Heidi Silvennoinen

Essays on household time allocation decisions in a collective household model
Abstract

This thesis considers the consequences of traditional division of labour in households in a setting where spouses are allowed to have distinct preferences. This approach leads to different results compared to the traditional unitary approach and is better equipped to take into consideration gender related issues of household decision making. The thesis consists of three theoretical essays where the household production theory is applied in the collective household model.

The first essay of the thesis considers joint consumption of the good produced in the household. The novelty of this essay is in simultaneous consideration of public consumption, non-marketable household production and corner solutions in the collective household model. For the model formulated in this essay the effects of exogenous changes in the female household member’s wage are analysed. The change in wage is assumed to affect the decision power in the household according to the bargaining view. Therefore the results obtained differ from those in the traditional framework. The contributions of the essay are threefold. First it is shown how the household consumption bundle adjusts as a result of a change in the decision power in the household. Secondly, it is shown how the time – market good – mix of the good produced in the household responses to the changes in the decision power in the household. Finally, it is shown that the change in the decision power in the household may induce shifts in household optimal time allocation regime.

The second essay of the thesis analyses the effects of social norms on female household member’s time allocation decisions. Both unitary and bargaining household models predict that due to the increase in female wages the female household member’s share of the household work should decline. However, according to time use studies this has not happened. Traditional gender roles seem to be persistent in many Western societies despite the fact that female participation in the labour market has increased considerably. I argue that it is possible that social norms and customs of the society in question have their effect on female time allocation decisions. This essay analyses the effects of traditional gender roles on female household member’s time allocation decisions in the collective household model. It is shown that the response of the female household member’s time allocation with respect to strengthening norm for tradition depends on the household members’ attitudes towards the social norm of the society and on the distribution of decision power in the household. The essay analyses as well the policy implications in the presence of norm for tradition. It is shown that family policy can, depending on the policy measure, either reinforce or mitigate the effect of tradition on female labour supply.

The third essay of this thesis extends the collective household production model into dynamic framework where time allocation decisions made in previous periods affect the decision making and allocation of resources in subsequent periods. It is widely argued in the literature that combining dynamics with endogenous decision power in the collective household model leads into solutions that fail Pareto efficiency. If decision making power in the household is driven by the household members’ actual earnings, the resulting labour supplies can be inefficiently high. This is because the household members recognize the decrease in their future bargaining power due to the time devoted into household work. Decreasing say in the household implies lower private consumption in the future for the individual in question. It is
shown here that household solution will be on the efficient frontier when the joint consumption of the domestic good is taken into consideration. If attention is given only on the household members’ private consumptions, the solutions will fail Pareto efficiency.

**Keywords:** collective household model, female labour supply, gender, household welfare, household production, household time allocation
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The initial spark for this thesis I got on an interdisciplinary postgraduate course concerning women and organizations. This course led me to think how I, as an economist, could analyse the questions of gender and career. I chose household time allocation decisions as the central theme of my research since gendered division of labour in the household is arguably an important determinant for the outcomes on women's career. As a result, this thesis is not about contagious currency crises, the theme I originally worked on.

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Heidi Silvennoinen
Essays on Household Time Allocation Decisions in a Collective Household Model

This dissertation consists of an introduction and the following three essays:

Essay I:
Interaction of Time and Goods in the Collective Household Model,
Heidi Silvennoinen, unpublished.

Essay II:
Social Norms and Female Labour Supply,
Heidi Silvennoinen, unpublished.

Essay III:
Collective Household Model with Endogenous Balance of Power and Household Production,
Heidi Silvennoinen, unpublished.
Essay II:
Social Norms and Female Labour Supply

1. Introduction
2. The model
3. Household utility – a formal example
   3.1 Time allocation responses with respect to stronger norm for tradition
   3.2 Household utility responses with respect to stronger norm for tradition
4. Policy implications
5. Discussion
6. Conclusions
   References
   Appendix

Essay III:
Collective Household Model with Endogenous Balance of Power and Household Production

1. Introduction
2. Household time allocation with endogenous decision power
   – one period model
3. Household time allocation with endogenous decision power
   – two period model
   3.1 Commitment
   3.2 No-commitment
4. Intertemporal efficiency
5. Conclusions
   References
Introduction:

Women and family on economists’ research agenda

Abstract

Neoclassical economics has been accused of neglecting gender aspect in its analysis. I argue here that by introducing household production and joint consumption of the domestic good into the collective household model allows one to consider many important gender related aspects of household behaviour.

The way economists see the household as an institution has varied through time. For the classics the Victorian principles determined how women and family were connected to the society. From the mid 1800s the rights of women rose into public debate. But the brake through of the status and role of women on economists research agenda was not until 1960s relating to the work of Jacob Mincer (1962), Theodore Schulzt (1961) and Gary Becker (1965). In this unitary approach for household behaviour households were seen as single utility maximizing agents. Recent work shows that treating the household as a single utility maximizing unit can lead into wrong welfare implications since nothing can be said about the intra household allocation of resources. An alternative modelling approach that allows distinct utility functions for the spouses has gained researchers’ attention since late 1980s. However, the research on household behaviour in multi decision maker framework has mostly grown without consideration of the time allocated into household production. Since household production can be seen as important determinant of household welfare and since women still perform the largest part of the household work the issues of allocation of time and household production should not be neglected in the analysis of household behaviour.

1. Introduction

This thesis considers the consequences of traditional division of labour in households in a setting where spouses are allowed to have distinct preferences. This approach leads to different results compared to the traditional unitary approach and is better equipped to take into consideration gender related issues of household decision making. The thesis consists of three theoretical essays where the household production theory is applied in the collective household model. Neoclassical economics has been accused of neglecting gender aspect in its analysis. I argue here that by introducing household production and joint consumption of the domestic good into the collective
The household model allows one to consider many important gender related aspects of household behaviour.

The way economists see the household as an institution has varied through time. For the classics the Victorian principles determined how women and family were connected to the society. Household and family were institutions that were considered as independent from the economy. This meant that issues concerning women and family were not relevant research topics for economists. This was partly due to the societal standing of women at that time. Women did not have right to the property or right to vote. From the mid 1800s the rights of women rose into public debate. But the brake through of the status and role of women on economists research agenda was not until 1960s relating to the work of Jacob Mincer (1962), Theodore Schulzt (1961) and Gary Becker (1965). In this unitary approach for household behaviour households were seen as single utility maximizing agents. Recent work shows that treating the household as a single utility maximizing unit can lead into wrong welfare implications since nothing can be said about the intra household allocation of resources. Further the empirical evidence against the predictions of the unitary model is quite impressive as well. Most convincing is the rejection of the so called income pooling hypothesis according to which the source of income should have no effect on resulting allocations in households (see e.g. Browning and Chiappori 1998; Fortin and Lacroix 1997; Lundberg, Pollak and Wales 1997; Browning, Bourguignon, Chiappori and Lechene 1994; Schultz 1990; and Thomas 1990). An alternative modelling approach that allows distinct utility functions for spouses has gained researchers’ attention since late 1980s. In multi decision maker framework the analysis is not limited to inter household distribution of welfare, it is possible to analyse intra household allocations as well.

However, the research on household behaviour in multi decision maker framework has mostly grown without consideration of the time allocated into household production. Since household production can be seen as important determinant of household welfare and since women still perform the largest part of the household work the issues of allocation of time and household production should not be neglected in the analysis of household behaviour. Further, the allocation of time in the household has consequences on individual earnings. The literature on motherhood and wages shows that the time devoted to childcare and household work has negative
effect on the earnings of mothers (see e.g. Bonke et al., 2003; Phips et al., 2001; Hersch&Stratton, 2000; and Waldfogel et al., 1999). Recent results with Finnish data show that career brakes due to child birth have negative effect on the earnings of mothers compared to childless women in Finland as well (Lilja et al., 2007; and Napari, 2007). On the other hand there is empirical evidence according to which individual earnings can be seen as an important determinant for decision power in households and this has consequences on how consumption is allocated in households (Blundel et al., 2007). Therefore, I argue that there is a rationale for analysing the time use and household production in a multi decision maker context. Strong argument for the inclusion of household production into multi decision maker household models can be found from Apps&Rees (1997) and Apps (2003).

This introductory section for the thesis is organized as follows. First, in section two, history of economic thought background is given on how the questions relating to women and family have been considered by economists. After this, in section three, the models of household decision making are formally presented in the context of the models formulated in the essays of this thesis. In section four, the questions relating to the applicability of household production approach are discussed. Finally summary of the essays is presented in section five and a short concluding note is given in section six.

2. Development of the theory of the household

2.1 The classics 1700–1800

Issues concerning women and family were excluded from the discussions of political economy in 1700s and 1800s. This reflected the way political economy was determined by then. The science of political economy concerned questions related to production and accumulation of wealth. Women had no right to vote, own property in their own names and therefore they had no role in the discussions of the political economy. Further, with the transition from home-centred form of economic production to an industrial economy, women became increasingly economically dependent and
isolated. As a result women were seen as non-persons in public world (Calas & Smircich, 1996).

Adam Smith (1723-1790) spoke for individual freedom and equality in the society and David Ricardo (1772-1834) was interested on the problems between social classes. According to Jefferson & King (2001) household and family were for Smith and Ricardo part of the institutional structure that provides the context for their discussions on other economic issues. Further, when Nassau Senior’s (1836) ‘An Outline of the Science of Political Economy’ determined the political economics as a science concentrating on production and accumulation of wealth, the questions relating to household and family were excluded from the relevant discussions of political economy (Hewitson, 1999).

Early advocates of women’s rights were John Stuart Mill and Harriet Taylor Mill writing in England in the period that covered the 1840’s to 1870’s. They criticized the exclusion of women from the public fields of the society and the idea that the proper sphere of women is private and domestic life. Mills argued that equal rights for women would also benefit men (Brue, 2000). However, Mills did not concern the specific problems of household production and time allocation as determinants of equal distribution of welfare in households. The main point of their writings was to gain the same rights for the women in the society as the men had (Jefferson & King, 2001). These were right to vote, access to employment and economic resources.

2.2 Marginalism and neoclassical economics 1870s

The emergence of marginal utility school in the 1870s further reduced the interest in the analysis of domestic labour. Mainstream economics was now focused upon market transactions. The marginal utility theory shifted attention from production of the goods to the way the goods were allocated through the market. Discussions of labour referred only to paid labour. The analysis did not specifically exclude household labour instead its existence was ignored. The public/private dichotomy continued to outline the discussions of neoclassical economics. The public sphere included market activity and was seen as male domain. The private sphere was that of household work generally performed by the women. This approach became
institutionalized through the categorization of individuals as breadwinners and dependants in official statistics (Jefferson&King, 2001).

The status of women in the society was seen through motherhood and the demands it posed to the women. Important discussants of neoclassical economics Jevons (1904) and Marshall (1930), were against female labour outside the household, they argued that female labour supply would have negative effect on the well-being of their children (Pujol, 1995). According to Jeffreson&King (2001) the first explicit step excluding the productive activities of domestic labour from the subject matter of economics may be attributed to Alfred Marshall. Marshall stressed the ‘exact money measurement’ as distinguishing feature of economic activity. Further, Pigou (1932) determined national income as the value of production of all the goods and services that can be valued at monetary measures. According to Pigou’s famous example the national income will decrease if the man marries his housekeeper (Jefferson&King, 2001).

Poppel et al. (2006) analyse the emergence of the housewife in the Netherlands 1812-1922. The main explanation offered has been the increase in knowledge about the causes and transmission mechanisms of infectious diseases which led to public plea for mothers to stay home for the sake of their children’s well being. This explanation is in line with the view of Jevons and Marshall on the role of women in the society. However, Poppel et al. (2006) show that the emergence of the housewife in the Netherlands started much earlier than the spread of useful health knowledge. Alternative hypothesis offered by the authors is that the division of labour within households is affected by ‘conspicuous leisure’ in terminology of Thorstein Veblen1, so that individual utility preferences can be understood in relation to the utility preferences of upper-class others. The authors argue that there was a social norm that women should withdraw from the labour market when entering to marriage in order to gain social esteem. By the end of 1910s entering marriage without an occupation was the dominant choice among every social class in Netherlands. It is not possible to be sure about the rationale to enter marriage as a housewife. Which ever of the proposed

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1 Veblen, Thorstein (1899) The theory of the leisure class – An economic study of institutions, New York.
explanations is right, the end result is the same, and reflects women’s relative position in the society at the end of 1800s and beginning of the 1900s.

2.3 Household economics

In the beginning of 1900s research on questions relating to domestic labour developed in two areas of economics; the national accounting framework and the sub discipline of home economics.

2.3.1 National account framework 1920s

The significance of household production for welfare was recognized especially in Sweden, the US and the UK. In Sweden Lindahl, Dahlgren and Kock (1937) evaluated that the value of domestic production was approximately 32 percent of GNP in 1929. In the US Kuznets (1941) estimated that the value of domestic production was 35 percent of the GNP in 1929 (Jefferson&King, 2001). The literature on the value of household production is by now very extensive. However, the literature does not develop a theory on household behaviour. Central in the determination of the value of domestic production is the evaluation of the time used in the production process. It is possible to evaluate the price for the time input on the basis of the income foregone or on the basis of market cost. In fact according to Jefferson&King (2001) Kuznets stressed the subjective nature of the estimates given from the value of domestic production, since the result is highly dependent on what is taken into consideration and how the inputs in the production process are valued. Despite these problems Kuznets argued for the importance of including household production into national accounts.  

The evaluation of non-market work has been slow. Domestic production still is not a standard component of national accounts. One of the explanations given for the exclusion of the questions relating to domestic sphere of women in the field of economics has been the lack of statistics describing the value and amount of non-market work. In fact MacDonald (1997) argues that important future challenge of

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2 Examples of Finnish research on the value of household production are Taimio (1990) and Heikkilä&Piekkola (2004).
economics research is to collect data that helps to analyse the gender related questions more specifically. Information about the allocation of time in the households is obtained from the time use studies. Although the first time use studies can be dated to the end of 1800s, the data between countries and times still is not always comparable and many countries have conducted their first time use surveys only very recently (Ruuskanen, 2004).

2.3.2 Early home economics 1930s and 1940s

In the sub discipline of home economics the path breaking work was done by Kyrk (1923), Reid (1934) and Hoyt (1938). They discussed domestic production in the context of consumer theory and specifically Reid stressed the importance of the time inputs. The three researchers recognized that the time available was allocated between market work, leisure and household production. The time allocation decisions of the household were seen to concern the whole family instead of the individuals independently. Besides household production, social issues like the distribution of income and the status of women were addressed. The close interaction of these three researchers permitted consumption economics to flourish during the 1930s and 1940s in the US (Jeffreson&King, 2001; and Grossbard-Schectman, 2001). The determination of the household production is still most often done after the definition of Reid (1934). It is assumed that household production satisfies the so called ‘third party rule’ meaning that it is possible to substitute market goods and services for one’s own time. It is possible to pay someone to perform these tasks but the household members are not paid for performing them.

The central problem in the application of consumer economics into household behaviour has been how to model the household as if it was a single utility maximizing consumer. Samuelson (1956) introduced the conditions under which family behaviour can be rationalized as the outcome of maximizing single utility function. Each member of the household has an individual utility function that depends on his or her private consumption of goods. By consensus the household members agree to maximize a social welfare function of their individual utilities, subject to a joint budget constraint that pools individual incomes. This optimisation problem generates household
demands that depend only upon prices and household total income. Therefore comparative statistics of traditional consumer demand theory apply directly to household behaviour. Samuelson did not explain how the household achieves a consensus regarding the joint welfare function and how the consensus is maintained. It is assumed that the intra-household allocation of resources is only affected by total household income. The share of income or wealth of individuals within the household will not affect the allocations.

2.3.3 New home economics 1960s

Besides the work done by Kyrk, Reid and Hoyt in 1930s and 1940s important spade work for the development for the theory of household was done by Theodore Schultz (1961) and Jacob Mincer (1963).

Schultz (1961) came to economics via research on agriculture and he stressed the importance of the labour input in his analyses. He argued that economists had concentrated too much on land as a factor of production and therefore their analyses had failed. According to Schultz the quality of labour depends on nutrition, health, child care and education. He explained how investment in education and health flow in productivity. This ‘human capital theory’ had a link into household production through the notion that in agricultural societies large part of the consumption came from the goods produced in the household.

Mincer (1962) was the first to consider the labour supply of married women in household context and to agree with the early home economists that time is divided not only between leisure and labour supply but as well in home production, child care and education. Mincer’s work gained a lot of attention since the earlier labour supply theory had been based on individual labour/leisure tradeoffs. This theory obviously failed to explain the specific nature of female labour supply.

It was, however, not until Becker’s (1965) article that household production was formally integrated for the first time into neoclassical economic theory. In Becker’s theory households combine time and market goods to produce commodities that directly enter their utility functions. Households were now seen as both producing and utility maximization units. Becker successfully combined the theory of human capital
as well into his models of household behaviour and allocation of resources. This new line of research was named as ‘new home economics’ as contrasted to the earlier work of Kyrk, Reid and Hoyt in 1930s and 1940s.

In 1970s the models concerning marriage, fertility, household behaviour and male female wage differentials were further developed (e.g. Gronau, 1973; Becker, 1973, 1974a, and 1981; and Mincer&Polachek, 1974). Becker offers a rationale for using a unified household model, even when household members have different preferences. Altruistic parents and their children maximize the same utility function, even if the kids are selfish. Resources are allocated in such a manner as to maximize household income. However, this ‘rotten kid theorem’ holds only if the altruist has the last move.

Household production and family economics became in 1960s and 1970s a large and growing research agenda. The emergence of the models of household behaviour and female allocation of time coincided with major social changes. The increasing participation of married women into labour market and the public debate on the role and status of women triggered economists interests on female allocation of time. However, according to Grossbard-Shechtman (2001) the researchers concerned on the social construction of gender lost their interest on Becker’s work after he published articles (1974b, 1976) that claim natural grounds for existing gender-based division of labour and authority structures.

2.3.4 Consensus and non-consensus models of the household 1980s

Since the work of Becker (1965) economists have considered households as single utility maximizing agents. Recent work shows that this is not necessarily the case. From a theoretical viewpoint it has been argued that many person households cannot be modelled as a single individual because that would contradict the neoclassical starting point that every individual should be characterized by his/her own preferences (Chiappori, 1988, 1992). Further, it has been argued that the traditional approach may lead to wrong welfare implications since nothing can be said about intra household resource allocation, although, comparisons between households are possible (Chiappori, 1988; and Apps&Rees, 1997). Empirical evidence against the unitary model is quite impressive as well (Browning and Chiappori 1998; Fortin and Lacroix
An alternative modelling approach that allows distinct utility functions for spouses has gained researchers’ attention during the past decades. These models can be divided into two main categories as consensus and non-consensus models of the household contrasted to the common preference framework considered earlier. In the consensus framework most attention is given to the Nash bargaining approach initiated by Manser and Brown (1980) and McElroy and Horney (1981) and to the collective approach initiated by Chiappori (1988).

In the Nash bargaining approach the distribution of utilities which results from a marriage is given by symmetric Nash bargaining solution. The Nash solution is the outcome that maximizes the product of the gains to cooperation under the household budget constraint. Depending on the threat points a specific Pareto efficient intra household allocation is obtained. The collective model has only one assumption which is that collective decisions are Pareto efficient. Rather than assuming that observed outcomes are solutions to a particular game, it is only assumed that observed outcomes are efficient.

Household models that allow inefficient outcomes are called non-consensus models to emphasise non-cooperative behaviour of the household members assumed in these models. In non-cooperative household models the multi decision maker framework is explicitly considered. Household members maximize their utility, subject to individual budget constraint and taking the other individual’s behaviour as given. This means that Pareto efficient intra household allocations do not necessarily occur. The non-cooperative models not only allow for individuals to have different preferences, but also allow for individuals to make consumption and production decisions based on their own labour and access to resources. Both Pareto efficient and non-Pareto efficient outcomes are consistent with these models. For the implications of non-cooperative household behaviour see, for example, Konrad&Lommerud (2000, 1997) and Lundberg&Pollak (1994). Chiappori (1988, 1992) argues that household is an example of repeated game and it can be assumed that each person knows the preferences of the other people in the household. Together these lead to the argument that agents find mechanisms to support efficient outcomes. Further, non-cooperative
models can be seen as somewhat unrealistic description of family behaviour even recognizing that not all outcomes of household resource allocation will be efficient. In fact Shubik (1989) argues that the non-cooperative game theory is not useful for analysing complex, loosely structured social interactions. This description certainly suits well to households. Therefore, in what follows non-cooperative household behaviour is not considered.

Next the cooperative models of household decision making are formally presented in the context of the models formulated in the essays of the thesis.

3. Household decision models

In this section I go formally through the main characteristics of the cooperative household models in order to be able to contrast the outcomes of the household utility maximization process of the different frameworks with each other. The alternative approaches to model household behaviour in cooperative setting are presented using the same notation as in the essays of this thesis. Therefore, before proceeding I present a short note on the specific features of the models formulated in the essays.

In all cases I assume that the household consists of two members who both participate in the labour market. The hourly wages do not depend on the hours worked. Both household members have well defined twice differentiable utility functions and they gain utility from private consumptions of the market goods \( x^i, i = f, m \), and from the joint consumption of the good produced in the household \( G \). In the following examples the most general form of preferences is assumed. That is individual utilities are defined as \( U^i(x^i, x^j, G), i \neq j \).

The household resource allocation problem with household production can be solved in two stages (see, for example, Gigno, 1991). In the first stage the cost minimization problem is solved for the time - market good - mix that minimizes the income forgone in order to attain desired level of the good \( G \) produced in the household. After this household consumption allocation problem is solved in stage two. The imputation of the domestic production into the collective household model is discussed in length in the first essay of this thesis. Therefore, the household time
allocation problem is not formally considered in this introductory section. Instead in the following formal presentations of the alternative household models, only the stage two of the household resource allocation problem is considered, given the inputted price \( p \) for the good \( G \) produced in the household. The price for the household good can be either exogenous or endogenous to the household depending whether the good produced in the household is marketable or not.

There are two special features in the models formulated in this thesis worth noting. First of which is that leisure demand is not considered. The household members are assumed to allocate all the time available between market work and household production. This non-standard modification is used in the essays of this thesis since the focus of the analysis is on the trade off between household members’ private consumptions versus joint consumption of the good produced in the household. Another rationale for excluding leisure from the analysis here is that time use surveys show that total work time (market work plus household work) between genders is equal on average in most Western countries (Burda et. al., 2006). The equality of total work between genders implies that the leisure time available between genders has to be equal on average as well. Household production is made at cost. Household members engaging in household production have to forego a certain amount of leisure and a part or totality of an income from market work. Here the absence of leisure implies that the cost of household production comes only from the loss in market income. However, it has to be noted that in the framework of the current study it would be possible to consider the case where the vectors of private consumption include individual leisure times.

A further special feature of the models constructed in this thesis is that non-labour income is not considered and therefore the household expenditure cannot exceed household earnings. In the essays I concentrate on the effects of exogenous wage changes (for example due to policy reforms) on household behaviour and therefore non-labour income is not explicitly taken into consideration. Non-labour income would be easily added though, into the models formulated here. This is done in the

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3 In the empirical literature non-labour income is often assumed to be exogenous to the household but in reality large part of household non-labour income can depend on past labour supply decisions of the household members.
essay II of this thesis where the effects of family policy on female labour supply are analysed.

3.1 Unitary model

A fundamental assumption of the unitary approach is that the household preferences are assumed to be representable by a unique well-behaved utility function. To aggregate individual preferences into household preferences, it has to be assumed that either all of the members of the household have the same utility function or it has to be assumed that some rule exists for aggregating the utility functions of the household members.\(^4\)

Assume that the household consist of two household members, male and female, that both work in the market. Household members gain utility from private consumptions \(x^i, i = f, m\), for female and male household member respectively and from the joint consumption of the good produced in the household \(G\). The household utility function in the unitary framework is the following:

\[
U = u\left(x^f, x^m, G\right)
\]

where \(u\) is a strongly quasi-concave, increasing and twice continuously differentiable function of its arguments. By setting the total time available for each household member equal to one, \(T = 1\), and arranging the uses of income on the left hand side of the budget constraint and the sources of income on the right hand side of the budget constraint we get the household full budget constraint as:

\[
x^f + x^m + pG \leq w^f + w^m \equiv Y
\]

Where \(w^f + w^m \equiv Y\) is the household potential income. That is, \(Y\) is the income that would occur if both household members allocate all the time available into market

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\(^4\) Examples are Samuleson’s (1956) common preference model and Becker’s (1974a) ‘rotten kid theorem’.
work. The prices of the market goods consumed privately are normalized to one. The inputted price of the household public good $G$ is denoted by $p$ which depends on the wages of the household members, that is $p(w_f, w_m)$. The household maximization problem outlined above produces Marshallian demand functions of the following form:

$$X^i = x^i(Y, p), \ i = f, m$$
$$G = g(Y, p)$$

It is seen that the increase in the market wage of either of the household members induces positive income effect for household consumption of the private goods as well as for the household public good. This effect does not depend on whether the wage of the female household member increases or whether the wage of the male household member increases. This result is referred in the literature as income pooling since it implies that the source of the income has no effect on consumption allocation in the unitary model for household behaviour. Further, as a result of increase in either of the wages, the cost of producing the household good increases, implying substitution away from the use of the time input of the household member in question. Price increase means as well that there is substitution away from the joint consumption of the household good. The net effect of the wage increase on the joint consumption of the household good depends on whether the positive income effect or the negative substitution effect dominates.

The above demand functions satisfy the well-known properties of adding up, homogeneity, Slutsky symmetry and negativity. When these conditions hold, observed demands are integrable to a rational preference ordering. Except for the adding up condition, each of these restrictions has been rejected in various studies. For evidence in labour supply models see Blundell&Meghir (1986) and Blundell&Walker (1986) for evidence in consumption allocation models see Blundell (1988). According to Vermeulen (2002) the rejections of the income pooling hypothesis led to a reinterpretation of consumer theory and development of the models taking into
consideration the possibility that the household members may have different preferences.

3.2 Nash-bargaining model

The initial work to develop a bargaining approach to modelling household behaviour was done by McElroy and Horney (1981) and by Manser and Brown (1980). They formulate a bargaining framework in which household decisions are made through a cooperative Nash game. The Nash solution is the outcome that maximizes the product of the gains to cooperation under the household budget constraint. Using the definitions and notation of the example presented in the case of the unitary household model the household consumption allocation problem is now the following:

$$\begin{align*}
\text{Max } N &= \left[ u^f(x^f, x^m, G) - d^f \right] \times \left[ u^m(x^f, x^m, G) - d^m \right] \\
\text{s.t} & \\
\ x^f + x^m + pG & \leq w^f + w^m \equiv Y
\end{align*}$$

where $d^i$, $i = f, m$, is household member $i$'s threat point or disagreement point. This is the outcome that results if the household members fail to cooperate. The Nash solution is the outcome that maximizes the product of the gains to cooperation under the household budget constraint. Figure (1) describes the Nash solution to household resource allocation problem. The utility possibility frontier is the locus of all Pareto optimal points corresponding to given prices and incomes. The pair of outcomes in the case of disagreement $(d^f, d^m)$, the threat point, lies inside the utility frontier. Geometrically the Nash solution is the tangential point of the utility possibility frontier and the hyperbola:

---

5 McElroy & Horney (1981) consider other bargaining concepts as well (the Kalai-Smorodinsky and the dictatorial solution concepts).
\[ N = \left[ u^f(x^f, x^m, G) - d^f \right] \times \left[ u^m(x^f, x^m, G) - d^m \right] = \text{constant} \]

farthest away from the conflict point \((d^f, d^m)\).

**Figure 1:** Household consumption allocation in the Nash-bargaining model.

In order to be able to apply Nash bargaining framework to household behaviour the threat points have to be properly defined. McElroy and Horney (1981) and Manser and Brown (1980) use utility levels when single, i.e. after divorce, as the threat points and thus the name ‘divorce threat model’ often used for this approach. Another example is the one applied by Lundberg and Pollak (1993) where the threat points are determined by the utility levels associated with non-cooperative household behaviour inside the household.

The Marshallian demands, resulting from the household problem \((N)\) are of the following form:
The cooperative bargaining model predicts that factors that influence the threat points of individuals may affect the distribution within households, even if the individual and total household level of resources are not altered. In a simple unitary model prices and household income are the only explanatory variables. In bargaining models every variable that can be expected to affect the threat points of the household members can be taken up in the analysis. There can be parameters affecting \( d^i, i = f, m \) and so causing shift in the threat points. McElroy and Horney (1990) name the factors potentially affecting the threat points of the household members as ‘extra environmental parameters’ (EEPs). Examples of EEPs are the ratio of males to females in the relevant marriage market, laws on alimony and child benefits, tax laws that differ according to marital status and divorce law. Further, an increase in wages for women can affect the allocation of resources within households, even in households where the women are not employed, through its effect on \( d^f \).

### 3.3 Collective model

In the collective model for household behaviour initiated by Chiappori (1988) the only assumption is that household decisions are Pareto efficient. In contrast to the Nash bargaining approach no restrictions is imposed on which point on the Pareto frontier is chosen by the household. Pareto efficiency only requires that chosen consumption bundles are such that an individual’s welfare cannot be increased without decreasing the welfare of the other household member.

The Pareto optimal allocation of consumption can be found as solution to the following maximization problem:

\[
X^i = x^i(Y, p, d^f, d^m), i = f, m
\]

\[
G = g(Y, p, d^f, d^m)
\]
\[
\begin{align*}
\text{Max} & \quad u^f(x^f, x^m, G) \\
\text{s.t} & \quad u^m(x^f, x^m, G) \geq \bar{u}^m \\
& \quad x^f + x^m + pG \leq w^f + w^m = Y
\end{align*}
\]

The first constraint is the Pareto constraint, where \( \bar{u}^m \) is some required utility level for the husband. Thus the wife’s welfare is maximized subject to some pre-allocated welfare level of the husband and household full budget constraint. By varying \( \bar{u}^m \), all Pareto efficient outcomes can be traced. As long as the household members’ individual utility functions are strongly concave and the household budget constraint defines a convex set, the utility possibility set, describing all the attainable outcomes, is strictly convex. This means that it is possible to characterize all the Pareto efficient allocations as stationary points of a linear social welfare function for some positive welfare weights for both household members (Chiappori, 1988, 1992; Vermeulen, 2002). In the collective model the household consumption allocation problem can therefore be defined as solution to the following problem:

\[
\begin{align*}
\text{Max} & \quad \Omega = \theta u^f(x^f, x^m, G) + (1-\theta) u^m(x^f, x^m, G) \\
\text{s.t.} & \quad x^f + x^m + pG \leq w^f + w^m = Y
\end{align*}
\]

where \( \theta \) is the weight given for the wife’s preferences in the household utility maximization process. The household full budget constraint defines the Pareto frontier for given utility functions for the spouses. The outcome of household’s utility maximizing process will be located on this frontier. The welfare weight \( \theta \) determines the final location on this Pareto frontier. The welfare weights are the normalized Lagrangian multipliers of the maximization problem in (P) and in general they will depend on prices and income. In the framework of the current example the welfare weight is a function of prices and household income. Since the prices for private
consumptions are normalized to unity and the imputed price for the domestic good depends on the household members’ hourly wages we have $\theta(w^f, w^m, Y)$ for the case considered here.

**Figure 2:** Household consumption allocation in the collective model.

The welfare weight $\theta(w^f, w^m, Y)$ for the wife is bounded between zero and unity and gives the influence of the wife on the household demands. For the extreme where $\theta=1$ the household utility is determined as $\Omega = U^f$ implying female dictator household. And when $\theta=0$ the household utility is determined as $\Omega = U^m$ implying male dictator household instead.\(^6\) For intermediate values, the household behaves as if each person has some decision power.

The Marshallian demands resulting from the household consumption maximization problem (C) are of the following form:

\(^6\) In these cases the collective model reduces into a unitary model. There are further three cases where the collective model reduces into the unitary model. These are the case when the welfare weights are constant, the case where the household members have identical preferences, and finally the case where the welfare weight does not depend on prices.
\[ X^i = x^i(Y, p, \theta(w^f, w^m, Y)), \quad i = f, m \]
\[ G = g(Y, p, \theta(w^f, w^m, Y)) \]

It is immediately seen that now there is, besides the usual income and substitution effects, an effect caused by the shift in the decision power in the household when the factors determining the welfare weights \( \theta \) and \( (1 - \theta) \) change.\(^7\)

Bourguignon et al. (1994) extend the general collective model by noting that other factors besides prices, wages and non-labour incomes may also affect the household allocation of consumption. These factors are called the distribution factors and are defined as variables that affect the decision power in the household but do not have direct effect on individual preferences or household budget constraint. Preference factors are characteristics that affect preferences directly. They can include features such as age, education, and the number of children. Distribution factors affect the division of consumption between the household members but they do not enter the budget constraint or utility function. Examples of distribution factors are the relative incomes of the members of the household, the difference in educational level between spouses, or the difference in age between the spouses. Further, Browning and Chiappori (1998) suggest that the so-called ‘extra environmental parameters’ (EEP’s) used in Nash bargaining framework can be categorized as distribution factors. Chiappori (2002) analysed the effect of the state of the marriage market and divorce laws on household labour supply with the collective model. The results support Becker’s idea that the state of the marriage markets is relevant when the household behaviour is considered. Distinction between variables that affect household behaviour and variables that affect preferences was developed in order to get empirical content into the collective model (Browning and Chiappori, 1998; and Chiappori, 2002).

If the presence of the distribution factors were assumed the vector of distribution factors \( z \) would enter only in function \( \theta(w^f, w^m, Y, z) \) determining the welfare weight.

It is important to note that the influence of prices and distribution factors in the

---

\(^7\) Note the analogy with the adjustment of the threat points in the Nash bargaining framework presented in the previous section.
household utility function enters only through the function $\theta(w', w^m, Y, z)$ which gives restrictions on how they can affect behaviour. Therefore these factors affect household behaviour only through their effect on the decision power in the household. The drawback of the collective model is that it gives no hint of the process whereby the members of a household might achieve a Pareto efficient outcome or where on the utility possibility frontier the household will be located (Basu 2001; Browning and Lechene 2001). More importantly, it gives no guidance as to what variables should appear in the set of distribution factors (Browning and Lechene, 2001).

Since the welfare weights $\theta$ and $(1-\theta)$ generally depend on prices, household preferences are price dependent. According to Chiappori (1988) household preferences in the collective household model can be seen as generalized version of the price dependent preferences considered by Pollak (1977). This implies that the Slutsky effects are no longer symmetric and the matrix consisting of these Slutsky effects is no longer necessarily negative semi definite. The failure of the Slutsky conditions can be seen as important distinguishing implication of the collective household model. Browning et al. (2004) consider how the assumptions made on the welfare weight $\theta$ affect on whether or not household demands satisfy the Slutsky conditions. The authors consider four different classes of models and propose that the term unitary should be used for any model that leads to demands that satisfy the Slutsky conditions, whether or not the model is independent from the distribution factors determining the decision power in the household. Thus, there may be unitary models that depend on the distribution factors. The term collective should be used for models that fail Slutsky conditions, whether or not they depend from the distribution factors. Therefore, the dependence of the welfare weight on prices (or wages) can be seen as a necessary condition for the collective household model.

### 3.3.1 The sharing rule result

Chiappori (1988, 1992) does not introduce any particular assumption on individual preferences, except that they can be represented by conventional utility functions. In the general version of the collective model, considered so far, there were no
restrictions on preferences. Intra-household consumption externalities, altruism or any other preference interaction is allowed. This most general form of preferences is known as the case of altruism $U^j(x^j, x^i, G)$, $i \neq j$, where household members’ private consumptions $x^j$, $i = f, m$ enter into individual utility functions together with the joint consumption of the household good $G$. In general both positive and negative externalities are allowed. According to Chiappori (1992) it would be reasonable to assume that $U^i$ is strictly increasing in $x^i$ and in $G$ but not necessarily in $x^j$. The most restrictive case for preferences is the case of egoistic preferences where $U^i(x^i, G)$, $i = f, m$. In this case, the household members care only on their private consumptions and the joint consumption of the household good. An intermediate case is Becker’s (1974a) notion on caring in which each household member has a welfare function that depends on both own and companion’s egoistic utilities $W[U^i(x^i, G)U^j(x^j, G)]$, $i \neq j$. Further, according to Browning and Lechene (2001) all combinations of preferences are possible. This means that one household member can be egoistic while the other may be altruistic.

