Publication II


Modeling of ferromagnetic semiconductor devices for spintronics

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We develop physical models for magnetic semiconductor devices, where a part of the device structure consists of a ferromagnetic semiconductor layer. First we calculate the effect of the exchange interaction between the charge carrier spins and the spins of the localized magnetic atoms on the electronic states, recombination processes, and charge transport in ferromagnetic semiconductors such as (Ga,Mn)As. Taking into account, e.g., the splitting of the conduction and valence bands due to the exchange interaction, we model the electrical characteristics of the basic magnetic semiconductor devices such as Schottky diodes consisting of a nonmagnetic metal/ferromagnetic semiconductor interface, pn diodes consisting of a ferromagnetic/nonmagnetic junction and bipolar transistors having a ferromagnetic emitter. The models predict that at temperatures close to the Curie temperature $T_C$ the electrical properties of the magnetic semiconductor devices become strongly dependent on the average spin polarization of the magnetic atoms. A feature in the models is that many device parameters such as diffusion lengths or potential barriers become spin dependent in magnetic semiconductor devices. In a ferromagnetic Schottky diode the sensitivity of the device current $I$ to the external magnetic field may be as large as $(dI/dB)_i^{-1} \approx 1/T$ at temperatures close to $T_C$. In a ferromagnetic $pn$ diode both the ideal and recombination currents become magnetic field dependent. In a ferromagnetic bipolar transistor the current gain shows the same sensitivity to the spin polarization as the dc current in the ferromagnetic $pn$ diodes. According to our model calculations optimal structures showing the largest magnetization dependence of the electrical characteristics in III–V ferromagnetic semiconductor devices would be those where the magnetic side of the junction is of $n$ type. © 2003 American Institute of Physics. [DOI: 10.1063/1.1575498]

I. INTRODUCTION

Magnetoelectronics, or “spintronics,” refers to the systematic use of electron spin, in addition to its charge, in the device applications for micro- and nanoelectronics. In the case of metallic multilayers, composed of alternating ferromagnetic and nonmagnetic metal thin films, there already exist commercialized discoveries such as the giant magnetoresistance (GMR) effect, as applied to magnetic information storage. The next product utilizing GMR is expected to be nonvolatile magnetic computer memory. However, although the existing metal-based magnetic devices may act successfully as switches or valves, a drawback is their inability to amplify signals.

In principle, a lot of important advantages over the fully metallic structures could be anticipated, if the spintronic devices are fabricated of magnetic semiconductors instead of magnetic metals. These advantages include, e.g., the following: (i) integration of spintronics with conventional semiconductor technology, (ii) large magnetoresistive effects, and (iii) the possibility to control the magnetic properties by charge injection and electric field in the case of carrier-induced ferromagnetism, (iv) semiconductor-based spintronic devices could provide amplification and serve as multifunctional devices, (v) possibility to fabricate magnetooptoelectronic devices, and (vi) possibility to reach 100% spin polarization for charge carrier spins.

Magnetic semiconductors such as Eu chalcogenides (e.g., EuO and EuS) and Cr-chalcogenide spinels (e.g., CdCr$_2$Se$_4$ and CdCr$_2$S$_4$) have been a topic of intensive scientific research since the 1960s, long before the concept spintronics was introduced. One of the most attractive features found in magnetic semiconductors was a strong exchange interaction between the spins of the itinerant charge carriers in the conduction band and the spins of the localized magnetic electrons at magnetic atoms. This interaction is manifested, e.g., as a strong dependence of the electrical and optical properties of the ferromagnetic semiconductors on the average spin polarization and on the spin fluctuations of the magnetic lattice. For instance, below the Curie temperature or in high magnetic fields ferromagnetic semiconductors display a strong redshift in the fundamental optical absorption edge. Also, in resistivity a prominent peak at $T_C$ has been found, which then disappears in sufficiently high mag...
nnetic fields, showing a large negative magnetoresistance. However, the extreme difficulty in growing these materials as well as the rather low Curie temperatures have hindered the application of the Eu chalcogenides and Cr-chalcogenide spinels in semiconductor devices.

In the 1980s interest was focused on diluted magnetic semiconductors (DMSs) such as Cd$_{1-x}$Mn$_x$Te and Zn$_{1-x}$Mn$_x$Te, where the magnetic properties can be controlled by altering the content of magnetic Mn ions. Although a lot of interesting findings have been made in these materials, no ferromagnetism has been reported in II–VI or IV–VI DMSs until very recently. The discovery of carrier-induced ferromagnetism in III–V DMSs such as In$_{1-x}$Mn$_x$As (Ref. 13) and Ga$_{1-x}$Mn$_x$As (Ref. 14) with $x \approx 0.01$–0.1 opened a route to the implementation of the ferromagnetic semiconductor devices. The synthesis of III–V alloys and transition elements like Mn is fully compatible with existing III–V semiconductor technology. Therefore, the integration of ferromagnetism with conventional or mesoscopic semiconductor devices is now possible. Also, the fabrication of magnetically asymmetric heterostructures consisting of alternating nonmagnetic and ferromagnetic semiconductor layers is feasible, allowing a more pronounced coupling between the magnetic lattice and the electrical properties of semiconductor devices. The fabrication of the ferromagnetic III–V devices offers an opportunity to study the physics of magnetic semiconductor materials in well-defined devices having well-known energy-band structures. Some initial success in using ferromagnetic semiconductor layers for injecting spin-polarized holes has been reported recently. The resistivity peak at $T_C$, negative magnetoresistance, and band splitting for spin-up and spin-down carriers due to the exchange interaction have been observed in Ga$_{1-x}$Mn$_x$As. However, there are still several open questions related, e.g., to the origin of ferromagnetism and to the strength of the exchange interaction in Ga$_{1-x}$Mn$_x$As. For example, the experimental values for the exchange parameter between the hole spins and the spins of the 3d electrons vary from 0.6 to 3.3 eV.

The highest $T_C$ observed in the ferromagnetic GaAs-based DMSs are still below 120 K whereas for practical applications room-temperature ferromagnetism is required. It has been predicted theoretically that room-temperature ferromagnetism should be present in wide-band-gap semiconductors such as Mn-doped GaN. Indeed, very recently some high-$T_C$ ferromagnetic semiconductors have been reported. Medvedkin et al. found room-temperature ferromagnetism in heavily Mn-doped CdGeP$_2$, which is a semiconductor having rather high mobilities both for holes and electrons. Also, in wide-band-gap semiconductors such as Mn-doped GaN, GaP, and TiO$_2$ (Ref. 28) high-$T_C$ ferromagnetism has been reported. Therefore, we may expect high-$T_C$ ferromagnetic semiconductor devices to be fabricated in the near future. Especially (Ga,Mn)N is interesting, since the recent development in growth techniques for wurtzite III nitrides has resulted in successful fabrication of GaN-based optical and electrical devices. Some simple device structures such as $pn$ junctions and light-emitting diodes, tunneling diodes have been made of III–V DMSs. Already decades ago experimental results for Schottky diodes based on ferromagnetic Eu chalcogenides have been presented.

So far, only minor effort has been put into the modeling of ferromagnetic semiconductor devices. Recently, we have presented a model for ferromagnetic resonant diodes based on a semiclassical transport theory. The same topic has also been treated by Petukhov et al. by using $k \cdot p$ perturbation theory and by Nonoyama et al. by using a recursive Green's function method. In the present article, we report the modeling results for some basic ferromagnetic semiconductor devices such as nonmagnetic metal/ferromagnetic semiconductor Schottky diodes, ferromagnetic/nonmagnetic $pn$ junctions, and bipolar transistors having a ferromagnetic emitter. Especially, we study the effect of the band splitting on the electrical transport in these devices. First, we derive the retarded Green's function for the charge carriers interacting with the magnetic atoms. The poles of the Green's function give the perturbed band energy and the band splitting, and from the imaginary part of the self-energy we get an estimate for the scattering rate due to spin disorder scattering processes. We also estimate the effect of band splitting on the conduction rate. Then, we calculate the $I-V$ characteristics of a ferromagnetic Schottky diode and estimate the sensitivity of the diode to the external magnetic field at various temperatures. We also model the ferromagnetic $pn$ diode by using both an ideal diode model based on the diffusion currents and a model for the recombination currents. Finally, we calculate the dependence of the current gain on the magnetic state of the ferromagnetic emitter in an ideal ferromagnetic bipolar transistor. We derive the modeling equations both for the accurate numerical simulations, where, e.g., the infinite order perturbation theory can be used to calculate the corrections to the band energies, and for the cases where simpler analytical results based on the first order perturbation theory for the conduction- and valence-band splittings is used. In the numerical calculations we typically use the material parameters for the (Ga,Mn)As material, but the model equations are general and applicable to many other ferromagnetic semiconductor materials as well.

II. EFFECT OF THE EXCHANGE INTERACTION ON DEVICE PARAMETERS

A. Retarded Green's function

Let us first consider the effect of the exchange interaction between the charge carrier spins and the localized magnetic moments on the energy-band structure and the transport properties in a model ferromagnetic semiconductor, where the free-charge carriers move in a broad isotropic conduction or valence band described by a single effective mass $m^*$. The dependence of the band edges on the magnetic order in ferromagnetic semiconductors can be estimated by using the ordinary first and second order perturbation theory, as shown by Haas. However, since the second order correction depends on the spin correlation functions, which diverge at $T = T_C$, Haas's result is not valid at temperatures close to the
magnetic transition temperature. Therefore, we have to apply an infinite order perturbation theory based on Green’s functions.\textsuperscript{37}

The total Hamiltonian describing the free-carrier and magnetic subsystems as well as their mutual interaction in a ferromagnetic semiconductor is given by

$$H_{\text{tot}} = H^{(1)} + H_{\text{exch}} + H_{\text{m}},$$

(1)

where

$$H^{(1)} = \sum_{k,\sigma} E_{k\sigma} d_{k\sigma}^\dagger d_{k\sigma},$$

(2)

$$H_{\text{exch}} = -\frac{\Omega}{2} \sum_{k,\sigma} \int J(r - R)|\langle S_R\rangle\psi^\dagger_\sigma(r)\psi_\sigma(r) + \langle S_R\rangle\psi_\sigma^\dagger(r)\psi_\sigma(r) - \psi^\dagger_\sigma(r)\psi_\sigma(r)] \, d^2r,$$

(3)

$$H_{\text{m}} = -\sum_{\mathbf{R},\mathbf{R}'} I(\mathbf{R}, \mathbf{R}') S_\mathbf{R} \cdot \mathbf{S}_{\mathbf{R}'} - \mu_B B \sum_{\mathbf{R}} \mathbf{S}_{\mathbf{R}}.$$

(4)

$$H^{(1)}$$

gives the charge carrier energies in the spin-polarized band

$$E_{k\sigma}^{(0)} = E_{k\sigma}^{(0)} + \frac{\hbar^2 k^2}{2m^*} - \frac{x}{2} J_{\text{exch}}(S^z)(\langle \delta_{\sigma\uparrow} \rangle - \langle \delta_{\sigma\downarrow} \rangle),$$

(5)

where $E_{k\sigma}^{(0)}$ is the energy of the band edge, $x$ is a mole fraction of the magnetic atoms, and $\langle S^z \rangle$ is the average spin polarization of the magnetic subsystem. The last term in Eq. (5) gives the first-order correction to the band energies due to the exchange potential, which is assumed to be rapidly varying over the unit cell, $J(r - \mathbf{R}) = J_{\text{exch}}(r - \mathbf{R})$. Equation (5) was obtained by inserting the identity $\langle S_R \rangle = \langle S^z \rangle = \langle S^z \rangle$ in the exchange interaction Hamiltonian (3), where

$$\psi^\dagger_\sigma(r) = \frac{1}{\sqrt{V}} \sum_{k} a_{k\sigma}^\dagger e^{-i\mathbf{k} \cdot \mathbf{r}} a_{k\sigma}^\dagger \psi_\sigma(r),$$

(6)

are the charge carrier field operators, $a_{k\sigma}^\dagger$ and $a_{k\sigma}$ being the creation and annihilation operators, respectively, for a band state $|k\sigma\rangle$, and $\sigma = (\uparrow, \downarrow)$ is the spin index. Here, a plane wave representation normalized to the volume $V$ is used for the band states $|k\sigma\rangle = V^{-1/2} \exp(i\mathbf{k} \cdot \mathbf{r}) |\sigma\rangle$. The spin raising and lowering operators are defined as usual by $S^+_R = S^z_R + i S^x_R$ and $S^-_R = S^z_R - i S^x_R$, respectively, and $S_R$ is the spin operator of the magnetic atom at a lattice site $\mathbf{R}$. $\Omega = V/N$ is the volume of the unit cell, whose number is $N$. $H_{\text{m}}$ is a Heisenberg-type Hamiltonian for the magnetic subsystem, $I(\mathbf{R}, \mathbf{R}')$ being the magnetic coupling constant between the localized spins $S_{\mathbf{R}}$. The last term in the Hamiltonian (4) gives the Zeeman energy when an external magnetic field $\mathbf{B}$ has been applied in the $z$ direction.

