Comparison of different MIMO-OFDM signal detectors for LTE

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**Abstract**

Long Term Evolution (LTE), is a standard for radio communications of high-speed data transmission for mobile phones and data business [1]. It is based on the Global System for Mobile communications (GSM)/ Enhanced Data rate for GSM Evolution (EDGE) and Universal Mobile Telecommunications System (UMTS)/ High-Speed Packet Access (HSPA) network technologies, increasing the capacity and speed using new modulation techniques [1, 2]. Orthogonal frequency-division multiplexing (OFDM) and multiple-input multiple-output (MIMO) are the key technologies of LTE. MIMO technology can significantly increase channel capacity and spectrum efficiency without occupying any more bandwidth. Signal detection algorithms are studied in this thesis: Zero-Forcing (ZF) detector, Minimum Mean Square Error (MMSE) detector, Maximum Likelihood (ML) detector, QR decomposition with M-algorithm maximum likelihood detector (QRM-MLD) and Sphere Decoding (SD) detector. The results of the study show that the optimal signal detector is superior to other signal detectors on bit-error-rate (BER) performance. The BER performance in a correlated MIMO-OFDM scenario of the detectors are also studied.

**Keywords:** LTE, OFDM, MIMO, ZF, MMSE, ML, QRM-MLD, Sphere Decoding, Spatial Correlation, BER
Comparison of different MIMO-OFDM signal detectors for LTE
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[Text]

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SHAO QIWEN

[date and place]

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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile communications</td>
</tr>
<tr>
<td>EDGE</td>
<td>Enhanced Data rate for GSM Evolution</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>HSPA</td>
<td>High-Speed Packet Access</td>
</tr>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>3G</td>
<td>3rd Generation</td>
</tr>
<tr>
<td>4G</td>
<td>4th Generation</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>BER</td>
<td>Bit Error Rate</td>
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<td>ZF</td>
<td>Zero-Forcing</td>
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<td>MMSE</td>
<td>Minimum Mean Square Error</td>
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<tr>
<td>SD</td>
<td>Sphere Decoding</td>
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<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
</tr>
<tr>
<td>QRM-MLD</td>
<td>QR decomposition with M-algorithm Maximum Likelihood Detector</td>
</tr>
<tr>
<td>UL</td>
<td>UpLink</td>
</tr>
<tr>
<td>DL</td>
<td>DownLink</td>
</tr>
<tr>
<td>HSDPA</td>
<td>High Speed Downlink Packet Access</td>
</tr>
<tr>
<td>E-DCH</td>
<td>Enhanced Dedicated Channel</td>
</tr>
<tr>
<td>IMT-2000</td>
<td>International Mobile Telecommunications-2000</td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter Symbol Interference</td>
</tr>
<tr>
<td>GI</td>
<td>Guard Interval</td>
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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>I-METRA</td>
<td>Intelligent Multi-Element Transmit and Receive. Antennas</td>
</tr>
<tr>
<td>SCM</td>
<td>Spatial Channel Model</td>
</tr>
<tr>
<td>LoS</td>
<td>Line-of-Sight</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise Ratio</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal-to-Interference plus Noise Ratio</td>
</tr>
<tr>
<td>MAP</td>
<td>Maximum A Posteriori</td>
</tr>
<tr>
<td>MML</td>
<td>Modified ML</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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CHAPTER 1

INTRODUCTION

1.1 MOTIVATION

For growing voice and data services, it raises a higher demand of the transmission rate, transmission performance and data throughput. To achieve this, it is not enough only to use more spectrum resources, the space resources of the wireless signal should be used as well. That is using multiple antennas to transmit and receive the signal. Due to scattering in the wireless communication environment, reflection and diffraction caused by the multipath fading is a major factor in the deteriorating performance of wireless communication systems. Diversity has been an effective technology. Common sources of diversity are time diversity, frequency diversity and space diversity [3]. Space diversity technique does not require extra time and bandwidth.

LTE is a standard for radio communications of high-speed data transmission for mobile phones and data business [1]. It is based on the GSM/EDGE and UMTS/HSPA network technologies, increasing the capacity and speed using new modulation techniques [1, 2]. OFDM and MIMO are key technologies of LTE. MIMO technology can significantly increase channel capacity and spectrum efficiency without occupying more bandwidth. MIMO communications system uses multiple antennas of both transmitting end and receiving end, the data throughput and the spectrum utilization can grow exponentially to meet the requirements of high transmission rate, high transmission performance and high data throughput, MIMO improves communications system performance by full use of space diversity. Meanwhile, OFDM has been widely considered in
Comparison of different MIMO-OFDM signal detectors for LTE

the academia and industry. OFDM is an efficient multi-carrier transmission technology. It converts high speed serial data streams to relatively low transmission rate of symbols on a group of sub channels by serial/parallel conversion. In OFDM, each subcarrier is orthogonal to each other. In frequency domain, the responses of the sub channels overlap. Thus OFDM can provide a higher spectrum utilization than normal frequency division multiplexing system.

As one of the key technologies of LTE, signal detection algorithms are studied in this thesis. After nearly a decade of efforts, MIMO systems and related technologies have developed greatly, but there are still some shortcomings. With the continuous improvement of transmission speed, with increasing numbers of antennas, and higher and higher modulation order, the tradeoff between complexity and performance of the MIMO system should be studied. Approaching the capacity of multi-antenna system needs to have a good detection technology. However, the MIMO system not only brings a huge capacity, but also produces great complexity for the received signal detection.

It is the objective of this thesis to investigate MIMO-OFDM detectors for LTE. Five signal detection algorithms are studied: ZF detection, MMSE detection, ML detection, QRM-MLD and SD detection. ZF [3] and MMSE [3] are simple and have low complexity but the performance is not ideal. ML [3] detection has the optimal detection performance. However, because of it searches through all possible points in the signal space, if the number of transmit antennas is greater than four or modulation constellation is larger than QPSK, the complexity will reach a very high level [3]. ML is thus difficult to implement in practical applications. The current study focused on the signal detection algorithm is in two areas: to enhance the performance of the linear detector and to reduce the complexity of ML algorithm. QRM-MLD [4] and SD [5] detector are the detectors reduce the complexity of ML algorithm. SD detector do not search all the points as ML detector. It limits the search area into a sphere for the center is the actual received point \( y \). Thus the points need to be searched are reduced. The range of search is far less than ML detector; therefore it significantly reduces the computational complexity. QRM-MLD is based on QR decomposition and ML algorithm. This detection algorithm approximately obtains the ML detection performance. Compared to ML detection, it greatly reduces the complexity of
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detection. The performance of QRM-MLD would decrease when the number of transmit antennas increase and when the value of parameter M of the detector reduce. The results of the study show that the optimal signal detector is superior to other signal detectors on BER performance, but it has a large computational complexity. This thesis also studied the performance on BER of five signal detectors if the MIMO-OFDM channel is spatial correlated with spatial correlation.

1.2 STRUCTURE OF THE THESIS

The thesis has been structured into the following chapters that are shortly outlined below:

Chapter 2: This chapter denotes the technology background of LTE, especially MIMO and OFDM technologies. This chapter also studies the Multipath Rayleigh fading channel, spatial correlation and QR decomposition.

Chapter 3: This chapter introduces the signal detection algorithms. Five detector algorithms are studied in this chapter. It analyzes these five detectors including algorithm and assuming performance.

Chapter 4: This chapter introduces the implementation of five different detectors in Matlab simulation.

Chapter 5: This chapter denotes the results of the simulation and analyzes the results on BER performance with a correlated MIMO-OFDM channel and with an uncorrelated MIMO-OFDM channel.

1.3 BASIC DEFINITIONS

In this thesis, the following tasks will be studied:

1. In the same wireless transmission environment with the same channel, the same transmitted signal, the same MIMO and OFDM parameters etc., and five different detectors will be used for signal detection while the signal-to-noise ratio (SNR) will be the variable but with the same SNR range. The BER will be
the result for the performance of these detectors. What are the performances of these detectors?

2. In the same wireless transmission environment, and the BER will be the result for the performance of these detectors. How does QRM-MLD behave while the value of parameter M changes from 4 to 16 compared to ML detector?

3. In the same wireless transmission environment, and the BER will be the result for the performance of these detectors. How does SD detector behave while the radius changes from small value to the value large enough compared to ML detector?

4. In the same wireless transmission environment, but the channel is spatial correlated, five different detectors will be used for signal detection. The BER will be the result for the performance of these detectors. What are the performances of these detectors with a correlated MIMO-OFDM channel?

5. For a signal detector, if the wireless transmission environment is the same except for the spatial correlation, what is the performance of the detector while the MIMO-OFDM channel is correlated compared to if it is uncorrelated?
CHAPTER 2

2.1 LTE

There has been a "data explosion" nowadays which means the use of voice and data service by cellular communications system increases rapidly and this increase will only become larger and larger. To achieve the increased demands for data transmission speeds and lower latency, it is not enough only using more spectrum resources, but also using of the space resources of the wireless signal, that is using multiple antennas to transmit and receive the signal.

Since November 2004, 3rd Generation Partnership Project (3GPP) has been working on the LTE for enhancements to the UMTS, and focus on adopting 4th Generation (4G) technology. Specs (Rel-8) were finalized and approved in January 2008 [1].

