Superconducting sensors have gained wide acceptance in various fields where high sensitivity is required, such as security, space technology, medical imaging and metrology. This thesis summarizes the development and use of superconducting sensors designated for two specific applications, falling into the two last-mentioned categories. The first application, a combination of ultra-low-field magnetic resonance imaging and magnetoencephalography, allows structural and functional imaging of the human brain with a single device. The second application is null detection related to a quantum metrology triangle experiment that aims at verifying the consistency of the three quantum standards of current, voltage and resistance. The operation and physics of the sensors relevant to the applications are studied, and the applications utilizing the sensors are constructed and tested.

Juho Luomahaara

Advances in superconducting sensors for medical imaging and metrology

Department of Micro- and Nanosciences
Advances in superconducting sensors for medical imaging and metrology

Juho Luomahaara

A doctoral dissertation completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Electrical Engineering, at a public examination held at the lecture hall TU1 of the school on 12 December 2014 at 13.

Aalto University
School of Electrical Engineering
Department of Micro- and Nanosciences
Abstract

In brain imaging, two complementary but technologically contradicting techniques are magnetoencephalography (MEG) and magnetic resonance imaging (MRI). MEG examines the function of the brain by measuring very weak magnetic fields, produced as a result of neuronal activity, with sensors based on superconducting quantum interference device (SQUID). MRI employs large magnetic fields and enables imaging of the structure of matter. The recent advances in ultra-low-field (ULF) MRI have made a medical instrument incorporating MEG and ULF MRI functionalities an attractive topic of research. The contradictions become evident when comparatively high fields of ULF MRI are subjected to SQUID magnetic field sensors, degrading their performance. In this thesis, the field tolerance of the sensors was improved. Special attention was paid to sensor response recovery and operation after a magnetic pulse. A hybrid MEG-ULF MRI instrument was constructed with the aid of new sensors. The instrument operation was verified, and results indicate that including ULF MRI in a MEG device is a viable concept.

In addition, a new type of magnetometer was developed, taking advantage of the nonlinear kinetic inductance of superconducting material. The experimental data, together with the theory, demonstrate a device with low noise and intrinsically high dynamic range. Furthermore, the kinetic inductance magnetometer is suitable for biomagnetic multichannel measurements, as only one amplifier is needed in the readout of multiple sensors. The simple design reduces costs in fabrication and enables higher tolerance of magnetic fields than achievable with SQUID sensors.

A new superconducting transformer design is introduced as a final step. Connecting to a SQUID results in a highly sensitive current detector. The device is a candidate for closing the quantum metrology triangle (QMT) experiment, a long-standing goal in metrology. The aim of the QMT is to improve confidence in the planned revision of the SI unit system by comparing the quantum standards of current, voltage and resistance. The device was characterized for the purpose by using it as a null current detector in the simulation of a QMT experiment. Results disclose the potential of the device and provide insight into some of the practical challenges relevant to null detection.

Keywords superconducting quantum interference device, SQUID, magnetoencephalography, MEG, ultra-low-field magnetic resonance imaging, ULF MRI, kinetic inductance, quantum metrology triangle, QMT, null detector, flux transformer

Avainsanat: suprajohtava kvantti-interferenssilaitte, SQUID, magnetoenkefalografia, MEG, ultramatalan kentän magneettikuvaus, ULF MRI, kineettinen induktanssi, kvanttimetrologiakolmio, QMT, nollailmaisin, vuomuuntaja

ISBN (painettu) 978-952-60-5955-6
ISBN (pdf) 978-952-60-5956-3
ISSN-L 1799-4934
ISSN (painettu) 1799-4934
ISSN (pdf) 1799-4942
Julkaisupaikka Helsinki
Painopaikka Helsinki
Vuosi 2014
Sivumäärä 121
Preface

The work leading to this thesis was carried out at VTT Technical Research Centre of Finland during the years 2010–2014. Since funding doctoral studies at VTT is not always self-evident, I have been fortunate to be participating in projects, which have supported and served as the basis for my studies. The financial support from the European Community’s Seventh Framework Programme (FP7/2007–2013) under Grant Agreement no. 200859, Technology Industries of Finland Centennial Foundation and the Academy of Finland is gratefully acknowledged.

On a personal level, I would first like to thank my instructor, Dr. Juha Hassel. After completing Master’s Thesis under his guidance, it was an easy choice for me to continue working within the field. Juha’s ideas, drive and constant support have been vital for this study. Besides Juha, other people of the former Quantronics group have formed an essential part of my working environment. I thank Dr. Panu Helistö for all the help with both bureaucracy and science. I am grateful to my ‘roommates’, Visa Vesterinen and Nikolai Beev, for the scientific and technical interaction we’ve had together. Knowing someone like Mikko Kiviranta with an extensive know-how has been a big plus. Leif Grönberg, Dr. Andrey Timofeev, Hannu Sipola, Dr. Kirsi Tappura, Dr. Mikko Leivo, Prof. Arttu Luukanen, Prof. Heikki Seppä and Dr. Timo Varpula, all deserve to be mentioned. Yet, one of the biggest credits belongs to Paula Holmlund, whose assistance in many practical issues has made my life much easier.

Outside VTT, the collaboration with Aivon Oy, in other words Dr. Jari Penttilä and Dr. Leif Roschier, has been very fruitful. Aivon’s role in designing and testing SQUIDs and related electronics has been important. During the MEGMRI project, I learned to know a group of new people, with whom I’ve had a great pleasure to work. Dr. Panu Vesanen, Dr. Jaakko Nieminen, Dr. Juhani Dabek, Andrey Zhdanov, Koos Zevenhoven,
Sarianna Alanko, Prof. Lauri Parkkonen, Dr. Juha Simola, Dr. Antti Ahonen, Prof. Risto Ilmoniemi and Dr. Jyrki Mäkelä, I thank you all for the time spent together. I am grateful to the individuals at MIKES for the collaboration in the Quantum Metrology Triangle project: Dr. Antti Kemppinen, Emma Mykkänen, Dr. Jaani Nissilä, Dr. Ossi Hahtela and Dr. Antti Manninen. I also want to thank my supervisor, Prof. Ilkka Tittonen from Aalto University for making things run smoothly during my studies.

Finally, I want to express my gratitude to the ones who are closest to me, my friends and family. Most of all, the silent support of my parents over the years deserves a huge recognition. And of course, thank you Jutta for sharing your life, but not all of it, with me as we walk side by side.

Espoo, 29 October, 2014,

Juho Luomahaara
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#### 4.3 Null detector

### 5. Conclusion

### Bibliography

### Publications
This thesis is based on the following original publications which are referred to in the text as I-V. The publications are reproduced with kind permission from the publishers.


IV J. Luomahaara, M. Kiviranta and J. Hassel, A large winding-ratio planar transformer with an optimized geometry for SQUID ammeter, Supercond. Sci. Technol. 25, 035006 (2012).

Author’s Contributions

I The author took part in the SQUID design and derived the theory for the flux penetration. The author had a major role in the majority of the measurements and data analysis. The author is the principal writer of the article.

II The author’s main contributions include the sensors and assistance during the development of other instrumentation. The author wrote the sensor section of the article and contributed in editing the rest of the text.

III The author carried out the measurements as well as the data analysis. He is the principal writer of the manuscript.

IV The author performed the measurements and analyzed the data. He is the principal writer of the article.

V The author designed the magnetic shield and performed the measurements. He also interpreted the results and is the main writer of the article.
# List of Abbreviations

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<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>ac</td>
<td>Alternating current</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-digital converter</td>
</tr>
<tr>
<td>APF</td>
<td>Additional positive feedback</td>
</tr>
<tr>
<td>BLB</td>
<td>Bean-Livingstone barrier</td>
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<tr>
<td>BOT</td>
<td>Bloch oscillating transistor</td>
</tr>
<tr>
<td>CCC</td>
<td>Cryogenic current comparator</td>
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<tr>
<td>dc</td>
<td>Direct current</td>
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<tr>
<td>EEG</td>
<td>Electroencephalography</td>
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<td>FLL</td>
<td>Flux-locked loop</td>
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<tr>
<td>GB</td>
<td>Geometrical barrier</td>
</tr>
<tr>
<td>HEMT</td>
<td>High-electron-mobility transistor</td>
</tr>
<tr>
<td>JVS</td>
<td>Josephson voltage standard</td>
</tr>
<tr>
<td>MEG</td>
<td>Magnetoencephalography</td>
</tr>
<tr>
<td>MSR</td>
<td>Magnetically shielded room</td>
</tr>
<tr>
<td>NC</td>
<td>Noise cancellation</td>
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<tr>
<td>NMR</td>
<td>Nuclear magnetic resonance</td>
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<td>PECVD</td>
<td>Plasma enhanced chemical vapour deposition</td>
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<td>QHR</td>
<td>Quantum Hall resistance</td>
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<td>QMT</td>
<td>Quantum metrology triangle</td>
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<tr>
<td>rf</td>
<td>Radio frequency</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>RIE</td>
<td>Reactive ion etching</td>
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<tr>
<td>RCSJ</td>
<td>Resistively and capacitively shunted junction</td>
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<tr>
<td>SIN</td>
<td>Superconductor - insulator - normal metal</td>
</tr>
<tr>
<td>SINIS</td>
<td>Superconductor - insulator - normal metal - insulator - superconductor</td>
</tr>
<tr>
<td>SIS</td>
<td>Superconductor - insulator - superconductor</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
<tr>
<td>SQUID</td>
<td>Superconducting quantum interference device</td>
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<tr>
<td>ULF MRI</td>
<td>Ultra-low-field magnetic resonance imaging</td>
</tr>
</tbody>
</table>
List of Symbols

\( \beta_c \)  Stewart-McCumber parameter
\( \beta_L \)  screening parameter
\( \gamma \)  gyromagnetic ratio
\( \Delta I \)  current in bias reversal measurement
\( \delta I \)  current uncertainty
\( \Delta \)  energy gap of superconductor
\( \delta \)  phase difference
\( \Delta_0 \)  energy gap of superconductor at zero temperature
\( \Delta_{\text{QMT}} \)  result of QMT experiment
\( \delta_{\text{QMT}} \)  uncertainty of QMT experiment
\( \epsilon \)  energy resolution per unit bandwidth
\( \theta \)  phase of Cooper pairs
\( \kappa \)  Ginzburg-Landau parameter
\( \Lambda \)  modified magnetic penetration depth
\( \lambda \)  magnetic penetration depth
\( \lambda_L \)  London penetration depth
\( \mu_0 \)  vacuum permeability
\( \xi \)  coherence length
\( \rho \)  resistivity
\( \sigma \)  complex conductivity
List of Symbols

