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Polarizabilities of nonreciprocal bi-anisotropic particles

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This thesis studies two known canonical particles introduced as possible realizations of nonreciprocal bi-anisotropic particles. They are called Tellegen-omega and moving-chiral. For each particle, firstly all the dipolar polarizabilities (electric, magneto-electric, electro-magnetic, and magnetic) are analytically derived by considering a uniform electric or magnetic field in the plane of the particle as the exciting fields. Furthermore, at the second step, the analytical results are compared to the numerical ones. Finally, appropriate measurement setups are suggested which make us able to measure and extract experimentally the electromagnetic and magneto-electric polarizabilities.

Keywords: Bi-anisotropic particles, electric dipole moment, magnetic moment, magnetization, polarizability
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Mohammad Sajjad Mirmoosa

Otaniemi, 2.8.2013
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<tr>
<td>AC</td>
<td>Alternating Current</td>
</tr>
<tr>
<td>C</td>
<td>Celsius</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>Ferric Oxide</td>
</tr>
<tr>
<td>GHz</td>
<td>GigaHertz</td>
</tr>
<tr>
<td>HFSS</td>
<td>High Frequency Structure Simulator</td>
</tr>
<tr>
<td>MHz</td>
<td>MegaHertz</td>
</tr>
<tr>
<td>mm</td>
<td>Millimetre</td>
</tr>
<tr>
<td>MOM</td>
<td>Method of Moment</td>
</tr>
<tr>
<td>NiO</td>
<td>Nickel Oxide</td>
</tr>
<tr>
<td>Oe</td>
<td>Oersted</td>
</tr>
<tr>
<td>PEC</td>
<td>Perfect Electric Conductor</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse Electric</td>
</tr>
<tr>
<td>YIG</td>
<td>Yttrium Iron Garnet</td>
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<table>
<thead>
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<tr>
<td>( \varepsilon )</td>
<td>Permittivity dyadic</td>
</tr>
<tr>
<td>( \varepsilon_0 )</td>
<td>Permittivity of free space</td>
</tr>
<tr>
<td>( \varepsilon_r )</td>
<td>Relative permittivity</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Permeability dyadic</td>
</tr>
<tr>
<td>( \mu_0 )</td>
<td>Permeability of free space</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Chirality dyadic (parameter)</td>
</tr>
<tr>
<td>( \tau_{tt} )</td>
<td>Tellegen dyadic (parameter)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Wave impedance of free space</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Wavelength</td>
</tr>
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<td>Gyromagnetic ratio</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Damping constant</td>
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<td>( \Delta H )</td>
<td>Linewidth</td>
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<tr>
<td>( \omega_0 )</td>
<td>Larmor frequency</td>
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<td>( \omega_r )</td>
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<td>( \omega )</td>
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<td>( \chi )</td>
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<td>( \bar{\chi} )</td>
<td>Electric polarizability dyadic</td>
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<td>( \bar{\alpha}_{ee} )</td>
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</tr>
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<td>( \bar{\alpha}_{me} )</td>
<td>Magneto-electric polarizability dyadic</td>
</tr>
<tr>
<td>( \bar{\alpha}_{mm} )</td>
<td>Magnetic polarizability dyadic</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Reflection coefficient</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Transmission coefficient</td>
</tr>
<tr>
<td>( \beta_w )</td>
<td>Phase constant in a waveguide</td>
</tr>
<tr>
<td>( a )</td>
<td>Ferrite sphere radius</td>
</tr>
<tr>
<td>( b )</td>
<td>Width of waveguide</td>
</tr>
<tr>
<td>( B )</td>
<td>Magnetic flux density</td>
</tr>
<tr>
<td>( c )</td>
<td>Speed of light</td>
</tr>
<tr>
<td>( d )</td>
<td>Height of waveguide</td>
</tr>
<tr>
<td>( D )</td>
<td>Electric flux density</td>
</tr>
<tr>
<td>( \mathbf{e}_{01} )</td>
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</tr>
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<td>( \mathbf{e}_{10} )</td>
<td>Unit electric field for rectangular waveguide TE_{10} mode</td>
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<tr>
<td>( \mathbf{E} )</td>
<td>Electric field vector</td>
</tr>
<tr>
<td>( \mathbf{E}_{inc} )</td>
<td>Incident electric field vector</td>
</tr>
<tr>
<td>( \mathbf{E}_{local} )</td>
<td>Local electric field vector</td>
</tr>
<tr>
<td>( \mathbf{E}_{scat} )</td>
<td>Scattered electric field vector</td>
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<td>Frequency</td>
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<td>Time-varying magnetic field vector</td>
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<td>( H_0 )</td>
<td>Internal bias field</td>
</tr>
<tr>
<td>( H_a )</td>
<td>Applied external bias field</td>
</tr>
<tr>
<td>( H_{dem} )</td>
<td>Demagnetizing field vector</td>
</tr>
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</table>
\( \mathbf{H}_{\text{inc}} \) Incident magnetic field vector
\( \mathbf{H}_{\text{local}} \) Local magnetic field vector
\( \bar{I} \) Unit dyadic
\( I \) Electric current
\( \mathbf{J} \) Angular momentum vector
\( k \) Wavenumber of free space
\( K_{\text{cut-off}} \) Cut-off wavenumber in a waveguide
\( l \) Length of metal wire
\( \mathbf{m} \) Magnetic moment vector
\( \mathbf{M} \) Magnetization vector
\( \mathbf{M} \) Time-varying magnetization vector
\( M_s \) Saturation magnetization
\( \mathbf{p} \) Electric moment vector
\( r_0 \) Radius of wire
\( S_{11} \) Input reflection coefficient of a two-port network
\( S_{21} \) Transmission coefficient of a two-port network
\( T \) Torque
\( V \) Ferrite sphere volume
\( Z \) Characteristic impedance in a waveguide
\( Z_{\text{in}} \) Input impedance of a linear wire antenna
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1 Introduction