The UMTS cellular technology upgrade has accordingly been dubbed LTE [1]. For growing voice and data services, it raises a higher demand of the transmission rate, transmission performance and data throughput. The LTE milestone in 3GPP standard evolution is shown in Figure 1 [1],

![Figure 1 LTE Milestone in 3GPP Standard Evolution](image)

For 3GPP LTE, there were many technological requirements [2]:

- **Rel’ 99**: UMTS FDD, DCH up to 2Mbps
- **Rel’ 4**: Core Network Evolution, FDD repeaters 1.28Mcps TDD
- **Rel’ 5**: HSDPA
- **Rel’ 6**: Multimedia sub-system
- **Rel’ 7**: HSUPA
- **Rel’ 8**: MBMS
- **Rel’ 9/10**: HSPA+ i.e. MIMO, CPC, DL 64QAM, UL 16QAM
- **LTE**
1. For spectrum efficiency, the downlink (DL) should be 3-4 times High Speed Downlink Packet Access (HSDPA) for 2×2 MIMO antenna arrays while the uplink (UL) should be 2-3 times Enhanced Dedicated Channel (E-DCH) for 1×2 MIMO antenna arrays.

2. A scalable bandwidth of 1.4, 3, 5, 10, 15, 20 MHz should be possible. All carrier frequencies of International Mobile Telecommunications-2000 (IMT-2000) from 450 MHz to 2.6 GHz should be covered.

3. The peak data rate in DL should be greater than 100Mb/s for 20MHz spectrum allocation while in UL it should be greater than 50Mb/s for 20MHz spectrum allocation.

4. The capacity should be 200 users for 5MHz and 400 users in large spectrum allocations.

5. The latency requirement is less than 100ms to establish U-plane in C-plane and less than 10ms from user equipment (UE) to server in U-plane.

6. The coverage requirements is full performance target for cell radius up to 5km while performance with slight degradation should be possible for cell radius up to 30km.

7. For mobility requirements, LTE is optimized for low speeds 0-15km/h but the connection should be maintained for speeds up to 350 or 500km/h. Handover time between 3rd Generation (3G) & 3G LTE in real time mode should be no more than 300ms and no more than 500ms in non-real time mode.

2.2 MIMO-OFDM

In the history of the development of wireless communication, the contradiction between the growing demands of the data throughput with limited radio spectrum resources has been an important force driving wireless communication technology innovation [1]. Next-generation wireless communication systems for higher demand on data transfer rate, requiring high-speed, high-performance data transfer with limited spectrum, is a huge
challenge for the 4G/3GPP LTE of wireless communication that must be faced [2].

2.2.1 MIMO SYSTEM

Generally, multipath propagation would cause channel fading, which is regarded as a harmful factor to wireless communication [3]. However, research shows that in a MIMO system, multipath transmission can be favorable to the wireless communication. Multiple antennas (or array antennas) and multiple channels are used in the transmitter and receiver of MIMO system [3]. In the transmitter, the serial data symbol stream after the necessary space-time processing is sent to the transmit antennas, and then transmitted to the receiver. In the receiver, the received data symbols are recovered through a variety of space-time detection technologies. In order to guarantee effective separation of the various sub-data symbol streams, the antennas must be separated with a sufficient distance (usually more than half a carrier wavelength) in order to prevent too much correlation between the received signals at the different antennas [3]. Figure 2 illustrates a MIMO system.

![Figure 2 MIMO system](image)

As shown in Figure 2, signals are transmitted by antennas, and after propagating over the wireless channel such as the urban channel, they are received at the receive antennas. Each receiving antenna receives a superposition sum of the signals from the transmitting antennas.
2.2.2 OFDM

OFDM is a multi-carrier modulation. In OFDM, the channel is divided into a number of orthogonal sub-channels and the high speed data signal is converted into parallel low speed sub-streams [3]. Those sub-streams are modulated on each sub-channel to be transmitted. Figure 3 illustrates the basic processing in an OFDM system.

![Figure 3 OFDM system](image)

OFDM is effective against frequency selective fading and Inter Symbol Interference (ISI) [3]. Since orthogonal sub-carriers are using as sub-channels, the spectral efficiency has been greatly improved.

Wireless data services are asymmetric, such that the transmission capacity requirement of downlink is greater than of uplink. While using OFDM, the number of sub-channels can be adjusted flexibly to meet the different rate of downlink and uplink transmission [3]. OFDM can be jointly used with other access methods that improves the reliability of signal transmission in physical layer [6].

Generally, in OFDM, a certain length of the guard interval (GI) should be added and it overcomes the ISI when the duration of GI is greater than the maximum multipath delay spread of the radio channel. Typically the GI is filled with a cyclic prefix (CP) [3].
A major advantage of OFDM technology is that fast Fourier transform (FFT)/inverse fast Fourier transform (IFFT) could be used for the implementation of modulation and demodulation of orthogonal sub-channels [6]. For the N point FFT operation, one needs $N \times \log(N)$ complex multiplications, instead of $N^2$, which would be required by a straight forward implementation.

OFDM is the key technology in LTE. There are various joint OFDM technologies: V-OFDM, W-OFDM, F-OFDM and MIMO-OFDM etc. In this thesis, we will focus on the study of five different detectors of MIMO-OFDM in LTE.

2.3 CHANNEL
2.3.1 CHANNEL MODEL
While the wireless signal travels from the transmit antenna to the receive antenna, the characteristics of the signal changes because of the following factors: 1) the distance between the two antennas, 2) the path(s) taken by the signal and 3) the environment (building and other objects) around the path [6].

The medium between the transmit antenna and the receive antenna is called the channel. The effects of the channel can be characterized by a linear response. For MIMO-OFDM in frequency domain, the received signal $y$ and the transmitted signal $x$ can be expressed as

$$y = Hx + n \quad (2.1)$$

Where $H$ denotes channel response, and $n$ denotes the noise.

To analyze performance of a communication system, we need a channel model [8]. There are three key components of the channel response: 1) path loss, 2) shadowing and 3) multipath. In this thesis, we concentrate on the multipath fading of the channel.

2.3.2 MULTIPATH RAYLEIGH FADEING CHANNEL
The objects around the wireless signal path reflect the signal and some of the reflected signal waves also received at the receiver [8]. Each received signals including the reflected ones takes a different path with different amplitude and phase.
Rayleigh distribution is the one-dimensional distribution of the envelope of a stationary narrowband Gaussian process with mean is 0 and variance $\sigma^2$. The probability density of the amplitude of a Rayleigh fading signal is [6]

$$f(z) = \frac{z}{\sigma^2} e^{\frac{-z^2}{2\sigma^2}}, \quad z \geq 0 \quad (2.2)$$

Rayleigh distribution is the most common model used to describe the statistics of the receiving envelope of a time varying flat fading signal or the receiving envelope of an independent multipath components [8]. Rayleigh fading can effectively describe a radio propagation environment with a large number of obstacles scattering the radio signals [8].

Rayleigh fading model is applicable to describe the wireless channels of built-up zone in city center [8]. There is no direct path between the transmitter and receiver due to the buildings and other objects. In this case, the radio signals is attenuated, reflected, refracted or diffracted. In experiments in Manhattan, it shows that the local environment is indeed close to the Rayleigh fading channel [8]. Rayleigh fading is small-scale fading and it is always superimposed on shadows, attenuation and other large-scale fading [8]. In this thesis, we only concentrate on the multipath Rayleigh fading channel.

In MIMO system, the diversity is increased of multi-antennas to overcome the channel fading. Signals contain the same information are transmitted through different paths and the receiver will obtain different independent fading replicas of the data symbols.

### 2.3.3 MIMO CHANNEL

MIMO system employs multiple antennas in the transmitter and/or receiver. The correlation between transmit and receive antennas is an important aspect of the MIMO channel. In the literature, two methods of modeling a correlated MIMO channel are used: a correlation matrix-based Intelligent Multi-Element Transmit and Receive Antennas (I-METRA) channel model [7] and a ray-based 3GPP spatial channel model (SCM) [8]. The correlation matrix-based channel model can be implemented with a spatial correlation matrix for the spatial channel. The ray-based channel model requires neither Doppler spectrum nor
spatial correlation matrix, as the mobility effects and the antenna correlation are directly calculated from the time development of the modeled rays.

Equation (2.1) can be represented as below for a $N_R \times N_T$ MIMO system as

$$
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_{N_R}
\end{bmatrix} =
\begin{bmatrix}
h_{11} & h_{12} & \ldots & h_{1N_T} \\
h_{21} & h_{22} & \ldots & h_{2N_T} \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_{R1}} & h_{N_{R2}} & \ldots & h_{N_{RN_T}}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_{N_T}
\end{bmatrix} +
\begin{bmatrix}
n_1 \\
n_2 \\
\vdots \\
n_{N_R}
\end{bmatrix}
$$

(2.3)

Where $N_T$ represents the number of transmit antennas and $N_R$ represents the number of receive antennas, $\mathbf{y}$ denotes $N_R \times 1$ received signal vectors, $\mathbf{x}$ denotes $N_T \times 1$ transmitted signal vectors and $\mathbf{n}$ denotes $N_R \times 1$ noise vectors. The element of the $N_R \times N_T$ channel matrix $\mathbf{H}$ in position $(i,j)$ denotes the channel from the $j$th transmit antenna to the $i$th receive antenna.

2.3.4 SPATIAL CORRELATION

Spatial correlation always appear in practice because the channels between different antennas are often correlated. In ideal radio communications, the path between the transmitter and the receiver is a line-of-sight (LoS) path. However, in practice urban cellular systems, the channel between the base station and the user is a multipath non-line-of-sight channel. The channel represents how the signal is reflected from the transmitter to the receiver. In this case, the received signal may be received from certain directions with strong spatial characteristics. The correlation between the received average signal gain and the angle of arrival (AoA) of a signal is the so called spatial correlation [9].

