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tion.
CARTOGRAPHIC PORTRAYAL OF TERRAIN
IN OBLIQUE PARALLEL PROJECTION

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Abstract
The visualization of three-dimensional spatial information as oblique views is widely regarded as beneficial for the perceiving of three-dimensional space. From the cartographic perspective, oblique views need to be carefully designed, in order to help the map reader to form a useful mental model of the terrain. In this study, 3D cartographic methods were applied to illustrate directions, distances and heights in oblique views. An oblique parallel projection was used for representing the view in a metrically homogeneous manner and for emphasizing the relative heights. The height was further visualized using directed lighting, hypsographic coloring and contours. In addition, an equilateral and equiangular grid was draped on the terrain model, for fixing the cardinal directions and for facilitating the estimation of distances and surface areas. These methods were combined and analyzed in a case study on a high-resolution DEM covering a national park area. The resulting oblique parallel views provide visual cues for the estimation of terrain dimensions, particularly heights. The results prepare a basis for further research on cartographic 3D visualization.

Keywords: 3D cartography, oblique parallel projection, map grid, DEM

1 INTRODUCTION

Outdoor leisure activities, such as forest hiking, have increasingly gained in popularity. These activities often benefit from navigational aids, such as route markings and maps. Recent technological developments have led to the successful implementation of personal navigation applications on mobile devices, which primarily focus on city life. On the contrary, outdoor leisure activities lack useful personal navigational solutions, although many user groups have a need for easy-to-use mobile guidance. For such solutions, oblique landscape views can offer an illustrative and attractive base.

1.1 Background
The research presented in this paper is a part of two projects: “Multi-publishing in supporting outdoor leisure activities” (MenoMaps) -project and the “Ubiquitous Spatial Communication” (UbiMap) -project. The focus of the MenoMaps project is on mobile map services that provide rich contents of information, and on the efficient utilization of
high-resolution spatial information. New contents, such as digital landscapes, have a potential for the development of a richer user-experience regarding outdoor leisure activities (Sarjakoski et al., 2008). The UbiMap project is a multi-disciplinary research project the goal of which is to develop theoretical understanding and research methodology related to ubiquitous spatial communication, and to explore new technologies and ways of presenting and interacting with geospatial information. A research prototype service for novel interactive ubiquitous maps linked to an outdoor leisure activity will be set up in the project The Finnish Geodetic Institute’s (FGI) test environment for ubiquitous geospatial services, covering the Nuuksio national park and adjacent areas in southern Finland, is utilized for both of the projects (Sarjakoski et al., 2007).

1.2 Previous research

The presentation of three-dimensional (3D) cartographic views on two-dimensional (2D) screens has emerged lately as an intense subject of geovisualization research. Wide interest has fallen on the domain of modern 3D cartography, and a thorough discussion is taking place in regards to the nature and aims of the 3D cartography (Dykes et al., 2005; Döllner, 2007; Jobst and Germanchis, 2007; Häberling et al., 2008). The key element of the discussion is the level of abstraction, which is characteristic to 3D cartography in contrast to other forms of 3D visualization, particularly photorealism and virtual reality environments.

Meng (2001) categorized the 3D visualization of geospatial data into the category of map-related presentations and differentiated them from traditional 2D maps. She stated that map readers are willing to view further information beyond the photorealistic appearance, i.e., the abstraction of the spatial objects. Together with the advice of minimizing the mental load of the map reader in regards to the user interface, these findings revealed the importance of 3D cartography and its basic concepts for 3D visualization.

Regarding the definitions of the map by many cartographic authors (e.g. ICA, 2003), 3D oblique views meet the specifications of a map precisely. Thus, it is possible to refer to convenient 3D geovisualizations as real 3D maps, as did Häberling et al. (2008), by developing cartographic design principles for representing all three dimensions. Döllner (2007) named his 3D maps as expressive virtual city models and confronted them with photorealistic geovisualizations. He stated that the latter was in many ways inadequate for cartographic use, due to e.g. the lack of visual abstraction.

Jobst and Germanchis (2007) demanded that 3D cartography should be developed as an independent domain, since state-of-the-art 3D environments are mostly targeted at the virtual reality and dismiss the cartographic principles of selection, generalization and symbolization. Cartwright's (2007) user experiment on the web-delivered urban 3D-world showed that not all image content needs to be aimed at an extreme detail level and that such detailed scenes can even make a virtual 3D view confusing. Kettunen (2008) divided 3D oblique views into three categories: 1) photorealistic, 2) expressive
and 3) sketch renderings. He qualified the expressive rendering as the most convenient visualization method for cartographic oblique views.

