation of the work.
Chapter 2

Background

2.1 Shunt and shuffle phenomenon

Vehicle powertrains are lightly damped oscillating systems. The purpose of a powertrain is to deliver the torque from the motor to the tractive wheels in order to produce longitudinal tyre forces that accelerate the vehicle forward. Rapid torque transients act like torque impulses to the powertrain. They excite the oscillations in the powertrain which affects directly to the longitudinal acceleration of the vehicle. In automotive field the torque transient and the following oscillation in the powertrain is called a shunt and shuffle phenomenon. This phenomenon is simulated in Figure 2.1.

![Figure 2.1: Shunt and shuffle phenomenon](image)

As seen in the Figure 2.1, the shunt describes the torque transient after the backlash traversal and the shuffle means the following oscillation in the
tractive driveshaft torque of the vehicle. The powertrain oscillations can be felt clearly in the longitudinal movement of the vehicle. Figure 2.2 shows measured longitudinal acceleration data of a real electric vehicle. During the measurement the vehicle was subjected to a similar torque step as was shown in Figure 2.1.

![Measured longitudinal acceleration of the vehicle after torque step](image)

Figure 2.2: Measured longitudinal acceleration of a test vehicle

As seen in the Figure 2.2, the longitudinal jerk is obvious. One must still notice that measuring the longitudinal acceleration of the vehicle suffers from the varying pitch angle of the vehicle due to flexible chassis. Also longitudinal weight transfer of the vehicle increases the pitching effect. More accurate longitudinal acceleration curve could be obtained with the help of sensor fusion techniques. However, the longitudinal jerk profile can still be recognized clearly from the acceleration curve. One should also notice that a part of the oscillation is damped by the tyres. Therefore the oscillation amplitude is suppressed before the driver feels the longitudinal jerk of the vehicle as reported in [2].

In order to avoid the oscillating behaviour, the possible rapid torque transients should be compensated away. Researchers in automotive field have designed controllers that compensate these powertrain oscillations. As a result driving performance and comfortability are improved as reported in [3–5].

In real world applications, the powertrain control becomes even harder due to the backlash found in gearboxes and joints of shafts. As noted in [1], the engine attachment can be considered to contribute to the total backlash gap size due to flexible rubber mountings. However, usually the electric motors are attached rigidly to the vehicle body and this removes the problem.
When controlling the powertrain the backlash introduces hard nonlinearity to the torque control loop [6]. If this nonlinearity is not taken into account, the powertrain control performance suffers. However, in some applications, the backlash is also wanted feature. The need for the backlash in gears is justified because of lubrication. In this case the backlash size is so small that it can be neglected in powertrain control. In real world, the backlash is usually much greater than needed for lubrication. This fact origins from manufacturing tolerances. Thus, by making more precise mechanical parts, there would not exist significant backlash. But, this would lead to more expensive production costs. Secondly, all mechanical parts wear out and probably backlash is introduced in long term anyway. More flexible way to compensate backlash is to design a control system that takes care of the backlash effects. This kind of control system can be adapted to mechanical wear also.

The problems with backlash appear when the driving torque changes its sign\(^1\). At that moment the backlash gap opens and, for a while, the motor is unconnected with the load. This cuts the torque delivery from the motor to the wheels and thus the motor loses a significant load. The motor inertia accelerates rapidly which results in a rotational speed difference of the motor compared to the load. The motor inertia hits hard to the load side when the contact is achieved after traversing the backlash gap. This results in a big shunt and the powertrain starts to oscillate harder than a system without backlash would do. This is shown in the Figure 2.3.

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\(^1\)Backlash gap does not open immediately when the sign of torque changes. This is due to shaft twist as seen later in this thesis.
As shown in the Figure 2.3, the backlash in the system results in much greater shunt and torque oscillation. The simulation result reveals also that, although backlash would be compensated, the system would still oscillate in rapid torque transients. For this reason, a separate controller for drivetrain oscillation damping purpose is needed. However, oscillation damping is out of the scope of this thesis and the focus is kept in the backlash compensation. As seen later in this thesis it is also possible that the backlash gap opens several times due to powertrain oscillations. This results in a series of mechanical hits which may even break the powertrain. This certainly wears the mechanical parts which is definitely not desirable in long term. Naturally this leads to increasing service costs. For the mentioned reasons the backlash compensation is highly recommended feature in electric vehicle powertrains. The same applies to other powertrain applications that are controlled by electric drives, such as windmill and crane applications.
CHAPTER 2. BACKGROUND

2.2 Review of backlash compensation methods

In order to reduce the shunt and shuffle phenomenon in a powertrain, the traversal of the backlash has to be controlled gently. The trade-off is that the response of the system due to gentle backlash traversal will be slower as discussed in [1, 2].

In automotive powertrain research, the backlash is reported to be very well known nonlinearity that should be definitely taken into account in powertrain control. Some methods to compensate the backlash are suggested but mostly based on pure simulations. The compensation methods can be categorized into three categories: Linear control, nonlinear passive control and nonlinear active control. This chapter reviews these methods found in literature. The review is largely based on [7].

2.2.1 Linear controllers

The simplest and mostly used method is to treat the powertrain as a linear system and apply linear control theory in order to compensate the backlash. In this approach the controller does not know anything about the hard nonlinearity in the torque control loop of the drivetrain. The role of the backlash is considered as a torque disturbance acting on the system. In this category, the controllers are usually basic PID -controllers [8] or state feedback controllers with full-state feedback or observer-based feedback. The basic idea is that the linear controllers are tuned robust enough such that they would not excite oscillations. Templin and Egardt [2] report that the state-of-the-art in production vehicles is to tune a simple linear controller such that in backlash traversal the backlash is not closed too rapidly. In order to achieve robust linear design the QFT- and $H_{\infty}$-design theories can be utilized. However as a trade-off to robustness, the system response will be slow. Brandenburg and Shaefer [9] report that the backlash can be considered as a torque disturbance which is estimated by a torque observer. This torque disturbance is then compensated by a compensator loop inside a PID-control loop. In this approach, the PID-controller sees a "backlash-free" system.

2.2.2 Nonlinear passive controllers

In nonlinear passive methods, the powertrain is controlled as a nonlinear system. When the backlash gap opens the controller becomes cautious. This is usually achieved by switching controller techniques where two separate con-
controllers are designed. The first controller is for the powertrain that operates in contact. The second controller is designed for the powertrain where the backlash gap is open. The latter controller is tuned such that it does not excite oscillations. In [10] two state feedback controllers are switched according to the operation mode of the system.

### 2.2.3 Nonlinear active controllers

Nonlinear active controllers are the most sophisticated controllers that provide the best response of the system. The idea is that, when the system enters the backlash region, the controller drives the system back to the contact region as fast as possible. However, this is done gently such that almost no shunt and shuffle occurs. These type of controllers are divided basically into two groups: Inverting controllers and switching controllers. Inverting controllers include an inverse model of the nonlinearity of the system and the controller utilizes this information in the control actions. However, the reported results of these controllers are not aimed for the feedback structured backlash such as in automotive powertrains. Switching controllers are just like in the group of nonlinear passive controllers but in this group they do not only get cautious, they also drive the system’s state out of the backlash back to the contact mode as fast as possible. The most promising results seem to follow optimal control approaches in this group. Lagerberg [1] gives value for results of Tao’s research group. This group has reported on optimal control strategy in [11]. This research has been extended by [12] and finally tested in a real-life application in [13].

In this thesis, the nonlinear active controllers are considered the most interesting category in order to reach the best performance of the backlash control. More detailed backlash control review can be found in [14] and [7].
2.3 Powertrain models in the literature

A good understanding of the powertrain dynamics and its individual parts is needed in order to control shunt and shuffle phenomenon in an automotive powertrain. For this reason, this section introduces several powertrain models in decreasing number of modelled inertias. First a ninth order powertrain model [5] is introduced. This model captures all the necessary inertias, frictions and flexibilities in a basic front-wheel driven vehicle. Then five analogous models are introduced in decreasing level of complexity. The last model will be the most lumped model that still captures the main phenomena what comes to dominating oscillation modes in the powertrain. All these models can be used for the powertrain simulation purposes according to the level of needed complexity. Note that this section is valid for both electric vehicle and conventional vehicle powertrains. Thus, it does not matter whether the actuator of the drivetrain is a combustion engine or an electric motor.

The working principle of a basic powertrain in a front-wheel driven electric vehicle is shown in Figure 2.4. The represented configuration describes also the real electric vehicle powertrain that is used later in this thesis for the testing purposes.

As seen in Figure 2.4, the electric motor is the actuator for the drivetrain. The combination of the electric motor and the flywheel can be considered to represent one lumped inertia $J_m$. This motor inertia is connected with a flexible shaft to the primary side of the gear box. The shaft inertia is neglected because it is so small compared to the other inertias in the drivetrain. The
primary side of the gear box is considered to have some inertia $J_{g1}$. The secondary side of the gear box is considered to have some inertia $J_{g2}$. The gear ratio $i_g$ defines the torque and speed relation between the primary side and the secondary side inertias. The gear box is connected to the final gear which is considered to have some inertia $J_{fg}$. Naturally the final gear has also some gear ratio $i_f$. From the final gear the torque is delivered through the flexible driveshafts to the hub of the wheels. The wheels are considered to have some inertia $J_w$. From the wheel the torque is delivered to the flexible tyres which are considered to have their own inertia $J_t$. Finally the contact patch between the tyre and the road is considered to allow slippage of the tyre. In order to model this powertrain chain dynamics precisely, a complex powertrain model is needed. This complex powertrain model is represented in the next section. Also more simplified powertrain models are reviewed.

2.3.1 Complex powertrain model

The powertrain in the Figure 2.4 can be represented equivalently as a rotating multiple mass-spring-damper model as shown in Figure 2.5. Note that often the mass of the vehicle, that affects to the longitudinal dynamics through Newton’s II law, is transformed into an equivalent rotational inertia. This inertia is shown in the end of the powertrain chain in the Figure 2.5. Usually, this inertia is called an “equivalent rotational inertia of a vehicle”. The vehicle inertia is multiple times larger than any other inertia in the powertrain as seen clearly in the Figure 2.5.

Figure 2.5: Powertrain as a rotating multi-mass-spring-damper model
Borodani and D’Ambrosio [5] report, that the model shown in Figure 2.5, can be described by ninth-order dynamics. Basically all the main inertias are included in this model and flexibilities of the shafts are modelled with individual springs or spring-damper combinations. The flexibilities are indicated by the blue symbols embedded into the shafts in the Figure 2.5. The authors report also, that the ninth-order model is too heavy model to be implemented in the controller design. For this reason, the powertrain models are often reduced to three-mass or two-mass models which provide a good behaviour over the frequency band of interest in the controller design.

From now on, all the inertias are drawn with identical size in the forthcoming powertrain models. The reader should still keep in mind that the vehicle’s equivalent rotational inertia is always much larger than the other inertias in the powertrain. Additionally, all the drawn shafts are considered flexible although there are no explicit blue spring-damper symbols in the shafts.

2.3.2 Three-mass powertrain model

Three-mass powertrain models are considered to have three lumped inertias in the powertrain. The first inertia represents the combination of the motor and the flywheel. The second lumped inertia represents the combination of the transmission and the final gear inertias. The third inertia represents the combined inertia of the wheel and the vehicle’s equivalent rotational inertia. A basic three-mass model is shown in Figure 2.6.

![Figure 2.6: Powertrain as rotating three-mass model](image_url)
As seen in the Figure 2.6, the shaft from the motor up to the final gear is considered to be flexible. The parameters $k_1$ and $c_1$ define the shaft stiffness and damping properties, respectively. Although there are several shafts along this path, the flexibilities are lumped together and represented by one equivalent spring-damper combination. Same kind of procedure is applied for the path between the final gear and the equivalent vehicle inertia. That is, all the flexibilities found between the driveshaft and the contact patch of the tyre are lumped into one equivalent flexible shaft. The lumped flexibility and damping properties are then defined by the parameters $k_2$ and $c_2$, respectively. Note that this model does lump all the inertias after the final gear to the wheel. No wheel-slip is considered here. The precise dynamics of this system are reported by Kiencke and Nielsen [15].

2.3.3 Two-mass powertrain model

By far the most used model for the powertrain simulation in controller design is the two-mass model that consists of two inertias connected with a flexible shaft. The inertia of the transmission is considered to be so small compared to the motor inertia and the load inertia that it can be neglected or lumped to one of these two inertias. The two-mass model is shown in the Figure 2.7.

![Powertrain as rotating two-mass model](image)

Figure 2.7: Powertrain as rotating two-mass model

As shown in Figure 2.7, the parameters $k$ and $c$ define the powertrain flexibility properties alone. As reported by Kiencke and Nielsen [15], this model is called a "Basic Driveshaft Model" and it can capture the main oscillation phenomena in a drivetrain. This model has been widely used in model-based powertrain control with success.

In this thesis one of the targets is to keep the model as simple as possible, still maintaining the model accuracy in a reasonable level. For this reason,
this model was chosen to be the base for the powertrain modelling.

2.4 Backlash models in the literature

Backlash is characterized to be mechanical slack between adjacent movable parts \(^2\) [14]. During the backlash traversal, the torque is not transferred from the otherside of the backlash gap to the otherside. As shown in the Figure 2.8, the backlash gap size is often represented with a size of \(2\alpha\). As a definition, there is a negative contact when the driving side and the driven side have a constant position angle error such that the driving side position angle is lagging the driven side by \(-\alpha\). In this position, only negative torque can be applied through the drivetrain. Controversely, when the driving side position angle leads the driven side by \(\alpha\), there is a positive contact that allows only positive torque to be applied through the path. If the driving side position angle is between \(-\alpha\) and \(\alpha\), there is no contact at all. Thus, no torque is transmitted to the otherside. All the backlash models reported in this thesis are based on these assumptions.

\[\begin{align*}
\theta_1 & \text{ (driving side)} \\
\theta_2 & \text{ (driven side)} \\
2\alpha & \text{ (backlash gap)}
\end{align*}\]

Figure 2.8: Backlash gap model

In order to model the backlash in the powertrain, there are few options available in the literature. Most of the models are basically different type of dead-zone models with different characteristics and corrections. Nordin, \(^2\)As in a series of gears
Galic and Gutman [16] have published also a more precise backlash model that takes into account the twisting of the shaft. This model is reported to be more accurate when compared to the traditional dead-zone models. Next, these models are introduced with their pros and cons.

### 2.4.1 Dead-zone backlash model

The very basic dead-zone model is the most widespread backlash model. This model is given below as a conditional equation.

\[
T = \begin{cases} 
  k(\theta_d - \alpha), & \theta_d > \alpha \\
  0, & |\theta_d| < \alpha \\
  k(\theta_d + \alpha), & \theta_d < -\alpha
\end{cases} \tag{2.1}
\]

The given model is basically a spring that respects the given rules for the backlash modelling. The \( \theta_d \) represents the total shaft displacement that is simply the measured position error between the angles \( \theta_3 \) and \( \theta_1 \), shown in the Figure 2.8. When the backlash contribution is subtracted from the total shaft displacement, one obtains the shaft twist \( \theta_s \). Thus, the shaft torque is directly proportional to the shaft twist \( \theta_s \). Having said that, the model simply considers only the spring dynamics without the damping term. If the shaft damping is included to this model, the model will result in a non-physical torque generation term. Karlsson [17] suggests if-else conditions for fixing this problem. For this reason, care must be taken, when basic dead-zone models are used. This can lead to erroneous simulation results.