The restrictions on preferences have produced important identification results for empirical work with collective household models. Chiappori (1988, 1992) shows that if household members’ preferences are egoistic or caring then the Pareto efficient household allocation problem can be decentralized into a two stage budgeting process. The decentralization results derived by Chiappori (1988, 1992) hold only if consumption is purely private.\footnote{For the case with public goods it is possible to determine the sharing rule conditionally on public consumption (Blundell et al. 2005 and Donni, 2006).}

Assume that the household potential income $Y$ is shared between the household members. Let $\phi(w^f, w^m, Y)$ be the amount received by the wife and $Y - \phi(w^f, w^m, Y)$ the amount received by the husband. If there were other sources of income than labour income in the model, the share $\phi(w^f, w^m, Y)$ could be negative implying that the wife is sharing income with her husband. Now the Pareto efficient household problem (P) is
equivalent to the existence of a function $\phi(w^f, w^m, Y)$ such that household consumption bundle is solution to the two maximization programs $i = f, m$ where each member individually chooses consumption of the market good and the good produced in the household:

$$\begin{align*}
\text{Max } & u^i(x^i, G^i), \quad i = f, m \\
\text{s.t. } & (S^i) \\
& x^i + pG^i \leq \phi^i(w^f, w^m, Y)
\end{align*}$$

Where $\phi^f(w^f, w^m, Y) = \phi(w^f, w^m, Y)$ and $\phi^m(w^f, w^m, Y) = Y - \phi(w^f, w^m, Y)$ denote the respective shares in potential household income of the spouses. Since both shares add up to one, the sharing rule itself can actually be recovered up to an additive constant. Knowing the rule allows to write down each member’s actual budget constraint. Preferences can then be computed in the usual way. For identification of the sharing rule without distribution factors see Chiappori (1992), and for identification of the sharing rule with distribution factors see Chiappori et al. (2002). Note that in general the sharing rule depends on the same factors as the weighting factor in the general version of the collective household model.

In the current example with household production the sharing rule is only partially identifiable in the case of non-marketable household goods. In this case the price $p$ for the good produced in the household is endogenous to the household and depends on wages, production technology and preferences. Therefore, it is not possible to retrieve all the partial derivatives of the sharing rule needed for identification. Instead only partial information of the sharing rule is available (Apps&Rees, 1997; and Aronson et al., 2001). Chiappori (1997) shows that with constant returns to scale technology it is possible to identify the sharing rule with non-marketable household good only up to an additive function of wages. Whereas in the case of marketable household production the price for the household good $p$ is exogenous to the household and it is possible to identify the sharing rule up to an additive constant. In this case testable restrictions are
generated on market labour supply functions as in the case without household production.

The two interpretations of the collective model, the general version versus the sharing rule, are equivalent. For the proof see Chiappori (1992). It has to be noted, however, that with the most general form of preferences \( u_i(x^f, x^m, G) \), \( i = f, m \) the collective decisions cannot generally be decentralized by the use of sharing rules.

### 3.3.2 On the applicability of the collective household model

There is no agreement on which model of household behaviour is appropriate. According to Browning et al. (2004) it may be that different models are relevant in different contexts. For example, Del Boca (1995) cannot reject the unitary household model for households with children less than six years of age with Italian data while the unitary model is rejected for households with no children or with children older than six years of age.

The collective model initiated by Chiappori (1988) contains as special cases all cooperative models (including the unitary model) and some solutions to some non-cooperative models. Thus this approach is more general than the most widely studied alternative for non-consensus family behaviour, the bargaining approach. Further many bargaining rules assume Pareto efficiency, examples are, Nash, Kalai-Smorodinsky, utilitarian and egalitarian solutions. Chiappori (1988) pleads for minimum assumptions on the household decision process. The bargaining model provides a rule to specify which Pareto efficient point will be chosen, while the collective model only assumes that the outcome will be Pareto efficient. When a specific bargaining model is empirically rejected it is impossible to say whether the rejection is due to the failure of the collective setting as such or to the particular bargaining concept. For these reasons the collective model is applied in the essays of this thesis. A thorough survey of collective household models is found in Vermeulen (2002).

The applications of the collective household model can be divided in two separate branches. In labour supply models prices are assumed constant and the only price
variation comes from individual wages. In consumption models prices are variable and labour supply is assumed to be fixed. Both of these approaches have been used in the extensions of the original labour supply model of Chiappori (1988, 1992). The extensions of the original model concern: household production (Apps&Rees, 1997; Chiappori 1997; and Donni, 2005a), non-participation and corner solutions (Donni, 2005b; and Blundell et al., 2007), expenditures on children and other public goods (Donni, 2006, consumption model; and Blundell et al., 2005, labour supply model), non-linear budget sets (Donni, 2003; and Beninger&Laisney, 2002), and intertemporal household allocations (Basu, 2006; and Mazzoco, 2004). It has to be noted, however that many of the applications of the collective model are still in a very preliminary stage and vast efforts remain for the implementation of collective models in all situations where unitary models have been used in practice.

Next I discuss the questions related to the applicability of household production theory since household production is in central role in the models formulated in the essays of this thesis.

4. Household production theory

In Becker’s (1965) theory on household production households combine time and market goods to produce commodities that directly enter their utility functions. Since according to time use studies women perform two thirds of household work in most Western countries, it is important to take household production into consideration when analysing female labour supply and distribution of welfare in the households.

There are, however, some serious problems relating to the household production function approach. Pollak and Wachter (1975) provide an insightful critical analysis of this approach. Since it can be argued that household members have preferences over different uses of time, the household equilibrium will be characterized by implicit internal prices. The implicit price of household production depends on the tastes of the individuals as well as on household technology. These prices may be household specific, i.e. dependent on preferences and productivity parameters that may differ across households. To avoid the problem of preference dependent implicit prices the assumption in household production models is that the household members are
indifferent between the uses of time. Assumption that utility comes from the consumption of commodities produced in household, with no direct utility (or disutility) provided by the time input itself, is a standard assumption in the models of time allocation and home production inspired by Becker (1965). Further Pollak & Wachter (1975) argue that the applicability of the household production theory requires separability of consumption and production process in the household and constant returns to scale technology.

According to Ruuskanen (2004) the main problem of the empirical application of the household production theory is that the outputs of the production process cannot be directly observable. Therefore the author argues that the most fruitful applications of the household production theory are in development economics, where the output from rural households can be measured and the inputs calculated. Although the original application of the household production theory concerned consumption in the agricultural households, the significance of taking household production into account when analysing distribution of welfare in non-agricultural households cannot be denied. Household production clearly brings additional welfare for all household members. Therefore, household production is an important determinant of household welfare. Apps & Rees (1997) and Chiappori (1997) show that it is possible to analyse household production, even when the outputs of the production process are not observable. With the standard constant returns to scale assumption it is sufficient to have the information on the time inputs.

Household production was central to the analysis for household behaviour in the 1960s. However, the multi decision maker models of household behaviour have developed mostly without the recognition of the household production as an important determinant of the household welfare and allocation of time and resources in the household. The few exceptions are Apps & Rees (1997), Chiappori (1997) and Donni (2005) in theoretical framework and Aronsson et al. (2001) in empirical framework. Further, Klaveren et al. (2006a,b) estimate the general version of the collective household model with household production. The main drawback is that the models considered in existing studies are based on unrealistic assumptions on domestic production. All the models concentrate on private consumption of the good produced in the household and robust results are obtained only for the case of marketable
domestic goods. A better way to model household production would be the one allowing public consumption of the non-marketable domestic good. The consumption of the good produced in the household can hardly be assigned between household members and sold to the market. Work has to be done in order to take these important aspects of household production into account. And this is precisely one of the goals of the first essay of this thesis.

Gronau (1997) concludes that one of the main points of the household production model is that one cannot separate the analysis of consumption behaviour from the analysis of time use. Both are jointly determined by the demand for home activities and by the technology for home production. He argues that the full potential of the theory has not yet been exhausted. Further, as it was emphasised in the beginning of this introductory section for this thesis neoclassical economics has been accused of neglecting gender aspect in its analysis (Nelson, 1994 and 1995). The methodology, language and research topics of economists have been criticised for being determined by the masculinity of economics profession. Strober (1994) calls for move away from mathematical modelling of abstractions and move towards complex verbal explanations of real economic problems. But as Ruuskanen (2004) points out the comparative advantage of economic research compared to other, less formal, scientific disciplines is in the formal analysis providing accurate responses of rational individuals to outside constraints. I argue here that by introducing household production into the framework with distinct utility functions for the spouses allows one to consider gender related aspects of household time allocation without throwing the baby out with the bath water.

5. Summary of the essays

My research on household time allocation decisions in the collective framework for household behaviour consists of three theoretical essays. The first part of the thesis considers interaction of time and goods in collective household model where the gains from marriage come from the joint consumption of the household public good. The second part of the thesis analyses the effect of social norm for traditional gender roles on the female household member’s time allocation decisions. The third part of the
thesis extends the collective household production model into dynamic framework where time allocation decisions made in previous periods affect the decision making and allocation of resources in subsequent periods. In this section the main questions and results of the essays are presented together with the description of the methodology used.

**Essay I: Interaction of Time and Goods in the Collective Household Model**

The first part of the thesis considers interaction of time and goods in general version of the collective household model where the gains from marriage come from the joint consumption of the good produced in the household. The question how the household members substitute for each other’s time and market goods in the production of the domestic good is of crucial importance in a modern welfare society where both spouses are likely to work fulltime. The growth of women’s paid employment has greatly increased the opportunity cost of domestic labour and thereby contributed to commoditisation of housework so that ever increasing part of needs is satisfied through the market in Western societies.

The general version of the collective model is flexible enough to allow for simultaneous consideration of household production, joint consumption of the domestic good and the possibility of corner solutions. Each of these questions has been addressed in earlier studies separately. The most serious problems in the existing literature are the unrealistic assumptions made on household production and on consumption of the domestic good. It is shown in the literature that the sharing rule in the collective household model can be identified for private consumption of the tradable domestic good (Chiappori, 1997; Apps&Rees, 1997; and Donni, 2005a). In reality the consumption of the good produced in the household is hardly assignable between the household members and it is unlikely that the good produced in the household is sold at the market. Therefore, in this essay the joint consumption of non-marketable household good is considered with the possibility of corner solutions to the household time allocation problem.
Four different time allocation regimes are possible. In interior solution both household members allocate their time between household and market work. Then there are two regimes where only one of the spouses allocates time between household and market work while his/her spouse allocates all the time available into market work. The cases of full specialization are obtained as special cases of these two regimes. In the extreme case it is possible that the household buys household services from the market and household members allocate all the time available into market work. From the policy point of view the consideration of the corner solutions is important since households’ responses with respect to policy reforms may vary between household time allocation regimes.

For the model formulated in this essay the effects of exogenous changes in female household member’s wage are considered. The wage change is assumed to affect the decision power in the household according to the bargaining view. Increase in the relative share of individuals’ hourly wage out of household income implies that the decision power of the individual in question increases. The paper shows that in the collective model there is, besides the usual income and substitution effects, a distribution effect that describes the adjustment of the household consumption bundle as the decision power in the household changes.

The contributions of this paper are threefold. First it is shown how the household consumption bundle adjusts as a result of a change in the decision power in the household. In interior solution the private consumption of the individual always increases with the increase in his/her say in the household. This result is in accordance with the results obtained in Blundel et al. (2005). But when the corner solutions are taken into consideration this result no longer holds. This is because in corner solutions private consumptions depend on the relationship between the household members’ marginal utilities from the consumption of the household public good. As a result of this it is possible that the private consumption of the individual gaining more say in the household decreases. Therefore, the conclusion is that the wife’s private consumption increases as her say in the household increases if and only if she does not value the joint consumption of the household good too much relative to her husband.
Secondly, it is shown how the demand and time – market good – mix of the good produced in the household responses to the changes in the decision power in the household. The availability, substitutability, and prices of the inputs used in production process, all have an influence to the final composition of the household service. In the collective model, besides these, the adjustment of decision power in the household affects as well the composition of the good produced in household, i.e. on how individual time inputs are substituted with each other and with market goods in the household production process. The distribution effect either magnifies or dampens the positive income effect for more household goods. The sign of the distribution effect is shown to depend on the household members’ marginal utilities from the consumption of the household good relative to each other. The magnitude of the distribution effect is shown to depend on the substitutability of the inputs in the household production process. The more important the input in question is the larger is the adjustment in the use of that input as the decision power in the household changes.

Finally, it is shown that the change in the decision power in the household may induce shifts in household optimal time allocation regime. The optimum conditions for each time allocation regime depends on the decision power of the household members. The shadow prices for household work change with the increase in the wife’s wage and adjustment of the decision power in the household. Therefore the change in the decision power in the household may induce the household to move from one time allocation regime to another earlier or later than implied by the wage change alone.

The household production process outlined in the current paper deserves more attention. Further work is required to gain identification results needed for empirical work with this more realistic description of household production where the corner solutions are possible and where the domestic good is not tradable and is jointly consumed by the household members. This essay can be seen as spade work into introduction of joint consumption of the good produced in the household to the collective framework. The work is done with the general version of the collective household model. In order to gain the identification results important for empirical work there are two possibilities to proceed. It would be possible to try to identify the
sharing rule conditional on public consumption of the good produced in the household. However, the possible identification results would be only partial due to the dependence of the full price of the domestic good on household characteristics. Alternatively it would be possible to try to estimate the general version of the collective model directly as in Klaveren et al. (2006a,b).

**Essay II: Social Norms and Female Labour Supply**

The second part of the thesis analyses the effect of social norm for traditional gender roles on the female household member’s time allocation decisions. Both the unitary and bargaining household models predict that due to increase in female wages the household work time of the female household member should decline. However, traditional gender roles seem to be persistent in many Western societies despite the fact that female participation in the labour market has increased considerably. According to time use studies European women perform two thirds of all household work and mothers are mainly responsible for child care (Eurostat, 2004). Further, in Finland the wife does nearly two thirds of the all household work even in two earner households (Piekkola & Ruuskanen, 2006 and Takala, 2005). I argue here that it is possible that social norms and customs of the society in question have their effect on female time allocation decisions. Thus, if there is a social norm towards traditional division of labour in households, the resulting household time allocations differ from that predicted by the theory.

There is now a growing new literature on the relationship between social norms and individual time allocation decisions. Burda et al. (2006) study the distribution of total work (market work and household work) in the US and EU. The results show that gender differences in total work within a country are smaller than variation across countries and time. They formulate a theoretical model where the coordination device that equalizes total work across agents is social norm for leisure. Fernandez & Fogli (2005) and Fernandez (2007) examine the work and fertility behaviour of women born in the US, but whose parents were born elsewhere. It is shown that the cultural proxies, describing the social norm, have positive and significant explanatory power for individual work and fertility outcomes. Maurin & Moschion (2006) show with
French data for the years between 1990 and 2001 that a mother’s decision to participate in the labour market is correlated with those of the other mother’s living in the same neighbourhood. Social norms are already recognized in the current economic literature as having an important effect on individual behaviour. However, as far as I know, the effect of traditional gender roles has not yet been explicitly studied in the context of household decision making models.

In order to be able to analyse the effect of tradition on household behaviour social payoff function describing the norm for traditional division of labour in households is introduced into the general version of the collective household model with household production. The social payoff function is formulated so that the average female labour supply for the society in question represents the normative standard for female allocation of time. It is possible that the household members hold different view on the role of women in the society. It is shown that the response of the female household member’s time allocation with respect to strengthening norm for tradition depends on the household members’ social preferences and on the distribution of the decision power in the household. Four different cases can be separated.

For the case where neither of the spouses values the traditional division of time the stronger norm has naturally no effect on behaviour. For the case where only the wife cares about tradition it is shown that when her decision power in the household increases the effect of tradition on her time allocation diminishes. While for the case where only the husband cares about the norm for tradition and the more say the wife has in the household, the larger is the effect of tradition on the wife’s time allocation decisions. This result reflects the conflict of interests of the spouses when only the husband is socially minded and when the wife has more say in the household. Finally in the case where both household members value traditional gender roles there is U-shape relationship between female household member’s relative earnings and her household work. This result is in line with empirical results obtained with Australian and US data (Bitman et al., 2003) and more recently with Spanish data (Fernandez&Sevilla-Sanz, 2006). These empirical results are in contrast to what the exchange theory in sociology and both unitary and bargaining theories in economics predict. In sociological literature this result is seen as evidence from ‘doing gender’. This means that women earning more than their husbands seem to compensate with a
more traditional division of household work. Here similar result is obtained for the first time from a microeconomic model based on rational behaviour and utility maximization. Further, it is shown that stronger norm may either increase or decrease household utility. Whether the household utility increases or decreases as a response to stronger norm for traditional gender roles depends on the relationship between the spouses’ earnings. The household can be hurt by the norm for tradition if the household deviates from the normative standard according to which men make more money than women.

The implications of family policy are analysed as well in the context of the model formulated in this essay. It is shown that family policy can, depending on the policy measure, either reinforce or mitigate the effect of tradition on female labour supply. The workings of a direct transfer for the home care of the children versus of a market substitute for maternal care are considered. The results show that the transfer from the government for the home care of the children may strengthen the existing social norms for traditional gender roles, and as a consequence lead to lower than anticipated average female labour supply in the society in question.

In light of the results obtained in this essay the conclusion is that the way the policymaker can minimize the effect of tradition is by introducing market substitutes for household production and introducing parental leave policy targeted on fathers instead targeting parental leave totally on mothers or just leaving the decision to the households.

Further, while family policy can be seen as a means to increase the labour force participation of mothers, there may be important boomerang effects on the position of women overall if the benefits are seen as limiting female employees’ commitment to work. The norm for traditional gender roles in households can have important effect on the labour market status of women overall. If employers hold the norm for traditional division of labour in households and treat all female employees as potential mothers, the issue of motherhood may affect the labour demand of women overall – married or single, with or without children. If this is the case then the policy measures aimed for altering the traditional gender roles in households benefit women in general. Similar arguments within different context can be found from Datta Gupta et al. (2008 and 2006), Albrecht et al. (2001) and Lommerud et al. (2000). It would be possible to
consider this mechanism with the model outlined in this essay in general equilibrium framework where employers’ decisions to hire female workers are affected by the norm for tradition.

Still another possibility for future work is related to the significance of cultural factors in explaining household behaviour in developing economies. The World Bank policy report (2001) concludes that a thorough understanding of local gender systems is critical in ensuring that that the political and development programs are designed and implemented in a way that foster greater gender equality. The rejection of the cooperative household model with African data (Udry, 1996 and McPeak&Doss, 2006) may actually be a consequence of strong cultural factors and gender roles in developing economies. Rather that being evidence from non-cooperative household behaviour the failure of the cooperative models may be a consequence of the externality created by the cultural factors. This opens a new research agenda where the collective household decision making with the extension for social concerns should be analysed.

Essay III: Collective Household Model with Endogenous Balance of Power and Household Production

The third part of this thesis extends the general collective household production model into dynamic framework where time allocation decisions made in previous periods affect the decision power and allocation of resources in the subsequent periods.

Introducing dynamics into collective household model allowing distinct preferences for the spouses is very important since many household decisions are dynamic in nature and a decision made today affects the choice set available in the future. Examples of decisions potentially affecting the choice set available in the future are decisions about labour supply and fertility. It is argued in the literature that individual decision power in the household depends on individual’s actual earnings. If this is the case then labour supply decisions are both a matter of household decision and a determinant of the balance of power in the household. Basu (2006) was the first to suggest (in his (2001) working paper version) a model where the household balance
of power is determined endogenously by the household decisions. Basu (2006) shows that introducing dynamics into a collective household model with endogenous welfare weight is likely to lead into solutions that fail Pareto efficiency. Similar inefficiency result is obtained in other studies as well (Browning et al., 2004; Lundberg et al., 2003; Iyigun & Walsh, 2007; and Konrad & Lommerud, 2000). The main conclusion in the literature so far is that one cannot restrict attention on the efficient frontier when the spousal choices affect decision power in the household.

However, in most cases the existing studies concentrate on the allocation of private consumption in the household. Ignoring household production and public consumption in the household may lead into misleading conclusions about the allocation of resources in the household. I argue here that if the household members gain utility from joint consumption of the domestic good besides their private consumptions, it is not clear, whether introducing dynamics leads into inefficient situations. The question is whether the gains in the terms of household public good balance out the losses in decision power and private consumption due to the negative effect of household work on actual earnings.

This essay analyses female household member’s time allocation decisions and the demand for household public good in a framework where household production is explicitly modelled and there is a link between household decisions and the balance of power in the household. I assume that there exists a process of accumulation of human capital in market work that links the decisions of subsequent periods together. The time allocation decisions made in the household thus affect future behaviour since earnings are assumed to depend on the labour supply decisions made in previous period.

The household dynamic efficiency is analysed with a two period model where the gains from marriage come from the possibility to have children in the second period. In the first period the household members gain utility only from private consumption. Devoting time into rearing and caring children implies decreasing say in the household. If children can be seen as household public goods enhancing the parents’ utility, then the presence of the children implies an outward shift of the household utility possibility frontier. If the first and second period utility possibility frontiers are parallel, then every point in the outer second period frontier is Pareto preferred to an
allocation in the inner first period frontier. In this case the decision not to have children would imply inefficiency in the sense that the potentially Pareto improving move has not taken place.

It is shown that in the case where the household members can commit to refrain from exploiting the future bargaining advantage there always is a solution where the time allocated into household is greater than zero and where the household is on the efficient frontier. For the case of no-commitment the conclusion is that the household will have children when the marginal value from having children is larger for the wife than for the husband and when the spouses have identical preferences. In the case of identical preferences the model reduces into full efficiency model where the joint demand for the household public good does not depend on the decision power in the household. Finally, it is shown that even in the case where the marginal utility from having children is lower for the wife than for the husband, it is possible that the couple will have children if the marginal utility for the husband from having children is large enough. It is shown that in the case of no-commitment even a small exogenous transfer from the government guarantees that the household solution will be on the efficient frontier.

6. Conclusions

As usual this study opens more questions than which it is able to answer, both theoretically and empirically. There obviously is a need for empirical work with collective household models encompassing household production. So far the only application of the collective household model with Finnish data is that of Ruuskanen (1997). He tests collective consumption model and his results indicate that the sharing rule cannot be found with Finnish data. However, since the household production is not taken into consideration the results may be biased.

Further, the results obtained from the essays of this thesis clearly show the significance of household production and public consumption in the household as a determinant of household welfare. Specialisation according to traditional gender roles is not problematic as itself. In the modern society the value of household skills is lower than in the past. Therefore, as a consequence of asymmetrical accumulation of
human capital the traditional gender roles lead to limited consumption possibilities in the future for the spouse specializing in household work. This obviously is a challenge for policy design and calls for empirical assessment of the questions analysed in this thesis with Finnish data.

References


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Abstract

This paper considers joint consumption of the good produced in the household in the collective model for household behaviour. The novelty of this paper is in simultaneous consideration of public consumption, non-marketable household production and corner solutions in the collective household model. For the model formulated in this paper the effects of exogenous changes in the female household member’s wage are analysed. The change in wage is assumed to affect the decision power in the household according to the bargaining view. Therefore the results obtained differ from those in traditional framework. The contributions of the current paper are threefold. First it is shown how the household consumption bundle adjusts as a result of a change in the decision power in the household. Secondly, it is shown how the time – market good – mix of the good produced in the household responses to the changes in the decision power in the household. Finally, it is shown that the change in the decision power in the household may induce shifts in household optimal time allocation regime.

1. Introduction

This paper considers joint consumption of the good produced in the household in the collective framework for household behaviour. The main questions are, how the joint demand for the good produced in the household adjusts in the collective household model with respect to exogenous wage changes, and how the family members substitute for each others time and market goods in the production of the household public good.

The novelty of this paper is in the simultaneous consideration of public consumption, non-marketable household production and corner solutions in the collective household model. Each of these questions has been addressed in earlier
studies separately. The most serious problems in the existing literature are the unrealistic assumptions made on household production and on consumption of the domestic good. It is shown in the literature that the sharing rule in the collective household model can be identified for the private consumption of the tradable domestic good (Chiappori, 1997; Apps&Rees, 1997; and Donni, 2005). In reality the consumption of the good produced in the household is hardly assignable between the household members and it is unlikely that the good produced in the household is sold in the market. Therefore, in this paper the joint consumption of non-marketable household good is considered with the possibility of corner solutions to the household time allocation problem. Four different time allocation regimes are possible. In interior solution both household members allocate their time between household and market work. Then there are two regimes where only one of the spouses allocates time between household and market work while his/her spouse allocates all the time available into market work. The cases of full specialization are obtained as special cases of these two regimes. Finally, in the extreme case it is possible that the household buys household services from the market and household members allocate all the time available into market work. From the policy point of view the consideration of corner solutions is important since households’ responses with respect to policy reforms vary between household time allocation regimes.

**Background**

*Household production in the collective household model* Despite of the obvious importance of household production as a determinant of household welfare, household production has only very recently been considered in the collective household literature. Chiappori (1997) and Apps and Rees (1997) were the first to consider household production in the context of the collective household model. They analyse whether the sharing rule in the collective household model can be identified when the household production is introduced into the model. The results differ according to whether or not the good produced in the household is substitute for market goods. Chiappori (1997) concentrates to the case of tradable household goods. In this case the price for the good produced in the household is exogenously fixed in the market
and it is possible to identify the sharing rule. In the case of non-tradable household good, more assumptions on the decision process are needed to identify the sharing rule. Donni (2005) shows that, as long as the domestic good is marketable and the time inputs of the household members are perfect substitutes, the results obtained from the collective household model ignoring domestic production are unbiased. For the case where the individual time inputs are not perfect substitutes, the bias from ignoring domestic production in the collective model depends on the complementarity/substitutability of the spousal time inputs. In the all above cases where domestic production is introduced into the collective household model, only private consumption of the good produced in the household is considered. However, this is very unrealistic assumption since the good produced in the household can hardly be assigned between household members (for example, clean house and well being of the children). A better way to model household production would be the one allowing public consumption of the non-marketable domestic good.

Public consumption in the collective household model Considerations of public consumption in the context of the collective household model are still very rare due to difficulties related to the decentralization of household decision process in the presence of public consumption (Blundel et al. 2005; and Donni, 2006). Blundell et al. (2005) note that it is by now widely accepted that intra household distribution of income and decision power matters. Therefore targeting a benefit to a particular household member may have important effect on how the resources are eventually used. For example, several empirical studies show that an increase in the mother’s power within the household results in more expenditure made for children (Thomas, 1990; Schultz, 1990; Browning et al., 1994; Lundberg et al., 1997; Phipps&Burton, 1998; and Hoddinot&Haddad, 1995). However, according to Blundell et al. (2005) the theoretical foundations for the targeting view are very weak. This is because the methodological tool for analysing household behaviour has been the unitary model where the unique household utility function is maximized. This implies that targeting cannot be effective. In the collective model targeting matters through its impact on the Pareto weights describing the decision power in the household. Blundell et al. (2005) derive identification results for the collective model with public consumption in the household and show that a shift in the decision power in favour of one household
member always boosts his/her private consumption but the demand for household public good increases if and only if the marginal willingness to pay of this member is more sensitive to increases in her private consumption than that of the other member. However, only the interior solution to the household time allocation problem is considered and household production is not explicitly taken into consideration. From the above considerations the conclusion is that it is important to consider the public consumption of the domestic good in the collective household model.

*Time and goods* Time use studies show that, although women still perform the larger share of household work, the time devoted to household production has declined. Besides the increase in female labour supply during the past decades, at least two distinct reasons can be given for this development. First because of smaller family size, less time is needed for household chores (Knowles, 2005). Secondly there are significantly more and better market substitutes for household production available today than just few decades ago (Greenwood et al. 2005, and Freeman 2005). The use of market goods in the production process for household (public) goods is not usually considered in the applications of household production theory. Instead household production is assumed to be a function of household members’ time inputs only. The household production process considered in the current paper follows Becker’s (1965) original ideas, where individual time and market goods are combined to produce utility enhancing commodities. On the basis of the time use studies it can be argued that in a modern Western society the use of market goods in the production process (and thus possibly affecting on the resulting time allocation decision) is of crucial importance.

Here the general version of the collective household model is extended to allow for joint consumption of the good produced in the household with the possibility of corner solutions. For the model formulated in this paper the effects of exogenous changes in female household member’s wage are considered. The change in wage is assumed to affect the decision power in the household according to the bargaining view. That is the increase in the relative share of individuals’ hourly wage out of household income implies that the decision power of the individual in question increases. The paper shows that in the collective model there are, besides the usual
income and substitution effects a distribution effect that describes the adjustment of the household consumption bundle as the decision power in the household changes.

The contributions of this paper are threefold. First it is shown how the household consumption bundle adjusts as a result of a change in the decision power in the household. In interior solution the private consumption of the individual always increases with the increase in his/her say in the household. This result is in accordance with the results obtained in Blundel et al. (2005). But when the corner solutions are taken into consideration this result no longer holds. This is because in corner solutions private consumptions depend on the relationship between the household members’ marginal utilities from the consumption of the household public good. As a result of this it is possible that the private consumption of the individual gaining more say in the household decreases. Therefore, the conclusion is that the wife’s private consumption increases as her say in the household increases if and only if she does not value the joint consumption of the household good too much relative to her husband.

Secondly, it is shown how the demand and time – market good -mix of the good produced in the household responds to the changes in the decision power in the household. The availability, substitutability, and prices of the inputs used in production process, all have an influence to the final composition of the household service. In the collective model, besides these, the adjustment of decision power in the household affects as well the composition of the good produced in household, i.e. on how individual time inputs are substituted with each other and with market goods in the household production process. The distribution effect either magnifies or dampens the positive income effect for more household goods. The sign of the distribution effect is shown to depend on the household members’ marginal utilities from the consumption of the household good relative to each other. The magnitude of the distribution effect is shown to depend on the substitutability of the inputs in the household production process. The more important the input in question is the larger is the adjustment in the use of that input as the decision power in the household changes.

Finally, it is shown that the change in the decision power in the household may induce shifts in household optimal time allocation regime. The optimum conditions
for each time allocation regime depend on the decision power of the household members. The shadow prices for household work change with the increase in the wife’s wage and adjustment of the decision power in the household. Therefore the change in the decision power in the household may induce the household to move from one time allocation regime to another earlier or later than implied by the wage change alone.

The paper is organized as follows. Section two presents the model, and section three provides the general lines of the solution process for household resource allocation problem in the collective model with household production. Explicit solutions for the special case of Cobb-Douglas family are presented in section four, and section five concludes.

2. The collective model with household production

The general version of the collective model is flexible enough to allow for simultaneous consideration of household production, joint consumption of the domestic good and the possibility of corner solutions. Therefore, the general version of the collective household model is considered here. The emphasis is on household production typical in a modern Western society, where households do not sell commodities produced in the household, but it is possible that household services are bought from the market. The household good can be produced by using various combinations of individual time inputs and market goods. Household members are assumed to be indifferent on how certain household good is produced. This means that utility is gained only from consumption of the household good and not from different uses of time.

Preferences

It is assumed here that the household consists of two members who may or may not participate in the labour market. Both household members have well defined twice differentiable utility functions and they gain utility from private consumptions of the
market goods \( x^i, i = f, m \), for the female and male household member respectively and from the joint consumption of the good produced in the household \( G \).

Chiappori (1992) does not introduce any particular assumption on individual preferences, except that they can be represented by conventional utility functions. Intra-household consumption externalities, altruism or any other preference interaction is allowed. Further, Chiappori (1992) argues that the household can be seen as an example of repeated game and it can be assumed that each person knows the preferences of the other people in the household. The most general form of preferences would be the case of altruism \( U^i(x^i, x^j, G) \), where household members’ private consumptions \((x^i, x^j)\) enter into individual utility functions together with the joint consumption of the household good \( G \). According to Chiappori (1992) it would be reasonable to assume that \( U^i \) is strictly increasing in \( x^i \) and in \( G \) but not necessarily in \( x^j \). The most restrictive case for preferences is the case of egoistic preferences where \( U^i(x^i, G) \). An intermediate case is Becker’s (1974) notion on caring in which each household member has a welfare function that depends on both own and companion’s egoistic utilities \( W^i[U^i(x^i, G)U^j(x^j, G)] \), \( i \neq j \). According to Browning and Lechene (2001) all combinations of preferences are possible. This means that one household member can be egoistic while the other may be altruistic.

Here it is assumed that the spouses have egoistic preferences. Individual preferences can be written as \( U^i(x^i, G) \), \( i = f, m \). In the case of egoistic preferences the results obtained depend on household members’ valuations on their own private consumptions versus their valuations on the joint consumption of the household good. The assumption of egoistic preferences implies that the consumption of the household member \( j \neq i \) has no effect on the utility of the household member \( i \), that is \( U^j = 0 \). The formulation of egoistic preferences is chosen because it makes the analysis of the
effects of individual wages and the decision power in the household to the final demands tractable. Further, assume that $U_i^i > 0, U_{x}^i < 0$ and $U_G^i > 0, U_{GG}^i < 0$, $i = f, m$. Applicability of the household production theory requires that household production must be separable from household consumption. Therefore it is assumed that $U_{xG}^i = U_{Gx}^i = 0$, $i = f, m$ which implies separability of private consumption versus joint consumption of the domestic good in individual utility functions.

In most applications of the collective household model, individual utility is determined over the individual private consumption of market goods $x^i$ and individual consumption of good G produced in the household. In this case individual utility is written as $U_i^i(x^i, G)$ and $G = G^i + G^j$. I want to stress the public good nature of household production and this is why I assume that individual i gains utility from joint consumption of the good G produced in the household. Since the formulation of egoistic preferences is used the gains from marriage result solely from the joint consumption of household public goods. The private goods of the spouses are assumed to be exclusive and the household public good is assumed to be non-exclusive. The examples can be clean, warm house and well behaving children with clean clothes. In reality many domestic goods are impure public goods and can be difficult to separate from leisure and private goods. For example, food preparation is very problematic. Meals can be seen as exclusive private goods and the time used for preparing the meals can be considered as leisure if the individual enjoys cooking. Instead house cleaning can be considered as work not giving pleasure in itself and it can be seen as non-assignable household public good. The assumption about pure public goods is made here in order the household production theory to be applicable.

1 If more general structure of preferences, for example caring, were allowed the results obtained would depend on household members’ valuations on their own private consumptions versus their valuations on the private consumptions of their spouse and joint consumption of the household good.

2 The main reason behind this assumption is that in this case the Pareto efficient household allocation problem can be decentralized into a two stage budgeting process. The decentralization results derived by Chiappori (1988, 1992) hold only if consumption is purely private.
Household production and joint budget constraint

Household production is defined here so that it satisfies the third party rule implying that it is possible to substitute market goods and services for one’s own time. It is possible to pay someone to perform these tasks but the household members are not paid for performing them. Note that leisure and tertiary activities do not satisfy the third party rule. Each domestic good can be thought to have a specific production function that reflects availability and/or substitutability of the inputs. The production function for household good \( G \) is:

\[
G = g(\tilde{H}^i, \tilde{H}^m, a^g x^g)
\]

(1)

where

\[
\tilde{H}^i = \mu^i + a^i H^i
\]

= individual intermediate product

\( \mu^i \) = stock of durable domestic goods or standard for household production

\( H^i \) = individual time input

\( a^i \) = individual productivity at household work

\( x^g \) = market goods used in household production

\( a^g \) = productivity parameter relating to the use of market goods

Spouses’ time inputs are combined with market goods in the production of the household good \( G \). When \( a^i \) increases the amount of individual time input \( H^i \) needed to produce the required amount of the intermediate product \( \tilde{H}^i \) and thus the final product \( G \) decreases. Similarly, the higher the value of the parameter \( \mu^i \) the less individual time, \( H^i \), is needed for the individual intermediate product \( \tilde{H}^i \). The graph describing the relationship between the components of the individual intermediate product is presented in the Figure 1.
Different consumer durables imply different levels for $\mu^i$. For example, $\mu^i$ for a vacuum cleaner is higher than that for a broom stick and $\mu^i$ for a dishwasher is higher than that for a sink. If instead the interpretation given on $\mu^i$ reflects the standard for household production (inherited from the childhood home, for example), then high $\mu^i$ would imply more market prone production for $G$ and low $\mu^i$ would imply self production.

The model at hand does not consider leisure demand, since the focus of the analysis is in the interaction of time and market goods in the production of household goods. The total time available $T$ is assumed to be allocated between market work and household production, therefore $T = L^i + H^i$, $i = f, m$. Since only the effects of exogenous wage changes are considered the other sources of household income are not considered here. With the assumptions made here the household budget constraint where the time budgets for the spouses are inserted into the household budget constraint is the following:
The prices for market goods consumed privately are normalized to 1 while the price for the market goods used in the production of G is denoted by p.

Individual’s time input in household production $H^i$ can’t exceed the total amount of time available $T$. The time input can be zero for both spouses in the extreme case where household services are bought from the market. Therefore we have $0 \leq H^i \leq T$, $i = f, m$. But if $H^i = T$ then $H^j < T$, $i \neq j$, the time input can be positive for both spouses but it can not equal $T$ for both unless there is non-labour income in the model. The case of full specialization is possible as well, implying $H^i = 0$ and $H^j = T$, $i \neq j$.

Normalize the total time available to one, $T = 1$, and rearrange (2) so as to isolate the uses of income, including expenditure on household time, on the left hand side of the budget constraint and the sources of income on the right hand side. We have:

$$x^f + x^m + px^g + w^f H^f + w^m H^m = w^f + w^m \equiv Y$$

This is the household full budget constraint. Where $w^f + w^m \equiv Y$ is the household potential income. The household potential income $Y$ would occur if both household members allocate all the time available into market work. Further, it is assumed that hourly wages do not depend on the hours worked.

**The welfare weight**

In the general version of the collective household model it is assumed that there exists a welfare weight $\theta$ belonging to $[0,1]$. The allocation of resources can be solved from the household utility function of the form:
\[ \Omega = \theta U^f + (1 - \theta)U^m \]

where

\[ \theta = \text{welfare weight or the Pareto weight} \]
\[ U^i = \text{individual utility, } i = f, m \]

The household joint budget constraint defines the Pareto frontier for given utility functions for the spouses. The outcome of household’s utility maximization process will be located on this frontier. Then \( \theta \) determines the final location on this frontier. In general the welfare weight depends on prices and income. In the labour supply model the price variation comes through wages and therefore the welfare weight is a function of the hourly wages of the household members. If non-labour income were considered it would affect the decision power in the household as usual in the collective labour supply model.

