Longevity Risk Securitization of Housing Reverse Mortgages in Future China

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China has gradually stepped into an aging society, and the living quality of the aged has been declining substantially due to the insufficient cash flow. The longevity risk is accelerated correspondingly. In this paper, housing reverse mortgage longevity risk securitization is introduced, and empirical analysis is conducted with the survivor bonds as the case. According to the longevity risk bond model, a sensitivity test is carried out on the survivor bonds by acknowledging the impact of mortality improvement on the investment income. This is seen to be of assistance for the design of future housing reverse mortgage longevity risk bonds.

The main body of this paper consists of six parts.

The first chapter is the introduction, covering a brief research background, research problems, assumptions and limitations.

The second chapter is the literature review. The development of the longevity risk market, management method, securitization products, longevity risk, pricing model of longevity risk derivatives and optimal allocation design have been reviewed.

The third chapter is the methodology including both qualitative and quantitative methods used in this study. The longevity risk bond type, structure and cash flow analysis of the current housing reverse mortgage loan is introduced with emphasis.

The fourth chapter is the overview of the background to study, with housing reverse mortgage, longevity risk securitization and the securitization of the housing reverse mortgage loan. Secondly, the meaning of longevity risk and mortality risk is illustrated broadly.

The fifth chapter is the empirical analysis of the impact of housing reverse mortgage loan longevity risk bond on the investment income. Sensitivity analysis is conducted for the natural mortality and average housing price growth rate.

The sixth chapter is the conclusion. It summarizes the main contribution, managerial conclusions, and puts forward some new prospects on future researches.

To conclude, throughout the sensitivity test, it can be found that the Lee-carter model is more dynamic, and can be modified to more forms in mortality prospect researches. The results also show that, there is a limited impact on investor’s returns along with the mortality improvement, which oversees that it fits Chinese market. At last, longevity risk bonds connected with enterprise annuity and equity incentive could be studied to meet the needs of financial markets in the near future.

**Keywords:** Longevity Risks, Housing Reverse Mortgage, Securitization, Longevity Risk Bonds
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CHAPTER 1 INTRODUCTION

1.1 Background and Research Problems

Compared with the data of 1990-1993, Chinese expected lifespan table of the life insurance industry during 2000 and 2003 showed that men lived 4.8 years longer and women lived 4.7 years longer (Wei Yingning, 2006). According to the data of China Statistical Annual Report of 2010, the average expected lifespan of the national population in 2000 was 71.4 years old, while it was only 68.55 years old in 1990. There was a 2.85-year-old improvement.

In addition, according to the 1% national population sampling survey data of 2005, there were more than 100 million people over the age of 65 in China, which accounts for 7.7% of the total population. The amount of the old people aged 60 and older made about 10.5% of China’s population, and the figure will climb to 28% or higher by 2050. According to the latest data published by China National Committee On Aging (2012, 1), by the end of 2011, the total population of elderly people (over 60) in China had been 184.99 million. Elderly dependency ratio to the total population was 19.67%. According to the UN statistical standards, if the over 60 elderly population reaches 10% of a country’s total population, or the over 65 elderly population reaches 7%, this country will be defined as a population aging country. Therefore, China is now a population aging country.

Meanwhile, however, China's average retirement age has not changed much since 1980s. The figure is 51.2 years old. The gap between average expected lifespan and retirement age has resulted a significant loss of wealth utility. The wealth utility loss includes a huge loss of China's social pension insurance expenditure, and also the intangible loss of life quality of the labor forces after their retirement. According to related documents of the Ministry of labor and social security, in 2000 "empty account" of China's pension is approximately 36 billion RMB, it has increased to 800
billion RMB till the end of 2005, which had made some great pressure on China's pension plan. On the individual level, the retirement period is too long. It is difficult for an individual to maintain reasonable life quality during this period, as the person lacks maintenance funds. As Liang & Emiliano’s (2007, 2) statement, the risk of outliving one's resources in old age, or longevity risk, is becoming increasingly relevant, both because of longer retirement and the global demographic shift towards older cohorts. Longevity risk is one of the main reasons leading to these conditions.

According to Richard et al. (2006, 551), longevity risk is a risk caused by a higher actual lifespan than expected.

In western countries, longevity risk has appeared for a long time due to earlier aging society. Many developed countries have launched a series of financial products to address longevity risks. These financial products include individual insurance, reinsurance products which aim at insurance companies and securitized products which lead to a better diversification of longevity risks. Most of these financial products were introduced at the beginning of 21st century. In the United States, the existing pension products have been unable to meet the needs of retirement community, given declining death and birth rates, accompanied by a lifespan extension and other factors. In this case, it follows that products designed by the insurance company which provide some measures of longevity insurance will enjoy increasing demands. Housing reverse mortgage is one class of such instruments.

According to the definition from Liang & Emiliano (2007,3), a housing reverse mortgage, or RM for short, is essentially a financial product that allows retirees to convert a proportion of the equity in their home into either a lump sum or annuity income and at the same time, to remain in their homes until they die, sell or vacate their homes to live elsewhere. Loans made through a reverse mortgage accrue with interest and are settled only upon the death of the borrower, sale of the property or on tenure surrender. There are no repayments made during the course of the loan, and no assets other than the home may be attached to debt repayment - the loan is
non-recourse. If at the time of settlement, the loan accrued with interest is larger than
the sale price of the property, then the provider (or lender), usually a bank or an
insurance company, recovers only up to the sale price of the property. The interest
rate charged on the accruing loan carries a premium to cover this risk.

As a financial instrument for pension, the reverse mortgage and related annuity
business have been increasingly popular since it improves the quality of retired life
around the globe. However, it also involves huge longevity risk at the same time.
More specifically, owing to the increase in average lifespan, the changes in interest
rates, inflation and the decrease in housing price, the housing value might be higher
than the terminal actual value so that insurance companies and the government will
face a range of pressure and loss. Therefore, western scholars have applied the
longevity risk securitization to the reverse mortgage market where investors share the
longevity risk in the market.

The financial market faces a series of urgent problems. The first problem in current
China is that there exists an unbalance between the demands and supplies in housing
reverse mortgage market. Actually there is yet no such mature insurance products for
the reverse mortgage in China; however, retirees do need them. The Baidu insurance
industry report in 2011 indicates that the daily search volume of life insurance and
endowment insurance in 2011 reached 46 thousand, which increased by 11.96%
year-on-year. The increase in demand for insurance products is mainly caused by the
improvements in network technology and marketing skills. Meanwhile, the population
is aging in China. By the middle of this century, there are expected 440 million old
people over 60. Since that China’s one child policy has been kept for more than 30
years, now most Chinese families are composed by 4 elders, 2 mid-aged persons and
1 child, which means young couples have to support 4 old parents. For an
individual, under the current situation, the burden of support elders is increasing
sharply. So elder people can not totally depend on their children. For China, the
pension mechanism is also facing severe challenges, which means that it is not
feasible only to depend on social or family pension insurance.
In contrast, by using the housing reverse mortgage, old people’s housing assets can be transferred into cash in advance, which provides them money to support themselves continuously, alleviating the pressure of national social security. Therefore, the reverse mortgage will gain its huge market in China because it can effectively solve the longevity risk brought by aging population.

Secondly, although we can design a range of RM products in China, on the perspective of providers, we have to consider the problem caused by mortality improvement. It is worth noting that the increasing lifespan will delay the settlement of the reverse mortgage loan and will therefore increase the rate of hitting the "crossover" mark. Thus we try to introduce a longevity risk securitization to avoid it.

Furthermore, to design a suitable RM product for China, a close sampling data and a longevity risk securitization model set up are essential. This paper demonstrates the correlation between the designed longevity risk securitization bonds and the Chinese RM market would be shown at last.

1.2 Research Objectives and Research Questions

1.21 Research Objectives

There is little comprehensive research on both reverse mortgage and securitization in China, but there is more research done individually on mortality prediction models, reverse mortgage and relative securitization. That being said, innovative and detailed research done overseas in longevity risk-linked field can be used for reference. Both domestic and overseas research will be reviewed in next chapter, which are strongly expected to support the empirical study in Chapter Five.

Secondly, it is hoped that the theories and empirical analysis can provide China’s financial market with some suggestions on the design and revision of housing reverse mortgage bonds in the near future. In the background of the reforms in China, the
government, insurance companies and capital market are looking forward to innovation and reconstruction on annuity production and pension mechanism.

Finally, the global visions on reinsurance and longevity risk securitization are demonstrated in the conclusion part. These visions involve only the macroeconomic levels because of indirect and remote acknowledgement. The limited visions and ideas are hopefully to be spread and taken advantage of essence.

1.22 Research Questions

Before discussing longevity risk securitization, there are some questions to ask, so as to achieve our goals. The questions are listed as below:

(1) Which advantages does the advanced Lee-carter model have, compared with simple prediction index model?

(2) How do investment revenues of housing reverse mortgage survivor bonds react to revised and predicted mortality improvement in Chinese market?

(3) Is it possible that survivor bonds can be optimized and derived to other kinds of products, like survivor swaps, mortality-index bonds or other innovative products?

1.3 Assumptions and Limitations

1.31 Assumptions

As discussed above, research questions should be resulted from deduction process on a basis of assumptions. Assuming that (1) The mortality development during 1997 and 2007 in China has consistent rate compared with future’s; (2) Insured group or borrowers’ lifespan coincides mortality improvement tendency; (3) Risk-free interest rate and growth rate of housing prices are constant; (4) Investment market is effective and efficient, it can be inferred that the mortality improvement makes limited influence on investment revenues, and housing reverse mortgage longevity risk bond is feasible and positively related to financial market in China.
1.32 Limitations

Above all, due to the weakness in the programming capacity and mathematical skill, the Wang Pricing Model of Convertible Bonds hasn’t been perfectly integrated in the paper. This restricts the ultimate results of sensitivity test.

Besides, the statistics data of domestic mortality in this study ranges from 1997 to 2007, which cannot be updated due to 10-year period. This maybe result in some mean errors for future mortality prediction, although correction has been considered via a prediction model.

On the other hand, the study involves in various academic sectors, such as insurance, asset securitization and financial bond pricing. The limited understanding of those knowledge may lead to errors and omissions, and for instance, there should have been more pricing methods and empirical study in bond pricing model especially.
CHAPTER 2 LITERATURE REVIEW

Globally speaking, current research on longevity risk is mainly focused on the following five aspects: longevity risk and market development of longevity risk, management tools for longevity risk, pension insurance and pension system longevity risk analysis, pricing model of longevity risk securitization derivatives and optimal configuration design, recommendations for pension system, longevity risk and capital accumulation.

2.1 Longevity Risk and Market Development of Longevity Risk

Insurance Risk Securitization is first introduced by American scholars Robert Goshay and Richard Sandor in a paper on derivatives published in 1973. Until the late 1980s and early 1990s, due to unexpected catastrophe, insurance company suffered huge losses and insurance securitization was then again taken seriously, in which case catastrophe risk securitization research has a higher propriety over longevity risk securitization. In the 1990s, the failure of Equitable Life Assurance Society largely shifted people’s attention on the importance of longevity risk. Since reinsurance did not effectively disperse the longevity risk and insurance and reinsurance companies suffered huge losses, studies of longevity risk securitization started to get attention.

Blake & Burrows (2001, 340) suggested that improved mortality is a huge challenge for the annuity and pension insurance providers, who will therefore suffer some losses as well as individuals. However, the issue of inflation-linked bonds by the U.S. government causes greater threat from improved mortality than the rate of inflation, and the Government indeed has a direct impact on improved mortality. Each year public healthcare and health programs help people live longer, making it more difficult for insurance actuaries predict future changes in mortality. The author defines such threat caused by difficulty to accurately estimate future mortality as mortality
Chapter 2 Literature Review

risk. Therefore, it is necessary for the government to help non-governmental insurance companies and individuals share risks posed by improved mortality; the government should also help providers to the introduction of a new type of bond, which can effectively inhibit mortality risk. Future coupons of the bond depend on the proportion of the still alive retirement population on expiration. The coupons will decline over the time, until the death of the last holder. Such type of bond has a groundbreaking name "survivor bonds".

Richard et al. (2006, 551) first pointed out that the cash-based national pension and company pension plans have put great financial pressure on the government and enterprises. The decline in birth rate and people's improved life span have, in many countries, induced the inversion phenomenon of age distribution. This proposed the definition of longevity risk, which refers to the risk that average life expectancy of the individual or the general population is higher than the expected life expectancy. The author has divided longevity risk into two parts: individual longevity risk and aggregate longevity risk. Individual longevity risk refers the over-spending of the actual accumulated wealth in one’s life time. management of the risk can be done through participation in pension insurance, such as participating in the government's social pension insurance, enterprise annuity, purchase annuity products from life insurance companies, etc.

Secondly, one of the authors, Blake, has done some research on the relationship between longevity risk and capital market, proposing that the capital market would provide a platform to transfer risk and capital for governments, businesses and individuals, shifting the risk to more individual investors. Similarly, individuals can use their income to buy some safe pension products in order to guarantee their own life after retirement. However, the existing financial tools are not effective in dispersion of aggregate longevity risk for institutions and enterprises. According to the Second International Longevity Risk and Capital Market Solutions Conference in 2006, David proposed some urgent problems, mainly in the following aspects: (1) To which grade does aggregate longevity risk influence the capital market? Who are the
main stakeholders? (2) How to make reasonable risk development? What are the better development for pricing and allocation of the risk? (3) Who should issue the tools for longevity risk in the capital market? Individuals? Government? Or enterprises? (4) Whether longevity risk is a data or mathematical problem to be solved? Or impossible to predict and control the problem? (5) If the insurance money will be paid to market participants, then in an imperfect competitive market, how to determine when the risk is at a premium?