### 2.4.2 Physical backlash model

As a preferred solution for backlash modelling, a so called physical model of the backlash by Nordin, Galic and Gutman [16] is introduced. The authors report that this model is valid also for flexible shafts with damping. This model represents backlash dynamics that includes a state variable that makes it possible to model backlash angle and the shaft twist at the same time. The physical backlash model is given below.

\[
T_s = k\theta_s + c\dot{\theta}_s = k(\theta_d - \theta_b) + c(\dot{\theta}_d - \dot{\theta}_b) \tag{2.2}
\]
\[ \dot{\theta}_b = \begin{cases} \max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = -\alpha \\ \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) & |\theta_b| < \alpha \\ \min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)) & \theta_b = \alpha \end{cases} \] (2.3)

As seen from the Equations (2.2) and (2.3), the shaft torque calculation is based on the backlash angle \( \theta_b \) and the total shaft displacement \( \theta_d \). Due to shaft damping also the derivatives of these angles are taken into account. That is, the shaft damping is modelled as a viscose damping that is directly proportional to the angular speeds. With the physical backlash model it is easy to represent the backlash phenomenon in the powertrain models by replacing the original shaft dynamics with the Equations (2.2) and (2.3). One superior thing is, that this model considers the fact that the shaft can be still twisted when the backlash gap opens. Thus the backlash position dynamics change considerably with different shaft stiffnesses. This behaviour is physically correct - hence the name of this model. The simulation results, shown later in this thesis, clarify more this phenomenon.

Next the combined powertrain and backlash models are introduced. All the forthcoming models are based on the physical backlash model discussed above. The detailed derivation of the physical backlash model can be found in [16].

### 2.5 Two-mass powertrain model with backlash included

When the flexible driveshaft and the backlash are combined with a basic two-mass powertrain model, the model is considered to be constructed as shown in the Figure 2.9.

As seen clearly in the Figure 2.9, the backlash is lumped to the end of the driveshaft and it represents the total backlash found in the powertrain. Even though the backlash is spread all over the powertrain in reality, it is reasonable to lump it to the end of the driveshaft. This is because, in this place the torque starts to affect to the wheel when all the backlash is traversed over in the powertrain. As reported by Lagerberg [1], some apparent backlash in conventional vehicles is introduced because of flexible mountings of an engine in engine compartment. However, with electric motors, this is no problem and can be neglected as they are often fixed rigidly to the vehicle’s body. Note that again, the load inertia does not take into account wheel slip in this two-mass model.
CHAPTER 2. BACKGROUND

Figure 2.9: Powertrain as a rotating two-mass model with backlash included

The gearbox in Figure 2.9 is not considered to have any inertia. It works only as a gear ratio and thus scales the torque and the rotational speed from the motor side to the wheel side.

2.5.1 Two-mass powertrain model with backlash and tyre slip

As an extension to the previously represented two-mass model, the Figure 2.10 represents a model that includes the wheel slip. Some authors argue that the wheel slip is a significant energy loss in a powertrain and thus it damps down the oscillation amplitudes felt by the driver. For this reason, the slip is modelled as an extra damper $c_v$ in the model.

Figure 2.10: Powertrain as a rotating two-mass model with the backlash and wheel slip included (picture from:[2])

The model shown in the Figure 2.10 differs from the previous models in
a way that now the load side inertia represents only the vehicle’s equivalent rotational inertia. This is necessary if the wheel slip is considered in the model. Because the model is a two-mass model, it means that the wheel inertia is neglected. The wheel inertia could be added to the model but then this model would be represented in the three-mass-model series. The precise dynamics of this model are reported by Templin and Egardt [2].

2.6 Summary of the powertrain models

As seen in the Figures 2.5 to 2.10, the literature offers plenty of models for the powertrain modelling. Different models take into account different things and share different level of complexity. The choice of a specific model in controller design depends on the available sensors in a specific target platform. Also the needed accuracy of the model affects to the choice. One should note, that all the models including backlash, are highly nonlinear in terms of torque delivery from the actuator to the wheels. If no backlash is present, then the powertrain can be modelled as a linear model. Almost every model treat the friction as a viscose friction and thus the friction torque is directly proportional to the speed difference of the two mechanical parts. In real life however, the friction is also a highly nonlinear effect. This thesis does not review different friction types but literature is full of information of friction modelling when needed. The problem with nonlinear friction models is that they are too complex for practical control design, where the model simplicity is one of the required key factors for successful controller implementation. In addition to friction models, the tyre dynamics are not taken into account in this thesis. For this reason, the tyre slip is ignored.
Chapter 3

Powertrain modelling in Simulink

The combined two-mass model with the physical backlash model was chosen to be implemented in this thesis. As stated earlier, the wheel slip was ignored. The choice was based on the fact that the model is able to capture the main oscillation modes of a powertrain. Furthermore, the minimum realization of the plant would need only three state variables which keeps the controller design and simulation simple. In addition to this, the physical backlash model would take the shaft twist into account and thus there would be more physically reasonable simulation results. There would be also a small amount of parameters in the parameter identification phase, which is discussed later in this thesis. This section describes how the model was implemented in Matlab Simulink environment.

3.1 Dynamic model equations

The chosen model includes two rotating inertias as shown in the Figure 2.9. The dynamic model can be derived according to the general Newton’s second law for rotating masses. The basic idea is, that this model has lumped motor and transmission inertia connected to the lumped wheel and vehicle inertia through a flexible driveshaft. The driveshaft is the most flexible component in the drivetrain as it is subjected to a large torsion especially with low gears. For this reason, this model is usually simulated with low gears because then the motor torque is amplified by a large gear ratio. The ratio can be 1:15 in passenger vehicles and even 1:60 in heavy vehicles. The dynamics of the powertrain is assumed to have stiff clutch. As proved by Kiencke and Nielsen [15], this assumption is fair and does not degrade the model accuracy.
3.2 Motor inertia dynamics

The dynamics of the motor inertia can be expressed:

\[ J_m \ddot{\theta}_m = T_m - \frac{T_s}{i_t} - b_m \dot{\theta}_m \]  \hspace{1cm} (3.1)

where

- \( J_m \) is the motor inertia [kgm²]
- \( \ddot{\theta}_m \) is the angular acceleration of the motor inertia [rad/s²]
- \( T_m \) is the motor torque [Nm]
- \( T_s \) is the driveshaft torque [Nm]
- \( i_t \) is the total gear ratio (including final gear)
- \( b_m \) is the motor viscose friction coefficient [Nm/(rad/s)]
- \( \dot{\theta}_m \) is the angular speed of the motor inertia [rad/s]

The motor torque \( T_m \) is modelled as a first-order process with time constant \( T_{const} \) and time delay \( \tau \). Thus, in time domain the dynamics are described as:

\[ \dot{T}_m = \frac{T_{req}(t - \tau) - T_m(t)}{T_{const}} \]  \hspace{1cm} (3.2)

3.3 Load inertia dynamics

The dynamics of the load inertia can be expressed:

\[ J_l \ddot{\theta}_l = T_s - b_l \dot{\theta}_l - T_l \]  \hspace{1cm} (3.3)

where

- \( J_l \) is the load inertia [kgm²]
- \( \ddot{\theta}_l \) is the angular acceleration of the load inertia [rad/s²]
- \( T_s \) is the driveshaft torque [Nm]
- \( T_l \) is the load torque [Nm]
- \( b_l \) is the load viscose friction coefficient [Nm/(rad/s)]
- \( \dot{\theta}_l \) is the angular speed of the load inertia [rad/s]

The driveshaft torque is modelled as a flexible shaft with damping. In this model the driveshaft torque is linearly proportional to the twist of the shaft and to the twist speed of the shaft as follows:
By including the effect of the backlash into the model, the model can be expressed in the following way:

\[ T_s = k\theta_s + c\dot{\theta}_s = k(\theta_d - \theta_b) + c(\dot{\theta}_d - \dot{\theta}_b) \]  

(3.5)

where

\[ \dot{\theta}_b = \begin{cases} 
  \max(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)), & \text{if } \theta_b = -\alpha \\
  \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b), & \text{if } \theta_b = |\alpha| \\
  \min(0, \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b)), & \text{if } \theta_b = \alpha
\]  

(3.6)

The used parameters in the previous models are correspondingly:

- \( T_s \) is the driveshift torque [Nm]
- \( k \) is the shaft stiffness [Nm/rad]
- \( c \) is the internal shaft damping [Nm/(rad/s)]
- \( \alpha \) is the size of half of the backlash gap [rad]
- \( \theta_d \) is the total shaft displacement [rad]
- \( \theta_b \) is the backlash angle [rad]

The backlash model is thus a state-space model with one state. The state can be interpreted as a limited integrator with the time derivative \( \dot{\theta}_d + \frac{k}{c}(\theta_d - \theta_b) \) and with a limit \( \alpha \) as described by Nordin, Galic and Gutman [16]. Also the derivation of the obtained model with the aid of a phase plane plot is reported there. The basic idea of the driveshift dynamics in Equation (3.5) is, that the positive torque can only be transmitted when the backlash is in the positive contact. Similarly, the negative torque can only be transmitted when the backlash is in the negative contact. When the backlash gap is open, no torque is transmitted from the motor to the wheels.

By looking at the Equation (3.6) more carefully, one can see how the backlash acts in the negative or positive contact. The backlash angle stays constant or then the backlash gap starts to open. It can be seen also that the shaft can remain twisted when the backlash gap opens. The moment when the backlash gap opens depends on the sum of the speed difference between the motor and the load inertias (\( \dot{\theta}_d \)) and the shaft twist (\( \theta_s = \theta_d - \theta_b \)) weighted with the factor of \( k/c \). For example, in the positive contact (\( \theta_b = \alpha \), the
backlash gap starts to open when negative speed difference becomes bigger than the shaft twist weighted with the term \( k/c \). Thus, if there is no shaft twist at all, the backlash gap opens immediately when there exists a negative speed difference between the motor and the load inertias.

### 3.4 Modelling load torque

The load torque represents the total resistive torque that is generated by driving resistances acting on the longitudinal dynamics of the vehicle. Lagerberg [1] treats the load torque as an unknown constant or slowly varying parameter. However, in order to simulate the vehicle powertrain in Simulink environment, the load torque was modelled according to physics. Next section treats the modelling of the load torque.

As shown earlier, there are two inputs in the described two-mass system models. The first input is the motor torque. The second input is the load torque. The load torque results from driving resistances of the driven vehicle. These driving resistances are shown in the Figure 3.1.

![Figure 3.1: Longitudinal driving resistances](image)

Figure 3.1: Longitudinal driving resistances
The represented driving resistances in the Figure 3.1 are calculated as follows:

- Air resistance $F_{air} = \frac{1}{2} \rho A_f C_D (V + V_w)^2$
- Rolling resistance $F_{roll} = m \cdot g \cdot (c r_1 + c r_2 \cdot v^2)$
- Climbing resistance $F_{climb} = m \cdot g \cdot \sin(\alpha_{road})$
- Inertia resistance $F_{inertia} = m \cdot a$

The inertia resistance is not drawn in the Figure 3.1, but it originates from the Newton’s II law. Simply put, the vehicle with mass $m$ needs a force $F$ in order to give an acceleration $a$ for the vehicle. Thus, when accelerating the vehicle, it is subjected to this force that must be overcome. The resistances are the forces working against the propulsive force of the vehicle. The total load force becomes:

$$F_{load} = F_{air} + F_{roll} + F_{climb} + F_{inertia} \quad (3.7)$$

The load force affects to the longitudinal dynamics of the vehicle but in this thesis the focus is on rotational dynamics of the powertrain. For this reason, the load torque instead of the load force is needed. The load torque is obtained from the total load force as shown below:

$$T_{load} = F_{load} \cdot r_{eff} \quad (3.8)$$

Equation (3.8) uses the effective tyre radius for transforming the load force to the corresponding load torque affecting in the hub of the wheel. The effective tyre radius lies somewhere between the static tyre radius and the uncompressed tyre radius which are shown in the Figure 3.2.

The effective tyre radius can be calculated as a function of a static and uncompressed tyre radius as expressed by Equation (3.9).

$$r_{eff} = \sin \left\{ \cos^{-1} \left( \frac{r_{static}}{r_w} \right) \right\} \cdot r_w \quad (3.9)$$

The derivation of this static formula is reported by Rajamani [18].
CHAPTER 3. POWERTRAIN MODELLING IN SIMULINK

Figure 3.2: Effective tire radius lies somewhere between $r_w$ and $r_{stat}$

3.5 Two-mass system in state-space form

By ignoring the nonlinear backlash element in the powertrain, the given system dynamics can be expressed in linear state-space form

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where $u$ is the requested motor torque by the driver. The state vector consists of six states as follows:

$$x = [\theta_m \ \omega_m \ \theta_l \ \omega_l \ T_l \ T_m]^T$$

The measurement vector consists of the motor and the load position measurements as follows:

$$y = [\theta_m \ \theta_l]^T$$

According to system dynamics the state-space has A and B matrices as:
\[ A = \begin{bmatrix}
\frac{-k}{J_m k} & -\frac{k + b_m}{J_m^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{k}{J_m} & \frac{c}{J_m^2} & 0 & \frac{1}{J_m} \\
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{k}{J_l} & \frac{c}{J_l} & -\frac{k}{J_l} & -\frac{c + b_l}{J_l} & -\frac{1}{J_l} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau}
\end{bmatrix} \] (3.14)

and

\[ B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}^T \] (3.15)

The motor position and load position are measurable states and thus C matrix takes the form

\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \] (3.16)

There is no feed through term in this system and thus the D matrix gets the form

\[ D = \begin{bmatrix} 0 \\
0 \end{bmatrix} \] (3.17)

The constructed state-space model is used as the basis for the state estimators in this thesis.
3.6 Verification of the constructed model

A complete system based on represented motor and load inertia dynamics was constructed in Simulink environment in order to verify the system behaviour. The verification was carried out in comparison to similar simulation results found in the literature. The block diagram structure of the constructed system is shown below in Figure 3.3.

As seen from the Figure 3.3, there are two inputs affecting to the system. The driver affects directly only to the motor inertia which affects to the rest of the powertrain if the backlash is in contact mode. In the backlash mode, the powertrain is decoupled and there is no connection through the driveshaft (green box). The state of the backlash is affected by both sides of the backlash. That is, the motor inertia dynamics affect to it and so does load dynamics. Hence the backlash is often said to have sandwiched structure.

In order to verify the rationality of the constructed model, some initial tests were carried out. During the tests open-loop control was used and thus no control system was applied to the system yet. This means, that the actuator was directly controlled by the driver through the acceleration pedal. In the first test, the system was initialized such that backlash gap was closed in positive contact and a torque step was given to the system in rest. The first gear was chosen in order to excite oscillations better in the powertrain. The test results can be seen in the Figure 3.4.
As seen in the Figure 3.4, the system is excited by stepwise torque at the time instant $t = 2s$. As expected, the motor speed starts to increase and because the backlash gap is closed the power is transferred to the wheel through the driveshaft. The wheel starts to accelerate. From the lower right corner can be seen that the backlash gap keeps closed in the positive contact all the time. As seen in the upper right corner, the motor speed has quite much oscillation before it reaches the steady acceleration. This naturally origins from the flexibility of the powertrain that acts as a rotational spring with damping.

In the Figure 3.5 more output data from the same simulation is shown. The driveshaft torque is represented in the upper left corner. The torque is very oscillative due to the driveshaft flexibility. As the driveshaft torque oscillates, it affects also to the longitudinal acceleration of the vehicle as expected. The shuffling is obvious. Also the load torque seems to act as expected. The stepwise load torque origins from rolling resistance that starts to affect immediately when the tyre starts to roll. As the speed increases the rolling resistance increases. Furthermore air resistance has a tiny effect to the total load torque. However, the rolling resistance is the most dominating resistance in the low speeds as in this simulation.

After having convincing simulation results from the constructed model, the backlash dynamics remain to be verified. In order to get the backlash gap open, the powertrain was intialized by constant deceleration torque in order to get the backlash position initially to the negative contact. After

\[ \theta_b = +\alpha = +0.041\text{rad} \]
CHAPTER 3. POWERTRAIN MODELLING IN SIMULINK

Figure 3.5: Torque step applied to system in contact mode

giving a positive torque command to the drivetrain the backlash should be traversed to the positive contact mode. The simulation results are shown in the Figure 3.6.