Chiappori (1992) gives two opposite examples on how the decision power in the household, implied by the welfare weight, is affected when the wage of one of the household member’s increases. The first interpretation emphasises redistribution. If there are transfers between the spouses to compensate the inequalities in wage incomes within the household, the increase in member \( i \)’s wage ameliorates the member \( i \)’s situation and therefore reduces the need for compensating transfer in his favour. This implies that the welfare weight for the member \( j \neq i \) should increase in this case. The other interpretation stresses the bargaining process and leads into inverse conclusion. Now the increase in the individual \( i \)’s wage increases his bargaining strength since he would be better off in the case of divorce than before. In this case the welfare weight for the individual \( j \neq i \) should decrease. The existing empirical literature suggests that individual’s decision power in the household depends on his/her earnings (Blundel et al., 2007; Browning & Lechene, 2001; and Browning et al., 1994). Therefore the interpretation given here on the relationship between the welfare weight and individual wages emphasises the bargaining view. In the analysis that follows I concentrate on the effects of exogenous changes in the female household member’s wage on household behaviour. Change in the wage of the wife implies that
the decision power in the household adjusts through the adjustment the welfare weight $\theta$ in the wife’s favour. The increase in the wife’s wage implies thus that $\frac{\partial \theta}{\partial w^f} > 0$ since $\theta$ is the weight given for the wife’s preferences. The bargaining view applied here implies further that $\frac{\partial \theta}{\partial w^m} < 0$, $\frac{\partial (1-\theta)}{\partial w^m} > 0$ and $\frac{\partial (1-\theta)}{\partial w^f} < 0$. This completes the model. Now the household problem outlined in this section can be written as follows:

$$
\begin{align*}
\text{Max}_{x^f, x^m, G} & \Omega = \theta U^f(x^f, G) + (1-\theta) U^m(x^m, G) \\
\text{s.t.} & \\
& x^f + x^m + px^g = (1-H^f)w^f + (1-H^m)w^m \\
& G \leq g(\mu^f + a^f H^f, \mu^m + a^m H^m, a^g x^g) \\
& H^f, H^m \geq 0 \\
& x^f, x^m, x^g > 0 \\
& 1 = H^f + L^f, i = f, m
\end{align*}
$$

3. The allocation of resources

The household resource allocation problem with household production can be solved in two stages (see, for example, Gigno, 1991). In the first stage the cost minimization problem is solved for the time-market good mix that minimizes the income forgone $\left( w^f H^f + w^m H^m + px^g \right)$ in order to attain desired level of the good produced in the household G. After this household consumption allocation problem is solved in stage two.
3.1 Time allocation problem

The cost minimization problem for the first stage can be written here as follows:

\[
\min_{H^f, H^m, x^g} w^f H^f + w^m H^m + px^g
\]

s.t.

\[
G \leq g(\tilde{H}^f, \tilde{H}^m, a^g x^g) \\
H^f \geq 0 \\
H^m \geq 0 \\
1 = H^i + L^i, i = f, m
\]  \hspace{1cm} (6)

The restrictions for the time inputs follow from the assumption that H's are not necessary for the production of the household good. It is possible that the household buys services from the market without using spouses’ time as inputs in the production process. Remember that individual intermediate products are determined as \( \tilde{H}^i = \mu^i + a^i H^i \), where \( \mu^i \) describes the existing stock of durable domestic production goods individual \( i \) has brought into this household or standard for household production. Thus \( H^i = 0 \) implies that \( \tilde{H}^i = \mu^i \) in household production function. The Lagrangian function for the problem above is:

\[
L = w^f H^f + w^m H^m + px^g - \lambda_g \left[ g(\mu^f + a^f H^f, \mu^m + a^m H^m, a^g x^g) - G \right] 
\]  \hspace{1cm} (7)

The Kuhn-Tucker conditions are:

\[
\frac{\partial L}{\partial H^f} = w^f - \lambda_g a^f \geq 0 \\
H^f \geq 0, H^f \frac{\partial L}{\partial H^f} = 0
\]  \hspace{1cm} (8)
\[ \frac{\partial L}{\partial H^m} = w^m - \lambda g^m a^m \geq 0 \]
\[ H^m \geq 0, H^m \frac{\partial L}{\partial H^m} = 0 \quad (9) \]

\[ \frac{\partial L}{\partial x^g} = p - \lambda g^g a^g = 0 \quad (10) \]

\[ \frac{\partial L}{\partial \lambda} = G - g(\mu^f + a^f H^f, \mu^m + a^m H^m, a^g x^g) \leq 0 \]
\[ \lambda \geq 0, \lambda \frac{\partial L}{\partial \lambda} \quad (11) \]

From (10) we have \( \lambda = \frac{p}{g^g a^g} = \frac{\partial C(G)}{\partial G} \) the marginal cost of the household good.

Setting \( \lambda = 0 \) would violate (10). Therefore, we have \( \lambda > 0 \) and \( \frac{\partial L}{\partial \lambda} = 0 \) by complementary slackness. Now the optimum conditions can be written as:

\[ w^f - \frac{g^f}{g^x} a^f \frac{p}{a^g} \geq 0 \quad (8') \]
\[ H^f \geq 0, H^f \frac{\partial L}{\partial H^f} = 0 \]

\[ w^m - \frac{g^m}{g^x} a^m \frac{p}{a^g} \geq 0 \quad (9') \]
\[ H^m \geq 0, H^m \frac{\partial L}{\partial H^m} = 0 \]

\[ g(\mu^f + a^f H^f, \mu^m + a^m H^m, a^g x^g) = G \quad (11') \]

It follows that there are four cases to consider depending on whether the constraints for individual time inputs \( H^i, i = f, m \) are binding or not. The first case is where both time inputs are zero i.e. \( H^i = 0, i = f, m \). In this case household members buy
services from the market instead of producing them by themselves. This case is likely to be relevant for couples without children concentrating on their careers. Examples are restaurant meals, house cleaning, and laundry services. Since \( H^i = 0, i = f,m \) complementary slackness implies that \( \frac{\partial L}{\partial H^i} > 0 \). The optimum conditions for this case are:

\[
\begin{align*}
    w^f &> \frac{g_{i^f}}{g_{x^f}} a^f \frac{p}{a^g} \\
    w^m &> \frac{g_{i^m}}{g_{x^g}} a^m \frac{p}{a^g} \\
    g(\mu^f, \mu^m, a^g x^g) &= G
\end{align*}
\]  

(12)

Both household members allocate all the time available into market work since their market wages exceed their shadow wages for household production. Put otherwise, the implicit prices of the spouses’ time inputs are larger than the marginal cost of household service that is \( \frac{w^i}{g_{i^i} a^i} > \frac{p}{g_{x^g} a^g} = \frac{\partial C(G)}{\partial G}, i = f,m \). The relevant cost function for the first case, where household services are bought from the market, is of the form \( C\left(G, \frac{p}{a^g}\right) \).

The second case, where \( H^f > 0 \) and \( H^m = 0 \), is possible if the following conditions are satisfied:

\[
\begin{align*}
    w^f &= \frac{g_{i^f}}{g_{x^f}} a^f \frac{p}{a^g} \\
    w^m &> \frac{g_{i^m}}{g_{x^g}} a^m \frac{p}{a^g} \\
    g(\mu^f, \mu^m, a^g x^g) &= G
\end{align*}
\]  

(13)

Now the wife’s market wage is equal to her shadow wage rate and she allocates her time between market work and household work. The husband’s market wage exceeds
his shadow wage and he allocates all the time available into market work. The cost function in this case is of the form $C_{III} \left( G, \frac{p}{a^g}, \frac{w^f}{a^f} \right)$.

The third case is symmetrical to the second one and we have $H^f = 0$ and $H^m > 0$. The husband allocates his time between market work and household work while the wife allocates all the time available into market work, when the following conditions are satisfied:

$$w^f > \frac{g m^f}{g_{x^g}} a^f \frac{p}{a^g}$$
$$w^m = \frac{g m^m}{g_{x^g}} a^m \frac{p}{a^g}$$
$$g(\mu^f, \mu^m, a^g x^g) = G$$

The cost function for this case is of the form $C_{III} \left( G, \frac{p}{a^g}, \frac{w^m}{a^m} \right)$.

The fourth possible case is the interior solution to the household time allocation problem. In this case both time inputs in the production of the household service are positive, $H^i > 0$, $i = f, m$. Complementary slackness implies $\frac{\partial L}{\partial H^i} = 0$, and the optimum conditions are:

$$w^f = \frac{g m^f}{g_{x^g}} a^f \frac{p}{a^g}$$
$$w^m = \frac{g m^m}{g_{x^g}} a^m \frac{p}{a^g}$$
$$g(\mu^f, \mu^m, a^g x^g) = G$$

From these it is immediately seen that in interior solution
The marginal rate of technical substitution of $H^f$ for $H^m$ is equated to the wage ratio. The spouses allocate their time between market work and household production. The associated cost function is of the form $C^IV(G, \frac{p}{a^g}, \frac{w^f}{a^f}, \frac{w^m}{a^m})$.

Besides the wage changes productivity changes in the household sector influence the demand for individual time inputs. Here fully cooperative model of household behaviour is analysed and the rise in individual productivity parameter $a^i$ implies, that the implicit price of that individual’s time input in household production decreases, and therefore the demand for household good and household time is higher. However, when the productivity is high less time is required to produce a given household service and, therefore more time is left for market work. Here the decision power in the household affects the net result since the demand for the joint consumption of the household good depends on the welfare weight determining the decision power in the household. All possible household time allocation regimes for the household production process outlined here with the special case of linear technology are presented graphically in the Appendix.

### 3.2 Consumption allocation

After having solved the household time allocation problem the household consumption allocation is solved in the stage two from:

$$\begin{align*}
\max_{x^f, x^m, G} \Omega &= \theta U^f(x^f, G) + (1 - \theta) U^m(x^m, G) \\
\text{s.t.} & \quad x^f + x^m + C(G) = w^f + w^m \equiv Y
\end{align*}$$

(17)

The household full budget constraint derived in (3) is presented in (17) in the form where $C(G)$ is the minimized cost of producing the desired amount of household
service $G$. The maximization problem in (17) generates the Pareto frontier. Note that
the relevant cost function $C(G)$ depends on the household time allocation regime. The
Lagrangian for the above maximization problem is:

$$ L = \theta U^f(x^f, G) + (1 - \theta) U^m(x^m, G) + \gamma(w^f + w^m - x^f - x^m - C(G)) $$

(18)

The first order conditions are the following:

$$ \frac{\partial L}{\partial x^f} = \theta U^f_x - \gamma = 0 $$

(19)

$$ \frac{\partial L}{\partial x^m} = (1 - \theta) U^m_x - \gamma = 0 $$

(20)

$$ \frac{\partial L}{\partial G} = \theta U^f_G + (1 - \theta) U^m_G - \gamma \frac{\partial C(G)}{\partial G} = 0 $$

(21)

$$ \frac{\partial L}{\partial \gamma} = w^f + w^m - x^f - x^m - C(G) = 0 $$

(22)

Solve (21) for $\gamma$ and insert the result into (19) and (20). The first order conditions can
be written as follows:

$$ \theta U^f_x = \theta U^f_G + (1 - \theta) U^m_G \frac{\partial C(G)}{\partial G} $$

(19')

$$ (1 - \theta) U^m_x = \theta U^f_G + (1 - \theta) U^m_G \frac{\partial C(G)}{\partial G} $$

(20')

$$ w^f + w^m = x^f + x^m + C(G) $$

(22')

The above system can be analysed for each possible time allocation regime by using
the relevant cost function derived in stage one time allocation problem. The cost
function can be written as \( C(G) = p^G G \), where \( p^G \) stands for the full price (time plus money) of the household service. This price depends on the household time allocation regime and is different for each case. Note as well that \( \frac{\partial C(G)}{\partial G} = p^G \).

The interest here is on how the composition of the household consumption bundle changes with respect to changes in individual hourly wage and thus in the decision power in the household. Since the welfare weight, determining the decision power in the household, is assumed to adjust with respect to household members’ wages according to the bargaining view, the increase in the wife’s wage implies that her say in the household increases. Special attention is given on the responses of the joint consumption in the household. How the demand and composition of the good produced in the household responses with respect to exogenous changes in female household member’s wage and decision power in the household. Similar analysis naturally applies for the increase in the male household member’s wage implying less say for the wife. By taking total differentials of the system in (19’)-(22’) and then applying Cramer’s rule we obtain the comparative static derivatives of interest here.

The adjustment of the household optimal consumption bundle with respect to changes in the decision power in the household is called from now on as ‘the distribution effect’. The distribution effect tells how the household consumption adjusts as a result of change in the decision power in the household. Since according to the targeting view (Blundell et al., 2005) increase in the resources in the individual’s control increases his/her consumption even in the case when there are no income effects we consider first separately the distribution effect. The reaction of female household member’s private consumption when her say in the household increases is found to be:

\[
\frac{dx^f}{d\theta} = \\
\begin{bmatrix}
-\frac{1}{p^G} (1-\theta) U^m_{x^f x^e} - (U^f_{x^f} + U^m) \left( \theta U^f_{x^f} G^G + (1-\theta) U^m_{x^f} G^G \right) \\
+ (1-\theta) U^m_{x^f x^e} (U^f_{x^f} - U^m_{x^f})
\end{bmatrix}
\frac{1}{|\varphi|} > 0
\]  

(23)
Where $|l| > 0$ is the system determinant. It is seen that the adjustment of the wife’s private consumption, when her say in the household increases, is the sum of three effects. First there is substitution away from the husband’s private consumption in favour of the wife’s private consumption. Secondly there is substitution away from the joint consumption of the household public good in favour of private consumption. These two effects are always positive. The third term describes substitution towards/away from the joint consumption of the household good. This can be either positive or negative depending on individual marginal utilities from the consumption of the household good. The response of the wife’s private consumption with respect to increase in her decision power in the household is guaranteed to be positive if and only if $U_G^f \leq U_G^m$. This means that the wife’s private consumption increases as her say in the household increases only if she does not value the joint consumption of the household good too much compared to her husband. If instead, $U_G^f > U_G^m$, it is possible the third effect is negative and that it outweighs the other two effects. If this is the case the wife’s private consumption decreases as a response to the increase in her say in the household.

For the responses of the husband’s private consumption when the wife’s say in the household increases we have:

$$
\frac{dx^m}{d\theta} = \frac{\theta U^f_{x^m} \frac{1}{p_G} U^m_{x^m} + (U^f_{x^m} + U^m_{x^m}) \frac{1}{p_G} (\theta U^f_{GG} + (1 - \theta)U^m_{GG})}{|l|} 
+ \theta U^f_{x^m} (U^f_G - U^m_G) < 0
$$

This result is of course the opposite of that obtained for $x^f$, since $\theta$ describes the weight given to the female household member’s preferences. Thus, the husband’s private consumption declines when $\theta$ increases unless his preference for the household service is relatively high (implying low $x^m$ in the first place). We see that
the derivative \( \frac{dx^m}{d\theta} \) is guaranteed to be negative if and only if \( U'_G \geq U'_G \). Otherwise the effect of \( \theta \) on \( x^m \) can be either positive or negative.

The joint consumption of the household good responses to the increase in the wife’s decision power in the household as follows:

\[
\frac{dG}{d\theta} = \left[ \left( U^f_{x^f} \left( 1 - \theta \right) U^m_{x^m, x^m} - U^m_{x^m, x^m} \theta U^f_{x^f, x^f} \right) - \left( \theta U^f_{x^f, x^f} + (1 - \theta) U^m_{x^m, x^m} \right) \frac{1}{p_G} \left( U^f_{G} - U^m_{G} \right) \right] < 0 \tag{25}
\]

When the wife’s say in the household increases there is negative cross substitution effect for the husband’s private consumption (first term in the numerator) and positive cross substitution effect for the wife (second term in the numerator). Together these two are positive indicating that the wife’s private consumption increases as her say in the household increases. The last term in the numerator describes substitution towards/away from the consumption of the household good. The sign of this effect depends on the relationship between individual marginal utilities from the joint consumption of \( G \). This is positive if the wife prefers relatively more the consumption of the household service than her husband and negative if she values \( G \) relatively less than her husband.

Blundell et al. (2005) show that a shift in the decision power in favour of one household member always boosts his/her private consumption. The results obtained here show that when the corner solutions are taken into consideration this result no longer holds. In the corner solutions the private consumptions depend on how the household members value the joint consumption of the domestic good relative to each other. As a result of this it is possible that the private consumption of the individual gaining more say in the household decreases. Therefore, the conclusion is that the wife’s private consumption increases as her say in the household increases if and only
if she does not value the joint consumption of the household good too much relative to her husband.

Consider next the total effect of an increase in the female household member’s wage on the joint consumption of the household public good. The effect of exogenous increase in the female household member’s wage for the demand of household service $G^*(\theta,Y,p^G)$ is found from the following:

$$\frac{dG^*}{dw^f} = \frac{\partial G^*}{\partial \theta} \cdot \frac{\partial \theta}{\partial w^f} + \frac{\partial G^*}{\partial Y} \cdot \frac{\partial Y}{\partial w^f} + \frac{\partial G^*}{\partial p^G} \cdot \frac{\partial p^G}{\partial w^f} > 0$$

The first term on the right hand side describes the distribution effect due to a change in the decision power in the household as a result of the increase in $w^f$. The sign of the distribution effect depends on the sign of the derivative $\frac{\partial G^*}{\partial \theta}$ since $\frac{\partial \theta}{\partial w^f}$ is always positive in the current setting. It was shown above that the sign of the derivative $\frac{\partial G^*}{\partial \theta}$ can be either positive or negative and the result depends on the relationship between individual marginal utilities from private consumption and from the joint consumption of $G$. The second term is the income effect, always positive if $G$ is normal good. The third term describes substitution away from the use of the female household member’s time in the production of $G$, this effect is always negative since $\frac{\partial G^*}{\partial p^G}$ is negative and $\frac{\partial p^G}{\partial w^f}$ is positive. However, it has to be noted that the full price (time plus money) of the household good varies with the household time allocation regime. The reason for this is that due to the limitations in input usage in corner solutions the full price of the good produced in the household depends on the level of the household good demanded. Therefore, the full price of the household good depends on the welfare weight $\theta$ in corner solutions to the household time allocation problem. This implies that the adjustment of the full price for the household good depends on how the household members’ value the joint consumption in the household relative to each other. In fact for the increase in $w^f$ we have:

$$\frac{dp^G}{dw^f} = \frac{\partial p^G}{\partial w^f} + \frac{\partial p^G}{\partial \theta} \cdot \frac{\partial \theta}{\partial w^f} < 0.$$
The derivative $\frac{\partial p^G}{\partial \theta}$ describes the response of the full price for the domestic good with respect to the adjustment of the decision power in the household. It is difficult to analyse in the general setting of the current section. Therefore here the response of the demand for household good with respect to the exogenous increase in the female household member’s wage is analysed only for the interior solution to the household time allocation problem. The corner solutions are considered in the next section where more structure is introduced into the model at hand.

The response of the joint consumption of the household public good with respect to the increase in female household member’s wage in interior solution is found to be:

$$\frac{dG}{dw} = \left[ \begin{array}{c} \frac{\partial \varphi}{\partial w} \left\{ \left( U_x f \left( 1 - \theta \right) \frac{U_{x,x^m} - U_x^m \theta U_{x,x^f}}{1 - \theta} \right) \right. \\
- \left. \left( \theta U_{x,x^f} + (1 - \theta) U_{x,x^m} \right) \frac{1}{p^G} \left( U_G^f - U_G^m \right) \right\} \\
+ \frac{\partial Y}{\partial w} \left( 1 - \theta \right) U_{x,x^m} \\
\frac{\partial p^G}{\partial w} \left\{ \left( \theta U_{x,x^f} + (1 - \theta) U_{x,x^m} \right) \frac{\theta U_G^f + (1 - \theta) U_G^m}{(p^G)^2} \right\} \\
- \theta U_{x,x^f} \left( 1 - \theta \right) U_{x,x^m} \frac{G}{\eta} \right] > 0 \quad \text{< 0}$$

It is seen that in the interior solution the total effect of the change in female household member’s wage consists of four effects. These are the distribution effect, the income effect, and two substitution effects. The distribution effect describes the adjustment of the household consumption bundle according to the female household member’s preferences. There is adjustment between the two private consumptions and adjustment between private consumptions and the joint consumption of the household good. This means that the optimal composition of household consumption bundle changes when $\theta$ adjusts. The income effect is always positive if the household good is a normal good. When the household income increases, more household good is
demanded. Finally the negative substitution effect consists of two parts. First there is substitution away from the use of now relatively more expensive female time input in the production process for the household service. Secondly household service is substituted for private consumption when the price of G increases. We can conclude that the adjustment of the welfare weight either magnifies or dampens the positive income effect for the demand for G depending on which one of the spouses is getting more say in the household the one preferring relatively more of G or the one preferring relatively less of it.

If the relationship between the household members’ marginal utilities is such that $U_G^f > U_G^m$, the distribution effect magnifies the income effect as the wage of the wife increases. Therefore, in this case the resulting demand for the household good is higher than in the case of the unitary household model. Alternatively when $U_G^f < U_G^m$ the negative distribution effect dampens the income effect and therefore the resulting demand for the household good is lower than in the case of unitary household model. With identical preferences the distribution effect is zero, and we get the familiar result from the unitary model where the wage effect is positive if income effect dominates the negative substitution effects. In this case less domestic good is demanded and the desired amount is produced by using less female time.

As noted earlier both the full price of the household good and the decision power in the household adjust as wages change. Therefore, the resulting time allocation regime may differ from that implied by comparative advantage alone. If the welfare weight, $\theta$, shifts in favour of the individual preferring more G the cut in demand for G due to higher price is smaller than it would be in the case of unitary model. Since more of the household good is demanded, the demand for the inputs used in the production process is higher as well. The key question is that does the adjustment of decision power in the household distort the time allocation structure based on comparative advantages? Is there more or less specialisation into different chores? These questions are adhered in the next section where more structure into the model at hand is introduced.
4. The example of Cobb-Douglas household

In order to get tractable results on how the joint consumption and the optimal time–market good–mix of the household good responses to changes in individual wages in corner solutions some structure into the model at hand has to be added. Here the household utility is formulated as spouses having Cobb-Douglas preferences with differing tastes towards private consumption versus the joint consumption of the good produced in the household. The household resource allocation problem is solved in two stages as in the previous section where the general version of the model was analysed. In the first stage the cost minimization problem is solved for the time-market good mix that minimizes the income forgone \((w^f H^f + w^m H^m + px^g)\) in order to attain desired level of household service \(G\). After this household consumption allocation problem is solved in stage two. The stage one cost minimization problem is solved with respect to constant returns to scale technology of the form:

\[
G = \left(\frac{H^f}{\bar{H}^f}\right)^\eta \left(\frac{H^m}{\bar{H}^m}\right)^{(1-\rho)} \left(\frac{a^g x^g}{\bar{a}^g x^g}\right)^{1-\nu}
\]  

(27)

Where, the inputs in production of the household good are specified as in the general version of the model at hand.

4.1 Time allocation problem

The solution process for the household time allocation problem goes exactly along the lines presented in the section three of this study where the general version of the model was analysed. The solution process is replicated here since the explicit forms for the cost functions and the optimum conditions for each possible household time allocation regime are needed.

The household time allocation problem is now solved from the following:
\[
\begin{align*}
\text{Min} & \quad w^f H^f + w^m H^m + px^g \\
\text{s.t.} & \\
\left(\mu^f + a^f H^f\right)^{\rho} \left(\mu^m + a^m H^m\right)^{\psi(l-p)} \left(a^g x^g\right)^{\psi} \geq G \\
H^f & \geq 0 \\
H^m & \geq 0 \\
1 & = H^f + L^i, \ i = f, m
\end{align*}
\] (28)

Individual time available for household and market work is normalized to one as in the general version of the model considered in the previous section of this essay.

Lagrangian function for the household problem in (28) is:

\[
L = w^f H^f + w^m H^m + px^g \\
- \lambda \left[\left(\mu^f + a^f H^f\right)^{\rho} \left(\mu^m + a^m H^m\right)^{\psi(l-p)} \left(a^g x^g\right)^{\psi} - G\right]
\] (29)

The Kuhn-Tucker conditions are:

\[
\frac{\partial L}{\partial H^f} = w^f - \lambda \nu \varphi \left(\mu^f + a^f H^f\right)^{\rho-1} \left(\mu^m + a^m H^m\right)^{\psi(l-p)-1} \left(a^g x^g\right)^{\psi-1} a^f \geq 0 \\
H^f \geq 0, \ H^f \frac{\partial L}{\partial H^f} = 0 
\] (30)

\[
\frac{\partial L}{\partial H^m} = w^m - \lambda \nu (1-p) \left(\mu^f + a^f H^f\right)^{\rho} \left(\mu^m + a^m H^m\right)^{\psi(l-p)-1} \left(a^g x^g\right)^{\psi-1} a^m \geq 0 \\
H^m \geq 0, \ H^m \frac{\partial L}{\partial H^m} = 0
\] (31)

\[
\frac{\partial L}{\partial x^g} = p - \lambda (1-\nu) \left(\mu^f + a^f H^f\right)^{\rho} \left(\mu^m + a^m H^m\right)^{\psi(l-p)} \left(a^g x^g\right)^{\psi} a^g = 0
\] (32)

\[
\frac{\partial L}{\partial \lambda} = \left(\mu^f + a^f H^f\right)^{\rho} \left(\mu^m + a^m H^m\right)^{\psi(l-p)} \left(a^g x^g\right)^{\psi} - G \leq 0 \\
\lambda \geq 0, \ \lambda \frac{\partial L}{\partial \lambda} = 0
\] (33)
It is seen that setting $\lambda=0$ would violate condition (32). Therefore we have $\lambda>0$ and $\partial L/\partial \lambda=0$ by complementary slackness. Solve $\lambda$ from (32) to get:

$$
\lambda = \frac{p(a^g x^g)^\nu}{(\mu^f + a^f H^f)^\rho (\mu^m + a^m H^m)^{(1-\rho)} (1-\nu)a^g} \tag{34}
$$

Insert this into (30) and (31) and rewrite the first order conditions as:

$$
\begin{align*}
    w^f &\geq p \frac{\nu}{1-\nu} \frac{\rho a^f}{\mu^f + a^f H^f} x^g \\
    H^f &\geq 0, \quad H^f \left. \frac{\partial L}{\partial H^f} \right|_\lambda = 0 
\end{align*} \tag{30'}
$$

$$
\begin{align*}
    w^m &\geq p \frac{\nu}{1-\nu} \frac{(1-\rho)a^m}{\mu^m + a^m H^m} x^g \\
    H^m &\geq 0, \quad H^m \left. \frac{\partial L}{\partial H^m} \right|_\lambda = 0 
\end{align*} \tag{31'}
$$

Since $\partial L/\partial \lambda=0$ the condition (33) has to be satisfied as an equality.

$$
G = \left(\mu^f + a^f H^f\right)^\rho (\mu^m + a^m H^m)^{(1-\rho)} (a^g x^g)^{-\nu} \tag{33'}
$$

We have four cases to consider depending on whether the constraints for the individual time inputs $H^i$ are binding or not.

**Case I: $H^f = 0$ and $H^m = 0$**

In case I time allocation regime both spouses specialize into market work and household services are bought from the market. Thus we have $H^i=0$, $i=f,m$, and by complementary slackness this implies that $\partial L/\partial H^i>0$, $i=f,m$. Therefore the optimal solution for this time allocation regime has to satisfy the following conditions:
\[ w^f > \frac{v}{1-v} \rho a^f \frac{x^g}{\mu^f} \]
\[ w^m > \frac{v}{1-v} (1-\rho) a^m \frac{x^g}{\mu^m} \]
\[ G = \left( \mu^f \right)^{\nu} \left( \mu^m \right)^{\nu(1-\rho)} \left( a^g x^g \right) \]

Solve the last equation in condition (35) for \( x^g \) to get:

\[ x^g = G^{\frac{1}{1-\nu}} \left( \mu^f \right)^{-\nu} \left( \mu^m \right)^{-\nu(1-\rho)} \left( a^g \right)^{-1} \]

By inserting \( x^g \) into the two first conditions the optimum conditions for the case I time allocation regime can be written as follows:

\[ w^f > \frac{v}{1-v} \rho a^f G^{\frac{1}{1-\nu}} (\mu^f)^{\frac{1}{1-\nu}-1} (\mu^m)^{\nu(1-\rho)(1-\nu)} \]

\[ w^m > \frac{v}{1-v} (1-\rho) a^m G^{\frac{1}{1-\nu}} (\mu^f)^{\frac{1}{1-\nu}} (\mu^m)^{\nu(1-\rho)(1-\nu)} \]

Both household members specialize into market work when their market wage exceeds their shadow wages in household production. Solve the above two conditions for \( G \) to get:

\[ G < \left( \frac{w^f}{a^f} \right)^{1-\nu} \left( \frac{a^g}{p} \right)^{1-\nu} \left( \frac{1-v}{v \rho} \right)^{1-\nu} (\mu^f)^{\nu(1-\nu)} (\mu^m)^{\nu(1-\rho)(1-\nu)} \]

\[ G < \left( \frac{w^m}{a^m} \right)^{1-\nu} \left( \frac{a^g}{p} \right)^{1-\nu} \left( \frac{1-v}{v(1-\rho)} \right)^{1-\nu} (\mu^f)^{\nu(1-\nu)} (\mu^m)^{\nu(1-\rho)(1-\nu)} \]
It is seen that the case I is possible when the desired level for household good G is low enough; individual wages are high and individual productivities in the household work are low; market price for $x^g$ is low and the productivity of $x^g$ is high; and the values of the parameters $\mu^f$ and $\mu^m$, describing the stock of consumer durables the household members have brought into this household, are high.

Since $H^f=0$ and $H^m=0$ the cost function for the case I time allocation regime is found from the following:

$$C(G)^I = px^g = \frac{p}{a^g} G^{\frac{1}{1-v}} (\mu^f)^{-\nu_p} (\mu^m)^{-\nu(1-\rho)}$$ (39)

It is immediately seen that in the case where both spousal time inputs are zero the household technology is not of constant returns to scale even if it was specified to be so for the case where all inputs are used in positive amounts. This implies that the full price of the household service depends on the level of the household service and it is therefore endogenous to the household. As a result of this the adjustment of the decision power will possibly affect the optimal household time allocation regime besides the adjustments in the optimal consumption bundle.

**Case II: $H^f > 0$ and $H^m = 0$**

Consider next the case where the wife divides her time between household and market work and the husband specializes into market work. Thus we have $H^f>0$ and $H^m=0$ these imply by complementary slackness that $\partial L/\partial H^f=0$ and $\partial L/\partial H^m>0$. The optimal solution for this case has to satisfy the following conditions:

$$w^f = \frac{p}{1-v} \frac{\rho a^f}{\mu^f + a^f H^f} x^g$$

$$w^m > \frac{p}{1-v} (1-\rho) a^m x^g$$

$$G = (\mu^f + a^f H^f)^\nu (\mu^m)^{\nu(1-\rho)} (a^g x^g)^{1-\nu}$$ (40)
In order the wife to allocate her time both in market and household work her market wage has to be equal with her shadow wage for household work. Solve the last condition for $x^g$ and insert the result into the two first conditions to get:

$$w^f = \frac{p}{a^g} \frac{v}{1-v} \rho a^f G^{1-v} \left( \mu^f + a^f H^f \right)^{\frac{v}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}}$$

$$(41)$$

$$w^m > \frac{p}{a^g} \frac{v}{1-v} (1-\rho) a^m G^{1-v} \left( \mu^f + a^f H^f \right)^{\frac{v}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}}$$

Next solve these conditions for G:

$$G = \left( \frac{w^f}{a^f} \right)^{1-v} \left( \frac{a^g}{p} \right)^{1-v} \left( \frac{1-v}{\nu} \right)^{1-v} \left( \mu^f + a^f H^f \right)^{\nu(1-v)} \left( \mu^m \right)^{(1-\rho)(1-v)}$$

$$(42)$$

$$G < \left( \frac{w^m}{a^m} \right)^{1-v} \left( \frac{a^g}{p} \right)^{1-v} \left( \frac{1-v}{\nu(1-\rho)} \right)^{1-v} \left( \mu^f + a^f H^f \right)^{\nu(1-v)} \left( \mu^m \right)^{(1-\rho)(1-v)}$$

By combining these we get the following optimum condition for the case II household time allocation regime:

$$\left( \frac{w^f}{a^f} \right)^{1-v} (\rho)^{(1-v)} \left( \mu^f + a^f H^f \right)^{(1-v)} < \left( \frac{w^m}{a^m} \right)^{1-v} (1-\rho)^{(1-v)} \left( \mu^m \right)^{(1-v)}$$

$$(43)$$

It is seen that the case II is possible when the implicit price of the female household member’s time in the production of household service is lower than that for the male household member. This condition can be solved for the female household member’s intermediate product in household production $\tilde{H}^f = \mu^f + a^f H^f$ as:

$$\tilde{H}^f < \left( \frac{w^m}{a^m} \right) \left( \frac{w^f}{a^f} \right)^{-1} \left( \frac{\rho}{1-\rho} \right) \mu^m$$

$$(44)$$
Thus the case II is possible when $\tilde{H}^f$ is low enough; male household member’s wage $w^m$ is high and his productivity in the household work $a^m$ is low; female household member’s wage $w^f$ is low and her productivity in household work $a^f$ is high; female household member’s time input in the production process is relatively more important than that for her husband, implying high $\rho$; and when the value of the parameter $\mu^m$, describing the stock of consumer durables the husband has brought into this household, is high.

Since the husband’s time input equals zero for this regime, the relevant cost function is obtained by solving $H^f$ as a function of $G$ from the optimum conditions for this time allocation regime and inserting the result together with $x^g$ into the formulation for the cost function for the case II. Thus, we have:

$$C(G)^{II} = w^f H^f + px^g$$

$$= G^{\frac{1}{\nu}} \left( \frac{w^f}{a^f} \right)^{\frac{\nu p}{\nu p + (1-\nu)}} \left( \mu^m \right)^{\frac{\nu (1-\nu p)}{\nu (1-\nu p) + (1-\nu)}} \left( \frac{\rho p}{a^g} \right)^{\frac{1-\nu}{\nu}} \times \left[ \frac{\nu p}{1-\nu} \right] \left( \frac{w^f}{a^f} \right) \mu^f$$

Since the level of husband’s time input in the production of the household service is fixed to zero and since the female time input cannot be increased above total time available, the household technology implies decreasing returns to scale. In this case the full price of the good produced in the household is endogenous to the household as in the previous case considered.

Note that a special case of the time allocation regime considered here is that of full specialisation where the wife allocates all the time available into household while the husband allocates all the time available into market work. For this case the cost function is the following:
\[ C(G)^{lb} = w^f + px^g \]
\[ = w^f + \frac{p}{a^g} \frac{1}{G^{1-v}} \left( \mu^f + a^f \right)^{\frac{v_p}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}} \]  
\[ (46) \]

Case III: \( H^f = 0 \) and \( H^m > 0 \)

Consider the case where the wife specializes into market work while the husband allocates his time between household and market work. This implies that \( H^f = 0 \) and \( H^m > 0 \) and by complementary slackness that \( \partial L / \partial H^f > 0 \) and \( \partial L / \partial H^m = 0 \). Thus, the optimal solution for this case has to satisfy:

\[ w^f > p \frac{v}{1-v} (1-\rho) a^f \frac{x^g}{\mu^f} \]
\[ w^m = p \frac{v}{1-v} (1-\rho) a^m \frac{x^g}{\mu^m + a^m H^m} \]  
\[ G = \left( \mu^f \right)^{\frac{v_p}{1-v}} \left( \mu^m + a^m H^m \right)^{\frac{v(1-\rho)}{1-v}} \]  
\[ (47) \]

Symmetrically with the case II here the husband’s market wage has to equal his shadow wage for household work in order him to allocate his time between market and household work. Solve the last condition for \( x^g \) and insert the result into the two first conditions to get:

\[ w^f > \left( \frac{p}{a^g} \right) \frac{v}{1-v} \rho a^f G^{1-v} \left( \mu^f \right)^{\frac{v_p}{1-v}} \left( \mu^m + a^m H^m \right)^{\frac{v(1-\rho)}{1-v}} \]  
\[ \]  
\[ w^m = \left( \frac{p}{a^g} \right) \frac{v}{1-v} (1-\rho) a^m G^{1-v} \left( \mu^f \right)^{\frac{v_p}{1-v}} \left( \mu^m + a^m H^m \right)^{\frac{v(1-\rho)}{1-v} - 1} \]  
\[ (48) \]

Solve the conditions in (48) for \( G \) to get:
By combining these we get the following optimum condition for the case III household time allocation regime:

\[
G < \left( \frac{w^f}{a^f} \right)^{1-v} \left( \frac{a^g}{p} \right)^{1-v} \left( \frac{1-v}{(1-\rho)} \right)^{1-v} (\mu^f)^{(1-\rho)} (\mu^m + a^m H^m)^{(1-\rho)} \]  

(49)

\[
G = \left( \frac{w^m}{a^m} \right)^{1-v} \left( \frac{a^g}{p} \right)^{1-v} \left( \frac{1-v}{(1-\rho)} \right)^{1-v} (\mu^f)^{(1-\rho)} (\mu^m + a^m H^m)^{(1-\rho)(1-v)}
\]

By combining these we get the following optimum condition for the case III household time allocation regime:

\[
\left( \frac{w^m}{a^m} \right)^{1-v} (1-\rho)^{1-v} (\mu^m + a^m H^m)^{(1-v)} < \left( \frac{w^f}{a^f} \right)^{1-v} (\rho)^{1-v} (\mu^f)^{(1-v)}
\]

(50)

The implicit price for the husband’s time input has to be lower than that for his wife in order only him to allocate time into household work. This relationship can be simplified further by solving it for the husband’s intermediate product in household production \( \tilde{H}^m = \mu^m + a^m H^m \):

\[
\tilde{H}^m < \left( \frac{w^f}{a^f} \right) \left( \frac{w^m}{a^m} \right)^{-1} \left( \frac{1-\rho}{p} \right) \mu^f
\]

(51)

From this we see that the case where the wife specializes into market work is possible only if \( \tilde{H}^m \) is low enough; the wife’s market wage \( w^f \) is high and her productivity in the household work \( a^f \) is low; the husband’s market wage \( w^m \) is low and his productivity in household work \( a^m \) is high; husband’s time input in the production process for household service is relatively more important than that for his wife, implying low \( \rho \); and the value of the parameter \( \mu^f \), describing the stock of consumer durables the wife has brought into this household, is high.