According to the authors presented above, the cartographic methods must be applied to 3D spatial models for creating 3D maps that could improve the spatial perception of the map readers. Numerous experiments have been conducted in order to tackle this task, including both terrain and building models (Angsusser and Kumke, 2001; Bleisch and Nebiker, 2007). Jenny and Patterson (2007) focused on an oblique parallel projection that they called a plan oblique projection and on which they created prototypes of 3D maps.

In this paper, composition of large-scale oblique views is studied in order to portray terrain dimensions. The oblique parallel projection is used in a detailed 3D oblique view. Information on distances and directions is embedded into the view by means of directed lighting, hypsographic coloring and contours, as well as an equilateral and equiangular grid. The creation of the view is incrementally conducted, by adding new content on top of the previous views. The cartographic value of the views is then discussed and eventually, conclusions are made.

2 METHODS

A map must communicate a tangible image of the reality to a user. The map reader should not only be able to associate the map with the surrounding terrain whilst navigating, but also be able to form a useful mental image of the terrain as a whole. For these purposes, it is necessary to comprehend the scale and the rotation of the map in relation to known spatial references, typically a unit of length and the cardinal directions. In oblique projections, the changes in scale and direction can be difficult for the map reader to perceive throughout the scene. Visual cues are required in order to interpret the distances and directions in the 3D oblique views. In this study, such cues consisted of oblique parallel projection, directed lighting, hypsographic coloring, contours and an equilateral and equiangular grid.

2.1 Oblique parallel projection

The parallel projection is a geometric mapping from three to two dimensions, using parallel projection rays. The projected image is created by intersecting the projection rays using a projection surface, which is usually a projection plane. The main character of the planar parallel projection is its property of mapping parallel lines in 3D to lines parallel in 2D (Graf and Nyström, 1958). Consequently, the scale of a projected image remains unchanged on the planes, which are parallel with the projection plane.

In the oblique parallel projection, the angle between the projection plane and the projection rays is less than 90°. The visibility of the planes perpendicular to the projection plane increases according to the decrease of the vertical viewing angle. The oblique parallel projection preserves the scales of the planes that are parallel and perpendicular to the projection plane throughout the image (Hearn and Baker, 1997). This feature makes it appealing for cartographic use: the projection plane can be placed...
parallel to the ground and thereby fix both of the horizontal and vertical scales to be homogeneous in the whole map area. The resulting scenes imitate the orthographic projection of conventional maps and simultaneously visualize the vertical dimension of the terrain in a relief-like manner (Kettunen, 2008).

The horizontal viewing angle of the oblique parallel projection which sides of the vertical objects are visible. Jenny and Patterson (2007) proposed that this angle should follow the viewing direction of the map for the most natural impression. For the vertical viewing angle, they, as many others (e.g. Bleisch and Nebiker, 2007), have suggested an angle of 45° that keeps the horizontal and vertical distances comparable with each other. However, this angle should be optimized based on the relative heights in the scene.

The geometric mapping of the oblique parallel projection from the terrain co-ordinates can be formulated as

\[
x = k(X - X_0) - \frac{Z}{\cot \alpha}
\]

\[
y = k(Y - Y_0 + (Z - Z_0)\cot \alpha)
\]

where \(X,Y,Z\) are the terrain co-ordinates in an orthographic coordinate system, \(X_0,Y_0\) the origin of the area to be imaged, \(Z_0\) a chosen reference elevation, \(\alpha\) the vertical viewing angle, \(k\) the scale factor and \(x,y\) the resulting coordinates on the projection plane. The horizontal viewing direction is aligned with the \(Y\)-axis.

### 2.2 Directed lighting

The lighting of a 3D scene is an essential depth cue to a map reader and should be carefully designed (Maass, 2007; Döllner, 2007). In an oblique parallel projection, directed lighting from one source should illuminate the visible sides of vertical objects. Thus, the horizontal lighting angle should be close to the horizontal viewing angle. Light from the south-west is primarily proposed and used for 3D oblique views, but in certain situations, the south-east may also be suitable (Jenny and Patterson, 2007). For the vertical lighting angle, there are no such clear recommendations. However, it can be modified to influence the shade-originated visibility of the terrain details and also the visual brightness of the rendered picture as a whole.