Figure 3.6: Backlash traversal test

As seen in the Figure 3.6, the upper left graph shows that the powertrain is initially decelerating due to the negative motor torque. The deceleration is clearly seen in the motor speed plot. In the backlash angle graph, the contact during the deceleration phase is in the negative contact side. This enables the negative torque transfer to the load which can be seen in the deceleration of the wheel speed. At the moment, when the positive motor
torque is introduced, the backlash gap opens as expected (first red ellipse in the Figure 3.6). When the backlash is traversed, the motor and load inertias are decoupled and this can be seen from the zero-level driveshaft torque plotted in the upper left corner of the Figure 3.7 (first red ellipse).

Figure 3.7: Backlash traversal test

All the red ellipses in the Figures 3.6 and 3.7 denote the causality between the backlash angle, driveshaft torque and the vehicle longitudinal acceleration plots. Simply put, when the backlash angle is between the negative and positive contacts, the driveshaft torque and correspondingly the longitudinal acceleration curve become zero because obviously no torque is delivered.

The load torque simulation works also well. The behaviour of the load torque is reasonable because during the deceleration phase the load torque decreases due to decreasing load forces. After the contact is achieved again, the vehicle accelerates and the load torque starts to increase as expected.

The system behaviour was compared to the simulation results found in the literature. Templin and Egardt [2] report similar results of a two-mass model with backlash. Additionally Lagerberg and Egardt [19] report similar system behaviour. However, one must notice that results published by Lagerberg [19] are obtained for closed-loop powertrain configuration. That is, an acceleration controller is applied to the vehicle powertrain. The driver introduces only a reference acceleration for the vehicle through the gas pedal. The PID controller regulates the vehicle acceleration then to this reference.

In this thesis, the system was driven in an open-loop configuration. This means that the driver applies a torque reference to the powertrain and this reference is not modified with any control laws (no feedback applied) before the command is applied to the powertrain. Based on the intuition and similar
results found in the literature, the system was considered to be a reasonable tool for simulation of a powertrain that includes backlash. Verification is considered thus successful.

3.6.1 Effect of load torque

The load torque is considered to be the second input to the system. This input cannot be controlled as it origins from the environment. Especially because the load torque is not in our hands it can open the backlash gap suddenly. In order to research the effect of sudden load torque changes, few simulations were carried out with the constructed Simulink model. First, a sudden load torque peak was tested. This test was considered to simulate a case where the tire of the vehicle decelerates suddenly for a while. This can happen for example when driving over a pothole or when the mechanical brakes are activated for a short time. To be precise, driving over a pothole is considered first as a disappeared load due to the wheel slip. However, during the wheel spin, it probably gets stuck to the pothole. This phase is seen as a sudden increased load torque. The simulation results are shown in the Figure 3.8.

As seen in the Figure 3.8, the first backlash opening is caused by the driver who lifts his leg off the acceleration pedal. The second opening is caused by the sudden change in the load torque which results from mechanical braking. It takes over 2.5 seconds, before the backlash position is recovered back to the positive contact mode.
Another scenario where the load torque has a big role is shown in the Figure 3.9.

Here the vehicle drives first with a constant torque request to uphill. Due to the increasing grade of the uphill the load torque increases from 80 Nm up to 200 Nm for a while. After the driver reaches the top of the hill, a downhill starts which gives acceleration to the vehicle due to gravity. The driver releases the acceleration pedal such that only a small positive torque is requested. The black dashed line in the Figure 3.9 shows how the backlash would recover back to positive contact almost after releasing the gas pedal if no downhill would exist. Due to the downhill and respective acceleration the time to recover back to the positive contact mode takes much more time with the given torque request. This is only a theoretical example how the load torque variation due to hilly driving course affects to the backlash position. In this thesis the vehicle is assumed to drive with 1. or 2. gear and low speed because this provides the biggest shunt and shuffle phenomenon due to flexible driveshaft that is twisted most under these conditions. Similar thoughts are reported by Kiencke and Nielsen [15] and Lagerberg [1]. With low speeds the most significant load torque variations originate from sudden road grade variation or obstacles that the vehicle meets on the road. The mechanical brakes are considered to contribute in the load torque in this thesis. When the road is flat, the rolling resistance is the main resistive force in low speeds. By assuming that the mechanical brakes are not used, the backlash opening is purely based on the torque request of the driver.
3.6.2 Backlash position vs total shaft displacement

In order to address the importance of the backlash position estimation, the simulation for stiff and loose driveshafts were carried out. The powertrain model with tracked angles are shown again in the Figure 3.10, for convenience.

![Powertrain model used in simulation](image)

The total shaft displacement $\theta_d$ is a quantity that can be measured directly from a basic vehicle that is equipped with a motor encoder and a wheel position sensor. If the shaft is considered to be extremely stiff, the position difference between the motor and the load inertias in the wheel co-ordinates reveal the backlash position directly. However, as discussed earlier, in this thesis the driveshaft is usually very flexible component in a drivetrain that certainly twists especially with low gear driving. When the shaft twist is taken into account the total shaft displacement does not equal the backlash position anymore. The total shaft displacement with a straight shaft equals the backlash position when the negative or the positive contact is achieved. After this the flexible shaft starts to twist which increases the apparent total shaft displacement. However, the backlash position angle stays in its maximum value limited by the backlash gap size. If the backlash position angle is driven suddenly to the other side of the backlash gap it is truly possible that the shaft is still twisted when the backlash gap opens. During the backlash gap traverse the shaft may straighten fully or partially depending on the shaft characteristics and the given torque impulses in the drivetrain. The first case where the shaft manages to straighten fully, is simulated in the Figure 3.11.
As seen in the Figure 3.11 the backlash position is in the positive contact initially. Additionally, the difference angle between the total shaft displacement (green) and the backlash position angle (blue) indicates that the shaft is twisted approximately 0.06705 radians. At time instant $t = 18.9 \text{ s}$ a negative torque request is applied to the system. The shaft starts to straighten clearly before the backlash gap starts to open. When the backlash starts to open the shaft is still a bit twisted. During the beginning of the backlash traverse the shaft straightens fully and the total shaft displacement equals the backlash position angle. When the negative contact is achieved the shaft starts to twist. This is seen from the total shaft displacement curve at time instant $t = 19 \text{ s}$. The same happens when the backlash is driven from the negative contact to the positive contact from the time instant $t = 19.1 \text{ s}$ forwards. When the backlash gap achieves the positive contact the shaft twist oscillates but still the backlash gap keeps closed.

The second case where the shaft does not straighten fully during the backlash traversal is simulated in the Figure 3.12.
As seen in the Figure 3.12, the loose shaft changes the results dramatically. In this case the backlash position $\theta_b$ never equals the total shaft displacement $\theta_d$. The shaft twists clearly more now and when the backlash is driven from the positive contact to the negative contact the shaft starts to straighten but the negative contact side is achieved before the shaft has straightened fully. The same happens with the traversal from the negative backlash contact to the positive backlash contact. The simulations show clearly, how to interpret the backlash gap behaviour and the contribution of the shaft twist. As a summary, the backlash position $\theta_b$ does not equal the $\theta_d$ although at first it seems so. In the next chapter the parameter identification is carried out in order to fit the constructed powertrain model to a real vehicle powertrain.
Chapter 4

Parameter identification

A model-based approach in estimation and control systems is based on accurate dynamic models that describe the system sufficiently well. In order to get the constructed two-mass model mimic the dynamics of a real vehicle powertrain, the unknown parameters have to be identified from the real system. From literature it can be seen that usually the models are parametrized with some physically reasonable parameters in order to simulate the general behaviour of a system. In these cases the model does not mimic any real system. However, if the model outputs will be compared to a real system measurements then the model needs to be parametrized with precise parameter values such that the model fits to the measured data. In some cases these parameter values are obtained directly from the manufacturer who has designed the system. Another way is to disassemble the powertrain and measure the needed parameter values manually. These values are then fed to the model. However, the model is always an assumption of the underlying system and thus the measured physical parameter values may not work directly with the constructed two-mass model. Furthermore, if the model has to be parametrized for different customers, there is no sense to approach the parametrization of the model by disassembling customers’ powertrains. Considering the fact that the relationships between the unknown parameters were available, it was natural to approach the parameter identification through grey-box modelling. This chapter deals with construction of the grey-box model for the powertrain. Then the needed parameter values are identified by utilizing motor position, wheel position and motor torque measurements of a real powertrain.
4.1 Grey-box modelling

Grey-box modelling is a powerful technique in parameter identification. It is based on the fact that the physical relationships between the identified parameters are known beforehand. That is, the system dynamics are well-known and they can be written explicitly such as in this thesis. The two-mass model relates the parameters to each other and thus the model can be directly used for identification. Matlab System Identification Toolbox provides a way to do parameter identification based on grey-box modelling. All needed is the system model in state-space form and logged data from the input and output signal(s) of a real powertrain. The state-space model and the measurements are converted to suitable objects required by the system identification toolbox before the identification process. The engineer can fix any parameters that are known initially such as the gear ratio. The upper and lower bounds for the identifiable parameters can also be set. This way there is a chance that the parameter values converge to reasonable values. One must understand that the identification process searches suitable parameter values such that the model fits optimally to the measurement data. Optimal fit here means that identified parameters minimize some optimization criterion used during the identification. Because the model is only a simplified assumption of the underlying physics in the system the parameter values may not be same as obtaining the parameters by laboratory measurements of the same quantities.

4.2 Construction of linear grey-box model

When constructing the grey-box model for the powertrain the decision of the model type between linear and nonlinear has to be done first. The powertrain model derived in Chapter 2 includes the backlash that is the only thing that makes the powertrain model nonlinear. Considering this fact a crucial epiphany helped to choose the linear grey-box framework for the system. That is, by ensuring that the backlash gap is closed all the time when the real powertrain measurements are taken, the model turns to be linear. Encouraging is that Tallfors [20] has ended up with same thoughts when identifying drivetrains with backlash.

In order to construct a linear grey-box model, the system identification toolbox requires that the system is provided in the state-space form. The state-space model is implemented in a separate function file which is later used in the identification process. This function returns state-space matrices that are dependent on user-defined parameters and information of the model.

The separate function file is written in a following form:
\[ [A, B, C, D] = myfunc(par_1, par_2, \ldots, par_N, Ts, aux_1, aux_2, \ldots) \]

where matrices are calculated with given parameters as shown below:

\[
\begin{align*}
A &= fcn(par_1, par_2, \ldots, par_N, Ts) \\
B &= fcn(par_1, par_2, \ldots, par_N, Ts) \\
C &= fcn(par_1, par_2, \ldots, par_N, Ts) \\
D &= fcn(par_1, par_2, \ldots, par_N, Ts)
\end{align*}
\]

However before use of the function file, the system dynamics had to be discretized. The discretization method is described in the next section.

### 4.3 Discretization of the process model

The real powertrain under research is a continuous-time process but it is sampled with constant measurement rate. In order to fit the model to the discrete measurement data, the model had to be discretized. Since the measurement device uses zero-order hold sampling the equivalent ZOH-system had to be derived for the continuous-time system model. In practice this means that equivalent discrete matrices for A and B in state-space representation had to be solved.

According to [21] the ZOH - equivalent discrete time state-space matrices can be obtained as follows:

\[
\Phi = e^{Ah} \quad (4.1)
\]

and

\[
\Gamma = \int_0^h e^{As} ds \quad (4.2)
\]

Thus the ZOH - equivalent discrete-time state-space system has the form:

\[
\begin{align*}
x(k + 1) &= \Phi x(k) + \Gamma u(k) \\
y(k) &= C x(k) + D u(k) \\
x(0) &= x_0
\end{align*}
\]
CHAPTER 4. PARAMETER IDENTIFICATION

The discretization method was implemented into the \textit{myfunc} -function such that it would output the numerically ZOH -equivalent discrete-time matrices $\Phi, \Gamma, C$ and $D$.

Next the function is handed over for the \textit{idgrey} -function provided by System Identification Toolbox. \textit{Idgrey} -function returns a discrete-time grey-box object that is used for identification later.

4.4 Constructing idgrey object

After defining the separate function file where the state-space model of the identifiable process was written, the next step is to create an \textit{idgrey} object of the system. This will be the final grey-box model of the system. In order to construct an \textit{idgrey} -object, the written \textit{myfunc} -file is handed over for a built-in function called \textit{idgrey}(). This function takes several input arguments such as:

- \textit{myfunc} -ODE-file
- initial guesses of the identifiable parameter values
- sample rate of the identifiable discrete-time system (= sample rate of measurements)
- definition whether the system is continuous / discrete

The form of this function is then:

$$\text{greySys} = \text{idgrey('ODE-file', par, systemType, aux, Ts)}$$

where

\text{ODE-file} is the handle to the earlier constructed \textit{myfunc} -function. \textit{Par} is the vector of initial guesses for the identifiable parameters. The \textit{systemType} tells whether the system is discrete or continuous. \textit{Ts} is the sample time. The function returns an \textit{greySys} -object that can be directly used with Matlab’s built-in system identification methods.

Before the identification process, real measurement data had to be gathered. Next section describes the measurement setup of a real electric vehicle used for gathering the necessary data. Also structured identification is discussed which leads to calculation of the already known parameter values that can be fixed during the identification process.
4.5 Measurement setup for parameter identification

The parameter identification needs real measurements of a real powertrain. These measurements were carried out in Metropolia University of Applied Sciences that has built a Fiat Doblo -electric vehicle conversion for prototype testing purposes. The vehicle is shown in the Figure 4.1.

![Figure 4.1: Fiat Doblo - electric vehicle](image)

This vehicle is equipped with HES880 -inverter unit by ABB Oy which makes the vehicle a unique test platform for ABB Oy. This section explains the measurement setup that was built around the vehicle in order to carry out parameter identification and research backlash in a powertrain.

4.5.1 Wheel position measurement

The test vehicle was initially equipped with inductive sensors utilized by the anti-lock braking system (ABS). However, getting the data from ABS -control unit would be too time consuming. Furthermore, reading directly the inductive signal needs signal processing such as amplifiers and a Smitt Trigger in order to get reasonable data from the wheel position. In order to solve the problem of wheel position measurement, the vehicle was equipped with new wheel-speed sensors. For this purpose the front driveshafts were
equipped with a toothed ring that was read by a back-biased hall -sensor. This sensor could be directly connected to the data logger.

The wheel position measurement is based on calculated number of by-passing teeth. The installed ring has 44 teeth per revolution. The operation principle of the wheel position measurement is shown in the Figure 4.2.

As seen in the Figure 4.2, the hall sensor produces a square pulse every time when a tooth by-passes the sensor. By knowing the number of teeth per revolution, the information of the wheel position is possible to be determined with the following formula:

$$\theta_{load} = N \cdot \frac{2\pi}{44} \ [\text{rad}], \quad (4.3)$$

where $N$ is the calculated number of pulses read from the toothed ring. One must notice that the worst resolution of the position measurement is achieved if only rising edges of the signal are read. By reading also the falling edge of the signal the resolution can be doubled. However, this must be taken into account in the denominator where the pulses per revolutions are given.

By utilizing the information of the pulse train frequency, the wheel speed can be calculated as:

$$\omega_{load} = \frac{\text{pulsefreq}[\text{Hz}]}{44} \cdot 2\pi \ [\text{rad/s}], \quad (4.4)$$

where

pulsefreq is the frequency of the pulses.
4.5.2 Motor position measurement

The motor was equipped with a quadrature encoder that produces 80 pulses per revolution. The quadrature encoder has two signal channels: A and B. These channels output a square wave just as the hall sensor in the wheel position measurements. However, the A and B channels have a 90° phase shift such that the rotation direction can be detected. By utilizing rising and falling edges of the encoder signal from both channels the resolution of the position measurement can be made four times more accurate. Encoder was connected directly to the data logger such that motor position could be measured along other measurements.

