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INTERFEROMETRIC AND ACOUSTIC MEASUREMENTS IN SUPERFLUID $^3$He-B AND WETTING STUDIES IN $^3$He/$^4$He MIXTURES

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References
1. Introduction

In Nature there exist systems, referred to as superfluids, which are characterized by radical changes in their behavior as the temperature is lowered below a transition point. Superfluids are known to exhibit vanishing viscosity; unlike ordinary liquids they are able to flow through narrow gaps and tubes without dissipation. Their macroscopic behavior is governed by quantum statistics, which becomes important at low temperatures near the absolute zero.

So far the only known superfluids are formed by the two isotopes of helium, $^3$He and $^4$He. Both remain as liquids down to $T = 0$. This is possible because (1) the small mass of He-atoms results in strong zero-point motion and (2) the high symmetry of the atoms weakens the attractive interaction between them. Helium solidifies only under a pressure of ca. 30 bar [1].

While superfluid $^4$He has been under active investigation for half a century, the superfluid A- and B-phases of $^3$He were discovered in 1972 by Osheroff, Richardson, and Lee [2]. A rapid surge of experimental and theoretical studies then followed. At present, the best theory in condensed matter physics is probably for $^3$He, and this anisotropic superfluid could be used as a model system in cosmology [3].

The behavior of $^3$He in its superfluid state is spectacular: In a single system, in addition to superfluidity, well known properties of magnets and liquid crystals are encountered [4]. Superfluid $^3$He has the most intricate broken symmetry in condensed matter physics. It is also the cleanest substance to study, being completely free of any impurities at low temperatures.

In this thesis work superfluid $^3$He was investigated by two methods. First, ultrasonic experiments on $^3$He-B were carried out using two coincident zero sound pulses. The second technique is optics, a novel method for ultra low temperatures. The developed method, two-beam interferometry, was employed successfully for studies of wetting phenomena in liquid $^3$He/$^4$He mixtures as well.

Publications 1–3 contain the results of acoustic spectroscopy on $^3$He-B. Most importantly, the real squashing collective mode (rsq) was excited by two simultaneous sound pulses yielding two phonon absorption (TPA). This nonlinear phenomenon was applied to study the dispersion relation of the rsq-mode. Zeeman splitting of the
nonlinearly excited rsq-mode was investigated in a magnetic field. By means of TPA, an anomalous behavior was found also near the pair-breaking edge.

Publications 4 and 5 describe technical preparations for optical studies at millikelvin temperatures. In Publication 6 the first optical observations on superfluid $^3\text{He}$ are described. Publications 7–9 report interferometric measurements in $^3\text{He}$-$\text{B}$, performed using an optical setup with improved resolution. Dynamic processes in a superfluid layer of $^3\text{He}$ were investigated by employing thermomechanical excitation. In rotation, two unique superfluid menisci were observed, produced by the rotation of either the normal fluid or the superfluid component. In Publications 10 and 11 optical experiments in $^3\text{He}^4\text{He}$ mixture films are described; unexpected nonwetting phenomena were found.

1.1. **Helium superfluids**

From a microscopic point of view, the helium atom is just a structureless spherical particle. The different behaviors of the two stable isotopes, $^3\text{He}$ and $^4\text{He}$, at low temperatures are due to different quantum statistics. In the case of $^4\text{He}$, the interacting atoms undergo a Bose-Einstein condensation into the superfluid state at $T_\lambda = 2.17$ K. In $^3\text{He}$ the atoms obey Fermi-Dirac statistics owing to their half-integer spin; condensation to a superfluid state, which is the consequence of Cooper-pairing of $^3\text{He}$ atoms, takes place at temperatures lower than 3 mK. Owing to the strong repulsive short-range force between the atoms, pairing in $^3\text{He}$ takes place into a state with orbital angular momentum $L = 1$. At the same time, the $^3\text{He}$-liquid is strongly paramagnetic which causes pairing into a spin-triplet state with $S = 1$ [4].

In $^4\text{He}$ there exists only one superfluid phase, $^4\text{He}$-II. In $^3\text{He}$ the rich variety of possible pairing results in several superfluids. In bulk $^3\text{He}$, three different superfluid phases, $^3\text{He}$-A, $^3\text{He}$-B, and $^3\text{He}$-A$_1$ are found to be stable. Only the B-phase (see Fig. 1) is present at zero pressure while at higher pressures also the A-phase can exist. The presence of a magnetic field $H$ induces the third superfluid phase $^3\text{He}$-A$_1$, which appears in the phase diagram as a narrow region between the normal fluid and the A-phase. At the same time, the field causes the A-phase to extend to lower pressures and temperatures while, at $H > 0.6$ T, no B-phase exists in the phase diagram.

At finite temperatures a superfluid can be considered as a mixture of two interpenetrating liquids: the normal component with density $\rho_n$ and the superfluid component with density $\rho_s$, so that the total density of the liquid $\rho = \rho_n + \rho_s$. At absolute zero the
whole condensate would consist of the superfluid component only. An increase in temperature yields a decrease of the superfluid density while the normal fluid contribution grows until at the superfluid transition $\rho_n = \rho$. However, the two-fluid model is only a phenomenological description. In reality, the individual atoms do not belong to the superfluid or to the normal fraction since all atoms are identical.

![Phase diagram](image)

**Figure 1.** Phase diagram of $^3$He as a function of temperature, pressure and magnetic field.

A complete theory of two-fluid hydrodynamics has been developed, and this model explains successfully the macroscopic behavior of a superfluid [5]. Owing to its peculiar nature, a superfluid can display both frictionless and viscous flow at the same time. Strong coupling between thermal and mechanical phenomena gives rise to unique sound modes in superfluids [6].

The superfluid component can be described by an order parameter which in $^4$He-II is a scalar quantity:

$$\Psi(\vec{r}) = C(\vec{r})e^{i\Phi(\vec{r})},$$  \hspace{1cm} (1)
where $C^2(\vec{r}) = \rho_s$, and $\phi$ denotes the phase of the condensate. In superfluid $^3$He the situation is much more involved because the order parameter is a $3 \times 3$ complex matrix which, for the isotropic B-phase, can be written in the form [4]

$$A_{\mu j} = \Delta(T)e^{i\phi}R_{\mu j}(\hat{n}, \theta).$$ (2)

Here $\mu$ and $j$ refer to the three spin and orbital degrees of freedom, respectively, $\Delta(T)$ is the superfluid energy gap, and $R_{\mu j}$ is a matrix which rotates the spin and orbital coordinates with respect to each other by an angle $\theta$ about an axis $\hat{n}$.

1.2. Superfluids in rotation

Superfluid flow is potential. This was first postulated by Landau [7] who introduced the irrotationality condition $\vec{V} \times \vec{v}_s = 0$. The superfluid velocity, both for $^4$He-II and for $^3$He-B, can be written as

$$\vec{v}_s = \frac{1}{m} \frac{\hbar}{n} \nabla \phi(\vec{r}).$$ (3)

where $m$ is the mass of the He atom and $n$ is an integer. For $^4$He $n = 1$, while $n = 2$ for $^3$He because of Cooper pairing. In order to satisfy the irrotationality condition, superfluid should stay at rest when the container is brought into rotation. Since the centripetal force would then act only on the normal component, the free surface of a rotating superfluid should be completely flat in the limit $T = 0$. However, by visual observations on the free surface of $^4$He-II it was found that the rotating superfluid displays the same parabolic meniscus as the normal liquid [8], viz.

$$z(r) = \frac{\Omega^2 r^2}{2g}.$$ (4)

Thus, contrary to the theoretical prediction, the superfluid component must rotate as well.