General bi-anisotropic particles have been attracting considerable attention because of their interesting physical properties, and they can be used for many applications. For example, the electrically small particles known as optimal particles are able to extract the maximum power from an incident electromagnetic field, and the artificial electromagnetic materials designed with the use of optimal particles are expected to interact most strongly with different kinds of electromagnetic fields, which causes increase in the efficiency of various devices [1].

Property of zero forward and backward electromagnetic scattering is another phenomenon leading to worthy applications of electrically small bi-anisotropic particles. In [2], it is explained how conditions for zero backscattering and zero forward scattering are satisfied for a general uniaxial bi-anisotropic particle and also for particles of all fundamental classes of bi-anisotropic particles. For instance, the particle introduced in Figure 1(a) (uniaxial chiral particle) corresponds to zero backscattering, and the particle shown in Figure 1(b) (active omega particle) corresponds to zero forward scattering [2]. The material of the particles in these models is perfect electric conductor (PEC).

![Figure 1: (a) Uniaxial chiral particle showing zero backscattering; (b) active omega particle showing zero forward scattering.](image)

In addition, another valuable application can be a thin-sheet isolator which is a layer containing inclusions of a specific class of bi-anisotropic particles [3]. From one side of the layer, the electromagnetic field is totally absorbed, while the incident field can be transmitted from the other side of the layer. In other words, the layer is acting as an isolator. Phase shifter [4], twist polarizer [5], perfect absorber [3] and transparent layer are other useful applications of bi-anisotropic particles. In a
transparent layer, we can have transparency feature from one side of the layer, while we can control the functionality of the layer for the waves coming from the other side by electro-magnetic coupling in particles.

The most important characteristic of bi-anisotropic particles is the available electro-magnetic couplings which create a variety of possibilities to achieve beneficial physical phenomena. Based on the electro-magnetic coupling effects, bi-anisotropic media (which are made of bi-anisotropic particles) are classified as reciprocal and nonreciprocal [6]. Generally, we can describe a bi-anisotropic material by the following equations:

\[
\begin{align*}
D &= \varepsilon \cdot \mathbf{E} + \sqrt{\varepsilon_0 \mu_0} \tilde{a}_{em} \cdot \mathbf{H}, \\
B &= \tilde{\mu} \cdot \mathbf{H} + \sqrt{\varepsilon_0 \mu_0} \tilde{a}_{me} \cdot \mathbf{E},
\end{align*}
\]  

(1)

in which \( \mathbf{E}, \mathbf{H} \) are the electric and magnetic fields, and \( D, B \) represent the flux densities. \( \varepsilon \) is the permittivity dyadic and \( \tilde{\mu} \) is the permeability dyadic. Parameters \( \tilde{a}_{em} \) and \( \tilde{a}_{me} \) are the electro-magnetic and magneto-electric coupling dyadics, respectively. \( \varepsilon_0 \) indicates the free-space permittivity and \( \mu_0 \) denotes the free-space permeability.

In a reciprocal material the permittivity and permeability dyadics are symmetric, and the coupling dyadics are related to each other as

\[
\tilde{a}_{me} = -\tilde{a}_{em}^T,
\]  

(2)

where \( T \) is the transpose operation. Considering (2), the material equation for reciprocal media can be expressed as

\[
\begin{align*}
D &= \tilde{\varepsilon} \cdot \mathbf{E} - j\sqrt{\varepsilon_0 \mu_0} \tilde{\kappa} \cdot \mathbf{H}, \\
B &= \tilde{\mu} \cdot \mathbf{H} + j\sqrt{\varepsilon_0 \mu_0} \tilde{\kappa}^T \cdot \mathbf{E}.
\end{align*}
\]  

(3)

The electro-magnetic coupling dyadic \( \tilde{\kappa} \) is called the reciprocal magneto-electric coupling parameter, and it can be decomposed as

\[
\tilde{\kappa} = \kappa \tilde{I} + \tilde{N} + \tilde{J},
\]  