4.5.3 Motor torque measurement

In order to utilize grey-box identification methods provided by Matlab, also the input signal to the powertrain had to be measured. In most cases electric vehicles do not have a separate motor torque sensor. However, this measure can be estimated indirectly from motor current measurements. HES880 - inverter by ABB Oy is able to provide estimated motor torque and send it to the CAN-bus. In order to get the best possible estimate of the motor torque, all the filters and ramps from the torque signal path were removed. The filters smooth the signal but also introduce delay in the signals. The inverter was configured to send the most recent motor torque request (just before it is realized by the power semiconductor switches) in order to have most reliable estimate of the injected motor torque. This signal was sent to the CAN-bus in a 1ms interval.

4.6 Structured Identification

In order to improve the parameter identification, some known parameters were fixed to known values beforehand. These parameters were gear ratio and motor inertia. Additionally, it was possible to determine limits for the unknown parameters. This kind of identification is called structured estimation in Matlab System Identification Toolbox [22].
According to the grey-box model the parameters to be identified were:

### Table 4.1: Parameters to be identified

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Unit</th>
<th>Fixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft stiffness</td>
<td>$k$</td>
<td>Nm/rad</td>
<td>No</td>
</tr>
<tr>
<td>Shaft damping</td>
<td>$c$</td>
<td>Nm/(rad/s)</td>
<td>No</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>$i_t$</td>
<td>-</td>
<td>Yes</td>
</tr>
<tr>
<td>Load inertia</td>
<td>$J_L$</td>
<td>kgm</td>
<td>No</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>$J_m$</td>
<td>kgm</td>
<td>Yes</td>
</tr>
<tr>
<td>Motor viscose friction factor</td>
<td>$b_m$</td>
<td>Nm/(rad/s)</td>
<td>No</td>
</tr>
<tr>
<td>Load viscose friction factor</td>
<td>$b_l$</td>
<td>Nm/(rad)</td>
<td>No</td>
</tr>
</tbody>
</table>

As shown in Table 4.1, there are five unfixed parameters. The minimum and maximum values were chosen to be in reasonable range based on powertrain simulation papers found in literature. The chosen ranges are shown in Table 4.2.

### Table 4.2: Chosen range limits for parameters under estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
<th>Initial value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft stiffness</td>
<td>5000-17000</td>
<td>10556</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>Shaft damping</td>
<td>10-200</td>
<td>90</td>
<td>Nm/(rad/s)</td>
</tr>
<tr>
<td>Load inertia</td>
<td>100-400</td>
<td>203.48</td>
<td>kgm</td>
</tr>
<tr>
<td>Motor viscose friction factor</td>
<td>0.001 - 0.01</td>
<td>0.001</td>
<td>Nms/rad</td>
</tr>
<tr>
<td>Load viscose friction factor</td>
<td>0.001 - 0.01</td>
<td>0.001</td>
<td>Nms/rad</td>
</tr>
</tbody>
</table>

In order to choose reasonable parameter range for the shaft stiffness, the driveshaft was disassembled from the target vehicle. The measures of the shaft were taken and shear modulus of $G = 80$ GPa was assumed. According to Airila, Mauri and Hautala [23], the elasticity of the shaft can be then determined by

\[
  k_{\text{shaft}} = \frac{(G \cdot \pi \cdot d^4)}{(32 \cdot l)} = 10556 \frac{Nm}{\text{rad}} \tag{4.5}
\]

where $d$ is the diameter of the shaft and $l$ is the length of the shaft. The shaft stiffness is very close to the parameter values used in other simulations found in literature. For this reason, it was validated to be a good initial guess. The range for shaft stiffness was chosen to be around this value as seen in Table 4.2. The range of the shaft damping coefficient was chosen around the value that was reported by Northcote [24] for Volkswagen Jetta.
passenger vehicle. Load inertia was estimated by measuring the static mass of the vehicle and transforming this to equivalent rotational inertia. The transformations between linear and rotational masses are explained thoroughly by Fajri, Ahmadi and Ferdowsi [25]. The frictional coefficient range was determined based on used values in the literature.

### 4.6.1 Pre-calculation of motor inertia

The motor inertia was pre-calculated such that it could be used in parameter estimation as a fixed value. The measurement was based on acceleration tests. As described in Chapter 3, the motor inertia was considered to be lumped inertia up to the backlash gap. For this reason, the motor inertia was measured such that the powertrain was ”cut” between the final gear and the driveshaft\(^1\). Then a torque step with known amplitude was applied to the motor inertia which gives an acceleration to the motor based on Newton’s second law of motion. By measuring the speed of the motor the acceleration can be defined from the derivative of the measured rotational speed. The Figure 4.3 clarifies the calculation procedure.

As seen in the upper plot of the Figure 4.3, a torque step with amplitude of 10% of nominal torque is applied to the motor inertia. The motor inertia starts to accelerate with almost constant slope. Because the slope is not exactly constant, a straight line was fitted on the speed curve by least squares

\(^1\)Driveshaft has negligible inertia
method. The lower plot in the Figure 4.3 shows the fitted line (red) from which the slope was finally calculated. By knowing the applied torque and the slope of the velocity from the fitted line, the motor inertia was calculated according to the Newton’s second law of motion:

\[ J_m = \frac{T_m}{\alpha_m} \]  

(4.6)

where \( T_m \) is the motor torque and \( \alpha_m \) represents the angular acceleration of the motor inertia.

In order to obtain the most precise value for motor inertia, the same measurement was repeated with motor torque steps beginning from 10% up to 100% of nominal torque by increasing the applied torque by 10% in every measurement round. As a result, an average value of \( J_m = 0.1451 \text{ kgm}^2 \) was obtained for the motor inertia with 1. gear selected. Correspondingly for the 2. gear, the averaged motor inertia \( J_m = 0.1462 \text{ kgm}^2 \) was obtained. The exact measurement results obtained with 1. gear and 2. gear are shown in Appendix A (Table A.1 and Table A.2).

As expected, the averaged motor inertia obtained with both 1. and 2. gears are very close to each other. In order to choose one motor inertia that can be used with both gears the average values from both tests were averaged one more time. Thus the motor inertia is considered to be \( J_m = 0.1456 \text{ kgm}^2 \) in this thesis.

### 4.6.2 Pre-calculation of gear ratio

The gear ratio could be also determined beforehand. In order to get correct results, the structure of an open differential gear has to be taken into account. The differential gear in a vehicle allows the outer wheel to travel faster compared to the inner wheel when cornering. This functionality has to be taken into account such that the correct wheel speed for gear ratio measurement is used. In order to make the measurement easy, the test vehicle was lifted up with a car lift. The test vehicle is front-wheel driven and, for this reason, the open differential is located between the front driveshafts. When the car was lifted up the other front wheel was locked mechanically such that it could not start spinning. Due to functionality of the open differential, the power is transferred now to the other front wheel that can freely rotate when the vehicle is lifted up from the ground. Normally when the vehicle is moving straight forward, both front wheels are rotating with the same speed. If now the other wheel is locked and the other is rotating freely, this corresponds to straight road driving if the speed of the rotating wheel is divided by two.
With this analogy the correct wheel speed is obtained for gear ratio calculation. Given the fact that the other wheel was locked mechanically, the gear ratio can be obtained as follows:

\[ i_g = \frac{\omega_m}{\omega_w} \]  

(4.7)

where \( \omega_w \) is the measured wheel speed and \( \omega_m \) is the measured motor speed. Due to capability of driving the electric motor in speed control mode the electric drive was set to maintain a constant speed. The wheel speed was measured by a handheld Monarch PLT 200 Tachometer which is based on laser measurement.

As an example, with the first gear the motor speed was set to spin 1000 rpm and the unlocked wheel was measured to have speed of 132 rpm. Thus the gear ratio for the first gear is obtained:

\[ i_{g1} = \frac{1000}{132/2} = 15.15 \]  

(4.8)

All the gears were measured with the same procedure and the gear ratios were obtained as shown in Table 4.3.

<table>
<thead>
<tr>
<th>Selected gear</th>
<th>Gear ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.151</td>
</tr>
<tr>
<td>2</td>
<td>8.695</td>
</tr>
<tr>
<td>3</td>
<td>5.633</td>
</tr>
<tr>
<td>4</td>
<td>3.984</td>
</tr>
<tr>
<td>5</td>
<td>3.076</td>
</tr>
<tr>
<td>R</td>
<td>15.151</td>
</tr>
</tbody>
</table>
4.7 Rolling resistance tests

The load torque can be seen originating mainly from rolling resistance when the vehicle is driven with low speed on a flat surface such as in this thesis. As the load torque is one of the system inputs, it should be definitely taken into account when doing parameter identification for the powertrain. In order to measure empirically the rolling resistance of the test vehicle, a coast down test was carried out. The test was done on the same surface and the same place where the parameter identification would be done in order to approximate the load torque as accurate as possible. In the test the vehicle is driven up to a specific speed and a coast down is started by shifting the gear to neutral position. The rolling resistance coefficient can be obtained then from the deceleration of the vehicle. Although the test surface was flat for an eye, the coast down test was done to both directions of the field. By averaging the results, the inclination effect of the surface is cancelled out thus producing a more reliable result. In the coast down test the vehicle was driven up to speeds of 10km/h, 15km/h and 20km/h before the coast down was started. In order to get the precise initial speed before the coast down section was started, the vehicle was driven in speed controlled mode and the maximum speed was limited to the given top speeds. A sample of the coast down test measurements is shown in the Figure 4.4.

Figure 4.4: Coast down from speed of 20 km/h.
As seen in the Figure 4.4, the vehicle is first accelerated to the test speed which is reached after $t = 5s$. The motor controller maintains the desired speed because the vehicle is driven in speed controlled mode. The green stepwise curve indicates when the clutch is pressed down and the coast down starts. As seen clearly the coast down results in linearly decreasing speed. When the load torque is assumed to be the only torque decelerating the vehicle, its value can be obtained directly from the Newton’s second law.

$$T_{roll} = J_{vehicle} \cdot a_{vehicle} \tag{4.9}$$

where

$$J_{vehicle} = 4 \cdot J_{wheel} + m \cdot r_{wheel}^{2} \tag{4.10}$$

The rolling resistance force is obtained from Equation (4.9) as follows:

$$F_{r} = \frac{T_{roll}}{r_{wheel}} \tag{4.11}$$

Finally the rolling resistance coefficient is obtained from the well known relation:

$$f_{r} = \frac{m \cdot g}{F_{r}} \tag{4.12}$$

Table 4.4 shows the results of the obtained rolling resistance coefficients:

<table>
<thead>
<tr>
<th>Initial speed (km/h)</th>
<th>Clutch pressed coast down</th>
<th>Neutral gear coast down</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.016</td>
<td>0.0144</td>
</tr>
<tr>
<td>15</td>
<td>0.0166</td>
<td>0.0144</td>
</tr>
<tr>
<td>20</td>
<td>0.0193</td>
<td>0.0166</td>
</tr>
<tr>
<td>Avg:</td>
<td>0.0173</td>
<td>0.0151</td>
</tr>
</tbody>
</table>

In Table 4.4 the obtained rolling resistance coefficients for different speeds are averaged from multiple coast down tests. The coast down test with 10km/h was done three times in both directions and the average value of
rolling resistance coefficient from these tests is given in Table 4.4. The table shows also the results for coast down tests that were carried out by shifting the gear to neutral position instead of pressing the clutch pedal in order to decouple the motor and load inertias during the measurement. As seen from the results, this has an affect. When the coast down is done only by pressing clutch there is more friction in the drivetrain that decelerates the vehicle thus increasing the apparent rolling resistance coefficient. Thus by using the rolling resistance coefficient obtained from the tests when the neutral gear is used for coast down, the results may be more accurate. As can be seen the rolling resistance coefficients seem to be same for speeds 10km/h and 15km/h. This thesis researches the cases where the vehicle is driven with low speeds. For this reason, the rolling resistance coefficient for the test vehicle was chosen to be 0.0144. As reported by Ehsani, Gao and Emadi [26], a typical rolling resistance value for passenger vehicles is around 0.013 on asphalt surface. This naturally varies with tire pressure and tire materials. Comparing to this reported value, the test result was considered to be successful.
4.8 System excitation signal

After obtaining the initial parameter values and fixed parameter values for the powertrain model under identification, the grey-box model was ready to be identified. However in order to achieve the best possible parameter identification result, the excitation signal should contain all the frequencies distributed evenly. Furthermore the amplitude of the signal should be as large as possible. As reported by Saarakkala and Hinkkanen [27], in electric drives the torque and speed are limited quantities. For the limited quantities the largest frequency variation is provided by binary signals. That is, the signal has only two possible values. Pseudorandom binary signal (PRBS) meets the requirement of this description and thus it can excite the system in the best possible way. The richness of frequencies in PRBS is based on the fact that it approximates a discrete-time white noise and thus has a broad spectral content. The signal looks like a rectangular pulse train that is modulated in width. The name comes from the fact that the signal is characterized by a sequence length and inside this length the width of the pulses changes randomly. However, in long horizon they are periodic with a period by the length of the sequence as indicated in [28].

After all, this kind of signal could be possibly unusable for the system with backlash. As seen already in the model verification section, the backlash opens when the torque request is reduced after a step input. In the PRBS signal there would be random rising and falling edges in the excitation signal that could probably open the backlash gap. This would ruin the identification process because the identification has to be done for a system where apparently no backlash exists.

In order to ensure the system behaviour the PRBS signal was constructed with shift registers in Simulink and this signal was injected to the powertrain model constructed in Chapter 3. The backlash gap behaviour is seen in the Figure 4.5. During the first simulation the shaft was considered to be sloppy and thus the stiffness coefficient was set to $k = 1000$ (Nm/rad).
As seen in the Figure 4.5, the simulation result encourages to use PRBS signal as a system excitation signal. The backlash gap does not open although the signal is varying rapidly with the amplitude of 90 Nm. One should still notice that in order to keep the backlash gap closed, the signal should stay in positive torque region all the time. However, if the pulses in the PRBS signal are widened too much there is a risk that backlash gap opens and ruins the identification process as seen in the Figure 4.6.

As seen clearly in the Figure 4.6, the backlash gap opens at the time instant $t = 5.1s$. This originates from the too long pulse length when the
signal goes down to 10Nm. Finding a proper PRBS signal seems to be more like a trial-and-error art. The pulse length depends naturally on the shaft stiffness that defines how much the shaft twists and thus how much it has to counter twist before the backlash gap opens. Thus the best pulse length for the PRBS signal used with systems with backlash is a system specific choice. Due to the risks of injecting PRBS signal into a real vehicle powertrain, the signal was not used for real parameter identification. However, in future this might be a good solution in order to cover rich spectrum of frequencies.

Finally the same simulation was done with a stiffer shaft coefficient $k = 10056$ (Nm/rad) that was obtained earlier based on Equation (4.5). The results change dramatically as shown in the Figure 4.7.

![Figure 4.7: PRBS as system excitation signal (short pulses, stiff shaft)](image)

As seen in the Figure 4.7, the backlash gap opens easier with the stiffer driveshaft. According to the final results, the PRBS signal was considered to be too uncertain input excitation signal with systems where significant backlash exists. In order to ensure a closed backlash gap during the identification measurements, the excitation signal was chosen to be a torque step. This way the backlash would not open at all during the measurements. Next section continues the identification process where the stepwise torque input is injected to the powertrain.
4.9 Identification based on motor position and wheel position measurement

The parameter identification test was carried out based on wheel position and motor position measurements as system output variables. The identification process was performed on a flat test field and during a still weather in order to minimize load torque originating from other sources than rolling resistance. This way the load torque could be estimated based on the results obtained from the rolling resistance tests performed on the same field. The vehicle was stopped initially on the test field and a small amount of positive motor torque was applied to the powertrain in order to ensure that the backlash gap was closed initially. The torque was kept so low that it did not move the vehicle, thus working only against static friction. In order to excite oscillations in the powertrain, a torque step was applied to the system. The measurement was continued until all the oscillations were certainly attenuated. The test was carried out with several torque steps ranging from 50% to 120% of the nominal torque in first gear. The same test was done in second gear with torque steps from 50% to 140% of the nominal torque. Some test results with the first gear were considered uncertain due to tire slip. For this reason, also the torque range was narrower compared to the tests done with the second gear. After having sufficient amount of measurement data, all the measurement sessions driven with the second gear were turned into \textit{iddata} objects as required by System Identification Toolbox. The \textit{iddata} objects were then merged together in order to provide more data for the identification routines in Matlab. In total 32 measurement sessions were merged together.