The cost function for the case III is obtained by solving \( H^m \) as a function of \( G \) from the optimum conditions for this time allocation regime and inserting the result together with \( x^g \) into the formulation for the cost function for the case III. Thus, we have:
The wife’s time input in production of the household service is fixed to zero as long as the input prices satisfy the optimum conditions for this time allocation regime. The household technology implies decreasing returns to scale due to the limitations in the input usage. Further, as in the two previous cases the full price for the household service is endogenous to the household and therefore depends on the decision power in the household.

The special case of this time allocation regime is the case of full specialisation where the wife allocates all the time available into market work while the husband allocates all the time available into household work. In this case the cost function is the following:

\[
C(G)^{III} = w^m H^m + px^g = \frac{1}{G^{v(1-\rho)+(1-\nu)}} \left( \frac{w^m}{a^m} \right)^{v(1-\rho)} \left( \mu^f \right)^{v(1-\rho)+(1-\nu)} \left( \frac{p}{a^g} \right)^{1-\nu} \times 
\]

\[
\left[ \left( 1 - \nu \right) \left( v(1-\rho) \right)^{v(1-\rho)+(1-\nu)} + \left( 1 - \nu \right) \left( v(1-\rho) \right)^{v(1-\rho)+(1-\nu)} \right] - \frac{w^m}{a^m} \mu^m
\]

\[
(52)
\]

The special case of this time allocation regime is the case of full specialisation where the wife allocates all the time available into market work while the husband allocates all the time available into household work. In this case the cost function is the following:

\[
C(G)^{IIIb} = w^m H^m + px^g
\]

\[
= w^m + \frac{p}{a^g} G^{v(1-\rho)} \left( \mu^f \right)^{v(1-\rho)+(1-\nu)} \left( a^m + a^g \right)^{v(1-\rho)+(1-\nu)}
\]

\[
(53)
\]

Case IV: \( H^f > 0 \) and \( H^m > 0 \)

Consider finally the interior solution to the household time allocation problem. Now we have \( H^f > 0 \) and \( H^m > 0 \) and therefore \( \partial L/\partial H^f = 0 \) and \( \partial L/\partial H^m = 0 \) by complementary slackness. The optimal solution for this case has to satisfy:
\[
\begin{align*}
    w^f &= \frac{p}{a^f} \frac{v}{1-v} \frac{\rho a^f}{\mu^f + a^f H^f} x^g \\
    w^m &= \frac{p}{a^m} \frac{v}{1-v} (1-\rho) a^m \frac{x^g}{\mu^m + a^m H^m} \\
    G &= (\mu^f + a^f H^f)^{\frac{1}{1-v}} (\mu^m + a^m H^m)^{\frac{1}{1-v}} (a^g x^g)^{\frac{v(1-\rho)}{1-v}}
\end{align*}
\]

(54)

Solve the last condition for \(x^g\) and insert the result into the two first conditions to get:

\[
\begin{align*}
    w^f &= \left(\frac{p}{a^g}\right) \frac{v}{1-v} \rho a^f G^{\frac{1}{1-v}} \left(\mu^f + a^f H^f\right)^{\frac{1}{1-v}} \left(\mu^m + a^m H^m\right)^{\frac{v(1-\rho)}{1-v}} \\
    w^m &= \left(\frac{p}{a^g}\right) \frac{v}{1-v} (1-\rho) a^m G^{\frac{1}{1-v}} \left(\mu^f + a^f H^f\right)^{\frac{1}{1-v}} \left(\mu^m + a^m H^m\right)^{\frac{v(1-\rho)}{1-v}}
\end{align*}
\]

(55)

Solve these for \(G\) to get:

\[
\begin{align*}
    G &= \left(\frac{w^f}{a^f}\right)^{\frac{1}{1-v}} \left(\frac{a^g}{p}\right)^{\frac{1}{1-v}} \left(\frac{1-v}{v}\right)^{\frac{1}{1-v}} \left(\mu^f + a^f H^f\right)^{\frac{v(1-\rho)}{1-v}} \left(\mu^m + a^m H^m\right)^{\frac{v(1-\rho)}{1-v}} \\
    G &= \left(\frac{w^m}{a^m}\right)^{\frac{1}{1-v}} \left(\frac{a^g}{p}\right)^{\frac{1}{1-v}} \left(\frac{1-v}{v(1-\rho)}\right)^{\frac{1}{1-v}} \left(\mu^f + a^f H^f\right)^{\frac{v(1-\rho)}{1-v}} \left(\mu^m + a^m H^m\right)^{\frac{v(1-\rho)(1-v)}{1-v}}
\end{align*}
\]

(56)

By combining these we get the familiar condition for the interior solution into household time allocation problem:

\[
\left(\frac{w^f}{a^f}\right) \left(\frac{\rho}{\tilde{H}^f}\right)^{\frac{1}{\rho}} = \left(\frac{w^m}{a^m}\right) \left(\frac{1-\rho}{\tilde{H}^m}\right)^{\frac{1}{\rho}}
\]

(57)

In interior solution the implicit prices for the time inputs have to be equal with each other. Further, we get:
The wage ratio has to be equal with the technical rate of substitution in order both household members to allocate their time between market work and household work.

Next solve $H^f$ and $H^m$ from the optimum conditions as a function of $G$ and insert the results together with $x^g$ into the formulation for the cost function for the case IV:

$$C(G)^{IV} = w_f H^f + w_m H^m + px^g =$$

$$G \left( \frac{w_f}{a^f} \right)^{vp} \left( \frac{w_m}{a^m} \right)^{(1-\rho)} \left( \frac{p}{a^g} \right)^{1-v} \times$$

$$\left[ \frac{vp}{1-v} \left( \frac{1-\rho}{\rho} \right)^{1-v} + \left( \frac{v(1-\rho)}{1-v} \right)^{1-v} \left( \frac{\rho}{1-\rho} \right)^{1-v} \right]$$

$$- \frac{w_f}{a^f} \mu^f - \frac{w_m}{a^m} \mu^m$$

(59)

It is immediately seen that the technology implies constant returns to scale in interior solution into household time allocation problem. As a result of this the full price of the household service is independent of the level of the household service in interior solution.

The full price of the household service in each possible time allocation regime can be obtained from the relevant cost function by taking the derivative of the cost function with respect to $G$. For the corner solutions the full price of the household service depends on the level of $G$ and is thus endogenous to the household. Further, the optimum conditions for each time allocation regime depend on the welfare weight $\theta$, determining the decision power in the household. Therefore the adjustment of the decision power in the household will affect the optimum conditions for the time allocation regimes. It may cause the household to move from one regime to another.
earlier or later than implied by the wage change alone. I will analyse this possibility after having solved the optimal consumption allocations and the adjustment of the household optimal consumption bundle for each possible household time allocation regime.

4.2 Consumption allocation

After having solved relevant cost functions for each time allocation regime we are ready to consider the household consumption allocation problem. With Cobb-Douglas preferences the household consumption allocation problem is solved from the following:

$$\begin{align*}
\max_{x^f, x^m, G} \Omega & \equiv \theta \{(\alpha \ln x^f + (1-\alpha) \ln G) + (1-\theta)(\beta \ln x^m + (1-\beta) \ln G) \} \\
\text{s.t.} \quad x^f + x^m + C(G) &= w^f + w^m \equiv Y
\end{align*}$$  (60)

The household budget constraint is presented as full budget constraint where $w^f + w^m \equiv Y$ is the household potential income.\(^3\) That is, $Y$ is the income that would occur if both household members allocate all the time available into market work. The prices of the market goods consumed privately are normalized to one as in the general version of the model at hand considered earlier. $C(G)$ represents the minimized cost of producing the desired level of household services $G$. The applicability of the household production theory requires separability of consumption and production in the household. Therefore an important feature of the form assumed here for individual preferences is that the preferences are additively separable in private consumption versus joint consumption of the domestic good. Lagrangian for the household consumption allocation problem is:

$$L(x^f, x^m, x^g, \gamma) = \theta \{(\alpha \ln x^f + (1-\alpha) \ln G) + (1-\theta)(\beta \ln x^m + (1-\beta) \ln G) \} + \gamma(w^f + w^m - x^f - x^m - C(G))$$  (61)

\(^3\) For the derivation of the household full budget constraint see equation (3) in the page 10.
The first order conditions are:

\[
\frac{\partial L}{\partial x^f} = \frac{\theta \alpha}{x^f} - \gamma = 0 \tag{62}
\]

\[
\frac{\partial L}{x^m} = \frac{(1 - \theta) \beta}{x^m} - \gamma = 0 \tag{63}
\]

\[
\frac{\partial L}{\partial G} = \frac{\theta (1 - \alpha) + (1 - \theta)(1 - \beta)}{G} - \gamma \frac{\partial C(G)}{\partial G} = 0 \tag{64}
\]

\[
\frac{\partial L}{\partial \gamma} = w^f + w^m - x^f - x^m - C(G) = 0 \tag{65}
\]

Solve (64) for \( \gamma \) and insert the result into (62) and (63) to get:

\[
\frac{\theta \alpha}{x^f} = \frac{\theta (1 - \alpha) + (1 - \theta)(1 - \beta)}{G} \frac{\partial C(G)}{\partial G} \tag{62'}
\]

\[
\frac{(1 - \theta) \beta}{x^m} = \frac{\theta (1 - \alpha) + (1 - \theta)(1 - \beta)}{G} \frac{\partial C(G)}{\partial G} \tag{63'}
\]

\[
x^f + x^m + C(G) = w^f + w^m \tag{65'}
\]

Each possible time allocation regime can be analysed separately by inserting the cost functions \( C(G)^{I-IV} \) derived in stage one cost minimization problem into the optimum conditions for the household consumption allocation problem.

Next the optimal consumptions are derived for each time allocation regime. After this the responses of the household optimal consumption bundle with respect to
changes in female household member’s wage and the decision power in the household are analysed.

**Case I: \( H^f = 0 \) and \( H^m = 0 \)**

Insert first the cost function derived for the case I time allocation regime into the optimum conditions for the consumption allocation problem. The optimal demands for the case where spouses allocate all the time available into market work are found to be the following:

\[
x^{f*} = \frac{\theta \alpha}{\theta \alpha + (1 - \theta) \beta + \{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\}(1 - \nu)} \left( w^f + w^m \right) \tag{66}
\]

\[
x^{m*} = \frac{(1 - \theta) \beta}{\theta \alpha + (1 - \theta) \beta + \{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\}(1 - \nu)} \left( w^f + w^m \right) \tag{67}
\]

\[
G^* = \left[ \frac{\{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\}(1 - \nu)}{\theta \alpha + (1 - \theta) \beta + \{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\}(1 - \nu)} \right]^{1 - \nu} \times \left( w^f + w^m \right)^{\nu} \left( \frac{p}{a^*} \right)^{(1 - \nu)} \left( \mu^f \right) \left( \mu^m \right)^{(1 - \rho)} \tag{68}
\]

Conditional demand for the market goods \( x^g \) can be obtained by Shepard’s lemma from the relevant cost function as \( \partial C(G) / \partial p \). Insert \( G^* \) into the result to get \( x^{g*} \):

\[
x^{g*} = \frac{\{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\}(1 - \nu)}{\theta \alpha + (1 - \theta) \beta + \{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\}(1 - \nu)} \left( w^f + w^m \right) \frac{1}{p} \tag{69}
\]

Remember that for this case \( H^f = 0, i = f, m \) implies that individual labour supplies are \( L^{i*} = 1, i = f, m \).
Case II: \( H^f > 0 \) and \( H^m = 0 \)

By using the relevant cost function for the case II time allocation regime the optimal demands for the case II time allocation regime are found to be the following:

\[
x_f^* = \frac{\theta \alpha}{\theta \alpha + (1-\theta)\beta + \{\theta(1-\alpha) + (1-\theta)(1-\beta)\}[vp + (1-v)]} \times \left( w_f + w_m + \frac{w_f}{\alpha} \mu_f \right) \\
\]

\[
x_m^* = \frac{(1-\theta)\beta}{\theta \alpha + (1-\theta)\beta + \{\theta(1-\alpha) + (1-\theta)(1-\beta)\}[vp + (1-v)]} \times \left( w_f + w_m + \frac{w_f}{\alpha} \mu_f \right) \\
\]

\[
G^* = \left( w_f + w_m + \frac{w_f}{\alpha} \mu_f \right)^{\gamma + (1-v)} \times \left[ \frac{\{\theta(1-\alpha) + (1-\theta)(1-\beta)\}[vp + (1-v)]}{\theta \alpha + (1-\theta)\beta + \{\theta(1-\alpha) + (1-\theta)(1-\beta)\}[vp + (1-v)]} \right]^{\gamma + (1-v)} \times \left( \frac{p}{a^g} \right)^{(1-v)} \left( \frac{w_f}{\alpha f} \right)^{-\gamma p} \left( \mu_m \right)^{(1-\rho)} \left[ \left( \frac{vp}{1-v} \right)^{\frac{(1-v)}{\gamma p + (1-v)}} + \left( \frac{vp}{1-v} \right)^{\gamma p - (1-v)} \right]^{(\gamma p + (1-v))} \\
\]

Conditional demands for \( x^g \) and \( H^f \) used in the production of \( G \) can be obtained by Shepard’s lemma from \( \partial C(G)^{H^f} / \partial p \) and \( \partial C(G)^{H^f} / \partial w_f \) respectively. By inserting the optimal consumption for \( G^* \) for the case II into the results we get the optimal levels for \( x^g \) and \( H^m \).
Note that for the household time allocation regime II the individual labour supplies are found from $L^{m^*} = 1$ and $L^{f^*} = 1 - H^{f^*}$.

For the case of full specialization where the wife allocates all the time available into household work while the husband allocates all the time available into market work we get the following demands:

$$
x^{f^*} = \frac{\theta \alpha}{\theta \alpha + (1 - \theta) \beta + \theta (1 - \alpha) + (1 - \theta) (1 - \beta)(1 - \nu)} w^m \tag{75}
$$

$$
x^{m^*} = \frac{(1 - \theta) \beta}{\theta \alpha + (1 - \theta) \beta + \theta (1 - \alpha) + (1 - \theta) (1 - \beta)(1 - \nu)} w^m \tag{76}
$$

$$
G^* = \left[ \frac{\theta (1 - \alpha) + (1 - \theta) (1 - \beta)(1 - \nu)}{\theta \alpha + (1 - \theta) \beta + \theta (1 - \alpha) + (1 - \theta) (1 - \beta)(1 - \nu)} \right]^{1-v} \times \left( \frac{p}{a^g} \right)^{-(1-v)} \left( \mu^f + a^f \right)^{\nu} \left( \mu^m \right)^{(1-\nu)} \left( w^m \right)^{v} \tag{77}
$$

The conditional demands for the inputs $H^{f^*}$ and $x^{q^*}$ in the case of full specialization are found to be:

$$
H^{f^*} = 1 \tag{78}
$$
Case III: \( H^{f} = 0 \) and \( H^{m} > 0 \)

By using the relevant cost function for the case III time allocation regime the optimal demands for this case are found to be the following:

\[
x^{f*} = \frac{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)}{\theta \alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} \left( \frac{1}{p} \right) (w^{m})
\]

(79)

\[
x^{m*} = \frac{(1 - \theta)\beta}{\theta \alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} \left( \frac{1}{p} \right) (w^{m})
\]

(80)

\[
x^{m*} = \left( \frac{w^{f} + w^{m} + \frac{w^{m}}{a^{m}} \mu^{m}}{\left( \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu) \right)^{\frac{1}{p}} \left( \frac{1}{a^{m}} \right)^{(1 - p)/(1 - \nu)}} \right)
\]

(81)

\[
G^{*} = \left( \frac{w^{f} + w^{m} + \frac{w^{m}}{a^{m}} \mu^{m}}{\left( \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu) \right)^{\frac{1}{p}} \left( \frac{1}{a^{m}} \right)^{(1 - p)/(1 - \nu)}} \right)
\]

(82)

Conditional demands for \( x^{g} \) and \( H^{m} \) can again be obtained by Shepard’s lemma from \( \partial C(G)^{III}/\partial p \) and \( \partial C(G)^{III}/\partial w^{m} \) respectively. By inserting the optimal consumption of \( G^{*} \) for the case II time allocation regime into these we get the optimal consumptions of \( x^{g*} \) and \( H^{m*} \).
Note that for the household time allocation regime III the individual labour supplies are found from $L^\alpha = 1$ and $L^\omega = 1 - H^\omega$.

For the case of full specialization where the wife allocates all the time available into market work while the husband allocates all the time available into household work the optimal demands are found to be the following:

\begin{equation}
    x^\omega = \frac{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)}{\theta \alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} \left( w^\omega + \frac{w^m}{a^m} \mu^m \right) (p)^{-1} \tag{83}
\end{equation}

\begin{equation}
    H^\omega = \frac{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)\nu(1 - \rho)}{\theta \alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)\nu(1 - \rho) + (1 - \nu)} \left( w^\omega + \frac{w^m}{a^m} \mu^m \right)^{-1} - \frac{\mu^m}{a^m} \tag{84}
\end{equation}

The conditional demands for the inputs $H^\omega$ and $x^\omega$ in the case of full specialization are found to be:

\begin{equation}
    H^\omega = 1 \tag{85}
\end{equation}

\begin{equation}
    x^\omega = \frac{\theta \alpha}{\theta \alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} w^\omega \tag{86}
\end{equation}

\begin{equation}
    G^* = \left[ \frac{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)}{\theta \alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} \right]^{1-\nu} \left( \frac{p}{a^g} \right)^{(1-\nu)} \left( \mu^\omega \right)^{1-\nu} \left( \mu^m + a^f \right)^{(1-\rho)} \left( w^\omega \right)^{1-\nu} \tag{87}
\end{equation}

\begin{equation}
    H^\omega = 1 \tag{88}
\end{equation}
\[ x_{g}^* = \frac{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)}{\theta \alpha + (1 - \theta) \beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} \frac{1}{p} \left( w^f \right) \]  

(89)

**Case IV: \( H^f > 0 \) and \( H^m > 0 \)**

By using the relevant cost function for the case IV time allocation regime the optimal demands for the interior solution to the household time allocation problem are found to be the following:

\[ x_f^* = \theta \alpha \left( w_f^f + w_m^m + \frac{w_f^f}{a^f} \mu^f + \frac{w_m^m}{a_m^m} \mu^m \right) \]  

(90)

\[ x_m^* = (1 - \theta) \beta \left( w_f^f + w_m^m + \frac{w_f^f}{a^f} \mu^f + \frac{w_m^m}{a_m^m} \mu^m \right) \]  

(91)

\[ G^* = \left( \theta(1 - \alpha) + (1 - \theta)(1 - \beta) \right) \left( w_f^f + w_m^m + \frac{w_f^f}{a^f} \mu^f + \frac{w_m^m}{a_m^m} \mu^m \right) \times \]

\[ \left( \frac{a^g}{a^f} \right)^{(1 - \nu)} \left( \frac{w_f^f}{a^f} \right)^{-\nu} \left( \frac{w_m^m}{a_m^m} \right)^{-\nu(1 - \rho)} \times \left( \frac{v^2}{1 - \nu} \right)^{1 - \nu} \left( \frac{1 - \rho}{\rho} \right)^{-\nu(1 - \rho)} + \left( \frac{v(1 - \rho)}{1 - \nu} \right)^{1 - \nu} \left( \frac{\rho}{1 - \rho} \right)^{\nu(1 - \rho)} \right]^{-1} \]  

(92)

Conditional demands for \( x^g \), \( H^f \) and \( H^m \) used in the production of can be obtained by Shepard’s lemma from \( \partial C(G)^{IV} / \partial p \), \( \partial C(G)^{IV} / \partial w^f \) and \( \partial C(G)^{IV} / \partial w^m \) respectively. By inserting the optimal consumption of \( G^* \) for the case IV into these we get the optimal levels of \( x^g^* \), \( H^f^* \) and \( H^m^* \).
For the interior solution the individual labour supplies are found from
\[ L^i = 1 - H^i, \quad i = f, m. \]

In this section the optimum conditions and optimal consumption bundles for each
time allocation regime were derived for the collective household model with
household production. Now it is possible to analyse how exogenous changes in the
female wage and in the decision power in the household affect household optimal
consumption bundle and household optimal time allocation regime.

### 4.3 Adjustment of the household optimal consumption bundle

Since the welfare weight determining the decision power in the household adjusts
as the individual wages change, there is besides the standard income and substitution
effects a distribution effect that tells how the composition of the household optimal
consumption bundle changes with respect to changes in the decision power in the
household. Further, besides the changes in the household optimal consumption bundle
the optimal time – market good – mix of the good produced in the household changes
as well. And finally the changes in individual earnings and decision power in the
household can induce shifts in the household optimal time allocation regime.

\[
x^e = \{\theta(1 - \alpha)(1 - \theta)(1 - \beta)(1 - \nu)\times
\left( w^f + w^m + \frac{w^f}{a^f} \mu^f + \frac{w^m}{a^m} \mu^m \right)(p)^{-1}
\] (93)

\[
H^f = \{\theta(1 - \alpha)(1 - \theta)(1 - \beta)\nu \rho \times
\left( w^f + w^m + \frac{w^f}{a^f} \mu^f + \frac{w^m}{a^m} \mu^m \right)(w^f)^{-1} - \frac{\mu^f}{a^f}
\] (94)

\[
H^m = \{\theta(1 - \alpha)(1 - \theta)(1 - \beta)\nu(1 - \rho) \times
\left( w^f + w^m + \frac{w^f}{a^f} \mu^f + \frac{w^m}{a^m} \mu^m \right)(w^m)^{-1} - \frac{\mu^m}{a^m}
\] (95)
In what follows I consider first the responses of the household optimal consumption bundle with respect to increase in the female household member’s wage. After this the adjustment of the optimal time–market good–mix of the good produced in the household is considered. Possible shifts induced in the household optimal time allocation regime by the increase in the female household member’s wage and her decision power in the household are considered separately in the section 4.4. of this paper.

**Case I: \( H^{f} = 0 \) and \( H^{m} = 0 \)**

Consider first the response of the wife’s private consumption with respect to increases in her wage in the case where both spouses allocate all the time available into market work.

The total effect is a sum of an income effect and a distribution effect. The distribution effect describes the adjustment of the household optimal consumption bundle as the decision power in the household changes. The distribution effect is presented by the term in the square brackets. The distribution effect consists of two parts and it can be either positive or negative depending on how the household members value private consumption versus joint consumption of the domestic good. The first term in the square brackets describes the direct increase in the wife’s private consumption as her say in the household increases. The second term in the square brackets describes the trade-off between the wife’s private consumption and the joint consumption of the household public good as the wife’s say in the household increases. It is seen that the wife’s private demand is guaranteed to increase as a result of a rise in her wage if and
only if $\alpha \leq \beta$. This means that there is substitution away from the joint consumption of the household public good if the husband does not value the joint consumption of the household public good too much relative to his wife. This is the case if the husband’s marginal utility from his private consumption is larger than that for his wife. The structure of the distribution effect found here with CD-preferences is exactly the same as in the general version of the model considered in the section three (see, the equation (23) on the page 63). In the general version of the model, the distribution effect was a sum of three components two of which are summarized by the first term in the square brackets in (96) representing the direct distribution effect for the special case of CD-preferences considered here. The third term in the general version describes the trade-off between private and joint consumption of the household good as in the current example with CD-preferences.

The response of the husband’s private consumption with respect to the increase in the wife’s wage is found to be the following:

$$\frac{\partial x^m}{\partial w^f} = (1-\theta)\beta$$

$$+ \frac{\partial \theta}{\partial w^f} (w^f + w^m)$$

$$\left[ -\beta + \frac{(1-\theta)\beta (\beta - \alpha)}{\theta \alpha + (1-\theta)\beta + \theta(1 -\alpha) + (1-\theta)(1 -\beta) (1-\nu)} \right] \geq 0$$

The total effect is a sum of an income effect and a distribution effect. The distribution effect is presented in the square brackets and it consists of two parts. The first term in the square brackets describes direct substitution away from the husband’s private consumption as the decision power of the wife increases. The second term in the square brackets describes the trade-off between the husband’s private consumption and the joint consumption of the household public good. The distribution effect is negative in the case of identical preferences or if the wife values relatively more her private consumption than the joint consumption of the household good, i.e. if $\alpha \geq \beta$. In this case the effect of the increase in the wife’s wage to the husband’s private
consumption is negative, if the negative distribution effect outweighs the positive income effect.

In order to see how the optimal demand and the optimal time – market good - mix for the good produced in the household varies with changes in female wage, decompose the total effect on the demand for $G$ into partial effects by using the derivatives of the components of $G$ with respect to $w^f$. By (27) the optimal joint demand for the household good can be found from:

$$G^* = \left( \tilde{H}^{f*} \right)^{\rho} \left( \tilde{H}^{m*} \right)^{(1-\rho)} \left( a^x x^g \right)^{-v}.$$

By differentiating the above with respect to female wage we get:

$$\frac{dG^*}{dw^f} = \nu \rho \left( \tilde{H}^{f*} \right)^{\rho-1} \left( \tilde{H}^{m*} \right)^{(1-\rho)} \left( a^x x^g \right)^{-v} \frac{\partial \tilde{H}^{f*}}{w^f} + \nu (1 - \rho) \left( \tilde{H}^{m*} \right)^{(1-\rho)-1} \left( \tilde{H}^{f*} \right)^{\rho} \left( a^x x^g \right)^{-v} \frac{\partial \tilde{H}^{m*}}{w^f} + (1 - \nu) \left( a^x x^g \right)^{(1-v)-1} \left( \tilde{H}^{f*} \right)^{\nu} \left( \tilde{H}^{m*} \right)^{(1-\rho)} a^g \frac{\partial x^g}{w^f}$$

$$= G^* \left[ \nu \rho \left( \tilde{H}^{f*} \right)^{-1} \frac{\partial \tilde{H}^{f*}}{w^f} + \nu (1 - \rho) \left( \tilde{H}^{m*} \right)^{-1} \frac{\partial \tilde{H}^{m*}}{w^f} \right] + (1 - \nu) \left( a^x x^g \right)^{-1} \frac{\partial x^g}{w^f}$$

(98)

In the time allocation regime I both household members specialise into market work. Therefore, $H^f = 0$ and $H^m = 0$, and these imply that $\tilde{H}^f = \mu^f$ and $\tilde{H}^m = \mu^m$. The response of the optimal time – market good- mix of the household good with respect to change in $w^f$ is now found from the following:
\[
\frac{dG^*}{dw'} = G^* (1 - \nu) \left( x^g \right)^{-1} \frac{\partial x^g}{\partial w'}
\]

\[
= G^* (1 - \nu) \times \left[ \frac{1}{w' + w''} \left( \frac{\beta - \alpha}{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)} \right) + \frac{(\beta - \alpha)\nu}{\theta\alpha + (1 - \theta)\beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \nu)} \right] \wedge 0
\]

The total effect is again the sum of an income effect and a distribution effect. The first term in the square brackets describes how the demand for \( x^g \) increases due to the increase in the demand for \( G \) as the household income increases. This income effect is always positive (assuming that \( G \) is a normal good). The more important \( x^g \) is (i.e. small \( \nu \)) the more positive is the income effect and the more market goods \( x^g \) are needed in order to have more joint consumption in the household.

The second term describes the distribution effect. This can be either positive or negative depending on the relationship between individual preferences. In the case of identical preferences (\( \alpha = \beta \)) the distribution effect is zero and there is only the positive income effect. Thus, demand for \( G \) rises when household income increases. In the case of differing preferences the distribution effect can be either positive or negative. If the female household member values relatively more her own private consumption than her husband, i.e. \( \alpha > \beta \) the distribution effect is negative and it dampens or outweighs the positive income effect of the rise in household income. This means that, the rise in the wife’s decision power in the household implies lower joint demand for the domestic good \( G \). Thus less market services are demanded compared to the case of identical preferences. Finally, if the female household member values her own private consumption relatively less than her husband, i.e. if \( \alpha < \beta \) the distribution effect is positive and it magnifies the positive income effect. More market services are needed in order to get more joint consumption in the household compared to the case with identical preferences.

The total effect of the female household member’s wage increase on household demands is always a sum of an income effect and a distribution effect. In the corner
solutions to the household time allocation problem the distribution effect for private
demands always consists of two parts. The first part of the distribution effect
describes the direct distribution towards the private consumption of the individual
whose decision power in the household increases. The second part of the distribution
effect describes the trade-off between private consumptions and joint consumption of
the household good. The total distribution effect can thus be either positive or
negative depending on how the household members value the joint consumption of
the domestic good relative to each other. As a result of this the distribution effect
either magnifies or dampens the positive income effect for private consumptions in
the corner solutions to the household time allocation problem.

In order to economize in space and to avoid repetition only the responses of the
optimal time – market good – mix for the household good G are analysed for the two
other possible time allocation regimes representing corner solution to the household
time allocation problem.

**Case II: \( H^f > 0 \) and \( H^m = 0 \)**

Since the husband allocates all the time available into market work in the time
allocation regime II we have \( \tilde{H}^m = \mu^m \) for this case. Therefore, when the female
wage increases the change in the demand and optimal time – market good – mix for
the household good are found from the following:

\[
\frac{dG^*}{dw^*} = G^* \left[ \frac{\partial \tilde{H}^f}{\partial w^f} \right]^{-1} \frac{\partial \tilde{H}^f}{\partial w^f} + (1 - \nu) \left( x^f \right)^{-1} \frac{\partial x^f}{\partial w^f} > 0
\]

\[
\Rightarrow
\]
Now there are three effects due to the rise in female wage. The first effect is the positive income effect, for more $x^f$ and $H^m$ as the demand for $G$ increases due to the rise in the household income. The second effect describes substitution away from the use of now relatively more expensive female time input in the production of the household service $G$. High $\rho$ means that the female time input in production of $G$ is not easily substituted with the male time input and the parameter $\nu$ reflects the relative time intensiveness of the production process for $G$. The high values of these parameters imply that $H^f$ is high. Therefore, when female wage increases there has to be larger cut in her home time $H^f$ than in the case where the female time input component is not so important in the production process compared to market goods used as inputs (i.e. when $\rho$ and $\nu$ are small). As a result of the increase in the wife’s wage the household service will be produced less time intensively.

The third term in the square brackets is again the distribution effect, which can be either negative or positive depending on how the household members value the domestic good relative to each other, as in the previous case. In the case of identical preferences the distribution effect for joint consumption is always zero. In this case the change in demand depends only on the usual income and substitution effects. While in the case where the wife values relatively more her own private consumption ($\alpha>\beta$) the distribution effect is negative and it dampens the positive income effect for the joint consumption of the household good. Therefore less female time and market goods are needed in the household production process compared to the case with
identical preferences. Finally, in the case where the wife values relatively more the joint consumption of the household good ($\alpha<\beta$) the distribution effect is positive and it magnifies the positive income effect for the joint consumption of the domestic good. As a result of this more female time and market goods are needed compared to the case of identical preferences.

The sign of the distribution effect depends on how the spouses’ value their private consumption versus the joint consumption of the household good. However, the magnitude of the distribution effect depends on the substitutability of the inputs in the household production process. The more important the input in question is the larger is the distribution effect. For the household time allocation regime II the distribution effect is actually sum of the distribution effect for the use of the female time input and the distribution effect for the use of market goods in the production process. The weight given for female time input is $\nu \rho (1 - \rho)$ which is growing in $\rho$ when $\rho \in \left[0, \frac{1}{2}\right]$ and decreasing in $\rho$ when $\rho \in \left[\frac{1}{2}, 1\right]$. Thus, when female time is not easily substituted with the male time in household production the larger are shifts in the use of $H^f$ implied by shifts in decision power. With respect to $v$ the distribution effect for the use of female time input is growing in $v$ over the whole domain of $v$, i.e. the more important the time component is compared to the market good component the larger is the distribution effect.

The weight given for the market goods is $(1 - v)\nu (1 - \rho)$. This is growing in $v$ when $v \in \left[0, \frac{1}{2}\right]$, i.e. when the market good component is relatively more important than the time component, the larger are the shifts in the use of $x^g$ induced by the change in decision power. When $v \in \left[\frac{1}{2}, 1\right]$, the weight given to market goods is decreasing in $v$. The distribution effect for the use of market goods is decreasing with $\rho$, i.e. the more important the female time component is the less the change in the decision power affects the use of market goods in the production process. The conclusion is that the more important is the input in question, the larger is the distribution effect resulting from the change in the decision power in the household.
For the case of full specialization where the wife allocates all the time available into market work while the husband allocates all the time available into market work there is only the distribution effect since in this case the household income is independent of the female wage.

**Case III:** $H^f = 0$ and $H^m > 0$

Since the wife allocates all the time available into market work in the time allocation regime III we have $\tilde{H}^f = \mu^f$ for this case. Therefore, when female wage increases the change in demand for the domestic good and the change in the optimal time – market good - mix in the production process are found from the following:

$$
\frac{dG^*}{d\mu^f} = G^* \left( v(1 - \rho) \left( \frac{\partial \mu^*}{\partial \mu^f} \right) + (1 - \nu) \left( x^g \right) \left( \frac{\partial x^g}{\partial \mu^f} \right) \right)
$$

$$
= G^* \left( v(1 - \rho) + (1 - \nu) \right) \times
$$

$$
\left[ \frac{1}{(w^f + w^m + \frac{w^m}{\mu^m})} \right] + \frac{\partial \theta}{\partial \mu^f} \times
$$

$$
\left\{ \frac{\beta - \alpha}{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)} \right\} + \left( \frac{\beta - \alpha}{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)} \right) (\nu(1 - \rho) + (1 - \nu))
$$

(101)

The total effect is again the sum of a positive income effect and a distribution effect. There are positive income effects both for the use of husband’s time and for the use of market goods in the production process of $G$. The weight for these depends on the technology parameters. The income effect for the use of market goods $x^g$ is decreasing in $v$, i.e. the more important the time component is, the smaller is the income effect for the use of $x^g$ induced by the rise in household income. The income effect for the
use of the male time component is increasing in υ and decreasing in ρ. Thus, the more important the time component is in the production process, the larger is the income effect for the use of the husband’s time due to the income rise. But if the male time input is easily substituted with the female time in the production process (i.e. large ρ) the smaller is the positive income effect for the husband’s time use in the production process.

As before, the distribution effect is a sum of two components. Both of these are zero in the case of identical preferences. In this case the responses of the joint consumption in the household with respect to changes in wages depend only on the income effect. Whereas if the wife values her private consumption relatively more than her husband that is, if $\alpha > \beta$, the distribution effect is negative. In this case the negative distribution effect dampens the positive income effect and less domestic good is demanded compared to the case of identical preferences. And if $\alpha < \beta$, implying that the wife values the joint consumption of the household good relatively more than her husband, the distribution effect magnifies the positive income effect and more household good is demanded compared to the case where $\alpha = \beta$. Therefore, more market goods and husband’s time are needed for the production of the household good. From the previous section we know that the magnitude of the distribution effect depends on the substitutability of the inputs in the household production process. The more important the input in question is in the household production process, the larger is the distribution effect for the use of that input.