(4)

where the dyadic \( \tilde{J} \) is presented as

\[
\tilde{J} = \mathbf{v} \times \tilde{I}.
\]  

(5)

\( \tilde{I} \) is the unit dyadic, \( \tilde{N} \) is a symmetric dyadic, and \( \tilde{J} \) is an antisymmetric dyadic. Also, \( \mathbf{v} \) is a complex vector. Based on (4), we can classify reciprocal media into seven groups which can be found in [6]. Two well-known classes of the reciprocal bi-anisotropic materials are the chiral medium (\( \kappa \neq 0, \tilde{N} = 0, \tilde{J} = 0 \)) and the omega medium (\( \kappa = 0, \tilde{N} = 0, \tilde{J} \neq 0 \)).
If for a bi-anisotropic medium the reciprocal dyadic \( \kappa = 0 \), we will have a non-reciprocal coupling. In this case the magneto-electric dyadic is the transpose of the electro-magnetic dyadic

\[
\alpha_{me} = \alpha_{em}^T.
\]

and the constitutive equation of the bi-anisotropic material can be written as

\[
\begin{align*}
D &= \varepsilon \cdot E + \sqrt{\varepsilon_0 \mu_0} \chi_t \cdot H, \\
B &= \mu \cdot H + \sqrt{\varepsilon_0 \mu_0} \chi_t^T \cdot E.
\end{align*}
\]

\( \chi_t \) is called the nonreciprocal magneto-electric coupling parameter. Similar to the dyadic \( \kappa \), we can express the nonreciprocal coupling dyadic \( \chi_t \) as the summation of three dyadics. Therefore,

\[
\chi_t = \chi_t^I + \chi_t^Q + \chi_t^S,
\]

in which \( \chi_t^Q \) and \( \chi_t^S \) are two symmetric and antisymmetric dyadics, respectively. According to (8), a classification of nonreciprocal bi-anisotropic materials is suggested in [6]. Tellegen medium (\( \chi_t \neq 0, \chi_t^Q = 0, \chi_t^S = 0 \)) and moving medium (\( \chi_t = 0, \chi_t^Q = 0, \chi_t^S \neq 0 \)) are two prominent groups of this classification.

The aim in this thesis is to investigate in detail the bi-anisotropic particles which possess nonreciprocal electro-magnetic coupling. Nonreciprocal bi-anisotropic particles can open a novel route to realize new microwave devices such as perfect absorbers and thin-sheet isolators. Hence, study, design and optimization of these particles are so necessary.

It is known that in order to get nonreciprocal coupling in bi-anisotropic particles, we need a system having both electrically polarizable and magnetizable subsystems which should be coupled.

The first method for creation of artificial nonreciprocal particles was presented by Tellegen in 1948. His idea was to distribute randomly the particles which possess fixed electric and magnetic dipole moments. F. Olyslager used the concept of metamaterials, and suggested that it could be possible to realize nonreciprocal coupling by connecting a short wire dipole antenna and a small loop antenna via a gyrator. This realization was simpler, but it required a gyrator for the desired frequency range.

In 1996, Kamenetskii used composition of a magneto-static ferrite resonators with partially metalized surfaces to achieve chiral and bi-anisotropic waveguide structures [7]. He showed that it is possible to obtain controllable chirality simply by tuning bias magnetic field. Dmitriev generalized the principle and introduced new possible media [8]. It was shown in [9] that the introduced transparent absorbing boundary for the termination of computational domains in finite methods [10] can be described by the constitutive relations of the same type as that of bi-anisotropic moving media which was described by Tai [11].

In 1998, Tretyakov, by properly choosing passive scatterers as their formative inclusions, introduced the first design of artificial composite media (stationary media at rest) showing nonreciprocal Tellegen and moving coupling effects [12]. The
geometry of the considered artificial nonreciprocal bi-anisotropic particles is shown in Figure 2. The magnetic bias field is along the \( z_0 \)-axis. The principle is that an external electric field would excite the metal wires, and so the magnetic field, formed by the electric current of the wires, induces the magnetic moment in the ferrite sphere. Also a high-frequency incident magnetic field present within the ferrite sample causes the high-frequency magnetization, which could induce electric currents and therefore, the electric dipole moment on the metal wires. It is important to consider that in the presence of the incident field, the ferrite inclusion and the metal wires couple to each other, so that the magnetic moment excites the wires, and the electric currents excite the ferrite inclusion.

The polarizabilities of such nonreciprocal bi-anisotropic particles have not been
yet found analytically. In this work, all the polarizabilities are derived theoretically, and then they are compared with simulated results. Also some proper measurement setups are introduced to allow one to measure experimentally the electro-magnetic and magneto-electric polarizabilities. This work is believed to be useful for theoretical studies of the electromagnetic properties of arrays composed of nonreciprocal Tellegen-omega and moving-chiral particles, design and realization of new nonreciprocal microwave devices.