The merged testdata was finally injected to the constructed grey box model i.e \textit{idgrey} -object of the system. The identification was then based on nonlinear least squares optimization method. The line search method decision was left for Matlab engine.
The identified parameters are finally given below in Table 4.5, for easy comparison.

Table 4.5: Identified parameter values vs. initial guesses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial value</th>
<th>Identified value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shaft stiffness</td>
<td>10556</td>
<td>1.103e+04</td>
</tr>
<tr>
<td>Shaft damping</td>
<td>90</td>
<td>108.9</td>
</tr>
<tr>
<td>Load inertia</td>
<td>203.48</td>
<td>154.7</td>
</tr>
<tr>
<td>Motor inertia</td>
<td>0.1462</td>
<td>0.1462</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>8.658</td>
<td>8.658</td>
</tr>
<tr>
<td>Motor viscose friction factor</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>Load viscose friction factor</td>
<td>0.001</td>
<td>0.001</td>
</tr>
</tbody>
</table>

As seen in Table 4.5, the identified parameters have reasonable values. The fixed parameters ($it, Jm$) remain the same as expected. The free parameters are identified such that certain optimization criteria is minimized. In this case the cost function is:

$$J = \sum_{k=1}^{N} e^2(k)$$  \hspace{1cm} (4.13)

where $e(k)$ is the error between the predicted output of the system and the measured output of the system. Hence the name prediction error -method for this optimization procedure. More details of the model fit and the optimization procedure are given in Figure A.1 in Appendix A (underlined with red color). Additionally, the identified state-space matrices are introduced there in the Figure A.2. In overall, the outputs are over 90% accurate compared to the real measurement data. The parameter identification of the real powertrain can be considered successful.
Chapter 5

Backlash estimation

State estimators are used in order to estimate internal states of a system that are not directly measurable. This chapter describes the construction of the estimators for backlash gap size and position estimation. The backlash in the powertrain makes the powertrain dynamics highly nonlinear. However, by considering the system in different contact modes the system turns out to be piecewise linear. By utilizing this fact, the estimators can be designed with linear dynamics that correspond to the system mode. The methods for backlash size estimation and position estimation were introduced originally by Lagerberg [1].

5.1 Kalman filter

In linear Kalman filter theory, the system is considered to be a stochastic linear system. The system is subjected to Gaussian process noise and measurement noise. The equivalent system can be mathematically modelled as

\[
\begin{align*}
    x(k+1) &= \Phi x(k) + \Gamma u(k) + v(k) \\
    y(k) &= C x(k) + D u(k) + w(k) \\
    x(0) &= x_0
\end{align*}
\]

The process noise \( v(k) \) is considered to be zero-mean Gaussian noise with covariance

\[
E[v(k)v(k)'] = Q(k)
\]

Similarly the measurement noise \( w(k) \) is considered to be zero-mean Gaussian noise with covariance


\[ E[w(k)w(k)'] = R(k) \]

The process noise and the measurement noise are considered to be mutually independent. That is, they do not correlate. Often the system matrices \( \Phi, \Gamma, C \) and \( D \) are time-invariant as in this thesis.

Given the above assumptions the Kalman filter is a minimum mean-square error (MMSE) estimator. The algorithm works in recursive form and it has Markov property. That is, the current system state summarizes probabilistically its past as discussed by Bar-Shalom, Li and Kirubarajan [29].

The Kalman filter has the following form

\[
\dot{x} = A\hat{x} + Bu + K(y - C\hat{x})
\]

Priori estimate Measurement update

The \( K \) matrix is chosen such that it minimizes the covariance of estimation error. The action of Kalman filter can be separated into two parts. First part is the state prediction. The second part is the update phase. The well-known Kalman filter algorithm is given below

**State and measurement prediction**

State prediction \( \dot{x}_{k|k-1} = \Phi x_{k-1|k-1} + Bu_k \) (5.2)

Predict state covariance \( P_{k|k-1} = \Phi P_{k-1|k-1} \Phi^T + Q_k \) (5.3)

Measurement prediction \( \hat{z}_{k|k-1} = C\hat{x}_{k|k-1} \) (5.4)

Innovation covariance \( S_k = R_k + C_k P_{k|k-1} C_k^T \) (5.5)

**State update**

Measurement residual \( v_k = z_k - \hat{z}_{k-1} \) (5.6)

Kalman gain \( K_k = P_{k|k-1} C_k^T S_k^{-1} \) (5.7)

Updated state estimate \( \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k v_k \) (5.8)

Updated state covariance \( P_{k|k} = P_{k|k-1} - K_k S_k K_k^T \) (5.9)

As seen from the equations above, the calculation of the state and innovation covariances is separated from the state prediction. The Kalman gain depends only on the state and innovation covariance and thus it is possible to
calculate the steady-state Kalman gain beforehand offline. The steady-state Kalman gain is obtained by running the covariance formulas in for-loop so long that the Kalman gain converges to some value. Another option is to use \textit{dlqe}-function in Matlab. The \textit{dlqe} – \textit{function} simply solves the discrete linear quadratic estimation problem by calculating the suitable Kalman gain based on the given system dynamics. Both ways result in the same steady-state Kalman gain.

5.2 Backlash size estimation

Early work in backlash compensation in literature has assumed that the backlash gap size is well-known by the controller. In reality this is rarely the case. For this reason, an estimator suggested by Lagerberg [1] was designed. The backlash size estimator is based on the state-space model introduced in Chapter 3.5. The idea is that the backlash size could be estimated in normal driving conditions without any special excitation signals to the system. The fundamental idea behind the backlash gap size estimation is to utilize motor position and load position measurements.

If a linear Kalman filter based on linear dynamics described by 3.14, 3.15, 3.16 and 3.17 is applied to a real vehicle powertrain that includes backlash, there will be constant offset between the estimated motor and wheel positions. This offset originates from backlash but also from initial zero offset \( \theta_0 \) in the sensors. The offset parameters for both contact modes in a powertrain can be mathematically formulated as

\[
\theta_{0+} = \theta_0 + \alpha \\
\theta_{0-} = \theta_0 - \alpha
\]

(5.10) and (5.11)

In order to construct the backlash size estimator the state-space representation introduced in Chapter 3.5 has to be augmented with these two offset parameters \( \theta_{0+} \) and \( \theta_{0-} \). More about the augmentation of dynamic models can be found in [30]. As a result, the size of the state vector increases and becomes

\[
x = \begin{bmatrix} \theta_m & \omega_m & \theta_l & \omega_l & T_l & T_m & \theta_{0+} & \theta_{0-} \end{bmatrix}^T
\]

(5.12)
The offset parameters are modelled as constant or slowly varying parameters. For this reason, the state transition matrix $A$ has to be extended by zero dynamics of these parameters. The augmented $A$-matrix becomes

$$
A_{\text{aug}} = \begin{bmatrix}
0 & \frac{1}{J_m} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{k}{J_m^2} & -\frac{k+b_m}{J_m} & \frac{k}{J_m} & \frac{c}{J_m} & 0 & \frac{1}{J_m} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{J_l} & 0 & 0 & 0 & 0 \\
\frac{k}{J_l} & \frac{c}{J_l} & -\frac{k}{J_l} & -\frac{c+b_l}{J_l} & -\frac{1}{J_l} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Similarly the $B$ matrix is augmented as follows:

$$
B_{\text{aug}} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{-1}{\tau} & 0 & 0 \end{bmatrix}^T
$$

The augmentation of the offset parameters affect also to the measurement vector as follows

$$
y = \begin{bmatrix} \theta_m \\
\theta_t - \theta_{0(+/\!-)} \end{bmatrix}
$$

The notation $\theta_{0(+/\!-)}$ in the Equation (5.15) is interpreted such that either $\theta_{0+}$ or $\theta_{0-}$ is used in this equation. In positive contact mode, $\theta_{0+}$ is used. Similarly in negative contact mode, $\theta_{0-}$ is used.

From estimated offset parameters, the backlash gap size can be solved by utilizing the information given by Equations (5.10) and (5.11) as follows:

$$
\theta_{\text{gap}} = \theta_{0+} - \theta_{0-} = \theta_0 + \alpha - \theta_0 + \alpha = 2\alpha
$$
5.2.1 Switched mode Kalman filter

The switched mode Kalman filter is used for backlash size estimation. In positive contact mode the positive offset parameter is estimated and updated by the filter. Similarly in negative contact mode the negative offset parameter is estimated and updated. When the contact mode is changed also the mode of the Kalman filter is changed. This is done by using different measurement matrices \( C_{\text{pos}} \) or \( C_{\text{neg}} \) and Kalman gain matrices \( K_{\text{pos}} \) or \( K_{\text{neg}} \) for respective contact modes. When the backlash gap is open the system mode is called a wait mode. In the wait mode the offset parameters are not updated at all. As a result the Kalman filter is of form

\[
\dot{\hat{x}} = A_{\text{aug}}\hat{x} + B_{\text{aug}}u + K_{\text{aug}}(y - C_{\text{aug}}\hat{x})
\]

where

\[
C_{\text{aug}} = \begin{cases} C_{\text{pos}}, & \text{in positive contact mode.} \\ C_{\text{neg}}, & \text{in negative contact mode.} \end{cases}
\]

\[
C_{\text{pos}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \end{bmatrix}
\]

\[
C_{\text{neg}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}
\]

\[
K_{\text{aug}} = \begin{cases} K_{\text{pos}}, & \text{in positive contact mode.} \\ K_{\text{neg}}, & \text{in negative contact mode.} \\ K_{\text{wait}}, & \text{in waiting mode.} \end{cases}
\]

\[
K_{\text{pos}} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{o1} & 0 \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{o2} & 0 \end{bmatrix}
\]

\[
K_{\text{neg}} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & 0 & k_{o1} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & 0 & k_{o2} \end{bmatrix}
\]
As mentioned before, the Kalman gain can be calculated offline beforehand. The suggested way by Lagerberg [1] is to calculate the Kalman gain for pair \((A_{aug}, C_{pos})\) which yields the Kalman gain for positive contact mode. Then by rearranging the gain terms for offset states as shown in 5.23, the Kalman gain for negative contact mode is obtained. In waiting mode the Kalman gain terms for offset parameters are set to zero such that they are not updated.

Observability of the system can be explored by calculating the rank of the system’s observability matrix and comparing it to the rank of the system’s \(A\)-matrix. As the estimator does not update the negative offset parameter in positive contact mode and vice versa the unupdated offset parameter can be dropped away in the state-space representation. The corresponding column in \(C\)-matrix is dropped away. The observability matrix of this ”reduced” system is obtained:

\[
M_{obs} = \begin{bmatrix}
C \\
CA \\
CA^2 \\
\vdots \\
CA^{n-1}
\end{bmatrix}
\]  (5.25)

The observability matrix is easily obtained with Matlab command \(obsv(A,C)\). From this matrix, the rank is under special interest. For the constructed observability matrix of the ZOH -discretized system the rank is 7. The system has 7 states and thus the system has the full rank. Based on this, the system is observable and thus all the linear algebraic state equations are solvable. This holds for the system in both contact modes. This refers to the fact that the estimator should work fine with the used measurements and selected states.
CHAPTER 5. BACKLASH ESTIMATION

5.3 Backlash position estimation

The backlash position estimation is based on the same switching mode idea than the backlash size estimation. The dynamics of the backlash shown in Equation (3.6) can be expressed in the vector notation form as follows:

$$\dot{\theta}_b = \begin{cases} \max(0, e x), & \theta_b = -\alpha \\ e x, & \theta_b < |\alpha| \\ \min(0, e x), & \theta_b = \alpha \end{cases}$$ (5.26)

where

$$e = \begin{bmatrix} \frac{k}{c} & \frac{1}{i} & \frac{-k}{c} & -1 & 0 & 0 & \frac{k}{c} \end{bmatrix},$$ (5.27)

if the state vector is chosen to be

$$x = [\theta_m \ \omega_m \ \theta_l \ \omega_l \ T_l \ T_m \ \theta_b]^T.$$ (5.28)

As seen from the coefficients of the vector $e$, the backlash dynamics are piecewise linear. This fact allows us to define two modes for the power-train: Contact mode and Backlash mode. Mathematically the modes can be determined by the following conditions:

$$\text{Mode} = \begin{cases} \text{co}, & |\theta_b| = \alpha \text{ and } \theta_b \cdot e \cdot x \geq 0 \\ \text{bl}, & |\theta_b| < \alpha \text{ or } \theta_b \cdot e \cdot x < 0 \end{cases}$$ (5.29)

With the help of the mode definitions, the backlash dynamics can be rewritten according to the modes:

$$\dot{\theta}_b = \begin{cases} 0, & \text{contact mode.} \\ e \cdot x, & \text{backlash mode.} \end{cases}$$ (5.30)

Finally this kind of mode structure leads to a system that switches between the two modes. This system is piecewise linear and described by the switching dynamics as shown below:
\[ \dot{x} = \begin{cases} A_{co}x + Bu & (contact) \\ A_{bl}x + Bu & (backlash) \end{cases} \] (5.31)

The measurement equation is not changed anyway and thus it remains:

\[ y = Cx \] (5.32)

The system matrices are given below:

\[ A_{co} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{k}{J_m \tau} & -\frac{1}{\tau} + \frac{b_m}{J_m} & 0 & 0 & 0 & 0 & 0 \\ \frac{k}{J_l \tau} & \frac{c}{J_l} & -\frac{k}{J_l} & -\frac{c+b_l}{J_l} & -\frac{1}{J_l} & 0 & -\frac{k}{J_l} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\tau} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \] (5.33)

\[ A_{bl} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{b_m}{J_m} & 0 & 0 & 0 & \frac{1}{J_m} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{b_l}{J_l} & -\frac{1}{J_l} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{k}{c} & \frac{1}{c} & -\frac{k}{c} & -1 & 0 & 0 & -\frac{k}{c} \end{bmatrix} \] (5.34)

\[ B = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{\tau} & 0 & 0 \end{bmatrix}^T \] (5.35)

\[ C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \] (5.36)
5.4 Modified backlash state estimator

When the system equations (3.1), (3.3), (3.5) and (3.6) are looked more carefully, it can be seen that all the equations are based on the position and speed differences between the motor and the load. None of the equations are dependent on the absolute values of the speed or position. Furthermore, in real applications there could be a risk that an absolute value of either positions increase so large that the datatypes of the algorithms overflow due to restricted number of bits. Due to these two facts a modified backlash state estimator was constructed. The estimator dynamics were not changed but more reasonable states were chosen for the model. The new states were chosen to be

- $x_1 =$ motor speed
- $x_2 =$ speed difference
- $x_3 =$ position difference
- $x_4 =$ backlash angle
- $x_5 =$ torque load
- $x_6 =$ torque motor

The speed difference is same as $\dot{\theta}_d = \frac{\omega_m}{i} - \omega_l$ as introduced in Chapter 3. Similarly the position difference is same as $\theta_d = \frac{\theta_m}{i} - \theta_l$.