$\sigma_1$ real conductivity

$\sigma_2$ imaginary conductivity

$\tau$ scattering time

$\tau_R$ lifetime of resonance

$\tau_r$ bias reversal time

$\nu_0$ Larmor frequency

$\Phi_p$ magnetic flux in pickup loop

$\Phi_T$ total magnetic flux

$\Phi_0$ flux quantum

$\Phi_a$ applied magnetic flux

$\Phi_{sq}$ magnetic flux in SQUID loop

$\Psi$ wave function of Cooper pairs

$\omega$ angular frequency

$\omega_J$ Josephson angular frequency

$\omega_p$ plasma angular frequency

$\omega_R$ resonance angular frequency

$A$ area

$a$ inner diameter of SQUID loop

$B$ magnetic field

$b$ exponent of flicker noise

$B_0$ measurement field

$B_{C1}$ lower critical field of type II superconductor

$B_{C2}$ higher critical field of type II superconductor

$B_C$ critical field of type I superconductor

$B_m$ critical field for flux expulsion

$B_{P,GB}$ penetration field for geometrical barrier

xii
List of Symbols

\[ B_p \] prepolarization field
\[ C \] (junction) capacitance
\[ C_p \] parasitic capacitance
\[ d \] strip height
\[ e \] elementary charge
\[ E_J \] Josephson coupling energy
\[ E_c \] charging energy
\[ F \] force
\[ f_c \] corner frequency of 1/f noise
\[ f_J \] Josephson frequency
\[ f_p \] pump frequency
\[ f_r \] bias reversal frequency
\[ G \] Gibbs free energy
\[ G_{APF} \] APF gain
\[ h \] Planck constant
\[ \hbar \] reduced Planck constant
\[ I \] current
\[ I^* \] parameter describing current nonlinearity
\[ I_0 \] critical current of Josephson junction
\[ I_{c,\text{sq}} \] critical current of SQUID
\[ I_{c,\text{su}} \] critical current of superconductor
\[ I_d \] displacement current
\[ i_n \] current noise generator
\[ I_p \] pump current
\[ I_{qp} \] quasiparticle current
\[ I_s \] supercurrent
List of Symbols

\[ J_s \] supercurrent density
\[ k \] coupling coefficient
\[ k_B \] Boltzmann constant
\[ l \] strip length
\[ L_g \] geometric inductance
\[ L_{in} \] input coil inductance
\[ L_{k0} \] kinetic inductance at zero current
\[ L_k \] kinetic inductance
\[ L_p \] pickup coil inductance
\[ L_{sq} \] SQUID inductance
\[ L_{tot} \] resonator inductance
\[ m_e \] electron mass
\[ N \] number of sensors
\[ n \] number of turns
\[ n_n \] density of normal electrons
\[ n_s \] Cooper pair density
\[ Q_i \] intrinsic quality factor
\[ Q_R \] quality factor of resonance
\[ R \] (junction) resistance
\[ r \] distance
\[ R_c \] cryoresistor resistance
\[ R_d \] dynamic resistance
\[ R_{H} \] quantum Hall resistance
\[ R_{opt} \] optimal source resistance
\[ S_\Phi \] flux noise power
\[ S_B \] magnetic field noise power
List of Symbols

$S_{f_1}$ current noise power of cryoresistor

$S_f$ current noise power

$S_V$ voltage noise power

$T$ temperature

$t$ time

$T_1$ spin-lattice relaxation time

$T_2$ spin-spin relaxation time

$T_c$ critical temperature of superconductor

$T_n$ noise temperature

$U$ voltage

$U_J$ washboard potential

$u_n$ voltage noise generator

$U_\Phi$ flux-to-voltage transfer coefficient

$V_J$ Josephson voltage

$w$ width

$x$ position
1. Introduction

Certain materials, when cooled below critical temperature $T_c$, lose electrical resistivity at low frequencies, allowing unrestricted flow of electrical current. The material is said to turn from normal to superconducting [1]. Charge transport as well as many other properties in superconductors are explained with the aid of so-called Cooper pairs. In the presence of an attractive force mediated by the electron-phonon interaction, two electrons may form such a pair if the thermal energy stays below the binding energy. Electrons are distributed into unpaired electrons (quasiparticles) and Cooper pairs carrying the supercurrent. Cooper pairs form a macroscopic ground state that can be described with a single wave function. The energy difference between the ground state and excitations is given by the energy gap $\Delta$.

In addition to the loss of resistance, superconductors exhibit several other physical phenomena not encountered elsewhere. In some cases, the small size of the structures under study, combined with the low operating temperature, make quantum effects visible. In superconductors, the highly correlated electron system gives rise to the quantum effects that can be observed at macroscopic level. The unique properties have made superconductors an interesting platform for low-noise sensors and electronics [2]. For example, zero resistance and low operating temperature reduce thermal noise and dissipation. Materials are classified into low- and high-$T_c$ groups based on material-dependent $T_c$, having a value either below or above the liquid nitrogen temperature, respectively. In either case, cryogenics is a necessity, increasing the costs and complexity of the measurement systems. Hence, the advantages provided by the low operating temperature must be well justified.

The basic physical phenomena in superconducting systems include Josephson tunnelling [3] and flux quantization [4, 5]. Josephson effect describes
the transport of supercurrent through a weak link between two superconductors, a Josephson junction, leading to nonlinear current-voltage characteristics across the junction. In a superconducting loop, on the other hand, total magnetic flux can only take a value that is a multiple of one flux quantum $\Phi_0 = \frac{h}{2e} \approx 2.07 \times 10^{-15}$ Wb, where $h$ is the Planck constant and $e$ the elementary charge. The combination of the two is utilized in a so-called superconducting quantum interference device (SQUID) [6]. SQUID refers to a superconducting loop that is interrupted by at least one junction. In the case of two junctions, the applied field through the loop transforms into a voltage over the junctions. In principle, any physical quantity which can be converted into a flux in the SQUID loop can be detected.

This thesis deals with the development of superconducting current and magnetic field sensors. To enhance the sensitivity and manufacturability, the sensors employ planar coils of large size fabricated mostly using thin-film techniques. The benefit of superconducting coils is that, due to the flux quantization, the magnetic coupling is frequency-independent. This is different from resistive coils [7], for which Faraday’s law of induction yields vanishing response at low frequencies.

A large part of the work was devoted to SQUID-based detectors developed for two low-frequency applications. Real-time measurement of human brain activity with a millisecond resolution is routinely performed in magnetoencephalography (MEG) [8]. The idea of incorporating ultra-low-field magnetic resonance imaging (ULF MRI) [9] with MEG in a single device stems from the need for improving the co-registration of anatomical images with functional data, respectively. The magnetic environment of the two imaging modalities is, however, very different, introducing extra requirements for the field tolerance of the magnetic field sensors. In MEG, to observe the tiny magnetic fields resulting from human brain activity, the instrument must be placed inside a magnetically shielded room (MSR). In ULF MRI, on the other hand, the sample magnetization with a prepolarization field having a magnitude usually above 10 mT interacts with the sensors as well as the MSR. This can lead to technical challenges in the imaging performed in a homogeneous measurement field (10–200 $\mu$T) following the prepolarization. Publication I describes the development of such sensors. In Publication II, the sensors are employed in practice as the design and the function of the hybrid MEG-MRI instrument is presented.
Besides the SQUIDs, superconductivity offers several other means for sensing. One category is kinetic inductance devices [10]. These detectors rely on Cooper pair breaking and quasiparticle production as a response to a change in external variable, such as temperature or dc current. The changes in the charge carrier densities modify the impedance of the superconducting line, where the resistive term is associated with quasiparticles and the reactive part consists of an inductive term related to the kinetic energy of the Cooper pairs. The impedance can be read out by making the superconducting structure a part of a resonator, which loads a transmission line and affects the transmitted power across the component near the resonance. Based on these ideas, Publication III introduces a novel magnetometer fabricated from a single thin-film layer.

The second application, a quantum metrology triangle (QMT) experiment [11, 12], relates to the upcoming revision of the SI unit system [13], in which all the seven basic units are defined through fixed constants of nature. In order to be successful, the constant values must be known exactly. The units of electrical quantities (current, voltage and resistance) can be reproduced with quantum standards that depend on \( e, h \) and frequency. The three quantum standards form an electrical triangle which can be used as a test for the underlying physical phenomena. The consistency is verified with a null detector provided Ohm’s law is satisfied. Practical reasons dictate the use of a low-noise ammeter operating at low frequencies and intermediate impedance levels. In Publication IV, such a detector is realized with the aid of a large thin-film flux transformer. Publication V discusses the practical null detection achieved with the device.
2. Background and Methods

In this section, the main physical principles relevant for this thesis are first briefly reviewed. This is followed by an introduction to SQUIDs and kinetic inductance devices. The last two subsections are devoted to applications, namely, combined ULF MRI and MEG, as well as the QMT experiment.

2.1 Fundamentals of superconductivity

2.1.1 Josephson junction

Superconductivity is described through a macroscopic wave function of the form \( \Psi(\vec{r},t) = \sqrt{n_s(\vec{r},t)} e^{i\theta(\vec{r},t)} \), where \( n_s \) is the density and \( \theta \) the phase of the Cooper pairs. When two superconductors are separated by a weak link, for example a thin insulating layer, the two wave functions of the superconductors begin to overlap, allowing flow of the supercurrent \( I_s \) through the barrier. The device is commonly known as a Josephson junction, described by relations \[ I_s = I_0 \sin \delta \] \[ \frac{\partial \delta}{\partial t} = \frac{2e}{\hbar} U - \frac{2\pi}{\Phi_0} U. \] According to the first Josephson equation, Cooper pairs tunnel through the junction, determined by the phase difference \( \delta = \theta_1 - \theta_2 \), without dissipation up to a maximum value, critical current \( I_0 \). The second Josephson equation states that when a voltage \( U \) is applied over the junction, \( \delta \) evolves in time and \( I_s \) begins to oscillate with so-called Josephson frequency \( \omega_J = \frac{2\pi U}{\Phi_0} \).

Practical junctions also contain resistive and capacitive components corresponding to quasiparticle \( I_{qp} \) and displacement \( I_d \) current through the
junction, respectively. Such a system is often described with a resistively and capacitively shunted junction (RCSJ) model shown in Figure 2.1(A).

Writing the total current \( I = I_s + I_{qp} + I_d \) leads to a second order differential equation for the phase difference

\[
\frac{\Phi_0}{2\pi R} \frac{\partial \delta}{\partial t} + \frac{\Phi_0 C}{2\pi} \frac{\partial^2 \delta}{\partial t^2} = I - I_0 \sin \delta = -\frac{2\pi}{\Phi_0} \frac{\partial U_J}{\partial \delta},
\]

(2.3)

where \( U_J = E_J(1 - \cos \delta - I \delta/I_0) \) is so-called tilted washboard potential and \( E_J = \Phi_0 I_0/2\pi \), the Josephson coupling energy. The washboard potential is depicted in Figure 2.1(B) illustrating the dynamics of the junction.