The perturbed Green’s function corresponding to the Hamiltonian (1) is given by (see Appendix A)

$$G_{\sigma\sigma'}(\mathbf{k}, E) = \frac{\hbar \delta_{\sigma\sigma'}}{E - E_{k\sigma}^{(1)} - \Sigma^{(2)}_{\sigma\sigma'}(\mathbf{k}, E)},$$

(7)

where $\Sigma^{(2)}$ is the second order self-energy

$$\Sigma^{(2)}_{\sigma\sigma'}(\mathbf{k}, E) = \frac{J_{\text{exch}}^2}{4N} \sum_{\mathbf{q}} \left[ \Gamma^{\uparrow\uparrow}(\mathbf{q}) \delta_{\sigma\sigma'} \left( \frac{E - E_{k+\mathbf{q}\downarrow}}{E - E_{k-\mathbf{q}\uparrow}} \right) + \Gamma^{\downarrow\downarrow}(\mathbf{q}) \delta_{\sigma\sigma'} \left( \frac{E - E_{k+\mathbf{q}\uparrow}}{E - E_{k-\mathbf{q}\downarrow}} \right) \right],$$

(8)

and $\Gamma(q) = \Gamma^{\uparrow\uparrow}(q) + \Gamma^{\downarrow\downarrow}(q) + \Gamma^{\uparrow\downarrow}(q)$ is the Fourier transform of the spin correlation function $\langle (\mathbf{S}_R - \langle \mathbf{S}_R \rangle \cdot (\mathbf{S}_{R'} - \langle \mathbf{S}_{R'} \rangle) \rangle$. By investigating the poles of the Green’s function (7) and the imaginary part of the self-energy (8) we get estimates for the band splitting and spin disorder scattering, respectively, in the ferromagnetic semiconductors.

### B. Band splitting and the shift of the band edge

The first-order correction to the band energies was given already in Eq. (5). This result predicts that the effect of the exchange interaction on the band energies vanishes when $\langle S^z \rangle = 0$. However, since the spin correlation function $\Gamma(q)$ is nonzero also in the paramagnetic region, the higher order terms given by the poles of the Green’s function (7) cause a band shift even at $T > T_C$. Taking into account the first order correction (5), the poles of the real part of the Green’s function (7) can be written as

$$E_{k\sigma} = E_{k\sigma}^{(0)} + \frac{\hbar^2 k^2}{2m^*} \frac{\Delta}{2} (\delta_{\sigma\uparrow} - \delta_{\sigma\downarrow}) + \frac{J_{\text{exch}}^2}{4N} \sum_{\mathbf{q}} \left[ \Gamma^{\uparrow\uparrow}(\mathbf{q}) \delta_{\sigma\sigma'} \left( \frac{E - E_{k+\mathbf{q}\downarrow}}{E - E_{k-\mathbf{q}\uparrow}} \right) + \Gamma^{\downarrow\downarrow}(\mathbf{q}) \delta_{\sigma\sigma'} \left( \frac{E - E_{k+\mathbf{q}\uparrow}}{E - E_{k-\mathbf{q}\downarrow}} \right) \right],$$

(9)

where $\Delta = x J_{\text{exch}}(S^z)$ is the first-order band splitting. Haas\textsuperscript{36} has obtained an energy correction similar to Eq. (9) by using the ordinary second-order perturbation theory. However, since only the first-order corrections (5) appear in the denominator of the $J_{\text{exch}}^2$ term, Haas’s result turned out to be divergent at $T_C$ due to the divergence of the spin correlation functions. Equation (9) shows the advantage of the infinite order perturbation theory based on the Green’s function technique: the perturbed band energy $E_{k\sigma}$ to be calculated appears in the denominator of the $J_{\text{exch}}^2$ term in Eq. (9), which removes the divergence at $T_C$. In the case $\langle S^z \rangle = 0$, Eq. (9) reduces to the result derived previously by Rys et al.\textsuperscript{28} However, their result is valid only in the paramagnetic region, whereas Eq. (9) is valid in the whole temperature range and also in the case of nonzero magnetic fields.

From Eq. (9) the perturbed band energies $E_{k\sigma}$ can be solved numerically by iterations. However, in order to calculate the higher order terms we have to know the wave-vector dependence of the spin correlation function $\Gamma(q)$. It can be derived, e.g., by using a procedure proposed by Sinkkonen.\textsuperscript{39}
as briefly summarized in the following: In the molecular field approximation the average spin polarization of the magnetic moments is given by

$$N^{-1} \sum_{\mathbf{R}} \langle S_{\mathbf{R}}^{z} \rangle = x \langle S_{\mathbf{R}}^{z} \rangle = xS B(\mathbf{y}),$$

(10)

where $B(\mathbf{y})$ is the Brillouin function for the spin quantum number $S$ with $y = g_{L} \mu_{B} S B_{\text{eff}} / k_{B} T$. The effective molecular field $B_{\text{eff}}$ acting on the spin $S_{\mathbf{R}}$ is obtained from the magnetic Hamiltonian (4)

$$B_{\text{eff}} = B + \frac{2I(q=0)}{g_{L} \mu_{B}},$$

(11)

where $I(q)$ is the Fourier transform of the magnetic coupling parameter $J(R, R')$ in Eq. (4), and in the case of a fcc lattice it is given by

$$I(q) = \frac{3k_{B}T}{2S(S+1)} \left[ 1 - \frac{q_{0}^{2}q^{2}}{12} \right],$$

(12)

where $a_{0}$ is the lattice constant. When calculating the spin correlation function for the diluted magnetic semiconductors by using the method of Slinkkonn,\textsuperscript{39} we have to take into account that the spin $S_{\mathbf{R}}$ is found at a lattice site with a probability $x$. Then, we have

$$F_{\parallel}[E_{\sigma}(\mathbf{k}=0)] = \frac{A}{c} \left[ \frac{\sqrt{b} + \text{Re} \left( \frac{2m^{*}}{k_{B}T} \right) \left[ E_{\sigma}(\mathbf{k}=0) + \Delta / 2 (\delta_{\sigma_{1}} - \delta_{\sigma_{2}}) \right]^{1} \right],$$

(15)

$$F_{\perp}[E_{\sigma}(\mathbf{k}=0)] = \frac{D}{f} \left[ \frac{\sqrt{c} + \text{Re} \left( \frac{2m^{*}}{k_{B}T} \right) \left[ E_{\sigma}(\mathbf{k}=0) - \Delta / 2 (\delta_{\sigma_{1}} - \delta_{\sigma_{2}}) \right]^{1} \right],$$

(16)

with $A = S^{2} B_{\delta}(\mathbf{y}) / y$, $D = S^{2} \partial B_{\delta}(\mathbf{y}) / \partial y$, and $\Delta = xJ_{\text{exch}}(S^{2}(T, B))$, and

$$b = 1 - \frac{2S^{2} I(q=0) B_{\delta}(\mathbf{y}) / y}{k_{B}T},$$

(17)

$$c = \frac{S^{2} I(q=0) \partial B_{\delta}(\mathbf{y}) / y}{6k_{B}T},$$

(18)

$$e = 1 - \frac{2S^{2} I(q=0) \partial B_{\delta}(\mathbf{y}) / y}{k_{B}T},$$

(19)

$$f = \frac{S^{2} I(q=0) \partial B_{\delta}(\mathbf{y}) / y}{6k_{B}T}. $$

(20)

Figures 1 and 2 show the temperature dependence of the band edge $E_{\sigma}(\mathbf{k}=0)$ at $B=0$ T and $B=4$ T in two cases: $J_{\text{exch}}^{f}=0.2$ eV (Fig. 1) corresponds to a typical value of the exchange interaction parameter for the conduction electrons in magnetic semiconductors\textsuperscript{4,16} such as EuO, whereas $J_{\text{exch}}^{pd}=1.4$ eV (Fig. 2) corresponds to a value for the holes, e.g., in (Ga,Mn)\textsubscript{As}.\textsuperscript{3,18-20} The other material parameters for EuO were $m^{*}=1.0m_{0}$, $T_{C}=110$ K, $S=7/2$, and $a_{0}=5.15$ Å, and for (Ga,Mn)\textsubscript{As} $m^{*}=0.5m_{0}$, $T_{C}=110$ K, $S=5/2$, and $a_{0}=5.65$ Å. The results shown in Figs. 1(a) and 2(a) have been obtained by using the first order result, Eq. (5), whereas the results of Figs. 1(b) and 2(b) have been calculated by including also the higher order corrections in Eq. (9). Figure 1 shows that in the case of the small exchange interaction parameter the first order result describes rather well the overall behavior of the band edge and band splitting: the higher order corrections simply cause a small shift $\approx -0.1$ eV in the band edge in the paramagnetic region and a redshift at temperatures close to $T_{C}$ already when $\langle S^{2} \rangle = 0$ at $T>T_{C}$. However, in the case of the large $J_{\text{exch}}^{pd}$ the higher order corrections in Eq. (14) start to dominate. Now, the temperature and magnetic field dependences of the band edge are dominated by the spin correlation function $\Gamma(q)$. Since $\Gamma(q)$ decreases with increasing magnetic field and with decreasing temperature at $T<T_{C}$, the band edge shows a blueshift in the ferromagnetic region. The $B$ and $T$ dependences of the band edge have not been studied experimentally in (Ga,Mn)\textsubscript{As}, but in EuO the redshift shown in Fig. 1(b) has already been confirmed experimentally decades ago.\textsuperscript{41} Figures 1 and 2 show that in the ferromagnetic semiconductors the band splitting can be large when it is compared to the thermal or Fermi energies of the charge carriers.

Equation (14) can be used in the numerical device simu-
lations for ferromagnetic semiconductor devices. However, in order to keep the models fairly simple and analytical, in the next sections we derive the device equations mainly based on the first order results. This can be justified by the results of Figs. 1 and 2, which show that our most important modeling parameters $\Delta_1 = E_{c1} - E_{c0}$ and $\Delta_2 = E_{v1} - E_{v0}$ are well described by the first order results, although the band edges are not. Since the device structures considered in the present work, typically, consist of a heterojunction between nonmagnetic and magnetic materials, a possible band discontinuity must be taken into account, when the band edges are modeled. Therefore, as the best first order approximation for the band edges we model them as follows:

$$E_{c0} = E_{c1} - \Delta E^0_c = \frac{\hbar^2 k^2}{2m_e^*} + \frac{\Delta_1}{2} (\delta_{\sigma \uparrow} - \delta_{\sigma \downarrow}),$$

(21)

$$E_{v0} = E_{v1} + \Delta E^0_v = -\frac{\hbar^2 k^2}{2m_h^*} - \frac{\Delta_1}{2} (\delta_{\sigma \uparrow} - \delta_{\sigma \downarrow}),$$

(22)

where the band splitting parameter $\Delta_1 = x J_{\text{exch}}^{\text{pd}}(S^z)$ is for the conduction electrons and $\Delta_2 = x J_{\text{exch}}^{\text{pd}}(S^z)$ for the holes. The band discontinuities of the conduction and valence bands are modeled through the parameters $\Delta E^0_c$ and $\Delta E^0_v$, respectively.