Milevsky et al. (2006, 673) elaborated on definition of aggregate longevity risk in their article, suggesting aggregate longevity risk refers to the life expectancy of an average group life exceeds the expected number of years. Such a systemic risk cannot be dispersed according to the law of large numbers. Whether it is life insurance companies, pension plans of enterprises or the government's social insurance scheme, it is difficult to effectively manage aggregate longevity risk.

Eric (2006, 576) analyzed the cause for mispricing and prediction models in the existing longevity risk market, noting the cause is mainly due to these pricing and prediction models failed to take natural factors and improved longevity of average life expectancy into consideration. Moreover, he amended assumptions for pricing models and presented his own point of view: the uncertainty range of the retiree group is controllable while the future of the youth group is unpredictable, more researches on models are needed to support prediction for youth group.

A large part of the longevity risk study is about the mortality prediction model. Chen and Zhu (2009, 171) summarized two categories: static mortality model and dynamic mortality prediction model, further introducing the dynamic mortality prediction model under three models: the extension of the generalized linear model; Lee-Carter model and mortality prediction model with birth year effects.
2.2 Management Tools for Longevity Risk

Swiss Reinsurance in its 2006 advisory report noted that the current management measures for longevity risk include four categories: First, the self-retained longevity risk, which means the institution recognizes longevity risk as a normal commercial risk, and assume relevant responsibilities; Second, reinsurance, signing reinsurance agreements with reinsurers to transfer part or all of longevity risk to reinsurance companies; Third, adjust product design. For example, such as annuities, providers offer participatory annuity to transfer part of the longevity risk to the still surviving parties, which means the adjusting of mortality rate from predicted mortality at purchased moment to actual mortality; Fourth, transfer of longevity risk to the capital market based on innovative financial insurance products, such as longevity risk securities, or mortality and longevity index related securities and derivatives, including longevity bonds, annuities futures, annuity options, mortality rate swaps and long-term mortality.

2.2.1 Insurance and Reinsurance

Milevsky’s (2004, 674) analysis and research on the life insurance market found the low market share of longevity insurance products in the life insurance industry. He explained the reasons: (1) adverse selection: the annuity price is less sensitive to the characteristics of one’s life and information asymmetry between buyers and providers, people who expected he would not live long found that the price is too high and opted out of the market, thus causing price to rise further, forming a spiral effect; (2) bequest motive: retirees are not willing to transfer property for annuity because it cannot be regarded as an inheritance; (3) public welfare: public pension reduced annuity market. However, as countries continue to tighten pension policy, the impact is gradually weakened.

Purcal et al. (2003, 5) used random models to study the relative importance of these three factors, drawing the following conclusions: inheritance ranks the highest, followed by the exclusion of government pension, extra charge raised by adverse
selection remains the last, even with when added to 30%. The author also gives another two possible reasons, one is precautionary savings mainly in order to meet future contingencies (such as medical expenses and a favorable purchase opportunity) or unanticipated income delay; the other is personal short-sightedness and lack of understanding of the pension insurance products. American Life Insurance Association noted that very few people know about the insurance annuity benefits.

Milevsky (2004, 674) proposed a new longevity insurance product Alda, which is a postponed annuity revised according to the consumer price index. Taking account of the mortality credit the price of the product is very low and can be purchased with very little money at a young age.

Blake et al. (2006, 154) further described other ways for managing longevity risk, including: reinsurance; resale of the pension plan; limit the age of annuity purchase, for example, postpone the purchase of pension plan to retirement date; replace traditional non-profit annuities with profit-sharing annuities.

Da (2008, 3) compared the longevity risk insurance markets at home and abroad and found that existing concepts and methods were unable to stimulate the market demand of longevity insurance products. Therefore, “the government trigger index for market demand of longevity insurance products” has become the object of many academic researches. Related innovative research and the effect of government decision-making on longevity insurance products market demand are the two main focus for scholars.

### 2.2.2 Longevity risk securitization products

The first longevity risk securitization of financial contract originated in 1970 when the American Government National Mortgage Association issued mortgage mechanisms certificate of guarantee. In the late 1970s, the securitization of private housing mortgage contracts began to appear in dealing with the problem of mortgage fund shortage in the market. At the same time, demand of house owners and potential house owners for mortgages has exceeded the amount of market borrowings. Thus, Cummins & Lewis (2003, 1) suggested that financial markets need to find a more
effective way of financing to supplement the shortage of wealth to the borrowing funds of investors. From 1970s to 2002, the newly issued mortgage-backed securities (MBS) and asset-backed securities (ABS) reached $ 1.5 trillion and $ 450 billion.

Since then, the longevity risk securitization practice was officially introduced to the U.S. financial market, emerging a variety of securitized products. Such a boom was mainly due to two reasons: One was the tax reform in 1986, the U.S. government introduced new legislation to simplify the complexity of those securitized loan products, relieving tax burden; The second reason was proposed by Gorvett. The rapid development of computer technology had greatly facilitated the pricing and calculation of the specific needs of securitized products.

Subsequently, the mortality risk bonds began to be welcomed in the market. In the insurance sector, a large number of scholars did research on models and pricing. Cox et al. (2005, 4) discussed securitization of mortality risk in annuity business, which was the early study on longevity risk.

Blake & Burrous (2001, 342) first proposed the concept of "survivor bonds". Survivor bonds is an innovative bond, its future coupon depends the proportion of the retired survivors of the population on the release date with an uncertain expiration date. The purpose of this bond is to transfer the mortality risk the capital market. Since the change in mortality happened to have the opposite impact in life insurance and annuity business, a combination of the two can well hedge mortality risk.

Andrew et al. (2006, 428) made a brief introduction to existing securitized products of longevity risk, such as in December 2003, Swiss Reinsurance company issued a three-year mortality bond to reduce the risk of mortality deterioration. The mortality bond totaled $ 400 million with a quarterly interest based on a 3-month U.S. dollar LIBOR plus 135 basis points. In designing the bond, the way to deal with mortality risk is based on the mortality index changes to accordingly adjust the reimbursement of the principal amount that an unexpected mortality increase will lead to reduction in
reimbursement of principal. The mortality rate in this case is based on the collected data of the United States, Britain, France, Italy and Switzerland.

Cox et al. (2005, 4) displayed the mechanism of survivor bond and with a cash flow diagram, noting that survivor swap had a counter-party risk due to lack of the guarantee of a principal capital in comparison to survivor bond.

Andrew et al.(2006, 42) had a in-depth discussion on survivor bond, saying the design of the bond should consider the following factors: whether the bond is used as a longevity risk hedging tool; types of life insurance products with longevity risk, such as annuity products; choice of mortality index; payment function and its relationship with survival function. In conclusion, they suggested the following different forms of bonds: zero-coupon survivor bond; classic survivor bond; survivor bond involving principal risk; reverse survivor bond and mortgage bond.

Cox & Lin (2007, 34) described how to utilize mortality swap to achieve natural hedging effect. Blake et al.(2006, 154) introduced the application of survivor swap: there was a certain number of OTC survivor swaps but they were difficult to keep tracks.

Yu (2006, 1) drew on the successful experience of catastrophe bond and mortality index bonds, introducing of the concept of longevity risk securitization to present two mortality options and design the survivor bond, and finally set pricing by the Markoff process and random numerical simulation.

Zetao (2007, 4) introduced various forms of life insurance asset securitization and liability securitization. Furthermore, he analyzed the necessity and feasibility on life insurance securitization in China the on the basis of foreign experiences combined with China's national conditions, pointing out the main obstacles.

From the perspective of the balance sheet, Chuanguang (2008, 44) made an analysis on mortality risk management, introducing the mortality duration as a measure of mortality risk, and analyzing mortality-linked securities pricing and sensitivity.
2.2.3 Hedging risk of financial derivatives

Assuming a same amount of buyers for annuity insurance and mortality insurance, Milevsk & Promislow (2001, 674) analyzed the hedging measures for the two under fixed interest rate and random interest rate, pointing out that the portfolio of mortality insurance and zero-coupon bond fit for annuity products hedging.

Chen & Zhu (2009, 171) explained the advantage of longevity risk hedging techniques in comparison with other longevity risk management tools, such as lower cost, more competitive with risks, matching contract for longevity risk in the same period, accurate risk-avoiding and more flexible.

However, Cowley & Cummins (2005, 195) pointed out that there were differences between the buyers and the insurance policy usually wasn’t designed not for hedging purposes. Also there were no mortality products to hedge against private pension plans or state pension plan. Therefore, spontaneous hedging for longevity risk was difficult to achieve.

Leilei (2007, 30-38) deduced a theoretical model from the life insurance risk securitization practice, constructing an exotic option for mortality. She made assumptions, utilizing classic derivative pricing theory to work out price formula.

Liu et al. (2007, 17) researched on classified studies of longevity risk management, with particular reference to the use of financial derivatives to hedge against risk, mainly include insurance futures, options, etc., which had not lower than 20 years empirical data, using indexation conditions to hedge in the dimension of time or space to help buyers avoid risks. The article also make a simple analysis of foreign longevity insurance market, describing the experience of the Chicago Board of Trade (CBOT) and other insurance companies to provide such financial products.

In 2009, Goldman Sachs analyzed and exemplified the current innovative longevity risk products and made a specific model to explain the longevity risk index. Two of the new products at Goldman Sachs and JP Morgan were mentioned in the article, one was Goldman Sachs’s survivor QxX. LS index, with a monthly publish of the
expected number of survivors, investors could invest according to the actual number of death; the other one is JP Morgan Chase’s Life Metrics index, and a financial tool called "q forward" or "conditional probability of death".

Taking the UK market as the background Biffis & Blake (2010, 186-190) conducted a detailed analysis of the historical development and pricing of derivatives associated with longevity risk. There were several perspectives: a. Longevity risk transfer products; b. capital market solutions and mortality bonds and their derivatives; c. recent mortality products; d. mortality-related annuity products in the future.

2.3 Longevity Risk Analysis of Pension Insurance Products and Pension System

2.3.1 Longevity risk analysis of reverse mortgage

Chinloy & Megbolugbe (1994, 367-386) analyzed the reverse mortgage market and pointed out that reverse mortgage products involve a number of risks. Most of the loans face floating interest rates and loan rates have to be adjusted monthly or yearly according to market interest rate. However, when the loan rate exceeds the actual increase in house price the lending party will face a loss. This is called Crossover Risk. Their paper also presents the pricing model of reverse mortgage products to avoid such risk. The article was published when reverse mortgage research is still in its infancy. The concept of insurance securitization was not introduced.

Friedberg & Webb (2007,4-5) thought that reinsurance and insurance as traditional insurance measure in the past had dispersed more risk but was limited by various factors. The securitization of longevity risk was then introduced to design longevity bond pricing models, like Valdez Model and Wang Model. The article exemplified survivor bond and survivor swap to detect whether the two products can be effective in dispersing market risks.
Domestically, there was few researches on longevity risk securitization of reverse mortgages, the majority of which were introducing foreign innovative financial products. In the status and prospects of longevity risk market, Da (2008, 3) pointed out that reverse mortgages was the innovative financial product introduced by many countries to cope with the aging society in recent years. It was mainly launched by banks and life insurance companies. He also pointed out that the problem of the reverse mortgages in the Chinese market: only a small number of wealthy retirees had enough deposits to buy longevity insurance annuity. Even under mature payment provisions, it was difficult for low-income workers to accumulate sufficient savings to buy an annuity of 20 years or more. Except for the United States, which provides government incentives for buyers, in Australia, Canada, UK, Japan and other countries, the reverse mortgages market is still very limited.

2.3.2 Longevity risk research on personal annuity

Milevsky (2004, 674) proposed a universal model, which offers two options for retirees, asset allocation and free annuity. This design can meet people's altruistic need to freely distributing the heritage. It is also as an early possession of the consumer bonds, allowing individuals to invest their own budgets in the insurance market for other people to figure out the best time for annuity investment. At this point the consumption difference (the difference between actual spending and expected amount) became the focus of the study.

Scott (2007, 3) analyzed the market situation of the longevity risk of annuity products in the context of an aging society and limited consumer buying behavior. He pointed out that, in the U.S. market, the amount of investment is a major factor in the retiree's purchase of securitized annuity products. Therefore, the article put forward the optimal configuration pricing model, in which the results show that putting 10% -15% of investment capital on longevity risk annuity products is the most appropriate.

Chen and Zhu (2009, 172) discussed the aggregate longevity risk in the individual annuity products based on revised new life table, using actuarial methods to analyze
the impact of improved mortality in the new life table on individual annuity products. From the point of view where the life insurance premium was underestimated, the two redesigned of the pricing of annuity products according to China's actual situation and life table data.

2.4 Pricing Model and Optimal Configuration Design of Longevity Risk Securitization Derivatives

Since entering the 21st century, the pricing research of longevity risk securitization products has always been the focus of foreign scholars, and in recent years this study has also been increasingly favored by domestic scholars.