The current \( I \) tilts the cosine-shaped potential. When \( I < I_0 \), \( \delta \) is confined in a local potential minimum oscillating back and forth with plasma frequency \( \omega_p \). Once \( I \) exceeds \( I_0 \), local minima vanish and \( \delta \) is free to move, providing a finite dc voltage across the junction, see Figure 2.1(C). As \( I \) is lowered, \( \delta \) is recaptured by a potential minimum at current \( I_r \) determined by the junction capacitance \( C \). In the underdamped case, the Stewart-McCumber parameter

\[
\beta_c = \frac{2\pi}{\Phi_0} I_0 R^2 C
\]

(2.4)

takes a value above one, leading to a hysteretic \((I_r < I_0)\) \( I-U \) curve of the junction. In practice, a stable (nonhysteretic, \( I_r = I_0 \)) device operation is often recovered by designing \( \beta_c < 1 \). The classical treatment given above applies for large junctions, i.e. \( E_J \gg E_c \), where \( E_c = e^2/(2C) \) is the charging energy of the capacitor. In the remaining text, unless stated otherwise, we also assume \( E_J \gg k_B T \), i.e. \( \delta \) remains confined in a local potential minimum \((I < I_0)\) even in the presence of thermal fluctuations. Here \( k_B \) is the Boltzmann constant and \( T \) the temperature. Junctions based on the insulating barrier, i.e. superconductor-insulator-superconductor (SIS) junctions, were used in this thesis.
2.1.2 Flux quantization

One of the consequences of the macroscopic wave function is the flux quantization in a superconducting loop described by the equation [2]

\[
\oint_C \lambda^2 \mu_0 \vec{J}_s \cdot d\vec{l} + \int_A \vec{B} \cdot d\vec{S} = \frac{\Phi_0}{2\pi} \oint_C \nabla \theta \cdot d\vec{l},
\]

(2.5)

where \( \lambda \) is the magnetic penetration depth and \( \mu_0 \) the vacuum permeability. To ensure \( \Psi \) is well defined, \( \theta \) can differ only by \( 2\pi m \), where \( m \) is an integer, and the right-hand side becomes \( m\Phi_0 \). In case the contour \( C \), enclosing the loop area \( A \), is chosen so that \( \vec{J}_s \) is constant within the cross-section and deviates from zero, Equation (2.5) reduces to

\[
\left( \frac{\mu_0 \lambda^2 l}{wd} + L_g \right) I_s - \Phi_a = m\Phi_0.
\]

(2.6)

Here \( L_g \) is the geometric inductance of the loop, \( I_s \) the circulating current and \( \Phi_a \) the magnetic flux applied through area \( A \). The dimensions of the superconducting strip are given by width \( w \), height \( d \) and length \( l \). In addition, we define kinetic inductance \( L_k \) as

\[
L_k = \frac{\mu_0 \lambda^2 l}{wd},
\]

(2.7)

which will be considered in more detail in section 2.3.

2.1.3 A superconductor in magnetic field

Superconducting materials can be classified on the basis of their behaviour under magnetic field. Type I superconductors are characterized by the Ginzburg-Landau parameter \( \kappa = \frac{\lambda}{\xi} \leq 1/\sqrt{2} \), where the coherence length \( \xi \) is the distance over which the correlation of Cooper pairs persists, and exhibit total diamagnetism up to a critical field \( B_C \). The material is said to be in the Meissner state, as screening currents, confined at a distance of \( \lambda \) on the superconductor’s surface, prevent external magnetic field from entering the superconductor. Above \( B_C \), superconductivity breaks down and the material becomes normal.

For type II superconductors (\( \kappa > 1/\sqrt{2} \)), the Meissner state is preserved similarly below the lower critical field \( B_{C1} \). Above \( B_{C1} \), in contrast to type I superconductivity, magnetic field penetrates only partly into the material in the form of small cylindrical filaments called vortices. Each vortex is surrounded by a circulating supercurrent and carries one \( \Phi_0 \) through the superconducting material. The material is said to be in the mixed (vortex) state.
At fields only slightly higher than $B_{C1}$, the relative volume occupied by the vortices is fairly small. When the magnetic field is raised, the amount of vortices increases and the magnetic properties of the superconductor become a function of the vortex physics and the nature of the material. In a pure (ideal) material, the vortex state is determined solely by the applied magnetic field and the mutual repulsion of neighbouring vortices, resulting in a regular flux line lattice [14, 15]. At a higher critical field $B_{C2}$, the adjacent vortices in the lattice begin to overlap and the material becomes normal.

When a transport current flows through a superconductor, it exerts a Lorentz force on vortices, compelling them to move. This movement creates a changing magnetic flux and, according to Faraday's law of induction, a voltage (and resistance) in the material. However, practical (non-ideal) materials contain inhomogeneities that introduce potential barriers for vortices. If the transport current remains below a critical depinning value, vortices are pinned by many such defect sites. Lorentz force remains smaller than corresponding pinning force, preventing flux motion and the breakdown of superconductivity. The system is often described by the phenomenological critical state model [16].

Even in the absence of the externally driven transport current, another form of flux motion becomes possible through thermally activated vortex hopping. Due to the intervortex repulsion, vortices often form bundles, each comprising flux lines even up to several hundreds, moving simultaneously in the material [17]. Any change in external magnetic pressure leads to a reorganization of the flux distribution, a phenomenon commonly known as flux creep. All this vortex motion creates more heat which, accompanied by a large transport current, can drive the superconductor normal.

### 2.1.4 Complete flux expulsion

So far, discussion has concerned bulk superconductors under magnetic field, but the situation becomes somewhat different when the sample geometry is changed to a large aspect ratio object. Pearl was the first one to consider vortices in very thin ($d \ll \lambda$) type II structures [18]. He proposed a modified penetration depth $\Lambda = \lambda^2/d$ to describe the decay of the magnetic field in this limit. This results in a long-range interaction between two isolated vortices ($F \propto 1/r^2$) and an increased penetration depth for decreasing $d$ [19].
When a superconducting strip is placed in a transverse magnetic field $B$, field lines are focused at the edge of the film, where vortices start nucleating to the film below the bulk value $B_{C1}$. The interaction of a vortex with the sample edge is described with an image vortex of opposite sign on the other side of the interface [20]. The Gibbs free energy $G(x)$ for a vortex can be given in the form [21]

$$G(x) = \frac{4\pi}{\mu_0} \left[ \frac{\Phi_0^2}{4\pi^2 \Lambda} \ln \left( \frac{2w}{\pi \xi} \sin \frac{\pi x}{w} \right) - \frac{\Phi_0 B}{2\pi \Lambda} x(w-x) \right], \quad (2.8)$$

where $x$ is the horizontal position of the vortex in the strip, measured from one end (Figure 2.2(A)). The first term represents the image force driving vortices towards the edge, while the second term describes the Lorentz force due to Meissner screening, pulling flux lines to the film centre. In Figure 2.2(B), $G(x)$ has been plotted for different $B$. At low fields, $G(x)$ has a dome-like shape with a maximum in the centre and vortices do not exist in the strip. As $B$ is raised, the contribution of the second term increases, and a local minimum emerges in the centre at $B < B_m$ with

$$B_m \approx \frac{\Phi_0}{w^2}. \quad (2.9)$$

The field $B_m$ is called a critical field for complete vortex expulsion. Below this field, it is favourable for the strip to be free from vortices, a fact that has also been confirmed experimentally [22, 23, 24]. The flux expulsion has been found to occur when the sample is cooled in a magnetic field $B \lesssim B_m$ through $T_c$. For $T \sim T_c$, the vortices are mobile and can find the lowest free energy of the system. At lower temperatures, pinning restricts the vortex motion and may lead to vortex freezing in the material. The latter scenario is investigated in Publication I, where SQUIDs were subjected to large magnetic pulses at $T < T_c$. 

**Figure 2.2.** (A) An isolated vortex sitting in a superconducting strip and (B) Bean-Livingstone potential computed for different perpendicular magnetic fields $B$. 

![Figure 2.2](image.png)
2.1.5 Surface barriers

When $B$ is raised, a potential barrier known as the Bean-Livingstone barrier (BLB) [20] appears in the vicinity of the edge. The vortex nucleation occurs when the Lorentz and image forces become equal. Assuming this happens within a distance of $\xi$ from the thin-film edge, Equation (2.8) is no longer strictly valid, and the complete description of the vortex formation requires use of Ginzburg-Landau theory [25]. Many of the derivations nevertheless rely on London equations computed in the limit $d \ll \Lambda$ [26, 27, 28]. Moreover, even apparently clean surfaces contain defect sites that can suppress the barrier height [29].

![Figure 2.3. The field lines bend and pass the thin superconducting strip. Sharp corners are in the mixed state where the local field exceeds $B_{C1}$ and vortices are flexed. At $B = B_{P,GB}$, the Meissner currents pull the vortices to the interior. Vortices generate an opposing field at the edge of the strip, as indicated with dashed lines.](image)

Another type of barrier arises from a competition between a line tension force of a penetrating vortex and a Lorentz force of screening currents [30, 31]. For a thin structure ($d \ll w$) of rectangular cross-section in perpendicular magnetic field, the vortices first cut through the sharp rims of the sample and become flexed, see Figure 2.3. The increasing field drags the vortices towards the superconductor but at the same time line tension energy, being proportional to the vortex length, also increases. At sufficiently high field, the vortex suddenly straightens, reducing the line tension, and the flux line is driven to the strip centre. It has been shown that for a superconductor of elliptical cross-section these forces have the same spatial dependence [32], and no barrier is formed. Thus, the barrier height only depends on the shape of the sample, and is therefore often referred to as the geometrical barrier (GB).

Computations of the GB have been conducted for an infinitely long strip [30, 31] and a circular disk [33] with similar results. For a rectangular superconductor ($d \ll w$), the GB penetration field is

$$B_{P,GB} = \frac{B_{C1}}{\sqrt{w/d}}$$

(2.10)
Both the pinning and surface barriers delay the flux penetration to the sample, and lead to magnetic hysteresis and irreversibility in superconductors. Furthermore, distinguishing GB from BLB is not straightforward in general, even though this can be accomplished by varying the shape of the sample [34].

2.2 SQUID

In the late 1960s, the advent of one junction SQUID [35], commonly known as rf SQUID, started the evolution that is nowadays considered mature technology. It was, however, the introduction of the Nb-AlO$_x$-Nb tunnel junctions and thin-film washers, coupled to a multi-turn input coil, that secured the dominance of dc SQUIDs [36], which they possess today. A general description of the dc SQUID operation and characteristics based on [6, 37, 38] is given below.

2.2.1 Operation

A dc SQUID contains a superconducting loop interrupted by two Josephson junctions, see Figure 2.4(A). Let us consider, for simplicity, a symmetrical case where the two junctions and arms of the loop are identical. The device is biased with current $I$ and magnetic flux $\Phi_{sq}$ is applied through the ring, creating a circulating current $I_s$. The current through the junctions becomes

$$\frac{I}{2} \pm I_s = I_0 \sin \delta_{1,2} + \frac{\Phi_0}{2\pi R} \frac{\partial \delta_{1,2}}{\partial t} + \Phi_0 C \frac{\partial^2 \delta_{1,2}}{\partial t^2}, \quad (2.11)$$

where $\pm$ in front of $I_s$ refers to junctions 2 and 1, respectively. Integration
of Equation (2.5) around the SQUID loop yields

$$\delta_1 - \delta_2 = \frac{2\pi}{\Phi_0}(\Phi_{sq} + L_{sq}I_s), \quad (2.12)$$

where $L_{sq}$ is a sum of the kinetic and geometric inductance of the loop. It determines the amount of screening characterized by a parameter

$$\beta_L = \frac{2L_{sq}I_0}{\Phi_0}. \quad (2.13)$$

Assuming a zero-voltage state ($\delta_1, \delta_2$) and requiring $\beta_L \ll 1$, Equations (2.11) and (2.12) give the critical current of the SQUID

$$I_{c,sq} = 2I_0 \left| \cos \left( \pi \frac{\Phi_{sq}}{\Phi_0} \right) \right|. \quad (2.14)$$

Therefore, the maximum supercurrent $I_{c,sq,\text{max}}$ of the SQUID is limited to $2I_0$ and $\Phi_{sq}$ modulates $I_{c,sq}$ with a period $\Phi_0$. Figure 2.4(B) illustrates current-voltage characteristics of a practical SQUID. When biased slightly above maximum $I_{c,sq}$, a dc voltage $U$ over the SQUID develops with $U$ at maximum, when $I_{c,sq}$ is at minimum and vice versa. It is the increasing screening parameter $\beta_L$ that decreases the amount of this modulation. For $\beta_L = 1$, $\Delta I_{c,sq} = I_0$ whereas for $\beta_L \gg 1$, $\Delta I_{c,sq}$ converges as $\sim \Phi_0/L_{sq}$. Thus, the SQUID acts as a flux-to-voltage transducer, a property which we denote with a quantity $U_\Phi = \partial U/\partial \Phi_{sq}$.