The carrier concentrations for the electrons and holes in the spin-polarized subbands can be estimated in the usual way:

$$n_\sigma = \frac{1}{2} \frac{N_\sigma}{\sqrt{\pi}} F^{1/2} \left[ \frac{E_{F\sigma} - E_{c0}}{k_BT} \right],$$

(23)

$$p_\sigma = \frac{1}{2} \frac{N_\sigma}{\sqrt{\pi}} F^{1/2} \left[ \frac{E_{v0} - E_{F\sigma}}{k_BT} \right],$$

(24)

where $N_\sigma = 2(2 \pi m_\sigma^* k_BT/h^2)^{3/2}$ and $N_{v\sigma} = 2(2 \pi m_v^* k_BT/h^2)^{3/2}$ are the effective density states for the conduction and valence bands, respectively, and $F_{\text{pd}}(x)$ is the Fermi integral. In the Maxwell–Boltzmann approximation ($|E_{F\sigma} - E_{c0}| \gg k_BT$), Eqs. (23) and (24) reduce to the following simple expressions:

$$n_\sigma = \frac{N_\sigma}{2} e^{-(E_{F\sigma} - E_{c0})/k_BT},$$

(25)

$$p_\sigma = \frac{N_{v\sigma}}{2} e^{-(E_{F\sigma} - E_{v0})/k_BT},$$

(26)

where the band edges $E_{c,v,\sigma}$ can be calculated either from the first order results (21) and (22), or more accurately from Eq. (14).

C. Mobility and diffusion coefficient

In the magnetic semiconductors the spin fluctuations of the total spins of the localized magnetic electrons cause strong scattering of the charge carriers at temperatures close to the critical temperature. Since the electrical properties of the semiconductor devices depend on the charge carrier mobility and diffusion coefficient, it is important in the ferromagnetic semiconductor devices to model the dependence of these quantities on the magnetic state of the device.

The spin disorder scattering is an inelastic scattering process for which a relaxation time $\tau_{\text{SD}}$ cannot be defined. How-
ever, an estimate for $\tau_{SD}$ is obtainable from the imaginary part of the self-energy $\Sigma^{(2)}_{\sigma\sigma}(k)$ in Eq. (8), which describes the carrier lifetime in a state $k\sigma$.

\[
\tau_{SD}^{-1}(k) = \frac{2}{\hbar} \text{Im} \Sigma^{(2)}_{\sigma\sigma}(k) = \frac{2\pi}{\hbar} \frac{J_{\text{exch}}}{4N} \sum_q \left[ \Gamma^{zz}(q) \delta(E_{k\sigma} - E^{(1)}_{k\sigma,\downarrow}) \delta_{\sigma\uparrow} + \Gamma^{yy}(q) \delta(E_{k\sigma} - E^{(1)}_{k\sigma,\uparrow}) \delta_{\sigma\downarrow} + \Gamma^{exch}(q) \delta(E_{k\sigma} - E^{(1)}_{k\sigma,\sigma}) \right].
\]

Here, the first and second terms describe the spin-flip processes $\uparrow \rightarrow \downarrow$ and $\downarrow \rightarrow \uparrow$, respectively, and the last term gives a contribution from the scattering events without spin flip. In the case $E_{k\sigma} = E^{(1)}_{k\sigma}$, Eq. (27) reduces to the relaxation time derived originally by Haas\textsuperscript{31} for magnetic semiconductors by using Boltzmann’s semiclassical transport theory.

In the degenerate ferromagnetic semiconductors we can estimate from Eq. (27) the charge carrier mobility limited by the spin disorder scattering as

\[
\mu_{SD}^* \approx \frac{\tau_{SD}^{-1}(k_F)}{m*},
\]

where $k_F$ is the Fermi wave vector. In the nondegenerate case we must replace $\tau_{SD}^{-1}(k_F)$ by the following average:

\[
\langle \tau_{SD}^{-1} \rangle = \frac{\int_0^{E_F} \frac{E^{3/2}}{e^2} \frac{(E - E_F)^{1/2}}{k_B T} \frac{\tau_{SD}(E)}{E} dE}{\int_0^{E_F} \frac{E^{3/2}}{e^2} \frac{(E - E_F)^{1/2}}{k_B T} dE}.
\]

Figure 3 shows the hole mobility vs. temperature in (Ga,Mn)As as calculated from Eqs. (13), (27), and (28) by using the following material parameters: $J_{\text{exch}} = 1.4$ eV, $x = 0.053$, $m^*_n = 0.5m_0$, $S = 5/2$, $T_C = 100$ K, $a_0 = 5.65$ Å, and $E_F = \hbar^2 (3\pi^2 n)^{2/3}/2m^* = 205$ meV for the hole concentration $p = 1.5 \times 10^{20}$ cm$^{-3}$. The mobility decreases strongly at temperatures close to $T_C$, and its value is very small, which means that the total mobility may even be dominated by the spin disorder scattering. On the other hand, since $\mu_{SD}^*$ for conduction electrons, the minority carrier mobility $\mu_{SD}^*$ is large and the total mobility for conduction electrons probably is dominated by other mechanisms than the spin disorder scattering. In heavily Mn-doped GaAs the resistivity peak at $T_C$ and the negative magnetoresistance predicted by Eqs. (27) and (28) have been observed experimentally.\textsuperscript{3} Even more pronounced mobility minimum at $T_C$ has been observed earlier, e.g., in EuO.\textsuperscript{4}

The diffusion coefficient for spin-up and spin-down carriers can be estimated from the total mobility $\mu_{tot}^*$. For instance, in the case of conduction electrons having concentrations $n_1$ and $n_1$ in the spin-polarized subbands, the diffusion coefficient is given by\textsuperscript{42}

\[
D_{\sigma} = \frac{\mu_{tot}^* k_B T}{e} \left[ 1 + 0.35355 \left( \frac{n_{\sigma}}{N_C^*} \right)^{2/3} - 9.9 \times 10^{-3} \left( \frac{n_{\sigma}}{N_C^*} \right)^{4/3} + \cdots \right],
\]

where $N_C^*$ is the effective density of states for the subband.

D. Effect of band splitting on recombination processes

In the cases of the semiconductor devices, such as $pn$ diodes or bipolar transistors, where the operation of the devices is based on the minority carrier diffusion, the proper treatment of the recombination processes for the charge carriers play an important role in the accurate modeling of the devices. Since the recombination between electrons in the conduction band and the holes in the valence band depends on the energy difference between the band energies, the band splittings given by Eqs. (21) and (22) are expected to affect strongly the total recombination rate.

Let us first consider the direct recombination of the charge carriers in the spin-polarized bands shown in Fig. 4(a) (notice that the spin directions drawn in Fig.4— and the figures below—are those of the electrons also in the valence bands: the directions of the hole spins are the opposite to those of the electrons). Since the total spin must be conserved in the recombination, only the transition process $1^0$ in Fig. 4(a) is allowed for the spin-up carriers, and the process $2^0$ for the spin-down carriers. The recombination rate for a direct band-to-band (bb) recombination process is given by\textsuperscript{43}

\[
R_{\text{bb}} = \frac{1}{\tau_{\text{bb}}} = \int_{E_{\text{bb}}} N_{\sigma}^p(E) f_{\sigma}(E) \left[ 1 - f_{\sigma}(E) \right] dE,
\]

where

\[
\frac{1}{\tau_{\text{bb}}} = \frac{e^2 n_{\sigma}}{6\pi \alpha e_0 m_0 c^3} \left( \frac{2\mu_{ee}^2}{m_0} \right) (E_{\sigma\sigma} - E_{\sigma\sigma}),
\]
and $f(E)(E)$ is the Fermi-Dirac distribution function for spin-up (spin-down) carriers, $n\rangle$, the refractive index of the semiconductor, and $p\rangle$, the matrix element of the momentum operator between the conduction- and valence-band states $E_{\text{c},\alpha}$ and $E_{\text{v},\alpha}$, respectively. $N_{\text{c},\alpha}^{\sigma}$ is the reduced density of states given by

$$N_{\text{c},\alpha}^{\sigma}(E)=\left(\frac{\sqrt{2}}{2\pi^2}\right)\left(\frac{m_\sigma^*}{\hbar}\right)^{3/2}(E-E_{\text{c},\alpha}^{\sigma})^{1/2},$$

(33)

where $E_{\text{c},\alpha}^{\sigma}=E_{\text{c},\alpha}-E_{\text{v},\alpha}$ is the band gap between the spin-polarized subbands, and $m_\sigma^*=(m_e^*+m_h^*+1)$ is the reduced effective mass for the charge carriers.

In magnetic semiconductors the energy difference $E_{\text{c},\alpha}^{\sigma}-E_{\text{v},\alpha}$ in Eq. (32) depends on the average magnetization of the magnetic lattice, and, consequently, also the recombination rate depends on the magnetic state of the system. If we use the first order results, Eqs. (21) and (22), we can estimate the energy difference between the band edges explicitly:

$$E_{\text{c},\alpha}^{\sigma}-E_{\text{v},\alpha}^{\sigma}=E_0^{\sigma}-\Delta E_0^{\sigma}-E_0^{\sigma}-\Delta E_0^{\sigma}-\frac{\Delta_e}{2}(\delta_{e\uparrow}-\delta_{e\downarrow})$$

$$+\frac{\Delta_h}{2}(\delta_{h\uparrow}-\delta_{h\downarrow}).$$

(34)

From Eqs. (31), (32), and (34) we can conclude that since the band splitting parameters $\Delta_e$ and $\Delta_h$ for conduction and valence bands, respectively, typically, are quite different [e.g., in (Ga,Mn)As $\Delta_e<\Delta_h$ (Ref. 18)], the band splitting should change the recombination rates significantly.

We can estimate $R_{\text{bb}}^{\sigma}$ explicitly in two special cases. Let us first consider a nondegenerate magnetic semiconductor. Then, the distribution function $f(E)$ is given by a Boltzmann approximation, $f_{\text{bb}}(E)$ is the trap state energy in the case $\langle S^z\rangle=0$, and

$$R_{\text{bb}}^{\sigma} \left(\frac{2\pi\hbar^2 m_\sigma^*}{k_B T}\right)^{3/2} n_\alpha p_\sigma.$$  

(35)

For the minority carriers (e.g., for electrons if $p_\sigma \gg n_\sigma$) and in the case of a low injection ($\Delta n \ll p_\sigma$) we obtain from Eq. (35) the usual expression for the recombination rate

$$R_{\text{bb}}^{\sigma} = \frac{n_\sigma - n_\alpha^0}{\tau_{\text{bb}}^{\sigma}}.$$  

(36)

where

$$\frac{1}{\tau_{\text{bb}}^{\sigma}} = \left(\frac{m_\sigma^*}{m_e^*}\right)^{3/2} p_\sigma^0.$$  

(37)

The hole concentration $p_\sigma^0$ at thermal equilibrium is given by Eq. (26), if the quasi-Fermi level $E_{Fp}$ is replaced by the real Fermi level $E_F$.

An analytical expression for $R_{\text{bb}}^{\sigma}$ can be found also in the case, where the minority carriers are injected into a heavily doped region. Let us consider the electron injection into a $p$-type degenerate magnetic semiconductor. In this case $f_\alpha \ll 1$ and $1-f_\alpha\approx 1$, and from Eq. (31) we get an equation similar to Eq. (36) with

$$\frac{1}{\tau_{\text{bb}}^{\sigma}} = \frac{1}{2\tau_{\text{bb}}^{\sigma}} \left(\frac{m_\sigma^*}{m_e^*}\right)^{3/2}.$$  

(38)

In the case of the indirect recombination through localized trap states in the forbidden band gap we can apply the well-known Shockley–Read–Hall (SRH) theory developed for nonmagnetic semiconductors. However, in the case of the ferromagnetic semiconductors we have to modify the SRH theory to some extent due to the band splitting. Figure 4(b) shows the indirect recombination processes through the gap states for the spin-up and spin-down carriers. Here, we have taken into account a possible spin splitting of the gap state

$$E_0^{\sigma}=E_0^{\sigma} - \frac{\Delta_e}{2}(\delta_{e\uparrow}-\delta_{e\downarrow})$$

(39)

where $E_0^{\sigma}$ is the trap state energy in the case $\langle S^z\rangle=0$, and $\Delta_3=xJ_{\text{exch}}^\alpha(S^z)$ is the splitting energy with the exchange interaction parameter $J_{\text{exch}}^\alpha$ between the electron spin in a trap state and the spins of the localized magnetic electrons.

The processes 1° and 2° shown in Fig. 4(b) describe the indirect recombination of the spin-up electrons, and the processes 3° and 4° are relevant for the spin-down electrons. Applying the SRH treatment to these processes we obtain for the net rate of indirect recombination the following expression (see Appendix B):
The spin-down carriers. Again, below $T_C$ the band splitting causes a difference between $\tau_{2\text{b}}^\downarrow$ and $\tau_{2\text{b}}^\uparrow$, even at $B=0$ T. The magnetic field and temperature dependences of the recombination processes in the ferromagnetic semiconductors have not been studied experimentally. Therefore, the model predictions of Figs. 5 and 6 remain unverified. Fortunately, the model predictions in the present article are not critically dependent on the models for the spin-dependent recombination processes. Instead, the models depend mainly, e.g., on the band splitting, which is an experimentally verified effect also in III–V DMSs.