The existence of spatial correlation has been experimentally validated [10, 11]. Spatial correlation will degrade the performance of MIMO systems. The gain from having independent channels decreases. In reference [6] it is assumed that the correlation matrices for the transmitter and receiver can be separated for the MIMO channel model of the correlation matrix model. In literature [10], the Kronecker model is introduced for modeling spatial correlation which is widely used. In Kronecker model, the correlation between the transmit antennas and receive antennas are assumed independent and separable [10]. Kronecker model has been validated both indoor and outdoor [11]. Consider a MIMO
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system as illustrated in Figure 2, with Rayleigh fading, the Kronecker model of channel correlation can be represented as

\[ H = R_{NR}^{1/2} H_\omega (R_{NT}^{1/2})^T \]  (2.4)

Where \( R_{NR} \) and \( R_{NT} \) are the correlation matrices for the receive antennas and transmit antennas, respectively, while \( H_\omega \) represents an i.i.d. (independent and identically distributed) Rayleigh fading channel.

Under the Kronecker model, the i.i.d. channel \( H_\omega \) is pre-multiplied by the receive-side spatial correlation matrix \( R_{NR} \) and post-multiplied by transmit-side spatial correlation matrix \( R_{NT} \) [6]. The spatial correlation directly depends on the eigenvalue distributions of the correlation matrices \( R_{NT} \) and \( R_{NR} \) where high spatial correlation is represented by large eigenvalue spread in correlation matrices and low spatial correlation is represented by small eigenvalue spread. High spatial correlation means that some spatial directions are statistically stronger than others and low spatial correlation means almost the same signal gain can be obtained from all spatial directions [6].

2.4 QR DECOMPOSITION

In linear algebra, QR decomposition is the most effective method for seeking the eigenvalues of a general matrix and is widely used. A general matrix is transformed to a Hessenberg matrix via an orthogonal similarity transformation. Then the eigenvalues and eigenvectors can be found with the QR algorithm. A matrix is decomposed into an orthogonal matrix \( Q \) and an upper triangular matrix \( R \) [7]. A real square matrix \( A \) may be decomposed as

\[ A = QR \]  (2.5)

Here \( Q \) denotes an orthogonal matrix (\( Q^T Q = I \)) and \( R \) denotes an upper triangular matrix (right triangular matrix) [7]. Generally, QR decomposition is quite useful in wireless applications.
CHAPTER 3

3.1 SIGNAL DETECTION

Signal detection technology is one of key technologies of LTE. This chapter will study five main signal detection techniques for MIMO-OFDM systems. ZF signal detection, MMSE signal detection will be studied as the linear signal detection algorithms. SD signal detection, QRM-MLD signal detection and ML signal detection algorithms will be studied in this Chapter as non-linear signal detection algorithms. MIMO system not only brings a huge capacity, but also produces great complexity of the received signal detection. The current study focused on the signal detection algorithm is in two areas: to enhance the performance of the linear detector and to reduce the complexity of ML algorithm [6]. QRM-MLD algorithm and SD algorithm are the algorithms reduce the complexity of ML algorithm.

3.2 SIGNAL DETECTION ALGORITHMS AND DETECTORS

Consider a $N_k \times N_r$ MIMO system in Figure 2. Let $\mathbf{H}$ denotes a channel matrix with it $(j,i)$ th entry $h_{ji}$ for the channel gain between the $i$ th transmit antenna and the $j$ th receive antenna, $j=1,2,\ldots,N_k$ and $i=1,2,\ldots,N_r$. The spatially-multiplexed user data and the corresponding received signals are represented by $\mathbf{x} = [x_1, x_2, \ldots, x_{N_x}]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_{N_y}]^T$, respectively, where $x_i$ and $y_j$ denote the transmit signal from the $i$ th transmit antenna and the received signal at the $j$ th receive antenna, respectively. Let $z_j$ denote the white Gaussian noise with a variance of $\sigma_z^2$ at the $j$ th receive antenna, and $\mathbf{h}_i$ denote the $i$ th column vector of the channel matrix $\mathbf{H}$. Now, the $N_k \times N_r$ MIMO system is represented as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{z} = \mathbf{h}_1x_1 + \mathbf{h}_2x_2 + \cdots + \mathbf{h}_{N_r}x_{N_r} + \mathbf{z}$$

(3.1)
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Where \( z = [z_1, z_2, ..., z_N] \) \(^T\) [6].

### 3.2.1 LINEAR SIGNAL DETECTION

Linear signal detection only treats desired stream from target transmit antenna as signals and all other transmitted signals would be treat as interferences [6]. Interference signals from other transmit antennas are minimized or nullified when the desired signal was detected [6]. The effect of the channel is inverted by a weight matrix \( W \) such that [6]

\[
\begin{align*}
\tilde{x} &= [\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_N] \quad \text{or} \\
\approx &= W y
\end{align*}
\]  

(3.2)

That is the symbol detected is given by a linear combination of the received signals [6]. The standard linear detection algorithms include ZF detection and MMSE detection.

#### 3.2.1.1 ZF Detection

Zero-Forcing detection is low complexity linear detection algorithm that gives the estimate of \( x \) as [6]:

\[
\tilde{x}_{ZF} = W_{ZF} y = x + (H^H H)^{-1} H^H z = x + \tilde{z}_{ZF}
\]

(3.3)

The detector thus forces the interference to zero. The matrix \( W_{ZF} \) nullifying the interference is [6]

\[
W_{ZF} = (H^H H)^{-1} H^H
\]

(3.4)

Thus the processed noise is \( \tilde{z}_{ZF} = W_{ZF} z = (H^H H)^{-1} H^H z \) [6]. Here \((\cdot)^H\) denotes the Hermitian transpose operation.

ZF detection algorithm is a linear detection algorithm since it behaves as a linear filter separating different data streams to perform decoding independently on each stream, therefore eliminating the multi-stream interference [6]. The drawback of ZF detection is retarded BER performance due to noise enhancement [6]. The additive white Gaussian noise (AWGN) \( z \) loses its whiteness property it is enhanced and correlated across the data streams [8].
As the SNR increases, ZF solution $\hat{x}$ becomes more likely to coincide with the ML solution vector [8].

### 3.2.1.2 MMSE Detection

MMSE detector estimates the transmitted vector $x$ by applying the linear transformation to the received vector $y$. It finds out the estimate $\hat{x}_{\text{MMSE}}$ of the transmitted symbol vector $x$ as [6]:

$$\hat{x}_{\text{MMSE}} = W_{\text{MMSE}}y = (H^H H + \sigma_n^2 I)^{-1} H^H y = \hat{x} + (H^H H + \sigma_n^2 I)^{-1} H^H z = \hat{x} + \tilde{z}_{\text{MMSE}}$$  \hspace{1cm} (3.5)

MMSE weight matrix $W_{\text{MMSE}}$ is to maximize the post-detection signal-to-interference plus noise ratio (SINR) [6]. And MMSE receiver requires the statistical information of noise $\sigma_n^2$. MMSE detectors balances the noise enhancement and multi-stream interference by minimizing the total error [6]. Its BER performance is superior to ZF detection due to mitigating the noise enhancement.

### 3.2.2 ML DETECTION

ML detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel $H$, and finds the one with the minimum distance [6]. Let $C$ and $N_T$ denote a set of signal constellation symbol points and a number of transmit antennas, respectively. Then, ML detection determines the estimate of the transmitted signal vector $x$ as [6]

$$\hat{x}_{\text{ML}} = \arg \min_{x \in C^{N_T}} \|y - Hx\|^2$$  \hspace{1cm} (3.6)

Where $\|y - Hx\|^2$ corresponds to the ML metric. The ML method achieves the optimal performance as the maximum a posteriori (MAP) detection when all the transmitted vectors are equally likely [14]. However, its complexity increases exponentially as modulation order and/or the number of transmit antennas increases [14]. The required number of ML metric calculation is $|C|^{N_T}$, that is, the complexity of metric calculation exponentially increases with the number of antennas. Even if this particular method suffers from computational complexity, its performance serves as a reference to other detection methods since it
corresponds to the best possible performance. It has been shown that the number of ML metric calculations can be reduced from $|C|^N_r$ to $|C|^{N_r-1}$ by the modified ML (MML) detection method [14]. In other words, it will be useful for reducing the complexity when $N_r = 2$. However, its complexity is still too much for $N_r \geq 3$.

3.2.3 SPHERE DECODING

SD algorithm intends to find the transmitted signal vector with minimum ML metric, that is, to find the ML solution vector. However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors [15, 16]. SD adjusts the sphere radius until there exists a single vector (ML solution vector) within the sphere. It increases the radius when there exists no vector within the sphere, and decreases the radius when there exist multiple vectors within the sphere.