### 2.2.2 Noise

A quantity often used in the context of SQUIDs is the equivalent flux noise power $S_\Phi = S_V/U_{\Phi}^2$, where $S_V$ is the voltage noise power over the SQUID. However, the inductance levels of different SQUIDs can vary, and a more fundamental figure for the noise is obtained with the energy resolution

$$\epsilon = S_\Phi/(2L_{sq})$$

defined as the minimum magnetic field energy per unit bandwidth that can be measured. SQUID noise is a sum of thermal and flicker noise.

#### Thermal noise

Thermal noise typically stems from the Johnson voltage noise of the shunt resistors having power spectral density $S_R = 4k_B T R$. This is formally treated by adding current noise terms to Equation (2.11) and placing corresponding current noise sources in parallel with the junctions in Figure 2.4(A). In the washboard picture, the extra current term stochastically tilts the potential, letting the phase to escape from local minimum. In an overdamped SQUID, thermal noise rounds the current-voltage characteristics around the point, where the voltage state emerges.
Johnson noise in shunt resistors generates both current noise around the loop and voltage noise over the SQUID. This can occur either directly or through down-mixing due to Josephson oscillations [39]. Several studies have addressed the problem by solving the modified version of Equation (2.11) numerically. A general result is that the energy resolution takes the form

$$\epsilon = \alpha k_B T \sqrt{L_{\text{eq}} C},$$

(2.15)

suggesting small values for the temperature, SQUID inductance and junction capacitance. Here $\alpha$ is a constant and, depending on the device parameters, may vary to some extent, but the typical value is 12 [38], 14 [40] or 16 [6] assuming $\beta \equiv \beta_c \equiv 1$.

1/f noise

At low-enough frequencies, the SQUID flux noise $S_\Phi$ scales as $1/f^b$, where a value close to a unity is usually found for $b$ [41]. Hence, it is commonly known as $1/f$ (flicker) noise. Moreover, the corner frequency $f_c$, at which $1/f$ noise starts dominating over its white counterpart, can extend below 1 Hz, making low-$T_c$ SQUIDs attractive for many low-frequency applications. The origin of $1/f$ noise in SQUIDs has been studied extensively. Below we discuss three main mechanisms.

One mechanism is the fluctuations of critical current in the Josephson junction. This may occur when an electron, tunnelling through the barrier, is trapped and released by a defect site, affecting the local height and critical current density of the junction. When there are many such defects, each exhibiting a characteristic lifetime for the electrons, critical current starts switching and produces $1/f$-type spectrum. The fluctuations of the two junctions can occur either in phase or out of phase, resulting in a voltage across the SQUID or in a circulating current around the loop, respectively. The noise contribution due to critical current fluctuations can be reduced with an electronic scheme called bias reversal [42, 43].

Another source of $1/f$ noise is thermally activated vortex motion in the superconducting structures of the SQUID. Consider a vortex sitting in a flux transformer that is coupled to the SQUID loop. This leads to two partially correlated mechanisms [44]. When the vortex moves towards the SQUID, the direct stray flux sensed by the SQUID loop changes. Alternatively, the motion along the width of the flux transformer generates a shielding current change that, again, couples to the SQUID loop. In analogy to critical current fluctuations, each pinning site sets a characteristic
time for the vortex hopping that together give flicker-type noise behaviour.

Due to the thermal activation, the role of both above-mentioned 1/f noise sources is expected to decrease when temperature is lowered from 4.2 K down to millikelvin regime [45]. At these temperatures, Wellstood et al. [46] discovered an excess noise mechanism that appeared to be true flux noise, i.e. of magnetic origin. Over the past few years, an attempt has been made to understand the origin of this low-frequency noise. Recent studies [47, 48] have confirmed that the noise increases with decreasing temperature. It has been proposed that flux noise stems from fluctuating spins of localized electrons at the interfaces between the superconductor and its substrate and surface oxide layer [49].

### 2.2.3 Coupled SQUIDs

![Figure 2.5. A simplified equivalent circuit of the coupled SQUID using either (A) one or (B) double transformer circuit.](image)

Equation (2.15) indicates that \( \epsilon \propto L_{sq}^{1/2} \propto a^{1/2} \), where \( a \) is the inner diameter of the loop. However, when used for measuring magnetic fields, small \( L_{sq} \) and SQUID loop lead to a device exhibiting a good spatial resolution, but only a moderate field resolution. To excel in the latter requires a loop with a larger flux capture area \( A \). This is somewhat improved when the outer dimension of the SQUID loop is increased [50], although a more efficient way of enlarging the sensing area is to arrange multiple high-inductance loops in parallel [51, 52]. Alternatively, the magnetic signal may be detected with a separate pickup coil \( L_p \), inductively matched to the SQUID loop with a so-called flux transformer, see Figure 2.5(A).

Energy resolution \( \epsilon_{in} = S_{\Phi, in}/(2L_{in}^{eff}) \) at the input coil is

\[
\epsilon_{in} = \frac{1}{2} L_{in}^{eff} S_I = \frac{L_{in}^{eff} \epsilon_{sq}}{L_{in} k^2},
\]

(2.16)

where \( S_I \) is the equivalent current noise power spectral density referred to the input coil and \( k \) is the coupling constant between inductances \( L_{in} \) and \( L_{sq} \). Quantity \( L_{in}^{eff} \) is the real inductance over the input coil, which is smaller than \( L_{in} \) due to the screening effect of the SQUID loop. Rearranging (2.16) yields

\[
S_I^{1/2} = \frac{1}{k} \sqrt{\frac{2 \epsilon_{sq}}{L_{in}}},
\]

(2.17)
Energy resolution at the pickup $\epsilon_p = S_{B,p}/(2L_p)$ on the other hand gives

$$\epsilon_p = \frac{S_B A^2}{2L_p} = \frac{(L_p + L_{in}^{eff})^2}{k^2 L_{in} L_p} \epsilon_{sq}, \quad (2.18)$$

where $S_B$ is the equivalent magnetic field noise. We get

$$S_B^{1/2} = \frac{L_p + L_{in}^{eff}}{kA} \sqrt{\frac{2\epsilon_{sq}}{L_{in}}}. \quad (2.19)$$

Equation (2.17) describes how the current sensitivity of the SQUID can be enhanced with a flux transformer. A low-noise current detector is achieved by constructing a flux transformer, having large $L_{in}$ and $k$, which can be coupled to the SQUID loop in a noise-free way. In practice, compromises must be made to reach this goal, as described later.

For a magnetometer, field sensitivity can in principle be improved indefinitely by increasing $A$, which raises pickup inductance approximately as $L_p \propto \sqrt{A}$. Depending on the application, increasing $A$ becomes impractical at some point and, in contrast to the current detector, infinitely high $L_{in}$ is no longer supported. The optimization of $S_B^{1/2}$ leads to a condition $L_{in} = L_p$, where the screening of the SQUID was neglected [53], i.e. $L_{in} \approx L_{in}^{eff}$. When an intermediate flux transformer is used (Figure 2.5(B)), the corresponding relations can be derived by noting $M = M_1M_2/(L_1 + L_{2}^{eff})$ and $L_{in}^{eff} = L_{in} - M_1^2/(L_1 + L_{2}^{eff})$.

High sensitivity at the input of the flux transformer requires tight coupling between $L_{sq}$ and $L_{in}$, i.e. high coupling coefficient $k$. A common way to achieve this is to construct the SQUID loop with $w \gg a$, know as a washer, and to pattern a multi-turn input coil on top of it, yielding $k = 0.8–0.9$. The downside of the approach is a large parasitic capacitance $C_p$ introduced by the flux transformer circuitry, both across the SQUID and along the input coil and washer.

The former creates a washer resonance with $L_{sq}$ that may reduce the energy resolution of the device if $C_p$ is of the order junction capacitance $C$ or larger [54]. The latter, on the other hand, forms a microwave transmission line (a microstrip) with the washer acting as a ground plane. The microstrip exhibits high-Q resonances that can extend well below the Josephson frequency. Also, the self-resonance of the input circuitry may impact the SQUID performance (Publication V). If near the operating point, resonances interfere with the SQUID dynamics, creating hysteresis and distortions in the current-voltage characteristics, and increase of noise [55]. As a result, Equation (2.11) becomes inadequate in describing the device dynamics. An explanation for the noise increase can be
given in terms of thermal fluctuations that induce hopping to the resonance states. The noise can be assumed to be proportional to the lifetime \( \tau_R = Q_R/\omega_R \) of the resonance state, where \( Q_R \) and \( \omega_R \) are the quality factor and the frequency of the resonance, respectively. Thus, the noise can be lowered by damping the resonances with (frequency-dependent) shunt circuits inserted in parallel with the SQUID loop [56] or the input of the flux transformer [57].

High \( L_{in} \) is realized by a large number of turns \( n \) for the input coil, i.e. \( L_{in} = n^2L_{sq} \). Input coils fabricated from narrow linewidths can be employed [58, 59] to facilitate the problems related to parasitic capacitance and inductance matching. In practice, \( L_{sq} \) is often increased as well. Enpuku et al. [60] have shown that the increase in the energy resolution can be reduced with a shunt resistor inserted in parallel with the SQUID loop. High \( L_{sq} \) may be circumvented if an intermediate flux transformer (Figure 2.5(B)) is used instead [53, 61] with a drawback of somewhat lower flux coupling. An effective small SQUID inductance is also obtained with a so-called multiloop SQUID, in which a common input coil is wound on top of multiple high-inductance loops connected in parallel [51, 52]. The last option is to place several identical SQUIDs in series [62], a SQUID array. A high input inductance is formed when the signal coils of individual SQUIDs are also connected in series.

### 2.2.4 Readout

The readout of the SQUID [6] calls for carefully designed electronics having two main tasks. The periodic response with low output voltage (typically below 100 \( \mu \)V) requires the amplification of the voltage across the SQUID, and linearization of the transfer function \( U_\Phi \).

The first challenge of the electronics is to have an amplifier that does not limit the system noise. Amplifier noise properties can be described conveniently with a noise temperature

\[
T_n = \frac{1}{2} T_{n,\text{opt}} \left[ \frac{R_{\text{opt}}}{R_d} + \frac{R_d}{R_{\text{opt}}} \right],
\]

where \( R_d \) is the dynamic resistance of the SQUID, \( T_{n,\text{opt}} = u_n i_n/(2k_B) \) and \( R_{\text{opt}} = u_n/i_n \) are optimal noise temperature and source resistance, respectively, given by the equivalent voltage \( u_n \) and current \( i_n \) noise generators at the amplifier input [63]. The low (output) noise temperature of a typical SQUID can be obtained with semiconductor amplifiers operating at room temperature, but only for relatively high source resistance values.
Background and Methods

The dynamic resistance $R_d$ usually takes a value of the order $10\,\Omega$, degrading $T_n$ according to (2.20).

A traditional way to match the source impedance to that of the amplifier has been to use a step-up transformer between the two [65]. This leads to a flux modulation readout scheme, in which a square-wave modulation of flux is applied to the SQUID in such a manner that the operating point switches between adjacent slopes of the SQUID voltage modulation curve. The magnitude of the flux modulation is chosen so that the SQUID is biased at points with identical voltage and minimal noise. Now, a change in the applied flux produces a square wave voltage at the output, of which phase and magnitude can be used to determine the signal.