Wang (2000, 18) applied distorted probability distribution to finance and insurance pricing, proposing conversed probability distribution model--Wang conversion, and a comparative analysis of the CAPM model, Black-Scholes option pricing formula. The answers showed that Wang conversion could copy the results of CAPM model and Black-Scholes option pricing formula, thus Wang conversion is applicable to the financial and insurance risk pricing.

Lin & Cox (2005, 1) used Wang conversion for the pricing of longevity risk bonds. This method unified the pricing theories of insurance and financial industries.

Friedberg & Webb (2007, 15) applied Lee-Carter mortality model to quantify the mortality risk, and they found that: if the premium increased by 4.3%, there would be a 5% drop in the lack of solvency caused by uncertainty of the mortality. Insurance companies could transfer mortality risk through new mortality or bonds to financial markets, using CCAPM (Consumption Capital Asset Pricing Model) pricing model to determine the required risk premium for investing such securities.

Chen et al. (2010, 374-380) described the Lee-Charter model with mortality jump point. They added jump diffusion process to the Lee-Carter model to further illustrate
the model with the example of the issue of mortality bonds by Swiss Reinsurance Company.

Choi & Kim (2009, 887) designed a survivor bond with Percentile Tranching, and used the traditional Lee-Carter model to generate the distribution of future survival data.


Andrew et al. (2005, 41-44) introduced a stochastic model, assuming mortality had obeyed a stochastic model, and applied financial asset pricing theory to mortality risk, and then assessed the impact of mortality factors on preservation liabilities to study various pricing theoretical framework of such securities. Relating models consist of short-term mortality model, long-term mortality model, a positive mortality model and mortality market model. The introduction to a stochastic model for the securitization theory was a big step. The randomness of mortality represented the uncertainty of the future. Whether it was life insurance or annuity, random mortality was a very important factor.

2.5 Recommendations for Pension System, Longevity Risk and Capital Accumulation

The standard framework for studying relations between population and economy was proposed by Samuelson (1958, 467-482) and Diamond (1965, 1267-1150) by overlapping generations model (OLG).

Later Blanchard (1985, 223-247) used Yarri's (1965, 137) framework to establish a continuous-time OLG model, and gained some useful conclusions, but the heavy dependence on constant mortality assumption in model processing failed to simulate
longevity factor (decrease in mortality of the elderly) in microeconomic foundation of an ageing society. However, Diamond (1965, 11267-1150) and Blanchard (1985, 223-247) study found the inverse relationship between population growth and capital accumulation, which was a great help for later research.

D'Albis (2007, 411-434) applied OLG model to analyze the impact of demographic changes on economy and the long-term effect of exogenous change in birth rate on capital accumulation. However, the result showed that the effect could be either positive or negative, which was not a robust conclusion. In this regard, Lau (2009, 555) gave up D'Albis’s (2007, 411-434) mathematical content, achieving quantitative assessment with steady-state equilibrium. The result showed that there might be a positive relationship between population growth and capital accumulation, but such a relationship only existed within the population growth range in average industrialized countries. Diamond (1965, 1267-1150) and Blanchard (1985, 223-247) finding of the traditional inverse relationship between the two still remained.

In recent years, domestic scholars have widely used OLG model in research on reform of the pension insurance system, such as Yuan & Song (2000, 24-32). He (2001, 10), Feng (2004, 55-63) and Jiang (2010, 1) have respectively, from the perspective of optimal savings, welfare economics model, deduced some important conclusions.

Hu & Xu (2011, 1-13) researched on the longevity risk in pension system and capital accumulation using continuously varying mortality random OLG mode. Considering combined account of the pension system, the authors rebuilt dynamic life table, drawing that the increased longevity risk will improve capital accumulation, which was not reflected in the existing pension system in China. Thus, a simulated reform program was proposed in the article to show that the increase of personal accounts percentage in the account system will greatly improve capital accumulation.

The existence of longevity risk has an impact on annuity, pension plan, capital accumulation, and many other social problems. Both domestic and foreign scholars have invested in this study. Because reinsurance was not effective and timely in
dispersing longevity risk, more researches now are concentrated in "spread longevity risk and interest risk in the capital market", trying to get more investors to more effectively disperse the risk. Most scholars focus on the pricing model of securitized longevity risk innovative products, which include: survivor bond, survivor swap and other financial risk hedging derivatives. Conventional insurance pricing models are further developed on the use of data associated with mortality. Overall, pricing study accounts for a large proportion in study, while other researches are in the following order: annuity market securitization, financial securitization innovative products launched by reinsurance company, longevity risk securitization future outlook and reverse mortgage market longevity risk analysis.

In the annuity research area, domestic scholars have numerous studies on longevity risk, most of which are in line with the mortality data of national conditions. However, models used in these studies are mostly traditional models from abroad. There is less innovation here and in longevity risk securitization of reverse mortgage.
CHAPTER 3  METHODOLOGY

3.1 Methodology

For the reason that some big cities in China are now facing the problem of aging population, and the average retirement time is long, so the social basic pension insurance funds are facing big pressure. Since the funds supporting one’s life after retirement are born by the retiree, the individual and the society both desire a new pension mechanism for the aged. As it is not sustainable to live depending on pensions any longer, it is not only necessary but also marketable to rebuild the reverse mortgage which has been generally popular overseas. It is worth noting that when introducing the product, we have to consider the situation of Chinese financial and insurance market, in order to mitigate the risks on the government or providers.

This thesis is aiming to analyze how the housing reverse mortgage loan improves the utility of wealth after retirement, and how the securitization of longevity risks can diversify the existing longevity risks. A sensitivity test is made on the existing longevity risk bonds using the mortality data of China from 1997 to 2007 in order to know how the mortality rate influences these bonds, and whether the changes in mortality rate cause great losses to investors. Simultaneously, the typical overseas survivor bond model is applied as the basic of longevity risk bond pricing model. Since the writer has only limited capacity and there hasn’t appeared a true securitized product of longevity risks in China, the model of Lee-Carter is used to predict the improvement of mortality rates, so as to project the survivor bond model of reverse mortgage loans. On the basis of modified growth rate of mortality, it could work out the influences of the improvement of mortality rates on the revenue of the bond via longevity risk securitization.

The thesis mainly adopts two research methods. One is qualitative research, by which the thesis elaborates the impact of longevity risk, the development of the
Chapter 3 Methodology

securitization of housing reverse mortgage, and the traditional and advanced pricing model around the globe. The other one is quantitative analysis. The prediction of China’s mortality rates follows a deduction of existing and elementary expectations model. After that, a sensitivity test could be done to find out the degree of relevancy between the improvement of mortality rates and cash flow of the survivor bond. Finally it summarizes and analyzes the empirical results through the method of quantitative analysis.

On the respect of technology, so as to calculate the response statistics resulted from the sensitivity test, the model of mortality rates’ improvement and the pricing model of housing reverse mortgage loan bonds are programmed into a software developed by JAVA.

The content and the frame of the research is shown in Figure 3.1.

![Figure 3.1](image-url)
Chapter 3 Methodology

In view of coherence with empirical analysis, the specific and particular derivation method of mortality prediction model, reverse mortgage pricing model and survivor bond pricing model can be found at Chapter 5.1.

3.2 Management Methods of Longevity Risk

Before controlling longevity risk, the researches on the quantification and the measurement of longevity risk is very important. According to the quantitative model of longevity risk built by Lee and other scholars in 2011:

\[ LR_{x,t} = F_{x,t} \text{ Pro } (U_t < 0) \]  

In the above formula, \( LR_{x,t} \) represents the current magnitude of the longevity risk faced by the group aged \( x \) in the year \( t \); \( F_{x,t} \) represents the survival probability of the group aged \( x \) in the year \( t \) (\( F_{x,t} = 1 - m_{x,t} \)); \( \text{Pro}(U_t < 0) \) represents the probability that the wealth utility is less than 0.

\( U_t \), the function of wealth evolution in the formula (2) can be further represented as \( W_t \):

\[ W_t = W_{t-1}(1 + \alpha) + I_t - P_t y - C_t \]  

In the above formula, \( \alpha \) represents the natural growth rate of wealth, assuming it obeys normal distribution; \( y \) is a parameter obeying Poisson distribution, which represents the number of additional spending caused by each year’s appearances of accidental events such as major diseases and natural disasters; \( P_t \) represents the additional spending caused by the appearances of accidental events; \( I_t \) is other income such as social pension welfare and annuity benefits; \( C_t \) is the target amount of consuming in the year \( t \), including the spending of living, medical, and other routine expenses, depending on the target amount of consuming \( C_{t-1} \) in the year \( t-1 \) and the consumer price index (CPI).

To simplify the analysis, the linear utility function is used, assuming \( U_t = W_t \). Through the analysis it can be seen that when the amount of wealth \( W_t \) is less than 0,
the financial risk will appear. Through each coming year’s wealth level of the group aged $x$ obtained by the equation (3), combined with the data of mortality obtained by the equation (1), it is able to carry out a simulated calculation of the probability of the longevity risk occurrence.

Theoretically, the time of the longevity risk occurrence is the key factor in the design of retirement plans, pension provisions and the pension system, so in the established consumption targets and other constraints, the estimation of the first occurrence time of the longevity risk is of great significance. By changing the values of the parameters in the model and making simulated calculation of the results under different parameter values, the sensitivity analysis can be made to test the influence direction and extent of the model parameters on the longevity risk. This will help to deepen the understanding of longevity risk, so as to carry out better methods and tools to manage the longevity risk.

Currently, the methods used to mitigate the longevity risk are as follows:

(1) to control the longevity risk through the design of provisions;

(2) to transfer the risk to a third party through reinsurance;

(3) to share the risk of mortality with annuity policyholders;

(4) to use a natural hedge method;

(5) to hedge the risk through the use of financial derivatives;

(6) to transfer the risk to the capital markets through the securitization of the longevity risk.

The first method can only play an extremely limited role. One of the provision modifications can be to raise the premium, but the premium which is too high will affect the buying enthusiasm of the policyholders, leading to lower sales. Another provision modification can be to raise the pension age, which is to design an index according to the average lifespan of a specific population, and after a certain period of time, if the change in the index is greater than the specific range, the pension can be
Chapter 3 Methodology

received, but this method can only be used to control the debt. The last is to control the total amount of pension, which is to stop the supply of annuity when the annuity recipients of the pension plan reach a certain age, controlling the total amount in a specific range. However, since the problems caused by the longevity risk has stood out at that time, the aged with a longer life will need government subsidies, and otherwise they will have no income and be caught in a difficult life.

The second and third methods are currently the most used, and is the most important way to diversify the risk, but they are only partly transferring the risk to realize the purpose of risk mitigation, together with the disappearance of profits.

The method of hedge is still a major object of use and research, both in China and abroad. For the pension plans operated by insurance companies, the longevity risk increased the debt of pensions, but at the same time reduces the debt of term insurance policies due to the improvement of mortality level. Therefore, the insurance company can develop the scale of corresponding term insurance business based on its scale of pension business, making the influences of the mortality risk to both businesses cancel each other out. The disadvantage is that such a method will be effective only when an insurance company’s own annuity business and term insurance business are at a reasonable proportion, obviously unsuitable to the commercial insurance companies which only offer annuity services.

The last method is to transfer the longevity risk to the capital market through insurance risk securitization, which is obtaining funds from the capital market by means of securitized tools, issuing the corresponding securitized products in the capital market, thereby expanding the insurance company's underwriting capacity. Such securities include longevity bonds, mortality rate swaps, mortality rate futures, mortality rate options and other financial derivatives related to the longevity risk. The financial derivatives mentioned above mainly refers to insurance futures, options, etc. They are used to hedge the risks on the dimension of time or space in the condition of indexation, based on the long-term empirical data, longer than 20 years, so as to help
the policyholders to mitigate risks. For the longevity risk, the mortality rate of a specific population can be used as an indicator.

Huge capacity and all-sized investors in the capital market can effectively diversify the longevity risk. In addition, due to the lower cost of securitization and greater degree of risk transfer, it will be a good way of insurance risk transfer in the future.
CHAPTER 4 STUDY BACKGROUND AND OVERVIEW

It is in Europe continent and the United States where the reverse mortgage market is relatively more developed. In US, the concept of reverse mortgage emerged in 1979. It had been almost a decade before the concept of reverse mortgage was developed into a financial product and established a market, but in recent years, the market had witnessed the rapid evolvement of such products. Since 1990, many countries including Australia and Canada have introduced reverse mortgage products into their financial market in order to solve social pension problems. Inspite of the financial crisis happened in year 2009, the reverse mortgage market still showed great potential of development. It not only became the source of pension for old people, but also is promising in the sight of financial investors. In China, the concept of reverse mortgage has been studied by professionals, scholars and government agencies employees. The main point includes longevity risk and the impact of the product on the pension system. Nevertheless, the reverse mortgage products will attract a huge number of people in the future, due to China’s greying population and the huge number of real estate they own.

From the point of view of insurance companies, reverse mortgage has many risks itself, of which the most serious is longevity risk, followed by interest rate, house price, inflation risk, and moral risk. When the borrower lives longer than his expected lifespan, the insurance company might offer more money than the house (the collateral) worth, which means the lender might bear the loss. Therefore, the longevity risk is associated with the improvement of mortality rate.