In fact, the observation of a classical meniscus in a rotating superfluid provided the first evidence for the existence of superfluid vortices. Onsager [9] and Feynman [10] realized that a superfluid is able to imitate solid body rotation on average by creating a lattice of quantized vortex lines as illustrated in Fig. 2. Vorticity is permitted, but it is constrained into line singularities, each with a quantized circulation $\kappa = \oint \vec{v}_s \cdot d\vec{l} =$
\( h \sim \hbar \), where the integral is calculated around a vortex line; \( h \) is the Planck constant. Therefore, in a superfluid, by having singularities at vortex cores, it is possible to combine the irrotationality condition and rotation on average.

Quantized vortices have been studied since the late 40's. In \(^4\text{He-II}\), there exists only one stable vortex structure, and the equilibrium behavior of vortices is quite well understood [11]. In \(^3\text{He}\), vortices have been investigated in our laboratory since the beginning of the 80's and many different structures have been found [12]. In addition to the methods that had been applied in studies on \(^4\text{He-II}\), viz., ion mobility, persistent current, and sound propagation, the powerful nuclear magnetic resonance (NMR) technique is possible on \(^3\text{He}\), thanks to the nonzero nuclear spin of the \(^3\text{He}\) atom.

![Figure 2](image)

**Figure 2.** Vortex lattice in a rotating superfluid: (a) Surface profile and (b) view from above. Arrows indicate superfluid circulation.

Before the work described in this thesis, the minimum temperature reached in optical experiments was about 25 mK. Optical methods have been used in measurements on the free surface of a rotating \(^4\text{He-II}\) [8, 13]. Positions of vortex lines in \(^3\text{He}/^4\text{He}\) mixtures have been imaged in semi-optical experiments [14]: Electron bubbles (ions) were trapped into the vortex cores and, by applying an electric field which pulled the ions through the liquid meniscus, electrons were accelerated to a phosphorous screen for detection.
1.3. Ultrasound propagation in $^3$He

In normal $^3$He density fluctuations can propagate as ordinary longitudinal sound waves in the hydrodynamic regime $\omega \tau \ll 1$; here $\omega$ is the sound frequency and $\tau$ is the quasiparticle lifetime. This "first sound" mode is a pressure wave, propagating with the help of collisions between particles. Attenuation of first sound is proportional to the viscosity of the $^3$He-liquid, which increases towards lower temperatures as $1/T^2$ owing to an increase in the mean free path of quasiparticles.

At lower temperatures, when $\tau$ becomes long (and $\omega \tau \gg 1$), first sound is attenuated strongly. However, under these conditions, a different oscillatory mode, collisionless "zero sound" is supported in the superfluid state down to $T = 0$. The restoring force on zero sound comes from the molecular fields caused by quasiparticle interactions whereas collisions between quasiparticles attenuate this mode. Crossover from first to zero sound occurs when $\omega \tau \approx 1$, i.e. at 10 mK for the ultrasonic frequency $\omega / 2\pi = 20$ MHz.

In the superfluid state there are, in addition to quasiparticle collisions, two extra attenuation mechanisms for zero sound: These are (1) breaking of the Cooper pairs in the condensate by phonons when the sound frequency $\hbar \omega > 2\Delta(T)$ and (2) coupling with collective modes which are oscillations of the superfluid order parameter around its equilibrium value. In $^3$He-B, there exist altogether 18 collective modes, which can be classified using the total angular momentum $J$ and its projection $m_J$ on a quantization axis [15]. In addition, an index "$+"$ or "$-"$ is needed to specify whether the real ($+$) or the imaginary ($-$) part of the order parameter oscillates.

In $^3$He, in the case of exact particle-hole symmetry, with no difference in the density of states above and below the Fermi surface, imaginary parts of the order parameter components should couple to density oscillations (i.e., sound) and the real parts to spin-density oscillations. However, in reality the particle-hole symmetry is weakly broken [16] and this allows oscillations of the real part to be excited by a zero-sound phonon as well. Only modes with even $J$ couple to ultrasound [17], and the most extensively studied collective modes in $^3$He-B are the squashing (sq, $J = 2^-$) and the real squashing (rsq, $J = 2^+$) modes.

The dispersion relation of zero sound is linear, and $\omega = 0$ at the zero wave vector limit ($q = 0$). The collective modes will be excited by a phonon when the frequency and the
wave number of sound coincide with those of the collective mode (see Fig. 3). There is
a finite frequency for the $J = 2$ modes at $q = 0$ given by

$$\omega_{sq,rsq} = a_{sq,rsq} \Delta(T) / \hbar,$$  \hspace{1cm} (5)

where $a_{sq} = \sqrt{12/5}$ and $a_{rsq} = \sqrt{8/5}$ when the Fermi liquid corrections are not taken
into account [18].

![Figure 3. Zeeman splitting of the $J = 2$ modes (on the left) and the dispersion curve
for zero sound ($\omega = cq$) and for the $J = 2$ modes (on the right).](image)

The degeneracy of the collective modes is lifted by dispersion due to the finite wave
vector $q$; for the $J = 2^+$ modes the dispersion relation becomes [19]

$$\omega_{sq,|m|}^2 = \omega_{0,0}^2 + c_0^2 q^2 + \frac{1}{6} (4 - |m|) c_0^2 q^2,$$  \hspace{1cm} (6)

where $c_0^2$ and $c_0^2$ are dispersion parameters, and the quantum number $|m|$ can have the
values 0, 1, and 2. The substates with $|m| = 1$ and $|m| = 2$ are two-fold degenerate.
Only the $m = 0$ substate couples to ultrasound in zero magnetic field.

The degeneracy of the different substates can be lifted also by an external magnetic
field, as illustrated on the left side of Fig. 3. Zeeman splitting of the $rsq$-mode into the
five substates is one of the fundamental properties of a $J = 2$ collective excitation; it was
observed first by Avenel et al. [20]. The splitting is almost a linear function of the
magnetic field up to about 70 mT [21], but stronger fields deform considerably the
uniform energy gap of $^3$He-B and yield behavior similar to the Paschen-Back effect [22].

Acoustic spectroscopy on superfluid $^3$He is, in many ways, similar to optical spectroscopy on molecules. Acoustic analogues to some nonlinear optical effects, such as saturation and self-induced transparency, were found first in ultrasonic studies on superfluid $^3$He [23, 24]. In nonlinear acoustic experiments it is possible to take advantage of the different selection rules for linear and nonlinear processes, as in molecular spectroscopy, where nonlinear studies have yielded additional spectral information on molecules or impurities in crystals [25].

![Figure 4](image)

**Figure 4.** Schematic illustration of (a) two-phonon absorption and (b) stimulated Raman scattering by the rsq-mode in $^3$He-B.

Nonlinear acoustic processes in superfluid $^3$He have been considered theoretically by Serene [26] and, more extensively, by McKenzie and Sauls [27]. The simplest nonlinear process is a three-wave resonance where two modes excite a third so that energy and momentum are conserved, i.e., $\omega_1 + \omega_2 = \omega_3$ and $\vec{q}_1 + \vec{q}_2 = \vec{q}_3$ (see Fig. 4). According to theory [27], in two-phonon process the size of the anomalies expressed by the attenuation $\alpha$ of the signal wave (which is monitored) is given by

$$\Delta\alpha \equiv \frac{\left|\tilde{A}\right|^2}{\left(1 + P_0^s\right)^2} \frac{\Delta(T) U_p}{U_c},$$

(7)

Here $q_s$ is the wave number of the signal wave, $\tilde{A}$ is the coupling strength, $P_0^s$ is a Fermi liquid parameter, $U_p$ is the energy density of the pump wave (which provides parametric excitation), and $U_c$ is the superfluid condensation energy density. It is evident from Eq. (7) that two-phonon processes are more pronounced at low pressures, where $P_0^s$ is smallest, and at low temperatures where the quasiparticle lifetime $1/\Gamma$ is longest. The fact that changes in attenuation ($\Delta\alpha$) are proportional to the pump wave energy density ($U_p$) is a unique feature, characteristic of two-phonon processes.
2. Nonlinear acoustic experiments in $^3$He-B

2.1. Experimental techniques

In acoustic experiments on liquid $^3$He by the pulsed sound transmission technique, typically two transducers are used: One to generate and another to detect the ultrasound. In our nonlinear ultrasonic studies two different experimental chambers were employed. The first, a cylindrical cell had two parallel quartz crystals acting as transducers. The $X$-cut crystals were 4 mm apart and operated at the odd harmonics of their fundamental frequency of 8.9 MHz. The construction of the other, cubic cell is shown in Fig. 5a. It consisted of four quartz crystals, each with a basic frequency of 5.0 MHz, mounted on a cubic frame of Macor-glass. The sides of the cube were 9.5 mm. The crystals of one pair faced the top and bottom of the cell, and those of the other pair were opposite to two vertical sides, respectively. The sample space was formed by three crossed holes of 6 mm diameter, bored through each side.