Direct readout schemes shown in Figure 2.6 emerged at the beginning of the 1990s to simplify the readout of biomagnetic multichannel systems. In a so-called additional positive feedback (APF) method [66], the SQUID is current-biased and the voltage across the SQUID is sampled and converted to feedback flux with a circuit consisting of a coil $L_{APF}$ and a resistor $R_{FET}$. For $U_\Phi > 0$, a positive change in applied flux increases the current in the coil, which enlarges $U_\Phi$ and makes the voltage modulation curve asymmetric. As a result, APF can be described as a separate voltage amplifier following the SQUID and having a gain $G_{APF} = 1/(1 - U_\Phi(M_{APF} - M_d)/R_{FET})$, where $M_d = R_d/U_\Phi$. Assuming $U_\Phi$, and $R_d$ increase with the same factor, the output noise temperature of the SQUID enhances to $T_{n, sq} = S_\Phi G_{APF} U_\Phi^2/(4k_B R_d)$ [67].

In the noise-cancellation scheme (NC) [68], the voltage over the SQUID is fixed and the current through the device detected. The voltage noise of the amplifier generates a current across the SQUID. However, on the positive slope, the feedback is utilized to create an opposing current, and the two terms cancel each other when $R_{FET}^{opt} = U_\Phi M_{NC}$. Unlike in APF, the SQUID remains stable beyond this condition, with only the amplifier noise becoming dominant again. Furthermore, the linear regime is

---

**Figure 2.6.** Direct readout methods: (A) Additional positive feedback (Publications IV and V) and (B) Noise cancellation (Publication II). The negative feedback (FLL) linearizes the SQUID response.
not reduced. The adjustable resistor $R_{\text{FET}}$ loads and generates flux noise to the SQUID, introducing constrictions for practical values of feedback components.

Both APF and NC were designed to reduce the effect of $u_{\text{in}}$, which is often the dominating noise source. When the noise contribution from $i_{\text{n}}$ becomes a problem, an inductor, magnetically coupled to the SQUID loop, may be placed in series with the SQUID. In analogy with APF and NC, readout concepts for both current [69] and voltage [70] bias have been developed.

To linearize periodic $U_{\Phi}$, SQUIDs are usually operated in a flux-locked loop (FLL). Figure 2.6 depicts a simplified FLL used with direct readouts [66]. The SQUID voltage is amplified, integrated and fed back to the SQUID loop through mutual inductance $M_{\text{FLL}}$. The negative feedback keeps the flux in the SQUID loop constant. When combined with APF, the voltage at the output $U_{\text{out}}$ becomes

$$U_{\text{out}} = \Phi_{\text{sq}} U_{\Phi} G_{\text{APF}} \frac{R_{\text{1}}}{R_{\text{2}}} A_{\text{int}}(\omega) \left( 1 - \frac{M_{\text{FLL}}}{R_{\text{FLL}}} U_{\Phi} G_{\text{APF}} \frac{R_{\text{1}}}{R_{\text{2}}} A_{\text{int}}(\omega) \right) \approx -\Phi_{\text{sq}} \frac{R_{\text{FLL}}}{M_{\text{FLL}}},$$

(2.21)

where $\Phi_{\text{sq}}$ is the flux in the SQUID loop without feedbacks and an infinite value has been assumed for integrator gain $A_{\text{int}}(\omega)$ in the last stage. Thus, the response is linearized and the sensitivity and the dynamic range determined solely by parameters $R_{\text{FLL}}$ and $M_{\text{FLL}}$. The integrator determines the bandwidth in the FLL and reduces the loop gain at high frequencies, ensuring the stable operation of the feedback.

### 2.2.5 Fabrication

SQUID fabrication has gained a great deal of benefit from process development carried out in the semiconductor industry. Today, the wafer-scale fabrication of low-$T_c$ SQUIDs, which is well controlled and commercially available, gives these devices an advantage over high-$T_c$ SQUIDs. The superior properties and the mature Nb-AlO$_x$-Nb junction technology have made thin-film niobium the most widely used material in superconducting circuits.

Over the last two decades, SQUID fabrication at VTT has relied on the so-called trilayer process [71, 72] employed to produce Josephson junctions. The two first sheets of the trilayer, Nb and Al, are deposited with sputtering. An insulating tunnel barrier is then grown on top of Al by oxidation, followed by a second Nb layer. Junction definition is achieved either with etching or anodic oxidation of the junction perimeter.
Although the main features remain the same, many of the techniques and material choices used to fabricate SQUIDs for this thesis varied to some extent. Altogether, three superconducting layers were deposited on top of each other, permitting flexible circuit design and tight flux coupling in transformer structures. The layers were separated by insulating SiO\textsubscript{2} that was grown with plasma-enhanced chemical vapour deposition (PECVD). Capacitors were formed between adjacent layers of Nb with anodized niobium (high permittivity) or SiO\textsubscript{2} serving as the dielectric material. Nb and SiO\textsubscript{2} were both etched with reactive ion etching (RIE) while contact and projection lithography were used in patterning components. A layer of molybdenum (Mo) or titanium tungsten (TiW) was sputtered to make resistors. Finally, a PbIn layer was evaporated on top to enable superconducting bonding.

### 2.3 Kinetic inductance devices

Unlike the geometric inductance $L_g$ that is related to the energy stored in magnetic field, kinetic inductance $L_k$, introduced in Equation (2.7), stems from the kinetic energy of the charge carriers. In other words, $L_g$ is derived from Faraday’s law of induction, while the inertia of charge carriers determines $L_k$. Thus, both quantities oppose the changes of current in a circuit. The kinetic inductance may dominate only at very high frequencies in normal conductors, but is already important at low frequencies in superconductors thanks to the vanishing dissipation.

A microscopic description of the conductivity in superconductors, taking into account excitations from superconducting condensate over the energy gap, was developed by Mattis and Bardeen [73]. However, to adopt the main features at low frequencies $\hbar \omega \ll \Delta$, where $\hbar = h/(2\pi)$ is the reduced Planck constant and excitations induced by the signal can be ignored, the two-fluid model [2] is adequate. The theory assumes that Cooper pairs, obeying the first London equation, and normal electrons form two isolated fluids. At low frequencies ($\omega \tau \ll 1$), the complex conductivity is given by

$$\sigma = \sigma_1 - j\sigma_2 \approx \frac{n_n e^2 \tau}{m_e} - j\frac{1}{\omega\mu_0 \lambda_L^2}, \quad (2.22)$$

where $\lambda_L = (m_e/2\mu_0 n_h e^2)^{1/2}$ is the London penetration depth, $m_e$ the electron mass, $\tau$ the scattering time of the normal carriers, and $n_n$ as well as $n_h$ are the densities of normal electrons and Cooper pairs, respectively. The system is described with an equivalent circuit, in which supercon-
ducting and normal channels, i.e. an inductor and resistor, are connected in parallel. Hence, the real part $\sigma_1$ due to unpaired electrons is responsible for the losses in the material, while most of the current is carried by the imaginary part $\sigma_2 = l/(\omega L_k w d)$.

The inductive term in Equation (2.22) follows directly from the London theory, which assumes $\lambda = \lambda_L$. Strictly speaking, this applies only for clean type II superconductors. In the dirty limit, where the coherence length $\xi$ is limited by the very small mean free path, Mattis-Bardeen theory gives $\lambda = \sqrt{\hbar \rho/(\mu_0 \pi \Delta_0)}$, where $\rho$ is the resistivity at room temperature and $\Delta_0$ the energy gap at zero temperature. Alternatively, the same result can be derived from the nonlocal theory of Pippard [74].

The kinetic inductance of a superconductor is exploited in many applications, such as radiation detectors [75, 76], parametric amplifiers [77], metamaterials [78], quantum bits [79] and delay lines [80]. Furthermore, it is the current and temperature dependence of $L_k$ [81], as well as the high inductance value available with the reduction of the cross-section (see Equation (2.7)), which has attracted the attention to these devices. The two-fluid model gives a general expression for $T$-dependence through that of $\lambda$, something that is rarely found valid in practice and has led to the development of a modified model [82].

Publication III utilizes the current dependency. Although the detailed theory is rather involved [81, 83], the phenomenon is intuitively described as follows. With the increase of current, the velocity of paired electrons increases. This enhances the kinetic energy and excitations to the normal fluid. Subsequently, in order to maintain the constant current, the decrease in $n_s$ elevates the velocity even further. The phenomenon becomes stronger when the kinetic energy approaches the depairing energy of Cooper pairs, leading to a nonlinear relation. As in the case of temperature, $\lambda$ (and $L_k$) diverges to infinity when approaching the critical value $I_{c,\text{su}}$. At low currents, kinetic inductance is approximated as [77, 80, 83]

$$L_k = L_{k0} \left(1 + \left(\frac{I_s}{I^*}\right)^2\right), \quad (2.23)$$

where $L_{k0}$ and $I^*$ are parameters.

### 2.4 MEG and ULF MRI

Two imaging modalities, namely MEG and ULF MRI, will be discussed next. MEG is a technique used to examine the function of the human
brain, as opposed to structural images that are acquired with MRI. Several interesting characteristics peculiar to MRI performed at microtesla fields, ULF MRI, are covered, including the combination with MEG and signal detection with SQUIDs.

### 2.4.1 MEG

MEG is a noninvasive imaging technique in which weak magnetic fields (10 fT–1 pT), produced by the electrical activity in the human brain are measured [8]. An analogous technique for the electric field is electroencephalography (EEG). MEG is utilized both in brain research and for clinical purposes, such as detecting responses to external stimuli or measuring epileptic activity, respectively. The weak magnetic fields from the brain are detected with SQUIDs. Large arrays of SQUID magnetometers and gradiometers are arranged in a helmet containing liquid helium and placed around the patient’s head. The system is located in the MSR to reduce the effect of external magnetic interference. In MEG studies, the magnetic field is sampled in a frequency band of several kilohertz, leading to a high temporal resolution of the method. The main issue in MEG deals with finding the current distribution that produces the measured magnetic field profile. Mathematically, this is an inverse problem with no unique solution. In practice, different source models are used to describe the origin of neuronal activity.

### 2.4.2 ULF MRI

MRI is a powerful clinical tool capable of producing images of matter with a millimetre resolution. MRI is based on the nuclear magnetic resonance of spins having nonzero spin angular momentum [84]. In conventional MRI, a strong magnetic field $B_0$ (0.1–7 T) is applied to align the nuclear magnetization of hydrogen atoms. An excitation pulse is used to modify the alignment of the original magnetization, for example by $90\degree$ ($\pi/2$-pulse) or $180\degree$ ($\pi$-pulse). As a result, the net magnetic field begins to rotate about $B_0$ with a frequency

$$\nu_0 = \gamma B_0,$$  \hspace{1cm} (2.24)

where $\nu_0$ is the Larmor frequency corresponding to the energy difference between spin up and down states. The gyromagnetic ratio $\gamma$ for hydrogen is 42.58 MHz/T.

The precession is described by two relaxation processes. Longitudinal
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relaxation (or spin-lattice relaxation) time $T_1$ determines the timescale for spins realigning with $B_0$. The rotation in transverse plane occurs first in phase but relaxes according to spin-spin relaxation time constant $T_2$. The spatial information is encoded in volume elements called voxels. Three gradient coils create temporal and spatial variations of field that generate a unique NMR phase or frequency to each voxel. Altogether, this creates a signal, measured with an induction coil, which contains the information on the location of the spins [85].