After modification the system matrix in contact mode becomes:

$$A_{coe} = \begin{bmatrix} a_{11} & -\frac{c}{J_{mi}} & -\frac{k}{J_{mi}} & \frac{k}{J_{mi}} & 0 & \frac{1}{J_{mi}} \\ a_{21} & -\frac{c}{J_{mi}^2} + \frac{c + b_l}{J_l} & -\frac{k}{J_{mi}^2} + \frac{k}{J_{mi}} & \frac{k}{J_{mi}^2} + \frac{k}{J_l} & 0 & \frac{1}{J_{mi}} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10^{-10} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\tau} \end{bmatrix}$$

(5.37)

where

$$a_{11} = -\left(\frac{\dot{\theta}_d + b_m}{J_m} - \frac{c}{J_{mi}^2}\right)$$

and

$$a_{21} = \left(\frac{\dot{\theta}_d + b_m}{J_{mi}^2} + \frac{c}{J_l} - \frac{c}{J_{mi}^2} + \frac{c + b_l}{J_l}\right)$$

for convenience.
Similarly in backlash mode the system matrix becomes:

$$A_{ble} = \begin{bmatrix}
-b_m & 0 & 0 & 0 & 0 & \frac{1}{J_m} \\
-\frac{b_m}{J_{mL}} - \frac{b_l}{J_{lL}} & 0 & 0 & \frac{1}{J_l} & \frac{1}{J_{mL}} \\
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & -\frac{k}{c} & 0 & 0 \\
0 & 0 & 0 & 0 & -10^{-10} & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{7}
\end{bmatrix} \quad (5.38)$$

The control matrix B stays the same as before. However, now in C -matrix the used measurements are the position difference and the speed difference as given below:

$$C_e = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix} \quad (5.39)$$

As seen in 5.39, there is also a third measurement in $C_e$ -matrix. The additional measurement is the motor speed $\omega_m$. The reason for this is that the load torque estimation as a slowly varying parameter needs at least one absolute measurement such that the estimator can solve its absolute value. Because the third measurement considers only a speed quantity, there is no problem with the datatypes.

### 5.5 Observability and stability of the models

Before testing the constructed position estimators, the model observability was explored more closely. Based on full rank of the system matrix in contact mode it was found out that all the states in contact mode are observable. However, in backlash mode there are two unobserved states which were tracked to be the load torque and the backlash angle. The load torque is naturally one of the unobserved states due to zero row found in $A_{bl}$ matrix in (5.34). In order to try to make backlash angle observable, a measurement of motor speed was added to the system. However, this results in 6 unobservable states, thus, making the result worse. By adding the load speed measurement to the system there are still same two unobservable states in the system. By adding both speed measurements there are again 6 unobservable states. The unobservability of the backlash position may be explained by the dependence of the shaft twist ($\theta_s = \theta_d - \theta_b$) which is seen from the backlash dynamics in Equation (3.6). As for shaft twist there is a dependence of the $\theta_2$ angle as
CHAPTER 5. BACKLASH ESTIMATION

shown in the Figure 2.9. This angle is not measurable or otherwise available in the model when the system goes to the backlash mode. Important thing to notice is that when the shaft gets straightened the backlash is exactly the same thing as position difference $\theta_d$ as discussed in Section 3.6.2. Naturally this is true only in the backlash gap region.

When the observability of the modified position estimator is explored the same behaviour is noticed. There are two unobserved states that are the backlash angle and the load torque. This is naturally true because both models share the same dynamics. Only different states are used between the state-space representations\(^1\). Another interesting thing was noticed when exploring the stability of constructed systems. By calculating the system eigenvalues for system matrices 5.38 and 5.33 an unstable pole was found. The pole-zero map of the powertrain in contact mode is shown in the Figure 5.1.

![Pole-Zero Map](image)

Figure 5.1: Powertrain in contact mode, pole-zero map

As seen in the Figure 5.1, the pole on the right half plane is very close to zero. It was noticed that when the shaft flexibility is changed the pole position moves. With stiffer shaft the pole moves away from the origin and respectively with looser shaft the pole goes towards the origin. However with the modified system representation shown in the Section 5.4, there are no unstable poles. This indicates a numerical uncertainty because of the model structure based on the absolute motor and load angles. For this reason, the modified backlash estimator structure is more encouraging. The modified

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\(^1\)As it is well known, the state-space representation is not unique what comes to choosing states.
structure is based on the position and speed differences rather than absolute values of individual inertias.
Chapter 6

Backlash compensation

In order to compensate the backlash actively in the powertrain, the motor needs to be position and speed controlled. This way the backlash gap can be closed quickly. The landing of the backlash must be as soft as possible in order to avoid shunt at the contact instant. This protects the mechanical parts such as gear box and prevents the possible damage. Thus a longer lifetime for mechanical parts is achieved in terms of durability. The soft landing is achieved by driving the speed difference of the motor and load inertias close to zero. In this section two different controllers are designed to achieve these two goals. The designed backlash state estimator informs the controller about the backlash angle which is not otherwise measurable. The first controller is a linear proportional-derivative (PD) controller based on custom control law. The second controller is based on optimal LQ -control theory.

6.1 Simple control law

A simple way to control backlash is to construct a basic P-controller that controls the backlash position to the positive or negative side of the backlash gap. Furthermore, this control law is modified to drive the speed difference of the motor and load inertias near to zero in order to achieve soft landing. The control law that calculates the control action according to these objectives is given below.

$$u = k_1 \cdot (\alpha - \hat{\theta}_b) + k_2 \cdot \left( \frac{\hat{\omega}_m}{I} - \hat{\omega}_l \right)$$  

(6.1)

As seen in Equation (6.1), the weighting terms $k_1$ and $k_2$ are used to tune the control action for a specific system. The hat notation in this control law
means that these measures are obtained from the backlash state estimator. If the estimated backlash angle is far away from the target angle then more torque is applied to the motor in order to reach the target angle faster. The closer the backlash angle becomes the target the less torque is applied to the motor. Same logic applies for speed difference. The more speed difference exists the more energy is used to bring this speed difference to zero. The position reference for backlash is chosen to be $+\alpha$ or $-\alpha$ depending on the sign of the torque that driver requests. As discussed in Section 3.6.2, the backlash position estimate $\hat{\theta}_b$ equals $\theta_d$ if the shaft is straightened fully in the backlash region. This reveals that the suggested custom control law in Equation (6.1) becomes simply a PD -controller. The derivative term is important in this controller, because it is responsible of matching the speed difference between the motor and the load side. This way a softer landing is achieved. Without the derivative term the motor position is controlled to the reference position as soon as possible without taking into account the speed difference at the contact instant. This would lead to significant shunt that must be avoided. One must notice that with different vehicle powertrains the tuning of the controller must be done iteratively. With two different powertrains the coefficients $k_1$ and $k_2$ are application specific tuning constants. They must be changed according to specific platform, because no two exact same powertrains exist.

### 6.2 LQ -control

After designing a simple PD -control law in last section, a model-based controller was also designed. In model-based control, the state-feedback is the simplest way to control the system states to zero i.e to origin. However, choosing the placements for the poles can be tricky. LQ -theory is an optimal control technique that helps the designer to choose the poles such that quadratic cost function is minimized. Often iterative tuning is still needed until the designer is satisfied to the response of the system. One should notice that the used optimal control technique does not provide the true "optimal" response of the system. It changes the tuning of the controller from iteration of pure pole-placement to iteration of weighting factors in the quadratic cost function that is minimized. Still, tuning the system response through the weighting factors is far more intuitive than changing the pole placements directly in the complex s-domain (continuous-time) or z-domain (discrete-time).
6.2.1 Infinite time LQ-control

As the digital controller works in discrete time also the LQ-problem is treated in discrete time in this thesis. In optimal LQ-problem the fundamental result is that the optimal control law is a pure state feedback. However, the optimal feedback gain has to be determined first. The gain is found by minimization of a quadratic cost function. The problem can be stated as follows:

Given a plant description in state-space form such as derived in Section 3.5:

\[ x(k+1) = Ax(k) + Bu(k) \]  \hspace{1cm} (6.2)
\[ y(k) = Cx(k) + Du(k) \]  \hspace{1cm} (6.3)

find an optimal control action \( u(k) = -K(k)x(k) \) that minimizes the quadratic cost function

\[ J = \frac{1}{2} \sum_{k=0}^{N} \left[ x^T(k)Qx(k) + u(k)^T R u(k) \right] \]  \hspace{1cm} (6.4)

The problem by finding the optimal control in this problem is that the result ends up to having time-varying feedback gain \( K(k) \). That is, the gain \( K(k) \) changes every time step, and according to Franklin [31], the gain should be pre-computed for the known length of the optimization problem.

In order to design the system such that it could be implemented in a real system, the problem can be stated as infinite time LQ-problem which is a special case of the problem stated above. In the infinite time LQ-problem, the cost function is minimized over infinite time instead of finite time. Thus the cost function becomes

\[ J = \frac{1}{2} \sum_{k=0}^{\infty} \left[ x^T(k)Qx(k) + u(k)^T R u(k) \right] \]  \hspace{1cm} (6.5)

In this approach, the optimal feedback gain turns out to be constant \( K_\infty \) that also can be pre-computed. Thus, although the gain of the finite-time problem is time-varying, it is still constant most of the time from the beginning. The gain changes dramatically only at the end of the problem. By refining the problem to consider infinite time, the gain can be seen to be constant over the time because the flat part of the gain is only extended...
towards infinity. The solution to infinite time problem is a linear quadratic regulator (LQR) because the control is applied to a linear system, the cost is quadratic as stated before and the control actions regulate the system states to origin. In order to find a solution to the problem, the weighting matrix that penalizes control actions \((R)\) must be positive definite. The weighting matrix that penalizes states must be positive semidefinite. Mathematically expressed that is:

\[
R > 0 \tag{6.6}
\]

and

\[
Q \geq 0 \tag{6.7}
\]

The steady-state gain \(K_{\infty}\) is calculated as shown below:

\[
K_{\infty} = (B^T S_{\infty} B + R)^{-1} B^T S_{\infty} A \tag{6.8}
\]

The gain depends on \(S_{\infty}\) which is a steady-state solution of Discrete Algebraic Riccati Equation (DARE):

\[
S_{\infty} = A^T A - (A^T S_{\infty} B)(R + B^T S_{\infty} B)^{-1}(B^T S_{\infty} A) + Q \tag{6.9}
\]

A numerical solution of the optimal feedback gain is obtained easily with command \textit{dlqr} in Matlab. In order to utilize LQR technique, the pair \((A, B)\) must be stabilizable. That is, all the uncontrollable modes in the system must be naturally stable because control action does not affect to them.

In practice, the tuning of the controller is done iteratively by changing the \(Q\) and \(R\) matrices. The easiest way is to choose \(Q\) and \(R\) to be diagonal. In this way the diagonal entries in \(Q\) are weightings to system’s states and correspondingly the diagonal entries in \(R\) are weightings to the system’s inputs. In the control configuration of the system there is only one input which means that \(R\) is a scalar. Choosing \(Q\) and \(R\) matrices is often done by trial and error. Additionally there are few procedures mentioned by Franklin [31] for helping to choose reasonable values to begin with. The first one is Bryson’s rule where the diagonal terms are chosen such that a fixed percentage change of each variable results in an equal contribution to the cost. Another suggested approach is to use Pincer procedure where all the closed-loop poles are required to be inside of circle that has a radius of \(1/r\)
where $r \geq 1$. This gives a freedom to control the settling time of the states. Both of these procedures are explained thoroughly in [31].

The benefit of LQR technique is found in the tuning procedure. It is much more intuitive for human to penalize states and inputs instead of thinking the placements for the poles. The optimal feedback gain places the poles such that the given cost function with given weights is minimized.
Chapter 7

Testing and results

The constructed state estimators were tested off-line \(^1\). First the switching rule generation was tested by injecting noisy, unfiltered torque input data from a real test drive into the powertrain model in Simulink. This would reveal how the estimator dynamics would be switched based on the noisy data. Then the estimators were tested in parallel with the powertrain model in Simulink by using the simulated measurement signals generated by the powertrain model. The powertrain and the estimators were parametrized based on the parameter identification results discussed in Chapter 4. In order to increase uncertainty to the simulated measurement signals, artificial Gaussian noise was added to them in Simulink. This way the estimators could be tested before applying them to the real powertrain output data. After this the estimators were tested with real measurement data obtained from a test electric vehicle provided by Metropolia University of Applied Sciences. The test vehicle is equipped with HES880 - frequency converter manufactured by ABB Oy. HES880 -inverter unit is shown in the Figure 7.1.

Off-line tests were the only way to test the estimators due to the fact that HES880 -inverter by ABB Oy does not support yet two encoder interfaces for simultaneous position measurements. However if the off-line estimation works, then also on-line estimation should work. The only difference is the location of the Kalman filter algorithms between these two approaches. Finally, the backlash compensators were also tested in Simulink environment by applying the closed-loop control laws to the simulated powertrain.

\(^1\)Estimators were applied in Matlab after the measurements were obtained and preprocessed.
7.1 Enhanced position measurements

The measurement setup was same as used for parameter identification in Section 4.5. The motor and wheel position data is obtained through the encoders and the motor torque data is obtained directly from the vehicle CAN-bus. All these measurements were gathered with Sirius -data logger provided by DEWESoft. The Sirius -data logger utilizes individual A/D -converters for the input channels which enables the synchronization of the measurements with high precision. Traditionally a pulse train signal is read by a counter input that is increased everytime when a rising or a falling edge of a signal is detected. A basic problem with position measurements based on encoder pulses is that the number of pulses per revolution determines the resolution of the position measurement. As discussed in Chapter 4, the motor encoder produces 80 pulses per revolution. Correspondingly the wheel position sensor produces 44 pulses per revolution. That means that if the motor position is calculated based on rising edges in the encoder signal the position jumps with $360^\circ/80 = 4.5^\circ$ steps every time when a rising edge of the signal is detected. For the wheel position sensor the jump is even higher, namely $360^\circ/44 = 8.18^\circ$. The resolution can be improved by reading additionally the falling edge of the signal. This doubles the resolution. With quadrature encoders that provide two pulse train channels, A and B, can utilize the rising and falling edges of the both signal which makes the resolution four times better. However, with the wheel position encoder based on a simple toothed
ring and a Hall -sensor there is no way to do this. Additionally, although the resolution can be improved by utilizing more parts of the pulse train the traditional counter is latched only at a sample rate interval. In theory there is a chance that the counter input is just updated at the sample instant and a new rising edge in the signal arrives just after this moment. This updates the counter value but the position is calculated only at the next sample instant. That means that the position measurement can have error of one tooth gap at a maximum. The better resolution is available the less error there will be. However, in the test vehicle the error in wheel position measurement can be even 8.18°. This is almost double the total backlash size that was measured manually from the vehicle (4.5°). Thus the available position measurements based on discrete counter values were considered to be too uncertain for the backlash estimation. Normally in industrial applications there can be encoders that produce even 1024 or 2048 pulses per revolution which reduces this problem significantly.

The estimators in Chapter 5 were designed such that a precise knowledge of the motor and wheel position was known at every millisecond. That is, the discretization time was \( h = 1 \text{ms} \). Additionally, all the measurements were assumed to be synchronized in time. In order to log data with these requirements from a real powertrain, the Sirius -datalogger provides a special counter input called "super-counter". This counter utilizes two separate counters. The second counter is able to measure exactly where the position of the pulse is between two consecutive samples. This enables to calculate the exact interpolated position of the counter value at the sample instant. That is, counter value can have now non-discrete values and thus exact position is obtained. Also exact frequency of the pulses can be obtained from which an exact rotation speed of the object can be calculated. After a recording session one has the time synchronized data of the motor position, wheel position and the motor torque that can be used for estimator verification purposes. The overall measurement system description is shown in Figure 7.2.
Figure 7.2: Measurement setup
7.2 Gathering measurement data

In order to gather real measurement data for the offline testing, the test vehicle was driven on a flat test area. The frequency converter was parametrized such that the maximum allowed positive torque would be 80% of the nominal motor torque. Respectively, the maximum negative torque was set to -40% of the nominal motor torque. The vehicle control unit (VCU) was programmed such that when the gas pedal was released, there was a negative torque reference that would activate the electric motor brake. During the test the vehicle was driven straight and the gas pedal was pressed down and released repeatedly. The measured data is shown in the Figure 7.3.