In the sequel, we sketch the idea of SD through an example [6]. Consider a square quadrature amplitude modulation (QAM) in a $2 \times 2$ complex MIMO-OFDM channel. The underlying complex system can be converted into an equivalent real system. Let $y_{jr}$ and $y_{jr}$ denote the real and imaginary parts of the received signal at the $j$ th receive antenna, that is, $y_{jr} = \text{Re}\{y_j\}$ and $y_{jr} = \text{Im}\{y_j\}$. Similarly, the input signal $x_i$ from the $i$ th antenna can be represented by $x_{ir} = \text{Re}\{x_i\}$ and $x_{ir} = \text{Im}\{x_i\}$. For the $2 \times 2$ MIMO-OFDM channel, the received signal can be expressed in terms of its real and imaginary parts as follows [6]:

$$
\begin{bmatrix}
    y_{1r} + jy_{1i} \\
    y_{2r} + jy_{2i}
\end{bmatrix} =
\begin{bmatrix}
    h_{11r} + jh_{11i} & h_{12r} + jh_{12i} \\
    h_{21r} + jh_{21i} & h_{22r} + jh_{22i}
\end{bmatrix}
\begin{bmatrix}
    x_{1r} + jx_{1i} \\
    x_{2r} + jx_{2i}
\end{bmatrix} +
\begin{bmatrix}
    z_{1r} + jz_{1i} \\
    z_{2r} + jz_{2i}
\end{bmatrix} \tag{3.7}
$$

Where $h_{jr} = \text{Re}\{h_j\}$, $h_{jr} = \text{Im}\{h_j\}$, $z_{jr} = \text{Re}\{z_j\}$, and $z_{jr} = \text{Im}\{z_j\}$. The real and imaginary parts of Equation (3.7) can be combined to yield the following expression [6]:
Comparison of different MIMO-OFDM signal detectors for LTE

For \( y, H, x, \) and \( z \) defined in Equation (3.8), SD method exploits the following relation [6]:

\[
\arg \min_x \|y - Hx\|^2 = \arg \min_x (x - \hat{x})^T H^T H (x - \hat{x})
\]  

(3.9)

Where \( \hat{x} = (H^T H)^{-1} H^T y \), consider the following sphere with radius of \( R_{SD} \) [6]:

\[
(x - \hat{x})^T H^T H (x - \hat{x}) \leq R_{SD}^2
\]

(3.10)

SD algorithm considers only the vectors inside the sphere defined by Equation (3.10). Figure 4 illustrates a sphere with the center of \( \hat{x} = (H^T H)^{-1} H^T y \) and radius of \( R_{SD} \) [6]. In this example, this sphere includes four candidate vectors, one of which is the ML solution vector. We note that no vector outside the sphere can be the ML solution vector because their ML metric values are bigger than the ones inside the sphere [15]. If we were fortunate to choose the closest one among the four candidate vectors, we can reduce the radius in Equation (3.10) so that we may have a sphere within which a single vector remains [16]. Note that the new metric in Equation (3.9) is also expressed as [6]

\[
(x - \hat{x})^T H^T H (x - \hat{x}) = (x - \hat{x})^T R^T R (x - \hat{x}) = \|R(x - \hat{x})\|^2
\]

(3.11)

Where \( R \) is obtained from QR decomposition of the real channel matrix \( H = QR \). In other words, the ML solution vector is now contained in this sphere with a reduced radius, as illustrated in Figure 5 [16].
Comparison of different MIMO-OFDM signal detectors for LTE

Figure 4 Illustration of the original sphere of SD

Figure 5 Illustration of the new sphere of SD for reduced radius

When \( N_T = N_R = 2 \), the metric in Equation (3.10) can be expressed as [6]:

\[
\sum_{i,j=1}^{4} |r_{ij}(x_i - \hat{x}_i)|^2 + |r_{22}(x_2 - \hat{x}_2)|^2 \leq R_{SD}^2
\]

(3.12)

Using Equation (3.12), SD method can be illustrated as the following four steps [6]:

**Step 1:** From Equation (3.12), we first consider a candidate value for \( x_4 \) in its own dimension, which is arbitrarily chosen from the points in the sphere \( |r_{22}(x_2 - \hat{x}_2)|^2 \leq R_{SD}^2 \). This point must be chosen in the following range:

\[
\hat{x}_2 - \frac{R_{SD}}{r_{22}} \leq x_2 \leq \hat{x}_2 + \frac{R_{SD}}{r_{22}}
\]

(3.13)

Let \( \hat{x}_4 \) denotes the point chosen in **Step 1**. If there exists no candidate point satisfying the inequalities, the radius needs to be increased. We assume that a candidate value was successfully chosen. Then we proceed to **Step 2**.
**Step 2**: Referring to Equation (3.12) again, a candidate value for \( x_3 \) is chosen from the points in the following sphere:

\[
|r_{44}(x_4 - \hat{x}_4)|^2 + |r_{33}(x_3 - \hat{x}_3) + r_{34}(x_4 - \hat{x}_4)|^2 \leq R_{SD}^2
\]  

(3.14)

Which can be expressed as equivalent as below:

\[
\hat{x}_3 = \frac{\sqrt{R_{SD}^2 - |r_{44}(\hat{x}_4 - x_4)|^2 - r_{34}(\hat{x}_4 - x_4)}}{r_{33}} \leq x_3 \leq \hat{x}_3 + \frac{\sqrt{R_{SD}^2 - |r_{44}(\hat{x}_4 - x_4)|^2 - r_{34}(\hat{x}_4 - x_4)}}{r_{33}}
\]  

(3.15)

Note that \( \hat{x}_4 \) in Equation (3.15) is the one already chosen in **Step 1**. If a candidate value for \( x_3 \) does not exist, we go back to **Step 1** and choose other candidate value \( \hat{x}_4 \). Then search for \( x_3 \) that meets the inequalities in Equation (3.15) for the given \( \hat{x}_4 \). In case that no candidate value \( x_3 \) exists with all possible values of \( \hat{x}_4 \), we increase the radius of sphere, \( R_{SD} \), and repeat the **Step 1**. Let \( \bar{x}_4 \) and \( \bar{x}_2 \) denote the final points chosen from **Step 1** and **Step 2**, respectively.

**Step 3**: Given \( \bar{x}_4 \) and \( \bar{x}_3 \), a candidate value for \( x_2 \) is chosen from the points in the following sphere:

\[
|r_{44}(\bar{x}_4 - \hat{x}_4)|^2 + |r_{33}(\bar{x}_3 - \hat{x}_3) + r_{34}(\bar{x}_4 - \hat{x}_4)|^2 + |r_{22}(x_2 - \hat{x}_2) + r_{23}(\bar{x}_3 - \hat{x}_3) + r_{24}(\bar{x}_4 - \hat{x}_4)|^2 \leq R_{SD}^2
\]  

(3.16)

Arbitrary value is chosen for \( x_2 \) inside the sphere of Equation (3.16). In choosing a point, the inequality in Equation (3.16) is used as in the previous steps. If no candidate value for \( x_2 \) exists, we go back to **Step 2** and choose another candidate value \( \hat{x}_3 \). In case that no candidate value for \( x_2 \) exists after trying all possible candidate values for \( \hat{x}_3 \), we go back to **Step 1** and choose another candidate value for \( \hat{x}_4 \). The final points chosen from **Step 1** through **Step 3** are denoted as \( \bar{x}_4 \), \( \bar{x}_3 \), and \( \bar{x}_2 \), respectively.

**Step 4**: Now, a candidate value for \( x_1 \) is chosen from the points in the following sphere:

\[
|r_{44}(\bar{x}_4 - \hat{x}_4)|^2 + |r_{33}(\bar{x}_3 - \hat{x}_3) + r_{34}(\bar{x}_4 - \hat{x}_4)|^2 + |r_{22}(\bar{x}_2 - \hat{x}_2) + r_{23}(\bar{x}_3 - \hat{x}_3) + r_{24}(\bar{x}_4 - \hat{x}_4)|^2 + |r_{11}(x_1 - \hat{x}_1) + r_{12}(\bar{x}_2 - \hat{x}_2) + r_{13}(\bar{x}_3 - \hat{x}_3) + r_{14}(\bar{x}_4 - \hat{x}_4)|^2 \leq R_{SD}^2
\]  

(3.17)
An arbitrary value satisfying Equation (3.17) is chosen for $x_i$. If no candidate value for $x_i$ exists, we go back to Step 3 to choose other candidate value for $\bar{x}_i$. In case that no candidate value for $x_i$ exists after trying with all possible candidate values for $\bar{x}_i$, we go back to Step 2 to choose another value for $x_i$.

Let $\bar{x}_i$ denote the candidate value for $x_i$. Once we find all candidate values, $\bar{x}_4$, $\bar{x}_3$, $\bar{x}_2$, and $\bar{x}_1$, then the corresponding radius is calculated by using Equation (3.17). Using the new reduced radius, Step 1 is repeated. If $[\bar{x}_1 \ \bar{x}_2 \ \bar{x}_3 \ \bar{x}_4]$ turns out to be a single point inside a sphere with that radius, it is declared as the ML solution vector and our searching procedure stops.

### Table 1 Complexity of sphere decoding in each step

<table>
<thead>
<tr>
<th>Step</th>
<th>Multiplications</th>
<th>Divisions</th>
<th>Square roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x} = (\mathbf{H})^{-1} \mathbf{y}$</td>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$R_{SD}^2$</td>
<td>14</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Step 1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Step 2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$R_{SD}^2$ update</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that in a $N_r = N_t = 2$ MIMO-OFDM system, the complexity of the ML signal detection corresponds to the ML metric calculation of $16^2 = 256$ times. Assuming that four real multiplications are required for each ML metric calculation, $256 \times 4 = 1024$ real multiplications are required in total. The main drawback of SD is that its complexity depends on SNR [15]. Furthermore, the worst-case complexity is the same as that of ML detection, although the average complexity is significantly reduced [15].