In Section 2, we briefly discuss the ferrite materials and their properties at microwave frequencies. Specifically, we provide the relation between the magnetization vector of a ferrite sphere sample and an external magnetic field. This relation is very important for us and we use it several times in the next sections. In Section 3 and Section 4, we study in detail Tellegen-omega and moving-chiral particles, respectively. We firstly derive analytically the polarizabilities, and then for confirming the results, we compare them with our numerical results. In Section 5, by applying the waveguide measurement technique, we study how one can measure the Tellegen, chiral, omega and moving parameters of a nonreciprocal bi-anisotropic particle which is positioned inside a rectangular or square waveguide. Finally, Section 6 concludes the thesis and suggests possible directions of future studies.
2 Basics of Ferrites

Ferrites are polycrystalline magnetic oxides that can be described by the general chemical formula

$\text{XO} \cdot \text{Fe}_2\text{O}_3$ (9)

in which X is a divalent metal ion such as iron, manganese, magnesium, nickel, zinc, cadmium, cobalt, copper, or a combination of these. The ferrites belong to the category of ferrimagnetic materials, and they are fabricated by conventional ceramic techniques. For instance, to manufacture nickel ferrite, NiO and Fe$_2$O$_3$ are thoroughly mixed in the form of powder, and then they are pressed to the required shape, and sintered at temperatures more than $1200^\circ\text{C}$ [13].

There has been a great deal of research and engineering activities in the microwave ferrite area. The characteristics of ferrites can be used to construct a number of nonreciprocal, nonlinear, and tunable microwave devices such as isolators, circulators, phase shifters, modulators, power limiters, switches, amplifiers, delay lines, and filters [14–16]. Generally, the significant progress in the development of the ferrite materials came with the work of Snoek and his co-workers after 1933 at the Philips Research Laboratories at Eindhoven in Holland. He published the magnetic and electrical properties of some ferrite compounds in the mentioned chemical composition XO · Fe$_2$O$_3$ in 1936 [17]. Snoek was researching the ferrite materials possessing low eddy-current loss, high resistivity and high permeability suitable for applying as magnetic cores up to a few MHz. In 1946, he announced his results about ferrite materials suitable for using in radio circuits [18, 19]. Smit and Wijn published a book on the lower frequency properties of ferrites in 1959 in the Philips’s Technical Library [20]. Low-loss ferrites for the higher radio frequencies led to the investigation of their properties at microwave frequencies. Polder first derived the microwave permeability tensor in 1949 which laid the ground work for the understanding of ferrite behavior at microwave frequencies [21]. In 1952, Hogan used Faraday rotation to construct the first workable ferrite microwave gyrator and showed how it could make an isolator, circulator, switch, variable attenuator or modulator [22]. Suhl and Walker investigated the exact theoretical analysis of ferrite-filled circular waveguide [23], and Fox, Miller and Weiss published an important paper giving a very large number of possible configurations of ferrite in waveguide and their effects [24]. This was only a short summary of the development in ferrite area subsequent to the work by Snoek in Holland. For a more comprehensive review of history of ferrite research and applications, the reader is referred to a number of books on microwave ferrites [25–30].

This section reviews the basic concepts behind the relation between the magnetization vector and an external magnetic field. The first part 2.1 is about this relation for an infinite ferrite medium. Damping effect, and its relation with the linewidth of ferrites are discussed in the second part 2.2, and the third part 2.3, respectively. Finally, the impact of the finite size of the ferrite sample on the relation of magnetization and external field is described in 2.4.
2.1 Susceptibility Tensor in Infinite Medium

At microwave frequencies, a magnetized ferrite has a scalar permittivity, while its permeability is a tensor which is found from the linearized equation of motion. If an electron as a magnetic dipole is placed in an external DC magnetic field $\mathbf{H}_0$, in a balanced condition (zero force resultant), the magnetic moment is in the direction of $\mathbf{H}_0$. Assuming that the magnetic dipole and the DC field make an angle $\theta$ by a small external force, the torque, which is applied on the magnetic moment, is \[ T = \mathbf{m} \times \mu_0 \mathbf{H}_0. \] (10)

The ratio between the magnetic moment $\mathbf{m}$, and the corresponding angular momentum $\mathbf{J}$ is called the gyromagnetic ratio which is denoted by $\gamma$. In other words,

$$\mathbf{m} = \gamma \mathbf{J}. \quad (11)$$

Torque is the time derivative of the angular momentum, therefore by using (11) the torque can be written as

$$T = -\frac{1}{\gamma} \frac{d\mathbf{m}}{dt}. \quad (12)$$

Equations (10) and (12) result in

$$\frac{d\mathbf{m}}{dt} = -\gamma (\mathbf{m} \times \mu_0 \mathbf{H}_0), \quad (13)$$

which is well known as the equation of motion for one magnetic dipole. If there were $N$ unbalanced dipoles per unit volume, the equation (13) would be rewritten as