![Diagram](image)

**Figure 5.** (a) Construction of the cubic ultrasonic chamber and (b) the low temperature parts of the experimental setup (the filling line and the Pt-thermometer are not shown).

The experimental chamber, mounted into an epoxy tower (see Fig. 5b), was placed on top of the copper cooling stage of our nuclear demagnetization cryostat [28]. A vertical magnetic field up to 1 T, generated by a home-made superconducting solenoid [29], could be applied to the experimental volume. Most of the $^3$He-liquid was in the heat
exchanger volume, which was located inside the copper nuclear stage as shown in Fig. 5b. The connection to the experimental space was made by a 4-mm diameter bore in the epoxy tower. The heat exchanger consisted of 14 silver plates, each of which was silver sintered on both sides. The large surface area of the sinter, nominally more than 100 m², allowed the use of high ultrasound intensities needed for nonlinear experiments without too severe heating of the sample.

The ³He filling line was connected directly to the nuclear stage whose temperature could be lowered by demagnetizing or raised by magnetizing. Thermometry was based on measuring the susceptibility of ¹⁹⁵Pt by pulsed NMR. Powdered platinum, immersed in ³He, was in a separate tower and had a connection to the main liquid volume in the heat exchanger through a 70-mm long Cu-Ni-tube of 3 mm diameter. As a secondary thermometer, we used the phase velocity of ultrasound which is sensitive to temperature.

In our nonlinear experiments two sound waves were produced which propagated simultaneously through the sample. Two parallel waves were created by employing only one crystal: Using two separate rf-generators, the signal pulse of lower intensity was sent, together with the pump pulse of higher intensity, to the transmitter crystal through a directional bridge as shown in Fig. 6. Hence each pulse consisted of two frequencies. In order to create two perpendicularly propagating sound waves, one of the horizontal and one of the vertical crystals in the cubic cell were excited simultaneously, i.e., the pump pulse was sent to another crystal (see dash-dotted line in Fig. 6).

Figure 6. Block diagram of the electronics used in the nonlinear ultrasonic experiments.
In our measurements the ultrasound pulses were a few tens of microseconds long. The transmitter circuit determined the length of the interval between the sound bursts. The receiver detected the signal wave after it had travelled through the sample; the signal was then fed via a high-frequency preamplifier to a receiver unit [30], which separated the in-phase (0°) and the out-of-phase (90°) components. The data were recorded by a high frequency digital oscilloscope and transferred to a computer for further analysis.

The amplitude of the received sound signal was obtained from

$$A = \sqrt{A_{in}^2 + A_{out}^2}$$  \hspace{1cm} (8)

where \(A_{in, out} = \int_{t_1}^{t_2} a_{in, out} dt\) and \(a_{in, out}\) are the in- and out-of-phase signals. The time interval \(t_1 - t_2\) was chosen such that the transmitted sound pulse was clearly separated from the first reflected echo.

The attenuation \(\alpha\), defined as \(A(x) = A_0 e^{-\alpha x}\) where \(A_0\) is the initial transmitted sound amplitude and \(x\) marks the distance from the transmitter, was determined with respect to the reference level at \(T = T_c\), viz.

$$\alpha(T) - \alpha(T_c) = -\frac{1}{L} \ln \left( \frac{A(T)}{A(T_c)} \right)$$  \hspace{1cm} (9)

Here \(L\) is the distance between the transmitter and the receiver crystals.

2.2. Results: Two phonon absorption

Nonlinear processes have several advantages when compared with linear phenomena in ultrasonic studies of \(^{3}\)He-B. In linear excitation the wave number of the excited collective mode is determined by the frequency which depends on the dispersion relation of zero sound (see Fig. 3). In nonlinear excitation the wave number and frequency are independent and thus one can measure directly the dispersion relation of the collective mode. Another advantage comes from the different selection rules for linear and nonlinear processes. For instance, in linear studies of the rsq-mode only the \(m = 0\) submode couples to ultrasound in zero magnetic field. In nonlinear processes, using two nonparallel propagating waves, other substates of the rsq-mode can couple to ultrasound as well.
2.2.1. Real squashing collective mode

The rsq-mode was excited parametrically by means of two phonon absorption (TPA) for the first time by Torizuka et al. [31]. Those experiments were performed in the cylindrical cell, described in Sec. 2.1, by sending two parallel ultrasound pulses through the superfluid $^3$He-B sample along the vertical axis of the cryostat. The resonance condition for this nonlinear process is

$$\hbar(f_s + f_p) = \hbar\omega_{rsq}(T),$$

where $f_s$ and $f_p$ are the signal wave and pump wave frequencies, respectively. It was found that, in addition to the main attenuation peak caused by the two-phonon resonance of the $m = 0$ submode, there was also a smaller satellite in the spectrum. This peak originated from interaction of the signal wave with the counter-propagating pump wave which had been reflected from the receiver crystal and gave the resonance with the rsq-mode at lower temperature. Thus both peaks in the attenuation spectrum were caused by TPA.

![Figure 7](image-url)

Figure 7. Attenuation spectra obtained in the cubic cell with an $f_s = 25.15$ MHz signal wave, in the presence of an $f_p = 15.1$ MHz pump wave at $p = 0$ and $H = 0$, when (a) parallel and (b) perpendicular pulses were sent through the sample. The pulses were 45 $\mu$s long. Arrows indicate mutual directions of the propagating signal and pump waves.
The nonlinear excitation of the rsq-mode was more systematically studied in the cubic ultrasound cell [P1, P2, P3]. As in the experiments with the cylindrical cell, the locations of the attenuation maxima caused by TPA were in good agreement with theory (see Table I in [P3]), and also the satellite peak was often present. Figure 7a illustrates our experimental data in the cubic cell: The $m = 0$ submode was excited at two different wave vectors, by parallel ($|\bar{q}| = |\bar{q}_a| + |\bar{q}_p|$) and antiparallel ($|\bar{q}| = |\bar{q}_a| - |\bar{q}_p|$) sound pulses which enabled us to study the dispersion of the $J = 2^+$ mode. By sending two sound pulses in directions orthogonal to each other through the sample, the rsq-mode was excited at the third wave vector $|\bar{q}| = \sqrt{|\bar{q}_a|^2 + |\bar{q}_p|^2}$ as well (see Fig. 7b). In addition, the $|m| = 2$ substates were also excited.