In conventional MRI, high magnetic fields are favoured since the signal in the induction coil scales as $\alpha B_0^2$. However, there are means to increase the sensitivity enabling MRI at microtesla fields [86, 87]. In ULF MRI, the sample is first polarized in a magnetic field $B_p$ of the order 10–150 mT, boosting the signal to be measured. The imaging is then performed in the measurement field $B_0$ (~ 10–100 μT), usually perpendicular to $B_p$, fixing the Larmor frequency to kilohertz range. The frequency-independent magnetic field detection with SQUIDs enhances the sensitivity. In addition, external magnetic interference can be reduced, for example by operating the system in the MSR. In this way, in contrast to high-field MRI, the measured signal strength depends on $B_p$ alone.

ULF MRI bears several advantages over its high-field counterpart. Reviews on the subject can be found in [9, 88, 89, 90, 91]. In MRI, the spectral resolution scales linearly with the absolute inhomogeneity, making detection at low fields less vulnerable to relative inhomogeneity in the measurement field. As a result, a severely simpler coil system, having an open geometry, lower cost and less weight, can be designed. Operation in weak magnetic fields also decreases the distortions due to the susceptibility variations in the material. The method allows more silent and safer operation as there is no danger of ferromagnetic projectiles [92], for example. Furthermore, imaging through and in the presence of metal is possible, allowing new patient groups to enter the device.

MRI at ULF regime opens up potential for several interesting applications. For example, due to the similar signal frequency of neuronal currents and MRI signal below millitesla region, the direct measurement of brain activity through the interaction of the resulting magnetic field and a spin population has been proposed [93]. So far, reliable demonstration of this method has not been reported. Another promising application could be the detection of cancer based on the enhanced $T_1$ contrast between the healthy and cancerous tissue at low fields [94]. Reported studies of ULF
MRI use also include the detection of liquid explosives [95], uranium [96] and static current densities [97], as well as thermometry [98].

The most obvious target for ULF MRI, however, is the combination with MEG. The analysis of MEG data requires comparison with MR images that currently need to be taken with a separate device. This creates increased workload, inaccuracy between the two coordinate systems, and movement and deformation of the sample between the two scans. A hybrid device was constructed in Publication II to facilitate these problems.

Sensor characteristics of SQUIDs relevant to detecting MEG and ULF MRI signals are considered next.

2.4.3 Sensor considerations

MEG signal is located at low frequencies, typically below 30 Hz, which has favoured the use of low-\(T_c\) SQUIDs. However, high-\(T_c\) SQUIDs have also been developed for MEG [99, 100], enabling operation at liquid nitrogen temperature. Although having higher noise, thermal insulation is less strict at elevated temperatures, and high-\(T_c\) SQUIDs can be brought closer to the patient’s head, improving measurement sensitivity. MEG with atomic magnetometers has also been demonstrated [101, 102]. For ULF MRI, low- and high-\(T_c\) SQUIDs [103], mixed sensors [104] and atomic magnetometers [105, 106] have been utilized in signal detection. However, large sensor arrays will eventually be needed in combined MEG-MRI. To date, this has not been accomplished with sensors other than low-\(T_c\) SQUIDs.

ULF MRI with prepolarization technique sets requirements for the field tolerance of low-\(T_c\) SQUIDs. Sensors must be able to measure MR signal at measurement field within milliseconds after the polarization field has been turned off. The main problem is the magnetic flux that penetrates to the thin-film structures of the SQUID during the prepolarization. The entered vortices create a net stray field through the Josephson junctions lowering the critical current \(I_0\) [107]. For in-plane field, the decrease is expected to follow Fraunhofer-type relation, i.e. \(I_0\) vanishes when the flux through the junctions equals \(\Phi_0\). Indeed, critical field applied perpendicular to the SQUID, which creates flux of such order, has been shown to scale as \(\propto \Phi_0/w\) [108], where \(w\) is the width of the junction. The magnetization may also be detrimental to homogeneity of the measurement field. If large enough, the precessing spins dephase and the spectral resolution of the signal is degraded. Changing magnetic fields also compel the
The most effective way of minimizing such problems is to prevent the flux formation in the superconducting material after field removal. The total field experienced by the SQUID can be considered as a sum of the direct field, i.e. the field without the pickup coil, and the field captured by the pickup coil and directed to the SQUID loop through input circuitry. Magnetic shielding can be employed in protecting the SQUIDs from the direct field. In fact, most ULF MRI systems [109, 110] have the SQUIDs in a separate superconducting housing, away from the imaging volume. In Publication I, partial shielding was used. Moreover, magnetic flux is expected to escape from a pinning-free superconducting strip when magnetic field is lowered below $B_m$, see Equation (2.9). This favours the use of narrow-linewidth structures in SQUID design.

Methods of eliminating the large pickup current include Josephson junctions [111], thermal switches [112] and even SQUIDs [113] connected in series with the pickup coil. The dynamics of the pickup loop equipped with a Josephson junction (a flux dam) now follows from that of the rf SQUID in the extreme hysteretic limit $2\pi L_p I_0/\Phi_0 \gg 1$ illustrated in Figure 2.7. When the flux in the pickup loop $\Phi_p$ is increased from point A, $I_s = -I_0$ at point B and the junction momentarily switches to resistive state. A flux quantum $\Phi_0$ enters the pickup loop, decreasing $I_s$ and $\Phi_{sq}$. The same phenomenon also occurs for decreasing $\Phi_p$; $I_s = I_0$ at point D, and $\Phi_0$ escapes the pickup loop, preventing excessive build-up of $\Phi_{sq}$. When applied field variations are moderate, $I_s$ stays below $I_0$ and the normal linear response is maintained.

An alternative method for reducing the focused field is to use a gradientic pickup geometry instead of a magnetometer. Since only magnetic gradients can be sensed, the detector remains insensitive to homogenous vortices to rearrange in the material, generating additional noise to the system.

![Figure 2.7. Operation of a flux dam. (A) Total flux in the pickup loop $\Phi_T = \Phi_p - L_p I_s$ and (B) flux in the SQUID loop $\Phi_{sq}$ as a function of applied flux in the pickup loop $\Phi_p$.](image)
fields. However, less signal is obtained from distant sources. Finally, if despite all the mentioned precautions magnetic flux remains pinned in the SQUID structures, one can heat the SQUID above $T_c$ and let it cool down (preferably) at zero magnetic field.

Due to the nature of the signal to be measured, the optimal pickup type and size vary for MEG and MRI. Despite the sensitivity increase with increasing pickup area, the size of MEG pickup coils is typically limited to a couple of centimetres since the spatial resolution of current sources is otherwise reduced. According to an analysis for magnetometers carried out by Burmistrov et al. [114], MRI favours larger pickup loops.

Besides coil size, the number of sensors $N$ and the nature and origin of the noise all have an impact on the signal-to-noise ratio (SNR) affecting image quality. To consider optimal sensor configuration, a simplified case with uniform magnetic field signal is assumed. If the noise between adjacent units in a sensor array is correlated, the SNR is independent of $N$. For uncorrelated noise, the sensor array improves the SNR with a factor of $\sqrt{N}$.

In the latter case, whether to construct a single magnetometer having a large sensing area, or an array of similar sensors but with smaller pickup loops, depends on the dominating sources of noise. If the noise comes from the sensor, the best SNR is obtained with a single sensor, assuming ideal inductance matching in (2.19) and keeping the total sensing area $A$ fixed. However, as discussed in section 2.2.3, increasing $A$ tends to deteriorate the energy resolution of the SQUID and, at some point, the sensor field noise $S_B^{1/2}$ is no longer improved. If on the other hand an external noise source, coupling like the signal, dominates, the sensor array finds favour as increasing the magnetometer area does not help.

In (ULF) MRI, sensor array performance is evaluated with the aid of the voxel SNR [114, 115], in which case the principles given above remain valid [116]. In fact, the presence of the external noise source may degrade the SNR of a large pickup coil below that of a smaller loop, especially if the voxel is located near the sensor. The effect of distant noise sources can be reduced with the aid of gradiometers. Reasonable signal levels have been measured with long-baseline gradiometers placed close to the imaging volume, although the strongest signal is still obtained with the magnetometer [114]. Finally, it may be noted that the SNR can be computationally improved in MEG [117] by taking advantage of the noise correlation between the different channels. It is still unknown whether the
method can be exploited in MRI.

2.5 Quantum metrology triangle experiment

The planned revision of the SI unit system [13] aims to redefine four of the seven base units with the aid of physical constants. For instance, kilogram is today still referenced to a manufactured prototype located in France that is known to drift in time. Moreover, some of the present base units depend on each other. The fundamental unit of electricity, ampere, is based on kilogram, making it vulnerable to any uncertainty related to the definition of kg. Furthermore, realizing ampere in practice is cumbersome and an accuracy better than a few parts in $10^7$ has not been achieved.

Outside the current SI unit system, quantum standards based on Josephson and quantum Hall effects have long been used to realize volt and ohm, respectively. When a Josephson junction is irradiated with a microwave signal (frequency $f_J$), voltage steps $V_J$ appear in current-voltage characteristics of the junction quantized as

$$V_J = n_J \frac{\hbar}{2e} f_J,$$  \hspace{1cm} (2.25)

where $n_J$ is an integer. The device is known as Josephson voltage standard (JVS). The quantum Hall effect takes place when 2D electron gas is subjected to a perpendicular high magnetic field at low temperatures. As a result, the transverse resistance $R_H$ in the film, the quantum Hall resistance (QHR), can take values of the form

$$R_H = \frac{\hbar}{n_H e^2},$$  \hspace{1cm} (2.26)

where $n_H$ is another integer. In a proposed new SI system [13], ampere is defined through a transport of elementary charges

$$I_p = n_p e f_p.$$  \hspace{1cm} (2.27)

Here, an integer number $n_p$ of electrons is driven by a current pump with a frequency $f_p$. Current pumps rely on the control of single electrons, and numerous devices have been proposed and realized in practice [120]. Thus, volt, ohm and ampere given by the above quantum standards are defined with fundamental constants of nature, i.e. elementary charge $e$ and Planck constant $\hbar$. However, further evidence is required to validate the accuracy of relations (2.25)–(2.27) and to bind JVS and QHR to the SI unit system.
One such item of evidence is the quantum metrology triangle (QMT) experiment [11, 12, 121, 122], see Figure 2.8(A). QMT experiments can be divided into two categories. In an indirect experiment, originally formulated in NIST, a single-electron transistor (SET), acting as a current pump, charges a cryogenic capacitor with capacitance traced back to the QHR standard. The voltage generated in the capacitor is then compared to that of JVS. The direct version of the experiment exploits Ohm’s law. For example, the current produced with a charge pump can be compared to the current produced with JVS and the QHR standard, i.e. difference $I_p - V_J / R_H$ is measured. This type of a scheme is under preparation at the Centre for Metrology and Accreditation (MIKES) in collaboration with VTT and Aalto University.

### 2.5.1 MIKES system

A schematic drawing of the MIKES system [123] is shown in Figure 2.8(B). The experiment is installed in a dry dilution refrigerator (Bluefors Cryogenics). The current is planned to be produced with a SINIS turnstile [124] operated at about 20 mK. It is a hybrid single-electron transistor having two superconductor-insulator-normal metal (SIN) junctions in series. When under a bias voltage, tunnelling of the electrons through the device is controlled with a single gate voltage applied to an electrode placed next to the normal island. For QMT, an output current of 100 pA is needed, which has already been demonstrated [125] by operating several pumps in parallel. The challenges are related to generating this current with low-enough uncertainty and pumps based on silicon quantum dots have therefore also been investigated [126].