![Figure 7.3: Recorded backlash size estimation data](image)

As seen from the Figure 7.3, the upper left corner represents the torque input (Nm) to the drivetrain. The motor torque travels between positive and negative values which forces the backlash angle to traverse between the positive and negative contacts. The measured motor position and wheel position are shown in the upper right corner and lower right corner, respectively. The lower left corner shows the position angle difference between the motor and the wheel in the wheel co-ordinates. In the wheel co-ordinates the absolute
CHAPTER 7. TESTING AND RESULTS

motor position angle is divided with the known gear ratio. This way the motor and the load positions are comparable. As seen clearly, the position difference changes between the positive and negative values which means that the motor position leads or lags the wheel position just as it should be. The measured data was exported from DeweSOFT Sirius -datalogger and imported to Matlab/Simulink. In Simulink the powertrain model was parametrized by the values obtained from the parameter identification results discussed in Chapter 4.

7.3 Testing the switching rule

The noisy, logged torque input data was injected to the powertrain model in order to test the switching rule expressed by the conditions in 5.29. The simulated results are shown in the Figure 7.4.

As seen in the Figure 7.4, the blue curve represents the real torque data that was logged from the real vehicle. The torque curve is very noisy which affects also to the simulated motor and load positions. For this reason the total shaft displacement \( \theta_d \), that is the difference between the motor and the wheel position, is also affected. This affects to the simulated switching mode rule that is switching very heavily as represented by the green curve. This is seen especially at time interval \( t = 5..10s \).

![Figure 7.4: Switching rules based on raw data](image)
In order to remove the contribution of the noise effects in the switching rule calculation, the torque signal was low-pass filtered with a tenth order Butterworth filter. The cut-off frequency was set to $1\,Hz$ in order to smooth the torque signal effectively. The switching rules were simulated then again based on the filtered torque input data. The results are shown in the Figure 7.5.

As shown in the Figure 7.5, the high frequency switching rule oscillation has almost disappeared. The rest of the oscillation really occurs due to the real backlash traverses. As seen clearly, every time the torque sign changes the estimator switches to "wait" -mode. That is, the switching state gets value of 0.05 or -0.05 whether the backlash is traversed towards positive contact or negative contact, respectively. However, the switching mode is not changed to "wait" -mode immediately at the time instant of the torque sign change. This is explained by the fact that after the torque sign changes the twist of the shaft starts to straighten out and it takes a short time before the backlash traverse starts. In overall, the switching modes look very reasonable.
7.4 Backlash size estimation of the simulation model

First the backlash size estimator was tested based on the measurement signals obtained from the powertrain model simulation. The size estimator was implemented in parallel to the powertrain model. The torque request was connected as an input $u$ to the estimator. The position measurements of the motor and the wheel were connected as measurements $y_1$ and $y_2$ to the estimator. The simulated contact mode switching rule was also connected to the estimator such that its dynamics would be changed according to the present contact mode. The simulation setup is shown in the Figure 7.6.

The estimation setup, shown in the Figure 7.6, is exactly the same as if the estimator would be used on-line in a real vehicle. Only exception is the contact mode signal that is not available in a real powertrain. Next the torque request data obtained from a real test drive was imported to the Simulink. The torque request was injected to the powertrain model and artificial Gaussian noise with variance 0.1 rad$^2$ was added to the simulated motor and wheel position signals in order to add uncertainty to the measurements utilized by the backlash size estimator. Then the powertrain was simulated with the injected torque profile. The signals seen by the backlash size estimator are shown in the Figure 7.7.
In the upper most graph in the Figure 7.7 the torque input signal from the real test drive is shown. The alternation of positive and negative torque ensures that the backlash is driven between the positive and negative contact modes. The added Gaussian noise is not seen very well in the motor and wheel position signals that are shown in the next two graphs below the torque input graph. However, in the last graph there is shown the total shaft displacement $\theta_d$ which reveals that the position difference signal between the motor and the wheel is very noisy. Finally, the backlash size estimation results based on the represented simulation data are shown in the Figure 7.8.
The blue curves in the Figure 7.8 represent the estimated offset parameters $\theta_0^+$ and $\theta_0^-$ in the size estimation. The red, dashed horizontal lines represent the known sides of the backlash gap in the simulated powertrain model. As seen clearly, the estimator works as intended. The blue curves converge to the level of the red lines. The green curve represents simply the absolute difference between the estimated offset parameters. Thus the green curve converges to the estimated backlash gap size value.

The powertrain simulation starts in positive contact mode as the input torque is positive as shown in the Figure 7.7. For this reason, the positive offset parameter is updated first which can be clearly seen in the Figure 7.8. Right after when the torque sign changes, the estimator switches to the "wait-mode" and none of the offset parameters are updated. After the backlash has traversed to the negative contact, the size estimator starts to update the negative offset parameter. In the Figure 7.9 the backlash size estimation graph is zoomed in order to show the different estimator modes more clearly.

![Backlash size estimator modes](image)

Figure 7.9: Backlash size estimator mode changes

The zoomed estimation curves in the Figure 7.9 show more clearly the different switching modes for the reader. Inside the green box, only negative offset parameter is updated and the positive offset parameter maintains its last value. Inside the purple box, both offset parameter estimates maintain their values. Inside the yellow box, only positive offset parameter is updated and the negative offset parameter maintains its last value. Between the colored boxes there are naturally also positive, negative and wait -modes,
but they are very short in time. Thus the colored boxes are not representing exactly consecutive mode changes. They are only representing the different modes that are easy to catch for an eye.

The effect of the measurement noise in the estimation was researched by setting the measurement noise to zero in order to have perfect measurements. The backlash size estimation results for pure data are shown in the Figure 7.10.

![Backlash size estimate convergence](image)

Figure 7.10: Backlash size estimate without noise

As seen clearly in the Figure 7.10, the offset parameter estimates are very accurate and they do not fluctuate as much as with the noisy data. Thus the more precise and filtered data is used, the better estimate of backlash size is available.
CHAPTER 7. TESTING AND RESULTS

7.5 Backlash size estimation of the real test vehicle

As the simulated motor and wheel position data was a success with and without the measurement noise, the backlash size estimator was verified also with a real position data obtained from a test drive with Fiat Doblo -electric vehicle. The logged measurement data from a real electric vehicle is shown in the Figure 7.11.

![Figure 7.11: Recorder input data from a test vehicle](image)

During the test drive, both the motor position and wheel position signals were logged but in the Figure 7.11 only the difference of these signals is shown. As seen clearly, there is quite much oscillation in the position difference signal. The measurement data file is only 30 seconds long but this does not affect to the results while the size estimation can be continued by feeding the measurement data for the estimator as many times as needed. The reason for short measurement interval is that the test field was limited to 200 m long straight road. The measurement test data was concatenated 50 times in a row in order to guarantee that the offset parameter estimates converge to the steady-state values. The results are shown in the Figure 7.12.
The ratio of the state covariance matrix $Q$ and measurement covariance matrix $R$ was reconfigured such that less effect was given for the new measurements thus making Kalman gain $K$ smaller. This makes the offset parameter estimates converge slowly but in controlled manner as seen in the Figure 7.12. The spikes in the positive offset parameter estimate result from the measurement data concatenation. When the measured position data ends and the new copy starts, there is no smooth transition between the consecutive data batches. If the logged data would be from a one, complete test drive, there would not be such spikes due to smooth position data. Due to short test field and limited test time, such a complete 1500 seconds long measurement data was not possible. In overall, the spikes are only cosmetic problem and they do not affect to the results.

The average value of both offset parameters were taken from the flat portion of the estimates. The average values are drawn as red horizontal lines in the Figure 7.12. By looking at the red lines, the positive offset parameter converges to value 0.01909 and the negative offset parameter converges to value -0.05778. According to the Equation (5.16), the backlash gap size can be calculated then by
\[ 2\alpha = \theta_0^+ - \theta_0^- \]
\[ = 0.01909 \text{ rad} - (-0.05778 \text{ rad}) \]
\[ = 0.0769 \text{ rad} \]
\[ = 4.406^\circ \]

The measured backlash size by a digital angle meter was 4.4°-4.7°. Thus the estimate of the backlash gap size can be considered very accurate. The estimation error compared to the manually measured value can result from different measurement equipment. In overall, the result is considered successful.

In addition to the backlash gap size, the backlash size estimator estimates also other states such as position and speed of the motor and wheel. Additionally the load torque is estimated as a slowly varying parameter. The Figure 7.13 shows the estimation results of these states.

![Figure 7.13: Rest of the states estimated by backlash size estimator](image)

Figure 7.13: Rest of the states estimated by backlash size estimator

In the shown results, all the estimated states (red) are compared to the simulated powertrain states (blue). Furthermore, the noise in the signals is artificially added Gaussian noise. In the Figure 7.13 the upper most graph represents the estimated position difference against simulated position difference in the wheel co-ordinates. As seen clearly, the estimator filters out the position difference well from the noisy measurements. The second graph below represents the speed difference. The validation is done here against
the known speed difference obtained from the simulated powertrain. When
the powertrain is in contact mode, the estimator tracks very well the speed
difference. When the backlash is traversed, the estimator loses the real speed
difference until the contact mode is achieved again. This results from the fact
that the backlash size estimator dynamics are based on the dynamics of a
powertrain that does not include any backlash. Actually, for this reason the
backlash size estimation works. That is, the offset parameters are updated
based on the position and speed differences between the estimator dynamics
and the real dynamics. The last graph represents the load torque that is
estimated as a slowly varying parameter. The estimated load torque tracks
quite well the real load torque. The small deviations from the real load torque
does not affect almost at all because the equivalent load inertia does have so
much kinetic energy that the estimated load torque decelerates the rotating
inertia apparently as much as the real load torque does.

7.6 Backlash position estimation of the sim-
ulation model

Similarly to the backlash size estimator testing, the backlash position esti-
mator was tested first against the simulation data that was produced by the
simulation model and then with the real world measurement data. The po-
sition estimation results based on simulated measurement data are shown in
the Figure 7.14.

Figure 7.14: Total shaft displacement and backlash position estimate
As seen in the upper graph of the Figure 7.14, the estimate of the total shaft displacement (dashed red) tracks very well the simulated total shaft displacement (black solid). Again the curves are represented in the wheel co-ordinates. The blue curve in the same graph represents the backlash position estimate. The position estimate tracks the total shaft displacement curve very well in the backlash gap region that is restricted between the two grey, dashed horizontal lines. The injected torque to the system is shown in the lower graph. The zoomed backlash position estimate at time instant $t=42.5s$ is shown in the Figure 7.15.

![Figure 7.15: Backlash position estimate](image)

As seen in the Figure 7.15, the backlash position estimator thinks that the backlash traversal (blue) is started slightly before the backlash gap opens based on total shaft displacement (black/red) simulation. As reported in this thesis, this is natural because the total shaft displacement does not equal the backlash position angle until the shaft is straightened fully. However, the backlash gap is closed very precisely. At the end of the backlash gap the shaft is straightened fully and the both quantities report the same backlash position angle. The backlash position estimate timing depends highly on the switching instant of the estimator modes. Furthermore, the accuracy of the estimator model has a real effect to the results. The mode switching conditions in 5.29 seem to work very well in the simulation. Next the backlash position estimation was applied to the test data obtained from the test vehicle.
7.7 Backlash position estimation of the real test vehicle

The backlash position is not measurable in a real vehicle and thus only way to validate the estimator behaviour is to compare the measured total shaft displacement of the real vehicle and the corresponding estimate provided by the backlash position estimator. The same idea was used by Lagerberg [32]. The estimation results based on the real vehicle data is shown in the Figure 7.16.

![Backlash position estimation based on real unfiltered data](image)

Figure 7.16: Backlash position estimate based on unfiltered real data

The results seen in the Figure 7.16 reveal that the total shaft displacement is estimated really well. That is, the positions of the motor and the wheel are very well detected and thus the red and blue curves go on the top of each other. This was expected as both of the states are observable and they are also measured. The estimator mode is also decided by the estimator by itself. The current mode is shown with black line. The value +0.1/-0.1 means that the estimator works in contact mode. Correspondingly the value 0.05/-0.05 means that the estimator works in backlash mode. What comes to backlash position estimation, it does not work as well as in the Simulink verification shown in the Figure 7.15. This makes sense because the backlash angle is not observable state as the early observability check of the model addressed in the section 5.5. Simply put, there is no way to detect the remaining shaft twist when the backlash gap opens. The backlash
The results in the Figure 7.17 show that the backlash bouncing does not occur anymore. The backlash is traversed only in the case of torque sign change. Especially backlash traverses after time instant $t = 20s$ reveal that the backlash position estimate is too late in time and a backlash controller cannot be based on this feedback data. The model parametrization should be probably more precise. One should still notice that based on the powertrain simulation in Chapter 3.6.2 the stiff shaft manages to straighten before the backlash gap is traversed. According to the simulation represented, the shaft twist is already straightened at the beginning of the backlash traverse which means that the backlash position equals the total shaft displacement. Thus with stiff shafts there would not be too much error to replace the backlash position with the total shaft displacement as a feedback information for the controller. As seen in the Figures 7.16 and 7.17, the total shaft displacement is very well tracked by the estimator and can be considered trusted feedback.
Finally, the estimation of the load torque was also compared between all the three estimators built in this thesis. The results are shown in the Figure 7.18.

Figure 7.18: Load torque estimates from different estimators

In all the cases the estimates are compared against simulated load torques. Thus all the results in the Figure 7.18 are based on powertrain simulation in Simulink. The x-axis in all the graphs represent time in seconds (s). The y-axis represents the load torque (Nm). In all the graphs the solid blue line represents the actual load torque and the red dashed line represents the corresponding estimate of that quantity. The upper most graph represents the load torque estimate given by the backlash size estimator. The graph in the middle shows the estimation result based on the basic backlash position estimator. The last graph is the load torque estimate given by the modified backlash estimator. As seen clearly, the load torque is estimated most accurately by the backlash position estimators. The load torque in all the cases is estimated as a slowly varying parameter.
7.8 Backlash compensation results

The backlash compensation was verified with simulation methods. The testing with real electric vehicle was not possible due to missing capability of measuring load position by HES880 -inverter. However, the designed compensation methods could be tested with the constructed powertrain model in Simulink.

7.8.1 Testing the custom control law

The constructed backlash compensation configuration is seen in the Figure 7.19.

As seen in the Figure 7.19, the backlash position estimate is used as a feedback signal. The motor speed estimate is also fed back to the controller as a reference value to be used in the speed equalization. The torque reference is used in order to determine the backlash position reference based on the sign of the requested torque. The tuning parameters \( k_1 \) and \( k_2 \) were tuned manually as long as the response of the system was found to be satisfying. The results are shown in the Figure 7.20.
The results shown in the Figure 7.20 reveal the trade-off that has to be done between the system response time and the soft landing. The upper plot represents the backlash position as a function of time. The lower plot represents the motor torque that is determined by the backlash controller. In both graphs the solid blue line represents a system where no backlash compensation is applied - the system is driven in open-loop. A fair assumption, when interpreting the results, is that the load speed does not change significantly during the backlash traverse. Thus, the backlash position changes can be considered to originate from the position and speed changes of the motor inertia.

Because of the assumption of the static load, the change of the derivative of the backlash can be considered originating from the increasing position derivative of the motor inertia. This is intuitive because the applied motor torque stays positive constant which accelerates the inertia. As seen from the blue solid curve, at time instant $t = 33.7s$, the backlash gap opens if no backlash compensation is applied. The reopening of the backlash gap results from the powertrain oscillations caused by the shunt at the contact instant at $t = 33.59s$. The four dashed lines with different colors represent the backlash position when the backlash controller is activated. As can be seen from the control signal plot, the controller accelerates the motor inertia first in order to close the gap as soon as possible. When the backlash position starts to approach the positive contact, the controller starts to decelerate the motor inertia such that the speed difference approaches zero.