Finally, let us consider a special case for the indirect

\[ R^\text{ind} = \frac{n_\sigma p_\sigma}{\tau_\sigma} \left( \frac{N_\sigma N_\pi}{4} \exp\left[ -\frac{(E_{c\sigma} - E_{v\sigma})}{k_BT} \right] \right), \]

where $\tau_{\sigma}^{-1} = N_\sigma v_{\text{th}} A_\sigma^e$ and $\tau_{\sigma}(A_\pi^e)$. The band edges can be calculated from Eqs. (21) and (22), or more accurately from Eq. (14). In the case where the net magnetization vanishes, $\langle S \rangle = 0$, Eq. (41) reduces to the conventional SRH result for $R^\text{ind} = R^\text{th} + R^\text{ind}$. A feature in the expressions for the recombination rates $R$ derived above is the dependence of $R$ on the band splitting $\Delta$. Since $\Delta$ is a sensitive function of the average spin polarization $\langle S \rangle$ (see Fig. 1), we may expect that — according to Eqs. (37), (38), and (41) — also, the recombination rates show a significant dependence on the magnetic state of the system. Figure 5 shows the recombination rate versus temperature in various magnetic fields in the case of the direct band-to-band recombination as calculated for a nondegenerate ferromagnetic semiconductor from Eq. (37). The material parameters used in the calculations were those of a $p$-type Mn-doped GaAs, $m_0^* = 0.5 m_0$ (heavy holes), $m_h^* = 0.07 m_0$, $T_C = 100$ K, $J_{\text{exch}} = 0.2$ eV (for conduction electrons), $E_{\text{exch}} = 1.4$ eV (for holes in the valence band), $\varepsilon_r = 13.1$, $\Delta E^{\text{exch}} = 0.1$ eV, and $2 p_{cv} / m_0 = 23$ eV. The acceptor and donor concentrations in the nondegenerate case were assumed to be $10^{16}$ cm$^{-3}$.

Figure 5 shows that the magnetic field dependence is largest at temperatures close to $T_C$, since the carrier concentration $p_\sigma^0$ in Eq. (37) depends strongly on the energy of the band edge, Eq. (22), and according to Fig. 2, the B dependence of the band splitting is largest at $T_C$. Below $T_C$ the band splitting causes a large difference between the recombination times for spin-up and spin-down carriers even at $B = 0$ T.

The nondegenerate case of Fig. 5 is not realistic in III–V DMSs, since ferromagnetism in these materials is carrier induced, and therefore, it always requires heavy doping. Figure 6 shows the recombination time versus external magnetic field at various temperatures in the case of the direct band-to-band recombination as calculated from Eq. (38) in a degenerate ferromagnetic semiconductor. The changes in the recombination time are much smaller than in the nondegenerate case. Due to the sign difference in the band splitting between spin-up and spin-down carriers [see, also, Eq. (34)], the recombination time decreases with increasing field in the case of the spin-up carriers, whereas the opposite is true for the spin-down carriers. Again, below $T_C$ the band splitting causes a difference between $\tau_{2\text{b}}^\downarrow$ and $\tau_{2\text{b}}^\uparrow$, even at $B=0$ T. The magnetic field and temperature dependences of the recombination processes in the ferromagnetic semiconductors have not been studied experimentally. Therefore, the model predictions of Figs. 5 and 6 remain unverified. Fortunately, the model predictions in the present article are not critically dependent on the models for the spin-dependent recombination processes. Instead, the models depend mainly, e.g., on the band splitting, which is an experimentally verified effect also in III–V DMSs.

Finally, let us consider a special case for the indirect

\[ \tau_\sigma = \frac{n_\sigma p_\sigma}{\tau_\sigma} \left( \frac{N_\sigma N_\pi}{4} \exp\left[ -\frac{(E_{c\sigma} - E_{v\sigma})}{k_BT} \right] \right), \]
where we shall meet below in the device modeling, i.e., a low injection of minority carriers (electrons) into a heavily doped $p$-type ferromagnetic semiconductor, where $p_0 \approx n_0$. Then, we get from Eq. (41) the following simple expression for the recombination rate:

$$R^\text{ind} = \frac{n_0 - n_0}{\tau_{n0}},$$

(42)

where $n_0$ is given by Eq. (25). Now, we can combine Eq. (42) with our previous result, Eq. (38), for direct recombination, and we obtain an expression for the total recombination rate:

$$R^\text{tot} = R^\text{bb} + R^\text{ind} = \frac{n_0 - n_0}{\tau_{n0}} + \frac{1}{\tau_{n0}},$$

(43)

where

$$\frac{1}{\tau_{n0}} = \frac{1}{\tau_{n0}^{\text{bb}}} + \frac{1}{\tau_{n0}^{\text{ind}}}.$$

(44)

These two equations are the starting point for the modeling of the ferromagnetic $pn$ diodes and bipolar transistors.

**III. FERROMAGNETIC SCHOTTKY DIODE**

The most simple ferromagnetic semiconductor device — at least from the modeling point of view — is a Schottky diode consisting of a junction between a nonmagnetic metal and a nondegenerate ferromagnetic semiconductor. Since the recombination, which we shall meet below in the device modeling, i.e., a low injection of minority carriers (electrons) into a heavily doped $p$-type ferromagnetic semiconductor, where $p_0 \approx n_0$. Then, we get from Eq. (41) the following simple expression for the recombination rate:

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These two equations are the starting point for the modeling of the ferromagnetic $pn$ diodes and bipolar transistors.

**III. FERROMAGNETIC SCHOTTKY DIODE**

The most simple ferromagnetic semiconductor device — at least from the modeling point of view — is a Schottky diode consisting of a junction between a nonmagnetic metal and a nondegenerate ferromagnetic semiconductor. Since the electrical current through the Schottky diode depends exponentially on the height of the potential barrier between the semiconductor and the metal, which in turn depends on the energy difference between the metal work function and the semiconductor band edge, the band splitting described above should be seen clearly in the $I-V$ characteristics of the diode.

Let us consider a junction between an $n$-type nondegenerate ferromagnetic semiconductor and a nonmagnetic metal having the metal work function clearly larger than the electron affinity of the semiconductor. The energy diagram of the structure is shown in Fig. 7, where also the change of the potential barrier $e\phi_B$ as a result of the conduction-band splitting is indicated qualitatively by the dashed curves. The quantity $eV_0 - eV$ is the built-in potential of the junction, which depends on the applied voltage $V$.

Next, we apply the standard thermionic emission theory to the junction shown in Fig. 7, but now taking into account the magnetization dependence of the conduction band-edge $E_{c\sigma}$. The electron density in an incremental energy range $dE$ in the spin-polarized conduction band is given by

$$dn_\sigma = \frac{2\pi(2m^*_e)^{3/2}}{h^3} \sqrt{E - E_{c\sigma}} e^{-(E - E_{c\sigma} + eV_0)/k_B T} dE,$$

(45)

where $eV_0 = E_{c\sigma} - E_F$. We can express Eq. (45) in velocity coordinates simply by using the relation $E - E_{c\sigma} = m^*_e v^2 / 2$, and then we get

$$dn_\sigma = \left( \frac{m^*_e}{h} \right)^3 e^{-eV_0/k_B T} e^{-m^*_e v^2 / 2k_B T} 4\pi v^2 d v_\sigma.$$

(46)

If we assume that the current flows perpendicular to the metal–semiconductor interface (the $x$ direction), we can calculate the current density from the semiconductor into the metal by multiplying Eq. (46) by $e v_\sigma$, and then integrating from the band edge to infinity.
\[ J_{\text{tot}}^{\downarrow\downarrow} = J_{\text{th}}^{\downarrow\downarrow} + J_{\text{sc}}^{\downarrow\downarrow} \]
\[ = \left( \frac{2 \pi e m^*_{\text{eff}} k_B^2}{h^3} \right) T^2 \left( e^{-e V_0^{\downarrow\downarrow}/k_B T} - e^{-e V_0^{\downarrow\downarrow}/k_B T} \right) \]
\[ + e^{-e V_0^{\downarrow\downarrow}/k_B T} \frac{\Delta_1}{2 k_B T} \tanh \left( \frac{\Delta_1}{2 k_B T} \right) \tanh \left( e^{-e V_0^{\downarrow\downarrow}/k_B T} \right) \]
where \( v_{\text{min}} \) is related to the minimum kinetic energy needed to surmount the potential barrier from the semiconductor side, and it is given by
\[ \frac{1}{2} m^* v_{\text{min}}^2 = e V_0^{\downarrow\downarrow} - e V. \]

The energy difference \( e V_0^{\downarrow\downarrow} = E_{\text{c}} - E_F \) in Eq. (47) depends on the conduction-band splitting (see Fig. 7). In order to see explicitly the dependence of the current density on the splitting parameter \( \Delta_1 \), we may use Eq. (21), and then \( e V_0^{\downarrow\downarrow} \) is given by
\[ e V_0^{\downarrow\downarrow} = e V_0^{\downarrow\downarrow} \frac{\Delta_1}{2} (\delta_{s^1} - \delta_{s^0}), \]

where \( e V_0^{\downarrow\downarrow} \) is the built-in potential in the case where both the average spin polarization \( \langle S \rangle \) and the bias voltage \( V \) vanish. The reverse current from the metal into the semiconductor is obtained by setting \( V = 0 \) in Eq. (47). Then, the total current density consists of four contributions, which are obtained by inserting Eqs. (48) and (49) into Eq. (47):
\[ J_{\text{tot}} = J_{\text{th}}^{\downarrow\downarrow}(V) - J_{\text{th}}^{\downarrow\downarrow}(V = 0) + J_{\text{sc}}^{\downarrow\downarrow}(V) - J_{\text{sc}}^{\downarrow\downarrow}(V = 0) \]
\[ = \left( \frac{4 \pi e m^*_{\text{eff}} k_B^2}{h^3} \right) T^2 e^{-e \phi_{\text{eff}}^{\downarrow\downarrow}/k_B T} \cosh \left( \frac{\Delta_1}{2 k_B T} \right) \left( e^{e V_0^{\downarrow\downarrow}/k_B T} - 1 \right), \]

where the potential barrier \( e \phi_{\text{eff}}^{\downarrow\downarrow} = e V_{\text{bi}} + e V_0^{\downarrow\downarrow} \) gives the barrier height before the band splitting. In the case \( \Delta_1 = 0 \), Eq. (50) reduces to the standard “text book” equation for the thermionic emission current.\(^{42}\) If the band splitting is large, \( \Delta_1 \gg k_B T \), the conductance of the ferromagnetic Schottky diode as calculated from Eq. (50) is given by
\[ G = \frac{1}{\partial T} (V = 0) = G_0 e^{-e \phi_{\text{eff}}^{\downarrow\downarrow}/(1/2) \Delta_1/k_B T}, \]

where \( G_0 = (2 \pi e^2 m^*_{\text{eff}} k_B T) / (h^3) \). Equation (51) is similar to the result mentioned previously by Thompson et al.\(^{32}\) (see, also, the review by Mauger et al.\(^{31}\)) when they modeled the effect of the band splitting on the conductance of a metal/ferromagnetic semiconductor junction.