From the locations of the four attenuation peaks in Fig. 7 we determined the dispersion parameters $c_a^2$ and $c_b^2$, and also the dimensionless coefficient $a^2$, using the dispersion relation (Eq. (6)) of the rsq-mode. Our results are presented in Table I. The theoretical values for $c_a^2$ and $c_b^2$ were calculated in the $T = 0$ limit, employing the first order Fermi-liquid corrections by using Eq. (80) of Ref. 32. Our measurements are the first to determine $c_a^2$ directly from an experiment, and our values for $c_b^2$ and $a^2$ agree well with previous measurements [18, 33].

| Table 1. Theoretical and experimental values for parameters $a^2$, $c_a^2$ and $c_b^2$. |
| --- | --- | --- |
| Quantity | Calculated | Measured |
| | $F_1^a = 0$ | $F_1^a = -0.57$ | |
| $a_{rsq}^2$ | 1.60 | - | 1.47 ± 0.03 |
| $(c_a/\nu_F)^2$ | 0.224 | 0.213 | 0.30 ± 0.07 |
| $(c_b/\nu_F)^2$ | 0.327 | 0.297 | 0.30 ± 0.06 |

We demonstrated also the Zeeman splitting of the parametrically excited rsq-mode, applying a magnetic field perpendicular to sound propagation. Theory [27] predicts three-fold splitting (quantum numbers $m_J = -2, 0, 2$), but a five-fold splitting of the main peak was observed (see Fig. 8). The $m_J = \pm 1$ substates are in the spectrum because of a nonuniform texture in the limited geometry of the experimental cell. The observed linear splitting, 9.5 MHz/T, is rather close to the value 7.5 MHz/T measured in the linear regime [21].
Figure 8. Zeeman splitting of the parametrically excited rsq-mode for the sound pulses with frequencies $f_s = 25.15$ MHz and $f_p = 15.1$ MHz at zero pressure.

2.2.2. Other two-phonon phenomena

Two phonon absorption is not limited to studies of the rsq-mode only. In our measurements an interesting anomaly was observed near the two-phonon pair-breaking edge ($hf = \Delta(T)$) when energetic single-frequency ultrasound pulses (0.002 ... 0.06 $U_c$) were transmitted through the superfluid sample [34, P3]. Figure 9 displays our experimental data obtained in the cylindrical cell using the sound frequency $f = 26.8$ MHz: Attenuation of ultrasound pulses is presented as a function of temperature. There is a clear anomaly at $T = 0.76$ $T_c$, which is in good agreement with the theoretically calculated value $T = 0.74$ $T_c$ for the two-phonon pair-breaking edge. In a magnetic field, applied parallel to the sound propagation, the observed anomaly split into a triplet at least. In our cubic cell an anomaly in the attenuation was seen at $T = 0.77$ $T_c$, using a sound frequency $f = 25.15$ MHz at zero pressure. The expected location of the edge is at $T = 0.78$ $T_c$ under these conditions. Similar studies have been performed by Peters and Eska [35], but in the absence of a magnetic field.

Attempts were made to observe the stimulated Raman scattering, which corresponds to the conversion of a high frequency phonon into a low frequency phonon and a $J = 2^+$ excitation (see Fig. 4b). Therefore, when the low frequency wave is used as the signal wave, Raman scattering should amplify the signal because the high-frequency pump wave phonons are converted to low-frequency signal wave phonons.
Figure 9. Attenuation of ultrasound pulses with $f = 26.8$ MHz and with the energy density $U/U_c = 0.06$ near the two-phonon pair-breaking edge under zero pressure.

We tried to detect Raman scattering in our cubic cell using several combinations of frequencies that were available in our setup. Extensive measurements were carried out employing a 5.0 MHz signal wave and a 25.15 MHz pump wave. Unfortunately, we did not detect any positive signal which could have been interpreted as resulting from stimulated Raman scattering. In the case of high frequency signal wave, strong attenuation due to linear coupling to the $q$-mode took place at the expected location of the stimulated Raman peak. When a pump wave of the higher frequency was used, the power density decreased quickly when the wave travelled through the sample. Therefore, Raman scattering and other nonlinear phenomena were possible only in the immediate vicinity of the transmitter crystal and could not be observed.
3. Optical experiments in $^3$He-B and in $^3$He/$^4$He mixtures

Optical studies of superfluid $^3$He were initiated in our laboratory with the intention to image directly vortices created by rotation. So far, no experimental knowledge exists on the structure of the vortex lattice in superfluid $^3$He. One method to detect individual vortices would be optical observation of the small depressions that these singularities produce on the free surface of the rotating superfluid. However, the depth of the dimples, which are the result of the negative Bernoulli pressure caused by the $1/r$ - velocity field of superfluid around each vortex line, is predicted to be only a few nm [36, 37]. It is quite possible that some additional enhancement (application of an electric field on a charged surface [38], for instance) must be used in order to see the vortex lattice of superfluid $^3$He.

3.1. Development of interferometric techniques for ultralow temperatures

Prior to our work the minimum temperature for optical experiments was above 10 mK [39] because of heat leaks caused by thermal radiation. In our studies we chose a different approach from the methods used in conventional optical cryostats. The sample was illuminated through a single mode optical fiber, and the image was either transferred by a fiber bundle to a regular video camera at room temperature or, as in our later optical scheme, detected directly by a cooled charge coupled device (CCD), which was installed inside the 4-K vacuum jacket of the nuclear demagnetization cryostat.

Our chosen optical technique was the standard two-beam interferometry. Coherent light was reflected from the free surface of liquid $^3$He and from a reference plane, and the created interference pattern was imaged. The surface shape of the liquid could then be extracted by inspecting the interreference fringes.

To check the feasibility of visual studies at ultra low temperatures, several tests on optical components were carried out in a small dilution refrigerator built just for that purpose. In these preliminary experiments we mainly studied how to reduce the heat leak to the sample caused by illumination and by thermal radiation [P4, 40]. The possibility of using a CCD-sensor as a detector of light inside the vacuum jacket of the cryostat was also tested during these studies.
In the first optical experiments on superfluid $^3$He, a coherent bundle of 30 000 fibers was employed to transfer the interference image from the low temperature parts of the cryostat to room temperature [P6]. In this setup extraction of quantitative data was somewhat difficult because of a 2-lens "telescope"-system and the convex shape of the reference plane. The latter problem was due to the fact that the upper surface of the bottom window (optical wedge) of the cell, acting as the reference plane in these experiments, was deformed because of tightening. Moreover, the reflection from the reference wedge (reflection coefficient $R_{RW} = 10^{-2}$) was much larger than that of the free surface of liquid helium ($R_{He} = 10^{-4}$) which resulted in a poor contrast of the interference images.

After our first observations on superfluid $^3$He several modifications were made in the optical setup. The main improvement was the installation of a CCD-sensor inside the vacuum jacket of the cryostat by extending the wires between the sensor and the control unit of the video camera (JVC, model TK-S 200). The experimental resolution increased almost by one order of magnitude: Instead of 30 000 single fibers in a coherent bundle there are $518 \times 582$ pixels in the CCD-sensor (Matsushita, MN 3745). Initially, our real time (25 frames/s) video camera induced a heat leak on the order of 1 $\mu$W to the sample due to rf-radiation. We solved this problem by constructing a "Faraday cage" around the inner parts of the cryostat [P5]. As a result, the heat leak due to the digital signals entering the cryostat was reduced below 1 nW.

In the optical scheme of our interferometer we employed a single lens for focusing the interference picture to the sensor, so that we recorded the real image. The contrast in the interference pattern was improved by employing a reference wedge, the upper surface of which had a reflection coefficient $R_{RW} = 3 \times 10^{-4}$; this is close to that of the reflection from the free surface of liquid $^3$He. We installed the fused silica reference wedge inside the cell, pressing it only slightly against small pieces of indium in order to avoid mechanical stresses.