The traditional methods of reducing the longevity risks are insurance and reinsurance. The US Home Equity Conversion Mortgage(HECM) is a typical example. In this project, the insurance companies who lend money are guaranteed by the government, which means that when the amount of loans excess the actual value of the house, the government will take the risks and loss instead of the insurance
company. In the long term, the government might face huge losses. This study analyzes the securitization of the longevity risks in reverse mortgage, and the main methods of securitization is issuing bonds in the capital market, which can transfer unpredictable mortality risk from the issuer (such as insurance companies) to the investors in the capital market, so as to reduce risks. Securitization as a form of financial innovation was originated from 1970s in the US. Securitization includes the packaging of capital pool or independent cash flow, and the transactions in the capital market. It broke the traditional financing channels of insurance and reinsurance. The successful issue of catastrophe risk bonds in 2000 proved the potential of risk securitization products.

In 2001, securitization of longevity risks was introduced into the reverse mortgage. Swiss Re and European Investment Bank issued mortality-linked bonds respectively in 2003 and 2004. Swiss Re issued a simple bond, which was designed for hedging the change towards disadvantageous direction of reverse mortgage revenue caused by the change of mortality. While, European Investment Bank issued an indexed survivor bonds linked with mortality.

Longevity risk bonds of the reverse mortgage can be divided into two types: the survivor bonds and the survivor swaps. Both kinds of bonds are related to the survival figure or index of the reverse mortgage borrowers. Cox & Lin (2007, 34) gave the pricing model of survivor bond in 2005. They did sensitivity analysis to each bond in order to test the impact the improvements of mortality had on bond prices, and it proved that securitization is an effective and economical method of hedging longevity risks.

This chapter discusses the background and gives an overview of Housing Reverse Mortgage Securitization.
4.1 Development of Housing Reverse Mortgage

Housing reverse mortgage means that the borrowers mortgage the house-ownership to the mortgage lender, converting the value of the house to cash without paying any interest or principal until the owner dies or permanently moves out, and then the house will be for sale, taking the proceeds to pay the loan principal and interest, with the remaining part inherited by the mortgagor’s children. Housing reverse mortgages provide the aged with a loan based on the value of their houses, allowing them to consume with cash. The concept and the nature of housing reverse mortgage determine the triangle connection of sales contract, mortgage agreement and mortgage contract. The mortgage can be paid in the form of a one-time payment, an annuity payment, a credential payment or a combination of the three forms above.

Housing reverse mortgage is different from traditional loan in the following aspects. Firstly, the principal and the interest of housing reverse mortgage do not need to be repaid, but the outstanding amount of the interest will generate and may generate the management fee until the owner dies or permanently moves out, with the returning funds coming from the sale of the house or assets. Secondly, although there is a repayment deadline on housing reverse mortgage, the more common practice is that the borrower lives in the house until death or an own initiative leaving. Thirdly, the loan is considered to be non-recourse, in which the funds cannot be repaid on the borrower’s other property. Fourthly, the loan is mainly provided to retirees instead of any age group. Fifthly, housing reverse mortgage is only guaranteed by the sale of the house. Typically, the maximum initial loan amount is indicated by the ratio of the loan amount and the value of the house. The ratio depends on the current value of the house. It is a deliberate contrast to traditional loan whose guaranteed amount often exceeds the value of assets.

At present, no matter which payment form is used, longevity risk should be considered as an important factor when the lender concludes the contract, because the expiration of the contract is based on the death time of the borrower.
4.1.1 Reverse Mortgage Market in America

Housing reverse mortgages originated in the United States in the 1970s. During the period of nearly 40 years, it developed from the initial theoretical research to the later reverse mortgage bond, from a policy to a financial product, to a great extent helping American society to solve the cash using difficulties during the pension period caused by the aging population. Its development and policies can provide references to China's future development of reverse mortgage.

In the 1970s, American reverse mortgage research was still in its early stages, mainly embodied as the theoretical research of this financial innovation, from the feasibility to the implementation. During the 1980s, A large number of "cash poor, property rich" appeared in the United States, which became a big pressure on the social pension security system of the U.S. government. In 1981, the U.S. government set up a special agency - the National Center of House Equity Conversion (NcHEc), which was a non-profit and independently-operated organization, whose main task was to popularize the financial product of the reverse mortgage to the citizens especially old people, and to carry out financial education in the whole country.

In 1987, home equity conversion mortgage (HECM) came out, which was a reverse mortgage guarantee program offered by the Federal Housing Administration (FHA) to...
Chapter 4 Study Background and Overview

The program eliminated some of the potential risks involved in lending parties, which had more advantages than other reverse mortgage plans. Other major forms of reverse mortgages included house owner funds provided by Fannie Mae and Cash Account Loans provided by Financial Freedom Senior Funding Corporation. Subsequently, the U.S. government passed the "National Housing Act of 1987", according to which the Federal Housing and Urban Development Administration jointly designed and developed a home equity conversion mortgage (HECM). HECM is a new product, whose target was the whole elderly group. To reduce the risk of the product, the Federal Housing Administration provided guarantees for the financial institutions which offered loans. In the selection of financial institutions, all the financial institutions which met the conditions had the right to operate such a business, greatly expanding the business range of reverse mortgage.

In 1989, Fannie Mae tried to securitize housing reverse mortgage assets and sold the securitized products in the secondary market, which increased the liquidity of housing reverse mortgage products and reduced the disadvantages of long period and high occupation of funds.

In the same year, in order to protect the interests of the borrower, the U.S. government laid down an optimized mechanism. To cancel the worries of the elderly people, the Federal Housing Administration provided HECM with government guarantees that if the financial institutions which offered loans collapsed, the borrowers could be compensated by this guarantee, which greatly enhanced the public's confidence in the product, making old people begin to actively participate in the housing reverse mortgage. The government also provided HECM product more protection. For example, the product was non-recourse, so in the maturity of the loan, if the borrower was still alive, the financial institutions were prohibited to sell the house for cash to recover the loan. No matter how long the borrower can live, he didn’t need to use other than the property mortgaged to repay housing loan. It also eliminated more concerns of borrowers, boosting the public’s demand for the product. To reduce the risk of financial institutions and minimize their unnecessary losses, the government
also provided guarantees for financial institutions. When the borrower lived longer than the loan maturity or the property devalued below the loan cost caused by interest rate increase or falling house price, the government would bear the loss. The government institutions charged premiums to both sides of the deal, in order to compensate for the losses of the borrower and the financial institution. When the funds are insufficient to cover the losses, the government would give supports.

The beginning of the 21st century was the period during which American reverse mortgage market experienced a high-speed development. At that time, the inflation had been effectively controlled, and the Federal Reserve was implementing a loose monetary policy, both of which promoted the development of the real estate market. At the same time, reverse mortgage market got a size effect to some extent through the integration of the industry. With the rapid development of reverse mortgage products, the number of credit issuers greatly increased, the market continued to mature, and the competitiveness strengthened. According to a data of the American Reverse Mortgage Association, the number of the financial institutions participating in reverse mortgages rose to more than 1300 in 2005 from less than 200 in the 1990s, still showing an amplifying trend. After that, 107,388 HECMs were issued in 2007, while in 2003 the number was less than 7000. The housing reverse mortgage market was developing rapidly.

Since the 2008 subprime mortgage crisis and the 2010 European debt crisis, reverse mortgage bond has become an overwhelming investment product, which has a character of low-risk, close to risk-free, and high-return, higher than the yields of demand deposits and US Treasuries. It is known that the number of the American families who participated in housing reverse mortgages in 2009 reached 1.1 million. The financial review of "Wall Street Journal" mentioned several times that reverse mortgages could still be favored by the market in a crisis and pointed out: In the context of European debt crisis, investors worry about the global economic slowdown and begin to seek the investment of safe assets, which boosts the market of reverse mortgage bonds which were considered as bad debts. Investors were looking for
mortgage bonds backed by the U.S. government which were considered as a safe haven from the global economic turmoil. This reversal of fortune helped to drive mortgage rates for consumers to record lows. The mortgage bonds issued by Freddie Mac and Fannie Mae and other government-backed agencies, which had the implicit government guarantees were considered by investors as a quasi-governmental investment tools which got better guarantees than before. Barclays Capital MBS Index showed that the price of mortgage bonds was close to the record high of 20 years ago.

The successful experience of the United States can be simply summarized as four points. Firstly, education and training: the concept of reverse mortgage was penetrating into people’s life, especially the elderly targeted education and training made the aged more receptive to the product. Secondly, government support: government support, especially in the start-up period, not only protected the borrowers but also help the institutions. Thirdly, financial professionals: the unique American financial professionals and mature financial markets made it possible to use financial engineering, actuary and other means, based on a set of quite complete data, so as to make out feasible options and develop a variety of products, which not only met the diversified needs of the public, but also made the risks under control in the stage of product design, greatly reducing the risks the financial institutions may face. Fourthly, complete financial market: a sound financial market offered a good platform to financial derivatives, provided a large amount of reference data for the development of housing reverse mortgage in different stages, produced various risk diversification tools, including securitization, as well as ensured the cooperation in an orderly way among the financial institutions which carried out reverse mortgage business.

4.1.2 Reverse Mortgage Markets in Other Developed Countries

The reverse mortgage in Canada is very similar to that in the United States, called the Canadian Home Income Plan (CHIP). It is the leading Canadian reverse mortgage
program, provided by the Canadian private company. CHIP does not offer the loan in the form of annuity, and the elderly homeowner can get a one-time cash flow which equals 10% to 40% of the value of the house, the loan collateral. The interest rate is 1.5 percentage points higher than the conventional five-year mortgage rate, with compound interest every six months. The loan needs to be repaid only when the borrower dies or the homeowner changes. In addition to CHIP, Canada has another two reverse mortgage plans, which are Home Fund of Credit Reverse Mortgage and Fixed Term Reverse Annuity Mortgage. One of the features of reverse mortgages in Canada is operating without government involvement.

The reverse mortgage market in Britain relatively is not better developed than the United States and Canada. The most important reverse mortgage product there is the Equity Release Mechanism. British reverse mortgages are one-time loans, without the form of annuity. If the loan is used for investment, the proceeds will be treated as taxable income. The interest rate is fixed, currently 7%, higher than traditional mortgage rates. Its clause of "guarantee exemption of negative pledge" eliminates the possibility of the elderly borrower being driven out of the house when the value of the house is less than the sum of the loan principal and interest, but the borrower need to bear a higher rate. British reverse mortgages are not under supervision of the financial services authority.

The form of reverse mortgage in France is basically a private contract between a buyer and a seller. Firstly the Buyer pays the down payment to the seller, and then monthly pay the living expenses. Once the seller dies, the buyer will get the right to the property. This form is called "Rente Viager" in France.

According to Hongwei’s study (2012, 2), in Asia, Japan has 30% of its aging population getting "House-For-Pension" realized. Japan's first housing reverse mortgage in Musashino City in 1981, called "long-life volunteer fund loan system", which is organized by the local government, trust banks and other market participants. According to different implementing subjects, there are three forms, which are autonomous body dominated by the government (government agencies combined with
Japanese housing reverse mortgage has its own characteristics, and the main product is trust-based reverse mortgage. In the pilot phase in 1981, trust services were involved. After that, various trust-based reverse mortgage services were gradually developed. Trust-based reverse mortgage is called "assets-utilized bank loan" or "fiduciary reverse mortgage". In order to develop the business further, the Japanese Trust Economic Research Association has attempted to put forward a new framework of trust-based reverse mortgage, in which the trust bank uses the real estate to produce annuity of financial assets, the lender offers the medical fund to the borrower, and at the same time securitizes the annuity of financial assets, introducing securitized products to the investors in the trading market.

In addition, trust benefits based on trust assets out can be divided into beneficial interest and usufruct. The beneficial interest means that the principal (the elderly) can still live in the house mortgaged to the institution in order to get living and medical expenses. The usufruct means that the beneficiary is entitled to the allocation of proceeds generated from trust property. Therefore, how to build the circulation market of the benefits is more important reverse mortgage.

4.1.3 Advantages and Risks of Reverse Mortgage

From the borrower's points of view, the most obvious advantage of the reverse mortgage is that the borrower doesn’t need to repay the loan until he dies or leaves the house. Compared with traditional mortgage loans, the reverse mortgage offers the aged with a tool to hedge the risk of old age through maintaining a stable retirement income level, and at the same time keeps the long-term right of habitation. The next advantage of reverse mortgage is non-recourse provisions. When the loan is terminated, the borrower only needs to obtain the funds by selling the property to repay the loan principal and interest.
Although the reverse mortgage provides the borrower with many attractive benefits, it contains many risks to the lender. As mentioned above, the risk of reverse mortgage can be summarized as "cross risk". If the total amount of loan exceeds the value of the mortgaged house, the difference between the loan and the value of the real estate is a loss to the lender. The interest rate is often higher than the rate of the increase in the home value, so the loan value will certainly exceed the home value at a future point, which is the intersection point. On the other hand, if the loan is terminated before the intersection point, the excess amount of home sale will be returned to the borrower (or his heirs), rather than becoming the lender’s profits. This feature of reverse mortgage is similar to the provisions of options.