For the case of full specialization where the wife allocates all the time available into market work while the husband allocates all the time available into household work the joint demand for the domestic good adjusts as follows as the female wage increases:
\[
\frac{dG^*}{dw^f} = G^* (1 - \nu)^{(x^r)}^{-1} \frac{\partial x^{fr}}{\partial w^f}
= G^* (1 - \nu) \times \left[ \frac{1}{w^f} + \frac{\partial \theta}{\partial w^f} \left( \frac{(\beta - \alpha)}{\theta (1 - \alpha) + (1 - \theta)(1 - \beta)} \right) \right] \wedge 1 \nu
\] (102)

**Case IV:** \(H^f > 0\) and \(H^m > 0\)

In the interior solution to the household time allocation problem the responses of the private demands with respect to the wife’s wage increase are found to be:

\[
\frac{\partial x^{fr}}{\partial w^f} = \theta \alpha \left(1 + \frac{\mu^f}{1^f}ight) + \frac{\partial \theta}{\partial w^f} \left( w^f + w^m + w^f \frac{\mu^f}{1^f} + w^m \frac{\mu^m}{1^m} \right) > 0
\] (103)

\[
\frac{\partial x^{mr}}{\partial w^f} = (1 - \theta) \beta \left(1 + \frac{\mu^f}{1^f}\right) - \frac{\partial \theta}{\partial w^f} \left( w^f + w^m + w^f \frac{\mu^f}{1^f} + w^m \frac{\mu^m}{1^m} \right) > 0
\] (104)

The total effect is again a sum of an income effect and a distribution effect. In interior solution with CD-preferences the distribution effect is seen to consist only of the direct distribution effect. The direct distribution effect describes the distribution towards the private consumption of the individual whose decision power in the household increases. Therefore, it is immediately seen that in the interior solution the increase in the wife’s decision power in the household always increases her private consumption. While the husband’s private consumption decreases if the negative distribution effect outweighs the positive income effect.

In interior solution both spousal time inputs are used in positive amounts and the total change of the demand for the domestic good with respect to increase in female household member’s wage is found from:
\[
\frac{dG^*}{dw^f} = G^* \left[ \nu p \left( \frac{\partial H^*}{\partial w^*} \right)^{-1} \frac{\partial H^*}{\partial w^f} + \nu (1 - \rho) \left( \frac{\partial H^*}{\partial w^m} \right)^{-1} \frac{\partial H^*}{\partial w^f} + (1 - \nu) \left( \frac{x^*}{w^*} \right)^{-1} \frac{\partial x^*}{\partial w^f} \right]
\]

\[
= G^* \times \begin{bmatrix}
1 + \frac{\mu^f}{\alpha^f} \\
\frac{w^m + w^* \mu^m}{w^m + w^* \mu^m} \frac{1}{w^f} \\
\frac{\partial \theta}{\partial w^f} \left( \beta - \alpha \right) \\
\frac{\partial \theta}{\partial w^f} \left( \theta(1 - \alpha) + (1 - \theta)(1 - \beta) \right)
\end{bmatrix}
\]

The total change of \( G^* \) with respect to increase in \( w^f \) is seen to be a sum of four effects as in the general version of the model considered in the section three of this study (see, the equation (26) on the page 67). First there are positive income effects for the use of male time input as well as for the use of market goods in the production of the household good \( G \). The weight for the husband’s time input is \( \nu(1 - \rho) \) which is increasing in \( \nu \) and decreasing in \( \rho \). For the market good component the weight is \( (1 - \theta) \) which is decreasing in \( \theta \). Secondly there is substitution away from the use of now relatively more expensive female time input in the production of \( G \). This effect is the more negative the higher is \( \theta \). Interpretation for this is that the more important the female household member’s time input is in the production process for the household good the higher is the level of it, and thus the more it has been reduced due to the price increase. Finally the distribution effect tells how the demand for the household good adjusts as the say of the female household member increases. As before this effect is zero in the case of identical preferences and as a consequence the results depend on the usual income and substitution effects. While in the case where \( \alpha < \beta \) the distribution effect is positive and magnifies the income effect for the use of market goods and male time in household production, while it dampens the negative substitution effect for the use of female time. In this case the female household
member’s labour supply does not increase as much as in the case where the interdependence of the household members’ behaviour in not taken into consideration.

When \( \alpha > \beta \), the wife values the joint consumption of the domestic good relatively less than her husband. In this case the distribution effect is negative and dampens the positive income effect for joint consumption in the household and magnifies the substitution away from the use of female time in the production process. In this case the female household member’s home time decreases more and her labour supply increases more than in the case where the interdependence of household members’ behaviour is not considered.

Note that since all the inputs are used in positive amounts in the interior solution the technology is constant returns to scale and therefore the magnitude of the distribution effect on the demand for the domestic good does not depend on the household technology as it does in the corner solutions.

The conclusion from this section is that in the interior solution the increase in the wife’s say in the household always increases her private consumption. While the husband’s private consumption decreases if the negative distribution effect outweighs the positive income effect. This same result is derived in Blundel et al. (2005). But, as we saw earlier, in the corner solutions to the household time allocation problem, the private demands depend on how the household members value the consumption of the domestic good relative to each other. It was shown that as a result of this it is possible that private consumption of the individual gaining more say in the household decreases in corner solutions. Therefore, the conclusion is that the wife’s private consumption increases as her say in the household increases if and only if she does not value the joint consumption of the household good too much relative to her husband. In the corner solutions the demand for the domestic must not be too high due to the limitations in the input usage and therefore in order to have more domestic good it is possible that the household member gaining more say and desiring more domestic good have to give up some of his/her private consumption.

The analysis presented in this section allows one to see the changes in the time versus good intensiveness of household production process within each household time allocation regime considered. However, nothing can be said about whether the
household will move from one time allocation regime to another as a result of the exogenous increase in the female wage and decision power in the household. This question is analysed in the following section.

### 4.4 Shifts in the household optimal time allocation regime

In this section I analyse what happens to the household optimal time allocation regime when the female household member’s wage and therefore her decision power in the household increases. There can be shifts from one household time allocation regime to another since the shadow prices for domestic production change.

**Case I:** \( H^f = 0 \) and \( H^m = 0 \)

The optimum condition for the time allocation regime where both spouses specialize into market work was shown to be:

\[
\begin{align*}
  \hat{w}^f &> \frac{v}{1-v} \rho a^f G^{1-v} \left( \mu^f \right)^{\frac{v}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}} \\
  \hat{w}^m &> \frac{v}{1-v} (1-\rho) a^m G^{1-v} \left( \mu^f \right)^{\frac{v}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}}
\end{align*}
\]

Both household members specialize into market work when their market wage exceeds their shadow wages for household production. Differentiate the above conditions with respect to female wage to get:

\[
\begin{align*}
  1 > w^{sf} &\frac{1}{1-v} G^{-1} \frac{\partial G}{\partial \hat{w}^f} \\
  0 > w^{sm} &\frac{1}{1-v} G^{-1} \frac{\partial G}{\partial \hat{w}^f}
\end{align*}
\]
Where the individual shadow prices for the household time allocation regime I are denoted by $w^{Si}, i = f, m$. By substituting $\frac{\partial G}{\partial w^f}$ into the conditions in (106) it is seen that the time allocation regime, where the spouses allocate all the time available into market work and buy all household services from the market, is viable after the increase in the wife’s wage if and only if the following conditions are satisfied:

$$1 > w^{Si} \left[ \frac{1}{w^f + w^m} + \frac{\partial \theta}{\partial w^f} \left\{ \frac{(\beta - \alpha)}{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)} \right\} \right]$$

(107)

$$0 > w^{Sm} \left[ \frac{1}{w^f + w^m} + \frac{\partial \theta}{\partial w^f} \left\{ \frac{(\beta - \alpha)}{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)} \right\} \right]$$

The result depends again on how the household members value the domestic good relative to each other. In the case of identical preferences $\alpha = \beta$ the optimum condition for the husband is violated and he starts contributing towards domestic good while the condition for the wife continues to hold and she allocates all the time available into market work, as before the increase in her wage. In this case the household moves into the time allocation regime III where only the husband contributes towards domestic good.

When $\alpha < \beta$ the distribution effect magnifies the positive income effect and the husband starts to contribute towards production of the household good $G$. This is because the demand for the household good is too high in order the condition for the husband’s time allocation to continue to hold. Further, if the distribution effect is large, it is possible that optimum condition for the wife no longer holds, and the wife
starts contributing towards the household good. The conclusion for the case where \( \alpha < \beta \) is that the husband starts to contribute while the wife may or may not to start to contribute. Household moves either into regime III or into regime IV.

Instead if \( \alpha > \beta \) both optimality conditions continue to hold, if the distribution effect towards the wife’s private consumption is large enough to outweigh the positive income effect for more household services. In this case there are no regime shifts as a result of the increase in the wife’s wage. If the distribution effect is not large enough it is possible that the husband starts to contribute towards household production. The conclusion is that for the case when the household initially is on the time allocation regime I and when \( \alpha > \beta \) the household either stays in the time allocation regime I or moves into the time allocation regime III.

In general the conclusion is that the increase in the demand for domestic good must not be too high in order the optimum conditions for the time allocation regime I to continue to hold.

**Case II:** \( H^f > 0 \) and \( H^m = 0 \)

The optimum condition for this time allocation regime was shown to be:

\[
\begin{align*}
  w^f &= \frac{P}{a^f} \frac{v}{1 - v} \rho \alpha^f G_{1-v}^{1} \left( \mu^f + a^f H^f \right)^{\frac{1}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}} \\
  w^m &= \frac{P}{a^m} \frac{v}{1 - v} \left( 1 - \rho \right) \alpha^m G_{1-v}^{1} \left( \mu^f + a^f H^f \right)^{\frac{1}{1-v}} \left( \mu^m \right)^{\frac{v(1-\rho)}{1-v}}
\end{align*}
\]  

(41)

The wife allocates her time between market work and household production as long as her market wage is equal to her shadow wage for household production. While the husband specializes into market work as long as his market wage exceeds his shadow wage in household production. Differentiate the above conditions with respect to female wage to get:
\[ 1 = w_{\text{III}}^{\text{III}} \left[ (\bar{x}^g)^{-1} \frac{\partial \bar{x}^g}{\partial \bar{w}^f} - (\bar{H}^f)^{-1} \frac{\partial \bar{H}^f}{\partial \bar{w}^f} \right] \]

\[ 0 > w_{\text{III}}^{\text{III}} \left[ (\bar{x}^g)^{-1} \frac{\partial \bar{x}^g}{\partial \bar{w}^f} - (\bar{H}^f)^{-1} \frac{\partial \bar{H}^f}{\partial \bar{w}^f} \right] \quad (108) \]

Where the individual shadow prices for the household time allocation regime II are denoted by \( w_{\text{III}}^{\text{III}}, i = f, m \). By substituting \( \frac{\partial \bar{x}^g}{\partial \bar{w}^f} \) and \( \frac{\partial \bar{H}^f}{\partial \bar{w}^f} \) into the conditions in (108) we get the following:

\[
1 = w_{\text{III}}^{\text{III}} \frac{1}{w^f} \iff w^f = w_{\text{III}}^{\text{III}}
\]

\[
0 > w_{\text{III}}^{\text{III}} \left[ \frac{1 + \frac{\mu^f}{a^f}}{w^f + w^m + \frac{w^f}{a^f} \mu^f} \right. \\
+ \frac{\partial \theta}{\partial w^f} \left\{ \frac{(\beta - \alpha)}{\theta (1 - \alpha) + (1 - \theta) \left( 1 - \beta \right)} + \frac{(\beta - \alpha) \rho (1 - \rho)}{\theta \alpha + (1 - \theta) \beta + [\theta (1 - \alpha) + (1 - \theta) (1 - \beta)] (\nu \rho + (1 - \nu))} \right\} \right]
\quad (109)
\]

It is seen that the wife continues to contribute towards household good. For the husband the result depends on the household members’ preferences towards joint consumption of the domestic good.

When \( \alpha < \beta \), the distribution effect magnifies the positive income effect for more joint consumption in the household. In this case the optimum condition for the husband’s time allocation no longer holds. The husband starts to contribute towards household good and the household moves into time allocation regime IV where both household members divide their time between household and market work. This is the case as well with identical preferences where \( \alpha = \beta \).
If instead we have $\alpha > \beta$ the optimum condition for the husband’s time allocation continues to hold if and only if the negative distribution effect outweighs the positive income effect. If the distribution effect is not large enough the husband starts to contribute to domestic production. Therefore for the case when the household is initially on the time allocation regime II and when $\alpha > \beta$ the household either stays in the time allocation regime II after the increase in the wife’s wage or moves into the time allocation regime IV.

If before the increase in the female wage the household is in the regime of full specialization, where the wife allocates all the time available into market work while the husband allocates all the time available into market work, it is seen that the after the increase in the female wage the wife starts to supply labour in the market work while the husband continues to allocate all the time available into market work. The household thus moves from the regime of full specialization into the time allocation regime II.

**Case III: $H' = 0$ and $H'' > 0$**

The optimum conditions for this time allocation regime were shown to be the following:

\[
\begin{align*}
    w^f >& \left( \frac{p}{a^g} \right) \frac{v}{1-v} \rho \alpha \left( \frac{1}{1-v} \right)^{1/v} \left( \mu \right)^{\frac{v}{1-v}} \left( H^m \right)^{\frac{v(1-\rho)}{1-v}} \\
    w^m =& \left( \frac{p}{a^g} \right) \frac{v}{1-v} (1 - \rho) \alpha \left( \frac{1}{1-v} \right)^{1/v} \left( \mu \right)^{\frac{v}{1-v}} \left( H^m \right)^{\frac{v(1-\rho)}{1-v}} \cdot \frac{1}{1-v}
\end{align*}
\]

(48)

The wife allocates all the time available into market work as long as her market wage exceeds her shadow wage for household production. While the husband divides the time available between market work and household work as long as his market wage is equal to his shadow wage for household production. Differentiate the above conditions with respect to female wage to get:
\[ 1 > w^{III}_{ij} \left( \alpha \right)^{-1} \frac{\partial x^g}{\partial w^f} \]

\[ 0 = w^{III}_{ij} \left( \alpha \right)^{-1} \frac{\partial x^g}{\partial w^f} - \left( \bar{H}^m \right) \frac{\partial \bar{H}^m}{\partial w^f} \]

Where the individual shadow wages for the household time allocation regime III are denoted by \( w^{III}_{ij}, i = f, m \). By substituting \( \frac{\partial x^g}{\partial w^f} \) and \( \frac{\partial \bar{H}^m}{\partial w^f} \) into the conditions in (110) we get the following:

\[ 1 > w^{III}_{ij} \times 
\[ \frac{1}{w^f + w^m + \frac{w^m}{M^m}} + \frac{\partial \theta}{\partial w^f} \times 
\[ \left( \begin{array}{c}
\frac{\beta - \alpha}{\theta(1 - \alpha) + (1 - \theta)(1 - \beta)} \\
\frac{(\beta - \alpha) \rho}{\theta \alpha + (1 - \theta) \beta + \theta(1 - \alpha) + (1 - \theta)(1 - \beta)(1 - \rho)}
\end{array} \right) \]

\[ 0 = w^{III}_{ij} \times 0 \iff 0 = 0 \]

It is seen that the husband’s time allocation does not change and he continues to contribute towards domestic production. While the response of the wife’s time allocation depends on how the household members value the domestic good relative to each other. When \( \alpha < \beta \), it is possible that the wife starts to contribute. Thus it is possible that the household moves into the regime IV where both household members allocate their time between market and household work. This is the case if the positive distribution effect magnifies the positive income effect so that the resulting demand for the domestic good, after the increase in the wife’s wage, is too high for the
optimum condition for the wife’s time allocation to continue to hold. When \( \alpha > \beta \),
the distribution effect dampens the income effect and the optimum condition for the
wife’s time allocation continues to hold. The conclusion is that the household stays in
the time allocation regime III where only the husband contributes towards domestic
good if \( \alpha > \beta \). It is seen that this is the case as well with identical preferences where \( \alpha = \beta \).

For the case of full specialization where the wife allocates all the time available
into market work while the husband allocates all the time available into household
work we have:

\[
1 > w^{SmIIIb} \left[ (x^g)^{1/2} \frac{\partial x^g}{\partial w^f} \right]
\]

\[
0 < w^{SmIIIb} \left[ (x^g)^{1/2} \frac{\partial x^g}{\partial w^f} \right]
\]

By inserting the values for \( x^g \) and \( \frac{\partial x^g}{\partial w^f} \) into (112) we get:

\[
1 > w^{SmIIIb} \left[ \frac{1}{w^f} \left\{ \frac{\beta - \alpha}{\theta(1-\alpha)+(1-\theta)(1-\beta)} + \frac{(\beta-\alpha)\nu}{\theta\alpha + (1-\theta)\beta + \theta(1-\alpha)+(1-\theta)(1-\beta))(1-\nu)} \right\} \right]
\]

\[
0 < w^{SmIIIb} \left[ \frac{1}{w^f} \left\{ \frac{\beta - \alpha}{\theta(1-\alpha)+(1-\theta)(1-\beta)} + \frac{(\beta-\alpha)\nu}{\theta\alpha + (1-\theta)\beta + \theta(1-\alpha)+(1-\theta)(1-\beta))(1-\nu)} \right\} \right]
\]
It is seen that when $\alpha = \beta$ both conditions continue to hold and there are no regime shifts. If $\alpha < \beta$ then the condition for the husband’s time allocation continues to hold. While the condition for the wife’s time allocation continues to hold unless the positive distribution effect magnifies the positive income effect so that the resulting household demand for the domestic good is too high. Thus the household either stays in the regime of full specialization or moves into the time allocation regime where the wife allocates her time between market work and household work while the husband allocates all the time available into household work. Finally if $\alpha > \beta$ the optimum condition for the wife’s time allocation continues to hold. The optimum condition for the husband’s time allocation continues to hold unless the negative distribution effect outweighs the positive income effect. If the negative distribution effect outweighs the positive income effect, the husband starts to allocate time into market work while still contributing towards household production. Thus the household either stays in the regime of full specialization or moves into the time allocation regime III.

**Case IV: $H' > 0$ and $H'' > 0$**

The optimum conditions for the interior solution to the household time allocation problem were shown to be:

\[
\begin{align*}
    w^f &= \left( \frac{p}{a^g} \right) \frac{\nu}{1-\nu} \rho \alpha^f G^{\frac{1}{1+\nu}} \left( \bar{H}^f \right)^{\frac{\nu \rho}{1-\nu}} \left( \bar{H}^f \right)^{\frac{\nu(1-\rho)}{1-\nu}} \\
    w^m &= \left( \frac{p}{a^g} \right) \frac{\nu}{1-\nu} (\rho^m G^{1+\nu} \left( \bar{H}^m \right)^{\frac{\nu \rho}{1-\nu}} \left( \bar{H}^m \right)^{\frac{\nu(1-\rho)}{1-\nu}}
\end{align*}
\]  

(55)

Both household members allocate the time available between market work and household work as long as their market wage is equal to their shadow wage for household work. Differentiate the above conditions with respect to the wife’s wage to get:
Where the individual shadow wages for the household time allocation regime IV are denoted by $w_{hi IV}^{i} = f, m$. By substituting $\frac{\partial \bar{x}^g}{\partial \bar{w}^f}$, $\frac{\partial \bar{H}^f}{\partial \bar{w}^f}$ and $\frac{\partial \bar{H}^m}{\partial \bar{w}^f}$ into the conditions in (114) we get the following:

$$1 = w^{gIV} \left[ (x^g)^{-1} \frac{\partial \bar{x}^g}{\partial \bar{w}^f} - (\bar{H}^f)^{-1} \frac{\partial \bar{H}^f}{\partial \bar{w}^f} \right]$$

(114)

$$0 = w^{mIV} \left[ (x^g)^{-1} \frac{\partial \bar{x}^g}{\partial \bar{w}^f} - (\bar{H}^m)^{-1} \frac{\partial \bar{H}^m}{\partial \bar{w}^f} \right]$$

The conclusion is that both household members continue to allocate the time available between market and household work. Thus there are no regime shifts in interior solution to the household time allocation problem.

In the all cases considered above the household member who allocates time into household work before the exogenous change in the wife’s wage and decision power in the household, continues to do so after the changes as well. It was shown that it is possible that the spouse not participating in domestic production starts to do so after the changes in the female wage and decision power in the household. Thus, the wife can induce her husband to increase his home time as her decision power in the household increases.

The results obtained in this section of the study possibly owe to the assumptions made on household preferences and technology. The assumption that the individual preferences are additively separable in private consumption versus joint consumption of the domestic good was made as well in the more general version of the model in the section three. The structure of the wage effects was seen to be the same with both specifications. The total effect consists besides the usual income and substitution
effects on the distribution effect that describes the adjustment of the household consumption bundle as the decision power in the household changes. However, with the more general version of the model analysed in the section three the responses of the household members’ shadow prices to the changes in female wage would have been difficult to derive. The assumption of CD-preferences and technology made in this section makes possible the considerations of the adjustment of the household optimal consumption bundle and time allocation regimes in detail and therefore is worth the cost in the loss of generality.

5. Conclusions

This paper considered joint consumption of the good produced in the household in the collective household model. The general version of the collective household model is flexible enough to allow for simultaneous consideration of household production, joint consumption of the domestic good and the possibility of corner solutions.

For the model formulated in this paper the effects of exogenous changes in female household member’s wage were considered. The change in wage is assumed to affect the decision power in the household according to the bargaining view. Increase in the relative share of individuals’ hourly wage out of household income implies that the decision power of the individual in question increases. The paper shows that in the collective model there are, besides the usual income and substitution effects a distribution effect that describes the adjustment of the household consumption bundle as the decision power in the household changes.

The contributions of the paper are threefold. First it was shown how the household consumption bundle adjusts as a result of a change in the decision power in the household when the corner solutions to the household time allocation problem are considered. In interior solution the private consumption of the individual always increases with the increase in his/her say in the household. This result is in accordance with the results obtained in Blundel et al. (2005). But when the corner solutions are taken into consideration this result no longer holds. This is because in corner solutions private consumptions depend on the relationship between the household members’
marginal utilities from the consumption of the household public good. As a result of this it is possible that the private consumption of the individual gaining more say in the household decreases. Therefore, the conclusion is that the wife’s private consumption increases as her say in the household increases if and only if she does not value the joint consumption of the household good too much relative to her husband. In the corner solutions the demand for the domestic must not be too high due to the limitations in the input usage and therefore in order to have more domestic good it is possible that the household member gaining more say and desiring more domestic good have to give up some of his/her private consumption.

Secondly, it was shown how the demand and time – market good – mix for the good produced in the household responses to the changes in the decision power in the household. The availability, substitutability, and prices of the inputs used in production process, all have an influence to the final composition of the household service. In the collective model, besides these, the adjustment of decision power in the household affects as well the composition of the good produced in household. That is on how individual time inputs are substituted with each other and with market goods in the household production process. The distribution effect either magnifies or dampens the positive income effect for more household goods. The sign of the distribution effect depends on how the household members value the household good relative to each other. If the wife values the household good relatively more compared to her husband, then the wife’s wage increase implies that the distribution effect magnifies the positive income effect for joint consumption of the household good. The magnitude of the distribution effect depends on the substitutability of the inputs in the household production process. The more important the input in question is the larger is the adjustment required in the use of that input as the decision power in the household changes.

Finally, it was shown that the change in the decision power in the household may induce shifts in household optimal time allocation regime. The optimum conditions for the time allocation regimes depend on the decision power of the household members. The shadow prices for household work change with the increase in the wife’s wage and adjustment of the decision power in the household. Therefore, the change in the decision power in the household may induce the household to move
from one time allocation regime to another earlier or later than implied by the wage change alone.

The household production process outlined in the current paper deserves more attention. Further work is required to gain identification results needed for empirical work with this more realistic description of household production where the corner solutions are possible and where the domestic good is not tradable and is jointly consumed by the household members. This paper can be seen as spade work into introduction of joint consumption of the good produced in the household to the collective framework. The work is done with the general version of the collective household model. In order to gain the identification results important for empirical work there are two possibilities to proceed. It would be possible to try to identify the sharing rule conditional on public consumption of the good produced in the household. However, the possible identification results would be only partial due to the dependence of the full price of the domestic good on household characteristics. Alternatively it would be possible to try to estimate the general version of the collective model directly as in Klaveren et al. (2006a,b).

Further, the results obtained here imply that there are two channels through which the policymaker can affect household behaviour. First by affecting wages the policymaker can affect decision power in households. Secondly by offering close substitutes for individual time in household production or by family policy the policymaker can affect on the substitutability of the inputs used in the household production process. Therefore the policymaker can affect on the magnitude of the distribution effect and through this on optimal household behaviour. Further work is required for the analysis of the policy implications in the framework of the model outlined in this paper. Household’s responses with respect to policy reforms depend on the household time allocation regime. There are four different types of households with respect to time allocation regimes in the model presented in this paper. This framework opens possibility for interesting policy analysis. The effect of policy reforms could be analysed with balanced government budget constraints and heterogeneous households that differ with respect to the time allocation regime.
References


Appendix:

Graphical presentation of the possible time allocation regimes

Here the possible time allocation regimes, for the household production process outlined in the section 3.1, are considered by using as a benchmark linear technology for household production. Assume that wages differ so that \( w^i > w^j \). Assume further that implicit prices for spousal time inputs differ so that \( \frac{w^i}{a^i} > \frac{w^j}{a^j} \). This implies that the individual \( i \) has absolute advantage in market work while individual \( j \) has comparative advantage in household work.

Figure A1: Household time allocation.

In the Figure A1 the straight line truncated at \( T \) is the household isocost line. The isocost line consists of three parts where the cost of household production is constant. These are the point \( a \), the segment from \( a \) to \( c \) and the segment from \( c \) to \( e \). The slope of the segments \( ac \) and \( ce \) depend on the full price (time plus money) of the household
good. The segment ac relates to the cheaper time input. This is person j in the current example since it was assumed that \( w^j > w^i \).

The household optimal time allocations are found from the tangency of the household isocost line with the isoquant representing the household technology available in each case.

Point a in the picture describes the case where both household members are specialised into market work and the household services are bought from the market. Thus \( H^i = H^j = 0 \) and \( \frac{P}{a^i} < \frac{w^j}{a^j} < \frac{w^i}{a^j} \) i.e. the implicit prices of individual time inputs are higher than that for the market good used as input in the production process. If the household is at the point where \( H^i = H^j = 0 \), it is stuck in there until the prices change enough to restore equality of \( \frac{P}{a^i} \) with both or either one of the implicit prices for individual time inputs.\(^4\)

Point b describes the situation where individual i is fully specialised into market work and individual j divides his/her time between market work and household production. Thus \( H^i = 0 \) and \( 0 < H^j \leq T \) which implies \( \frac{P}{a^i} = \frac{w^j}{a^j} < \frac{w^i}{a^j} \).

Further, point c is the case of full specialisation where \( H^i = 0 \) and \( H^j = T \) while the relationship with the implicit prices is \( \frac{w^j}{a^j} < \frac{P}{a^i} < \frac{w^i}{a^i} \).

Point d describes a situation where individual j is fully specialised into household production while individual i divides his/her time between market and

\(^4\) It is possible that for some household chores the household members’ time inputs are not used in the household production process due to technological development. Technological progress can make the price of the market substitutes for household production relatively lower compared to that of individual time. As a result of this the time intensity of household production declines. This development is documented by Gronau and Hammermesh (2003), and Greenwood et al. (2005).
household work. Thus we have \(0 > H^i > T\) and \(H^i = T\) which implies \(\frac{w^i}{a^i} < \frac{w^j}{a^j}\)

\[
= \frac{p}{a^e}.
\]

Point e describes a situation where both household members are fully specialised into household production. This case is ruled out from the model at hand since \(x^e\) is assumed to be necessary in the production of household services and there is no non-labour income in the model.

Interior solution into the time allocation problem implies that the implicit prices for the spousal time inputs are equal with each other. With linear technology this implies that \(\frac{w^i}{a^i} = \frac{w^j}{a^j}\) has to hold. It can be seen that when \(w^j > w^i\) then it also has to be that \(a^j > a^i\) i.e. in order the implicit prices of the time inputs to be equal with each other, it has to be that the more expensive input, in terms of market wage, is used more efficiently and this implies higher productivity parameter for that input. If the wage rates are equal then the productivities have to be equal as well. With the terms of the previous picture we would have straight line from point a into point e. Linear technology implies perfect substitutability of spousal time inputs. In reality it is possible that the substitutability of spousal time inputs is less than perfect for some household chores.
Social Norms and Female Labour Supply

Abstract

Both unitary and bargaining household models predict that due to the increase in female wages the female household member’s share of the household work should decline. However, according to the time use studies this has not happened. Traditional gender roles seem to be persistent in many Western societies despite the fact that female participation in the labour market has increased considerably. I argue here that it is possible that social norms and customs of the society in question have their effect on female time allocation decisions. This paper analyses the effect of social norm for traditional gender roles on the female household member’s time allocation decisions in the collective household model. It is shown that the response of the female household member’s time allocation with respect to strengthening norm for tradition depends on the household members’ social preferences and on the decision power in the household. The paper analyses as well the policy implications in the presence of norm for tradition in the context of the collective household model. It is shown that family policy can, depending on the policy measure, either reinforce or mitigate the effect of tradition on female labour supply.

1. Introduction

Both unitary and bargaining household models predict that due to the increase in female wages the female household member’s share of the household work should decline. However, according to the time use studies this has not happened. Traditional gender roles seem to be persistent in many Western societies despite the fact that female participation in the labour market has increased considerably. According to time use studies European women do two thirds of all household work and mothers are mainly responsible for child care (Eurostat, 2004). Further, in Finland the wife does nearly two thirds of all household work even in two earner households (Piekkola&Ruuskanen, 2006 and Takala, 2005). I argue here that it is
possible that social norms and customs of the society in question have their effect on female time allocation decisions. Thus, if there is a social norm towards traditional division of labour in households the resulting household time allocations differ from that predicted by the theory.

The female labour supply is studied here in the context where spouses may hold different view on the role of women in the society. The general version of the collective household model is extended to allow for social preferences on the division of labour in households. The study analyses as well how family policy of the society in question may either reinforce or mitigate the effect of tradition on female labour supply.

**Related literature**

Economic incentives imply material rewards whereas social norms imply social rewards or punishments. Social norms are shared by the members of the society and are sustained by their approval or disapproval. Violating a norm can involve, for example, the loss of reputation within society and self-punishment through the feelings of guilt and shame. According to Elster (1989), to accept a social norm as a motivational mechanism is not to deny the importance of rational choice. Individual actions can be thought to be influenced both by rationality and social norms. Surveys for the implications of social preferences for economic analysis can be found from Brennan&Pettit (2005), Fehr&Fishbacher (2002), Becker&Murphy (2000), Weiss&Fershtman (1998), Lindbeck (1997) and Elster (1989).

There is now a growing new empirical literature on the relationship between social norms and individual time allocation decisions. Burda et al. (2006) study the distribution of total work (market work and household work) in the US and EU. The results show that gender differences in total work within a country are smaller than variation across countries and time. The European norm is to perform a larger share of total work on weekdays than on weekends so that weekends are free for personal care and leisure. The American norm is to mix work and non-work time more between weekdays and weekends. Americans work more in market, in total and at unusual times of the day and on weekends than Europeans. Burda et al. (2007) further analyse the fact that while there are substantial differences in total work across countries there is essentially no difference by gender in total
work. Men work more in the market, women work more in the household, and these balance out. The authors formulate a model where the coordination device that equalizes total work across agents is social norm for leisure. Fernadez & Fogli (2005) and Fernandez (2007) examine the work and fertility behaviour of women born in the US, but whose parents were born elsewhere. The authors use female labour force participation and total fertility rates from the country of ancestry as cultural proxies. These variables should capture the beliefs commonly held about the role of women in society from where the parents come from. It is shown that these cultural proxies have positive and significant explanatory power for individual work and fertility outcomes. The effect of these cultural proxies is amplified the greater is the tendency for ethnic groups to cluster in the same neighbourhoods. Maurin & Moschion (2006) show with French data for the years between 1990 and 2001 that a mother’s decision to participate in the labour market is correlated with those of the other mothers living in the same neighbourhood.

Social norms are already recognized in the current economic literature as having an important effect on individual behaviour. However, as far as I know, the effect of traditional gender roles has not yet been explicitly studied in the context of household decision making. This paper analyses the effect of social norm on traditional gender roles on the female household member’s time allocation decisions in the general version of the collective household model introduced by Chiappori (1988, 1992). In this setting the spouses may hold different view on the women’s role in the society. The interest here is on a question how a prevailing norm for traditional division of labour may distort the efficient time allocation in households and how this norm interacts with family policy.

The paper shows that the response of the female household member’s time allocation with respect to strengthening norm for tradition depends on the household members’ social preferences and on the decision power in the household. In the case where both household members value traditional gender roles the result is U-shape relationship between relative female earnings and her household work. This result is in line with empirical results obtained with US and Australian data (Bittman et al., 2003; and Akerlof & Kranton, 2000) and more recently with Spanish data (Fernandez & Sevilla-Sanz, 2006). Further, it is shown that the norm for tradition may either increase or decrease household utility. Whether the house-
hold utility decreases or increases as a response to stronger gender roles depends on the relationship between spouses’ earnings. The household can be hurt by the norm for tradition if the household deviates from the normative standard according to which men make more money than women. The paper analyses as well the policy implications in the presence of norm for tradition in the context of the collective household model. It is shown that family policy can, depending on the policy measure, either reinforce or mitigate the effect of tradition on female labour supply.

The paper proceeds as follows. Section two lays out the general model of the collective household with social preferences for traditional gender roles. In section three more structure is introduced into the general model in order to analyse the interdependence of the balance of power in the household and social reward parameters. Policy implications of the model at hand are discussed in section four. Discussion and concluding comments are presented in sections five and six respectively.

2. The model

Models of social preferences make various common assumptions. A first assumption is that the utility function of an individual has a ‘material’ part, which shows how much he or she values the monetary payoff, and a ‘non-material’ part, which shows how much the individual values the opinions of others. A second assumption is that individuals can differ regarding to the intensity with which they care about the non-material part of the utility function, relative to the material part. A third assumption is that, although individuals can differ with respect to their valuation to the non-material part, everyone shares the same structure of preferences and this fact is common knowledge. This means that everyone knows what the norm of the society is, but not everyone cares as much about it. The individual utility with social concerns can be written in the following additive form:

\[ U^i + \varepsilon^i S \]

Where \( U^i \) is individual \( i \)’s utility from consumption, \( S \) is the social payoff function describing the social norm of the society, and \( \varepsilon^i \) denotes the type of the
individual \( i \), determining how much weight the individual gives on the opinions of others.

The interest here is on the consequences of traditional division of labour in households. In order to analyse the effect of tradition on female labour supply, social payoff function describing the norm for tradition is introduced into the general version of the collective household model. I follow Fersthman and Weiss (1997) and determine the norm as the average action in the society. The individual gains a positive social reward, in terms of self esteem, if the personal performance is above the average and a negative social punishment, in terms of feelings of shame or guilt, if the personal performance is below the average. The social payoff function is modelled here to describe the norm for tradition instead of the willingness to work, usually considered in the framework of social preferences (e.g. Fershtman & Weiss, 1997 and Lindbeck et al., 1999). Therefore, the effort yielding social status equals here the household work of the female household member. This effort determines whether the individual faces social reward or punishment. The social norm for tradition influences but does not mandate female home time. The individuals have the choice of the extent to which they stick to the norm, and optimally balance the costs and benefits.

The social payoff function \( S \) for the norm for tradition where the average female labour market participation rate represents the socially accepted level for female participation in labour market can be written as:

\[
S = q \left[ L^a - L^f \right] = q \left[ L^a - (1 - H^f) \right] = q L^a - q H^f \tag{1}
\]

where

\[
L^a = \text{average female labour supply in the society} \\
L^f = \text{labour supply of the female household member} \\
H^f = \text{household work of the female household member} \\
q = \text{the population share of the individuals adhering to the norm}
\]
Total time available is set equal to one and the female household member allocates her time between market work $L^f$ and household work $H^f$. If there are $n$-households in the society, the average female labour supply is found from:

$$L^a = \frac{1}{n} \sum_{i=1}^{n} L^f_i, \, i = 1...n$$

From the above formulation it is seen that $L^a$ is consistent with social preferences and the distribution of types in the population through the labour supply of all females in the society. Each individual in the society treats $L^a$ as given and each female household member chooses her labour supply, $L^f$. For the society as a whole the average female labour supply $L^a$, is determined endogenously by the aggregate behaviour of all females in the society. Since the total time available for each individual is set equal to one, the average female labour supply varies between zero and one as well. It is assumed that $L^a \in (0, 1)$, to exclude the extremes where all females in the society are either fully specialized into household work or market work.

Individuals are allowed to differ with respect to the weight $\varepsilon^i$ given for social concerns. There can be a continuum of types each giving different non-negative weight for the non-material part of the utility function. For simplicity, it will be assumed here that $\varepsilon^i \in \{0, 1\}$, thus there are only two types of individuals so that an individual either cares about the opinion of the others or does not care at all about the others’ opinions. From now on I shall call the individual with $\varepsilon^i = 1$ as socially minded and the individual with $\varepsilon^i = 0$ as asocial in accordance with the terminology used in Fershtman & Weiss (1997). I assume that it is possible that individuals of differing types, i.e. with differing social preferences, form a household. Together with the assumption that $\varepsilon^i \in \{0, 1\}$ this implies that there can be four different types of households according to social preferences. These are: the households where both spouses are socially minded; households where only the wife is socially minded; households where only the husband is socially minded; and households where both spouses are asocial. Individuals in the society are assumed to be equally divided by gender, and they all belong to a household made up of two individuals of different gender.