If high \( T_C \) ferromagnetic Schottky diodes could be fabricated, one of the first applications might be a sensitive magnetic field sensor. Therefore, it is interesting to estimate the sensitivity of the diode to the external magnetic field. By using Eq. (50) and the usual definition of sensitivity,\(^{48}\) we obtain
\[ S = \frac{\partial J_{\text{tot}}}{\partial B} \bigg|_{J_{\text{tot}} = \text{constant}} = \tanh \left( \frac{\Delta_1}{2 k_B T} \right) \left( \frac{\partial \Delta_1}{\partial B} \right), \]

At temperatures close to \( T_C \), a typical value for \( \partial \Delta_1 / \partial B \) is 25 meV/T (see Fig. 1), which according to Eq. (52) results in an estimate \( S = 1 / T \). This value is almost two orders of magnitude larger than the values \( S = 0.03 - 0.05 / T \) in typical magnetotransistors.\(^{46}\)

Figure 8 shows the calculated magnetic field dependence of the current ratio \( I(B)/I(B = 0) \) at various temperatures in a ferromagnetic Schottky diode, when \( T_C = 100 \) K. The results have been calculated from Eq. (50) by using the band splitting parameter \( \Delta_1(T = 0) = \chi J_{\text{exch}} S = 0.19 eV \), which results in a colossal magnetoresistance at temperatures near \( T_C \).\(^{32}\)

The thermionic emission theory above is based on an assumption that there occurs no scattering of charge carriers, when they travel through the depletion region of a Schottky diode. When the scattering processes are taken into account, Eq. (50) has to be modified slightly:\(^{42}\)
\[ J_{\text{tot}} = e N_c \left( \frac{v_0^{\uparrow} + v_0^{\downarrow}}{2} \right) e^{-e \phi_{\text{eff}}^{\downarrow\downarrow}/k_B T} \cosh \left( \frac{\Delta_1}{2 k_B T} \right) \left( e^{e V_0^{\downarrow\downarrow}/k_B T} - 1 \right), \]

where \( v_0^{\uparrow(\downarrow)} \) is the effective diffusion velocity of the spin-up (spin-down) carriers in the depletion region. Since \( v_0^{\uparrow(\downarrow)} \) is proportional to the mobility of the majority carriers, which according to Fig. 3 is strongly temperature and magnetic field dependent, these dependences — in addition to the band splitting effects — should be seen in the \( J - V \) characteristics of the ferromagnetic Schottky diodes in the cases where Eq. (53) is valid.

The model predictions for the ferromagnetic Schottky diodes are compared to experimental data in Sec. VI below.

IV. FERROMAGNETIC PN DIODE

A. Diffusion theory

As compared to the treatment of the Schottky diode above, it is a much more difficult task to model a ferromag-
Since the hole concentration doping with magnetic atoms is taken into account by the \( p - H_{\text{GaAs}} \) on top of a ferromagnetic semiconductor, and the characteristics has been fabricated of ferromagnetic Mn-doped \( n - \text{type} \) semiconductor, Sec. II D must be taken into account. We may neglect from the magnetic side of the \( p - H_{\text{GaAs}} \) and the magnetic side of the \( n - \text{type} \) semiconductor. Therefore, let us consider a magnetically asymmetric \( p - n \) junction, where the \( p \) side is made of a ferromagnetic semiconductor, and the \( n \) side of an ordinary nonmagnetic semiconductor. This kind of diode structures has been fabricated of ferromagnetic Mn-doped \( p - \) GaAs on top of \( n - \) type nonmagnetic GaAs.\(^{17,29,30}\)

Figure 9 shows schematically the effect of the band splitting on the energy-band diagram of a nondegenerate ferromagnetic \( p - n \) junction. The dashed curves describe the spin-polarized bands on the ferromagnetic \( p \) side. A possible band-gap narrowing on the magnetic side due to a heavy doping with magnetic atoms is taken into account by the parameters \( \Delta E^0_p \) and \( \Delta E^0_n \) in Fig. 9 (see, also, Eqs. (21) and (22)). As shown in Fig. 9, the conduction-band splitting decreases (increases) the built-in potential \( \bar{e} V^0_{\text{bi}} \) between the \( n \) and \( p \) sides seen by the spin-up (spin-down) electrons coming from the \( n \) side to the \( p \) side. On the other hand, the splitting of the valence band increases (decreases) the potential barrier seen by the spin-up (spin-down) holes.

If we neglect the possible series resistances and assume that the voltage drop due to the applied voltage \( V \) occurs only in the depletion region \(( -x_p, x_n) \), we can couple the quasi-Fermi levels on both sides of the junction simply by the relation \( E_{\text{FQ}} = E_{Fp} + eV = E_{Fp} + eV \). Then, we get from Eqs. (25) and (26) the following boundary conditions for the minority carriers:

\[
\begin{align*}
n_{p,0}(x_p) &= \frac{N_e}{2} e^{-\frac{{\Delta E^0_p}}{k_BT}} e^{V/k_BT} e^{-eV/k_BT}, \\
p_n(x_n) &= \frac{N_e}{2} e^{-\frac{{\Delta E^0_n}}{k_BT}} e^{V/k_BT} e^{eV/k_BT}.
\end{align*}
\]

Since the hole concentration \( p_n(x_n) \) is calculated on the nonmagnetic side of the \( p - n \) junction, Eq. (55) does not depend on the average spin-polarization \( \langle S^z \rangle \). In order to see explicitly the dependence of the electron concentration \( n_{p,0}(x_p) \) on the band splitting, we can insert Eq. (21) into Eq. (54), and then we get

\[
n_{p,0}(x_p) = \frac{N_e}{2} e^{-\frac{{\Delta E^0_p}}{k_BT}} e^{V/k_BT} e^{-eV/k_BT} \\
= \frac{N_e}{2} e^{-\frac{{\Delta E^0_p}}{k_BT}} e^{V/k_BT} \\
	imes e^{-\left[ e\frac{V^0_{\text{bi}}}{k_BT} - (e\Delta E^0_p/k_BT) e^{V/k_BT} - (e\Delta E^0_p/k_BT) e^{V/k_BT} \right]} \\
= \frac{1}{2} n_{p,0} e^{-\frac{\Delta E^0_p}{k_BT}} e^{V/k_BT} (e\Delta E^0_p/k_BT - 1),
\]

where

\[
n_{p,0}^* = N_e \exp\left[ -\frac{{\Delta E^0_p}}{k_BT} \right] \\
\times \exp\left[ \frac{\Delta E^0_p}{2k_BT} (\delta_{\sigma_1} - \delta_{\sigma_2}) \right] \exp\left[ -\frac{eV^0_{\text{bi}}}{k_BT} \right]
\]

\[
= n_{p,0} \exp\left[ \frac{\Delta E^0_p}{2k_BT} (\delta_{\sigma_1} - \delta_{\sigma_2}) \right] \exp\left[ -\frac{eV^0_{\text{bi}}}{k_BT} \right]
\]

\[
= n_{p,0} \exp\left[ \frac{\Delta E^0_p}{2k_BT} (\delta_{\sigma_1} - \delta_{\sigma_2}) \right].
\]

From Eqs. (56) and (57) we see how the band splitting decreases (increases) the number of the minority carriers in the spin-up (spin-down) subband, i.e., the band splitting decreases (increases) the built-in potential \( eV^0_{\text{bi}} \) seen by the spin-up (spin-down) carriers in the junction. Equation (57) is valid in the case when \( \Delta E^0_p \ll eV^0_{\text{bi}} \), i.e., when we can neglect the thermionic emission over the barrier \( \Delta E^0_p \).

A diffusion equation for the minority carriers (electrons) on the \( p^+ \) side can be written in the usual way\(^{42}\)

\[
\frac{\partial^2 n_{p,0}}{\partial x^2} = \frac{n_{p,0} - n^*_{p,0}}{D_{\alpha\sigma} \tau_{\alpha\sigma}} = 0,
\]

where \( D_{\alpha\sigma} \) and \( \tau_{\alpha\sigma} \) are given by Eqs. (30) and (44), respectively. By using Eq. (54) and the condition \( n_{p,0}(\infty) = n_{p,0}^* \) [Eq. (57)] as the boundary conditions we can solve Eq. (58) and get an expression for the position dependence of the minority carrier concentration:

\[
n_{p,0}(x) = n_{p,0}^* + [n_{p,0}(x_p) - n_{p,0}^*] e^{(x + x_p)/L_{\alpha\sigma}},
\]

where \( L_{\alpha\sigma} = \sqrt{D_{\alpha\sigma} \tau_{\alpha\sigma}} \) is the spin-polarized diffusion length. The total diffusion current for the electrons is then given by

\[
J^0 = J^0_n = eD_n \left( \frac{\partial n}{\partial x} \right)_x = -x_p + eD_n \left( \frac{\partial n}{\partial x} \right)_x = -x_p
\]

\[
= \frac{eD_n [n_{p,0} e^{\Delta E^0_p/k_BT} - e^{eV/k_BT} - 1]}{L_{\alpha\sigma}}
\]

\[
+ \frac{eD_n [n_{p,0} e^{\Delta E^0_p/k_BT} - e^{eV/k_BT} - 1]}{L_{\alpha\sigma}}.
\]
In the case where $\Delta E_c^0 = 0$ and $(S') = 0$, and consequently, $n_t = n_{t0} = n_{p0}/2$, Eq. (60) reduces to the ordinary Shockley equation for electrons in an ideal pn junction.\cite{42}

The $I-V$ characteristics of the ferromagnetic pn junction can be calculated by adding to Eq. (60) the diffusion current for the minority carriers (holes) on the $n$ side. Since the $n$ side was assumed to be nonmagnetic, we get from the boundary condition (55) the following expression for the hole current:

$$J_p = \frac{eD_p n_{p0} e^{-(\Delta E_c^0/k_BT)}}{L_p} (e^{E/k_BT} - 1).$$

Again, in the case $\Delta E_c^0 = 0$ Eq. (61) reduces to the Shockley equation for the holes in an ideal pn junction.

If we solve numerically the band edge and the band splitting in the ferromagnetic layer from Eq. (14), we can estimate their effect on the diode current from Eq. (60). However, as in the case of the ferromagnetic Schottky diode above, we get a more explicit expression for the dependence on the band splitting parameter, if we apply the first order result, Eq. (21), to Eq. (60). Then, we have

$$J_{tot} = J_p + J_p = \frac{eD_p n_{p0} e^{-(\Delta E_c^0/k_BT)}}{2L_p} (e^{E/k_BT} - 1)$$

$$+ \frac{eD_p n_{p0} e^{-(\Delta E_c^0/k_BT)}}{2L_p} (e^{E/k_BT} - 1)$$

$$+ \frac{eD_p n_{p0} e^{-(\Delta E_c^0/k_BT)}}{2L_p} (e^{E/k_BT} - 1).$$

We see from Eq. (62) that the conduction-band discontinuity $\Delta E_c^0$ (see Fig. 9) lowers the built-in potential seen by the electrons diffusing from the $n$ side to the $p$ side, and consequently, the current increases by a factor $\exp(\Delta E_c^0/k_BT)$. On the other hand, the valence-band discontinuity $\Delta E_v^0$ increases the potential barrier seen by the holes on the $p$ side, and therefore, the hole diffusion current decreases by the factor $\exp(-\Delta E_v^0/k_BT)$.

Figure 10 shows the current ratio $I(B)/I(B=0)$ versus temperature in various magnetic fields in an ideal ferromagnetic pn junction. The results have been calculated from Eq. (62) in two cases: (a) $\Delta_1(T=0K)=0.19$ eV, and (b) $\Delta_1(T=0K)=0.025$ eV by using the following material parameters: $\mu_e=\mu_n = 10$ cm$^2$/V s, $E_g = 1.4$ eV, $2p^2/m^* = 23$ eV, $N_d = 10^{20}$ cm$^{-3}$, $N_p = 10^{19}$ cm$^{-3}$, $\Delta E_v = 0.1$ eV, $5.6$ $\Delta E_c^0 = 0.01$ eV, and $T_C = 100$ K. We see that at $T=T_C$ there appears a large magnetoresistance effect even in the case of the small band splitting parameter in Fig. 10(b), which is valid in Mn-doped GaAs.

Before we compare the theoretical results to experimental data let us consider next some nonidealities in the ferromagnetic pn junction.

B. Recombination current

In the above treatment of the ideal ferromagnetic $pn$ junction we assumed that there occurs no recombination of the carriers in the depletion region. However, typically in real $pn$-diodes the recombination current dominates at low bias voltages.\cite{42} We can estimate this contribution to the dc current using the results of Sec. II D.