The sphere radius $R_{SD}$ will impact on the performance of SD detector as is indicated above. If the radius $R_{SD}$ is chosen too small, there would be no points inside the search area in the sphere and the algorithm would fail. Meanwhile, if the radius $R_{SD}$ is chosen too large, there would be too many points would be searched and the complexity would increase while the efficiency would decrease.

#### 3.2.4 QRM-MLD

An approach for MIMO detection that applies QRM-MLD was proposed in [13]. It reduces the computational complexity compare with the computational
Comparison of different MIMO-OFDM signal detectors for LTE

complexity of ML detection in MIMO OFDM systems. Assuming that the number of the transmit and receive antennas are equal, the ML metric in Equation (3.6) can be equivalently expressed as [6]

\[ \|y - Hx\| = \|y - QRx\| = \|QR^\dagger (y - Rx)\| = \|\tilde{y} - Rx\| \]  

(3.18)

We note that QR decomposition also has been applied to sphere decoding in Equation (3.11) where it is for an equivalent real system. In Equation (3.18), however, it is for a complex channel matrix. We will illustrate QRM-MLD method for a $2 \times 2$ MIMO OFDM system, Equation (3.18) can be expressed as

\[ \|\tilde{y} - Rx\|^2 = \left\| \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\|^2 = |\tilde{y}_2 - r_{22}x_2|^2 + |\tilde{y}_1 - r_{11}x_1 - r_{12}x_2|^2 \]  

(3.19)

QRM-MLD method could be described as the following two steps:

Step 1: Among $|C|$ candidates for $x_2 \in C$, select M smallest values of $f_1(x_2) = |\tilde{y}_2 - r_{22}x_2|^2$ [6].

Step 2: Among $M \times |C|$ candidates for $x_2 \in C$, select M smallest values of $f_1(x_1, \tilde{x}_2) = |\tilde{y}_2 - r_{22}\tilde{x}_2|^2 + |\tilde{y}_1 - r_{11}x_1 - r_{12}\tilde{x}_2|^2$ [6].

For M candidates found above, select the one could minimize the metric in Equation (3.19) as the detected symbol. The performance of BER of QRM-MLD depends on the parameter M. If M increases, its performance approaches the performance of ML detector while the complexity will also increases. For 16QAM modulation, if M is equal to the value of 16, its performance should be almost the same as ML detector.
CHAPTER 4

4.1 SYSTEM MODEL

This thesis studies the five MIMO-OFDM signal detectors theoretically. It is complex to get the results from theoretical analysis. In practice, it is impossible to analyze the different MIMO-OFDM detectors with the same propagation configuration. The noise is randomly. It can be basic modeled as an AWGN [3]. A Matlab simulations is widely used for analyzing the performance of signal detection. It help us to better understand the MIMO-OFDM system, multipath Rayleigh channel, spatial correlation and the signal detection algorithms. There are many mature propagation model in Matlab simulation. In a simulation, all five detectors could be simulated as transmitted in the same propagation configuration. The performance is measured in terms of BER versus SNR. In the simulation, a MIMO-OFDM system will be built as the Figure 6 shows as below:

Figure 6 A MIMO-OFDM system with 2 transmit and 2 receive antennas

In this simulation, Rayleigh fading channel is selected. The spatial correlation is also considered in the simulation. In case that the ideal Rayleigh fading channel without spatial correlation and with spatial correlation are considered in the simulation simultaneously, the results of the detectors with/without spatial
correlation are calculated of the same simulation environment except the spatial correlation.

4.2 DETECTOR MODEL

In this section, the detectors will be modeled as shown in section 3.2.1 to 3.2.4.

4.2.1 ZF DETECTOR

According to Equation (3.3), ZF detector is designed as the figure shown as below:

\[ \tilde{x}_{ZF} = W_{ZF} y \]

\[ W_{ZF} = (H^H H)^{-1} H^H \]

Figure 7 ZF detector illustration
ZF detector is a linear detector, in addition, it gives \( N_R - N_T + 1 \) diversity order in a \( N_T \times N_R \) MIMO system with \( N_R \) the maximum possible diversity order. From Equation (3.3), we note that the BER performance is related to the power of \( \tilde{z}_{ZF} \). When the noise power of \( z \) increases, the power of \( \tilde{z}_{ZF} \) increases.

In Matlab, we use an independent function denotes ZF detector which only needs two variables \( Y \) (TrSymbol_data or TrSymbol1_data in Matlab code) and \( H \) (Hfreq or H1freq in Matlab code), and \( \tilde{x}_{ZF} \) (Tr_zf or Tr_zf1 in Matlab code) is the output of the function which denotes the estimate of transmitted signal \( x \) (Tx_OFDM_Symbol in Matlab code). However, as illustrated in Figure 6, \( x \) denotes the signal that transmitted by the transmit antennas, and \( y \) (TrSymbol_temp1 or TrSymbol1_temp1 in Matlab code) denotes the signal received at the receive antennas. Actually, \( s \) (TxBits in Matlab code) denotes the transmitted data (the original signal only contains the data) and \( Y \) denotes the received data. \( \tilde{s} \) (TrBits or TrBits1 in Matlab code) denotes the estimate of
s. The program of ZF detector in the simulation of Matlab is shown as Program 4.1 in appendix.

### 4.2.2 MMSE DETECTOR

According to Equation (3.5), MMSE detector is designed as the figure shown as below:

\[
\begin{align*}
W_{MMSE} &= (H^H H + \sigma_z^2 I)^{-1} H^H \\
\hat{x}_{MMSE} &= W_{MMSE} y
\end{align*}
\]

**Figure 8 MMSE detector illustration**

Due to Equation (3.5) the weight matrix \( W_{MMSE} \) can be expressed as below:

\[
W_{MMSE} = (H^H H + \sigma_z^2 I)^{-1} H^H = (H^H H + \frac{1}{SNR} I_{N_r})^{-1} H^H
\]

(4.1)

MMSE is also a linear detector, it balances the noise enhancement and multistream interference by minimizing the total error. Its BER performance is superior to ZF detection due to mitigating the noise enhancement. Its computational complexity is dominated by the matrix inversion in Equation (4.1), which is cubic order \( O(n^3) \).

In Matlab, we use an independent function denotes MMSE detector which only needs four variables \( Y, H \), the noise power (noisepower in Matlab code) of the channel and number of receive antennas \( N_R \) (Nr in Matlab code), and \( \hat{x}_{MMSE} \) (Tr_mmse or Tr_mmse1 in Matlab code) is the output of the function which denotes the estimate of transmitted signal \( x \) (Tx_OFDM_Symbol in Matlab code). The program of MMSE detector in the simulation of Matlab is shown as Program 4.2 in appendix.
4.2.3 ML DETECTOR

According to Equation (3.6), ML detector is designed as the figure shown as below:

![ML Detector Flow Chart](image)

**Figure 9 ML detector flow chart**

ML detector will search through the entire vector constellation set $\Lambda_x$ for $\hat{x}_{ML}$ the value of the transmitted vector $x$ that maximizes the probability of the received vector $y$ given a certain channel propagation coefficients $H$ [12].

The number of possible candidates involved in the minimization problem is $N_x = \text{Card}(\Lambda_x)$. This shows that for high modulation schemes such as 16QAM and a number of transmit antennas $N_T = 2$, the number of possible candidates for $s$ is $N_x = 16^2 = 256$. Due to such high values, if the modulation schemes become much higher, i.e. 64QAM, and the number of transmit antennas are greater like $N_T = 4$, the implementation of ML detectors becomes unaffordable from a complexity point of view.

In Matlab, we use an independent function denotes ML detector which only needs three variables $Y$, $H$ and 16QAM constellation values, and $\hat{x}_{ML}$(outdata in Matlab code) is the output of the function which denotes the estimate of
transmitted signal $x$ (Tx_OFDM_Symbol in Matlab code). The program of ML detector in the simulation of Matlab is shown as Program 4.3 in appendix.

## 4.2.4 SD DETECTOR

SD detector should be designed according to the **Step 1** to **Step 4** and Equation (3.13) to Equation (3.17) in Chapter 3. However, as the simulation is to study the performance as BER of the detectors, the complexity is not considered in the simulation, there is a simplified Matlab code to implement SD algorithm by not update the radius automatically. The radius value is set in the code, and we change this value to investigate the BER performance of SD detector of different radius. The algorithm can be implemented as the flow chart shown as below:

![SD detector flow chart](image)

*Figure 10 SD detector flow chart*

SD is a type of Branch and Bound tree search algorithms [13]. A tree constructed by SD algorithm for $2 \times 2$ 4-QAM MIMO system is illustrated in Figure 11.
Comparison of different MIMO-OFDM signal detectors for LTE

Figure 11 Sphere Detector algorithms tree structure illustration

From the tree, the points are not fulfilling the constraints are pruned from the tree and the lattice points are within the sphere are \( x_1 = (0,1,1,0) \) and \( x_2 = (0,0,1,1) \).

The worst case computational complexity of SD remains exponential; however it’s expected is regarded as cubic over certain SNR and problem dimensions [17].

In Matlab, we use an independent function denotes SD detector which only needs four variables \( \bar{x}, R \) (R for upper triangle matrix of QR decomposition), the radius of the sphere (d or d_sd in Matlab code) and 16QAM constellation values, and \( \bar{x}_{SD} \) (outdata in Matlab code) is the output of the function which denotes the estimate of transmitted signal \( x \) (Tx_OFDM_Symbol in Matlab code). The program of SD detector in the simulation of Matlab is shown as Program 4.4 in appendix.