A bellows system, operated using liquid $^3$He, was built in order to be able to adjust the orientation of the reference plane during experiments. The optical chamber was supported by two bellows (only one, B1, is shown in Fig. 10) from above, and a third corrugated tube (B2) made the connection from below to the heat exchanger volume. Thus, the orientation of the reference plane, together with the whole cell, could be inclined up to 0.5 degrees by changing the pressure in the two upper bellows.
Figure 10. Schematic illustration of our optical setup. Symbols: CCD, sensor; M1 ... M5, mirrors; BS, beam splitter; MC, mixing chamber of the dilution refrigerator; WU, WL, upper and lower windows; HXC, heat exchanger; BA, beam absorber; NS, nuclear stage; FC, Faraday cage. For other abbreviations, see text.

Figure 10 displays the optical setup which was used in the experiments described in Publications [P7–P9]. The source of illumination is a He-Ne laser (LA, $\lambda = 632.8$ nm). The light beam was guided into the cryostat via a single mode optical fiber (OF), expanded by a set of lenses (C and L) to 9 mm diameter, and passed through the sample cell (a copper cylinder with an inner radius $R_c = 10$ mm), which was sealed from both ends by means of fused silica windows. Main part of the illumination was absorbed by the still radiation shield after being reflected from the mirror (M5) located below the cell. Only about 0.05% of light participated in the creation of the interference pattern which was formed owing to reflections from the free surface of liquid helium and from the reference wedge (RW). The interference image was focused by a bi-convex lens ($L, f = 125$ mm) to the CCD-sensor, located above the still plate inside a tight cylindrical copper box. Thermal filters made of sapphire (F1, $d = 2$ mm) and CaF$_2$
(F2, $d = 8$ mm) were installed to the optical path; they prevented thermal radiation, emitted by the sensor at 70 K, to heat up the sample.

Our interferograms are snapshots photographed using 20 ms light pulses. The pictures were recorded on a video tape and further analysis was performed after the recordings had been transferred by a commercial program "SM-Camera" (ProficomP GmbH, Germany) to a Macintosh computer. Figure 11a shows an interferogram from a stationary superfluid sample. The straight interference lines are contours of equal liquid depth above the deliberately tilted (about 0.02 degrees) reference glass; adjacent fringes indicate a change in the liquid depth by $\lambda/2$, where $\lambda = 620$ nm is the optical wavelength in the liquid.

![Interferograms](image)

(a) (b)

**Figure 11.** Interferograms from (a) a stationary and (b) a rotating ($\Omega = 1.55$ rad/s) $^3$He-B sample. Side view is schematically illustrated in the insets. The displayed area in the interferograms is $5 \times 5$ mm$^2$.

Figure 11b presents an interferogram of a rotating superfluid sample at $\Omega = 1.55$ rad/s. The surface profile of the liquid could be obtained, in principle, directly from the recordings. However, at low rotational speeds, when the curvature of the liquid meniscus was small, it was difficult to reconstruct the parabolic form from the fringes. Thus, in our experiments we followed the change in the liquid layer thickness at the nadir of the rotation-induced paraboloid (see Sec. 4.1 in [P9]). We were able to determine spatial shifts of the interference fringes within an accuracy of about 1/30 of their width, which corresponds to a vertical resolution of 10 nm in the liquid depth. This was
achieved by calculating the lateral shift of fringes, which minimized the square sum of differences between two pictures.

3.2. Interferometry on $^3$He-B

3.2.1. Optical observations on superfluid $^3$He

In the first optical experiments on superfluid $^3$He [P6] the huge viscosity change at the superfluid transition temperature $T_c$ was seen directly. The cryostat was rotated slowly, at 0.1 rad/s, about an axis tilted by 0.1 degrees with respect to gravity, and the light beam which was reflected from the bottom window of the optical cell and refracted by the free surface of the liquid was followed. Above $T_c$, where the liquid is very viscous (comparable to light machine oil), its meniscus was determined by the cell walls, and no change was seen during rotation. Cooling into the superfluid phase caused a dramatic change: A precession of the light beam immediately started with the frequency of rotation. This occurred because the free surface of the highly mobile superfluid was able to adjust itself perpendicular to gravity.

![Image of interferograms](image)

**Figure 12.** Fountain effect in a 0.5 mm layer of superfluid $^3$He-B. From left to right, pictures were taken at 20-ms intervals. The shrinking rings (seen best by following the inner dark ring) indicate an increase in the layer thickness.

The fountain (or thermomechanical) effect of superfluid $^3$He was demonstrated in a neat way by these experiments. A special superleak was not necessary because the normal fluid component of a thin superfluid layer was locked to the bottom of the cell by its viscosity. Absorption of the illuminating light power in the cell windows, about 1% of the total power of 60 μW, induced a temperature rise in the liquid, which caused a rush of the superfluid component into the illuminated area. In the interferograms this was seen as shrinking of the rings towards their center (see Fig. 12). The increase of the
thickness, by about 10 \( \mu m \), took place during a few seconds of illumination, while the superfluid \(^3He\) layer warmed from 0.75 \( T_c \) to \( T_c \). After reaching the normal state, the liquid started to relax slowly back, which was seen as expanding fringes in the interferograms.

In these measurements the meniscus of the rotating superfluid was compared with that of the rotating normal liquid. No differences were detected between the normal and the superfluid states which implied the presence of an equilibrium number of vortices in the superfluid.

3.2.2. Superfluid dynamics in a thin layer of \(^3He\)-B

The main difference in the superfluid dynamics of \(^3He\), compared with \(^4He\)-II, originates from the high viscosity of the normal component in \(^3He\); the difference is more than three orders of magnitude (the kinematic viscosity \( \nu \) of \(^4He\) is roughly \( 10^{-4} \) cm\(^2\)/s at \( T = 1 \) K while that of \(^3He\) is about \( 1 \) cm\(^2\)/s at \( T = 1 \) mK). In addition to the shear viscosity, the second viscosity, which is the result of normal-superfluid conversion, represents an additional dissipative mechanism. It is well known that in superfluid \(^3He\) the second viscosity, together with the large thermal conductivity, strongly damp the collective modes based on hydrodynamic counterflow, \( i.e., \) second sound [41] and \( U \)-tube oscillations [42].

We studied the dynamics of a thin layer of superfluid \(^3He\)-B by measuring an impulse response of a liquid sheet utilizing thermomechanical excitation [P7]. The level rise in the superfluid sample, induced by a 50 ms long light pulse, was about 0.5 \( \mu m \) which is comparable to the wavelength of the laser light used for illumination. We were able to follow the height of the liquid surface during the relaxation of the level by applying another light pulse after a variable (0.2 ... 3.0 s) time delay.

Some of the reconstructed impulse responses for different temperatures are presented in Fig. 13. As can be seen, the damping is large in the liquid layer causing fast relaxation of the liquid level; no overshoot, which could have been due to fifth sound phenomena [43] was observed. Below 0.7 \( T_c \), the response of the reference glass was clearly slower than that of the liquid itself. Thus, we have taken the impulse response at our lowest measurement temperature 0.66 \( T_c \), shown with a dashed line in Fig. 13, as the response of the reference wedge itself.
In order to interpret our data we derived, using two-fluid hydrodynamics, the following formula for adiabatic thin-film oscillations (see Sec. 2.2 in [P9]):

$$\frac{d^2 q_k}{dt^2} + \left( \zeta_{\text{eff}} k^2 + D_T k^2 \right) \frac{dq_k}{dt} + \zeta_{\text{eff}} D_T k^4 q_k = 0,$$

(11)

where the effective viscosity $\zeta_{\text{eff}}$ stands for

$$\zeta_{\text{eff}} = \frac{\rho_s}{\rho} (\rho \zeta_3 + \frac{4}{3} \frac{\rho_n}{\rho} \nu),$$

(12)

and $\zeta_3$ is the second viscosity coefficient. In Eq. (11), $k$ is the wave number, $q_k$ represents the $k$-dependent vibration modes, and $D_T$ is the thermal diffusivity.