$$\frac{dM_0}{dt} = -\gamma (M_0 \times \mu_0 \mathbf{H}_0), \quad (14)$$

where $M_0$ is the total magnetization. A large external static field $\mathbf{H}_0$ causes that all spins are firmly coupled, and saturation of magnetization occurs, $M_0 = M_s$. Therefore, when no time-varying magnetic field is applied to the ferrite medium, the magnetization per unit volume is $M_s$. In the microwave case, the total magnetic field consists of both DC and RF parts. The time-varying magnetic field generates an AC component of the magnetization. Hence, for an infinite ferrite medium, the equation of motion based on (14) is

$$\frac{d(M_s + M)}{dt} = -\gamma ((M_s + M) \times \mu_0 (\mathbf{H}_0 + \mathbf{H})), \quad (15)$$

where $M$ and $\mathbf{H}$ represent the AC signals of the magnetization and the external magnetic field, respectively. Considering that the AC signal is much smaller than the DC one, and also the saturation magnetization is parallel to the external static field, from (15) we find

$$\frac{dM}{dt} = -\gamma \mu_0 ((M_s \times \mathbf{H}) + (M \times \mathbf{H}_0)). \quad (16)$$
We assume that the time dependence is in form $e^{j\omega t}$. If $\mathcal{M}$ and $\mathcal{H}$ were represented by the phasors $\mathbf{M}$, and $\mathbf{H}$, equation (16) would be simplified as

$$j\omega \mathbf{M} = -\gamma \mu_0 \left( (\mathbf{M}_s \times \mathbf{H}) + (\mathbf{M} \times \mathbf{H}_0) \right), \quad (17)$$

where

$$\mathbf{M} = M_x \mathbf{x}_0 + M_y \mathbf{y}_0 + M_z \mathbf{z}_0, \quad \mathbf{H} = H_x \mathbf{x}_0 + H_y \mathbf{y}_0. \quad (18)$$

Substituting (18) in (17), and solving this equation give easily the components of the magnetization vector as

$$M_x = \frac{\omega_0 \omega_m H_x + j\omega \omega_m H_y}{\omega_0^2 - \omega^2},$$

$$M_y = \frac{\omega_0 \omega_m H_y - j\omega \omega_m H_x}{\omega_0^2 - \omega^2}, \quad (19)$$

$$M_z = 0,$$

where

$$\omega_0 = \gamma \mu_0 H_0 \quad (20)$$

is usually called the Larmor frequency, and

$$\omega_m = \gamma \mu_0 M_s. \quad (21)$$

A susceptibility tensor $\overline{\chi}$ can be defined which relates the RF magnetization to the applied magnetic field:

$$\mathbf{M} = \overline{\chi} \cdot \mathbf{H}, \quad (22)$$

where

$$\overline{\chi} = \begin{pmatrix} \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & \chi_{yy} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (23)$$

and

$$\chi_{xx} = \chi_{yy} = \frac{\omega_0 \omega_m}{\omega_0^2 - \omega^2},$$

$$\chi_{xy} = -\chi_{yx} = \frac{j\omega \omega_m}{\omega_0^2 - \omega^2}. \quad (24)$$
2.2 Damping

As it is seen in (24), when the frequency $\omega$ and the Larmor frequency $\omega_0$ have the identical values, the susceptibility tensor becomes infinite, which is known as the gyromagnetic resonance effect. In reality ferrite materials have magnetic losses which damp such singularities. In the equations of motion, the loss is phenomenologically taken into account by two forms: the first one is called Bloch-Bloembergen, and the second one is Landau-Lifshitz. In practice, for most ferrites the modified form of the Landau-Lifshitz form, which is known as the Gilbert form, is often used, and it is given by [33]

$$\frac{dM}{dt} = -\gamma\mu_0 ((M_s + M) \times (H_0 + H)) + \frac{\delta}{M_s} M_s \times \frac{dM}{dt}, \quad (25)$$

where $\delta$ is the dimensionless damping constant. By analyzing equation (25) similarly to solving equation (15), the components of the susceptibility matrix are derived as [33,34]

$$\chi_{xx} = \chi_{yy} = \chi' - j\chi'', \quad \chi_{xy} = -\chi_{yx} = j(K' - jK''), \quad (26)$$

where

$$\chi' = \frac{\omega_0\omega_m (\omega_0^2 - \omega^2) + \omega_m \omega_0^2 \omega^2 \delta^2}{(\omega_0^2 - \omega^2 (1 + \delta^2))^2 + 4\omega_0^2 \omega^2 \delta^2};$$

$$\chi'' = \frac{\omega_0 \omega m (\omega_0^2 + \omega^2 (1 + \delta^2))}{(\omega_0^2 - \omega^2 (1 + \delta^2))^2 + 4\omega_0^2 \omega^2 \delta^2};$$

$$K' = \frac{\omega_0 \omega m (\omega_0^2 - \omega^2 (1 + \delta^2))}{(\omega_0^2 - \omega^2 (1 + \delta^2))^2 + 4\omega_0^2 \omega^2 \delta^2};$$

$$K'' = \frac{2\omega^2 \omega_0 \omega m \delta}{(\omega_0^2 - \omega^2 (1 + \delta^2))^2 + 4\omega_0^2 \omega^2 \delta^2}. \quad (27)$$