4.2.5 QRM-MLD

The basic idea of QR decomposition based M-branch search algorithm is that when doing the branch search for a complete search tree, not all branches are retained. Only fixed M smallest branches are selected in each layer in the search tree for reservation until the last layer of the search tree. According to the matrix theory, the channel matrix \( \mathbf{H} \) can be decomposed of a QR decomposition, that \( \mathbf{H} = \mathbf{QR} \). For a \( 2 \times 2 \) MIMO OFDM system, on the 1st antenna, which is the last row of the \( \mathbf{R} \) matrix, it only has one value and it will
not be affected by other transmitting antennas. For 16QAM, there are 16 possibilities substituting one by one like ML detector, but it only reserves M minimum distance symbols for the next antenna. The traversing range goes from $16 \times 16$ to $M \times 16$. If M is equal to 16 for 16QAM, the calculations are the same as ML detector and the results are almost the same as ML detector.

In Matlab, we use an independent function denotes SD detector which only needs four variables $\bar{x}$, $R$ (R for upper triangle matrix of QR decomposition), the M value of QRM-MLD (m_value in Matlab code) and 16QAM constellation values, and $\bar{x}_{QRM\_MLD}$ (outdata in Matlab code) is the output of the function which denotes the estimate of transmitted signal $x$ (Tx_OFDM_Symbol in Matlab code). The program of QRM-MLD in the simulation of Matlab is shown as Program 4.5 in appendix.

4.3 SIMULATION PARAMETERS

In the simulation for this thesis, we choose the following parameters as the simulation environment for LTE transmission.

<table>
<thead>
<tr>
<th>Table 2 Simulation Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transmission Bandwidth</strong></td>
</tr>
<tr>
<td>MIMO antenna arrays</td>
</tr>
<tr>
<td>Data length</td>
</tr>
<tr>
<td>Number of IFFT/FFT points</td>
</tr>
<tr>
<td>Cyclic prefix length</td>
</tr>
<tr>
<td>Data modulation/Spreading</td>
</tr>
<tr>
<td>Symbol timing detection</td>
</tr>
<tr>
<td>Multipath fading</td>
</tr>
<tr>
<td>Sample rate</td>
</tr>
<tr>
<td>Spatial correlation</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>
A frequency selective channel is selected in this simulation. Frequency selective channels that are widely used are e.g. the ITU Pedestrian B channel of “ITU Channel Model for Vehicular Test Environment” is used as shown in Table 3 [8]. For 6-taps Rayleigh channel, arrive time of each tap is [0 300 8900 12900 17100 20000] nanoseconds. The power of each tap is [-2.5 0 -12.8 -10.8 -25.2 -16] dB. Transfer the taps from time to the delay spread, so the delay spread should be [0 12 25 37 49 61]. To calculate the power normalized to 1 of each tap, the power should be [0.61 0.24 0.092 0.037 0.015 0.006].

<table>
<thead>
<tr>
<th>Tap</th>
<th>Channel A Relative Delay (ns)</th>
<th>Channel A Average Power (dB)</th>
<th>Channel B Relative Delay (ns)</th>
<th>Channel B Average Power (dB)</th>
<th>Doppler Spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0</td>
<td>0</td>
<td>-2.5</td>
<td>Classic</td>
</tr>
<tr>
<td>2</td>
<td>310</td>
<td>-1.0</td>
<td>300</td>
<td>0</td>
<td>Classic</td>
</tr>
<tr>
<td>3</td>
<td>710</td>
<td>-9.0</td>
<td>8900</td>
<td>-12.8</td>
<td>Classic</td>
</tr>
<tr>
<td>4</td>
<td>1090</td>
<td>-10.0</td>
<td>12900</td>
<td>-10.0</td>
<td>Classic</td>
</tr>
<tr>
<td>5</td>
<td>1730</td>
<td>-15.0</td>
<td>17100</td>
<td>-25.2</td>
<td>Classic</td>
</tr>
<tr>
<td>6</td>
<td>2510</td>
<td>-20.0</td>
<td>20000</td>
<td>-16.0</td>
<td>Classic</td>
</tr>
</tbody>
</table>

For spatial correlation, the environment is Macro and channel has a high correlation. In the simulation, the spatial correlation matrix $R_{NR}$ in Equation (2.4) can be expressed as Equation (4.2)

$$R_{NR} = \begin{bmatrix} 1 & -0.3043 \\ -0.3043 & 1 \end{bmatrix} \quad (4.2)$$

And the spatial correlation matrix $R_{NT}$ in Equation (2.4) can be expressed as Equation (4.3)

$$R_{NT} = \begin{bmatrix} 1 & 0.97 \\ 0.97 & 1 \end{bmatrix} \quad (4.3)$$

It means that some spatial directions are statistically stronger than others for large eigenvalue [6].

Then the transmitted signal is convolved with the Transmitter-Receiver antenna specific impulse response, and then summed together, and then the specific time domain received signal at the receive antenna is obtained.
In Matlab, we use an independent function denotes the multipath Rayleigh channel generator which uses three variables $\mathbf{x}$, FFT length (FFT_length in Matlab code) and channel parameters(Para in Matlab code) which denotes whether the channel is correlation or non-correlation. There are 2 outputs, one is the received signal at the receive antenna $\mathbf{y}$ (outdata in Matlab code) and the other is the channel response $\mathbf{H}$. The program of MMSE detector in the simulation of Matlab is shown as Program 4.6 in appendix.
CHAPTER 5

5.1 SIMULATION RESULTS

In the simulation, all the parameters for LTE are initialized. As shown in Table 2, a LTE down link with bandwidth 20MHz and sampling rate 30.72 MHz is selected. For MIMO system, we use an antenna array with 2 transmit antennas and 2 receive antennas. In LTE, there are 2048 subcarriers used for OFDM with 144 Cyclic Prefix length. The channel model in the simulation is the ITU Channel Model for Vehicular Test Environment of B channel that is a 6-taps multipath Rayleigh fading channel. For spatial correlation, we assume that the receiver correlation matrix is according to METRA while the transmitter correlation matrix is according to METRA Macro cell with 5° PAS. In the simulation, we assume the synchronization is ideal and the channel is ideal, too. When initialized, the simulation is following the following steps:

1. Data Generation. In the simulation, the transmit data for each transmit antenna is generated randomly as a binary bit stream.

2. 16QAM Modulation. The transmit data is modulated with 16QAM.

3. IFFT for OFDM. The modulated data is converted to the data streams for OFDM subcarriers and normalized to 1.

4. Cyclic Prefix. A CP length of 144 is added and the signal length becomes to 2192. The dimension of the transmit signal is 2.

5. Channel Generation. A 6-taps multipath Rayleigh channel is generated. For simulation, we assume there are two channels, one is without spatial correlation and one is with spatial correlation. According to Equation (2.4), these two channels have the same parameters of the 6-taps multipath Rayleigh channel that is generated, but they have different spatial correlation coefficients.

6. Get the Received Signal. Let the transmit signal pass the channel and get the received signal.
7. Add AWGN Noise. The noise always exists, and in the simulation, we assume the noise model is AWGN.

8. Receiver Conversion. In the simulation, we use “Receiver.m” to get the received data symbols. We remove Cyclic Prefix at this stage and also do the FFT transformation that transfer the data from OFDM subcarriers to the data symbols.

9. Signal Detection. We use five different detectors to detect the received data symbols. And we store five different detected data symbols. In the simulation, there are some variables needs to compare. These variables would influence the results of BER. The M values for QRM-MLD and the radius for SD are the variables of the detectors. If the radius is big enough so the result of SD is approximate the ML. And if the M value is equal to 16 which means the size of the 16QAM constellation, the result and complexity of QRM-MLD is the same as the ML.

10. Demodulation. Since the signal is modulated of 16QAM, at this stage, the detected data symbols will do the demodulation of 16QAM.

11. BER Calculation. To compare the demodulated detected data symbols with the origin transmit data, we calculate the BER as the evaluation of the performance of the detector.

For the basic definitions defined in Chapter 1, section 3, we did the following simulations to find out the BER performance of five different MIMO-OFDM signal detectors. ZF detector and MMSE detector are linear detectors. ML detector has the optimal detection performance theoretically. QRM-MLD and SD detector are non-linear detectors, both of them reduce the complexity and scarify the performance as well.

Firstly, we compared the BER performance of five different MIMO-OFDM signal detectors of the same environmental configuration with an uncorrelated 2×2 MIMO-OFDM channel and with a correlated 2×2 MIMO-OFDM channel. In this case, we assume that the QRM-MLD and SD detector has low complexity with the configuration of \( M = 4 \) for QRM-MLD and the radius of SD is half of the noise power. The results are shown in Figure 12.
Secondly, in order to investigate the impact on the BER performance of the complexity change of QRM-MLD and SD detector, we assume that the QRM-MLD and SD detector has higher complexity with the configuration of $M = 8$ for QRM-MLD and the radius of SD is twice of the noise power. The BER performance of QRM-MLD and SD detector are compared with MMSE detector as a linear signal detector and with ML detector as the optimal signal detector for BER performance while the MIMO-OFDM channel is spatial correlated or the MIMO-OFDM channel is uncorrelated. The results are shown in Figure 13.