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{figure13.png}
  \caption{Impulse response of a 0.1 mm thick film of $^3$He-B, measured at $T/T_c = 0.99$ (○), 0.86 (●), and 0.66 (---); $\delta h$ marks the change of the liquid height in the cell. The solid curves are the calculated impulse responses (obtained using Eq. (11)), fitted to the experimental data points.}
\end{figure}

In the data analysis we employed the analogy with the impulse response of second order systems in the overdamped case (see, e.g., Ref. 44). Applying the solution of Eq. (11) for fitting our experimental data points we were able to extract $\zeta_3$ in the temperature range 0.86 ... 0.99 $T_c$; the result is displayed in Fig. 14. Our data are in
nice agreement with the theoretical calculations of Einzel [45]. However, our measured values for $\zeta_3$ are about 40% smaller than those obtained from the vibrating wire measurements of Carless et al. [46]. This discrepancy may be explained by the uncertainty in the $k$-vector values and by the not-strictly-adiabatic conditions in our measurements.

![Graph showing $\zeta_3$ vs. $T/T_c$](image)

**Figure 14.** Second viscosity as a function of temperature obtained from our experimental data. The curve displays the theoretical behavior calculated by Einzel [45] using the quasiparticle relaxation time $\tau_0 = 3.2 \cdot 10^{-7}$ s at $T_c$ and the density of states $N_c = 1.1 \cdot 10^{37}$ (Jm$^3$)$^{-1}$.

### 3.2.3. Rotating $^3$He-B: Observations on the superfluid meniscus

Besides the equilibrium rotation, during which the superfluid accommodates to solid body behavior by creating a vortex lattice, two other rotational states are possible. Initially, while bringing the experimental cell into rotation, the superfluid component stays at rest and no vortices are created. Because the centrifugal force acts then only on the rotating normal component, the meniscus of such a state is given by

$$
\xi_{nf}(r) = \frac{\rho_n \Omega^2 r^2}{\rho \cdot 2g},
$$

(13)
i.e., the classical meniscus, given by Eq. (4), is scaled down by the factor $\rho_n/\rho$. This reduced meniscus is difficult to observe in $^4$He-II because vortices appear in the bulk liquid at very low angular velocities. In $^3$He-B, since the thermal activation of vortices is substantially reduced, the vortex-free state (also known as the Landau state) exists up to much higher rotational speeds [47, 48].

![Graph](image)

**Figure 15.** Level change $\delta = |z(0)|$ measured during acceleration for a 0.1 mm thick layer of $^3$He-B. The solid and dashed curves mark the calculated behavior for equilibrium rotation and for a vortex-free superfluid at $T = 0.68 \ T_c$, respectively. In the inset the maximum counterflow velocity $v_c$ at the cylinder wall is presented as a function of the film thickness $h$. The size of the data points is approximately equal to the experimental error bars.

The Landau state has been studied extensively in NMR experiments [47]. Our optical measurements were performed after the $^3$He sample had been cooled into the superfluid state at rest. The speed of the cryostat was then increased step-wise using a slow acceleration of 0.01 rad/s$^2$. After each acceleration period, the angular velocity was stabilized for 60 s at least before recording an image. Figure 15 displays our experimental results for a 0.1 mm thick layer of $^3$He-B at $T = 0.68 \ T_c$. The absolute change $\delta$ in the liquid thickness at the nadir of the rotation-induced parabola is plotted as a function of the rotation speed $\Omega$. The parabolic behavior of $\delta$ undergoes a jump at about $\Omega_c = 0.20$ rad/s, which is an indication of the appearance of vortices into the rotating sample. The meniscus thus becomes suddenly deeper. At higher speeds the experimental data follow the calculated behavior for classical fluids given by Eq. (4).
The points, measured at lower speeds $\Omega_c \lesssim 0.20$, fit well to the reduced depth of the vortex-free state, obtained from Eq. (13) using $\rho_n/\rho = 0.625$ [49].

Some of our results on the Landau state are shown by the inset of Fig. 15. The critical velocity $v_c = \Omega_c R$ for vortex nucleation was measured as a function of the layer thickness. The data points for the two thicker layers were obtained at $T = 0.74 \, T_c$, whereas the data point for the thinnest layer ($h = 0.1$ mm) was measured at a slightly lower temperature $T = 0.68 \, T_c$. There was a clear decrease in the critical velocity as the layer thickness was increased. The contact area of the liquid with the surrounding rough copper wall increases with the thickness of the layer, which enhances the probability of vortex nucleation.

In $^3$He one can also observe another unique meniscus which is produced by the rotation of the superfluid component only. This situation can be realized after a quick halt of a rotating container filled with the superfluid. In these so called "rapid stopping"-experiments the normal component decelerates quickly because of its high viscosity. The superfluid circulation, carried by vortices, decays over a much longer time interval.

![Interferograms from a rapid stopping - experiment, performed on a 0.4 mm thick layer of $^3$He-B: Snapshots (a) before deceleration, at $\Omega = 1.55$ rad/s, (b) at $t = 0.2$ s, and (c) at $t = 0.8$ s after deceleration was started.](image)

**Figure 16.** Interferograms from a rapid stopping - experiment, performed on a 0.4 mm thick layer of $^3$He-B: Snapshots (a) before deceleration, at $\Omega = 1.55$ rad/s, (b) at $t = 0.2$ s, and (c) at $t = 0.8$ s after deceleration was started.

In our experiments the cryostat was first rotated at a high angular speed ($\Omega >> \Omega_c$) with the equilibrium number of vortices in the experimental chamber. The apparatus was then brought to rest within 1 ... 1.5 s taking, at the same time, snapshots of the sample. Figure 16 displays a set of interferograms from a rapid stopping sequence on a 0.4 mm thick layer of $^3$He-B at $T = 0.73 \, T_c$. The initially circular interference fringes
change first to a curved pattern and then to moving, almost straight lines which are displaced from their equilibrium positions.

Our experimental data on the relaxation of the meniscus are presented in Fig. 17. The absolute level change $\delta$ at $r = 0$ is plotted as a function of time, counting from the moment when deceleration of the cryostat was started. The initial decrease in the depth of the meniscus is caused mostly by the normal liquid. Our analysis indicates that there is no contribution to the meniscus from the normal fluid component after $t = 2.5$ s, i.e., 1.3 s after the stop of the cryostat.

![Graph](image)

**Figure 17.** Relaxation of the meniscus in a 0.4 mm thick layer of superfluid $^3$He-B from the equilibrium vortex state at $\Omega = 1.55$ rad/s after the rotation is stopped; $T = 0.73 \ T_c$. The absolute shift $\delta$ of the nadir is presented as a function of time. The dash-dotted curve represents $\delta$ for the instantaneous speed of the cryostat during the linear deceleration; for explanations of the other curves, see text. Note that the left and right hand scales are in $\mu$m and in nm, respectively.

At $t > 2.5$ s the depth of the meniscus is governed solely by the rotating superfluid component. The momentum conservation law for the whole liquid (see Sec. 2.1 in [P9]) yields for the meniscus of a state when only the superfluid component is rotating
\[ z_{sf}(r) = \frac{\rho_s \Omega_s^2 r^2}{\rho 2g}. \] (14)

The dashed line in Fig. 17 marks a curve which is calculated using Eq. (14) and the theoretical time dependence of the decaying circulation [50], viz.

\[ \frac{\partial \Omega_s}{\partial t} + \frac{\rho_n}{\rho} B(\Omega_s^2 - \Omega(t) \Omega_s) = 0, \] (15)

where \( \Omega_s = \langle \vec{v} \times \vec{v}_s \rangle \), \( B \) is the dissipative coefficient of mutual friction (which characterizes the interaction between the superfluid and normal fluid components during their relative motion); at \( t = 0 \) the speed \( \Omega_s(0) = \Omega(0) \). This model can be used when the viscous relaxation is quick compared to changes in the angular velocity of the cryostat, \( \Omega(t) \) then gives the angular velocity of the normal liquid as well. We employed the value of \( B \rho_n / \rho = 2.85 \), measured by Bevan et al. [51] at 1.6 bar and \( T = 0.73 \ T_C \), using an oscillating diaphragm technique. The calculated curve is clearly below our data which indicates that the observed meniscus is deeper than predicted by Eqs. (14) and (15).