2.3 The Linewidth

At a certain frequency $\omega$, the imaginary part of the component $\chi_{xx}$ ($\chi''$) has its maximum value when the Larmor frequency becomes equal to this certain frequency $\omega_0 = \omega$. Therefore, by considering (27), we can find that

$$\omega_0 = \omega \longrightarrow \chi''_{\text{max}} \approx \frac{\omega_m}{2\omega \delta}. \quad (28)$$

Now, we can assume that the Larmor frequency is changed, such that $\chi''$ gains a value that is half of its value at the resonance. Hence,

$$\chi'' = \frac{\omega_0 \omega m \delta (\omega_0^2 + \omega^2 (1 + \delta^2))}{(\omega_0^2 - \omega^2 (1 + \delta^2))^2 + 4\omega_0^2 \omega^2 \delta^2} = \frac{1}{2} \left( \frac{\omega_m}{2\omega \delta} \right). \quad (29)$$
The solution to equation (29) is
\[ \omega'_0 \approx \omega \pm \omega \delta. \] (30)

As it is seen, \( \chi'' \) decreases to half its peak at two values for the Larmor frequency. The corresponding values for the static field, and the difference between them can be written as
\[ H'_0 \approx \frac{\omega \pm \omega \delta}{\gamma \mu_0} \rightarrow \Delta H = \frac{2 \omega \delta}{\gamma \mu_0}. \] (31)

\( \Delta H \) is called the full linewidth of the ferrite material. As (31) shows, the damping constant is related to the linewidth.

### 2.4 Demagnetization Factors

When a ferrite medium sample is finite in size, the external applied magnetic field (DC+AC) is not equal to the field inside the ferrite. In this case, the field known as the demagnetizing field \( \mathbf{H}_{\text{dem}} \) is introduced to account for this effect as [31]
\[ \mathbf{H}_{\text{internal}} = \mathbf{H}_{\text{external}} + \mathbf{H}_{\text{dem}}. \] (32)

Here \( \mathbf{H}_{\text{internal}} \) and \( \mathbf{H}_{\text{external}} \) are the total internal and external fields, respectively. In Cartesian components,
\[ \mathbf{H}_{\text{internal}} = H_0 z_0 + H_x x_0 + H_y y_0; \]
\[ \mathbf{H}_{\text{external}} = H_a z_0 + H_x x_0 + H_y y_0. \] (33)

In (33) it is assumed that the DC parts \( H_0 \) for the internal field, and \( H_a \) for the external field) are along the \( z_0 \)-axis. The demagnetizing field is expressed as
\[ \mathbf{H}_{\text{dem}} = - (N_x M_x x_0 + N_y M_y y_0 + N_z M_z z_0), \] (34)
in which \( N_x, N_y \) and \( N_z \) are called the demagnetization factors such that
\[ N_x + N_y + N_z = 1. \] (35)

Each shape has its own demagnetization factors. For a ferrite sphere which we use in this work, \( N_x = N_y = N_z = \frac{1}{3} \) due to the spherical symmetry. Hence, for a sphere, equation (32) can be rewritten as
\[ H_{xi} = H_{xe} - \frac{1}{3} M_x, \]
\[ H_{yi} = H_{ye} - \frac{1}{3} M_y, \] (36)
\[ H_0 = H_a - \frac{1}{3} M_s. \]
Equation (22) indicates the relation between the magnetization vector and the internal RF field as

\[
M_x = \chi_{xx} H_{xi} + \chi_{xy} H_{yi},
\]
\[
M_y = \chi_{yx} H_{xi} + \chi_{yy} H_{yi}.
\]

(37)

Substituting (36) into (37) gives

\[
\left(1 + \frac{1}{3} \chi_{xx}\right) M_x + \frac{1}{3} \chi_{xy} M_y = \chi_{xx} H_{xe} + \chi_{xy} H_{ye},
\]
\[
\frac{1}{3} \chi_{yx} M_x + \left(1 + \frac{1}{3} \chi_{yy}\right) M_y = \chi_{yx} H_{xe} + \chi_{yy} H_{ye}.
\]

(38)

If we define the coefficients \(\alpha\) and \(\beta\) as

\[
\alpha = 1 + \frac{1}{3} \chi_{xx}, \quad \beta = \frac{1}{3} \chi_{xy},
\]

(39)

the solution to equation (38) for \(M_x\) and \(M_y\) reads

\[
M_x = \left(\frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy}\right) H_{xe} + \left(\frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} - \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx}\right) H_{ye},
\]
\[
M_y = \left(-\frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx}\right) H_{xe} + \left(\frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy}\right) H_{ye}.
\]

(40)