The distribution of types in the population determines the effectiveness of the
social reward/punishment in manipulating the aggregate behaviour. The larger is
the population share of individuals adhering to the norm $q$, the more effective the
norm is. I assume non-degenerate social preferences so that there always are both
socially minded and asocial individuals in the population. The population share
of the socially minded individuals ($\varepsilon^i = 1$) is denoted by $q \in (0, 1)$. Therefore
$(1 - q) \in (0, 1)$ denotes the population share of asocial individuals ($\varepsilon^j = 0, i \neq j$).
The average action gets affected by the norm, and as a result the average female
labour supply in the society in question is the lower the more there are individuals
adhering to the norm for traditional division of labour. The intuition behind this
is that if the female household members in the socially minded households work
more in the household and less in the market than the female household members
in asocial households then the average female labour supply $L^a$ has to be low when
the population share of the socially minded individuals $q$ is high, and vice versa.
Thus, there is negative relationship between the population share of the socially
minded individuals and the average female labour supply. Further, it is assumed
that there is one to one mapping between $q$ and $L^a$. Each level of $q$ on the interval
$(0, 1)$ corresponds to a unique value of $L^a$ on the interval $(0, 1)$, and vice versa.
This means that the population share of socially minded individuals $q$ is decreasing
monotonic function of the average female labour supply $L^a$. This mechanism is
comparable to that in Linbeck et al. (1999) where the deviation from the norm to
live off one’s own income instead of transfers is analysed. There the more there are
deviators from the work norm the lower is the social embarrassment from living
on transfers. Here the higher is the average female labour supply of the society
in question the lower is the social embarrassment from exceeding this normative
standard since there are more asocial individuals than socially minded individuals
in the society.

On the basis of the above considerations the population share of the socially
minded individuals can be written as a function of the average female labour supply
in the society as follows:

$$q(L^a), \quad q'(L^a) < 0$$ (2)
The population share of the socially minded individuals $q$ depends only on the average female labour supply $L_a$ of the society in question which is determined endogenously by the average behaviour of all the female members of the society. The social payoff function can now be written as:

$$S = q(L_a) [L_a - L_f] = q(L_a) [L_a - (1 - H_f)]$$ (3)

Note, that the above formulation implies that the marginal social reward or punishment is now determined by $q(L_a)$. Therefore the higher is $q(L_a)$ the stronger the norm is in manipulating behaviour. A simple special case for the interdependence between the average female labour supply and the population share of the socially minded individuals is considered in the Appendix.

Consider next the household problem with the social norm for traditional division of labour in households. Assume that only the female household member works in the household and that the household good is produced by using the time input only. This means that household production equals the home time of the female household member $H_f$, in the current model. Note, that this implies that the male household member allocates all the time available into market work.\(^1\)

This is in line with the so called 'iso-work fact' according to which there is no difference by gender in total work (household work plus market work). Burda et al. (2006) formulate a model of social norm for leisure to explain the 'iso-work fact'. In their model the norm for leisure works as a coordination device so that total work between individuals in a society is equal, while the level of total work between societies can vary. However, the composition of total work between genders differs. Men work more in the market, women work more in the household, and these balance out (Burda et al., 2007 and 2006). The model formulated here aims to explain why the composition of the total work between genders differs.

It is assumed here that the household members gain utility from their private consumption $x^i$, $i = f, m$, for the female and male household member respectively, and from joint consumption of the household good $H_f$. Thus individual utility is

\(^1\)Since the social norm for traditional division of labour in the households affects only the time allocation of the female household member, the male household member’s home time is not considered in the model. The introduction of male home time would not affect the results in the sense how the female labour supply responds to stronger norm for tradition.
of the form:

\[ U^i(x^i, H^f) + \varepsilon^i S \] (4)

Assume that \( U^i_x > 0, U^i_{xx} < 0 \) and \( U^i_H > 0, U^i_{HH} < 0, i = f, m \). Further assume that \( U^i_{2H} = U^i_{HH} = 0 \) in order the household production approach to be viable. In the collective household model the household maximizes the weighted average of the household members’ individual utilities. The collective household problem with social norm for traditional division of labour is the following:

\[ \Omega \equiv \theta \{ U^f(x^f, H^f) + \varepsilon^f q(L^a) [L^a - (1 - H^f)] \} 
+ (1 - \theta) \{ U^m(x^m, H^f) + \varepsilon^m q(L^a) [L^a - (1 - H^f)] \} \] (5)

s.t.

\[ x^f + x^m = (1 - H^f) w^f + w^m \]
\[ x^f, x^m, H^f > 0 \]

where

\[ \theta = \text{welfare weight, } \theta \in (0, 1) \]
\[ U^i = \text{individual utility, } i = f, m \]
\[ x^i = \text{individual consumption of market goods, } i = f, m \]
\[ H^f = \text{household work done by the female household member} \]
\[ \varepsilon^i = \text{type of an individual, } \varepsilon^i \in \{0, 1\}, i = f, m \]
\[ q(L^a) = \text{population share of the socially minded individuals, } q \in (0, 1) \]
\[ L^a = \text{average female labour supply of the society in question, } L^a \in (0, 1) \]
Lagrangian for the above problem is:

\[
L = \theta \{ U^f(x^f, H^f) + \varepsilon^f q(L^a) [L^a - (1 - H^f)] \} \\
+ (1 - \theta) \{ U^m(x^m, H^f) + \varepsilon^m q(L^a) [L^a - (1 - H^f)] \} \\
+ \lambda \left( (1 - H^f) w^f + w^m - x^f - x^m \right) 
\]

(6)

The first order conditions are:

\[
\frac{\partial L}{\partial x^f} = \theta U^f_{xx} - \lambda = 0 
\]

(7)

\[
\frac{\partial L}{\partial x^m} = (1 - \theta) U^m_{xx} - \lambda = 0 
\]

(8)

\[
\frac{\partial L}{\partial H^f} = \theta \left( U^f_{H} + \varepsilon^f q(L^a) \right) + (1 - \theta) (U^m_{H} + \varepsilon^m q(L^a)) - \lambda w^f = 0 
\]

(9)

\[
\frac{\partial L}{\partial \lambda} = (1 - H^f) w^f + w^m - x^f - x^m = 0 
\]

(10)

The formulation used for the social payoff function implies that there is a feedback effect from the average female labour supply to individual behaviour. It is seen that when the average female labour supply of the society in question increases the female household member’s home time decreases as a result. We have:

\[
\frac{d H^f}{d L^a} = -\left\{ \theta U^f_{xx} + (1 - \theta) U^m_{xx} \right\} \left\{ \theta \varepsilon^f q'(L^a) + (1 - \theta) \varepsilon^m q'(L^a) \right\} \frac{1}{|I|} \leq 0
\]

(11)

where

\[
|I| = \theta^2 U^f_{xx} U^f_{HH} + \theta U^f_{xx} (1 - \theta) U^m_{HH} + (1 - \theta) U^m_{xx} \theta U^f_{xx} (w^f)^2 \\
+ (1 - \theta) U^m_{xx} \theta U^f_{HH} + (1 - \theta)^2 U^m_{xx} U^m_{HH}
\]

is the system determinant which is positive by the second order conditions. The derivative in (11) is zero in the case where both spouses are asocial, i.e. if \(\varepsilon^f = \)
\( \varepsilon^m = 0 \) then \( dH^f/dL^a = 0 \). But if at least one of the spouses is socially minded then (11) is negative by the second order conditions and since \( q'(L^a) \) is negative. The negative relationship between \( q \) and \( L^a \) implies that the higher is the average female labour supply for the society in question, the lower will the marginal social reward be.

Consider next the reactions of female household member’s time allocation with respect to stronger norm for traditional gender roles. The increase in the population share of the socially minded individuals implies that the norm for tradition gets stronger. The comparative static result for the female household member’s home time with respect to the population share of the socially minded individuals is found to be:

\[
\frac{dH^f}{dq} = \frac{-\{\theta U^{xf} + (1 - \theta) U^{xm}\} \times \theta \varepsilon^f \left(1 + q'(L^a) \frac{\partial L^a}{\partial q}\right) + (1 - \theta) \varepsilon^m \left(1 + q'(L^a) \frac{\partial L^a}{\partial q}\right)}{|I|} \geq 0 \quad (12)
\]

The expression in (12) is zero in the case where both spouses are asocial, i.e. if \( \varepsilon^f = \varepsilon^m = 0 \) then \( dH^f/dq = 0 \). But if at least one of the spouses is socially minded then (12) is positive by the second order conditions and by the inverse function rule. Since it was assumed that the population share of the socially minded individuals \( q \) is a decreasing monotonic function of the average female labour supply \( L^a \), we have by the inverse function rule that

\[
q'(L^a) \frac{\partial L^a}{\partial q} = q'(L^a) \frac{1}{q'(L^a)} = 1
\]

The conclusion is that the strengthening social norm has a positive effect on the household work of the female household member in socially minded households. This implies that female market labour supply for these households decreases whenever the norm for tradition gets stronger. It can be seen that the positive effect of stronger gender roles on the female household member’s home time consists of the direct effect due to the higher population share of the socially minded individuals and of the indirect effect through the decrease in the average female labour supply.
of the society. By applying the inverse function rule in (12) it simplifies into the following:

$$\frac{dH_f}{dq} = -2 \left( \theta U_{xx}^f + (1 - \theta) U_{xx}^m \right) \frac{\{ \theta \varepsilon_f + (1 - \theta) \varepsilon^m \}}{|I|}$$  \hspace{1cm} (13)$$

The magnitude of the positive effect of stronger norm on female household member’s home time depends on the types of the household members \( \varepsilon^i \), and on the decision power in the household. The decision power in the household is determined by the welfare weight \( \theta \in (0, 1) \). The welfare weight describes how much weight individual preferences get in the household utility function \( \Omega \). The welfare weight adjusts when the exogenous factors determining it change. Generally the welfare weight is assumed to depend on prices and on household income. For the current purposes nothing else is assumed about the welfare weight except that it is determined on the open interval from zero to one. This formulation rules out the cases of female or male dictatorship. In collective models for household behaviour it is thought that \( \theta \) summarizes the earlier decisions of the household members although the decision process itself is not modelled. For a survey of collective household models see Vermeulen, 2002. When \( \theta \) increases the wife’s preferences get more weight in the household utility function and consequently the husband’s preferences get less weight. For the adjustment of the welfare weight \( \theta \) the comparative static analysis for the wife’s home time yields:

$$\frac{dH_f}{d\theta} = \frac{-w^f \theta U_{xx}^f \left( U_f^f + U_x^m \right) - \left( \theta U_{xx}^f + (1 - \theta) U_{xx}^m \right) \times \\
\left\{ U_f^f + \varepsilon_f q (L^a) - w^f H_f - U_m^m - \varepsilon^m q (L^a) \right\}}{|I|} \quad \forall \theta > 0$$  \hspace{1cm} (14)$$

It is seen that the sign of the derivative \( dH_f/d\theta \) depends on the term

$$\left\{ U_f^f + \varepsilon_f q (L^a) - w^f U_x^f - U_H^m - \varepsilon^m q (L^a) \right\}$$

in the numerator. If this expression is greater that zero then \( dH_f/d\theta > 0 \). This means that the more say the wife has in the household the more she allocates her
time in household production. If instead

\[
\left\{ U_H^f + \varepsilon^f q(L^u) - w_x^f U_x^f - U_H^m - \varepsilon^m q(L^u) \right\} < 0
\]

then \( dH^f / d\theta < 0 \), which means that the more say the wife has the less she allocates her time in household work.

Consider now the meaning of the expression

\[
\left\{ U_H^f + \varepsilon^f q(L^u) - w_x^f U_x^f - U_H^m - \varepsilon^m q(L^u) \right\}
\]

It describes the distribution effect on the demand for household good when the welfare weight \( \theta \) adjusts, i.e. when the decision power in the household changes. The sign of the distribution effect depends on the spouses’ marginal utilities from the joint consumption of the household good \( H^f \), and on the wife’s marginal utility from her private consumption \( x^f \). More time allocated to the household work implies less time left for market work for the female household member and therefore less private consumption \( x^f \). In the context of social preferences the social marginal reward/punishment also affects this net result depending on the types of the household members. There are four cases to consider depending on the spouses’ social preferences.

Suppose that both household members are asocial, i.e. \( \varepsilon^f = \varepsilon^m = 0 \). In this case the social norm has no effect on household behaviour. The distribution effect for this case is

\[
\left\{ U_H^f - w_x^f U_x^f - U_H^m \right\}
\]

and it is positive only if the wife’s marginal willingness to pay for for the household good after the adjustment of the welfare weight is large enough. This means that we have more \( H^f \) when the wife’s say in the household increases, only if she prefers relatively more the consumption of the household good than her private consumption and this net effect has to be large enough to compensate for the decrease in the weight given on her husbands’s marginal utility from joint consumption of the household good.

If both household members are socially minded, i.e. \( \varepsilon^f = \varepsilon^m = 1 \), then the term describing marginal social reward/punishment for the spouses \( q(L^u) \) cancel

131
out in the expression for the distribution effect and we have the same result as above. This is because the increase in the weight given to the wife’s preferences equals the decrease in the weight given to the husband’s preferences.

Consider next the case where only the wife is socially minded, i.e. \( \varepsilon^f = 1 \) and \( \varepsilon^m = 0 \). There is no direct social reward or punishment for the husband but the wife’s utility depends on the social payoff. The distribution effect is now

\[
\left\{ U^f_H + q(L^a) - w^f U^f_x - U^m_H \right\}
\]

It is seen that there is marginal social reward equal to \( q(L^a) \) for the wife if she allocates more time into household work. In this case it is possible that even if the wife prefers more \( x^f \) to \( H^f \) the result may be that she allocates more time into household due to the norm for tradition when her say in the household increases. This result differs from the case without conformity to the social norm for traditional gender roles. Without conformity the household work done by the female household member always decreases when her say in the household increases and she values relatively more her private consumption than the joint consumption of the household good.

Finally, consider the case where only the husband is socially minded, i.e. \( \varepsilon^f = 0 \) and \( \varepsilon^m = 1 \). For this case the distribution effect is

\[
\left\{ U^f_H - w^f U^f_x - U^m_H - q(L^a) \right\}
\]

This naturally is just the opposite of the case where only the wife is socially minded. When the wife’s say in the household increases the socially minded husband’s preferences get less weight. In this case it is possible that the household time of the asocial wife decreases more than in the case without conformity towards traditional gender roles.

We have seen that for the cases where only one of the spouses is socially minded, the responses of the wife’s household time can differ from that indicated by the marginal rates of substitution between household good and private good. For example, if the derivative \( dH^f/d\theta \) were negative without social preference for female home time, it may turn to be positive in the presence of social norm for traditional gender roles.
division of labour. The conclusion is that the norm for traditional gender roles may either strengthen or weaken the distribution effect, due to the changes in the decision power in the household. The net effect is ambiguous in the general case considered here. Therefore the relationship between the parameters describing the social norm and decision power in the household is discussed in more detail in the next section where more structure is introduced into the model at hand.

3. Household utility - a formal example

Assume that the individual utility, $i = f, m$ is of the following form:

$$U_i(x_i, H^f) + \varepsilon_i S = \ln(x^f H^f) + \varepsilon_i q(L^a) \left[L^a - (1 - H^f)\right]$$

(15)

this implies that, $U_{xx}^i > 0$, $U_{xH}^i < 0$, $U_{HH}^i > 0$, $U_{HH}^i < 0$, and $U_{xH}^i = U_{Hx}^i = 0$ as required. All parameters and variables are specified as in the general version of the model in the section two of this paper. Note, that the above specification for individual preferences implies that the household members have identical additively separable preferences towards consumption of the private good and joint consumption of the household good. Since the focus here is on the effect of social norm for traditional gender roles, the spouses are allowed to differ only with respect to their social preferences. The collective household problem with individual preferences determined as in (15) can be written as:

$$\max \Omega \equiv \theta \left\{ \ln \left(x^f H^f\right) + \varepsilon f q \left(L^a\right) \left[L^a - (1 - H^f)\right]\right\}$$

$$+ \left(1 - \theta\right) \left\{ \ln \left(x^m H^f\right) + \varepsilon m q \left(L^a\right) \left[L^a - (1 - H^f)\right]\right\}$$

(16)

s.t.

$$x^f + x^m = (1 - H^f) w^f + w^m$$

$$x^f, x^m, H^f > 0$$

The optimal consumptions are found to be:
From (17)-(19) it can be seen that social norm affects optimal consumptions through the interaction of the parameter describing the decision power in the household $\theta$ and the parameters describing the social norm $\varepsilon^i, i = f, m$ and $q(L^a)$. As in the general version of the model, studied in the previous section, the average female labour supply of the society has a negative feedback effect on the home time of the female household member. It is seen that:

$$\frac{\partial H^*}{\partial L^a} = \frac{q'(L^a)}{2\theta\varepsilon^f q^2} + \frac{q'(L^a)}{2(1-\theta)\varepsilon^m q^2} < 0 \quad (20)$$

The higher is the average female labour supply of the society, the lower will the home time of the female household member be in the representative household. This negative effect on female home time comes through decreasing marginal social reward when the average female labour supply for the society in question increases. At the optimum the average level of female labour supply is found from:

$$L^a = \frac{1}{n} \sum_{i=1}^{n} L^*_i$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left[ 1 - \frac{1}{2} \left( \frac{w^f_i + w^m_i}{w^f_i} - \frac{1}{\theta_i\varepsilon^f_i q(L^a)} - \frac{1}{(1-\theta_i)\varepsilon^m_i q(L^a)} \right) \right] \quad (21)$$

This is difficult to solve explicitly for $L^a$, but $\partial L^a/\partial q$ is obtained as:

$$x^f* = \frac{\theta}{2} \left( w^f + w^m + \frac{w^f}{\theta\varepsilon^f q(L^a)} + \frac{w^f}{(1-\theta)\varepsilon^m q(L^a)} \right) \quad (17)$$

$$x^m* = \frac{(1-\theta)}{2} \left( w^f + w^m + \frac{w^f}{\theta\varepsilon^f q(L^a)} + \frac{w^f}{(1-\theta)\varepsilon^m q(L^a)} \right) \quad (18)$$

$$H^* = \frac{1}{2} \left( \frac{w^f + w^m}{w^f} - \frac{1}{\theta\varepsilon^f q(L^a)} - \frac{1}{(1-\theta)\varepsilon^m q(L^a)} \right) \quad (19)$$
\[
\frac{\partial L^a}{\partial q} = -\frac{1}{n} \sum_{i=1}^{n} \frac{1}{2} \left( \frac{1}{\theta_i \varepsilon_i^l q^2} + \frac{1}{(1 - \theta_i) \varepsilon_i^m q^2} \right) - \frac{1}{n} \sum_{i=1}^{n} \frac{1}{2} q' \left( L^a \right) \frac{\partial L^a}{\partial q} \left( \frac{1}{\theta_i \varepsilon_i^l q^2} + \frac{1}{(1 - \theta_i) \varepsilon_i^m q^2} \right)
\] (22)

Since it was assumed that the population share of the socially minded individuals \( q \) is decreasing monotonic function of the average female labour supply \( L^a \), we have \( q' \left( L^a \right) \frac{\partial L^a}{\partial q} = q' \left( L^a \right) \frac{1}{q' \left( L^a \right)} = 1 \) by the inverse function rule and (22) simplifies into:

\[
\frac{\partial L^{a*}}{\partial q} = -\frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{\theta_i \varepsilon_i^l q^2} + \frac{1}{(1 - \theta_i) \varepsilon_i^m q^2} \right) < 0
\] (23)

It is seen that the more there are socially minded individuals in the society in question the lower will the resulting average female labour supply be. Consider next the responses of the optimal demands with respect to stronger norm for tradition. We have the following:

\[
\frac{\partial x^{f*}}{\partial q} = - \frac{w^f \left( 1 + q' \left( L^a \right) \frac{\partial L^a}{\partial q} \right) - \theta w^f \left( 1 + q' \left( L^a \right) \frac{\partial L^a}{\partial q} \right)}{2 \varepsilon^f q^2} - \frac{2(1 - \theta) \varepsilon^m q^2}{2 \varepsilon^m q^2} < 0
\] (24)

\[
\frac{\partial x^{m*}}{\partial q} = - \frac{(1 - \theta) w^f \left( 1 + q' \left( L^a \right) \frac{\partial L^a}{\partial q} \right) - w^f \left( 1 + q' \left( L^a \right) \frac{\partial L^a}{\partial q} \right)}{2 \varepsilon^f q^2} - \frac{2 \varepsilon^m q^2}{2 \varepsilon^m q^2} < 0
\] (25)

\[
\frac{\partial H^{f*}}{\partial q} = \frac{1 + q' \left( L^a \right) \frac{\partial L^a}{\partial q}}{2 \theta \varepsilon^f q^2} + \frac{1 + q' \left( L^a \right) \frac{\partial L^a}{\partial q}}{2 (1 - \theta) \varepsilon^m q^2} > 0
\] (26)

From (24)-(26) it is seen that besides the direct effect, the increase in the population share of the socially minded individuals affects optimal demands indirectly through the decrease in the average female labour supply in the society in question. By applying the inverse function rule, the derivatives in (24)-(26) can be rewritten
as:

\[
\frac{\partial x^f}{\partial q} = -\frac{w^f}{\varepsilon^f q^2} - \frac{\theta w^f}{(1 - \theta) \varepsilon^m q^2} < 0 \tag{27}
\]

\[
\frac{\partial x^m}{\partial q} = -\frac{(1 - \theta) w^f}{\theta \varepsilon^f q^2} - \frac{w^f}{\varepsilon^m q^2} < 0 \tag{28}
\]

\[
\frac{\partial H^f}{\partial q} = \frac{1}{1 - \varepsilon^f q^2} + \frac{1}{(1 - \theta) \varepsilon^m q^2} > 0 \tag{29}
\]

The strengthening social norm has a positive effect on household work done by the female household member but a negative effect on private demands since there is trade off between household income and household production.

### 3.1 Time allocation responses with respect to stronger norm for tradition

As in the general version of the model, in section two, the magnitude of the positive effect of \( q \) on \( H^f \) depends on the interaction of the parameters describing household members’ social preferences and on the parameter describing decision power in the household. There are four possible cases depending on the household members’ social preferences. When both household members are asocial strengthening norm has no effect on behaviour. Therefore, the first case to consider here is the one where only the female household member is socially minded, i.e. if \( \varepsilon^f = 1 \) and \( \varepsilon^m = 0 \), we have:

\[
\frac{\partial H^f}{\partial q} = \frac{1}{\theta q^2} > 0 \tag{30}
\]

It is seen that the value of the derivative \( \partial H^f/\partial q \) is decreasing in the welfare weight \( \theta \). The closer to one the welfare weight \( \theta \) gets, that is the more say the female household member has in the household, the smaller is the effect of the norm for tradition on female household member’s home time. The female household member’s optimal labour supply is found from \( L^f = 1 - H^f \). Therefore we have:

\[
\frac{\partial L^f}{\partial q} = -\frac{1}{\theta q^2} < 0 \tag{31}
\]
It is seen that the higher is the welfare weight $\theta$, the less negative is the effect of strengthening social norm on her labour supply. We can conclude that when the welfare weight $\theta$ gets close to one the effect of social norm on the wife’s time allocation decisions diminishes in the case where only the wife is socially minded. The graph describing this process is presented in the Figure 1.

**Figure 1:** Only the wife is socially minded.

\[ \frac{\partial H^f}{\partial q} \]

Interpretation for the process presented in the Figure 1 is that when the weight given for the wife’s preferences is high (close to one) and only she is socially minded, her home time is at a relatively high level already before the norm for tradition gets stronger. Therefore the effect of stronger norm on her time allocation is small. Whereas when the weight given for the wife’s preferences is low (close to zero) and only she values traditional gender roles, her home time is at a relatively low level due to the trade-off between private consumption versus joint
consumption of the household good. Therefore the stronger norm has significant effect on the wife's time allocation with low levels of the welfare weight.

The second case to consider is that where only the husband is socially minded, that is when $\varepsilon^f = 0$ and $\varepsilon^m = 1$, we have:

$$\frac{\partial H^{f^*}}{\partial q} = \frac{1}{(1 - \theta) q^2} > 0$$

(32)

Now the value of the derivative $\partial H^{f^*}/\partial q$ is increasing in the welfare weight $\theta$. We can conclude that the closer to one the welfare weight $\theta$ gets, the larger is the effect of social norm to the home time of the wife. For responses of the the wife's labour supply we have:

$$\frac{\partial L^{f^*}}{\partial q} = -\frac{1}{(1 - \theta) q^2} < 0$$

(33)

From this it is seen that the closer to one the welfare weight $\theta$ gets, the more negative is the value of the derivative $\partial L^{f^*}/\partial q$. The conclusion is that when only the husband is socially minded, and the more say the asocial wife has in the household, the larger is the effect of strengthening social norm on the wife's time allocation decisions. The graph describing this process is presented in the Figure 2.
Figure 2: Only the husband is socially minded.

The interpretation for the process presented in the Figure 2 is that when the weight given to the wife’s preferences is high and she does not value traditional gender roles, her home time is at a relatively low level and therefore the stronger norm has a large impact on her behaviour. In a collective setting, even if the wife does not care for the norm for tradition, the fact that her husband cares about other’s opinions on the role of women in society implies that the wife’s home time increases as a response to a stronger norm. This result reflects the conflict of interests of the spouses when only the husband cares about the norm and when the wife has more say in the household.

Finally, the third case to consider is the one where both spouses are socially minded, that is when $\varepsilon^i = 1$ ($i = f, m$), the effect of stronger norm for tradition on the household work done by the female household member decreases when the welfare weight is between the range $0 < \theta < 1/2$ and increases when the welfare weight is between the range $1/2 < \theta < 1$. Thus, before the point where $\theta = 1/2$ the
effect of stronger norm on the wife’s time allocation is getting smaller as the wife’s say in the household increases. But after the point where \( \theta = 1/2 \) the conflict of interests of the spouses starts to dominate and the effect of stronger norm on the wife’s time allocation grows with \( \theta \). The graph describing this process is presented in the Figure 3.

**Figure 3:** Both household members are socially minded.

It is seen that there is U-shape relationship between the responses of the female household members home time with respect to stronger norm and her say in the household. The only assumption made here about the welfare weight, determining the decision power in the household, is that \( \theta \) is determined on the open interval from zero to one. The existing empirical literature both in economics and in sociology suggests that individual’s decision power in the household depends on his/her relative earnings. According to the bargaining view the increase in the share of wife’s earnings out of household total income increases her decision power in the household. If this is the case then the process presented in the Figure 3 leads to the interpretation according to which the response of the wife’s home time
with respect to stronger norm decreases when her relative earnings increase up to a point where the spouses earnings are equal. After this the response of the wife’s home time with respect to stronger norm increases with her relative earnings. This result is in line with empirical findings. When the wife provides more than half of the household income, her household time increases with further increases in her earnings. Fernandez&Sevilla-Sanz (2006) find U-shape relationship between female relative earnings and time devoted to household work with Spanish time use data 2002-2003. They conclude that this relationship could be due to 'doing gender' as a result of social norms associated to the division of housework. Earlier evidence for the U-shape relationship between female wages and household work can be found from Bittman et al. (2003) and from Akerlof&Kranton (2000). These empirical results are in contrast to what the exchange theory in sociology and both unitary and bargaining theories in economics predict.\textsuperscript{2} The conclusion in the sociological literature is that the couples that deviate from the normative standard, where men make more money than women, seem to compensate with a more traditional division of household work. Here similar result is obtained for the first time from a microeconomic model based on rational behaviour and utility maximization.

Takala (2005) has studied with Finnish time use data the existence of the U-shape relationship between female household time and relative female earnings. The U-shape pattern is not found with Finnish data. Instead it is found that female home time decreases with female earnings up to a point where the spouses’ earnings are equal. After this point female home time stays at level higher than the half of all household work. Therefore, Finnish data seem to be consistent with the case two of the current model where only the wife is socially minded. The result probably reflects women’s preferences and beliefs about women’s role. How women conceive their role in the household, do children benefit or suffer from having a working mother and how is she treated by friends and neighbours as a result of her choices. Since the average female labour supply gets affected by the family policy measures, the family policy of the society in question arguably strongly shapes the beliefs about women’s role. Family policy can be seen as a means to increase the

\textsuperscript{2}The exchange theory in sociology suggests that power flows from bringing resources into a relationship and that a spouse can use economically based bargaining power to get the other partner to do the household work.
participation rate of mothers. On the other hand, family policy can be interpreted as sustaining the norm for traditional division of labour, if family policy implies long absence of female workers from the labour market due to childbirth. I will return to this question in the section four of this paper where the implications of the family policy are analysed in the context of the current model.

3.2 Household utility responses with respect to stronger norm for tradition

When the population share of the socially minded individuals increases it implies stronger norm for traditional division of labour. This may either increase or decrease household utility. Before discussing on the effect of social norm on household utility $\Omega$, have a closer look on the social payoﬀ function $S$ at the optimal levels of consumption. Continue to assume that there are $n$ households in the society in question and consider the responses of the household $j$. At the optimum the social payoﬀ function for the household $j$ is:

$$S_j^* = q(L^a) \left[ L^a - L^a_j^* \right] = q(L^a) \left[ L^a - (1 - H_j^*) \right] \quad (34)$$

At the optimal level of $H_j^*$ the social payoﬀ function for the household $j$ responds to the increase in the population share of socially minded individuals as follows:

$$\frac{\partial S_j^*}{\partial q} = \left[ L^a - L^a_j^* \right] + q'(L^a) \frac{\partial L^a}{\partial q} \left[ L^a - L^a_j^* \right] + q(L^a) \left[ \frac{\partial L^a}{\partial q} - \frac{\partial L^a_j}{\partial q} \right] \quad (35)$$

By using (19), (21), (23) and (26) we can rewrite (35) as:
This reduces into:

\[
\frac{\partial S^*_j}{\partial q} = \frac{1}{n} \sum_{i=1}^{n} \left[ \frac{1}{2} \left( \frac{w_i^f + w_i^m}{w_i^f} - \frac{1}{\theta_i \varepsilon_{i}^f q (L^{a*)} - \frac{1}{(1 - \theta_i) \varepsilon_{i}^m q (L^{a*)}} \right) \right] \\
+ \left( \frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{\theta_i \varepsilon_{i}^f q^2} + \frac{1}{(1 - \theta_i) \varepsilon_{i}^m q^2} \right) \right) - \frac{1}{(1 - \theta_j) \varepsilon_{i}^m q^2} \\
\approx 0
\]

It is seen that the response of the household $j$’s social payoff with respect to stronger norm can be positive or negative. The average female wage rate in the society of $n$ households is:

\[
w^{af} = \frac{1}{n} \sum_{i=1}^{n} w_i^f
\]

And similarly the average male wage rate is:

\[
w^{am} = \frac{1}{n} \sum_{i=1}^{n} w_i^m
\]

Using these in (37) the expression for the response of the household $j$’s social payoff with respect to the increase in the population share of the socially minded individuals simplifies into:
From (38) it can be seen that the response of the household j’s social payoff to the increase in the population share of the socially minded individuals depends on the relationship between average male and female wages in the society and on the relationship between male and female household member’s wages in the household j. Since in most societies the relationship between average male and female wages is \( w_{am} > w_{af} \), the second term in the square brackets is greater than 1. This implies that if we have \( w_{mj} > w_{fj} \) for the household j, then the first term in the square brackets is greater than 1 and (38) is positive if and only if:

\[
\frac{w_{mj}^m}{w_{fj}^m} > \frac{w_{am}^m}{w_{af}^m} \tag{39}
\]

This means that if the wage difference between male and female household member is larger in the household j than in the society on average, the social payoff for the household j increases with the population share of the socially minded individuals. In this case there is positive social reward for household j. If instead we have \( w_{mj}^m < w_{fj}^m \) for household j, then the first term in the square brackets in (38) is less than 1. This means that the response of the social payoff for the household j with respect to the increase in the population share of the socially minded individuals is negative, implying social punishment. The conclusion is that the model implies social punishment for the households deviating from the normative standard where men make more money than women.

We are now ready to consider what happens to household utility when the social norm for traditional gender roles gets stronger. The household optimum for the household j is:

\[
\Omega_j^* = \theta_j \left( U_j^{f*} + \varepsilon_j^f S_j^* \right) + (1 - \theta_j) \left( U_j^{m*} + \varepsilon_j^m S_j^* \right)
\]

Differentiate this with respect to the population share of the socially minded
individuals to get:

\[
\frac{d\Omega^*_j}{dq} = \theta_j \left\{ \frac{\partial U^f_{j*}}{\partial x^f_{j*}} + \frac{\partial U^m_{j*}}{\partial H^f_{j*}} \right\} + (1 - \theta_j) \left\{ \frac{\partial U^m_{j*}}{\partial x^m_{j*}} + \frac{\partial U^m_{j*}}{\partial H^m_{j*}} \right\} + \varepsilon^f_j \frac{\partial S^*_j}{\partial q}
\]

(40)

In order to have a closer look on the responses of the household utility with respect to strengthening social norm, insert the comparative static derivatives derived earlier into the formulation given in (40). Now we have:

\[
\frac{d\Omega^*_j}{dq} = \theta_j \left( \frac{w^f_j - \varepsilon^f_j q}{\varepsilon^f_j q^2} - \frac{\theta_j w^f_j}{(1 - \theta_j) \varepsilon^m_j q^2} \right) + (1 - \theta_j) \left( \frac{(1 - \theta_j) w^f_j}{\theta_j \varepsilon^f_j q (L^a)} - \frac{w^f_j}{\varepsilon^m_j q^2} \right)
\]

(41)
This reduces into:

\[
\frac{d\Omega_j^*}{dq} = \frac{-w_j^f}{\theta_j \varepsilon_j^f q^2} \frac{w_j^f}{(1 - \theta_j) \varepsilon_j^m q^2} + \frac{w_j^m}{\theta_j \varepsilon_j^m q^2} + \frac{w_j^f}{(1 - \theta_j) \varepsilon_j^m q^2}
\]

\[
= \frac{1}{2} \left( \frac{w_j^f}{\theta_j \varepsilon_j^f q(L^a)} + \frac{w_j^f}{(1 - \theta_j) \varepsilon_j^m q(L^a)} \right) + \left[ \frac{\varepsilon_j^f \theta_j + \varepsilon_j^m (1 - \theta_j)}{w_j^m - w^{am}} \right] \left[ \frac{w_j^m}{w_j^f} - \frac{w^{am}}{w^{af}} \right]
\]

The first term on the right hand side relating to the private demands is always negative and the second term relating to the demand for the household good is always positive (the denominator of this expression is greater than zero since \( H_j^{f*} > 0 \)). The third term describing the response of the household's social payoff with respect to stronger norm can be either positive or negative as shown earlier.

There are again four cases to consider depending on the household member's social preferences. For asocial households we naturally have \( d\Omega/dq = 0 \). For the households where both spouses are socially minded the expression in (42) reduces into:
It can be seen that the positive effect of stronger norm on the demand for household good $H_f^j$ outweighs the negative effect of stronger norm on private demands. Therefore the total effect of stronger norm on household utility is positive if the expression

$$\left[ \frac{w_j^m}{w_j^f} - \frac{w_{am}}{w_{af}} \right]$$

is positive. It was shown earlier that this is always the case when the male wage relative to female wage is higher in the household $j$ than the average male wage relative to average female wage for the society in question. Therefore, when $w_j^m > \frac{w_{am}}{w_{af}}$ the social payoff for the household $j$ increases with $q$ and it implies that the household utility increases as well with $q$. But if $w_j^m < w_j^f$ the social payoff for the household $j$ decreases with $q$, and in this case it is possible that the household utility decreases with $q$, if the social punishment outweighs the positive effect from the increase in the home time of female household member. That is if she does not increase her home time enough to avoid the social punishment.

For the case where only the wife is socially minded it is seen as well that the positive effect of stronger norm on household joint consumption of $H_f^j$ outweighs the negative effect of stronger norm on the private consumptions. Therefore the
response of household utility with respect to the increase in the population share of the socially minded individuals depends again on the relationship between the male and female wages for the household $j$ and the average male and female wages for the society in question. In this case the welfare weight $\theta_j$ for the household $j$ determines how much weight the social reward or punishment gets in the household utility. The more say the socially minded wife has the larger will be the effect of norm on the household utility. Whereas in the case where only the husband is socially minded the weight given for the social reward or punishment is $(1 - \theta_j)$. The more say the socially minded husband has, the larger is the effect of the norm for tradition on the household utility. It is possible that there are households where the household members are hurt by the existence of social norm towards traditional division of labour. These households end up at a lower utility level as a consequence of stronger norm for traditional division of labour. The weight given in household utility on the social payoff is the largest for the case where both household members are socially minded. For the case where only one of the spouses is socially minded the weight given on the social payoff depends on the decision power in the household.