![Diagram showing cross risk and loan amounts over years.](image)

**Figure 4.2 Cross Risk (Wang & Li, 2008, 42)**

To illustrate the result of the lender’s loss, next give a simple example. Assuming the annual interest rate is 6%, the annual housing appreciation rate is 3%, and the ratio of the loan to the home value is 50%. A 62-year-old man obtains an one-time loan of 2
Cross risk is a combination of three basic risks which are longevity risk, interest rate risk and house price risk.

Longevity risk, also called occupancy risk. Occupancy risk is a situation of the accumulated loan value exceeding the house value caused by the borrower’s living
too long in the mortgaged house. The total repayment rate is a combination of mortality and turnover rate. Although the decision to move or repay can be influenced by the real estate market and the interest rate, the real attraction of reverse mortgage is that the product allows the borrower usually in the condition of low income or long-term residence to live in his house until death. Thus, the duration of the loan depends on the mortality. Due to the significant improvement in mortality since the 1970s, occupancy risk has become the most important risk of reverse mortgage products.

Interest rate risk and house price growth risk. Since the loan repayment is capped at the house value, high interest rate environment and sluggish real estate market will undoubtedly exacerbate the cross risk. The difference between interest rate risk and house price risk is that, interest rate risk can be diversified, while the house price risk can be partly diversified by holding a large number of mortgage loans in different regions.

Other risks. In addition to the three main risks, other major risks include maintenance risk and expenses risk. Maintenance risk is a risk of moral hazard, which means that in a reverse mortgage, although the borrower knows that the lender bears the risk of falling house price in the resale of the house, he doesn’t try his best to maintain the house price. The discussion of maintenance risk can be found in the researches of Miceli & Sirmans (1995, 83), as well as Shiller & Weiss (1998, 23). In addition, due to inflation, the expenses of the reverse mortgage portfolio management may be higher than the original expected costs, which causes the expenses risk.

4.2 China’s Housing Future Reverse Mortgage Market

4.2.1 Demand for Longevity Risk Securitization

The data of the sixth National Census shows that, up to 2010, China’s average life expectancy had reached 74.83 years old, 3 years older than that in 2000. In 2010, the average life expectancy of the world's population was 69.6 years old, in which that of
high-income countries or regions was 79.8 years old, while that of middle-income countries or regions was 69.1 years old. Thus, the average life expectancy of China is not only significantly higher than that of middle-income countries or regions, but also much higher than the world average, but is still about 5 years lower than that of high-income countries or regions.

In Western countries, due to the earlier retirement and the longer life expectancy, the majority of people meet the problem of "Asset-rich and Cash-poor" when they get old, which is that, the individual fixed asset is abundant while the accumulated wealth and cash is inadequate, resulting in worse quality of living after retirement. These phenomena appeared earlier in Western developed countries, so they started to think earlier how to solve the problem, called "longevity risk". According to the definition proposed by Ricard et al. (2006, 6), longevity risk is the risk of a shortage of wealth due to the individual’s or the general population’s longer actual life span than the life expectancy in the future. That is, longevity risk mainly comes from two aspects: one is the increase in life span due to the decline in mortality; the other is the change of the individual’s or the social group’s self wealth utility.

In China, due to the long-term implementation of the one-child policy, the traditional family structure has changed radically in recent years. The inverted-pyramid-shaped family structure of 4: 2: 1 and 8: 4: 2: 1 has become the pressure most families have to meet. In addition to the elderly, the wealth effect of the next generation will suffer losses.

On the other hand, with the increase in the average life expectancy, a dramatic aging of the population is exacerbating the impact of longevity risk to the social and financial system. For example, the annuity insurer may face the risk of paying beyond the expected level. American Life Insurance Company has ever suffered heavy losses because of neglecting the longevity risk. Between 1957 and 1988, the company sold a large number of participating endowment insurance products based on fixed interest rates, but was led to a serious financial crisis due to the decline of the interest rate and
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the improvement of the mortality in the 1990s. Obviously, the longevity risk gave the insurance company a heavy blow.

Longevity risk will not only affect the personal life of the elderly, but also bring huge challenges to a country’s social endowment insurance fund. According to "Global Financial Stability Report" released by International Monetary Fund on August 11, 2012, if the average life span is 3 years longer than the current expectation in 2050, the huge costs of aging population will meet a 50% increase further. The financial impact of longevity risk will cover most of the major countries. even if the mitigation measures are taken, the effects will be visible in a few years.

In China, due to the differences of urban development among different cities, the aging of population in some developed regions is more severe. Yangtze River Delta region is one of the most serious areas of aging in China. "Monitoring Statistics of Shanghai’s Aging Population and Aging Business 2011" shows that, up to December 31, 2011, in the registered population of Shanghai, people aged 60 and above reached 3.4776 million, an year-on-year increase of 5.1%, accounting for 24.5% of the total population. In addition, the coefficient of old age support of Shanghai’s working population aged 15 to 59 supporting people aged 60 and above increased by 2.2 percentage points over the previous year, which means further pressure of maintenance.

As another example, up to the end of 2011, Hangzhou’s population aged 60 and above reached 1.2219 million, accounting for 17.53% of the total population. The number old people living in pure elderly households reached 201.8 thousand.

Local governments have been taking active measures to solve the resulting pension problem such as the pension property. The pension houses have been built in some remote areas in Jiangsu Province and Zhejiang Province, but they are only targeted at certain groups of people, especially those who have the ability to afford the houses. For the general population that are unable to afford the houses, the government can only take the traditional method of pension funds. But currently, the problem of
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Pension gap in China is very serious. According to the Human Resources and Social Security Ministry, in 2010, rejecting the financial subsidies of 195.4 billion Yuan, there were 15 provinces, including Xinjiang Production and Construction Corps, whose corporate sectors’ current basic pension insurance funds met deficits, with a gap of up to 67.9 billion Yuan. It means that the potential financial risk of Chinese pension system has become more and more explicit, even to the point of being difficult to leave the financial subsidies. Deutsche Bank and Bank of China predicted that, China's pension gap would reach 18.3 trillion Yuan. Currently, the population aged over 60 account for 13% of the total population of China, and the World Bank predicts that the proportion will rise to 34% by 2050.

To mitigate the financial impact of longevity risk, the available measures that can be taken currently are: raising the retirement age (statutorily or voluntarily), increasing the proportion of pension plan contributions, and reducing the amount of benefits payment. However, these measures cannot fundamentally change the existing problems, on the contrary can increase the burden on individuals and the society, resulting in the deterioration of people’s quality of life, and at the same time the stress put on enterprises and the government cannot be reduced.

One of the implementing conditions of reverse mortgage is the house, generally requiring the borrower to have the absolute property right. In the market economy, China's urban elderly people have an optimistic situation of housing, and both the development of real estate and urbanization make it possible for China to implement reverse mortgage in the future.

There was a conflict between supply and demand in China’s housing allocation model, more and more unsuitable for the market economy environment, putting the government under the financial pressure. Since China began the economic reform in 1979, the government has introduced a reform of the housing system aimed at the housing problems with numerous contradictions. The housing reform plan starting from 1998 abolished the existing welfare housing distribution system to promote China's housing investment. The main achievements of the reform include: firstly,
through substantial sales at discounts, about 50 percent of the government’s public housing property rights were transferred to the urban residents in five years from 1998 to 2003, making the urban residents obtain a 22% net increase in income in the five years and changing the asset structure of urban residents; secondly, the number of urban residents who had home-ownership rose nearly 100% from 1998 to 2003, with a proportion of 80% in many big cities, even higher than the average level of about 65% in the United States and other industrialized countries.

The stock of public houses in 2002 was about 3 billion square meters, accounting for 64% of the total area of urban houses, 90% of which were sold. According to the expectation of Qu Hongbin in 2003, the housing property with a market value of more than 3.2 trillion Yuan had been sold at a very favorable price from the government to the public. In addition, while the public houses were sold, the commercialization of housing were promoted, and in the sales of the commercial houses, the proportion of individuals who purchase houses was more than 95%. The proportion of residents owning private houses reached more than 80%, and the value of housing property became the most important form of assets instead of bank deposits. The share of private housing assets in the urban household assets reached 47%, and the rapid accumulation of urban wealth promoted the growth of China's total urban consumption.

The comprehensive housing marketization opened the floodgates of housing demand. The house prices of different areas rose up in varying degrees, and Beijing, Shanghai, Shenzhen and other cities achieved double-digit annual rises. The development of secondary housing market also followed. The transactions of housing stock became more and more active, and the real estate agencies and property management developed rapidly, with continually increasing service items, resulting in the basic establishment of the real estate market service system.

The reform of the housing system, the establishment and prosperity of the real estate market, and the increase of property values made it possible to develop housing reverse mortgages.
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Firstly, urbanization provided a wide market for reverse mortgages.

Since the reform and opening up, China's economy has turned around completely. Urbanization is one of the most significant matters in China's economic and social development. According to the related statistics, the urbanization rate rose from 17.9% in 1978 to 33.35% in 1998, with an average annual increase of 0.77 percentage points. During the “Tenth Five-Year-Plan”, the level of urbanization in China has been greatly improved. By the end of 2004, China's urban population reached 542.83 million, accounting for 41.76% in the total population, and the built-up area expanded to 30,000 square kilometers. Zhou Yixing, Professor of Environmental School, Peking University, pointed out that, the pace of future urbanization should be: the proportion of the urban population would be about 46.50% in 2010, maybe above 50% in 2014, 57% in 2020 and above 60% in 2023.

With the development of urbanization, the rural population has continued to flood into cities, causing a big growth in the urban population, which means that the demand for urban housing will not reduce for a long period of time in the future, and the value of urban housing will keep a trend of steady appreciation.

One of the mortgage terms of reverse mortgage is that the real estate as a collateral must have a market value. On the current situation, to determine the market value of the house, only the market value of the houses in cities can be evaluated, because the houses in cities have the liquidity and the rental value, and urban constructions are in line with the standards established by the relevant departments, guaranteeing the housing quality. Therefore, the implementation of reverse mortgages at the present stage is more suitable in cities, and the urbanization provides a broad space for reverse mortgages.

Secondly, the big number of elderly people living alone made it possible to implement reverse mortgages.

The number of elderly people, especially those who have undisposed houses, basic condition of the reverse mortgage promotion.
Due to frequent job changes, accelerated population movements, improved living standards, pursuits of high-quality spiritual life and other factors, the structure of families is changing from the traditional extended family to the nuclear family, the number of elderly people living alone is increasing consistently. According to the statistics of 2003, in China's urban elderly people aged over 60, households with elderly couples accounted for 20.35% of the total number of households, and households with elderly singles accounted for 11.97%, for a total of 32.32%. According to the data of the fifth national census, in China's urban elderly people aged over 60, the number of living alone accounted for 52.95%, often called empty nest families. The empty nest family is the family in which the children move out when growing up, separating from their parents, leaving the old parents living alone.

The increase of empty nest families indicates that most elderly people over the age of 60 have fixed residences, and the living conditions have generally improved. Meanwhile, due to the reform of the housing system, the elderly can dispose their houses alone, which provides the possibility for reverse mortgages.

Since the longevity risk threatens various insurance institutions, both in western countries and in China, many scholars have made in-depth researches on the methods for managing the longevity risk. First of all, the measurement and identification of the longevity risk require more accurate forecasts, which have the most direct relationship with mortality. Currently, the most intuitive data mainly comes from the results of census and the insurance mortality tables. However, these two sources of data have some limitations. The results of census reflect the historical data, while the insurance mortality tables reflect the empirical data of the insured, neither of which has a wide representation, as well as reflects the future development trend of mortality. To solve this problem, foreign researchers have started researches on the prediction model of mortality.

Moreover, the traditional official data of mortality are often found to have underestimated the mortality decline, and on the contrary, overestimated the increase in life expectancy, especially in low-mortality countries.
At present, the most widely used prediction model of future mortality is the Lee-Carter model. This method is mainly used among OECD countries. For the United States and the G7 countries, this method is more often used to predict the life expectancy than the traditional methods. But for the countries with limited data, such as Third World countries, it is difficult to use the Lee-Carter model. For example, the mortality data of a certain age in China are relatively limited.

The main idea of the Lee-Carter model is to decompose the changes in mortality into time factor $t$ and age factor $x$. If $m_{x,t}$ represents the median mortality of the group aged at $x$ when the time is $t$, then $m_{x,t}$ satisfies the following functional relationship:

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{x,t}$$  \hspace{1cm} (1)

In the above formula, $\alpha_x$ is an age-factor-dependent parameter, reflecting the average level of natural logarithm of a specific age group’s mortality; $k_t$ is a time-factor-dependent parameter, usually called Mortality Index, reflecting the changing rate of mortality over the time; $\beta_x$ represents the sensitivity of the age factor to $k_t$; $\varepsilon_{x,t}$ is an error term with the mean of 0 and the variance of $\sigma^2$.

The prediction of mortality is the basis of the accuracy of longevity risk measurement, and is also the data base of various models of longevity risk securitization.

The first securitized product contract of longevity risk was born in the United States in 1970, when the Government National Mortgage Association began to sell mortgage transfer certificates. In the 1970s, private mortgage securitization appeared, aimed at the shortage of housing loan supply in the U.S.. The demand of house owners and potential owners for housing loans exceeded the ability of banks and other institutions, leading to a need of the financial market to find a more efficient way in order to transfer funds from the investors in the capital market to those who needed loans.