As it is seen, (40) relates the magnetization to the external RF field for a ferrite sphere. One important issue is the resonance frequency which can be achieved if the denominator of (40) becomes zero, in other words, \(\alpha^2 + \beta^2 = 0\). Hence,

\[
\left(1 + \frac{1}{3} \chi_{xx}\right)^2 + \left(\frac{1}{3} \chi_{xy}\right)^2 = 0.
\]

(41)

By assuming that the ferrite material is lossless (\(\delta = 0\), and applying (24), equation (41) reduces to

\[
1 + \frac{2\omega_0 \omega_m}{3 (\omega_0^2 - \omega^2)} + \frac{\omega_0^2 \omega_m^2}{9 (\omega_0^2 - \omega^2)^2} - \frac{\omega^2 \omega_m^2}{9 (\omega_0^2 - \omega^2)^2} = 0,
\]

(42)

which can be simplified as

\[
9 \omega^4 - \left(18 \omega_0^2 + 6 \omega_0 \omega_m + \omega_m^2\right) \omega^2 + \left(9 \omega_0^4 + 6 \omega_0^3 \omega_m + \omega_0^2 \omega_m^2\right) = 0.
\]

(43)

Solving the equation (43) for \(\omega\) gives the resonance frequency \(\omega_r\) as

\[
\omega_r = \omega_0 + \frac{1}{3} \omega_m.
\]

(44)
where
\[ \omega_0 = \gamma \mu_0 H_0 = \gamma \mu_0 \left( H_a - \frac{1}{3} M_s \right) = \gamma \mu_0 H_a - \frac{1}{3} \omega_m. \]  \hspace{1cm} (45)

Substituting (45) into (44) finally results
\[ \omega_r = \gamma \mu_0 H_a. \] \hspace{1cm} (46)

Therefore, for knowing the resonance frequency of a magnetized ferrite sphere, it is enough to know only the value of the external bias field.
3 Tellegen-Omega Particle

This section studies meticulously the particle called Tellegen-omega particle. This is one of those two particles which have been introduced as artificial nonreciprocal bi-anisotropic particles. It is clear from the name of the particle that simultaneously two electro-magnetic couplings: Tellegen and omega exist. Geometry of the particle is shown in Figure 3. As it is seen, two metal wires and a ferrite inclusion constitute the Tellegen-omega particle.

The polarizability dyadics of Tellegen-omega particle are expressed in 3.1. Subsequently, the polarizabilities are analytically derived in the second part 3.2. To confirm the theoretical results, we extract the polarizabilities from numerical results and the comparison between the theoretical and simulated results is shown in 3.3.

![Figure 3: Geometry of Tellegen-omega particle. The bias magnetic field is directed along z_0-axis.](image)

3.1 Polarizability Dyadics

The general polarizability relation for a bi-anisotropic particle whose electric and magnetic moments are induced by both electric and magnetic fields \( \mathbf{E} \) and \( \mathbf{H} \) can be written as [35]

\[
\begin{bmatrix}
\mathbf{p} \\
\mathbf{m}
\end{bmatrix} = \begin{bmatrix}
\overline{\alpha}_{ee} & \overline{\alpha}_{em} \\
\overline{\alpha}_{me} & \overline{\alpha}_{mm}
\end{bmatrix} \cdot \begin{bmatrix}
\mathbf{E} \\
\mathbf{H}
\end{bmatrix},
\]

where \( \mathbf{p} \) and \( \mathbf{m} \) are the induced electric and magnetic dipole moments, respectively, and \( \overline{\alpha}_{ij} \) are the polarizability dyadics. The studied Tellegen-omega particle is uni-
axial. Therefore, we write the polarizability dyadics of the particle in the form

\[ \overline{\alpha_{ee}} = \alpha_{ee}^{co} \overline{I}_t + \alpha_{ee}^{cr} \overline{J}_t, \]
\[ \overline{\alpha_{mm}} = \alpha_{mm}^{co} \overline{I}_t + \alpha_{mm}^{cr} \overline{J}_t, \]
\[ \overline{\alpha_{em}} = \alpha_{em}^{co} \overline{I}_t + \alpha_{em}^{cr} \overline{J}_t, \]
\[ \overline{\alpha_{me}} = \alpha_{me}^{co} \overline{I}_t + \alpha_{me}^{cr} \overline{J}_t. \]

(48)

Here, \( \overline{I}_t = \overline{I} - z_0 z_0 \) is the transverse unit dyadic, \( \overline{I} \) is the 3D unit dyadic, and \( \overline{J}_t = z_0 \times \overline{I}_t \) is the vector-product operator.

### 3.2 Analytical Polarizabilities

We usually derive the polarizabilities of a particle in two steps. The first step is achieving the electric and magneto-electric polarizabilities. To do this, we consider an electromagnetic plane wave such that only the incident electric field can excite the particle and the incident magnetic field is not able to induce any moment. For the present geometry of the Tellegen-omega particle shown in Figure 3, the incident electric field should be parallel to the \( x_0 y_0 \)-plane and the magnetic field must be in the \( z_0 \)-direction.