The results reveal also that the fundamental trade-off between the back-
lash traverse time and the speed difference between the motor and the load at the contact instant must be done by the designer. Also, if the backlash controller is tuned too careful there is a chance that the controller drives the speed difference to zero but the motor position does not reach the load position. This is naturally not acceptable in terms of system response where the driver wants that the requested torque is transmitted to the wheels. The controller is able to make the response much faster compared to the system that is not controlled as can be seen from the backlash position curves colored with red and magenta. In addition to faster response, the controller is still able to perform softer landing of the contact than powertrain without no backlash control. This is justified by the fact that the derivative of the backlash position in the contact instant is much lower with the controlled system (magenta, red) compared to the uncontrolled backlash position represented by the blue curve. When tuning the controller, one must consider the maximum available torque of the electric motor. The controller was tuned based on the known torque limits of the test vehicle.

As discussed in the results of the backlash position estimation, the estimation is not very accurate with real vehicle data. However, the total shaft displacement is observable quantity. In the cases where the shaft is straightened fully, the total shaft displacement equals the backlash position when the backlash is traversed. For this reason, a backlash compensation based on total shaft displacement feedback to the controller was tested. The results are shown in the Figure 7.21.
The results look promising as seen in the upper graph of Figure 7.21. The backlash position is traversed in a controlled manner also with the modified controller configuration. The controller was tuned in the same way, as in the results, shown in the Figure 7.20. The curves with same color can be compared against each others. The biggest difference can be seen from the settling time of the backlash position. When total shaft displacement is used as a feedback, the settling time decreases clearly. The reason for this is found from the lower plot. The controller uses much higher motor torque as a control action. However, these torque values go over the acceptable limits and the motor in the test vehicle would definitely saturate. The higher torque action is intuitive while the total shaft displacement is much greater than the backlash angle deviation from the reference point $\alpha$ due to shaft twist. The higher deviation results in higher injected motor torque. When total shaft displacement is used as a feedback, the controller gain parameters $k_1$ and $k_2$ should be assigned with lower values. One should also recall that using total shaft displacement as a feedback is only valid when the shaft straightens itself fully during the backlash gap traverse.


7.8.2 Testing the LQ-regulator

At last the backlash compensation was tested with a linear quadratic regulator. In backlash region, the motor position and speed are the only states that one can affect through the input of the system. Thus, the model of the motor inertia dynamics was used in order to calculate an optimal state feedback gain. All the represented backlash estimators in this thesis are capable of estimating the position and the speed of the load inertia when the system enters the backlash region. This knowledge was then utilized as a state reference for the motor position and speed control. The constructed state feedback configuration is shown in the Figure 7.22.

![Figure 7.22: LQ regulator configuration](image)

The torque request signal in the Figure 7.22 is used for the sign detection of the torque that the driver wants to deliver. This determines the side of the backlash gap where the backlash position is driven to. The reference state for the motor speed is the load speed. Additionally, the reference position for the motor is the load position where an offset of $\alpha$ is added or subtracted depending on the requested sign of the torque. The controller was tuned by penalizing the states and the input signal with the corresponding matrices $Q$ and $R$ that were introduced in Chapter 6. The backlash control results with LQR are shown in the Figure 7.23.
As seen in the Figure 7.23, the controller was tuned to be very gentle at the contact instant. For this reason, the settling of the backlash position to the referenced side takes long. The upper plot shows the increasing settling times due to the controller tuning where the speed difference is penalized more with the curves that take longer time to reach the reference. This only demonstrates how intuitive the tuning of the linear quadratic regulator can be. If one is not satisfied with the control action that are too high then one can penalize the control by increasing the control penalty matrix $R$.

Unfortunately, the linear quadratic regulator does not provide more accurate and faster control than the earlier represented custom control law. Thus, there will be no better performance with the model-based controller in this particular application. This can be seen from the state feedback control law that becomes exactly the same as the custom control law. This can be proven quickly. The first developed control law was stated as:

$$ u = k_1 \cdot (\alpha - \hat{\theta}_b) + k_2 \cdot (\frac{\hat{\omega}_m}{\hat{I}} - \hat{\omega}_l) \quad (7.1) $$

Due to the fact that the backlash position angle is not observable nor controllable it can be replaced by the total shaft displacement $\theta_d$ in the cases where the shaft straightens during the backlash traverse. Thus the simple control law can be modified to

$$ u = k_1 \cdot (\alpha - \hat{\theta}_d) + k_2 \cdot (\frac{\hat{\omega}_m}{\hat{I}} - \hat{\omega}_l) \quad (7.2) $$
CHAPTER 7. TESTING AND RESULTS

On the other hand, the state feedback is based on the states of motor position and motor speed. Thus, the state feedback can be written as:

\[ u = Kx = [k1 \ k2] \begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \end{bmatrix} \] (7.3)

When the reference states are introduced, the control law becomes:

\[ u = K(x_{ref} - x) = [k1 \ k2] \begin{bmatrix} \dot{\theta}_{m_{ref}} - \dot{\theta}_m \\ \dot{\omega}_{m_{ref}} - \dot{\omega}_m \end{bmatrix} \] (7.4)

Considering a case where the backlash is driven from the negative contact to the positive contact, a reasonable reference \( \theta_{m_{ref}} \) in wheel co-ordinates would be to drive the motor position \( \alpha \) radians over the current load position \( \hat{\theta}_l \) in order to close the backlash gap. In order to aim at soft landing, the motor speed and load speed whould equal at the contact instant. Thus, the speed reference \( \omega_{m_{ref}} \) for the motor is chosen to be equal to the load speed \( \hat{\omega}_l \) that is obtained from one of the constructed backlash state estimators. Finally the control law becomes:

\[ u = K(x_{ref} - x) = [k1 \ k2] \begin{bmatrix} \hat{\theta}_l + \alpha - \frac{\dot{\theta}_m}{i} \\ \hat{\omega}_l - \frac{\dot{\omega}_m}{i} \end{bmatrix} \] (7.5)

When the control law is multiplied open it becomes:

\[ u = k_1 \cdot (\hat{\theta}_l + \alpha - \frac{\dot{\theta}_m}{i}) + k_2 \cdot (\hat{\omega}_l - \frac{\dot{\omega}_m}{i}) \] (7.6)

By the fact that \( \theta_d = \frac{\dot{\theta}_m}{i} - \hat{\theta}_l \) the equation can be expressed:

\[ u = k_1 \cdot (\alpha - \dot{\theta}_d) + k_2 \cdot (\frac{\dot{\omega}_m}{i}) \] (7.7)

which is exactly the same control law as the custom control law recalled in Equation (7.2).
Chapter 8

Conclusions

Backlash compensation in electric vehicle powertrain was studied. The uncontrolled backlash traversal results in greater shunt and shuffle phenomenon. Additionally, mechanical parts wear out due to uncontrolled backlash traversal and the impacts can be audible in rapid torque sign changes. This may break the mechanical parts e.g in the gearbox. To prevent these problems, active backlash compensation was designed and tested. The service costs may be reduced significantly with the compensation.

This thesis introduced several powertrain models from which a two-mass model was used for active backlash compensation. This model was combined with a physically correct, one-state backlash model. The behaviour of the constructed model was verified against similar simulations found in the literature.

The powertrain model was fitted for a real electric vehicle powertrain through parameter identification. This was carried out by grey-box identification methods. Several step response tests were done with a real electric vehicle. The wheel and motor position measurements were logged as a system output data. The constructed two-mass model was fitted to the measurements based on prediction error method (PEM). The parametrized model achieved over 90% accuracy compared to all measured test data.

The parametrized model was used as a base for three backlash state estimators based on Kalman filtering theory. The first estimator was designed for the backlash gap size estimation. The second estimator was designed for tracking the backlash position angle. The third estimator was a modification of the second estimator and it was re-designed for practical use, avoiding datatype overflowing. The backlash size estimator was applied to the measurement data obtained from a real electric vehicle. The estimator found the backlash gap size to be 4.4°. This was very close to manually measured backlash gap size that was 4.4°-4.7°. The size estimator seems to be a successful
The backlash position angle estimator works well if the model of the powertrain is precisely parametrized. However, this is rarely the case in real world applications. Due to unobservability of the backlash position angle, the estimator cannot extract the backlash angle from the shaft twist. Thus, this feedback signal cannot be recommended in real life applications. Instead, the total shaft displacement can be used as a feedback. However, this is valid only if the shaft straightens fully during the backlash gap traverse. Luckily, this is often the case in real life vehicle applications due to stiff shafts.

Two active backlash compensation methods were designed and tested in Simulink environment. The first method was based on a custom control law. This resulted in an output feedback controller, namely a PD-controller. The backlash position angle, observed by the Kalman estimator, was used as a feedback signal to the controller. Thus, the motion control of the motor inertia was based on the knowledge of the backlash angle. This strategy worked well in simulations where the backlash position angle estimator was designed based on precise dynamics of the system. However, in practice the precise dynamics are not known and the model is only an assumption of the underlying physics. Thus, the unobservability of the backlash position angle results in uncertain feedback data. For this reason, this feedback strategy probably would not work in real world application. The backlash position angle signal as a feedback was replaced with the total shaft displacement signal. According to the simulation results, the controlled system achieves softer landing. Thus, the impact at the contact instant is avoided. However, the trade-off between the system response time and the soft landing has to be made when the actuator has certain torque limits.

The second compensation method was based on linear quadratic regulator (LQR). It was proven that the constructed model-based state-feedback control law resulted in exactly the same control law as obtained with the basic PD-controller. Thus, the PD-controller is truly the optimal controller for the backlash compensation. The tuning of the LQR is much more intuitive compared to PD-controller. For this reason, this controller configuration is highly recommended in real life powertrain applications.

Even though this thesis was aimed for electric vehicle powertrains the same algorithms and ideas can be utilized in versatile engineering applications. For example in crane applications, windmill applications and electric working machines where the backlash increases the powertrain oscillation phenomenon and may cause damage for any mechanical components. All these applications can be described by lumped two-mass model with flexible shaft. The fundamental requirement is that the powertrain is actuated by electric motor which enables fast and precise torque response.
The objectives of this thesis are considered fulfilled. According to the simulation results, the electric drive is able to compensate the backlash even though there would be multiple inertias in the powertrain. This is achieved with the lumped two-mass model. It does not matter how many inertias in the powertrain exist because the lumped two-mass model can still capture the main oscillation phenomena in the powertrain. All the inertias from the motor up to the hub of the wheel are lumped together and considered as a one motor inertia. The backlash is considered as a lumped total slack between the driveshaft and the hub of the wheel. Additionally, all the inertia after the backlash gap is lumped together and considered as a load inertia. Furthermore, the compensation is possible even though springback factors exist as seen in this thesis. If no spring back factors exist, the shaft stiffness can be considered infinite large in the constructed two-mass model. As with the inertias, also several spring back factors and damping coefficients are lumped together and considered as a one twisting shaft with damping.

The tuning of the control system is done manually. Either suitable pole positions are searched by tuning the PD-controller or then LQR-technique is used which is far more intuitive for practical engineers. However, both controllers need iterative tuning. Automatic controller tuning was finally considered to be outside of the scope of this thesis.

Thanks to model-based techniques, the represented solutions in this thesis can be used in versatile powertrain applications. This is done by fitting the two-mass model into specific powertrain through parameter identification as shown in this thesis. The unknown parameters are identified from the plant with simple step response tests.
Chapter 9

Future work

This thesis covered the theory of the active backlash compensation based on model-based techniques. This theory was applied to a real system, but the testing was done off-line due to missing measurement capabilities of HES880-inverter by ABB Oy. Furthermore, the implementation of these features for on-line use would have been time consuming. The on-line implementation brings certainly new hardware and software related problems that has to be solved. In order to stay within the given time limits of this thesis, the on-line implementation was forwarded to the future.

There are few suggestions for the future work in order to improve the results achieved in this thesis. The parameter identification was done with special equipment that takes care of the synchronization and interpolation of the position measurements between the pulses obtained from the encoders. This kind of functionality should be definitely implemented into the encoder interfaces provided by ABB Oy. This way, no special equipment would be needed. The identification process could be carried out with the HES880-inverter and no off-line processing would be needed. The parameter identification could be done during the commissioning of the drive or even during a normal use of the vehicle.

In this thesis, only the left front wheel position was measured and interpreted as a load position. The reason for this is that the measurement equipment supported only two super-counters. Third measurement should be available in order to utilize also the right front wheel position data. Measuring only the other front wheel position leads to erroneous load position measurement due to open differential functionality. There is no way to drive the vehicle perfectly straight which definitely makes the other wheel to travel a slightly different distance during the tests. This error affects directly to the identification results. However, by measuring both wheel positions and calculating the average position between them, this error can be cancelled. During
this thesis, the vehicle was driven several hours in order to get reasonable
data that was not affected significantly by the road bank and other environ-
mental factors that would lead to erroneous load position measurements. For
this reason, the second wheel position measurement is highly recommended
in order to decrease the time taken by the parameter identification tests.

In commercial vehicles the wheel speed data is often available due to
mandatory ABS-system. The wheel speed data is delivered into the CAN-
bus. The HES880 -drive could utilize this data directly from the CAN-
bus. However, one must notice that the data is not necessary synchronized
anymore. Additionally the fieldbus communication introduces delays which
should be taken into account in the state estimators. However, the utilization
of the CAN-bus interface should be definitely researched and tested in this
application.

Last but not least, this research could be extended by oscillation damping
control based on the constructed two-mass model. As seen in this thesis, the
backlash compensation does not remove the shuffle from the powertrain. In
contact mode, the oscillation damping algorithm could be applied to the
system. When the system goes into the backlash region, the active backlash
compensation would be activated.
Appendix A

Identification related data

Table A.1: Motor inertia measurements with 1. gear

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<th>Applied motor torque (% of nominal)</th>
<th>Motor inertia (kgm$^2$)</th>
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Avg: 0.1451
Table A.2: Motor inertia measurements with 2. gear

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APPENDIX A. IDENTIFICATION RELATED DATA

Estimation data: Time domain data Boxertrain
Data has 2 outputs, 2 inputs and [675 5326 6360 5350 4755 4263 3853 3567 3214 2900] samples.

ODE Function: GreyLagerbergStateModelDiscrete_twoinputs
Function type: 'cd'
Number of parameters: 7

Nonlinear least squares with automatically chosen line search method

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Result

Termination condition: Maximum number of iterations reached.
Number of iterations: 20, Number of function evaluations: 565

Status: Estimated using PEM with Focus = "prediction"

File to estimation data: [93.13 94.13 97.34 98.09 97.74 99.16 96.06 92.1 95.44 95.41:92.76 93.7 97.15 97.99 97.66 88.26 96.75 91.48 95.2 95.22] & TFE: 0.0409637

Figure A.1: Parameter identification process
Discrete-time linear grey-box model defined by
\[ x(t+T_s) = A \, x(t) + B \, u(t) \]
\[ y(t) = C \, x(t) + D \, u(t) \]

\[ A = \]
\[
\begin{array}{cccc}
  x_1 & x_2 & x_3 & x_4 \\
  x_1 & 0.9995 & 0.0009947 & 0.004339 & 4.429e-05 \\
  x_2 & -1 & 0.9892 & 8.662 & 0.08986 \\
  x_3 & 4.1e-06 & 4.185e-08 & 1 & 0.0009996 \\
  x_4 & 0.008185 & 8.49e-05 & -0.07087 & 0.9993 \\
\end{array}
\]

\[ B = \]
\[
\begin{array}{cc}
  Motor\; torque & Load\; torque \\
  x_1 & 3.408e-06 & -9.472e-11 \\
  x_2 & 0.006804 & -2.362e-07 \\
  x_3 & 9.472e-11 & -3.23e-09 \\
  x_4 & 2.862e-07 & -6.46e-06 \\
\end{array}
\]

\[ C = \]
\[
\begin{array}{cccc}
  x_1 & x_2 & x_3 & x_4 \\
  Motor\; pos & 1 & 0 & 0 & 0 \\
  Wheel\; pos & 0 & 0 & 1 & 0 \\
\end{array}
\]

\[ D = \]
\[
\begin{array}{cc}
  Motor\; torque & Load\; torque \\
  Motor\; pos & 0 & 0 \\
  Wheel\; pos & 0 & 0 \\
\end{array}
\]

Figure A.2: Identified state-space matrices
Bibliography