The rapid growth of the financial market securitization bonds made a variety of bonds emerge. The reasons for this phenomenon were attributed to two factors: the first was the change of the tax system in 1986, when the United States adopted a new
legislation to simplify the tax structure referred to complex mortgage securitization; the other was the rapid development of computer technology, the development and application of which made all kinds of bonds have the opportunity to grow, because the pricing models required complex calculations. After mortgage-backed bonds appeared, people began to buy a lot of bond contracts. By 2002, newly issued mortgage-backed securities (MBS) and asset-backed securities (ABS) respectively reached $1.5 trillion and $450 billion. In the second quarter of 2003, new bonds of $6.6 trillion were issued. Obviously, the insurance securitization began to boom in the financial market.

European governments and the U.S. government launched an unprecedented reform of financial services and risk securitization. Some non-traditional issuers, including multinational investment banks, commercial banks, intermediary business companies and private investors, were also keen to join into this market. This phenomenon also made life insurance product issuers have a sense of crisis, because the cake belonging to their own industry was divided up, with the sharp drop in product prices and profits. The financial services were continually consolidated and squeezed, making some financial groups finance more efficiently in the capital market. At the same time, the enterprise reform of the traditional financial industry promoted the securitization of the insurance industry. The insurers under this pressure were forced to focus on the optimization of the capital structure so as to maximize the shareholder value, and securitization could provide a good mechanism to improve the capital efficiency.

In China, the longevity risk securitization also needs to be considered and developed. The data from Ministry of Human Resources and Social Security showed that, by the end of 2009, the number of people participating in the urban basic endowment insurance, basic medical insurance and unemployment insurance were 235 million, 400 million and 127 million respectively. In other words, nearly 70% of China's population were not included in the pension insurance system, which indicated on one hand that the social security of Chinese people was still in a very low level, with a very limited coverage, and on the other hand that the financial
products aimed at the longevity risk would have a broad market space, such as annuity products, life insurance products and securitized products, etc.

Huge pension deficits makes the government and enterprises have to reform the existing pension policy, such as reducing the allowance standard, encouraging delayed retirement and increasing the pension age, etc. In order to get out of the trouble, many companies gradually transfer defined benefit plans into defined contribution plans, or directly buy out the pension plans. These measures will reduce the protection efforts of the social security system. However, due to the improvement of the level of social life, the original basic pensions and the supplementary pensions from companies will no longer meet the needs of the elderly. Coupled with the current trend of aging population which is more and more serious, the markets of individual annuity and group annuity will have a great potential for development.

The duration of longevity risk is very long, which usually lasts for 20-30 years, and some even for 50 years. Given that the longest duration of bonds in the current capital market is no more than 30 years, and the risk of stock investment is too great, only a small part of the annuity funds can be invested in the stock market. Therefore, a large part of the annuity funds cannot be matched with the current capital market, making the insurance companies and the pension providers bear a big tail risk. Currently, there are two ways to control the longevity risk: balancing the product structure and reinsurance. Due to the different product characteristics of life insurance and survival insurance, insurance companies usually control the longevity risk by adjusting the portfolio of a variety of products. However, life insurance and survival insurance face different customers, so the methods of controlling the product ratio have the basis risk and the adverse selection risk. Moreover, reinsurance companies are not keen on the reinsurance of the longevity risk, partly because the duration of the longevity risk is too long and the future mortality is unpredictable, and on the other hand, reinsurance companies must withdraw reserves according to the rules for the future compensations, having only a limited capacity of longevity risk control, so reinsurance is not an effective way to solve the problem of longevity risk, either.
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The investors who have the potential demand for longevity risk bonds contain hedgers, general investors, speculators, governments, regulators and other investors. Typically, investors buy longevity risk bonds mainly because mortality risk has little correlation with other risks in the financial market, so as long as the expected yield is reasonable, there will be many investors choosing mortality related securities (including longevity risk bonds) into their portfolios, in order to realize the risk diversification of diversified investments. Speculators hope to profit from abnormal price movements in the related securities. The government participates in the securities market related to mortality usually for the following reasons: the government can support the mortality risk securitization, regarding it as a positive behavior, or as "the last insurer", offer guarantees to insurance companies and annuity companies, reducing the probability of default to a certain extent.

4.2.2 Existing Problems and Risks

Compared with American reverse mortgage problems and phenomena at the exploratory stage, the problems of reverse mortgages, China may face are as followed:

Firstly, the publicity of housing reverse mortgages is unreasonable or the effect of publicity is disappointed. Reverse mortgage is a new kind of financial product. Most people know little about it, and it is difficult for people to understand a new concept, especially the elderly. In contemporary China, the elderly people have an inadequate reserve of knowledge, and do not understand the innovative financial products as well as their operational procedures, so it is very difficult for them to accept new things.

Secondly, the current trend of China's aging of population is more and more serious, and the social old-age security system is taking more and more responsibility for the poor elderly people. To develop a reverse mortgage program, the initial capital cost is very high, and the liquidity difficulty may be easily faced. But with the gradual opening up of pension funds to financial institutions, reverse mortgages will become objects of trying wealth management by more institutions.
Thirdly, the number of talents who master financial knowledge reserved in Chinese financial institutions is small, and the researchers with actuarial capabilities are very limited. Meanwhile, the financial managers who popularize the knowledge to the elderly need to have the relevant knowledge and accurate understanding of risks, and remind the elderly to pay attention to the risks of reverse mortgage products.

Fourthly, the policy level. At present, China's financial system is implementing strict separate operations, so insurance companies are still not allowed to run reverse mortgage business. However, reverse mortgage longevity risk bonds are issued in the capital market, and financial institutions are actively involved in this market, competing and sharing the benefits.

Fifthly, in Britain and France, the reverse mortgage business is mainly operated by insurance companies, and in the United States, this kind of business is more often taken by banks. In China, commercial banks still cannot undertake this task, because reverse mortgages require commercial banks to operate real estate business, which has more difficulties and risks.

Sixthly, elderly people are difficult to change the deep-rooted thought of "living on pension" model, so it is difficult for them to accept the fact that the houses become collaterals. It will take time to do propaganda work and change their mind.

### 4.3 Securitization of Reverse Mortgage’s Longevity Risk

Insurance industry’s interaction with the capital market has provided a great opportunity to the securitization of longevity risk, and therefore a variety of longevity risk securitized products have appeared. Longevity risk securitized products include the products similar to "mortality bonds" and "longevity bonds". If the time of death is earlier than expected, the values of "mortality bonds" will fall, but the values of "longevity bonds" will change in the opposite direction. Deutsche Bank, BNP Paribas and other financial institutions are in the research and development of so-called "mortality derivatives". At present, these products contain mortality index bonds,
survivor bonds, survivor swaps, etc.

4.3.1 Types of Longevity Risk Bonds

1. Mortality Index Bond

Mortality index bond is structured as follows: a standard bond is issued, and the mortality index is taken as a reference to determine the payment of bond principal and interest. When the mortality rate is below a certain value, the coupon will be reduced or to zero; when the bond matures, the principal will be reduced or to zero if the mortality rate is below a certain value. The reduced amount of coupon and principal will be used to compensate for the loss of the insurance company, thereby reducing the impact of longevity risk.

2. Survivor Bond

Survivor bond is generally considered as a mortality-related insurance product. Its payment is related with the proportion of the specific population who still survive in the future. In order to hedge the risk, the actual survival rate of customers is higher than originally expected, so the coupon will be honored only when more people survive.

This kind of bond is similar to the mortality index bond. According to the design of Blake & Burrows (2001, 340), relating the coupon payment of the survivor bond after 20 years to the proportion of the elderly people aged over 85 in the elderly people aged over 65 in that year, the higher the proportion, the less the coupon payment. The less interest payment is used to compensate for the excess pension payment.

In addition, the coupon of the annuity bond is related to the periodically published annual survivor index. Cox & Lin (2007, 34) have given the pricing formula of the survivor bond, making the longevity risk of annuity products securitized. In order to determine the specific risk premium, they use the Wang conversion to adjust the expected proportion of the survivors. The formula is as follows:

\[ q^*(x,t) = \Phi\left[\Phi^{-1}\left(q(x,t)\right) - \lambda\right] \]
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\( \Phi(*) \) satisfies the normal distribution. \( q \) and \( q^* \) represent the adjusted and unadjusted mortality rates of a specific age group. Renshaw’s mortality model is based on the generalized linear model and the mortality experiences of the United States, and the critical value of the survival rate at each stage is projected by Renshaw’s model. However, this model has a problem, which is that, the risks of the investors may be greater than the risks of life tables, because the risks of the investors are linked with the mortality experiences and annuities. Moreover, the Lee-Carter model is also used to predict the mortality index.

The cash flow analysis is as follows:

\[
\begin{align*}
\text{reinsurers} & \quad \overset{\text{Premium } P}{\longleftrightarrow} \quad \text{SPC} \quad \overset{\text{sales revenue of bonds } V}{\longleftrightarrow} \quad \text{investor} \\
& \quad \downarrow \text{claim amount } B_t \quad \downarrow \text{coupon } D_t
\end{align*}
\]

Figure 4.4 Cash Flow of Survivor Bonds

The insurers or the reinsurers buy reinsurance from a special purpose company (SPC). The SPC issues the survival bonds to investors. According to the reinsurance contracts, the SPC pays to the reinsurers according to the variation of the mortality each term, and the amount of payment is \( B_t \). According to the calculation method of the mortality index which has been agreed on, based on the variation of the mortality index, the SPC pays the varying coupons to the investors, and the amount of payment is \( D_t \). The principal \( F \) is repaid at the maturity date.

3. Survivor Swap

Survivor swap is a future cash flow swap agreement based on the performance of a certain survival index signed by trading partners. This kind of bond is an index bond of cash flow swap based on that there is least one survivor in the future. The trading partners are often the insurers of life insurance and pension insurance. Since the impacts of longevity on both sides are in the opposite directions, so the swap can diversify the risks of both sides. Therefore, survivor swap can avoid unfavorable
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mortality risks and longevity risks.

Some researchers in the United States said that, compared with survivor bond, survivor swap has many advantages, such as efficiency and flexibility. Due to the use of the principle of natural hedge, the critical value of future mortality doesn’t need to be projected, and survivor swap or mortality swap bonds can avoid the problem of basic risks. In 2006, Dowd et al. (2005, 41-44) gave two examples of the products related to mortality which can be explained, and both products have been used to hedge longevity risks and mortality risks.

The cash flow analysis is as follows:

The insurers or the reinsurers pay a fixed amount x to a special purpose company (SPC) each year, and the investors pay a fixed amount y to the SPC. The meaning of Bt and Dt is similar to that of survivor bond, with no repayment of principal at the maturity date. Compared with survivor bond, due to the lack of principal guarantee, survival swap has the counter-party risk. Dowd et al. (2005, 41-44) explored further the survivor swap in 2006. Dowd et al. (Ibid, 41-44) introduced the application of the survivor swap, and pointed out that a number of survival swaps existed in the over-the-counter transactions. Due to the characteristics of OTC, it is difficult to gain the transaction information of survivor swaps.

4. Examples of Current Longevity Risk Bonds

Product 1: Swiss Re Short-term Risk Bond

In December 2003, Swiss Re issued a guaranteed mortality-related bond to hedge mortality risk. Swiss Re set up a special purpose company (SPC), to raise $ 400 million from capital investors. This was the first time for Swiss Re to issue a floating
rate bond whose principal repayment was associated with mortality. The term was 4 years, and the investors would receive floating coupons, whose coupon rate was 135 basis points above the American labor force index. The floating rate was higher than the rate for fixed rate bonds, but if the weighted average of the five countries’ (the United States, Britain, France, Italy and Switzerland) mortality rates exceeded the 130% of that in 2002, the repayment of the principal would be at risk. Since the mortality rate is gradually improving (declining), such a high mortality rate is almost impossible, and the investors could get considerable returns, at the same time hedging the longevity risk.

Product 2: EIB Survivor Bond

In November 2004, the European Investment Bank (EIB) issued a longevity risk bond. The coupon of time T was related to the payout rate of £ 50 million annuity at the time T, based on the survival rate of the males aged over 65 in England and Wales after the period of time. Unlike the short-term bond issued by Swiss Re, the coupon payout rate of EIB survivor bond was associated with the survival index, in order to resist the long-term longevity risk. The total value of EIB survivor bond issued reached $ 540 million, primarily designed and released for UK pension funds.

4.3.2 Longevity Risk Securitization Structure of Reverse Mortgage

Longevity risk securitization has greatly improved the ability of the insurance market to bear the risks. Mortality bonds and other derivatives associated with the variations of mortality can effectively diversify the longevity risk caused by mortality through longevity risk securitization, thereby reducing the losses of insurance companies. The difference between the longevity risk securitization of reverse mortgage and general longevity risk bonds is that, the longevity risk bond product of reverse mortgage is more complex with an extra part of mortgaged houses. This part of the article will give a detailed description of the structure and process of longevity risk securitization of reverse mortgage loans.

The basic structure of asset securitization generally consists of four components:
retailers, the contract issuer, the special purpose company (SPC) and investors. This structure has been adopted by Swiss Reinsurance, USAA catastrophe bond, Winterthur storm bond and California earthquake bond.