### 2.3 Hypsographic coloring and contours

The hypsographic coloring is a color-coding method used to visualize the height levels of the terrain. It can provide a valuable depth cue to the 3D oblique views and prevent the relief inversion fallacy. A conventional green-to-brown coloring (Imhof, 1965), with equal intervals, portrays the altitude relations in the scene clearly. Hypsographic coloring can be complemented with contours that have smaller height intervals. This way, the contours provide a more precise height level depiction, where the hypsographic coloring provides a scene-wide illustration of height distribution.

### 2.4 Equilateral and equiangular grid

Square grids are commonly used to bring metric rigor into conventional maps. In
addition to showing the cardinal directions and co-ordinates on the map, the grid supports the visual approximation of distances and surface areas.

Besides the square grids, other equilateral grids also have the potential to form a useful metric frame on a map. Equilateral means that all of the line segments in a grid are of the same length. The appropriate regular arrangement of the equidistant segments enables the estimation of path distances. Furthermore, the structure of the grid organizes the perception of the map image.

In an equiangular grid all angles between the grid line segments are equal. In this study, we want grids to be simultaneously equilateral and equiangular. This leads to three possible grids: square, triangular and hexagonal grids. It should be noted that triangular and hexagonal grids may be oriented in two legible ways: a set of parallel lines aligned vertically or horizontally.

The use of a grid is informative in 3D oblique views, because the grid becomes deformed due to the projection when it is draped on a 3D terrain model and viewed at an oblique angle. Still, the grid serves as a similar georeference frame as on conventional 2D maps.

3 RENDERING EXPERIMENTS

This study combined an oblique parallel projection, directed lighting, hypsographic coloring, contours and an equilateral and equiangular grid into 3D oblique views. These methods were evaluated visually one by one and the suitable parameters for them were sought for.

The experimental renderings were based on a DEM, derived from the data sets of the FGI's test environment covering the Nuuksio National Park area (Sarjakoski et al., 2007). The 1 m resolution with a height accuracy of about 20 cm provided a detailed outlook of the terrain surface. The 3D rendering was made by Natural Scene Designer Pro 5.0, due to its capability for using the oblique parallel projection (Jenny and Patterson, 2007).

3.1 Configuration of the oblique parallel projection

At first, the DEM was rendered with 2 m contours in the default lighting of the software, in order to find the appropriate viewing configuration for the test region. Only vertical viewing angle was experimented with, since the software fixed the horizontal viewing angle to 180° (towards north). A number of vertical angles were tested to study the perception of the hills in the scene. It was found that the occlusion of back-facing slopes was an important factor for a satisfactory perception of elevations. The vertical angle of 45° adopted from the literature was too high for a sufficient amount of occlusion in this scene, and therefore lower values were tested. Finally, the vertical viewing angle of 23° was found to be appropriate (Figure 1).
3.2 Set-up for directed lighting

Next, the appropriate lighting angles in both the horizontal and vertical directions were experimented with. In the first test the default values (south-west for the horizontal and 45° for the vertical lighting angle) of the software were used, as proposed by Jenny and Patterson (2007) and by Bleisch and Nebiker (2007) (Figure 1). Due to the orientation of the hills in the scene, the horizontal lighting angle was also set to the south-east. For the vertical angle, significantly lower values were tested in search of the optimal depiction of terrain detail (Figure 2).

The south-east horizontal lighting angle appeared to be the most suitable option, as the hills of the region are oriented in a south-west–north-east direction. On one hand, the illumination perpendicular to the orientation of the hills lightens their prolonged slopes, and on the other hand, it creates enough shading on the opposite foreshortened slopes to clearly distinguish the hills.

The vertical lighting angle of 25° was found to be the most appropriate in depicting the DEM details. Decreasing the value darkened the rendered image notably and the terrain details on the slopes opposite to the light were hidden. On the other hand, higher values hid the details on the slopes that were facing the light.

3.3 Addition of hypsographic coloring and contours

Hypsographic coloring was applied to the scene, in order to prevent the relief inversion fallacy (Jenny and Patterson, 2007) and to provide information on the scene-wide height distribution (Figure 3a). In order to provide a coloring easy to perceive, a height interval of 10 meters was used. This resulted in five elevation classes in the test area. The

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Figure 1. Comparison of vertical viewing angles a) 45° b) 23° c) 15°
classes were represented with a color scale from dark green to dark brown. In order to create a more precise height level depiction, 2 m contours were added into the rendering (Figure 3b).