1. Direct recombination in a nondegenerate ferromagnetic $pn$ diode

Let us first consider the nondegenerate $pn$ junction in the case of a radiative (band-to-band) recombination. The current is obtained by integrating the recombination rate, Eq. (35), over the whole depletion region $x_0$. Assuming that the $pn$ product is constant in the depletion region (this can be verified by using numerical simulations\cite{43}), we obtain

$$J_{bb} = e \int_{-x_0}^{x_0} (R_{bb} + R_{bb}) dx$$

$$= \frac{e x_p}{4} \left[ \frac{2\pi h^2 m^*_p}{k_BT m^*_n x_0^3} \right]^{3/2} \left[ \frac{n_t(-x_p)p(-x_p)}{\tau_p^0} \right]$$

$$+ \frac{e x_p}{4} \left[ \frac{2\pi h^2 m^*_n}{k_BT m^*_p x_0^3} \right]^{3/2} \left[ \frac{n_t(x_p)p(x_p)}{\tau_n^0} \right].$$

$$= \frac{e x_p}{4} \left[ \frac{2\pi h^2 m^*_p}{k_BT m^*_n x_0^3} \right]^{3/2} \left[ n_t(x_p)p(x_p) \right].$$

FIG. 10. Current ratio $I(B)/I(B=0)$ vs temperature in various magnetic fields in an ideal ferromagnetic $pn$ junction. The results have been calculated from Eq. (62) in two cases: (a) $\Delta_1(T=0K)=0.19$ eV, and (b) $\Delta_1(T=0K)=0.025$ eV.
where the \(pn\) product on the \(p\) side can be calculated by using Eq. (34)

\[
\frac{n^2_p}{4} e^{-(E_x-p-E_F)p/k_BT} e^{(E_x-p-E_F)/k_BT} = \frac{n^2_n}{4} e^{(E_x-n-E_F)/k_BT} e^{(E_x-n-E_F)/k_BT}.
\]

and \(E_{FP} - E_{Fn} = eV\), and \(n^2_0 = N_c N_v \exp[-(E_0^p - E_0^n)/k_BT]\). On the \(n\) side the \(pn\) product \(n(x_n)p(x_n)\) in Eq. (63) is obtained from Eq. (64) by setting the band splitting and the band edge discontinuity parameters equal to zero. Then, we get from Eq. (63) the total recombination current

\[
J_{bb} = J_{bb}(V) = J_{bb}(V = 0) = \left(\frac{eX_p n^2_0}{16}\right) \times \left(\frac{2 \pi \hbar^2 m^*}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_n)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_n)/k_BT} - 1}{\tau_n}\right) + \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right) + \left(\frac{2 \pi \hbar^2 m^*_n}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_p)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right).
\]

From Eq. (65) we see that in the case of the band-to-band recombination the voltage dependence of the current is the same as in the case of diffusion current, Eq. (62). However, in Eq. (65) there also appears the band splitting parameter \(\Delta_2\) for the valence band, which is much larger than \(\Delta_1\) for the conduction band. Therefore, if the direct recombination dominates in a nondegenerate \(pn\) junction, the effect of the average spin polarization \(S^Z\) on the current given by Eq. (65) should be much stronger than in the case of the ideal diffusion current given by Eq. (62).

2. Indirect recombination current in a ferromagnetic \(pn\) diode

The case of indirect recombination can be treated in the same way by using Eq. (41). However, due to a more complicated dependence of the recombination rate \(R^\text{ind}_{\sigma}\) on the carrier concentrations \(n_p\) and \(p_p\), we cannot integrate \(R^\text{ind}_{\sigma}\) in a closed form. As an approximation we apply here the standard approach for calculating the recombination current in a forward biased \(pn\) junction by replacing the position dependent \(R^\text{ind}_{\sigma}\) by its maximum value in the depletion region. First, by using Eq. (34) we can write Eq. (41) for the spin-up carriers as

\[
R^\text{ind}_1 = \frac{n_0 p_1}{\tau_0} \frac{e^{(E_x - E_{p0})/k_BT} e^{(E_x - E_{p0})/k_BT}}{F_1(\Delta_1, \Delta_2, \Delta_3)}.
\]

where

\[
F_1(\Delta_1, \Delta_2, \Delta_3) = \frac{n_0 p_1}{\tau_0} e^{(E_x - E_{p0})/k_BT} e^{(E_x - E_{p0})/k_BT} + n_0 p_1 e^{-(E_x - E_{p0})/k_BT} e^{(E_x - E_{p0})/k_BT} - \frac{F_1(\Delta_1, \Delta_2, \Delta_3)}{\tau_0}.
\]

Then, in order to maximize \(R^\text{ind}_1\) we must minimize the sum \(\tau_0 R_1 + \tau_0 R_1\) in the denominator of Eq. (66) together with the constraint given in Eq. (64). Then, we get

\[
n_1 = \frac{n_0}{2} \sqrt{\frac{\tau_0 e^{(E_x - E_{p0})/k_BT} e^{(E_x - E_{p0})/k_BT} + F_1(\Delta_1, \Delta_2, \Delta_3)}{\tau_0}},
\]

and

\[
p_1 = \frac{n_0}{2} \sqrt{\frac{\tau_0 e^{(E_x - E_{p0})/k_BT} e^{(E_x - E_{p0})/k_BT} + F_1(\Delta_1, \Delta_2, \Delta_3)}{\tau_0}}.
\]

The recombination rate \(R^\text{ind}_{\sigma}\) for the spin-down carriers is obtained from Eqs. (66)–(69) simply by changing the sign of the parameters \(\Delta_1, \Delta_2\), and \(\Delta_3\) from + to −. Then, we can estimate the recombination current for the spin-up carriers by inserting Eqs. (68) and (69) into Eq. (66). Deriving similar equations for the spin-down carriers we finally get

\[
J_{bb} = J_{bb}(V) = J_{bb}(V = 0) = \left(\frac{eX_p n^2_0}{16}\right) \times \left(\frac{2 \pi \hbar^2 m^*}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_n)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_n)/k_BT} - 1}{\tau_n}\right) + \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right) + \left(\frac{2 \pi \hbar^2 m^*_n}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_p)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right).
\]

\[
J_{bb} = J_{bb}(V) = J_{bb}(V = 0) = \left(\frac{eX_p n^2_0}{16}\right) \times \left(\frac{2 \pi \hbar^2 m^*}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_n)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_n)/k_BT} - 1}{\tau_n}\right) + \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right) + \left(\frac{2 \pi \hbar^2 m^*_n}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_p)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right).
\]

\[
J_{bb} = J_{bb}(V) = J_{bb}(V = 0) = \left(\frac{eX_p n^2_0}{16}\right) \times \left(\frac{2 \pi \hbar^2 m^*}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_n)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_n)/k_BT} - 1}{\tau_n}\right) + \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right) + \left(\frac{2 \pi \hbar^2 m^*_n}{k_BT m^*_n m^*_p}\right)^{3/2} e^{(E_x - E_p)/k_BT} \left(\frac{e^{(\Delta_x - \Delta_p)/k_BT} - 1}{\tau_p}\right).
\]
Equation (70) reduces to the standard textbook\textsuperscript{42,43} equation for the SRH recombination current, if we set all the parameters $\Delta_1$, $\Delta_2$, $\Delta_3$, $\Delta E_0^n$, and $\Delta E_0^p$ equal to zero and $\tau_\infty = \tau_{\infty 0}$. From Eq. (70) we can see that instead of the usual voltage dependence of the diffusion current, $\sim \exp(eV/k_BT)$ [see Eq. (62)], the effective voltage dependence of the recombination current is proportional to $\sim \exp(eV/2k_BT)$, if $eV \ll k_BT$. As in the case of the direct recombination current, Eq. (65), the large valence-band splitting parameter $\Delta_3$ appears in Eq. (70) resulting in a strong dependence of the current on the voltage $V$. However, in the case of the direct recombination, $\Delta_3$ is valid also, e.g., in a $p^+\!n$ junction.

3. Direct recombination in a degenerate ferromagnetic pn diode

In a $pn$ junction the indirect recombination rate is highest in the region close to $x=0$ (see Fig. 9), where the Maxwell–Boltzmann statistics is valid, even when the other side of the junction is made of a degenerate semiconductor.\textsuperscript{42} Therefore, Eq. (70) is valid also, e.g., in a $p^+\!n$ junction. However, in the case of the direct recombination we must use Eq. (36) together with Eq. (38) when calculating the recombination current on the degenerate $p^+$ side. By using the position dependence of the minority carrier concentration given by Eq. (59), we can calculate the recombination current for the recombination rate $R_{bb}^{\text{exch}}$ in Eqs. (36) and (38):

![FIG. 11. Current ratio $I(B)/I(B=0)$ vs temperature in various magnetic fields in a ferromagnetic $pn$ junction.](image)

![FIG. 12. Current density vs temperature at $V=1.4$ V as calculated from Eq. (71) in two cases: (a) $\Delta_3(T=0\,\text{K})=0.025\,\text{eV}$, and (b) $\Delta_3(T=0\,\text{K})=0.19\,\text{eV}$.](image)
\(V = 1.4 \text{ V and } B = 0 \text{ T as calculated from Eqs. (71) and (72) in two cases: (a) } \Delta_1(T = 0 \text{ K}) = 0.19 \text{ eV, and (b) } \Delta_1(T = 0 \text{ K}) = 0.025 \text{ eV.}
\)

The other parameters were the same as in the case of the ideal \(pn\) junction (Fig. 10), excluding the values of the dopant concentrations, which were \(N_d = N_p = 10^{19} \text{ cm}^{-3}\). Curve (b) in Fig. 12 shows that in the case of the large band splitting parameter the temperature dependence of the current changes strongly below \(T_C\): Due to the lowering of the built-in potential for spin-up carriers the current increases with decreasing temperature in contrast to the usual behavior shown in the curve (a) in Fig. 12.

Figure 13 shows the current ratio \(I(B)/I(B=0)\) versus temperature in various magnetic fields in a ferromagnetic \(p^-n\) junction, when the direct recombination current dominates. The results have been calculated from Eqs. (71) and (72) in two cases: (a) \(\Delta_1(T = 0 \text{ K}) = 0.19 \text{ eV, and (b) } \Delta_1(T = 0 \text{ K}) = 0.025 \text{ eV.}
\)

The other parameters were the same as those in the case of Fig. 12. In the case of the small band splitting parameter the magnetoresistance is smaller than in an ideal ferromagnetic \(pn\) junction (see Fig. 10), as shown in Fig. 13(b). On the other hand, in the case of the large splitting parameter [Fig. 13(a)] the increase in the current caused by the external magnetic field can be on the same order of magnitude as in the case of the ferromagnetic Schottky diode (see Fig. 8). The model predictions for the ferromagnetic \(pn\) diodes are compared to the existing experimental data in Sec. VI below.

\(\text{FIG. 13. Current ratio } I(B)/I(B=0) \text{ vs temperature in various magnetic fields in a ferromagnetic } p^-n \text{ junction. The results have been calculated from Eq. (71) for the direct recombination current in two cases: (a) } \Delta_1(T = 0 \text{ K}) = 0.19 \text{ eV, and (b) } \Delta_1(T = 0 \text{ K}) = 0.025 \text{ eV.}
\)

\(\text{FIG. 14. Energy-band diagram for a } p^-np \text{ bipolar transistor having a ferromagnetic emitter. The dashed curves show the spin-polarized band edges in the emitter region.}
\)

\(\text{V. FERROMAGNETIC BIPOLAR TRANSISTOR}
\)

A natural extension of the modeling of the ferromagnetic \(pn\) junction is to consider a bipolar transistor structure, where the emitter is ferromagnetic and the base and collector regions are nonmagnetic. Based on the \(pn\) diodes made of (Ga,Mn)As\(^{5,29}\) so far, we can state that such ferromagnetic bipolar devices could be designed and fabricated by using the modern III–V DMS technology. Figure 14 shows the energy band structure of a \(p^-np\) bipolar transistor having a ferromagnetic emitter. The dashed curves indicate the spin-polarized bands. It is now straightforward to apply the theory of ferromagnetic \(pn\) junctions presented above to the emitter–base junction. Then, we get the following solution for the diffusion equation:

\[n_{p0}(x) = n_{p0}^E = n_{p0}^E e^{\frac{\Delta_0}{kBT}}(\frac{V_{EB}}{kBT})^{-1} e^{(x + x_E)/L_E}, \quad (73)\]

for \(x < -x_E\) (see Fig. 14). Here, \(V_{EB}\) is the emitter–base voltage and

\[n_{p0}^E = \frac{N_c}{2} e^{-(E_{\sigma \uparrow} - E_{\sigma \downarrow})/kBT}, \quad (74)\]

Similarly, in the case of the holes in the base region \(0 < x < W\) (see Fig. 14), we get

\[p_0(x) = p_{00} = \frac{\left[p(x, W) - p(x, 0) e^{-W/L_B}\right]}{2\sinh(W/L_B)} e^{x/L_B}, \quad (75)\]