This article will apply this structure into the longevity risk securitization of reverse mortgage in disguised form. The main participants include:

- borrowers (homeowners)
- loan originator (retailers such as insurance companies, etc.)
- special purpose company (SPC)
- lenders (investment banks, etc.)
- investors (capital markets)

Graph 4.6 illustrates the overall structure and cash flow distribution of the longevity risk securitization of reverse mortgage.
The contract starts from a retailer of the reverse mortgage. The retailer negotiates with the borrower (the house owner), and then the retailer raises a one-time fund from an investment bank or other institutions, followed by a payment of a certain proportion to the borrower. Accordingly, the borrower repays the loan through the sale of the house. In the process, in order to protect the interests of the lender (an investment bank or other institutions), a special purpose company (SPC) is introduced to the entire securitization process, to participate in the issuance of the security. The two sides sign the insurance contract, which is non-traditional but forms a risk protection system of longevity risk. The retailer of the reverse mortgage entrusts the debt to the SPC, and the reverse mortgage lender pays the premium as a commission to the SPC.
Meanwhile, the contract will stipulate a trigger value (or some trigger values). When the lender's loss reaches the pre-set trigger value, the SPC will pay a cap compensation to the lender.

In exchange, the SPC packages the premiums obtained from the lenders to design bond products, and according to the different preferences and needs of institutional and individual investors to bond products, the SPC designs different levels of bonds in order to satisfy the needs of the investors in different levels of interest rates, interest types and terms, and then entrusts the underwriting agencies (big insurance brokers can be chosen to play the role) to issue them in the market. Within the contract period, the coupons are paid to the investors each year, and the payments of the principal and coupons end when the contract expired.

The bond will be sold at a price lower than the average market price, because if the loss of the lender exceeds the trigger condition, some or all of the coupons will be transferred to the lender by default to compensate for the loss. But this structure is just a basic structure, and in reality there will be more complex changes. For different purposes, the structure of the product will be more diversified, and more constitutions will participate in it. For example, in order to protect the interests of bond investors, the credit rating agency will be added between the SPC and the bond investors to conduct the bond rating.

4.3.3 Cash Flow Analysis of Longevity Risk Securitization of Reverse Mortgage

1. Retailer of Reverse Mortgage

As an independent service constitution, the retailer provides services for the lender, and supervises the one-time loan payment of the lender, while maintaining the integrity and normalization of the cash flow payment process. After the loan begins, the retailer transfers the collected money to the people who need the loan. From the retailer's perspective, the cash flow of the lender and the borrower should remain the
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same, so as to ensure almost no loss of risk. China's current forms of retailers are limited, mainly insurance companies and banks, and more forms are in foreign countries.

2. Lender

Before the loan, the lender evaluates the number of survivors who need loans, the possible losses due to mortality improvements, interest rate floats and price changes of the real estate market. After the loan begins, the cash inflow of the lender and the loan amount withdrawn from the retailer are the same, and the amount of cash outflow is the accumulating capital cost. If the net value of both cash flow is below the trigger condition, the SPC will not offer compensation to the lender, and on the contrary, it will make up for the loss.

3. Special Purpose Company

The special purpose company is a passive entity, whose purpose of existence is to conduct the longevity risk securitization for the bonds and sell the bonds in the capital market. It raises the insurance premiums from the lender, designs survivor bonds and issues them. Assuming the fund of the sale is an investment of the risk-free rate. When the loan begins, the cash inflow of the SPC is the total amount of the investment, and the cash outflows are the compensation with priority to the lender and the coupons paid to the investors (bondholders). When the bond matures, SPC will repay the principal to the investors. The net cash flow for the SPC is always zero.

4. Investors

The investors of the survivor bond buys it at a price lower than the purchase price of a bond with a fixed rate, but they need to take greater risks. For example, the coupons received in the future will reduce because of the rise in the proportion of the mortality improvement, and that is, the maturity year and the increase of the death proportion in the initial year will reduce the coupons of the investors, and the reduced part is used to compensate for the lender who pays extra pensions. When the loan begins, the cash inflow for the bondholders is the coupon paid with an uncertain interest rate according to the SPC. However, when the bond matures, the investors will receive the full
amount of the principal, and depending on the types of bonds, the payment solutions will vary.
CHAPTER 5  EMPIRICAL ANALYSIS

5.1 Setting of Reverse Mortgage Longevity Risk Bond Pricing Model

5.1.1 Mortality Prediction Model

In the 19th century, British actuary Gompertz tried to find a simple way to estimate mortality of all ages, which was very important for life insurance. In the statistical analysis, he found an interesting law: babies had a higher mortality rate, which would decline over the years until reaching its lowest point in 10-to-15-year-old adolescents. After puberty, the mortality rate increased dramatically, approximately doubled every 10 years (which means the mortality rate at the age of 25 is twice as much as at the age of 15, 35 as much as 25, exponentially). This trend continued until after the age of 80. This rule is known as the law of mortality, suitable for different countries in different historical periods.

Altman mortality model developed is based on the life insurance ideas, methods and ideas of the model similar to the life insurance premiums. Based on the combination of defaults of bonds or loans in a specific time frame, according to the classification of credit, the model developed a mortality table which would predict a year-long MMR (marginal mortality rate) and CMR (cumulative mortality rate,) to measure the default rates a particular credit bonds or loans. Therefore, the model was often used for the credit rating of commercial banks.

Deviation of the actuarial pricing of mortality assumptions and the average future mortality the insured group, and future mortality fluctuation around the average value, make large effects on forward predicted life span, therefore measure for future changes in mortality is the base of longevity risk measurement. At present, the most used mortality model for longevity risk bond is the Lee-Carter model. From Lee (2000, 80-93) and Lee & Miller (2001, 537-549)’s study, Lee-Carter model based on
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U.S. mortality data is more simple, fitting in mortality changes, better in forecast compared with the official and capable of analysis of uncertainty in forecast, which had become one of the hotspots of the prediction model, and the predictions of the Lee-Carter model of mortality indicators were set as a benchmark by the U.S. Census Bureau as the U.S. population mortality long-term projections, as Hollmann et al. (2000, 25-27) stand.


In view of this, this article will apply the mortality data of the National Bureau of Statistics of China from 1997 to 2007 (see Appendix I), using the Lee-Carter model for insured group (over 60 years old) of China's future housing reverse mortgages longevity risk bonds, to forecast mortality.

5.1.2. Reverse Mortgage Pricing Model

This section will focus on the pricing method of one-time payment of reverse mortgages. Assuming a retiree pays a one-time reverse mortgage of $Q_0$ while the value of the house is $H_0$ at the time. After $t$ days, the loan amount is $Q_t$, the value of the house becomes $H_t$ and the cost of capital $M_t$. According to the definition of the value of housing reverse mortgage, picking the lower one between $Q_t$ and $H_t$, we have the formula:

$$V_t = \min (Q_t, H_t) \quad \text{Formula (5.1)}$$

The loss of the lender is the difference between $M_t$ and the value of the reverse mortgage:

$$L_t = M_t - V_t = M_t - \min (Q_t, H_t) \quad \text{Formula (5.2)}$$
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Taking into account for $Q$, there will be a risk-free interest rate $r$, and a risk of premium price $\lambda$, and for $H$, an average increase of $\delta$, for $M$, an interest rate $\eta$:

\[ Q_t = Q_0 \exp \left( \int_0^t (r_s + \lambda) \right) ; \]
\[ H_t = H_0 \exp \left( \int_0^t \delta ds \right) ; \]
\[ M_t = Q_0 \exp \left( \int_0^t \eta ds \right) ; \]

Therefore Formula (5.1) can be transformed to:

\[ V_t = \min \left[ Q_0 \exp \left( \int_0^t (r_s + \lambda) \right), H_0 \exp \left( \int_0^t \delta ds \right) \right] ; \quad \text{Formula (5.3)} \]

Formula (5.2) can be transformed to:

\[ L_t = Q_0 \exp \left( \int_0^t \eta ds \right) - \min \left[ Q_0 \exp \left( \int_0^t (r_s + \lambda) \right), H_0 \exp \left( \int_0^t \delta ds \right) \right] ; \quad \text{Formula (5.4)} \]

Assuming that the reverse mortgage borrower’s life cycle is a random variable $T$, the simulation of the future life of the borrower can be made. Thus, the lender’s loss $L_T$ will be expressed as:

\[ L_T = Q_0 \exp \left( \int_0^T \eta ds \right) - \min \left[ Q_0 \exp \left( \int_0^T (r_s + \lambda) \right), H_0 \exp \left( \int_0^T \delta ds \right) \right] ; \quad \text{Formula (5.5)} \]

Suppose the total expected loss is 0 at the time of $T$:

\[ E \left( e^{-rT} L_T \right) = 0 ; \]

Transform Formula (5.5), equaling total expected revenue and total expected loss, and drawing reverse mortgage pricing equation:
Chapter 5 Empirical Analysis

\[ E \left[ Q_0 \exp \left( \int_0^T \eta_s ds \right) \right] = E \left[ \min \left( Q_0 \exp \left( \int_0^T (r_s + \lambda) \right), H_0 \exp \left( \int_0^T \delta ds \right) \right) \right] \]

Formula (5.6)

Based on the pricing model, given the lender's risk of premium price \( \lambda \), \( Q_0 \) can be calculated. On the contrary, given an initial loan amount \( Q_0 \), lenders' actuarial risk of premium price \( \lambda \) can also be deduced.

5.1.3. Reverse Mortgage Survivor Bond Pricing Model

This article will set reverse mortgage survivor bond as the object of study to illustrate the impact of longevity risk securitization on revenues for investors and the capital market.

Assuming the interest rate \( r \) and the increase rate of house are constant, and the lender has a loan portfolio of \( l_0 \) people, at time 0, all borrowers are of the same age, for example 62-year-old, every time a lump sum loan \( Q_0 \), the value of the house \( H_0 \). In order to hedge against longevity risk, the lenders buy insurance at the premium of \( P \) from SPC. Under the contract, if the survivors \( l_t \) of the borrowers exceeds the expected number of survivors \( \hat{l}_t \) (trigger value), through cross-risk, SPC will pay lenders insurance claim \( A_t(l_t - \hat{l}_t) \) until it reaches the maximum compensation limit \( C \). During the time period \( t \), the loss of each loan \( i \) is \( L_{i,t} \), the interest rate \( r \) and the increase rate of house are constant, thus \( L_{i,t+1} = L_{i,t} \). Then the balance between the discount expected loss at time \( t+1 \) and loss at time \( t \) can be expressed:

\[ A_t = \frac{L_{i,t+1}}{1 + r} - L_{i,t} \]

If the actual number of survivors is greater than the expected number of survivors, the lenders can claim from the SPC at every stage of insurance claims \( B_t \) to make up for the loss of unknowable:
SPC issued a face value of F bond, the coupon and the number of survivors is associated and the issuance price is V. When the expected number of survivors is less than the actual number of survivors, the coupon will be part deducted until the deduction reaches a pre-set maximum C. Obtained at each stage, bondholder’s final coupon $C_t$ can be expressed as:

$$B_t = \begin{cases} 
0 & \text{if } h \leq \hat{h} \\
A_t \left( h - \hat{h} \right) & \text{if } \hat{h} < h < \frac{C}{A_t} \\
C & \text{if } h > \frac{C}{A_t}
\end{cases}$$

**Formula (5.7)**

Pricing is done using balanced interests pricing at first. Each year, SPC has to pay coupons $C_t$ to investors and $B_t$ to lenders. The annual total expenditure $W$ is a combination of $C_t$ and $B_t$, also $C_t + B_t = C$. T years later, the payment of principal is F. In order to balance the cash flow, at time 0, SPC issues risk-free bonds of premiums $P$ and issue price $V$ of the survivor bonds, which is the present value of its total revenues. SPC’s normal operation of cash, as shown in Figure 5.1, in the initial stage, the aggregate expenditure should be less than the present value of the total revenues, assuming a discount rate of $\nu = \frac{1}{1 + r}$, then:

$$P + V \geq W = F\nu^T + \sum_{t=1}^{T} \nu^t C_t$$
If \( P+V-W > 0 \), the benefit of issuing bonds goes to the insurance company (lender), who is the bond promoter. However, if the \( P+V-W \) is too large a sum, then the lending party would obtain too much profit to compromise buyers interests; if \( P+V-W < 0 \), then the lending party would not be willing to make the pricing. Therefore, in practice, the best condition is \( P+V-W = 0 \). By doing so \( V \) also can be added with a minor profit for lenders in the issuance of the bond. Be noted that, due to the lack of consideration of longevity risk in \( P \), the actual \( V \) will be greater than it is in the formula.