### 3.4 Draping an equilateral and equiangular grid

To compare the usefulness of the grids, the three equilateral and equiangular grids were draped on the DEM (Figure 4). The orientations of the grids were set in accordance with the cardinal directions in the scene. Grid lines in the east-west direction were found to be particularly useful, since they depict the east-west height profiles and give an important visual reference to the projection. Oppositely, lines in the north-south direction were found to be redundant.
The line segment lengths of the square, triangular and hexagonal grids were chosen to be 33, 50 and 20 m, respectively. These lengths resulted in relatively dense grids with a sufficient amount of segments, but with enough empty space for the actual map image. The square grid was tested in two orientations: aligned with the cardinal directions and rotated by 45°. The triangular grid was set in such a way that one set of parallel lines was oriented east-west, resulting in profile lines of the elevation. The hexagonal grid was arranged so that one set of the constructing lines was oriented north-south, which provides a reference of the cardinal directions.

The triangular grid was found to be the most suitable for presenting the orientation and dimensions of the image. While the continuous lines explicitly show the east-west direction, also the north-south direction is well represented by the “arrowhead” shapes of the triangles. Furthermore, there are no redundant north-south lines. In terms of estimating the distances, the triangular grid provides a means for estimating lengths of e.g. paths; every node is a start point for six line segments to follow.

4 DISCUSSION

The rendering experiment, applied in this study, resulted in novel integrations of rarely combined cartographic methods. The resulting views became rich and clear in illustrating the three-dimensional distances and directions. Such graphically loaded views could not be used with additional contents, but rather, choices should be made between the representation methods.

Figure 4. Equilateral and equiangular grids a) square b) 45˚ c) triangular d) hexagonal
The oblique parallel projection produced illustrative large-scale views of the terrain with a perceivable 3D impression. The occlusion of the back-facing slopes was found to be essential for creating this impression and for making the height depiction of the oblique parallel projection concrete. The occlusion is controlled by the vertical viewing angle that must be set according to the gradient of the slopes in the scene. The occlusion has the negative effect of hiding terrain information behind the hills, but this could be regarded as a minor issue.

The south-east lighting provided optimal views in our case, due to the orientation of the hills in the scene. Jenny and Patterson (2007) criticized this direction for its tendency to induce relief inversion, but here, the hypsographic coloring with contours prevented the fallacy appearing. Only one light source was considered here, even though the visual impression of the terrain forms changes significantly depending on the direction of the light (Imhof, 1965). Improvements to the shading could be achieved by using a light direction that is locally adapted (Jenny, 2001) or multi-band shading (Hobbs, 1999; Oksanen, 2003).

The equilateral and equiangular grids brought metric rigor into oblique parallel views and clarified the geometric properties of the projection. Their use in representing the cardinal and scene directions was illustrative and they also provided a valuable scale reference throughout the view. When draping a grid, one should focus on the line segment lengths and the grid color, to prevent the grid obscuring the underlying terrain image.

The different grids studied were each considered to be potentially usable. The ordinary square grid is the most common and probably the best known, however its north-south lines were redundant in visualizing the terrain forms. Rotation of the grid removed the redundancy, but dismissed the cardinal directions. The hexagonal grid has an illustrative unit shape that could be valuable in steering the eye of the map reader, but its nodes only have three line segments to follow in the case of the line segment path follow-up and it does not contain any continuous lines throughout the scene. The triangular grid was seen to combine the benefits of the square and hexagonal grids: in the used orientation, its horizontal lines form clear height profiles and every node has six possible line segments to follow during distance estimations.

5 CONCLUSIONS

The present study resulted in a novel integration of rarely combined cartographic methods: an oblique parallel projection, a hypsographic coloring with contours and an equilateral and equiangular grid. These were combined into an oblique parallel view of a natural park area, in order to find the means to support the map reader's perception of the terrain dimensions.

In the configuration of the projection, a slight occlusion of the back-facing slopes was found to be essential for the 3D impression. This led to the selection of the vertical viewing angle of 23°. For the optimal illumination of the scene, the directed light was set to originate from the south-east and the vertical lighting angle was set to 25°. The
hypsographic coloring and complementing contours were applied to the scene, which proved to provide easily perceivable elevation representation. The three equilateral and equiangular grids of square, triangular and hexagonal unit shapes were draped on the DEM, and the triangular one was found to be the most suitable, due to its capabilities of depicting the oblique parallel projection as well as being comprehensive in distance estimations.

In the future, the presented portrayal methods are to be combined with other forms of spatial data, to study their usefulness. The use of the oblique parallel projection is to be developed further and the effect of changing the horizontal viewing angle will be studied.

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