Then, to find out the result in defined in Chapter 1, section 3 as definition 2, we compared the BER performance of QRM-MLD with different values of $M$ parameter ($M = 4, M = 8$ and $M = 16$ respectively) with an uncorrelated $2 \times 2$ MIMO-OFDM channel and with a correlated $2 \times 2$ MIMO-OFDM channel of the same wireless propagation configuration. In this case, different $M$ value is denoted as the different complexity of QRM-MLD algorithm. The BER performances are compared with MMSE detector as a linear signal detector and with ML detector as the optimal signal detector. The results are shown in Figure 14.

Lastly, to find out the result in defined in Chapter 1, section 3 as definition 3, we compared the BER performance of SD detector with different values of the radius $R_{SD}$ of the sphere (half of the noise power denotes a small radius, twice of the noise power as a large radius and four times of the noise power plus one as a large enough radius respectively) with an uncorrelated $2 \times 2$ MIMO-OFDM channel and with a correlated $2 \times 2$ MIMO-OFDM channel of the same wireless propagation configuration. In this case, different radius $R_{SD}$ is denoted as the different complexity of SD algorithm. The BER performances are compared with MMSE detector as a linear signal detector and with ML detector as the optimal signal detector. The results are shown in Figure 15.

In all figures, the horizontal axis denotes the SNR intervals and the vertical axis denotes the BER distribution. The lines in red color are the BER performance of the detectors with spatial correlation and lines in black color are the BER performance of the detectors without spatial correlation.
Comparison of different MIMO-OFDM signal detectors for LTE

Figure 12 Simulation results for five MIMO-OFDM signal detectors \((M = 4\) for QRM-MLD, radius is half of the noise power for SD) with an uncorrelated \(2 \times 2\) MIMO-OFDM channel for LTE

Figure 13 Simulation results for four MIMO-OFDM signal detectors \((M = 8\) for QRM-MLD, radius is twice of the noise power for SD) with an uncorrelated \(2 \times 2\) MIMO-OFDM channel and with a correlated \(2 \times 2\) MIMO-OFDM channel for LTE
Comparison of different MIMO-OFDM signal detectors for LTE

Figure 14 Simulation results for QRM-MLD with different $M$ values ($M = 4$, $M = 8$, $M = 16$, respectively) with an uncorrelated $2 \times 2$ MIMO-OFDM channel and with a correlated $2 \times 2$ MIMO-OFDM channel for LTE compared to MMSE and ML.

Figure 15 Simulation results for SD detector with different radius values (half of the noise power, twice of the noise power and four times of the noise power plus one, respectively) with an uncorrelated $2 \times 2$ MIMO-OFDM channel and
Comparison of different MIMO-OFDM signal detectors for LTE

*with a correlated $2 \times 2$ MIMO-OFDM channel for LTE compared to MMSE and ML*

## 5.2 RESULT ANALYSIS

Initially, an uncorrelated scenario has been simulated to evaluate the BER performance of five different MIMO-OFDM detectors while a correlated scenario has been simulated simultaneously for LTE. The modulation is 16QAM. Figure 12 shows that in an uncorrelated scenario, ZF detector has the worst BER performance in these five detectors while the ML detector has the optimal BER performance. As linear MIMO-OFDM signal detectors for LTE, MMSE detector has a better BER performance than ZF detector. The theoretical analysis in Chapter 3 for linear signal detection algorithms has been validated. As the way to reduce the complexity of ML detector, QRM-MLD and SD detector with low complexity ($M = 4$ for QRM-MLD and the radius $R_{SD}$ is the half of the noise power respectively) have better BER performance than the basic linear detectors. However, to reduce the complexity means to scarify the BER performance, compared to ML, QRM-MLD or SD detector have a greater BER which means worse BER performance. Figure 12 also shows in a correlated scenario, the comparison of the BER performance of five different MIMO-OFDM detectors is relatively the same as in an uncorrelated scenario. Thus the BER of the detector degrades dramatically.

Then, with the same scenario of a correlated MIMO-OFDM channel and an uncorrelated MIMO-OFDM channel, we increase the complexity of QRM-MLD and SD detector by increasing the value of parameter $M$ of QRM-MLD from 4 to 8 and increasing the radius $R_{SD}$ of SD detector from half of the noise power to twice of the noise power. Compared the four MIMO-OFDM detectors (ZF detector is regardless), Figure 13 shows that QRM-MLD has a better BER performance than the basic linear signal detector as well as SD detector. ML detector always has the optimal BER performance in the same wireless propagation scenario. Figure 13 also shows in a correlated scenario, the comparison of the BER performance of four different MIMO-OFDM detectors is relatively the same as in an uncorrelated scenario. Thus the BER of the detector degrades dramatically.
To evaluate the impact on the different values of parameter $M$ of QRM-MLD in an uncorrelated scenario or in a correlated scenario for LTE, QRM-MLD for different $M$ values (we assumed the value of parameter $M$ is $M = 4$, $M = 8$ and $M = 16$ respectively) has been simulated simultaneously with ML detector as the optimal signal detector and MMSE detector as the basic linear signal detector. Figure 14 shows that the greater the value of parameter $M$ of QRM-MLD, the better the BER performance in the same spatial correlation scenario. When QRM-MLD has a low complexity (e.g. $M = 4$), it has a lower BER than MMSE detector while it has the greatest complexity ($M = 16$ for the 16QAM constellation size), it has the same BER performance as the ML detector.

To evaluate the impact on the different radius $R_{SD}$ of SD detector, an uncorrelated scenario has been simulated to compare the BER performance of different radius $R_{SD}$ of SD detector for the radius $R_{SD}$ is half of the noise power, the radius $R_{SD}$ is twice of the noise power and the radius $R_{SD}$ is four times of the noise power plus one while a correlated scenario has been simulated simultaneously for LTE. Figure 15 shows that the greater the radius $R_{SD}$, the better the BER performance in the same spatial correlation scenario. When SD detector has a low complexity (e.g. $R_{SD}$ is half of the noise power), it has a lower BER than MMSE detector while it has the greatest complexity ($R_{SD}$ is equal to four times of the noise power plus one for a large enough value), it has the same BER performance as the ML detector.

From all the results, it is obviously to see that for each detector, the BER decreases when SNR increases. It means that the greater the SNR, the more accuracy the data transmission.

To compare the impact on BER performance of the signal detector in a correlated MIMO-OFDM scenario to that in an uncorrelated MIMO-OFDM scenario, from Figure 12 to Figure 15, it is illustrated that the detector in an uncorrelated MIMO-OFDM scenario has better BER performance than that in a correlated MIMO-OFDM scenario. It means the spatial correlation has an impact on the BER performance of MIMO-OFDM signal detectors that the BER grows dramatically in a correlated MIMO-OFDM scenario. In the simulation results, it is obviously to see that the spatial correlation has a great impact on
BER performance that the BER of the detector increases if the MIMO-OFDM channel is spatial correlated compared to the detector in an uncorrelated MIMO-OFDM scenario.

As linear signal detectors, MMSE detector or ZF detector always has a greater BER than non-linear signal detectors in an uncorrelated scenario or in a correlated scenario in the simulation. However, in linear signal detection, MMSE detector has a better BER performance than ZF detector in an uncorrelated scenario or in a correlated scenario.

ML detector always has the optimal BER performance of the five detectors in an uncorrelated scenario or in a correlated scenario. While the value of parameter M of QRM-MLD is equal to the maximum size of the constellation (16QAM in this simulation) which means all nodes will be searched as ML dose, it has the same performance as ML detector. To reduce the M value is to reduce the complexity of the detector and scarify the performance at the same time. If the value of the radius of SD detector in large enough which means all points are in the sphere and will be searched, it has approximately the same performance as ML detector. If the radius of the sphere is reduced in SD detector, the candidates may be reduced. If the candidates reduce, then the complexity of SD detector reduces. Meanwhile, the BER performance will degrade as well.

CONCLUSIONS

For growing voice and data services, it raises a higher demand of the transmission rate, transmission performance and data throughput. LTE as a standard for radio communications of high-speed data transmission which is based on the GSM/EDGE and UMTS/HSPA network technologies is a solution and widely used worldwide. OFDM and MIMO are key technologies of LTE. MIMO technology can significantly increase channel capacity and spectrum efficiency. Meanwhile, OFDM is an efficiency multi-carrier transmission
technology as it can provide much higher spectrum utilization than the average frequency division multiplexing system. As one of key technologies of LTE, signal detection algorithms are studied in this thesis. Five MIMO-OFDM signal detectors are studied: ZF detector, MMSE detector, ML detector, QRM-MLD and SD detector. MIMO system not only brings a huge capacity, but also produces great complexity of the received signal detection. The current study focused on the signal detection algorithm is in two areas: to enhance the performance of the linear detector and to reduce the complexity of ML algorithm. QRM-MLD algorithm and SD algorithm are the algorithms reduce the complexity of ML algorithm. QRM-MLD and SD detector are also studied in this thesis. This thesis also studied the BER performance of five different MIMO-OFDM signal detectors in a $2 \times 2$ MIMO-OFDM channel of different spatial correlation configurations.

For the basic definition in Chapter 1, section 3, we assume all the parameters of a MIMO-OFDM system are the same and the channel is an uncorrelated $2 \times 2$ MIMO-OFDM channel, the BER performance of five detectors are studied. We also studied how does the value of parameter $M$ of QRM-MLD will affect the BER performance of the detector as well as how does the value of radius $R_{SD}$ of SD detector will impact on the BER performance of the detector. Then we assume the channel of the MIMO OFDM system is changed to be a correlated $2 \times 2$ MIMO-OFDM channel. The BER performances of the five detectors with a correlated $2 \times 2$ MIMO-OFDM channel are studied. In this case, how does the spatial correlation will impact on the BER performance of the detectors is studied with the comparison of the BER performance of a detector if the MIMO-OFDM channel is spatial correlated and if the MIMO-OFDM channel is uncorrelated. Besides, we compared the BER performance of five different signal detectors whether the MIMO-OFDM channel is spatial correlated or the MIMO-OFDM channel is uncorrelated. In this thesis, only the BER performance of the detector is considered, the complexity is not considered in this thesis.