Here, we have used the usual boundary conditions for the bipolar transistor\(^{42}\)}
Similarly, we get for the collector current the following expression:

\[ I_C = qA \left[ D_E^b \left( \frac{\partial p_n}{\partial x} \right)_{x=-x_E} + D_E^e \left( \frac{\partial n_p}{\partial x} \right)_{x=-x_E} \right] \]

\[ - qAD_B \frac{\partial p_n}{\partial x} \bigg|_{x=0} = \frac{qAD_E^b n_{p0}^b}{L_E^b} (e^{V_{EB}/kT} - 1) \]

\[ + \frac{qAD_E^e n_{p0}^e}{L_E^e} (e^{V_{EB}/kT} - 1) + \frac{qAD_B p_{n0}^b}{L_B} \coth \left( \frac{W}{L_B} \right) \]

\[ \times \left[ (e^{V_{CB}/kT} - 1) - \frac{1}{\cosh \left( \frac{W}{L_B} \right)} (e^{V_{CB}/kT} - 1) \right]. \]  

(78)

Similarly, we get for the collector current the following expression:

\[ I_C = -qAD_B \frac{\partial p_n}{\partial x} + qAD_C \frac{\partial n_p}{\partial x} \bigg|_{x=x_C} \]

\[ = \left( \frac{1}{\sinh \left( \frac{W}{L_B} \right)} \right) (e^{V_{EB}/kT} - 1) \]

\[ - \coth \left( \frac{W}{L_B} \right) (e^{V_{CB}/kT} - 1) + \frac{qAD_C n_{p0}^e}{L_C} \]

\[ \times (e^{V_{CB}/kT} - 1). \]  

(79)

In modern bipolar transistors \( W/L_B \approx 1 \) and in the normal operation point \( V_{CB} \approx 0 \), and therefore we can simplify Eqs. (78) and (79) as follows:

\[ I_E = qA \left( \frac{D_E^b n_{p0}^b}{L_E^b} + \frac{D_E^e n_{p0}^e}{L_E^e} \right) (e^{V_{EB}/kT} - 1) \]

\[ + \frac{qAD_B p_{n0}^b}{W} (e^{V_{EB}/kT} - 1), \]  

(80)

\[ I_C = \frac{qAD_B p_{n0}^b}{W} (e^{V_{CB}/kT} - 1). \]  

(81)

Hence, we get a compact expression for the base current:

\[ I_B = I_E - I_C = qA \left( \frac{D_E^b n_{p0}^b}{L_E^b} + \frac{D_E^e n_{p0}^e}{L_E^e} \right) (e^{V_{EB}/kT} - 1). \]  

(82)

Equation (82), which reduces to the conventional expression for the base current if the spin polarization vanishes, indicates that \( I_B \) is determined mainly by the minority carrier injection from the base to the emitter. A feature in the bipolar transistor having a ferromagnetic emitter is the magnetic field dependence of the base current, which according to Eqs. (60) and (80) is the same as in a ferromagnetic \( pn \) junction. This results in a magnetic field dependent dc current gain \( \beta \) defined as

\[ \beta = \frac{I_C}{I_B} = \frac{D_B p_{n0}^b}{W} \left( \frac{D_E^e n_{p0}^e}{L_E^e} + \frac{D_E^b n_{p0}^b}{L_E^b} \right)^{-1} \]  

(83)

As in the case of the ferromagnetic \( pn \) junction treated in Sec. IV, the strongest magnetic field dependence would occur, if instead of the \( p \)-type ferromagnetic emitter we could have an \( n \)-\( p \) transistor with a ferromagnetic emitter of \( n \) type. Then, we would have the large valence-band splitting parameter \( \Delta_2 \) in Eq. (83) [and Eq. (82)] instead of the small parameter \( \Delta_1 \).

The nonidealities such as recombination current can be added to the ferromagnetic bipolar transistor model just as in Sec. IV in the case of the ferromagnetic \( pn \) diode.

**VI. COMPARISON TO EXPERIMENTAL DATA AND DISCUSSION**

Although the required technology already exists, no one has fabricated the ferromagnetic bipolar transistors yet. However, some simple ferromagnetic diodes \(^{2,29,32}\) have been made, which allows us to compare the model predictions of Secs. III and IV to the existing experimental data to some extent.

**A. Ferromagnetic Schottky diodes**

The barrier lowering predicted by Eq. (51) has been observed experimentally in a ferromagnetic Schottky diode consisting of indium and Eu\(_{1-x}\)Gd\(_x\)S. \(^{32}\) The observed lowering \( \Delta V \approx 0.24 \text{ eV} \) was in good agreement with the value 0.27 eV deduced from the measured redshift of the optical absorption edge in Eu\(_{1-x}\)Gd\(_x\)S. Therefore, we may conclude that the main features predicted by Eq. (50) have been verified experimentally. However, since III–V ferromagnetic semiconductors are heavily doped \( (\text{hole concentration} \approx 10^{19} - 10^{20} \text{cm}^{-3}) \) and, typically, of \( p \) type, which always results in low Schottky barriers, it has not yet been possible to fabricate ferromagnetic Schottky diodes of these materials. The best candidate for a ferromagnetic Schottky diode based on the III–V DMSs is perhaps Mn-doped GaN, which in addition to having a high \( T_C \) also has been reported to be of \( n \) type.\(^{35}\)

**B. Ferromagnetic \( pn \) diodes**

By using the standard relationships, \( n_{p0} = n_{p0}^2 / N_A \) and \( p_{n0} = n_{n0}^2 / N_D \), we see from Eq. (62) that in the case of an ideal ferromagnetic \( pn \) junction the magnetic part \([= \text{the first two terms in Eq. (62)}]\) contributes to the current density, if the following condition is fulfilled:

\[ N_D \gg N_A \sqrt{\frac{\tau_p D_P}{\tau_n D_n}} e^{-(\Delta E_v^0 + \Delta E_e^0)/kT}. \]  

(84)

In a \( p^+ \) \( n \) junction \( N_D \gg N_P \), but the large band discontinuities could help to reach condition (84). The experimentally observed valence band discontinuity in (Ga,Mn)As is \( \Delta E_v^0 = 40 - 100 \text{ meV} \),\(^{5,6}\) so that at low temperatures close to \( T_C \),
condition (84) may be fulfilled in GaMnAs/GaAs $p^+n$ junction, even though the $p$ side is always heavily doped. Arata et al.\textsuperscript{18} and Ohno et al.\textsuperscript{19} have studied experimentally the $I–V$ characteristics of a hybrid ferromagnetic/nonmagnetic $pn$ junction consisting of $p$-type (Ga,Mn)As and $n$-type GaAs layers, also including an InGaAs quantum well inside the $i$-GaAs layer. The effect of the barrier $\Delta E_0^0$ on the $T$ dependence of the current in Eq. (62) was identified in the experimental results, but the effect of the external magnetic field was not studied. Since the $n$ side was lightly doped (or intrinsic), condition (84) was certainly not fulfilled. Therefore, in the diode structure,\textsuperscript{6,29} where also the recombination occurred in the nonmagnetic quantum well outside the magnetic $p^+$ region, we do not expect that the effect of the band splitting could be seen in the $I–V$ characteristics. Also, the small conduction band splitting parameter\textsuperscript{18} $\Delta_1$(max) $\approx 25$ meV appearing in Eq. (62) points to a minor magnetic field dependence in a (Ga,Mn)As $p^+n$ junction. The effect of the large band splitting of the valence band on the electrical behavior, since only the heavily doped $n$ side is ferromagnetic and the $p$ side nonmagnetic, we see that the parameter $\Delta_2$ will appear in the current equation instead of the parameter $\Delta_1$. Since $\Delta_2 \gg \Delta_1$ we could expect much stronger dependence of the diode current on the magnetic state of the structure. In (Ga,Mn)As only the $p$ side can be ferromagnetic, but Mn-doped GaN, which is even a high $T_c$ ferromagnetic semiconductor, has been reported\textsuperscript{25} to be of $n$ type. Also, the methods to dope GaN as $p$ type have been developed. Therefore, it is not totally unrealistic to expect a fabrication of the diode structure, where the $n$ side is ferromagnetic, by using the existing GaN technology.

VII. CONCLUSIONS

We have studied theoretically how the electrical dc characteristics change when a part of the otherwise nonmagnetic device structure is replaced by a ferromagnetic semiconductor layer in some basic semiconductor devices such as $pn$ diodes or bipolar transistors. The general aim was to see how the device physics is changed when the magnetic subsystem is added to the device structure. We have developed simple analytical model equations, but some of the results for the model parameters, such as those for the mobilities or the higher order corrections for the band edges, can also be used, e.g., in two-dimensional numerical device simulations. According to our models for various ferromagnetic semiconductor devices the splitting of the energy levels between the spin-up and spin-down carriers can be seen in the temperature and magnetic field dependences of the $I–V$ current, which differ a lot from the behavior of the conventional devices, especially at temperatures close to the critical temperature. The model equations for the devices can be improved in a straightforward manner by taking into account the nonidealities such as high injection, series resistances, nonisotropic bands, etc. Our modeling results indicate that since the valence-band splitting is much larger than the one in the conduction band, the most favorable case in the ferromagnetic $pn$ junctions made of III–V ferromagnetic semiconductors would be the one where the $n$ side is ferromagnetic and the $p$ side nonmagnetic. Some of the model predictions (such as those for Schottky diodes) could be verified by comparing the calculated results to the experimental ones, but many of the theoretical results in the present article remain unverified so far. There are still severe technological difficulties to be surmounted before the magnetically asymmetric and optimized Schottky diodes or $pn$ junction structures can be made of III–V DMSs. The best candidate is perhaps heavily Mn-doped GaN, which is not only a high $T_c$ ferromagnetic semiconductor with a well-known device technology, but it also
can be doped as an \( n \)-type ferromagnetic semiconductor.\(^{25}\) The further development in these device technologies could open a field of magnetically controlled semiconductor devices and sensors, which then requires a lot of modeling activities similar to that discussed in the present article.

**APPENDIX A: DERIVATION OF THE RETARDED GREEN’S FUNCTION**

The Green’s function in Eq. (7) can be derived by using various methods. Here, we derive it by using two quite different approaches, which, fortunately, lead exactly to the same final result, and thereby guarantee the correctness of the expression. Let us first describe a method based on Zubarev’s double-time Green’s functions.\(^{45}\) The retarded Green’s function for operators \( A \) and \( B \) is defined by

\[
G(t,t') = \langle A(t); B(t') \rangle = \frac{1}{\hbar} \theta(t-t') \times \langle [A(t),B(t')] \rangle ,
\]

where \( \theta \) is the unit step function, the brackets \( \langle \cdots \rangle \) denote the thermal average, and \( A(t) \) and \( B(t) \) are the operators in the Heisenberg picture. The index \( “-“ \) in \( [\cdots] \) denotes the anticommutator. By using the Heisenberg equation of motion for the operators, it can be shown\(^{37}\) that if \( H \) is the Hamiltonian of the system, the time-Fourier-transformed Green’s function \( \langle A; B \rangle \) satisfy the following equation of motion:

\[
E \langle A; B \rangle = \langle [A,B] + \langle AAH - HA \rangle \rangle ; B \rangle = E \langle A,B \rangle . \quad (A2)
\]

Now we can insert the Hamiltonian (1) into Eq. (A2), which yields for the propagator \( G_{\sigma \sigma'}(k, h, E) = \langle a_{k\sigma}; a_{h\sigma'}^\dagger \rangle \) the following equation of motion:

\[
EG_{\sigma \sigma'}(k, h, E) = \delta_{kh} \delta_{\sigma \sigma'} + E_{\sigma} G_{\sigma \sigma'}(k, h, E) + \frac{J_{\text{exch}}}{2N} \sum_{p,k,R} \langle \hat{G}_{\sigma \sigma'}^{(p-k)}(p, h, E; S_{R}) \rangle (\delta_{\sigma R} - \delta_{\sigma \sigma'}) + G_{\sigma \sigma'}(k, h, E; S_{R}) \delta_{\sigma \sigma'} + \frac{G_{\sigma \sigma'}^+(k, h, E; S_{R}^+)}{\delta_{\sigma \sigma'}}, \quad (A3)
\]

where the higher order Green’s functions are defined as \( G_{\sigma \sigma'}^{(p-k)}(p, h, E; S_{R}) = \langle (S_{R}^z - (S_{R}^{z+}a_{p\sigma}^\dagger a_{p\sigma}^\dagger)) \rangle \), and the same kind of definitions for \( G^{-}(S_{R}) \) and \( G^{+}(S_{R}) \), where \( S_{R} \) is replaced by the spin lowering and raising operators \( S_{R} \) and \( S_{R}^+ \), respectively. By using Eq. (A2), we can derive the equations of motion also for the higher order Green’s functions. By applying a decoupling procedure introduced by Sinkkonen,\(^{30}\) where the products of the spin operators are replaced by their thermal averages, we obtain, e.g., for \( G^{+}(S_{R}) \) the following equation:

\[
\langle E - E_{\text{exch}}^{(1)} \rangle G_{\sigma \sigma'}^{+}(p, h, E; S_{R}) = \frac{J_{\text{exch}}}{2N} \sum_{k', R} \langle \hat{G}_{\sigma \sigma'}^{+}(k', h, E; S_{R}) \rangle \times G_{\sigma \sigma'}(k', h, E). \quad (A4)
\]