The conclusion from this section is that when the population share of the socially minded individuals increases it implies stronger norm for traditional division of labour. This may either increase or decrease household utility. It is possible that both household members are worse off in the case the norm for tradition gets stronger. The net result depends here on the interaction of the parameters describing the social preferences and the balance of power in the household. In order to stronger norm for tradition to have a positive effect on household utility it has to be that the positive effect on the joint consumption of the household good plus possible social reward are large enough to outweigh the negative effect on private demands plus the possible social punishment. It was shown that this is always the case for the household in question when the male wage relative to female wage rate is higher than the average male wage relative to average female wage for the society. If instead the wife’s wage is higher than the husband’s wage, it is possible that the household utility decreases as a result of stronger norm for traditional division of labour in households.
4. Policy implications

The role of family policy has to be discussed in the context of the current paper since family policy affects the average female labour supply. Family policy can be seen as a means to increase the participation rate of mothers. On the other hand, family policy can be interpreted as sustaining the norm for traditional division of labour, if family policy implies long absence of female workers from the labour market due to childbirth. For example, parental leave decreases the labour supply of the group entitled and it is likely that individuals who do not use this opportunity face social cost. On the other hand, providing subsidized market substitutes for household work, such as the public day care for the children, can increase the labour supply of the mothers with small children. In Finland only 22% of children under three years of age are in day care outside the home and 66% of children from three to six years of age are in day care outside the home (Piekkola & Ruuskanen, 2006). This tells that the Finnish mothers are absent from the labour market due to childbirth for a long time.

I will analyse next the implications of two specific family policy measures used in Finland in the setting formulated in this paper. These are the child home care allowance and the public day care of the children. After parental leave families can choose between public day care and home care allowance. Home care allowance is paid to families with children under three years of age and not in public day care. Home care allowance relates to child care leave system according to which the mother or the father or both in turn can be on a child care leave from their job until the child is three years old.

The analysis here is made with illustrative examples and many important aspects are not considered. One of the most important ones is the employment status of the female household member when making the decision for the child care mode after the parental leave. If the mother does not have a job where return to after the parental leave, it certainly affects the decision that is made. In the following examples it is assumed that there is full employment in the society in question. The children are not explicitly modelled, instead I make the assumption that the

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3Parental leave consists of two parts, maternity and parental leave. The maternity leave is about 4 months and can be used only by the mother. The parental leave is about 6 months and can be used by either parent.
spouses gain utility besides their private consumption from the well being of their children. Thus individual utility is defined as $U^i(x^i, c), i = f, m$ with the idea that the joint consumption of the household good now equals the child welfare denoted by $c$. The aim is to describe the implications of the two family policy measures used in Finland within the context of the current model.

Consider first the household problem if it chooses the public day care of the children instead of the home care allowance. Public day care is introduced into the basic model by assuming that there is less than perfect market substitute for the maternal care. Denote the public day care by $d$ and the maternal care by $H^f$. Child welfare is produced by constant returns to scale technology as follows:

$$c = (H^f)^\alpha (d)^{1-\alpha}$$

where

\begin{align*}
c & = \text{total child welfare produced and consumed by the household} \\
H^f & = \text{child care performed by the mother} \\
d & = \text{public day care}
\end{align*}

The household maximizes:

\begin{align*}
Max \Omega_d & \equiv \theta \left\{ \ln (x^f (H^f)^\alpha (d)^{1-\alpha} + \varepsilon^f q (L^a) [L^a - (1 - H^f)]) \right\} \\
& + (1 - \theta) \left\{ \ln (x^m (H^f)^\alpha (d)^{1-\alpha} + \varepsilon^m q (L^a) [L^a - (1 - H^f)]) \right\}
\end{align*}

s.t.

\begin{align*}
x^f + x^m + d & = (1 - H^f) w^f + w^m \\
x^f, x^m, d & > 0
\end{align*}
Where the subsidized price of public day care is set to unity. The resulting optimal demands are:

\[ H_d^{fs} = \frac{\alpha}{2} \left\{ w^f + w^m + \frac{\alpha - 2}{\theta \varepsilon f q (L^a)} + \frac{\alpha - 2}{(1 - \theta) \varepsilon m q (L^a)} \right\} \]  

(46)

\[ d^s = \frac{1 - \alpha}{2} \left\{ w^f + w^m + \frac{\alpha w^f}{\theta \varepsilon f q (L^a)} + \frac{\alpha w^f}{(1 - \theta) \varepsilon m q (L^a)} \right\} \]  

(47)

\[ x_{d}^{fs} = \frac{\theta}{2} \left\{ w^f + w^m + \frac{\alpha w^f}{\theta \varepsilon f q (L^a)} + \frac{\alpha w^f}{(1 - \theta) \varepsilon m q (L^a)} \right\} \]  

(48)

\[ x_{d}^{ms} = \frac{(1 - \theta)}{2} \left\{ w^f + w^m + \frac{\alpha w^f}{\theta \varepsilon f q (L^a)} + \frac{\alpha w^f}{(1 - \theta) \varepsilon m q (L^a)} \right\} \]  

(49)

From these we can deduce the consequences of stronger norm for tradition on the use of the mother's time versus the market substitute in the production of the child welfare. The response of the female household member's home time with respect to stronger norm for the case where both household members are socially minded is now found to be:

\[ \frac{\partial H_d^{fs}}{\partial q} = \alpha \left\{ -\frac{\alpha - 2}{\theta q^2} - \frac{\alpha - 2}{(1 - \theta) q} \right\} > 0 \]  

(50)

The above expression is positive since \( \alpha - 2 < 0 \). When the scale parameter \( \alpha \) is close to zero it means that the market substitute, here the public day care \( d \), is relatively more important in the production of the child welfare than the mother's home time \( H^f \). In this case the social norm for tradition has smaller impact on the time allocation decision of the female household member than in the absence of market substitute for her home time. This can be seen by comparing the value of the derivative in (50) to the value of the derivative in (29) when \( \alpha \) is close to zero. If instead \( \alpha \) is close to one, implying that the mother's home time is relatively more important in the production of the child welfare than public day care, the value of the derivative in (50) is close to that in the case studied in the section three of this paper without a market substitute for \( H^f \). The conclusion is that the problem for the policymaker is to introduce a market substitute close enough for self production if the burden of the tradition is to be eliminated. This relates
to the questions of availability, quality and price of the market substitute for self provision.

For the demand for the public day care the effect of stronger norm is seen to be:

\[ \frac{\partial d^*}{\partial q} = 1 - \alpha \left\{ - \frac{\alpha w^f}{\theta q^2} - \frac{\alpha w^f}{(1 - \theta)q} \right\} < 0 \] (51)

As expected the stronger is the norm for traditional division of labour the lower is the demand for the public day care of the children.

Consider now the household problem in case it chooses the home care allowance instead of the public day care. Since home care allowance is paid only to the households with children under three years of age and not in public day care, choosing the home care allowance implies that \( c = H^f = 1 \), that is the mother allocates all the time available to child care. \(^4\) The home care allowance denoted by \( t \) affects only the household joint budget. Thus in this case the household maximizes:

\[ \text{Max} \Omega_t \equiv \theta \{ \ln x^f + \varepsilon^f q (L^n) L^n \} + (1 - \theta) \{ \ln x^m + \varepsilon^m q (L^n) L^n \} \] (52)

s.t.

\[ x^f + x^m = w^m + t \]
\[ x^f, x^m > 0 \]
\[ c = H^f = 1 \]

\(^4\)It is possible to use home care allowance to partly cover the fees for private day care for the children under three years of age. But since this option does not theoretically differ from the use of public day care of the children, it is not considered separately. The only difference in choosing private day care instead of public day care is that the net cost of the private day care is probably higher for the household. The aim here is to consider the workings of a direct transfer for the home care of the children versus a substitute for maternal care in the setting allowing social preferences for what is good for the childrens’ well being.
The optimal demands are:

\[ c^* = H_{i}^{fs} = 1 \]  
\[ x_{i}^{fs} = \theta \{ w^m + t \} \]  
\[ x_{i}^{m*} = (1 - \theta) \{ w^m + t \} \]  

Denote the share of the households that choose the home care allowance by \( \delta \) and the share of the households that choose the public day care by \( 1 - \delta \). The average female labour supply for the society in question is now found from:

\[ L^a = \frac{1}{n} \sum_{i=1}^{n} \left( \delta L_{i}^{fs} + (1 - \delta) L_{di}^{fs} \right) \]  

Since choosing the home care allowance implies that \( L_{di}^{fs} = 0 \), the average female labour supply for the society in question is:

\[ L^a = \frac{1}{n} \sum_{i=1}^{n} (1 - \delta) L_{di}^{fs} = \frac{1}{n} \sum_{i=1}^{n} (1 - \delta) \left( 1 - H_{di}^{fs} \right) \]  
\[ = \frac{1}{n} \sum_{i=1}^{n} (1 - \delta) \left[ 1 - \frac{\alpha}{2} \left\{ \frac{w_f^i + w_m^i}{w_f^i} + \frac{\alpha - 2}{\theta_i} \frac{\varepsilon_f^i q (L^a)}{q (L^a)} + \frac{\alpha - 2}{(1 - \theta_i)} \frac{\varepsilon_m^i q (L^a)}{q (L^a)} \right\} \right] \]  

From (57) it is immediately seen that the larger is the share of the households choosing home care allowance \( \delta \) the lower will the average female labour supply for the society in question be. The higher is the transfer payment the more there are households that choose the home care allowance instead of public day care everything else equal. This means that the transfer payment \( t \) affects \( L^a \) through \( \delta \). When \( L^a \) for the society in question decreases implying lower normative standard for female labour supply for the society in question it has negative feedback effect on labour supply of the female household member in the representative household. As a result the home time of the female household member in the representative household increases trough the feedback effect. Therefore, a transfer from the government can decrease the average labour supply of mothers of children under three
years old. In this case there is extra cost in terms of social punishment for those households who do not choose home care allowance. Transfer from the government, such as the home care allowance, may thus lead larger than anticipated negative effect on female labour supply in the presence of social norm for traditional gender roles.\footnote{According to Ilmakunnas (1997) majority of families with children less than three years of age chose home care allowance instead of public day care after the introduction of the home care allowance.}

When the two family policy measures are taken into consideration the average female labour supply responds with respect to stronger norm for traditional gender roles as follows:

\[
\frac{\partial L^a}{\partial q} = \frac{1}{n} \sum_{i=1}^{n} (1 - \delta) \alpha \left\{ \frac{\alpha - 2}{\theta_i \varepsilon_i q^2} + \frac{\alpha - 2}{(1 - \theta_i) \varepsilon_i^m q^2} \right\} < 0 \tag{58}
\]

The above is the more negative the more important is the mother’s time in production of the child welfare, i.e. the closer to one is the value of the scale parameter $\alpha \in (0, 1)$. Further the negative response of the average female labour supply is the larger the more there are households choosing the public day care of the children. This counter intuitive result is due to the fact that the households that have chosen the home care allowance $\delta$ are already using all the time available for the mother into child care and therefore the time use in these households does not respond to stronger norm. Whereas those households that use both mother’s time and market substitute in production of child welfare can respond to stronger norm by decreasing the use of the market substitute in favor of mother’s time. And as a result of this the average female labour supply for the society in question decreases as shown in (58).

**Household utility responses**

It is possible to analyse the responses of household utility with respect to stronger norm for traditional gender roles in the presence of the family policy similarly as in the section three without family policy. In order to stronger norm for tradition to have a positive effect on household utility it has to be that the
positive effect on the joint consumption of the household good plus possible social
reward are large enough to outweigh the negative effect on private demands plus
the possible social punishment. It was shown that this is always the case for the
household in question when the male wage relative to female wage is higher than
the average male wage relative to average female wage for the society.

The utility of the household that chooses public day care responses to stronger
norm for traditional gender roles as follows:

\[
\frac{\partial \Omega^*_d j}{\partial q} = \frac{(\alpha - 2) \left\{ \frac{\omega_{dj}^f}{\theta_{dj}^f q^2} + \frac{\omega_{dj}^f}{(1 - \theta_{dj}^f) q^2} \right\}}{\frac{1}{2} \left\{ w_{dj}^f + w_{dj}^m + \frac{\omega_{dj}^f}{\theta_{dj}^f q^2} \right\} + \frac{\omega_{dj}^f}{(1 - \theta_{dj}^f) q^2}} + (2 - \alpha) \left\{ \frac{\omega_{dj}^f}{\theta_{dj}^f q^2} \right\} + \frac{\omega_{dj}^f}{(1 - \theta_{dj}^f) q^2}
\]

The response of household utility with respect to stronger norm consists of
three terms. The first term on the right hand side of (59) describes the effect of
stronger norm on private demands and on the demand for public day care. This
is always negative since \( \alpha - 2 < 0 \). The second term describes the response of
the mother’s home time with respect to stronger norm. This is always positive
since \( 2 - \alpha > 0 \). It is seen that for the household choosing the public day care
the positive effect of stronger norm on the home time of the wife outweighs the
negative effect of tradition on private demands and on the demand for the public
day care with all possible values for the scale parameter \( \alpha \in (0, 1) \). Finally the
third term describes the social payoff for the household choosing the public day
care when the norm for tradition gets stronger. This can be either positive or
negative. Thus, the net response of the household utility depends whether there is social reward or punishment. The social payoff for the household that chooses the public day care instead of the home care allowance adjusts with respect to stronger norm as follows:

$$\frac{\partial S^*_d}{\partial q} = \alpha \left[ \frac{w^{m}_d}{w^{f}_d} - \frac{w^{am}}{w^{af}} \right] + \delta \left[ (\alpha - 2) + \alpha \frac{w^{am}}{w^{af}} \right] \geq 0 \quad (60)$$

In order the socially minded household to choose public day care instead of home care allowance, the gain in child welfare has to be large enough to outweigh the social punishment from the female household member’s market work. It is seen that there is no social reward nor punishment for socially minded household choosing public day care when the following condition holds:

$$w^{m}_d w^{f}_d = w^{am} w^{af} + 2 \left[ \frac{\alpha}{\alpha} - \frac{w^{am}}{w^{af}} \right] \quad (61)$$

The term in the square brackets in (61) is negative when $\alpha$ is close to one and $w^{am} > w^{af}$. Therefore the male wage relative to female wage in household choosing the public day care does not have to be as high as in the case without market substitute for the mother’s home time. If the household chooses public day care it faces social punishment as the norm for traditional gender roles gets stronger in the case where the value of the parameter $\alpha$ is low. This is because the closer to zero the value of the scale parameter $\alpha$ gets the larger will the term in square brackets be implying that in order the household to avoid social punishment the male wage relative to female wage for the household in question should be unrealistically high.

Consider next the utility responses of the household choosing home care allowance. Mothers use all the time available into child care in households that choose home care allowance. Therefore in these households the time use does not respond to stronger norm. For these households the response of the household utility with respect to stronger norm is always equal to the response of the social payoff. The response is seen to be:
\[
\frac{d\Omega_{ij}^*}{dq} = \frac{\partial S_{ij}^*}{\partial q} = \left\{ \theta_{ij} \varepsilon_{ij}^f + (1 - \theta_{ij}) \varepsilon_{ij}^m \right\} 2 (1 - \delta) > 0
\]  

The above is the more positive the lower is the share of the households that have chosen home care allowance \( \delta \). This counter intuitive result means that when the share of the households choosing home care allowance is high, the average female labour supply for the society in question is low, and this implies low social marginal reward \( q(L^a) \) for behaving according to the norm.

It has to be noted here that the household members’ social preferences do not dictate the decision for the child care mode. Socially minded households will buy day care outside the home as long as the gain in utility outweighs the possible social cost. In order the socially minded household to choose the public day care instead of the home care allowance, the gain in child welfare has to be large enough to outweigh the social punishment from the female household member’s market work.

Social policies may be interpreted as sustaining the norm for traditional division labour, if family policy implies long absence of female workers from the labour market due to childbirth. On the basis of the analysis presented in this section of the paper the conclusion is that the problem for the policymaker is to introduce a market substitute close enough for self production if the burden of the tradition is to be eliminated. This relates to the questions of availability, quality and price of the market substitute for self provision. Family policy measures directed to fathers instead of mothers can as well increase the labour supply of mothers and therefore lighten the cost of tradition.

5. Discussion

While family policy may be seen as a means to increase the labour force participation rate of mothers, there may be important boomerang effects on the position of women overall if the benefits are seen as limiting female employees’ commitment to work. The norm for traditional gender roles in households can have important effect on the labour market status of women overall. If employers hold the norm of the traditional division of labour in households and treat all female employees as potential mothers, the issue of motherhood may affect the labour demand of
women overall - married or single, with or without children. If this is the case then the policy measures aimed for altering the traditional roles in households benefit women in general. Similar arguments within different context can be found from Datta Gupta et al. (2008 and 2006), Albrecht et al. (2001) and Lommerud et al. (2000).

Katav-Herz (2003) analyses the incidence of child labour in the context of social preferences. However, the author does not consider the effect of tradition relating to household work done by daughters in developing economies when their mothers work outside the household. This has consequences for the school attendance of these girls. The World Bank policy research report (2001) highlights the consequences of increased female labour supply on the education of their daughters unless the day care system is improved simultaneously with participation of mothers into labour market in developing countries. The fact that daughters, instead of sons, take care of the home while the mother is away arguably reflects the norm for traditional gender roles. The World Bank policy research report (2001) concludes that a thorough understanding of local gender systems is critical to ensuring that the political and development programs are designed and implemented in way that foster greater gender equality.

The rejections of the cooperative household model with African data (Udry, 1996; and McPeak&Doss, 2006) may actually be a consequence of strong cultural factors and gender roles in developing economies. Rather than being evidence from non-cooperative behaviour in households the failure of the cooperative models may be a consequence of the externality created by the cultural factors. This opens a new research agenda where the collective household decision making with the extension for social concerns should be analysed.

6. Conclusions

This paper studied the effect of social norm for traditional gender roles on the female household member’s time allocation decisions in the collective household model. Household time is positively affected by strengthening social norm and consequently the female labour supply is negatively affected. The results show that the magnitude of the effect of tradition on the female household member’s time allocation decisions depends on the interaction of the social preferences and
decision power in the household.

For the case where only the wife cares about tradition it was shown that when her decision power in the household increases the effect of tradition on her time allocation diminishes. While for the case where only the husband cares about the norm for tradition and the more say the wife has in the household the larger is the effect of tradition on her time allocation decisions. This result reflects the conflict of interests of the spouses when only the husband is socially minded and when the wife has more say in the household. Finally, in the case where both household members value traditional gender roles the result is U-shape relationship between relative female earnings and her household work. This result is in line with empirical results obtained with US and Australian data (Bittman et al., 2003) and more recently with Spanish data (Fernandez & Sevilla-Sanz, 2006). These empirical results are in contrast to what the exchange theory in sociology and both unitary and bargaining theories in economics predict. In sociological literature this is seen as an evidence of 'doing gender'. This means that women earning more than their husbands seem to compensate with a more traditional division of household work. Here similar result is obtained from a microeconomic model based on rational behaviour and utility maximization. Further, it was shown that the norm for tradition may either increase or decrease household utility. Whether the household utility decreases or increases as a response to stronger norm for traditional gender roles depends on the relationship between spouses' earnings. The household can be hurt by the norm for tradition if the household deviates from the normative standard according to which men make more money than women.

The paper analyses as well the policy implications in the presence of norm for tradition in the context of the collective household model. It is shown that family policy can, depending on the policy measure, either reinforce or mitigate the effect of tradition on female labour supply. The workings of a direct transfer for the home care of the children versus a market substitute for maternal care was considered in the setting allowing social concerns for what is good for the well being of the children. The results show that the transfer from the government for the home care of the children may strengthen the existing social norm for traditional gender roles and as consequence lead to lower than anticipated average female labour supply in the society in question.
Specialization according to traditional gender roles is not problematic at itself. But if as a consequence there is asymmetrical accumulation of human capital and since in the modern society the value of household skills is lower than in the past this process leads to limited consumption possibilities in the future for the spouse specializing in household work. In light of the results of the current paper the conclusion is that the way the policy maker can minimize the cost of traditional gender roles, besides introducing market substitutes for household production, is introducing parental leave policy targeted on fathers instead of targeting parental leave fully on mothers or just leaving the decision to the households. As a result of this social norms could evolve into direction of more equal responsibilities for household chores. It has to be noted that the model formulated in this paper implicitly implies a norm of full time work for the male household member. This norm may be even stronger that the norm on female labour supply. Therefore, the family policy is in the key role in reshaping the gender roles in the society as whole.

References


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Appendix:
Interdependence between average female labour supply and the population share of the socially minded individuals

Both \( q \) and \( L^a \) are assumed to vary on the open interval between zero and one. The relationship between \( q \) and \( L^a \) is negative, so that the higher is the average female labour supply for the society in question the lower the population share of socially minded individuals has to be. Simplest example satisfying these properties for the relationship between \( q \) and \( L^a \) is the following:

\[
q = 1 - L^a
\]

This would imply

\[
\frac{\partial q}{\partial L^a} = -1 < 0
\]

and

\[
\frac{\partial L^a}{\partial q} = -1 < 0
\]

and therefore

\[
\frac{\partial q}{\partial L^a} \cdot \frac{\partial L^a}{\partial q} = 1
\]

thus it is seen that the inverse function rule for monotonic functions applies for this specification. Consider next the interaction of \( L^a \) and \( q \) with some heuristic examples. First assume that the average female labour supply for the society in question is very high, let say \( L^a = 0.9 \). This implies that the population share of the socially minded individuals has to be \( q = 0.1 \). In this case the social reward/punishment for the different levels of female household time is found from the following:
\[ S = 0.1 \left[ H - 0.1 \right] = 0.1H - 0.01 \]

It is seen that the norm for tradition has only negligible impact on behaviour. When the population share of the socially minded individuals is low and the average female labour supply for the society in question is high there is practically no social sanctions and only a moderate social payoff for those adhering to the norm for tradition. When \( L^a \to 1 \), the norm for tradition fades away since the marginal social reward/punishment \( q(L^a) \) describing the effectiveness of the norm diminishes. Social norms emerge and evolve or disappear when the society in question develops. Becker (2000) uses divorce rates as an example of this. He refers to strong stigma against divorce that prevailed in Western countries until a few decades ago. When the labour force participation of married women rose it induced a sustained rise in the break up of families since the 1960s. In terms of the current paper the higher is the average female labour supply in the society of question, the less likely is social punishment relating to market work done by the female household member.

Consider next the more realistic case where \( L^a = 0.6 \) and therefore \( q = 0.4 \). For this case the social payoff varies with different levels of female household time
as follows:

\[ S = 0.4 [H - 0.4] = 0.4H - 0.16 \]

It is seen that there is significant social punishment from exceeding the average female labour supply representing the normative standard for female labour supply. The social reward for those exerting effort above the average is significant as well.

Finally consider the extreme where the average female labour supply is very low, let say \( L^a = 0.1 \) and therefore \( q = 0.9 \). For this case the social payoff varies with different levels of female household time as follows:
\[ S = 0.9 \left[ H - 0.9 \right] = 0.9H - 0.81 \]

It is seen that there is considerable social punishment at very low levels of market work. The payoff for those following the norm does not need to be high since this behaviour is already a custom of the society. For those violating the norm the social punishment increases with the deviation from the normative standard implied by \( L^a \). The conclusion is that the effectiveness of the norm relates to the population shares of the socially minded versus asocial individuals in the society. When \( L^a \) is high the norm for tradition is not effective since in this case only a moderate punishment for deviating from the norm is possible. And when \( L^a \) is low the norm is effective since there can be considerable punishment for deviating from the norm.
Collective Household Model with Endogenous Balance of Power and Household Production

Abstract

This paper analyses the effect of introducing dynamics into a collective household model with domestic production where the balance of power in the household is determined endogenously. It is widely argued in the literature that combining dynamics with endogenous balance of power in the collective household model will lead into solutions that fail Pareto efficiency. If decision making power in the household is driven by the household members’ actual earnings, the resulting labour supplies can be inefficiently high. This is because the household members’ recognize the decrease in their future bargaining power due to the time devoted into household work. Decreasing say in the household implies lower private consumption in subsequent periods for the individual in question. I argue here that household solutions will be on the efficient frontier when the joint consumption of the domestic good is taken into consideration. If attention is given only on the household members’ private consumptions, the solutions will fail Pareto efficiency.

1. Introduction

Introducing dynamics into a household model allowing distinct preferences for the household members is very important since many household decisions are dynamic in nature and a decision made today affects the choice set available in the future. Examples of decisions potentially today affects the choice set available in the future are the decisions about labour force participation and fertility. It is argued in the literature of household behaviour that individual decision power in the household depends on individual’s actual earnings. If this is the case, then labour supply decisions are both matter of household decision and a determinant of decision power in the household.

The current paper analyses the female household member’s time allocation decisions and the demand for household public good in a framework where household production is explicitly modeled and there is a link between household decisions
and the balance of power in the household. It is shown that when household pro-
duction and public consumption of the domestic good is taken into consideration
introducing dynamics into a collective household model with endogenous balance
of power does not necessarily lead into solutions that fail Pareto efficiency.

**Related literature**

The existing studies of dynamic household behaviour in multi decision maker
framework consider, for example, location decisions, retirement decisions, educa-
tion, and female labour supply. Lundberg & Pollak (2003) analyse the location
decisions of two-earner couples. They show that when current location decisions
affect future bargaining power, inefficient outcomes are likely to occur. If the
spouses could make binding commitments to refrain from exploiting the future
bargaining advantage, the inefficiency would disappear.

Lundberg et al. (2003) study the retirement consumption puzzle in marital
bargaining model. They do not model an endogenous sharing rule. They as-
sume instead that since the Pareto weight depends on actual earnings and status,
the retirement decision implies that the decision making power of the individual
in question declines relative to his/her spouse. Wives have longer life expectan-
cies than husbands, thus they prefer lower per period consumption in order to
spread resources over their longer life. The authors show that in the case of
no-commitment the household consumption declines at the husband’s retirement.
While in the case of commitment consumption remains unchanged on the hus-
band’s retirement. Aura (2005) considers the effect of a change in US pension
law on the household demand for survivor annuities and life insurance in dynamic
bargaining framework. The predictions of the model are contrasted with the pre-
dictions of the standard unitary model. Aura’s (2005) results reject the standard
unitary model in favor of the bargaining model.

Iyigun & Walsh (2007) take into account how pre-marital education decisions
affect marital power and the share the spouses extract from household resources
in the future. Their model predicts that the wives invest more than is Pareto
efficient in their education in order to increase their bargaining power in mar-
riage. As a consequence, the couples will have fewer children and consume more
when exogenous structural changes lead women to invest more in education. Kon-
rad&Lommerud (2000) analyse education decisions in a two-stage model and show that family members who bargain cooperatively in the second stage will overinvest in their education in the first stage in order to improve their bargaining positions. The authors conclude that reduction in all education levels would be Pareto improving.

Iyigun (2004) introduces a process of spousal matching into a household labour supply model. He shows that large marriage markets help to generate maritally sustainable Pareto efficient levels of labour supply in a collective model in which intra-marital allocations are determined by an endogenous sharing rule that depends on the household members’ actual earnings. Further, Iyigun (2005) analyses bargaining and specialization in marriage in a standard Nash bargaining framework. There are potential gains from specialization but specializing in home production lowers market wages. The author shows that matching in large and asymmetric marriage markets induces spousal cooperation and specialization. But, when there are equal numbers of men and women in the marriage markets, spousal specialization may not occur unless there exists a commitment mechanism.

Basu (2006) was the first to suggest (in his (2001) working paper version), a model where the household balance of power is determined endogenously by household decisions. Basu (2006) discusses female labour supply in the context of collective household model where decision power in the household is determined endogenously. He shows that introducing dynamics into a collective household model with endogenous balance of power is likely to lead into solutions that fail Pareto efficiency. The main conclusion of the literature so far is that one cannot restrict attention on the efficient frontier when the household members’ choices affect the decision power in the household (Browning et al., 2004). In most cases the existing studies concentrate on the division of private consumption in the household. Ignoring household production and public consumption in the household may lead into misleading conclusions about the allocation of resources in households as pointed out by Apps (2003). If the household members gain utility besides their private consumptions from the joint consumption of the good produced in the household, it is not clear whether introducing dynamics leads into inefficient results. The question is whether the gains in terms of the household public good balance out the losses in bargaining power due to the negative effect of household
work on actual earnings.

This essay formulates a collective household model where the decision power in the household is driven by the household members’ actual earnings. The gains from marriage come from the joint consumption of the good produced in the household. Devoting time into household production implies decreasing say in the household. It is shown that in the case of commitment where the household members can commit to refrain from exploiting the future bargaining advantage there always is a solution where the time allocated into household is greater than zero and the solution will be on the efficient frontier. For the case of no-commitment it is shown that it is likely that there is a household equilibrium where the time devoted into household is greater than zero and the household is on the efficient frontier. Further, it is shown that in the case of no-commitment even a small exogenous transfer from the government guarantees that the household solution will be on the efficient frontier.

The paper is organized as follows. In section two the existence of household equilibrium in one period collective model with endogenously determined decision power and public consumption of the good produced in the household is analyzed. In section three dynamics is introduced into the model with endogenous decision power and joint consumption of the good produced in the household. Section four addresses the question of household intertemporal efficiency. Section five concludes.

2. Household time allocation with endogenous decision power
   - one period model

In this section I analyze the existence of household equilibrium in the presence of public consumption of the domestic good when the household decision power is determined endogenously. The collective labour supply model initiated by Chiappori (1988, 1992) is extended to allow for endogenous decision power and domestic production. In the general version of the collective household model the household maximand can be written as a weighted average of the household members’ individual utilities.

\[ \Omega = \theta U^f + (1 - \theta) U^m \]  

(1)
Where $\theta$ is the Pareto weight determining the decision power of the wife in the household. The Pareto weight generally depends on prices and household income (Chiappori, 1988, 1992). The endogeneity of decision power in the household relies on the interdependence of household decisions and the resulting Pareto weight. To capture this relationship Basu (2006) formulates the Pareto weight as a function of the female household member’s actual earnings. Thus, the Pareto weight depends on her labour supply decision. Pollak (2005) argues that it is the potential earnings instead of actual earnings that determine the balance of power in marriage. His argument goes that a spouse whose earnings are high because he or she chooses to allocate more hours to market work, and correspondingly less to household production, does not have more bargaining power. But a spouse whose earnings are high because of a high wage rate does have more bargaining power. However, both actual and potential earnings get affected by the time allocation decisions. According to human capital theory both education level and on the job training affect wages. And since specialization into household work implies less on the job training it can be argued that specialization affects the potential wage rate as well.

In order to have a tractable formulation for the link between household decisions and the balance of power in the household I determine the Pareto weight as a function of the female household member’s actual earnings. Total time available is set equal to one and is divided between market work and household work. Thus, I define the female household member’s say in the household as follows:

$$\theta = L_f^f w_f^f = (1 - H_f^f)w_f^f$$

(2)

where

$\theta =$ the Pareto weight given on the wife’s preferences

$L_f^f =$ labour supply of the wife

$w_f^f =$ market wage for the wife

$H_f^f =$ household work of the wife

Assume that only the wife contributes into household production while the
husband supplies all the time available into market work.\footnote{Note that according to the formulation given in (2) the Pareto weight varies on the interval from zero to \( w^f \). If it were assumed that the female wage rate for 1 unit of work is 1, then the Pareto weight would vary between zero and one as is standard in the general version of the collective household model. In the current setting \( \theta = 1 \) would mean that the wife allocates all the time available into market work while \( \theta = 0 \) would mean that she is fully specialized into household work.} This assumption is based on empirical findings according to which in Western societies men nearly always work full-time while female hours of work vary widely, and that the total work (market work plus household work) between genders is equal on average. Men work more in the market and women work more in the household and these balance out (Burda et al., 2006; Donni, 2005). The household good \( G \) is produced by concave household technology such that:

\[
G = g(H^f) \tag{3}
\]

\[
g'(H^f) > 0 \text{ and } g''(H^f) < 0
\]

The spouses gain utility besides from their private consumption, from the joint consumption of the household good \( G \) determined above. Household members are assumed to have egoistic preferences, thus the individual utilities \( i = f, m \) are defined as follows:

\[
U^i(x^i, G) \tag{4}
\]

where

\[
x^i = \text{private consumption}
\]

\[
G = \text{good produced in the household}
\]

Assume \( U^i_x > 0, U^i_{xx} < 0 \) and \( U^i_G > 0, U^i_{GG} < 0 \). Further, in order the household production approach to be viable it has to be that consumption and production in the household are separable. Therefore, it is assumed that the cross deriva-
tive $U^i_{xG} = U^j_{Gx}$ is zero. The household budget constraint, where individual time budgets are inserted into the household joint budget constraint, is the following:

$$x^f + x^m = (1 - H^f)w^f + w^m$$  \hspace{1cm} (5)$$

There is no non-labour income and the prices for the market goods $x^i, i = f, m$ are normalized to unity. Now the household maximization problem in collective framework can be solved from:

$$Max \Omega \equiv \theta U^f(x^f, G) + (1 - \theta)U^m(x^m, G)$$  \hspace{1cm} (6)$$

s.t.

$$x^f + x^m = (1 - H^f)w^f + w^m$$

$$G = g(H^f)$$

Insert $x^m$ and $G$ from the constraints into the household household maximand in order to obtain the derived household utility function of the form:

$$Max \Omega \equiv \theta U^f(x^f, g(H^f)) + (1 - \theta)U^m((1 - H^f)w^f + w^m - x^f, g(H^f))$$  \hspace{1cm} (7)$$

By differentiating (7) with respect to $x^f$ and $H^f$, taking $\theta$ as fixed, we get the first order conditions:

$$\theta U^f_x - (1 - \theta)U^m_x = 0$$  \hspace{1cm} (8)$$

$$\theta U^f_{Gg}(H^f) - (1 - \theta)U^m_xw^f + (1 - \theta)U^m_G g'(H^f) = 0$$  \hspace{1cm} (9)$$

By combining the equations (8) and (9) the following relationship is obtained.

$$\theta U^f_x w^f = \left[ \theta U^f_G + (1 - \theta)U^m_G \right] g'(H^f)$$  \hspace{1cm} (10)$$
This can be rewritten as:

\[
\frac{w^f}{g'(H^f)} = \frac{\theta U^f_G + (1 - \theta) U^m_G}{\theta U^f_G}
\]  (11)

Equation (11) tells that the implicit price for household production has to be equal to the wife’s marginal willingness to pay for the household public good in interior solution. If the wife’s willingness to pay for G is less than the implicit price for it then the wife does not allocate time into household production and the amount of the household good G is zero in the current model. This implies that she allocates all the time available into market work. Thus, the equation (10) can be given the interpretation of the incentive constraint for the wife determining whether or not she allocates time into household work. Further, from (11) it can be seen that in the collective setting individual’s willingness to pay for the household public good depends on the Pareto weights. By differentiating the right hand side of (11) by \( \theta \) we get:

\[
-\frac{U^f_G U^m_G}{\left(\theta U^f_G\right)^2} < 0
\]  (12)

Thus it is seen that due to the trade-off between private consumption and the consumption of the household public good, the wife’s willingness to pay for G is decreasing in her say in the household.

The household equilibrium has to satisfy the following two conditions:

\[
\theta = (1 - H^f)w^f
\]  (1)

\[
\theta U^f_G w^f = \left[\theta U^f_G + (1 - \theta) U^m_G\right] g'(H^f)
\]  (10)

The equation (1) represents the wife’s power earnings relationship as in Basu (2006). Here this relationship is presented in the terms of the wife’s household work. Household equilibrium can be found from the intersection of the wife’s
incentive constraint and the power earnings curve. Next differentiate the equations (1) and (10) in order to derive the slopes for them in the \((\theta, H^f)\) plane. Differentiate the equation (1) with respect to \(\theta\) and \(H^f\) to get:

\[
\frac{dH^f}{d\theta} = -w^f < 0
\]  

(13)

The power earnings curve in the terms of household work has negative slope equal to the wife’s wage rate. This means that the more time the wife allocates into household production the less weight her preferences get in the household resource allocation process. Next differentiate (10) with respect to \(\theta\) and \(H^f\) and solve for \(d\theta/dH^f\) in order to see the slope for the incentive constraint.

\[
\frac{d\theta}{dH^f} = \frac{\left[ \theta U^f_G + (1 - \theta) U^m_G \right] g''(H^f) + \left[ \theta U^f_{GG} + (1 - \theta) U^m_{GG} \right] g'(H^f)}{U^f_{\theta} w^f + (U^m_{\theta} - U^f_{\theta}) g'(H^f)} \geq 0
\]  

(14)

Since the numerator in (14) is always negative by the second order conditions the sign of the slope for incentive constraint depends on the sign of the denominator in (14). Thus, it is seen that the slope for the wife’s incentive constraint depends on the sign of the following:

\[
\rho = U^f_{\theta} w^f + (U^m_{\theta} - U^f_{\theta}) g'(H^f)
\]  

(15)

There are three cases to consider depending on the sign of \(\rho\).

(i) The sign of \(\rho\) is negative, and thus, the slope for the wife’s incentive constraint in \((\theta, H^f)\) plane will be positive when:

\[
\frac{w^f}{g'(H^f)} < \frac{U^f_G - U^m_G}{U^f_{\theta}}
\]  

(16)
The wife’s marginal willingness to pay for the household good after the adjustment induced by the change in the decision power is larger than the implicit cost of producing the household good. This is possible if the wife’s marginal utility from $G$ is large relative to that for the husband, i.e. if $U^m_G < U^f_G$ and the implicit price for $G$ is low, i.e. when the wife’s productivity in household work is high and her wage rate is low. In this case the relationship between $\theta$ and $H^f$ can be described by upward sloping curve and only the interior solution to the household time allocation problem is possible. Upward sloping incentive curve implies here that the more say the wife has in the household the more she will contribute towards production of the household good $G$ due to her large marginal value on it.