In the following passage, the author will be using discount on bond pricing, taking longevity risk and the improved situation of mortality into account through Lee-Carter model, so that accuracy of the data and prediction of the dynamic trend can be improved. Introducing the general pricing method and actuarial equivalence principle of financial bonds, the sum of the present value of the coupons plus due present value of the principal shall be equal to the initial price. \( C_t \) being a random variable varies according to \( \hat{h} \), then:

\[
V = Fv^T + \sum_{t=1}^{T} v' C_t
\]

Maintain the previous assumptions, Formula (5.8) can be further expressed as:

\[
C_t = C - \left[ A_t \left( l_t - \hat{h} \right), 0 \right]_+ + \left[ A_t \left( l_t - \hat{h} \right) - C, 0 \right]_+
\]

Add expectations to both sides:

\[
E(C_t) = C - E\left[ A_t \left( l_t - \hat{h} \right), 0 \right]_+ + E\left[ A_t \left( l_t - \hat{h} \right) - C, 0 \right]_+
\]
Reverse mortgage survivor bond issuance pricing model:

\[ V = F_v^T + \sum_{k=1}^{T} \nu^k \left\{ C - E \left[ A_t \left( l_t - \hat{l}_t \right), 0 \right]_+ + E \left[ A_t \left( l_t - \hat{l}_t \right) - C, 0 \right]_+ \right\} \]

Formula (5.9)

5.2 A Sensitivity Analysis of China's Mortality on Investment Income of Housing Reverse Mortgages Survivor Bond

5.2.1 Data Setting

In this paper, the model designed for insurance starts at the age of 60, before which there is few participating in reverse mortgages, and the fact that slow growth in mortality has made most insurance companies to set the starting age at 60.

The initial setting of a reverse mortgage insurance has a scale of 1,000 people, the initial house price is 100 million yuan \((H_0=1000000)\); the total value of the reverse mortgages is half a million yuan \((Q=500000)\); annual risk-free interest rate refers to 2012 Treasury bonds 5-year interest rate \((r=5.32\%)\); average rate of increase in house price (see Appendix 2) \(c=0.0753\); insurance lenders risk of premium price is set \(\lambda_1=3\%\), investors expected risk of premium price \(\lambda_2=1.5\%\); expected mortality rate 0.7%; the initial mortality improvements variance 1.0.

5.2.2 Mortality Projection

This section will use the Lee-Carter model to make dynamic prediction of China's mortality improvement and then capitalize the results to have a sensitivity analysis of the impact on the pricing model of reverse mortgage survivor bond, in order to further study the impact of longevity risk on investor's gain.

The Lee-Carter model of mortality was put forward in 1992:
\[ \ln(\mu_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{x,t} \]  

Formula (5.10)

In which \( \mu_{x,t} \) stands for central mortality; \( \alpha_x \) and \( \beta_x \) are parameters of age \( x \); \( k_t \) mortality trend; \( \varepsilon_{x,t} \) random item with a mean of 0 and standard deviation \( \sigma_{\varepsilon} \).

Due to an array of \( \alpha_x, \beta_x \) and \( k_t \) which meet the requirements of Formula (5.10), in order to estimate the parameters, Lee and Carter standardized \( \beta_x \) and \( k_t \) \( \sum_x \beta_x = 1, \sum_t k_t = 0 \) and get \( \hat{\alpha}_x = \frac{1}{T} \sum_t \ln \mu_{x,t} \) (T is the total number of a calendar year). Since the right end of the formula had no explanatory variable, according to standardization restrictions based on the above parameters, Lee and Carter uses of singular value matrix decomposition to derived the definite \( \hat{\alpha_x}, \hat{\beta_x}, \) and \( \hat{k_t} \).

According to the reverse mortgage survivor bond pricing Formula (5.9), project the natural mortality data from 1998 to 2007 in China (see Appendix 1) and apply JAVA to program improving mortality prediction model and housing reverse mortgage survivor bond pricing model, inputting the initial sets of data: insurance years, each year's natural mortality, average increase in house price and improving rate in mortality. Assuming 1000 60-year-old insurance buyers, with each passing year, each survival will be provided a random number by the computer to decide whether or not he is to survive. According to the Lee-Carter model, the candidate will survive if the number is less than the mortality; he won't if the number is greater than the mortality. Computer will first generate the expected mortality and the corrected mortality until no one survive, and in accordance with the expected mortality and the corrected mortality to generate expected average number of survival and the actual average number of survival. Then, utilize actual survival number bond model to calculate the difference between average annual discount loss and current loss \( A_t \). In \( t \) years, the
cumulative difference simulates 1000 times to obtain the average value, the computer then calculates $V$ through programmed formula.

Thus every change in mortality rate of improvement, the computer will get: 1. the expected mortality, 2. the analog corrected mortality, 3. the expected average number of survival, 4. the analog number of average survival, 5. the difference between average annual discount loss and current loss, $A_t$. Total simulation year is $t$, running until the corrected actual number of survival probability (the current number of survival divided by the number of initial survival) is less than the calculated precision.

The number of survival $l_t$ is subject to binomial distribution, termination rate is $q_{x+t}^*$. If the number of loans is huge, the central limit theorem will apply, the average $l_t$ is $\mu_t = l_t(1-q_{x+t}^*)$, and define variance of the mortality rate of improvement $\sigma_t^2 = l_t(1-q_{x+t}^*)q_{x+t}^*$. When the value equals 1, which means that the actual mortality matches the expected mortality; When the value is greater than 1, it indicates that the ratio of the actual mortality compared to the expected is increasing; When the value is less than 1, then the ratio is decreasing. It is assumed that the mortality rate of improvement at 1.001, $T$ tends to infinity. See Table 5.2:

<table>
<thead>
<tr>
<th>Year</th>
<th>Expected Mortality</th>
<th>Analog Corrected Mortality</th>
<th>Expected Average Survival</th>
<th>Analog Average Survival</th>
<th>$A_t$</th>
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<tbody>
<tr>
<td>0</td>
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<td>1000</td>
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## Chapter 5 Empirical Analysis

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<th>Analog Average Survival</th>
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### Chapter 5 Empirical Analysis

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<th>Expected Average Survival</th>
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### Chapter 5 Empirical Analysis

#### Table 5.4 Mortality rate of improvement at 0.950

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<th>Year</th>
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<th>Analog Corrected Mortality</th>
<th>Expected Average Survival</th>
<th>Analog Average Survival</th>
<th>$A_t$</th>
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### Table 5.5 Mortality rate of improvement at 1.050

At the end of each calculation, 4 average values will be obtained: 1. Calculated in accordance with the expected mortality, the average expected age; 2. Under fluctuations in mortality, the corrected average expected age; 3. According to the model, obtaining the survivor bond issue price when actual mortality matches the expected mortality. Per 10,000 yuan face value, the issue price of the bond is $1.0E9(8103.083/10000)$ yuan; 4. Rate of change in loss reflects the investor's principal and coupon income change and volatility, the data is achieved: (actual issue price - expected issue price) / expected issue price.

Table 5.6 is the results of the four average values when mortality rate of improvement is 1.001, 0.995, 0.950 and 1.050, respectively. In the case of different mortality change rate, the corrected average expected age decreases as mortality change rate increases. And the results also show that the investor's rate of change in loss in the respective sections is small, thus the impact is very limited.

<table>
<thead>
<tr>
<th>Mortality rate of improvement</th>
<th>1.001</th>
<th>0.995</th>
<th>0.950</th>
<th>1.050</th>
</tr>
</thead>
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<tr>
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<td>75.518837</td>
<td>75.518837</td>
<td>75.518837</td>
</tr>
<tr>
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<td>Survivor bond issue price</td>
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<td>$1.0E9$</td>
<td>$1.0E9$</td>
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</tr>
<tr>
<td>investor's rate of change in loss</td>
<td>-0.030394</td>
<td>0.1</td>
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</tbody>
</table>

**Table 5.6 Output values under different mortality rate of improvement**
5.2.3 The Impact of Longevity Risk Bonds on Investor's Returns

The results show that, 15 years later, during the entire process of investing in bonds, the mortality rate of improvement has little impact on investor's returns. When the rate is 1.001, the corrected average expected age is 75.43, the rate of change in loss is -3.03%. Investors have no loss and returns from coupons can be ensured. When the rate is 1.050, the corrected average expected age is 75.43, investors keep a good trend in returns. The rate of change in loss is -0.5205%, less fluctuations in returns, investors have an unlikely possibility of loss in principal and coupons.

When the rate is 0.950, the corrected average expected age is 84.52. Investors gain profits, the rate of change in loss is -0.309%. Still low possibility of loss in principal and coupons. When the rate is 0.995, the corrected average expected age is 76.01, the rate of change in loss is 0.1%. Investors may lose, but in a controllable sector.

Data has shown that when the rate changes between 0.950 to 1.050, the rate of change in loss is negative, meaning positive investment income. Also the rate of investment loss change is steadily under 3%, which means less volatility and in a controllable sector. When the rate reached 1, the corrected average expected age is higher, which means less longevity risk, the rate of change in loss is about twice the mortality rate of improvement. Such fluctuation range is acceptable.

When the mortality rate of improvement exceeds 0.950~1.050, expected death year will grow beyond normal figures(over 100). Thus, it should be kept within the range.
CHAPTER 6 CONCLUSIONS AND PROSPECTS

6.1 Conclusions

This study takes longevity risk into consideration by using Lee-Carter mortality prediction model and discount cash flow pricing method. Mortality improvement rate is concluded from the average growth rate of house price and human mortality from 1998 to 2007 in China, which is used to do the sensitivity testing for the longevity risk bond pricing model. We conclude from three respects in order to answer the research questions in chapter 1.22.

Firstly, this study makes improvements on the mortality prediction model based on China’s national conditions, and aims to make more accurate prediction on the human mortality trend in China. Compared with traditional index mortality prediction model, the Lee-Carter model produces more dynamic and more sensitive prediction of the mortality changes for future. It reflects the underlying trend even better, which before reacts in the securitization process. The average expected ages obtained from Lee-carter model appear automatically when we input pre-set variables in JAVA. Based on these figures, we conclusively got more close to the truth. The Lee-carter model or other fixed types can be applied to mortality related researches.

Secondly, it shows that the loss rate of the investors is basically negative, which means the ROI is positive. The variation range is about 0.01% to 3%, that is under control. To make a conclusion, in the longevity risk bond pricing model of this study, the mortality change has little impact on the investment market, which means this model is feasible in China’s financial market. What’s more, as we assumed above, China’s financial market is efficient. In this context, the housing reverse mortgage survivor bonds are easily available in commercial banks and insurance companies.

Currently, Chinese people hold positive and open attitude on most of financial products, no matter defined benefit or not. They welcome creative products which
connect with their welfare, like annuity funds, QDII products or overseas reinsurance products. If normal people including some retail organizations accept housing reverse mortgage survivor bonds, they participate in the market and share the longevity risks and credit risks with house providers and governments. Meanwhile, the housing reverse mortgage longevity risk bonds can not only help providers to spread the longevity risk, but also help investors to gain profits from a new point of view. It brings financial product types to the bond market, which means the investors can have more choices.

Finally, the highlight of this study lies in the combination of housing reverse mortgage with longevity risk securitization. The housing reverse mortgage is a feasible and practical initiative that can become a real solution, because aging and intensive retirement issues to date have emerged in an unpredictable fashion. Previous pension products cannot meet people’s needs. More designs are desired, just like housing reverse mortgage survivor bonds. Accordingly, the other relevant longevity risk securitization can be created. Depending on diverse variables, we can derive more types of longevity risk bonds. Survivor swaps are successful in some countries, which are designed for no cash flow between buyer and seller during the securitization. Although survivor swap may face the credit problem, we can take advantage of its deal structure, and fix its contract languages. We can take enterprise annuity and equity incentive securitization into consideration with fixed mortality prediction in near future.

6.2 Prospects

Housing Reverse Mortgage Loans have emerged much earlier throughout the world, especially in the developed countries with serious aging society problems. Given the aging problem trends and social insurance funds pressure in China, Housing Mortgage Loan will be welcome and accepted by borrowers and providers if only: 1) Appropriate guidance is offered by government, insurance companies and relevant commercial banks, 2) Elaborate mechanism is built as soon as possible, and 3)
Instruments are well designed according to accurate data and actuary models.

As we discussed, there has been various longevity risk bonds of housing reverse mortgage in the United States, whose performance has still been strong under the subprime crisis since 2009, and extremely popular as an investment. It is a less risky investment product, and is a marketing tool helping the issuer to diversify the risk in the economic downturn. Corresponding and urgent needs for longevity risk securitization will expend in those developed countries with high aging society. The mortality projection are only proved to be practical in China, because the variables and constant data are not typical throughout the world. However the pricing model and the securitization theory can be used with local mortality data.

To conclude, the idea that we combine longevity risk bonds of housing reverse mortgage with enterprise annuity or stock-linked incentives before one’s retirement can be researched and studied. We can diversify longevity risks with the help of enterprises and communities as well. This topic is hoped to be explored in the near future around the globe.
REFERENCES


[38] Lin Zetao, 2007. Securitization of Life Insurance Assets, Southwestern University of Finance and Economics, Vol.4


http://www.stats.gov.cn/


## APPENDIX I

**The birth rate, mortality and natural growth rate**

(Data from: China’s national bureau of statistics web site, 2012)

<table>
<thead>
<tr>
<th>Year</th>
<th>Birth Rate</th>
<th>Mortality</th>
<th>Natural Growth Rate</th>
<th>Total Population (100 million)</th>
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APPENDIX II

**Average Growth Rate of Three Types of House Price (1998--2007) (Unit: %)**

(Data from: China's national bureau of statistics web site, 2012)

<table>
<thead>
<tr>
<th>Year</th>
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