The momentum conservation law, however, is not valid in the decaying vortex state owing to a strong coupling of the thin normal-liquid layer with the cell bottom. Instead of momentum conservation one has to start from the equation of superfluid motion; the meniscus profile of such a state is also affected by the reactive mutual friction coefficient \( B' \). Therefore, instead of Eq. (14), one obtains for the relaxation of the pure superfluid meniscus the following formula (see Sec. 2.1 in [P9]):

\[ z'_{sf}(r) = \frac{1}{g} \left[ \left( 1 - \frac{\rho_n}{2\rho} B' \right) - \frac{\rho_s}{2\rho} \right] \Omega_s^2 r^2, \] (16)

which is valid under isothermal conditions. Using Eqs. (15) and (16), together with the values of \( B \rho_n / \rho = 2.85 \) and \( B' \rho_n / \rho = 1.05 \) [51], we obtain good agreement with the observed relaxation of the meniscus (see solid curve in Fig. 17).

We performed rapid stopping experiments on liquid layers of different thicknesses, \( h = 0.1, 0.4, \) and \( 1.0 \) mm. We did not observe any dependence on \( h \), which indicates negligible pinning effects. This suggests that optical-grade polished glass does not provide good sites for pinning of vortices. Studies on the relaxation of the superfluid meniscus close to \( T_C \), where the mutual friction force increases, were impossible because the enhanced friction caused the superfluid to decay faster while the relaxation
of the normal component slowed down at the same time. As a result, mixed behavior was observed close to $T_c$.

3.3. Nonwetting behavior in $^3$He/$^4$He mixtures

Figure 18 illustrates the phase diagram of liquid $^3$He/$^4$He mixtures at zero pressure. The superfluid transition of $^4$He ($T_\lambda = 2.17$ K) is depressed as the molar concentration of $^3$He is increased. Below the phase-separation curve the liquid separates into two phases: (1) the diluted phase which is rich in $^4$He below and (2) the concentrated phase which is rich in $^3$He and floats on top. These two phases are separated by an interface with a small, temperature-dependent surface tension. In the $^4$He-rich superfluid phase there is a finite solubility of $^3$He, which is 6.5% at $T < 10$ mK.

![Figure 18. Phase diagram of $^3$He/$^4$He mixtures. $c_3 = n_3/(n_3 + n_4)$ is the concentration of $^3$He.](image)

We tried to image a vortex lattice in liquid $^3$He/$^4$He mixtures using our interferometric technique, developed for optical studies on superfluid $^3$He (see Sec. 3.2). It had been predicted that the small surface tension of the $^3$He/$^4$He interface would deepen considerably the vortex dimples at the interface compared to the depressions on the free superfluid surface [37]. The depth of the dimples on the $^3$He/$^4$He interface, caused by $^4$He vortices, has been estimated to be about 60 nm, which is within the resolution of our interferometer.
Our experimental chamber for studying $^3$He/$^4$He mixtures is similar to the optical $^3$He-cell described in Sec. 3.1. It is a copper cylinder ($R_c = 6.5$ mm) which is sealed by two fused silica windows. The normal of the upper window is inclined by 2 degrees from the cylinder axis while the upper surface of the lower window ($0.5^\circ$ optical wedge) served as a reference plane. The filling capillary was made of a 0.1 mm Cu-Ni tube, which was well heat sunk to the 1-K plate but only weakly to the mixing chamber of the dilution refrigerator. The optical imaging scheme was otherwise the same as in the experiments with superfluid $^3$He, which is illustrated in Fig. 10.

![diagram](image)

(a) (b)

**Figure 19.** Interferograms measured in $^3$He/$^4$He mixture ($c_3 = 15\%$) films: (a) Above the phase separation temperature, at $T = 0.35$ K; the liquid $^3$He/$^4$He layer is wedge shaped as illustrated in the inset. (b) Below the phase separation temperature, at $T = 0.32$ K, the stripe-like feature in the middle of the picture is identified as a pool of the $^3$He-rich phase (with a contact angle $\theta$, see the inset) floating on top of the $^4$He-rich superfluid. Each fringe denotes an equal depth contour (multiples of $0.31 \mu$m) with respect to the $0.12^\circ$ inclined reference wedge. The covered area in the interferograms is $7.0 \times 4.5 \text{ mm}^2$.

In the first experiments a gas mixture of $c_3 = 13\%$ was condensed to the cell, until a film of a few tens of micrometers thick was covering the bottom window. Slow (10 ... 50 $\mu$K/s) temperature sweeps across the phase separation temperature $T_5$ were then performed with a non-rotating sample. To our surprise, we observed that the nucleated $^3$He-rich phase preferred to accumulate into floating pools on top of the heavier $^4$He-rich phase (see Fig. 19). These pools were stable over several hours, which was an indication of a steady state. For the contact angle $\theta$ we obtained about 10 mrad. In our
experiments it was found that the nonwetting behavior of liquid $^3$He was due to the nonequilibrium vapor phase in the sample cell [P10, P11].

It is well known that superfluid $^4$He covers all surfaces inside the experimental volume by forming a thin mobile film [52]. In our measurements this film climbed up along the filling capillary towards the warmer regions where it evaporated causing a reflux of $^4$He atoms to the sample. According to our measurements, the heat load from the re-condensing gas to the liquid in the sample cell was about 50 nW, which corresponds to the flow on the order of $10^{15}$ atoms/cm$^2$s.

![Graph](image.png)

**Figure 20.** Contact angle $\theta$ of the $^3$He-rich pools measured as a function of temperature at different $^3$He concentrations: $c_3 = 13\%$ (Δ), 15\% (●), 21\% (○), and 24\% (▽). The filled and open symbols refer to the 1 m and 4 m long filling capillaries, respectively.

We varied the condensation rate of $^4$He atoms by using two different lengths ($L = 1$ m and $L = 4$ m) of the filling capillary. No measurable dependence of the contact angle on the length of the capillary was detected. Our results on $\theta$ as a function of temperature are presented in Fig. 20. The data show a decrease from 16 mrad at low temperatures towards zero at the tricritical point ($T = 0.87$ K).

The gas flow was eliminated completely in some of our experiments by installing a sintered piece to the inlet at the mixing chamber. With this configuration, no anomalous
nonwetting behavior was observed when the temperature was swept slowly over $T_5$.

The interface was covered completely by the upper $^3$He-rich phase.

The finite contact angle, formed under a small flux of $^4$He atoms to the sample cell,
was presumably due to a modification in the surface tension of the upper $^3$He-rich
phase. This explanation is based on the large difference in the mass diffusion constant
between the superfluid and normal phases [53]. In the superfluid $^4$He-rich phase,
second sound can effectively eliminate concentration gradients while the normal $^3$He-
rich phase has a poor mass diffusivity. Strong modifications of the surface structure are
thus plausible under a continuous flow of $^4$He atoms. By employing the formula [54]

$$1 - \cos \theta = \left( \frac{(\sigma_3 + \sigma_{34})^2 - \sigma_4^2}{2 \sigma_3 \sigma_{34}} \right),$$

(17)

where $\sigma_3$ and $\sigma_4$ are the surface tensions of the $^3$He-rich and $^4$He-rich phases, and
$\sigma_{34}$ is that of the $^3$He/$^4$He-rich interface, it is evident that the relative increase of $\sigma_3$
has to be only $10^{-5}$ in order to explain our results. Another possibility is that there is a
small temperature drop $\Delta T = T_3 - T_4$ across the interface owing to the Kapitza
resistance between the $^3$He-rich and $^4$He-rich phases. However, in this case it is
difficult to explain why the contact angle is independent of the length of the filling
capillary, i.e., of the flux of the recondensing $^4$He atoms.