(ii) The sign of $\rho$, and thus, the slope for the wife’s incentive constraint equals zero when we have:

$$\frac{w^f}{g^f (H^f)} = \frac{U^f_G - U^m_G}{U^f_{x^f}} \quad (17)$$

The wife’s willingness to pay for the household good after the adjustment in $\theta$ equals the cost of producing it. In this case the incentive constraint is a straight horizontal line at the adjusted level of the Pareto weight $\theta$. The wife is indifferent between household work and market work after the adjustment in $\theta$, and therefore, changes in the level of household work do not lead to further adjustment in $\theta$ along the curve representing the incentive constraint for the wife. There will be an unique household equilibrium where the household time of the wife is positive but less than one.

(iii) The sign of $\rho$ is positive, and thus, the slope for the wife’s incentive constraint in $(\theta, H^f)$ plane will be negative when:

$$\frac{w^f}{g^f (H^f)} > \frac{U^f_G - U^m_G}{U^f_{x^f}} \quad (18)$$

The wife’s marginal willingness to pay for the household good after the adjustment induced by the change in the decision power is less than the implicit
price for the household good. The condition in (18) is guaranteed to be satisfied when the husbands marginal utility from G is large relative to that for the wife, i.e. if $U^I_G < U^W_G$ implying that the right hand side of (18) is negative while the left hand side is always positive. The negative slope for the wife’s incentive constraint implies that the higher is the value of the Pareto weight $\theta$, the less time the wife will contribute towards producing the household good $G$, since this means further downward adjustment in the her decision power along the curve depicting her incentive constraint. When both the power earnings curve and the incentive constraint for the wife are downward sloping the existence of household equilibrium depends on the curvature of the incentive constraint. If the two curves do not intersect the interior solution, where the wife allocates her time between market work and household work, is not possible. In this case we have $H^f = 0$ for all powerful female household and $H^f = 1$ for all powerful male household. The result obtained here differs from that in Basu (2006) where female labour supply was analysed in the context where only private consumption in the household was considered. There the result was that there is possibility of multiple equilibria for female labour supply. Thus, identical households may end up at different levels of female labour supply. Here the solutions obtained depend on household characteristics and the solution for each case is unique.

Here the existence of household equilibrium in a collective household model with endogenous decision power and household production was considered. Since only the wife is assumed to allocate time into household production the household equilibrium will depend on the wife’s marginal willingness to pay for the household good after the adjustment in the decision power. It was shown that as long as the wife’s marginal willingness to pay for the household good after the adjustment in the Pareto weight is greater than or equal to the implicit cost of producing the household good, there will exist a household equilibrium where she allocates time into household production. In the case where the wife’s marginal willingness to pay for the household good after the adjustment in the Pareto weight is less than the cost of producing household good, it is possible that interior solution where the wife allocates time between household and market work does not exist.
3. Household time allocation with endogenous decision power
- two period model

It was shown in the previous section with one period model that household equilibrium where the wife allocates time into household production is possible even when this implies that her decision power in the household decreases as a result. It has been argued in the literature that introducing dynamics into the household model where the decision power is determined endogenously leads into solutions that fail Pareto efficiency (Basu, 2006; and Browning et al., 2004) Therefore it is necessary to introduce dynamics into the household model with joint consumption of the good produced in the household and endogenous decision power considered in the previous section to analyse whether household solution will be on the efficient frontier.

In this section a two period version of the model at hand is formulated. In order to be able to analyse the effect of endogenous decision power in a context of a two period model there has to be some discrete decision in the first period that affects the wife’s decision power in the second period. One example of a discrete decision affecting time use is the decision to have children. Therefore the household public good is assumed here to be the number or well being of the children in the second period. When a child is born into a household, the time has to reallocated between market and household work. The gains from marriage come from the possibility to have children in the second period. In the first period both household members gain utility solely from their private consumption and they are assumed to supply all the time available into market work. Total time available is set equal to one and it is assumed that only the mother will allocate time into caring work. The individual utilities $i = f, m$ in the first period are:

\[ U_i(x_i^1) \]  

(19)

In the second period the household members can gain utility from the number or well being of their children, N, besides their private consumption. Thus, in the
second period the individual utilities $i = f, m$ are:

$$U_i^f(x_i, N)$$

(20)

where

$$N = \text{the number or well-being of the children}$$

The husband continues to inelastically supply one unit of labour to the market while the wife is assumed to divide her time between market work and rearing and caring children. The child welfare production function is:

$$N = \begin{cases} a^f H^f & \text{if } H^f > 0 \\ 0 & \text{if } H^f = 0 \end{cases}$$

(21)

where

$$a^f = \text{the wife’s productivity in caring work}$$

$$H^f = \text{the wife’s child rearing and caring time}$$

The Pareto weight depends on the wife’s actual earnings in each period, similarly as in the one period model considered in the previous section of this paper. This implies that the wife’s actual earnings, and therefore, her say in the second period is lower than in the first period in the case the couple decides to have children. The first period Pareto weight is determined as follows:
\[ \theta_1 = L_1^f w_1^f = w_1^f \quad (22) \]

where

\[ L_1^f = \text{the wife’s labour supply in the first period} \]
\[ w_1^f = \text{the wife’s market wage in the first period} \]

The first period Pareto weight is equal to the wife’s wage since it is assumed that in the first period the wife inelastically supplies one unit of labour into the market work. In case the couple decides to have children the wife spends \( H_f^2 \in (0, 1) \) units of time into rearing and caring children and supplies \( L_2^f = (1 - H_f^2) \) units of time into market work in the second period. Therefore the second period Pareto weight is found from the following:

\[ \theta_2 = (1 - H_f^2) w_2^f \quad (23) \]

The two periods are tied together by the process of accumulation of human capital. It is assumed that current wages depend on the previous period wages and labour supply decisions. This formulation stresses the positive effect of on the job training on wages and the negative effect of the career brakes on wages. Individual wage in the period two will be determined as follows:

\[ w_2^i = L_{2-1}^i w_{2-1}^i = L_1^i w_1^i, \quad i = f, m \quad (24) \]

Since both spouses are assumed to inelastically supply all the time available into market work in the first period, equation (24) implies that \( w_1^i = w_2^i \) (\( i = f, m \)). Further, from (24) it can be seen that specialization will affect future earnings since lost work experience has negative effect on the accumulation of
market specific human capital. For example, if the female household member starts to supply all the time available into market work after the second period, her third period wage would be lower than in the two first periods due to her career break. Thus, we would have \( w^f_1 = w^f_2 > w^f_3 \) if further periods after the second period were considered. This relationship between motherhood and wages is widely documented in the literature (Koorenman & Neumark, 1992; Hersch & Stratton, 1997; and Joshi et al., 1999 and Napari, 2007, for Finnish data).

The process of accumulation of human capital defined above implies that the second period Pareto weight can be written in terms of the first period Pareto weight as:

\[
\theta_2 = (1 - H^f) w^f_1 = (1 - H^f) \theta_1
\]

Next I will analyze household dynamic behaviour, for the given couple, in the case of commitment and in the case of no-commitment. In the literature of household behaviour no-commitment is often taken to mean the possibility of divorce or non-cooperative behaviour in the household. Here no-commitment is taken to mean only that the household members are not able to make binding contracts so that the allocation of resources would not change as the decision power in the household changes. Therefore, commitment means in the current setting that household members are able to make binding contracts so that the household allocation is determined by the first period Pareto weight \( \theta_1 \) on both periods.

**3.1 Commitment**

In this section the household problem is considered in the case of commitment. It is assumed that the household members can commit to initial allocation determined by the first period Pareto weight \( \theta_1 = w^f_1 \). Thus it is assumed that the husband can commit to refrain from exploiting the future bargaining advantage resulting from the decrease in the wife’s actual earnings. The analysis is made here with illustrative examples where it is assumed that household members’ individual utilities are additively separable in private consumption and in the number or well being of children. If the couple decides to have children the Lagrangian for the household second period problem in the case of commitment is:
\[ L_2 = \theta_1 \left[ \alpha \ln x_f^2 + (1 - \alpha) \ln N \right] + (1 - \theta_1) \left[ \beta \ln x_m^2 + (1 - \beta) \ln N \right] \]  
\[ + \lambda_2 \left[ \left( 1 - \frac{N}{a^f} \right) w_f^m + w_m^m - x_f^m - x_m^m \right] \]  

Note that from (21) we have \( H^f = \frac{N}{a^f} \). The first order conditions are:

\[ \frac{\partial L_2}{\partial x_f^2} = \frac{\theta_1 \alpha}{x_f^2} - \lambda_2 = 0 \]  
\[ \frac{\partial L_2}{\partial x_m^2} = \frac{(1 - \theta_1) \beta}{x_m^2} - \lambda_2 = 0 \]  
\[ \frac{\partial L_2}{\partial N} = \frac{\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)}{N} - \lambda_2 \frac{w_f^m}{a^f} = 0 \]  
\[ \frac{\partial L_2}{\partial \lambda_2} = \left( 1 - \frac{N}{a^f} \right) w_f^m + w_m^m - x_f^m - x_m^m = 0 \]

Using the first order conditions and the budget constraint the second period optimal demands when the household decides to have children in the case of commitment are found to be:

\[ x_f^* = \theta_1 \alpha \left( w_f^m + w_m^m \right) \]  
\[ x_m^* = (1 - \theta_1) \beta \left( w_f^m + w_m^m \right) \]  
\[ N^* = \left[ \theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta) \right] \frac{a^f}{w_f^m} \left( w_f^m + w_m^m \right) \]

Consider next the first period problem given the second period optimal behaviour. Assume intertemporally additive household utilities, and no discounting nor interest rates. The first period household problem is to maximize the sum of the period utilities \( \Omega_1 + \Omega_2 \) subject to the household life time budget constraint. The Lagrangian for this problem is the following:
\[ L_1 = \theta_1 \left[ \alpha \ln x^f_1 + \alpha \ln x^f_2 + (1 - \alpha) \ln N \right] \]
\[ + (1 - \theta_1) \left[ \beta \ln x^m_1 + \beta \ln x^m_2 + (1 - \beta) \ln N \right] \]
\[ + \lambda_1 \left[ w^f_1 + w^m_1 + \left( 1 - \frac{N}{a^f} \right) w^f_2 + w^m_2 - x^f_1 - x^m_1 - x^f_2 - x^m_2 \right] \]

The first order conditions for the household problem in the case of commitment are:

\[ \frac{\partial L_1}{\partial x^f_1} = \frac{\theta_1 \alpha}{x^f_1} - \lambda_1 = 0 \] (35)
\[ \frac{\partial L_1}{\partial x^m_1} = \frac{(1 - \theta_1) \beta}{x^m_1} - \lambda_1 = 0 \] (36)
\[ \frac{\partial L_1}{\partial x^f_2} = \frac{\theta_1 \alpha}{x^f_2} - \lambda_1 = 0 \] (37)
\[ \frac{\partial L_1}{\partial x^m_2} = \frac{(1 - \theta_1) \beta}{x^m_2} - \lambda_1 = 0 \] (38)
\[ \frac{\partial L_1}{\partial N} = \frac{\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)}{N} - \lambda_1 \frac{w^f_2}{a^f} = 0 \] (39)
\[ \frac{\partial L_1}{\partial \lambda_1} = w^f_1 + w^m_1 + \left( 1 - \frac{N}{a^f} \right) w^f_2 + w^m_2 - x^f_1 - x^m_1 - x^f_2 - x^m_2 = 0 \] (40)

From the first order conditions the following conditions for the optimal demands are obtained:

\[ x^f_1 = x^f_2 \] (41)
\[ x^m_1 = x^m_2 = \frac{(1 - \theta_1) \beta}{\theta_1 \alpha} x^f_1 \] (42)
\[ N = \frac{\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)}{\theta_1 \alpha} \frac{a^f}{w^f_2} x^f_1 \] (43)

Insert the second period optimal demands derived earlier into these conditions
to get the optimal demands for the first period. The household consumptions in
the case of commitment are found to be:

\[ x_{2}^{f*} = \theta_1 \alpha \left( w_{2}^{f} + w_{2}^{m} \right) = x_{1}^{f*} \] (44)

\[ x_{2}^{m*} = (1 - \theta_1) \beta \left( w_{2}^{f} + w_{2}^{m} \right) = x_{1}^{m*} \] (45)

\[ N_{com.}^{*} = [\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)] \frac{q_{f}}{w_{2}} \left( w_{2}^{f} + w_{2}^{m} \right) \] (46)

The private demands are equal between periods in the case of commitment
and the household consumes its endowment in each period. (Remember that
the second period wages are equal to the first period wages.) Since the child
welfare production function is determined as \( N = a^f H^f \) the wife’s optimal time
endowment into household or caring work in the second period is seen to be:

\[ H_{com.}^{f*} = \frac{N_{com.}^{*}}{a^f} = [\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)] \frac{q_{f}}{w_{2}} \left( w_{2}^{f} + w_{2}^{m} \right) \] (47)

In the case of commitment the household will have children in the second period
since both spouses get the same level of private consumption as in the first period
plus the utility gain from having children. The wife’s time allocation and thus
optimal number of children \( N_{com.}^{*} \) depends on the decision power in the household.
The following comparative static derivative is obtained for the increase in the wife’s
initial say \( \theta_1 \), in the household:

\[ \frac{\partial H_{com.}^{f*}}{\partial \theta_1} = [(1 - \alpha) - (1 - \beta)] \frac{\left( w_{2}^{f} + w_{2}^{m} \right)}{w_{2}} \geq 0 \] (48)

The above derivative is positive if the wife values relatively more having children
than her husband i.e. \( (1 - \alpha) > (1 - \beta) \). If instead the wife values relatively
more her own private consumption than her husband i.e. \( (1 - \alpha) < (1 - \beta) \) then
the more say the wife has the less children the couple will have. Finally in the case
of identical preferences \( (1 - \alpha) = (1 - \beta) \), the demand for children is independent on the decision power in the household. Further, from (46) it is seen that in the case of commitment and with identical preferences we get the same result for the demand for children as in the standard unitary household model. The demand for children increases with the wife’s productivity in the household work \( a^f \), while with respect to increase in the wife’s wage the effect on the demand for children is ambiguous.

Next turn attention to the case where the spouses cannot commit to the initial allocations determined by \( \theta_1 \).

### 3.2 No-commitment

If the household members cannot commit to the initial allocation determined by \( \theta_1 = w^f_1 \), then the wife’s say will decrease in the second period in the case the couple decides to have children. This is because the time that has to be devoted to caring and rearing children is larger than zero. Thus, the wife cannot continue to supply all the time available into market work if there is children in the household.

Assume that the individual preferences are additively separable in private consumption and in the well being of the children. The Lagrangian for the household second period problem when the couple decides to have children in the case of no-commitment is:

\[
L_2 = \theta_2 \left[ \alpha \ln x^f_2 + (1 - \alpha) \ln N \right] + (1 - \theta_2) \left[ \beta \ln x^m_2 + (1 - \beta) \ln N \right] + \lambda_2 \left[ \left( 1 - \frac{N}{a^f} \right) w^f_2 + w^m_2 - x^f_2 - x^m_2 \right]
\]

(49)

The first order conditions are:
\[
\frac{\partial L_2}{\partial x_f^2} = \frac{\theta_2 \alpha}{x_f^2} - \lambda_2 = 0 \tag{50}
\]
\[
\frac{\partial L_2}{\partial x_m^2} = \frac{(1 - \theta_2) \beta}{x_m^2} - \lambda_2 = 0 \tag{51}
\]
\[
\frac{\partial L_2}{\partial N} = \frac{\theta_2 (1 - \alpha) + (1 - \theta_2) (1 - \beta)}{N} \left( w_f^2 + w_m^2 - x_f^2 - x_m^2 \right) - \lambda_2 \frac{w_f^2}{a_f} = 0 \tag{52}
\]
\[
\frac{\partial L_2}{\partial \lambda_2} = \left( 1 - \frac{N}{a_f} \right) w_f^2 + w_m^2 - x_f^2 - x_m^2 = 0 \tag{53}
\]

The optimal demands for the second period in the case of no-commitment are.

\[
x_f^2 = \theta_2 \alpha \left( w_f^2 + w_m^2 \right) \tag{54}
\]
\[
x_m^2 = (1 - \theta_2) \beta \left( w_f^2 + w_m^2 \right) \tag{55}
\]
\[
N^* = [\theta_2 (1 - \alpha) + (1 - \theta_2) (1 - \beta)] \frac{a_f}{w_f^2} \left( w_f^2 + w_m^2 \right) \tag{56}
\]

Next turn to the first period problem given the second period optimal demands. Since I assume intertemporally additive household utilities, and no discounting nor interest rates, the Lagrangian for the household first period problem in the case of no-commitment is the following:

\[
L_1 = \theta_1 \alpha \ln x_f^1 + \theta_2 \left[ \alpha \ln x_f^1 + (1 - \alpha) \ln N \right] + (1 - \theta_1) \beta \ln x_m^1 + (1 - \theta_2) \left[ \beta \ln x_m^1 + (1 - \beta) \ln N \right] + \lambda_1 \left[ w_f^1 + w_m^1 + \left( 1 - \frac{N}{a_f} \right) \left( w_f^2 + w_m^2 - x_f^1 - x_m^1 - x_f^2 - x_m^2 \right) \right] \tag{57}
\]

The first order conditions for the above problem are:
\[
\frac{\partial L_1}{\partial x_1^f} = \frac{\theta_1 \alpha}{x_1^f} - \lambda_1 = 0
\]  
(58)

\[
\frac{\partial L_1}{\partial x_1^m} = \frac{(1 - \theta_1) \beta}{x_1^m} - \lambda_1 = 0
\]  
(59)

\[
\frac{\partial L_1}{\partial x_2^f} = \frac{\theta_2 \alpha}{x_2^f} - \lambda_1 = 0
\]  
(60)

\[
\frac{\partial L_1}{\partial x_2^m} = \frac{(1 - \theta_2) \beta}{x_2^m} - \lambda_1 = 0
\]  
(61)

\[
\frac{\partial L_1}{\partial N} = \frac{\theta_2 (1 - \alpha) + (1 - \theta_2) (1 - \beta)}{N} - \lambda_1 \frac{w_2^f}{a^f} = 0
\]  
(62)

\[
\frac{\partial L_1}{\partial \lambda_1} = w_1^f + w_1^m + \left(1 - \frac{N}{a^f}\right) w_2^f + w_2^m - x_1^f - x_1^m - x_2^f - x_2^m = 0
\]  
(63)

From these the following conditions for the optimal demands are obtained:

\[
x_1^m = \frac{(1 - \theta_1) \beta}{\theta_1 \alpha} x_1^f
\]  
(64)

\[
x_2^f = \frac{\theta_2}{\theta_1} x_1^f
\]  
(65)

\[
x_2^m = \frac{(1 - \theta_2) \beta}{\theta_1 \alpha} x_1^f
\]  
(66)

\[
N = \frac{\theta_2 (1 - \alpha) + (1 - \theta_2) (1 - \beta)}{\theta_1 \alpha} \frac{a^f}{w_2} x_1^f
\]  
(67)

Insert the second period optimal demands derived earlier into these conditions to get the optimal demands for the first period in the case of no-commitment. Thus:
\begin{align*}
x_f^* &= \theta_1 \alpha \left( w_{f1} + w_{m1} \right) > x_2^* = \theta_2 \alpha \left( w_{f2} + w_{m2} \right) \quad (68) \\
x_m^* &= (1 - \theta_1) \beta \left( w_{f1} + w_{m1} \right) < x_2^* = (1 - \theta_2) \beta \left( w_{f2} + w_{m2} \right) \quad (69) \\
N_{no-com.}^* &= [\theta_2 (1 - \alpha) + (1 - \theta_2) (1 - \beta)] \frac{\alpha_f}{w_2^*} \left( w_{f2} + w_{m2} \right) \quad (70)
\end{align*}

Remember, that the second period wage depends on labour supply and wage in the first period by equation (24) describing the accumulation of market specific human capital. Therefore, \( w_{i1} = w_{i2} \) (\( i = f, m \)) since both household members inelastically supply one unit of labour into the market work in the first period. From (68) it is seen that the wife’s private consumption decreases in the second period due to her diminished say in the household. While from (69) it is seen that the husband’s private consumption in the second period is larger than in the first period since his preferences get larger weight in the household utility maximization process in the second period. This result is in line with Blundell et al., (2005). They show with a static version of the collective model allowing for public consumption in the household that private consumption of an individual always increases with his/her decision power in the household.

Now the demand for children depends on the value of the second period Pareto weight \( \theta_2 \) while in the case of commitment the demand for children depends on the first period Pareto weight \( \theta_1 \). As shown earlier in (25) the second period Pareto weight can be written in the terms of the first period Pareto weight as follows:

\[ \theta_2 = (1 - H^f) \frac{w_{f1}}{w_2} = (1 - H^f) \theta_1 \]

Insert (25) into (70) and solve for \( H^f \) to get \( H_{no-com.}^{fs} \) in order to fully solve the optimal time the wife will contribute towards rearing and caring children, in the
case of no-commitment.

\[
H_{no-com.}^f = \frac{[\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)] (w_f^f + w_m^m)}{w_2^f} - \frac{\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)}{\theta_1 [(1 - \beta) - (1 - \alpha)]} \quad (71)
\]

From (71) it is immediately seen that the wife’s optimal time into household production in the case of no-commitment may be equal to or deviate from that in the case of commitment depending on the spouses’ marginal utilities from having children. The second term in the right hand side of (71) dictates whether the wife’s home time is equal to or less/greater than in the case of commitment. There are three cases to consider depending on the spouses’ marginal utilities from having children.

(i) If the wife values relatively more having children than her husband, we have \((1 - \alpha) > (1 - \beta)\) which implies that \(H_{no-com.}^f > H_{com.}^f > 0\). When the marginal utility for the wife from having children is higher than that for her husband, the wife will contribute more time into household than in the case of commitment.

(ii) If the spouses have identical preferences we have \((1 - \alpha) = (1 - \beta)\) which implies that \(H_{no-com.}^f = H_{com.}^f > 0\). In this case (71) reduces into the following:

\[
H^f = (1 - \alpha) \frac{(w_f^f + w_m^m)}{w_2^f} > 0 \quad (72)
\]

It is seen that in the case of no-commitment with identical preferences the wife’s time allocation, and the demand for children are independent of the decision power.
in the household. Noteworthy result here is that even in the case of no-commitment identical preferences lead into the solution where the joint consumption of the household good is independent of the decision of power in the household. However, the private demands do depend on the decision power and since the wife has less say in the household in the second period her private demand will be lower than in the first period.

(iii) If the husband values relatively more having children than his wife, we have \((1 - \alpha) < (1 - \beta)\) which implies that \(H_{no-com}^f < H_{com}^f\). In this case it is possible that the wife’s time in household work is greater than zero only if the following condition is satisfied. From (71) it can be seen that \(H_{no-com}^f > 0\) when

\[
\left[\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)\right] \left(\frac{w_2^f + w_2^m}{w_2^f}\right) > \frac{\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)}{\theta_1 [(1 - \beta) - (1 - \alpha)]} (73)
\]

Divide both sides of (73) by \(\theta_1 (1 - \alpha) + (1 - \theta_1) (1 - \beta)\).

\[
(1 - \beta) > \frac{w_2^f}{\theta_1 (w_2^f + w_2^m)} + (1 - \alpha) (74)
\]

Since \(\theta_1 = w_1^f\) by (22) and \(w_1^f = w_2^f\) by (24) then \(\theta_1 = w_1^f = w_2^f\) and therefore (74) further reduces into:

\[
(1 - \beta) > \frac{1}{(w_2^f + w_2^m)} + (1 - \alpha) (75)
\]

From (75) it is seen that, if the husbands marginal utility from having children is sufficiently large, then it is possible that the household will have children even if the wife values relatively more her own private consumption in the case of no-commitment. If the equation (75) above is satisfied as an equality then the wife will not allocate her time into household work and the optimal number of children...
is zero. This is because if (75) is satisfied as an equality it implies that (71) equals zero. Further, it is seen that the larger is the household income $Y = w^f + w^m$ the smaller is the first term on the right hand side of (75). While the lower is the household income the larger is the first term on the right hand side of (75). With low household income the husbands’s preference for children has to be significantly larger than that for the wife in order the couple to have children. Thus, when the household income is low it is more likely that $H^f = 0$ when $(1 - \alpha) < (1 - \beta)$, while when the household income is high it is possible that $H^f > 0$ even when $(1 - \alpha) < (1 - \beta)$.

The conclusion from this section is that it is possible that in the case of no-commitment the wife allocates time into rearing and caring for children in the second period despite the reduction in her decision power in the household. The wife’s private consumption in the second period is always lower than in the first period when the couple decides to have children. While the time devoted to caring children may be equal to or deviate from that in the case of commitment. The conclusion is that in the case of no-commitment the household will have children when the marginal value from having children is larger for the wife than for the husband and when the spouses have identical preferences. In case of identical preferences the model reduces into full efficiency model where the joint demand for the household public good does not depend on the decision power in the household. Finally, even in the case where the marginal utility from having children is lower for the wife than that for the husband it is possible that the couple will have children if the marginal utility for the husband from having children is large enough. These results are in line with those obtained in the section two of this paper, where one period version of the collective household model with endogenous balance of power and household production was analyzed. The analysis presented here shows that introducing dynamics into a collective household model with endogenous balance of power and household production does not change the conditions under which the household equilibrium is determined. Consider next the conditions under which the household second period solution will be on the efficient frontier.
4. Intertemporal efficiency

In the current model the gains from marriage come from the possibility to have children in the second period. In the first period the household members gain utility only from private consumption. If children can be seen as household public goods enhancing the parents utility, then \( N^* > 0 \) implies an outward shift of the household utility possibility frontier. In the case the household decides to have children the household second period utility possibility frontier lies outside the first period utility possibility frontier. If the two frontiers are parallel then every point in the outer frontier is Pareto preferred to an allocation in the inner frontier. In this case \( N^* = 0 \) would imply inefficiency in the sense that the potentially Pareto improving move has not taken place.

In the collective household model the decision process is not explicitly modelled, it is assumed instead that the result is Pareto efficient and that the Pareto weight 'summarizes' the process leading to this specific efficient outcome. In this section the allocations between the two periods are compared in order to see whether the household will have children in the second period. Household allocations in the two periods are compared by using the concepts of Pareto criterion and compensating criterion from welfare economics. According to Pareto criterion the second period allocation with children \( \Omega_2 \) Pareto dominates the first period allocation \( \Omega_1 \) if both spouses prefer \( \Omega_2 \) to \( \Omega_1 \). If one of the spouses prefers \( \Omega_1 \) to \( \Omega_2 \) while the other spouse prefers \( \Omega_2 \) to \( \Omega_1 \), it is possible that \( \Omega_2 \) is potentially Pareto preferred to \( \Omega_1 \) if there is some way to reallocate \( \Omega_2 \) so that both spouses prefer the reallocation of \( \Omega_2 \) to the original allocation \( \Omega_1 \). In order the second period allocation \( \Omega_2 \) to be Pareto preferred to the first period allocation \( \Omega_1 \) it has to be that:

\[
U^i_1 \leq U^i_2, \quad i = f, m
\]  

(76)

It is easy to see that in the case of commitment this condition is always satisfied. In the case of commitment the household members’ individual utilities in each
period are:

\[ U_{1}^{f*} = \theta_{1}\alpha \ln x_{1}^{f*} \quad (77) \]

\[ U_{2}^{f*} = \theta_{1}\left[\alpha \ln x_{2}^{f*} + (1 - \alpha) \ln N_{com}^{*}\right] \quad (78) \]

and

\[ U_{1}^{m*} = (1 - \theta_{1})\beta \ln x_{1}^{m*} \quad (79) \]

\[ U_{2}^{m*} = (1 - \theta_{1})\left[\beta \ln x_{2}^{m*} + (1 - \beta) \ln N_{com}^{*}\right] \quad (80) \]

From the section 3.1 we know that in the case of commitment the private demands of the household members are equal between the two periods. Thus, we have \( x_{1}^{f*} = x_{2}^{f*} \), and \( x_{1}^{m*} = x_{2}^{m*} \). In the case of commitment the household will have children in the second period since both spouses get the same level of private consumption as in the first period plus the utility gain from having children. Thus in the case of commitment the Pareto criterion is satisfied for both spouses. The conclusion is that the solution to the full efficiency model is always on the efficient frontier which, of course, is an obvious result. For similar results see Lundberg & Pollak (2003) and Mazzocco (2004).

What can be said about intertemporal Pareto efficiency in the case of no-commitment. From section 3.2 we know that in the case of no-commitment the private demands satisfy \( x_{1}^{f*} > x_{2}^{f*} \), and \( x_{1}^{m*} < x_{2}^{m*} \). Further, it was shown that the demand for children \( N_{no-com}^{*} \) can deviate from that indicated by the commitment (full efficiency) model depending on the preferences of the household members.

In the case of no-commitment the wife’s individual period utilities are found from the following:
The second period allocation is Pareto preferred by the wife if \( U_{1}^{f*} \leq U_{2}^{f*} \). In the case of no-commitment the wife’s private consumption is lower in the second period than in the first period when the couple decides to have children. Therefore the decrease in the wife’s private consumption in the second period has to be compensated by the gain in utility obtained from the number of well being of children in the second period. \( N^{*} > 0 \) would imply welfare gain at the household level and an outward shift of the utility possibility frontier, but this will not occur unless the wife is compensated for her welfare loss. Insert the the wife’s period utilities into the Pareto criterion to get:

\[
\theta_{1} \alpha \ln x_{1}^{f*} \leq (1 - H^{f}) \theta_{1} \left[ \alpha \ln x_{2}^{f*} + (1 - \alpha) \ln N_{no-com}^{*} \right]
\]  

(83)

Solve (83) for \( H^{f} \). It is seen that the wife Pareto prefers the second period allocation \( \Omega_{2} \) to the first period allocation \( \Omega_{1} \) if the level of her household time satisfies the following condition:
\[ H^f \leq \frac{\theta_1 \left[ \alpha \ln x_2^* + (1 - \alpha) \ln N_{\text{no-com.}} \right] - \theta_1 \alpha \ln x_1^*}{\theta_1 \left[ \alpha \ln x_2^* + (1 - \alpha) \ln N_{\text{no-com.}} \right]} \]

\[ \Rightarrow \]

\[ H^f \leq 1 - \frac{\alpha \ln x_1^*}{\alpha \ln x_2^* + (1 - \alpha) \ln N_{\text{no-com.}}} = 1 - \frac{U_1^*}{U_2^*} \quad (84) \]

From (84) it is seen that the higher is the wife’s utility in the first period compared to her utility in the second period, the less time she optimally allocates into household work. While the lower is the wife’s first period utility compared to her utility in the second period the more she will allocate time into household. Further, it is seen that full specialization is not possible in the case of no-commitment. From (84) we have \( 0 \leq H^f < 1 \). This result slightly differs from that obtained in Iyigun (2005) where even full specialization is possible. Iyigun (2005) introduces a process of spousal matching into a household labour supply model in a Nash bargaining framework. There are potential gains from specialization but specializing in household production lowers market wages. Iyigun shows that assortative mating in large and asymmetric marriage markets induces spousal cooperation so that full specialization is possible. Marriage market competition ensures that each spouse is compensated according to his/her marginal contribution to the marriage. But when there are equal numbers of men and women in the marriage markets, spousal specialization may not occur unless there exists a commitment mechanism.

For the husband the period utilities in the case of no-commitment are:

\[ U_{1}^{m*} = (1 - \theta_1) \beta \ln x_{1}^{m*} \quad (85) \]

\[ U_{2}^{m*} = (1 - \theta_2) \left[ \beta \ln x_{2}^{m*} + (1 - \beta) \ln N_{\text{no-com.}} \right] \quad (86) \]

\[ = (1 - (1 - H^f) \theta_1) \left[ \beta \ln x_{2}^{m*} + (1 - \beta) \ln N_{\text{no-com.}} \right] \]
It was shown that in the case of no-commitment the husband’s private consumption is larger in the second period than in the first period if the couple decides to have children, i.e. \( x_{1}^{pz} < x_{2}^{pz} \). The conclusion is that the husband always Pareto prefers \( \Omega_2 \) to \( \Omega_1 \) when \( N^* > 0 \), since this always implies welfare gain for him. For the husband even a small level of \( H^f \) and thus of \( N \) will be Pareto improvement to \( \Omega_1 \). This result, of course, is due to the stereotypical assumption according to which only the wife allocates time into rearing and caring children. This simplifying assumption on the spouses’ time use is made here in order to pinpoint the functioning of the collective household model when the decisions taken by the household members affect the future decision power in the household. Further, another rationale behind the assumption made is the fact that this type of time allocation structure depicts reality quite well (see, for example, Burda et al., 2006; Blundell et al., 2005; and Donni, 2005).

From the analysis presented in the section 3.2 of this paper it is known that there is one case where the household will not have children in the second period. This is in the case of no-commitment where the husband’s marginal utility from having children is larger than that for his wife, but his marginal utility is not large enough. This is when the equation (75) is satisfied as an equality:

\[
(1 - \beta) = \frac{1}{w_{2}^{f} + w_{2}^{m}} \cdot (1 - \alpha) \tag{87}
\]

In this case the wife prefers \( \Omega_1 \) to \( \Omega_2 \) while the husband prefers \( \Omega_2 \) to \( \Omega_1 \). Does this imply inefficiency? The question is that, is there a reallocation of \( \Omega_2 \) such that the wife would Pareto prefer it to the first period allocation \( \Omega_1 \). From (87) it is seen that if there is an exogenous increase in the household income \( Y = w_{2}^{f} + w_{2}^{m} \), the above relationship does no longer hold as an equality. The right hand side of the equation (87) decreases when \( Y = w_{2}^{f} + w_{2}^{m} \) increases. This is just what is required in order the household to have children in the case of no-commitment where the wife’s marginal utility from having children is lower than that for her husband. Thus, the conclusion is that, if there is even a small transfer from the government to the households with children then it is guaranteed that the second period allocation \( \Omega_2 \) is Pareto preferred to the first period allocation \( \Omega_1 \) and the
household solution will be on the efficient frontier.

Gains from marriage come from the possibility to have children in the second period. Therefore, $N^* > 0$ implies that household second period utility possibility frontier lies outside the first period utility possibility frontier. If the two frontiers are parallel then every point in the outer frontier is Pareto preferred to an allocation in the inner frontier. In this case $N^* = 0$ would imply that the solution fails Pareto efficiency in the sense that the Pareto improving move does not take place. Further, if attention is given only on private consumptions, the conclusion would be that in the case of no-commitment the wife would always prefer $\Omega_1$ to $\Omega_2$. This would imply that the potentially Pareto improving move does not take place and the household would be stuck into an inefficient situation. When the welfare gain from the home time is taken into consideration in the form of household good it is possible that the wife will be willing to allocate time into household even when this implies that her say in the household will decrease. However, it has to be noted that, if the two utility possibility frontiers representing the first and the second period are intersecting then every point in the second period frontier is not Pareto preferred to the allocation in the first period frontier.

5. Conclusions

This paper considered a collective household model where the decision power in the household is driven by the household members’ actual earnings. It is widely argued in the literature that combining dynamics with endogenous decision power in the collective household model will lead into solutions that fail Pareto efficiency. The household dynamic efficiency was analysed in this paper with a two period model where the gains from marriage come from the possibility have children in the second period. Devoting time into rearing and caring children implies decreasing say in the household. It was shown that in the case where the household members can commit to refrain from exploiting the future bargaining advantage there always is a solution where the time allocated into household is greater than zero and where the household is on the efficient frontier. For the case of no-commitment the conclusion is that the household will have children when the marginal value from having children is larger for the wife than for the husband and when the spouses have identical preferences. In case of identical preferences the model reduces into
full efficiency model where the joint demand for the household public good does not depend on the decision power in the household. Finally, even in the case where the marginal utility from having children is lower for the wife than that for the husband, it is possible that the couple will have children if the marginal utility for the husband from having children is large enough. Further, it is shown that in the case of no-commitment even a small exogenous transfer from the government guarantees that the household solution will be on the efficient frontier. The one period version of the collective household model with endogenous balance of power and household production was analysed as well, and the results are in line with those obtained for the two period version of the model. Thus the results show that introducing dynamics into a collective household model with endogenous balance of power and household production does not change the conditions under which the household equilibrium is determined.

The main conclusion of this paper is that introducing dynamics into a collective household model with endogenous decision power does not necessarily lead into solutions that fail Pareto efficiency when the joint consumption of the household good is taken into consideration. It was shown that increase in an individual’s say in the household always leads to increase in his/her private consumption. Thus, decrease in say implies less private consumption. If only private consumption is taken into consideration, the conclusion is that the household is stuck into an inefficient situation where the potentially Pareto improving move does not take place, since the individual losing his/her say in the household is not compensated for his/her diminished private consumption. In the current model the gains from marriage come from the possibility to have children in the second period while in the first period the household members gain utility only from private consumption. If children can be seen as household public goods enhancing the parents utility, then the presence of the children implies an outward shift of the household utility possibility frontier. If the first and second period utility possibility frontiers are parallel then every point in the outer second period frontier is Pareto preferred to an allocation in the inner first period frontier. In this case the decision not to have children would imply inefficiency in the sense that the potentially Pareto improving move does not take place.
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