Inserting Eq. (A4) and the same kind of equations for the higher order Green’s functions \( G^{+}(S_{R}) \) and \( G^{-}(S_{R}) \) into Eq. (A3), and using the familiar sum rule \( \sum \delta_{k,p} \langle \hat{G}_{\sigma \sigma'}^{+}(k, h, E; S_{R}) \rangle \equiv \delta_{k,p} \rho N \) over the lattice sites \( R \), we obtain

\[
G_{\sigma \sigma'}(k, h, E) = \langle a_{k\sigma}; a_{h\sigma'}^\dagger \rangle G_{\sigma \sigma'}(q, h, E) + \sum_{q} G_{\sigma \sigma'}^+(q, h, E) \langle \hat{G}_{\sigma \sigma'}^{+}(q, h, E; S_{R}) \rangle \times \langle a_{q\sigma}; a_{h\sigma'}^\dagger \rangle G_{\sigma \sigma'}(q, h, E), \quad (A5)
\]

where \( G^{+}(q) = \delta_{k,q} \delta_{\sigma \sigma'} (E - E_{\text{exch}}^{(1)})^{-1} \) is the first order propagator, and \( \Sigma^{(2)} \) is the second order self-energy:

\[
\Sigma^{(2)}(k,q,\sigma) = \frac{J_{\text{exch}}^2}{4N} \delta_{kh} \delta_{\sigma \sigma'} \left[ \Gamma^{(q)}(k') \delta_{\sigma \sigma'} + \Gamma^{+}(q') \delta_{\sigma \sigma'} \right] + \frac{\Gamma^{(q)}(k') \delta_{\sigma \sigma'}}{E - E_{\text{exch}}^{(1)}} \times \frac{\Gamma^{+}(q') \delta_{\sigma \sigma'}}{E - E_{\text{exch}}^{(1)}}. \quad (A6)
\]

Finally, since the self-energy is proportional to \( \delta_{k,q} \), we easily can solve \( G_{\sigma \sigma'}(k, h, E) \) from Eq. (A5) and obtain Eq. (7).

Since all the decoupling procedures in Zubarev’s method are arbitrary to some extent, it is desirable to check the results by using an alternative method, if possible. A quite different approach to the infinite order perturbation theory is to use Matsubara’s temperature Green’s functions and an \( S \)-matrix expansion for the perturbation Hamiltonian.\(^{37}\) In this method we need the field operators in the interaction picture, which are obtainable from Eq. (6) as follows:

\[
\Psi_{\sigma}^{+}(r,\tau) = e^{i\hat{H}_{0}\tau} \Psi_{\sigma}^{+}(r), \quad \Psi_{\sigma}(r,\tau) = e^{i\hat{H}_{0}\tau} \Psi_{\sigma}(r), \quad (A7)
\]

where \( \hat{H}_{0} \) is the Hamiltonian of the system without the exchange interaction (3), which in the interaction picture now reads

\[
\hat{H}_{\text{exch}}(\tau) = -\frac{\Omega}{2} \sum_{R} \int J(r-R) S_{R}^{\sigma} \Psi_{\sigma}^{+}(r,\tau) \Psi_{\sigma}(r,\tau) + S_{R}^{\sigma} \Psi_{\sigma}^{+}(r,\tau) \Psi_{\sigma}(r,\tau) + \langle S_{R}^{\sigma} - \langle S_{R}^{\sigma} \rangle \rangle \times \langle \Psi_{\sigma}^{+}(r,\tau) \Psi_{\sigma}(r,\tau) - \Psi_{\sigma}^{+}(r,\tau) \Psi_{\sigma}(r,\tau) \rangle d^{3}r. \quad (A8)
\]

Matsubara’s temperature Green’s function is defined as\(^{37}\)

\[
G_{\sigma \sigma'}(r, r', \tau') = -\langle T_{\tau} \Psi_{\sigma}(r, \tau) \Psi_{\sigma'}^{+}(r', \tau') \rangle_{\text{con}}, \quad (A9)
\]

where the brackets \( \langle \cdots \rangle_{\text{con}} \) denote the thermal average, when retaining only terms corresponding to connected diagrams in the series expansion. \( T_{\tau} \) is a time ordering operator, and the \( S \) matrix is defined as

\[
\hat{S} = \exp \left[ -\frac{\beta}{\hbar} \int_0^{\tau} \hat{H}_{\text{exch}}(\tau) d\tau' / \hbar \right], \quad (A10)
\]

where \( \beta = 1/k_{B}T \). If the spin-polarized band energies (5) are taken as the electronic ground state, the corresponding Green’s function is given by\(^{37}\)
Finally, the perturbed Green’s function is obtained by using the Fourier transform of \(G^{(1)}_{\sigma\sigma'}(\mathbf{r}, \tau, \mathbf{r}', \tau')\).

\[G^{(1)}_{\sigma\sigma'}(\mathbf{r}, \tau, \mathbf{r}', \tau') = \frac{1}{\hbar V} \sum_{\mathbf{k} \mathbf{q}} e^{i(\mathbf{r} - \mathbf{r}') - iE_{\mathbf{q}}(\tau - \tau')/\hbar} G^{(1)}_{\sigma\sigma', \mathbf{k}, \mathbf{q}} \times \langle \mathbf{k}, E_n \rangle = \frac{\delta_{\sigma\sigma'}}{\hbar V} \sum_{\mathbf{k} \mathbf{q}} e^{i(\mathbf{r} - \mathbf{r}') - iE_{\mathbf{q}}(\tau - \tau')/\hbar} E_n - E_n^0, \tag{A11}\]

where \(E_n = (2n + 1) \pi/\beta (n \text{ is an integer})\), and \(G^{(1)}_{\sigma\sigma'}(\mathbf{k}, E_n)\) is the Fourier transform of \(G^{(1)}_{\sigma\sigma'}(\mathbf{r}, \tau, \mathbf{r}', \tau')\).

The second order self-energy is obtained by inserting the \(S\) matrix into the Green’s function \(G^{(1)}\), and taking into account only the terms proportional to \(J_{\text{exch}}^2\), which yields

\[G^{(2)}_{\sigma\sigma'}(\mathbf{r}, \tau, \mathbf{r}', \tau') = - \frac{1}{2\hbar^2} \int_0^{\beta \hbar} d\tau_1 \int_0^{\beta \hbar} d\tau_2 \left[ \hat{H}_{\text{exch}}(\tau_1) \hat{H}_{\text{exch}}(\tau_2) \right] \times \Psi_r(\mathbf{r}, \tau) \Psi_r^+(\mathbf{r}', \tau') d\tau_1 . \tag{A12}\]

The relevant thermal average can be estimated by using the Hamiltonian (A8) and retaining only the terms that are linear in spin correlation function \(\propto (S_R \cdot S_R)\). Using the Green’s function (A11), we can write the terms appearing in the Fourier transform of Eq. (A12) in a compact form in the momentum space:

\[G^{(2)}_{\sigma\sigma'}(\mathbf{k}, \omega_n) = G_{\sigma\sigma'}^0(\mathbf{k}, \omega_n) \left[ \frac{J_{\text{exch}}^2}{4N} \sum_q \frac{\Gamma^{\text{exch}}(q)}{i\hbar \omega_n - E_{\mathbf{k} - \mathbf{q}}^0} G_{\sigma\sigma'}^0(\mathbf{k}, \omega_n) + G_{\sigma\sigma'}^0(\mathbf{k}, \omega_n) \right] \frac{\Gamma^{\text{exch}}(q)}{i\hbar \omega_n - E_{\mathbf{k} + \mathbf{q}}^0} G_{\sigma\sigma'}^0(\mathbf{k}, \omega_n) + G_{\sigma\sigma'}^0(\mathbf{k}, \omega_n) \frac{\Gamma^{\text{exch}}(q)}{i\hbar \omega_n - E_{\mathbf{k} - \mathbf{q}}^0} G_{\sigma\sigma'}^0(\mathbf{k}, \omega_n). \tag{A13}\]

Now, we can identify the second order self-energy from Eq. (A13), which is exactly the same as given by Eq. (A6). Finally, the perturbed Green’s function is obtained by using the Dyson’s equation \(G = G^0 [1 - G^0 \Sigma]^{-1}\) in momentum space.

**APPENDIX B: SRH MECHANISM IN FERROMAGNETIC SEMICONDUCTORS**

We derive the expression for the net rate of recombination in the case of indirect recombination of the electrons and holes in the spin-polarized valence and conduction bands, respectively, by following (and properly modifying) the original treatment published first by Shockley et al.\(^{44,45}\) Let us first consider the processes \(1^\circ\) and \(2^\circ\) for spin-up carriers in Fig. 4. The electron capture rate \(R^1\) from the band edge \(E_{\text{c1}}\) to the trap state \(E_{\text{t1}}\) is given by

\[R^1_{\text{e}} = n_i N_{\text{t1}}^i [1 - f(E_{\text{t1}})] v_{\text{th}} A_{\text{e}}^p . \tag{B1}\]

where \(A_{\text{e}}^p\) is the capture cross section for electrons. On the other hand, the emission rate \(R^2\) from the trap state to the band edge is given by

\[R^2_{\text{e}} = N_i f(E_{\text{c1}}) P_{\text{e}}^p . \tag{B2}\]

where \(P_{\text{e}}^p\) is the electron emission probability from the trap state to the conduction band. Similarly, the hole capture rate \(R^1\) from \(E_{\text{c2}}\) to \(E_{\text{t2}}\) is given by

\[R^1_{\text{h}} = p_i N_{\text{t2}}^i f(E_{\text{c2}}) v_{\text{th}} A_{\text{h}}^e , \tag{B3}\]

where \(A_{\text{h}}^e\) is the capture cross section for holes. For the hole emission rate from the trap state to the valence band we get

\[R^2_{\text{h}} = N_i [1 - f(E_{\text{t2}})] P_{\text{h}}^e . \tag{B4}\]

where \(P_{\text{h}}^e\) is the hole emission probability. At thermal equilibrium the net rate of recombination vanishes, \(R^1 = R^2 = 0\), and by using this condition we can solve \(P_{\text{e}}^p\) and \(P_{\text{h}}^e\) from Eqs. (B1)–(B4):

\[P_{\text{e}}^p = n_i v_{\text{th}} A_{\text{e}}^p (E_{\text{e}} - E_i)/k_B T , \tag{B5}\]

\[P_{\text{h}}^e = p_i v_{\text{th}} A_{\text{h}}^e (E_{\text{h}} - E_i)/k_B T . \tag{B6}\]

When equilibrium is disturbed by low-level excitation, the net rate of recombination in the case of the indirect recombination is given by

\[R_{\text{ind}} = R^1 - R^2 = R_3 - R_4 . \tag{B7}\]

Inserting Eqs. (B1)–(B4) into Eq. (B7), we easily can solve \(f(E_{\text{c}}:\)

\[f(E_{\text{c}}:] = \frac{n_i N_{\text{t1}} v_{\text{th}} A_{\text{e}}^p + N_i P_{\text{h}}^e}{n_i N_{\text{t1}} v_{\text{th}} A_{\text{e}}^p + N_i P_{\text{h}}^e + p_i N_{\text{t2}} v_{\text{th}} A_{\text{h}}^e + N_i P_{\text{h}}^e} . \tag{B8}\]

Finally, inserting Eq. (B8) into Eqs. (B1) and (B2), we get \(R_{\text{ind}} = R^1 - R^2\), i.e., Eq. (A40).

In the same way, an equation for the recombination rate of the spin-down carriers can be derived.

46 D. N. Zubarev, Nonequilibrium Statistical Thermodynamics (Consultant Bureau, New York, 1974).