In rotation, no fingerprints from vortices on the $^3$He/$^4$He interface were seen in our
experiments. Under equilibrium conditions, the weak interference pattern from the
phase separation interface was hidden behind the stronger fringes created by the
reflection from the free surface. When the cell was filled completely, so that there was
no liquid-vapor interface present, the $^3$He/$^4$He interface was vibrating too much for
accurate observations even when the cryostat was at rest. Attempts to detect vortex
dimples were thus unsuccessful.
4. Summary

In this thesis, superfluid $^3$He-B was investigated by two methods, ultrasound and optics. In acoustic studies, the nonlinear excitation of the real squashing mode (rsq) enabled us to study the dispersion of the rsq-mode in zero magnetic field which is not possible in linear experiments. Zeeman splitting of the parametrically excited rsq-mode was demonstrated in a magnetic field perpendicular to the sound propagation. In our measurements, nonlinear acoustic phenomena were also detected near the two-phonon pair-breaking threshold.

In optical experiments, the free surface of superfluid $^3$He was visualized for the first time ever, using standard two-beam interferometry. This was made possible by developing a new non-conventional method in which there is no direct optical path from room temperature to the sample at low temperatures. In our optical studies on the dynamics of superfluid $^3$He-B, the impulse response of a thin superfluid layer was recorded by employing thermomechanical excitation. As a result, the second viscosity coefficient $\zeta_3$ was determined close to $T_c$.

Two superfluid menisci, never seen before, were observed in rotation. At low rotational speeds, $\Omega \lesssim 0.20$ rad/s, a reduced parabolic meniscus was formed which signalled the absence of rotation of the superfluid component (the Landau state). After a rapid halt of rotation from an equilibrium vortex state, a novel enhanced meniscus was found which was induced by the decaying vorticity. The depth of this unique meniscus was observed to be governed by the reactive mutual friction arising between the superfluid and normal components.

Using the interferometric technique, developed for studies of superfluid $^3$He, unexpected nonwetting phenomena were discovered in phase separated $^3$He/$^4$He mixture films. A small flux of $^4$He gas (on the order of $10^{15}$ atoms/cm$^2$s) to the liquid sample induced pools of the $^3$He-rich phase which floated on top of the heavier $^4$He-rich phase; the contact angle was 10 mrad.
5. Publications

This thesis consists of the following original publications.


Nonlinear ultrasonic investigations on the real squashing (rsq) collective mode in superfluid $^3$He-B are reported. Besides excitation of the rsq-mode by two simultaneous ultrasound pulses, yielding two phonon absorption, the Zeeman splitting of the parametrically excited rsq-mode, in a magnetic field perpendicular to the sound propagation, was demonstrated. The dependence of the mode frequency on the magnitude of the wave vector was studied as well.


Dispersion of the real squashing collective mode in $^3$He-B was investigated by means of two phonon absorption. From the observed peaks in the attenuation spectrum, measured while two simultaneous sound pulses propagated either in parallel, antiparallel, or perpendicular directions, the collective mode velocities were extracted. Qualitatively these results are in agreement with the theoretical dispersion relation of the $J = 2^+$ mode.


This paper is a comprehensive review of the nonlinear acoustic experiments in $^3$He-B, including the experiments described in [P1] and [P2]. In addition to studies on the parametrically excited real squashing collective mode, measurements near the two-phonon pair-breaking edge are presented: An anomaly in the attenuation spectrum split into a triplet in an applied magnetic field.


An experimental feasibility study for optical investigations at ultra low temperatures is described. Mainly heat leak measurements were made in order to determine how to reduce thermal radiation from a video-camera sensor and from the illumination to the sample at low temperatures.

A millikelvin temperature imaging system was constructed by employing a cooled video camera sensor inside the 4-K vacuum can of a nuclear demagnetization cryostat. The techniques for reducing the heat leak from the camera to a tolerable level (less than 1 nW) are described. The limits of operation of the whole system are discussed.


The first optical experiments on superfluid $^3$He are described. The dramatic change in viscosity of the liquid at the superfluid transition was seen directly. The meniscus of rotating superfluid was compared with the normal liquid behavior: The average solid body rotation of the superfluid component was observed, implying the existence of a state with the equilibrium number of vortices. The fountain effect of superfluid $^3$He was demonstrated as well.


The first optical studies on the dynamics of superfluid $^3$He are reported. The impulse response of a $^3$He-B layer with 0.1 mm thickness was recorded after thermomechanical excitation. The measured behavior is compared with the derived formula for thin-film oscillations, obtained using two-fluid hydrodynamics. As a result, the second viscosity coefficient $\zeta_3$ was deduced from the experimental data in the range $0.86 \ldots 0.99$ $T_C$. Good agreement with the theoretical results of Einzel was obtained.


Interferometric measurements on rotating $^3$He-B resulted in observations of two superfluid menisci never seen before. At small rotational speeds, $\Omega \lesssim 0.2$ rad/s, a reduced temperature dependent parabolic meniscus demonstrated the absence of rotation of the superfluid component. Rapid halt of the cryostat from the equilibrium vortex state at high rotational speeds yielded a unique enhanced meniscus which was induced by decaying vorticity. A theory is presented, which relates the enhanced depth of this meniscus with the reactive mutual friction on the liquid pressure.

This paper is a review of interferometric measurements performed on $^3$He-B. In the theoretical part, the meniscus of a rotating superfluid is examined by considering the momentum conservation law. Using two-fluid hydrodynamics, the formulae for the modes of oscillation are derived for a thin superfluid layer. The experimental setup and the adjustment procedures are presented, and the measurements described in Publications [P7] and [P8] are discussed in more detail.


Nonwetting behavior of liquid $^3$He was discovered in phase separated $^3$He/$^4$He mixtures. A small continuous flow of $^4$He gas (on the order of $10^{15}$ atoms/cm$^2$s) to the vapor phase of the mixture film caused the $^3$He-rich phase to form well-defined pools floating on top of the heavier $^4$He-rich superfluid. For the contact angle the measurements yielded about 10 mrad, which suggests a fractional change in the $^3$He surface tension by a factor of $10^{-5}$ from the equilibrium value.


A more detailed description of the experiments reported in Publication [P10] is given. It is shown that, for the phase separated mixtures in equilibrium, Antonow’s rule is valid within an accuracy of $10^{-6}$, which is four orders of magnitude better resolution than what has been achieved in regular measurements of surface tension. A continuous flux of condensing $^4$He atoms to the sample induced slight, 10 ppm deviations from this rule and caused difficulties in the spreading the $^3$He-rich phase on top of the $^4$He-rich superfluid.
Author's own contribution:

All work presented in this thesis is the result of a group effort. I joined the ROTA 2 project in the autumn of 1990 and took active part in the acoustic experiments, which include the last measurements performed in the cylindrical ultrasonic chamber and all experiments in the new cubic cell. At that time, I operated and maintained our nuclear demagnetization cryostat and carried out the first data analysis of the experiments in the cubic cell; this work is presented in Publications [P1]–[P3]. In technical preparations for optical studies, my contribution was larger in the advanced stage of work which is described in Publication [P5]. During the measurements reported in Publication [P6] I was mostly involved in another project, namely preparing for optical experiments on the liquid-solid interface. I was the senior graduate student in the ROTA 2 group when the experiments described in [P7]–[P11] were carried out and performed most of the data analysis described in Publications [P8]–[P11]. I have written Publications [P8], [P9] and [P11], and the first version of Publication [P5].
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