From the theory of the signal detection algorithms of the five detectors studied in this thesis, ZF detection and MMSE detection have advantages of simple and low complexity but the performance is not ideal. ML detection has the optimal
Comparison of different MIMO-OFDM signal detectors for LTE

detection performance. Sphere decoding detection reduces the complexity, and it reduces the BER performance as choose a certain sphere radius $R_{SD}$. For a $2 \times 2$ MIMO OFDM system with 16QAM modulation, QRM-MLD detection reduces the complexity from $16 \times 16$ to $M \times M$ according to the value of parameter $M$, as well as the performance is reduced. However, when the radius of SD detector is large enough or the value of parameter $M$ is equal to 16 of QRM-MLD, the detectors will performance almost the same as ML.

In the simulation, we assumed the radio communications environment is a DL of 3GPP LTE of a bandwidth 20 MHz and the subcarriers are 2048, the channel is modeled as an “ITU Channel Model for Vehicular Test Environment B Channel”, MIMO antenna array is $2 \times 2$, however the modulation is 16QAM. Meanwhile, to compare the BER performance of the five signal detectors if the MIMO-OFDM channel is uncorrelated with that if the MIMO-OFDM channel is spatial correlated, we assumed the spatial correlation for receiver is “METRA” and for transmitter is “Macro”. The detectors are ZF detector, MMSE detector, ML detector, SD detector and QRM-MLD. For SD detector, we assumed the radius is a large enough value (four times of the noise power plus one), a large value (two time of the noise power) and a small value (half of the noise power or twice of the noise power) respectively. For QRM-MLD, we assumed the value of parameter $M$ is $M = 4, M = 8$ and $M = 16$ respectively. The results of the performance of the detectors are BER. The SNR range is from 0 dB to 20 dB as the interval is 2 dB.

From the results, we can see that for a single signal detector, the BER decreases while the SNR increases. And for a single signal detector, the BER performance is worse while the channel is a correlated MIMO-OFDM channel. For SD detector, the BER performance is better of the radius value is 4 time of the noise power plus one than that of the radius value is half of the noise power. That means the larger the radius value, the better the BER performance. For QRM-MLD, the BER performance is better of the $M$ is equal to 8 than that of the $M$ is equal to 4 and the BER performance is better of the $M$ is equal to 16 than that of the $M$ is equal to 8. That means the larger the $M$ value, the better the BER performance. For the comparison of five signal detectors if the MIMO-OFDM channel is uncorrelated, the performance of ZF detector is the worst in
the five signal detectors and the ML is always the best. As linear signal detection, ZF or MMSE signal detector has higher BER than SD detector and QRM-MLD which means the worse performance. For the comparison of five signal detectors if the MIMO-OFDM channel is spatial correlated, it is almost the same situation. ML signal detector always has the best BER performance, at the same SNR, it has the lowest BER. ZF signal detector has the worst BER performance. In linear signal detection algorithms, MMSE signal detector has a better BER performance than ZF signal detector.

Regardless of the value of parameter M, QRM-MLD has a better BER performance than the linear signal detectors, e.g. ZF signal detector or MMSE signal detector. If the radius value of SD signal detector is a valid value, regardless of the radius, SD has a better BER performance than the linear signal detectors. If the M value is equal to 16 as the size of the 16QAM constellation, then QRM-MLD has the same performance of ML signal detector. If the radius value of SD is large enough, then SD signal detector has approximately the same BER performance of ML signal detector. It is hard to compare the performance of SD signal detector with the performance of QRM-MLD, the result is depending on the value of parameter M in QRM-MLD and the radius of SD signal detector.

REFERENCES

Comparison of different MIMO-OFDM signal detectors for LTE


APPENDIX-A

%-----------------------------------------
% Input:
%    H: channel
%    Y: received value
% Output:
%    Tr_zf:X
%-----------------------------------------
function Tr_zf = ZFDetection(H,Y)
    W_ZF = ((H'*H)^(-1))*H';
    Tr_zf = W_ZF*Y;
%X = WZF*Y

%-----------------------------------------
% Input:
%    H: channel
%    Y: received value
%    Nr: number of receiver antennas
%    noisePower: noise power
% Output:
%    Tr_mmse:X
%-----------------------------------------
function Tr_mmse = MMSEDetection(H,Y,noisePower,Nr)
    W_mmse = ((H'*H+eye(Nr)*noisePower)^(-1))*H';
    Tr_mmse = W_mmse*Y;

Program 4.1 “ZFDetection”

Program 4.2 “MMSEDetection”
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Program 4.3 “MLDetection”

```matlab
% Input: for Y=HX+N
%       h: H
% Tr_data: Y
% Qsymbol: all possible X values
%Output:
%       outdata: the estimated X value

function outdata = MLDetection(h,Tr_data,Qsymbol)

s1 = zeros(2,256);  %256=16^2, each antenna has 16 possibilities of 16QAM
r1 = zeros(1,256);
i = 1;
for i1 = 1:16  % each antenna has 16 possibilities of 16QAM
    for i2 = 1:16
        s1(:,i) = [Qsymbol(i1);Qsymbol(i2)];  % Traverse the possible combinations of transmitted symbols, 1 of 256 possibilities
        i = i + 1;
    end
end
for i1 = 1:256
    r1(i1) = (h*s1(:,i1)-Tr_data)'*(h*s1(:,i1)-Tr_data);  % ||Y-HX||^2
end
b_idx = find(r1==min(r1));  % find the minimum value, min(r1) is the minimum value
outdata = s1(:,b_idx);  % s1 saved 256 possibilities
```
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%-----------------------------------------------------------------
% Input: for Y=HX+N
%   rx: Q'*rx
%   h: R for QR Decomposition
%   d: search range, the radius of the search
% Qsymbol: all possible X values
%Output:
%   outdata: the estimated X value
%-----------------------------------------------------------------
function outdata = SDDetection(rx,h,d,Qsymbol)

s1 = zeros(1,16);
for i1 = 1:16
    s1(i1) = abs(rx(2)-h(2,2)*Qsymbol(i1)).^2;
end

%get the candidate symbol of the radius
idx1 = find( s1 <= d);
%last layer's reserved send value as candidate
if isempty(idx1)  %if there is no candidate in the radius
    idx1 = 1:16;
end

y1 = Qsymbol(idx1);  % This is the candidate set of all the
i = 1;  % reserved symbols on the final antenna
stemp = zeros(2,16*length(y1));
for i1 = 1:length(y1)
    for i2 = 1:16
        stemp(:,i) = [Qsymbol(i2);y1(i1)];
        i = i + 1;
    end
end

r1 = zeros(1,16*length(y1));
%get the norm collection
for i1 = 1:16*length(y1)
    r1(i1) = (h*stemp(:,i1)-rx)'*(h*stemp(:,i1)-rx);
end
%get the minimum output
b_idx = find(r1==min(r1));
outdata = stemp(:,b_idx);

Program 4.4 “SDDetection”
Comparison of different MIMO-OFDM signal detectors for LTE

Program 4.5 “mMLDetection”

```matlab
%------------------------------------------------------------
% Input: for Y=HX+N
%      rx:
%          h: H
%      m_value: M value, no more than 16 for 16QAM
%      Qsymbol: all possible X values
% Output:
%      outdata: the estimated X value
%------------------------------------------------------------
function outdata = mMLDetection(rx,h,Qsymbol,m_value)

if m_value > 16
    fprintf('m_value can not greater than max symbol number 16!');
    return;
end
s1 = zeros(1,16);
% All possible Euclidean distance
for i1 = 1:16
    s1(i1) = abs(rx(2) - h(2,2)*Qsymbol(i1)).^2;
end
%on the 1st antenna—which is the last row of the R
matrix—if only has one value—it will not be affected by other
transmitting antennas
%in 16 possibilities of 16QAM—substituting one by one like
ML—but only reserve m minimum distance symbols, for the next
antenna—traversing range went from 16X16 to mX16

%select the m symbol value from s1
b1 = sort(s1); % sort s1 from min to max
b2 = b1(1:m_value); % get m minimum value
y = zeros(1,m_value); % to save m possible transmitted signal

for i1 = 1:m_value
    mm = find(s1==b2(i1));
    y(i1) = Qsymbol(mm(1)); % a greater value
    s1(mm(1)) = 100;
end
% Get the possible solution set of the penultimate layer
stemp = zeros(2,16*m_value); %the possible set—m for
antenna 2E=16 for antenna 1 of 16QAM
for i1 = 1:m_value
    for i2 = 1:16
        stemp(:,(i1-1)*16+i2) = [Qsymbol(i2);y(i1)]; % Possible transmission symbols
    end
end
% Get the norm collection
r1 = zeros(1,16*m_value);
for i1 = 1:16*m_value
    r1(i1) = (h*stemp(:,i1)-rx)'*(h*stemp(:,i1)-rx);
end
% get the minimum output
b_idx = find(r1==min(r1));
outdata = stemp(:,b_idx);
```
Comparison of different MIMO-OFDM signal detectors for LTE

Program 4.6 “multipath_rayleigh”