The voltage supplied with a JVS, containing one Josephson junction, is fed through a stable cryoresistor $R_c$ [127, 128] of 1 MΩ. In the final experiment, the cryoresistor will be calibrated against the QHR standard.
Thus, Josephson voltage $V_J \sim 100 \, \mu V$ generates an opposing current of appropriate magnitude. When integers $n_J, n_H$ and $n_p$ are fixed, the currents are balanced by tuning the frequencies $f_J$ and $f_p$ and a current close to zero flows through the ammeter, commonly known as a null detector.

### 2.5.2 Null detection

In null detection, the gain of the detector does not need to be known exactly. Due to the $1/f$ processes and drifts, the null signal exhibits correlations over long averaging times, increasing the uncertainty of the measurement. The polarities of the JVS and current pump are therefore reversed in phase with a frequency $f_r = 1/\tau_r$. Consequently, one measurement is defined as

$$\Delta I_i = \frac{1}{2} (I_{2i-1} - I_{2i}),$$

where $I_i (i = 1\ldots m)$ is the current integrated over one bias reversal time $\tau_r$ and the total measurement time is given by $t = 2m\tau_r$.

The ultimate figure of merit for the measurement system is the equivalent current noise power referred to the input of the null detector. In an ideal case, this is limited by the thermal noise of the cryoresistor $S_{It} = 4k_B T/R_c$. If above this limit, the noise of the null detector must also be taken into account. In the presence of white noise, the relative uncertainty for the bias reversal measurement becomes

$$\frac{\delta I}{I_p} = \sqrt{\frac{i_n^2 + (u_n/R_c)^2 + 4k_B T/R_c}{2I_p^2 t}},$$

where $i_n$ and $u_n$ are equivalent current and voltage noise generators of the null detector.

The result of the QMT measurement is expressed as $\Delta_{QMT} \pm \delta_{QMT}$, where $\Delta_{QMT}$ is the relative deviation from the null and $\delta_{QMT}$ is the corresponding uncertainty. If $\Delta_{QMT} > \delta_{QMT}$, a correction term for at least one of the identities (2.25)–(2.27) is expected. For $\Delta_{QMT} < \delta_{QMT}$, the triangle is said to be closed at a confidence level given by $\delta_{QMT}$. At present, a QMT result with an uncertainty of $10^{-6}$ is significant. Eventually a few parts in $10^7$ will be required.

The expected low bias reversal frequency combined with a high source impedance determines the optimal noise properties of the null detector. For SQUID-based detectors, the current resolution is enhanced with a flux transformer having high input inductance as described before, in which case the voltage noise $u_n$ can be ignored. Recent advances in SQUID technology [108] and flux transformers (Publication IV) have made current
sensors with a sensitivity close (Publication V) and even below 10 fA/Hz$^{1/2}$ possible [129].

An important variant of SQUID current detectors is the cryogenic current comparator (CCC) [130]. CCC comprises a superconducting tube with two wires fed through and wound around in an axial direction. The currents in the wires create a shielding current inside the tube to maintain zero magnetic flux density. The returning current outside, being a measure for the two applied currents, generates magnetic field that is measured with a SQUID magnetometer. Advantages of CCC include exact calibration of current ratio [131] and high sensitivity achieved by winding thousands of turns of wire around the tube. CCCs are therefore often impractical for purposes other than metrology. Corner frequency $f_c$ lies well below 1 Hz [132] and thermal noise as low as 0.5 fA/Hz$^{1/2}$ may be achieved [133].

Another candidate for null detection is the Bloch oscillating transistor (BOT) [134, 135]. This is a mesoscopic device, in which the supercurrent of a Josephson junction is controlled with a quasiparticle tunnelling, occurring through a normal tunnel junction from the base electrode to the superconducting island. BOT has a reasonable gain and a current noise approaching 1 fA/Hz$^{1/2}$ with low $f_c$, making it an ideal choice for QMT. However, due to the problems in fabrication and operational reliability, a demonstration of actual null detection is yet to be shown. Finally, the progress made recently with cryogenic HEMTs [136] can open up new possibilities for semiconducting amplifiers to be used at low frequencies and intermediate impedance levels.
3. Overview of Publications

A brief summary of each publication is given in the following subsections.

3.1 Publication I: "All-planar SQUIDs and pickup coils for combined MEG and MRI"

In Publication I, thin-film SQUID sensors and detection geometry of a commercial MEG system (Elekta Oy) were taken as a starting point in developing magnetic field sensors for a combined MEG and MRI system. These types of sensors become handy when large quantities are manufactured and placed into a sensor helmet. This deviates from the other reported ULF MRI systems that rely on large axial second-order gradiometers with SQUIDs enclosed inside a superconducting shield [109, 110, 112, 113].

![Figure 3.1](image)

**Figure 3.1.** (A) The module in perpendicular magnetic field of 50 mT. The niobium shields protect the SQUID (without significantly altering the signal to be measured) which otherwise would not stand operable after the magnetic pulse (B), as discovered with different SQUID models (A–F).

However, the selected approach led to challenges in MRI sensor behaviour. Several SQUID designs were fabricated and tested in magnetic field pulsing experiments, after which a SQUID design having an intermediate flux transformer was connected to a pickup coil to form a magnetic field sensor. All SQUIDs suffered from flux trapping at threshold...
fields ranging from hundreds of microtesla to above 10 mT. Even SQUIDs designed using narrow linewidths, although performing better, failed to operate after such pulses, implying that no spontaneous flux expulsion occurred in discrepancy with (2.9). A theory for the flux penetration inside the washer was introduced, taking into account the geometrical barrier at the edge of thin film [137]. Moreover, magnetic field sensors were found to be magnetized, creating extra magnetic fields in the sample volume. After the removal of prepolarization field (8 mT), transient-like noise, decaying with a time constant of the order 1 s, was observed in the sensors.

To improve the field tolerance, sensors were packaged in modules with Nb shields on both sides and flux dams inserted in series with the input coil (Figure 3.1). As a result, a spontaneous response recovery up to 50 mT was achieved with the sensors. The module contains a magnetometer and two planar gradiometers.

Sensor magnetization was found to originate from thin shields and wide thin-film pickup coils. The problem was solved by increasing the aspect ratio of these superconducting components.

The origin of the transient noise was traced to the pickup coil. Different pickup materials and geometries were studied, and an explanation was given according to which the vortices rearrange themselves after the switch-off, generating $1/f^2$-type noise. The effect was smallest for pickups fabricated from Pb wire and 6-μm wide thin-film Nb, allowing sensors to be used for both MEG and ULF MRI.

3.2 Publication II: "Hybrid ultra-low-field MRI and magnetoencephalography system based on a commercial whole-head neuromagnetometer"

Publication II describes a hybrid MEG-MRI device built around a commercial MEG system from Elekta. The instrument, located inside a magnetically shielded room, comprises a dewar with a sensor array placed inside and a set of coils for generating fields required by MRI, see Figure 3.2. In total 16 sensor modules (Publication I) were fabricated and inserted near the visual cortex. A superconducting polarizing coil was wound around the insert holding the SQUIDs. The coil was designed to have minimum stray field outside the imaging volume, reducing eddy currents in the conductive layers of the MSR during the field ramping [138]. In addition, coils for producing the measurement field $B_0$, gradients, and
Figure 3.2. (A) The MEG-MRI system at Aalto University. Wooden frames support three orthogonal gradient coils, $B_0$ coil and excitation coil. (B) The dewar insert holds the superconducting coil and sensor modules equipped with Nb shields. (C) ULF MRI slices were acquired during a 92-minute measurement session, in which a 3D spin-echo sequence with $B_p = 22 \text{ mT}$ in the imaging volume was employed, assuring sensor operation at elevated field levels. The image resolution is $4 \times 4 \times 6 \text{ mm}^3$.

the excitation field, were constructed on the sides of the dewar.

Both MEG and MRI functionalities of the device were verified in separate measurements. Figure 3.2(C) shows coronal slices of the human brain acquired with the instrument. Indeed, comparison to images obtained with a high-field scanner confirm that several anatomical features can be distinguished from low-field images. MEG recordings carried out with a standard MEG device and Aalto MEG-MRI prototype after a visual stimulus revealed no significant difference in MEG performance between the two devices.

3.3 Publication III: "Kinetic inductance magnetometer"

Publication III presents a new type of magnetometer utilizing the nonlinearity of kinetic inductance with respect to the dc current, see Equation (2.23). Figure 3.3(A) shows the design of the device. Magnetic flux applied through a superconducting ring induces a current, which in turn increases the inductance of the loop through the described nonlinearity. The inductance change is read out by installing the loop as a part of a resonator and measuring the transmission through the component, see Figure 3.3(B) and (C).

The device was shown to have numerous advantages over competing technologies. The magnetometer is fabricated from a single thin-film layer of NbN to compare with a typical 10-layer process used for fabricating SQUIDs in Publication I. The detection scheme enables the use of one
amplifier in the readout of multiple sensors. Furthermore, the absence of Josephson junctions is expected to improve the tolerance against ambient magnetic fields. Measurements indicated that the intrinsic dynamic range of kinetic inductance magnetometer is several hundreds of nanotesla for a given bias field. Theoretical noise level was calculated to be 2 fT/Hz$^{1/2}$ while measurements yielded field sensitivities of 32 fT/Hz$^{1/2}$ in the white part of the spectrum and 150 fT/Hz$^{1/2}$ at 10 Hz, figures that were found to be dominated by the electronics.

3.4 Publication IV: "A large winding-ratio planar transformer with an optimized geometry for SQUID ammeter"

A new type of planar flux transformer aimed for QMT is introduced in Publication IV. The transformer is arranged as a gradiometer and has a winding ratio of 1284:2 (Figure 3.4(B)), with the secondary matched to an input inductance of the order 100 nH. The primary ($w = 3 \mu m$) treads in two layers with adjacent loops designed to overlap each other. The secondary ($w = 50 \mu m$) is wound around the primary. This sort of config-
Overview of Publications

Figure 3.4. (A) An equivalent circuit of the current detector and (B) the transformer design.

uration reduces the parasitic capacitance and emergence of transmission line resonances in the flux transformer, enabling a large number of turns without degrading the coupling too much. Experiments yielded $L_{IN} = 39 \text{ mH}$ and $k = 0.39$ for the transformer. Furthermore, when connected to a SQUID, a current resolution of $25 \text{ fA Hz}^{-1/2}$ was reached with the device in the white part of the spectrum.

3.5 Publication V: "Characterization of SQUID-based null detector for a Quantum Metrology Triangle experiment"

Figure 3.5. Absolute value of averaged current $|\langle \Delta I \rangle|$ and estimates for the measurement uncertainty $\delta I$ obtained from the 13-h measurement as a function of bias reversal frequency $f_r$. The expectation value of $\Delta I$ is zero with an uncertainty given by the estimates.

In this last publication, the current detector demonstrated in Publication IV was applied in null detection. A number of devices were first characterized at liquid helium temperature 4.2 K, and an improved current noise down to $14 \text{ fA Hz}^{-1/2}$ was measured with the best device. One de-
vice was then installed in a MIKES cryostat. Due to the magnetic interference introduced by the cryostat, a magnetic shield was designed for the detector operating at 0.9 K.

Null detection was simulated by leaving the transformer input open and sampling the SQUID output. The zero signal at the input mimics the realistic situation of the fully balanced QMT experiment. For a 13-hour measurement and assuming bias reversal frequency $f_r = 15$ Hz, an uncertainty of $3 \times 10^{-16}$ A was measured, corresponding to $\delta I/I_p = 3$ ppm with $I_p = 100$ pA. A general formula was presented for the measurement uncertainty applicable for different noise types.
4. Discussion

4.1 SQUID-detected ULF MRI

The most substantial improvement needed for ULF MRI is the image quality, which can be accomplished either by increasing $B_p$, or by lowering the system noise. The limiting factor for enhancing $B_p$ above 22 mT found in Publication II was the trapped flux in the superconducting coil causing extra magnetic gradients and, subsequently, distortions in the image. References [88, 89] list additional shimming coils, the modification of the reconstruction algorithm or the degaussing of the polarizing coil after the ramp-down as possible means of avoiding the problem. So far, the degaussing has enabled increasing $B_p$ by some tens of per cent [139]. In case the proposed methods are found inadequate, a new resistive coil requiring active cooling must be designed and $B_p$ up to 100 mT can be anticipated.

In Publication II, system noise of the order 5 fT Hz$^{-1/2}$, a sum of several noise sources, was measured with magnetometers. The impact of environmental noise and current fluctuations in MRI coils were minimized through the use of MSR and batteries as a current supply, respectively. The main contributions come from the sensors and thermal noise originating from the conductive radiation shield [140]. The impact of the latter can be decreased by redesigning the cryostat [141]. The intrinsic sensor noise can also be lowered down to 1 fT Hz$^{-1/2}$ or below, which is currently the level achieved with the best sensors. Thus, with these improvements alone, the voxel SNR $\alpha B_p/\sqrt{S_B t}$ affecting the image quality increases with a factor of 25, or the measurement time $t$ is decreased by $25^2$ for the given image quality. The signal levels both in ULF MRI and MEG are more or less comparable. Thus, the nonidealities, for example
magnetic transients in the MSR, determine the necessary dynamic range.

Transient-like sensor noise after the polarizing field was also discovered in Publication I. Although the problem was solved with the aid of new pickup coils, later experiments have revealed increasing noise with increasing $B_p$, even with 3-μm-wide thin-film Nb and wire-wound Pb pick-ups. It remains somewhat unclear what causes this. Pb is a type I superconductor and should not trap flux, as demonstrated in a recent study [142]. Furthermore, calculating cross-correlation spectra between two neighbouring Pb channels over multiple cycles gives no correlation in noise, implying that the source is located close to the sensor units. In fact, a similar phenomenon with $S_{B}^{1/2} \propto 1/f$ has also been encountered at PTB using wire-wound pickups fabricated from Nb and NbN [113]. However, the measured lower critical field $B_{C1}$ of Nb wire was found to be significantly higher than the polarizing field at which the transient noise emerged. Furthermore, different $B_{C1}$ values have been measured for Nb wires having different size and origin [142].

Another problem identified in the sensors was caused by Nb plates, when placed perpendicular to $B_0$, creating inhomogeneities in the measurement field. The problem was circumvented by aligning the modules in parallel with $B_0$. However, future improvement in the field-of-view and SNR will require the use of the whole sensor array available in the helmet. This sets constraints for the shields. Possible solutions could be the reduction of the shield size, aligning all the shields in parallel with $B_0$ or removing the shields completely. In the second approach, a module with the SQUID and pickup coil adjusted in different planes would need to be constructed. In the last case, the field tolerance of SQUIDs would need to be improved further. In Publication I, a threshold value of $\sim 10$ mT was measured with the best design. This can be enhanced above 50 mT by reducing the junction size and, hence, the flux coupled to the junction [108].

Heating after a polarizing pulse is also an option. However, an extensive heating pulse targeted at a large area, combined with a short cooling time, makes the practical realization of the heating sequence challenging. Until now, an excess-noise-free sensor operation without Nb shields after a 10-mT polarizing pulse has been verified with a heating sequence. Indeed, including the pickup coil and the SQUID in the same module has advantages, such as the compact design, improved field coupling and the ease of manufacturing large quantities. At the same time, it has also
Discussion

led to challenges in performance, as described above, which will probably become more severe as the strength of the polarizing field is raised. If unsuccessful in solving them, one can always place the SQUID away from the imaging volume, which has been the strategy in other existing ULF MRI systems [109, 110, 112, 113].

4.2 Kinetic inductance magnetometer

Many of the problems encountered in SQUID-detected ULF MRI are potentially solved with the kinetic inductance magnetometer introduced in Publication III. The absence of junctions is expected to lead to a larger field tolerance, eliminating the need for shielding plates. Subsequently, recovery from the polarizing pulse, and the operation in the measurement field, would be improved. The large dynamic range is also an advantage, since the removal of the polarizing field may cause magnetic transients to appear in the imaging volume as a result of, for example the eddy currents in conductive layers of MSR. Assuming $I^*$ scales with respect to the critical current $I_{c,su}$, the large screening current needed for biasing the device, and potentially interfering with the rotating spins, could be made smaller by decreasing the width and height of the thin-film coil.

At the same time, further work is required to thoroughly determine the intrinsic noise properties of the device. The alternatives are either to find ways of increasing the device gain and/or minimizing the noise due to the electronics. Several electronics-related fluctuation mechanisms affecting the noise level were identified. One such mechanism is the white background noise originating from the readout amplifier and excitation chain. In addition, the low source impedance at resonance seen by the room-temperature preamplifier elevates the white noise of the system. The excess noise at low frequencies was found mainly to be a result of mixing. Cryogenic amplifiers with low $T_n$ and alternative mixing schemes can be employed to bring down the noise at high and low frequencies, respectively.

Thermal noise from unpaired electrons and the feeding line was identified as the dominating noise mechanism. Evaluating Equation (6) of Publication III yielded $S_{B,th}^{1/2} \sim 2 \text{ fT/Hz}^{1/2}$ at liquid helium temperature 4.2 K.
Discussion

performance gives

\[ S_{B,\text{th}}^{1/2} \propto \sqrt{\frac{TL_{\text{tot}}}{Q_i I_s A}} \left( 1 + 3 \left( \frac{I_s}{I_{k0}} \right)^2 + \frac{L_{k0}}{L_{k0}} \right), \]  

(4.1)

where \( L_{\text{tot}} \) and \( Q_i \) are the inductance and the intrinsic quality factor of the resonator, respectively. Thus, \( S_{B,\text{th}}^{1/2} \) can be improved by lowering their ratio, for example by material selection. Since both the geometric and kinetic inductances scale approximately as \( \sqrt{A} \), the same outcome is achieved by increasing the pickup area. Furthermore, the kinetic inductance should dominate over its geometric counterpart. To achieve this, kinetic inductance devices are commonly fabricated from impure materials, where the mean free path approaches zero and \( \lambda \) is substantially increased compared to its clean-limit value.

The measured dynamic range (600 nT) with respect to the theoretical field noise (2 fT/√Hz) amounts to 170 dB, presuming a 1-Hz bandwidth. The electronic system, designed to read and digitize the signal, must be able to handle such a large figure. In fact, the bottleneck is often the analog-to-digital converter (ADC). The true dynamic range of the electronic system is determined through the bandwidth, and it is advantageous to use a high sampling rate. Using the bandwidth of 100 kHz attainable with the kinetic inductance magnetometer, the dynamic range reduces to 120 dB, corresponding to a 20-bit ADC.

### 4.3 Null detector

In Publication V, the applicability of the SQUID-based null detector was assessed in a dry dilution refrigerator environment. The magnitude and type of the noise sources together determine the averaging time \( t \) needed for reaching a given uncertainty. For \( 1/f^2 \)- and \( 1/f \)-noise processes, increasing the bias reversal frequency \( f_r \) lowers the measurement uncertainty while, for white noise, the uncertainty becomes independent of \( f_r \). The low-frequency noise was found to emerge below 40 Hz, highlighting the importance of selecting optimal \( f_r \). For instance, to achieve a measurement uncertainty of \( 10^{-16} \) A, averaging times \( t = 14 \) h and \( t = 123 \) d can be estimated assuming \( f_r = 100 \) Hz and \( f_r = 1 \) Hz, respectively, and the measured current spectral density \( S_I \). With a pumped current of 100 pA, this would lead to a relative uncertainty of 1 ppm. The ultimate goal, 0.1 ppm, would require 100 times longer averaging. At the moment, it remains somewhat unclear how fast the polarity of the current pump and
JVS can be reversed.

The simulation of the bias reversal measurement yielded a larger uncertainty than expected, both with low and high values of $f_r$. At high frequencies, the deviation is probably explained with the interference peaks in the spectrum. The reason for the difference at low frequencies, although smaller, is unknown. It seems that, for some reason, the current noise spectrum was not completely stationary over the long-term measurement.

The high current resolution of the device was achieved specifically due to the new flux transformer. The design enables the use of a large number of turns, resulting in high input inductance and a minimal increase in device flux noise. Increasing $n$ and optimizing the secondary inductance with only one turn should yield an even higher mutual inductance. However, it is the SQUID itself that can provide substantial improvement to device performance. Indeed, current noise comparable to that of CCC has been reported with thin-film flux transformers using cross-type Josephson junctions [129]. Later experiments have also pointed out that the system connected to the input must be optimized since objects such as vibrating wires can induce additional noise to the detector [143].
5. Conclusion

The constant rise in the impact of neuroscience [144] is reinforcing the demand for new imaging methods and instrumentation. In this thesis, a significant amount of effort was put into developing hardware and, in particular, sensors for a hybrid MEG-MRI scanner. As a result, structural images of the human brain were acquired with a system constructed around a commercial MEG device. Altogether, the measurement of MEG and ULF MRI signals in a single device appears to produce information that is clinically relevant. However, several challenges must be overcome before this can become commercially feasible. Besides the image quality, the long imaging time remains a problem. A way of reducing the latter would be an interleaved measurement, where ULF MRI and MEG signals are recorded simultaneously after the prepolarization. The possible commercialization of MEG-MRI would be supported by the planar geometry of the sensors, with components fabricated from thin-film materials. Furthermore, many aspects of MRI at low fields remain unexplored and the focus should be on finding the 'killer application(s)', unreachable with conventional MRI.

The kinetic inductance magnetometer may become a worthy competitor for SQUIDs in the future. The many advantages, specifically the simple design and readout, make it well suited for biomagnetic measurements, where large sensor arrays are used. On the downside, the current lack of low-noise readout may present some delays before the device can be operated in such systems. Also, the use of external coils, if not circumvented, in biasing each device to its optimum operating point may become impractical and introduce additional sources of noise.

Before closing 'the Finnish QMT' with a low uncertainty can become a reality, further work is required in several fields. One fundamental issue remaining to be solved is the generation of the quantum current with
both high output value (~ 100 pA) and low uncertainty (< 1 ppm). The other two legs of the triangle appear to be less problematic, although only the integration of all three legs together at a system level will yield the final answer. The noise performance of the current detector studied in this thesis also needs further improvement if it is to be used in the final QMT experiment. In the end, it is the system noise at low frequencies and the (preferably high) bias reversal frequency that will fully determine the applicability of the null detector.


[64] INA121 (FET-Input Low Power Instrumentation Amplifier), Texas Instruments.


Superconducting sensors have gained wide acceptance in various fields where high sensitivity is required, such as security, space technology, medical imaging and metrology. This thesis summarizes the development and use of superconducting sensors designated for two specific applications, falling into the two last-mentioned categories. The first application, a combination of ultra-low-field magnetic resonance imaging and magnetoencephalography, allows structural and functional imaging of the human brain with a single device. The second application is null detection related to a quantum metrology triangle experiment that aims at verifying the consistency of the three quantum standards of current, voltage and resistance. The operation and physics of the sensors relevant to the applications are studied, and the applications utilizing the sensors are constructed and tested.