The magnetic and electro-magnetic polarizabilities are derived in the second step in which only the incident magnetic field excites the particle. Therefore, the incident magnetic field is assumed to be in the \( x_0 \)- or \( y_0 \)-direction. In this case the incident electric field should be parallel to the bias magnetic field which is directed along the \( z_0 \)-axis.

To get the analytical polarizabilities of the Tellegen-omega particle, we combine the theory of electrically small wire antennas and the concepts mentioned in Section 2 about the ferrite materials.

#### 3.2.1 Electric and magneto-electric polarizabilities

We assume that the Tellegen-omega particle is excited by an electromagnetic plane wave with the \( x_0 \)-directed electric field. This external field induces electric current on the metal wire which is in the direction of the electric field. Assuming that the wire length is much smaller than the wavelength \( l \ll \lambda \), the current distribution on the wire is approximated as [36]

\[ I_x = I_{0x} \cos(kx) - \cos(kl) \approx I_{0x} \left( 1 - \frac{x^2}{l^2} \right), \]

(49)

where

\[ I_{0x} = \frac{2 \tan \left( \frac{kl}{2} \right)}{k Z_{in}} E \approx \frac{l}{Z_{in}} E. \]

(50)

\( E \) is the peak value of the incident electric field, \( k \) is the free-space wavenumber,
and $Z_{in}$ represents the input impedance of a linear electric dipole antenna. For such an antenna, the input admittance can be expressed as [37]

$$Y_{in} = 2\pi j \frac{k l}{\eta \Psi} \left[ 1 + k^2 l^2 \frac{F}{3} - j k^3 l^3 \frac{1}{3(\Omega - 3)} \right],$$

$$F = 1 + \frac{1.08}{\Omega - 3},$$

$$\Omega = 2 \log \frac{2l}{r_0},$$

$$\Psi = 2 \log \frac{l}{r_0} - 2,$$

(51)

in which $\eta$ is the free-space wave impedance, $l$ is half of the length of the metal wire, and $r_0$ represents the wire radius.

The induced electric current on the wire generates magnetic field. By applying the Biot-Savart law in the magneto-static approximation [38–40], the magnetic field close to the wire can be written as

$$H = \int_{-l}^{+l} \frac{I_x dl \times r'}{r'^3} =$$

$$\frac{I_{0x} (l^2 - x^2)}{4\pi R l^2} \left( \frac{l + x}{\sqrt{R^2 + (l + x)^2}} + \frac{l - x}{\sqrt{R^2 + (l - x)^2}} \right) \left( -\sin \phi y_0 + \cos \phi z_0 \right).$$

(52)

As shown in Figure 4, $r'$ is the distance vector from a differential element to the observation point A. $R$ and $\phi$ are the cylindrical coordinates in the $y_0z_0$-plane.

The $y_0$-component of the magnetic field, generated by the induced current on the wire, excites the ferrite sphere. As it is seen from equation (52), this component is not uniform over the ferrite sphere volume. Hence, it is necessary to take its volume average to find the equivalent uniform external magnetic field exciting the ferrite sphere. Therefore,

$$H_{y-\text{average}} = \frac{1}{V} \int_V H_y dv =$$

$$\frac{I_{0x}}{4\pi R l^2 V} \int_0^{2\pi} \int_0^\pi \int_0^{-2a \sin \theta \sin \phi} f(r, \theta, \phi) dr d\theta d\phi,$$

$$f(r, \theta, \phi) = (l^2 - r^2 \cos^2 \theta) \left( \frac{l + r \cos \theta}{\sqrt{r^2 + l^2 + 2rl \cos \theta}} + \frac{l - r \cos \theta}{\sqrt{r^2 + l^2 - 2rl \cos \theta}} \right) (-r \sin \phi),$$

(53)
Figure 4: The wire along the $x_0$-axis excited by the incident electric field.

where $V$ is the ferrite sphere volume. Although it seems that solving (53) is not easy, by assuming that the ferrite sphere is small, the problem can be simplified and the averaged $y_0$-component of the magnetic field can be written as

$$H_{y\text{-average}} = \frac{2a^2}{3V} I_{0x}. \quad (54)$$

In Section 2, Eq (40), it was shown that the magnetization vector of a ferrite sphere, with the saturation magnetization along the $z_0$-axis, is related to the external RF magnetic field as

$$M_x = \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right) H_{xe} + \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} - \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right) H_{ye},$$
$$M_y = \left( \frac{-\alpha}{\alpha^2 + \beta^2} \chi_{xy} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right) H_{xe} + \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right) H_{ye}. \quad (55)$$

Here

$$\alpha = 1 + \frac{1}{3} \chi_{xx}, \quad \beta = \frac{1}{3} \chi_{xy}, \quad (56)$$

and $H_{xe}, H_{ye}$ are the $x_0$- and $y_0$-components of the external magnetic field. Susceptibility elements are given by (26) and (27) for a lossy ferrite material. The intensity
of magnetization is the magnetic moment per unit volume, in other words

\[ \mathbf{m} = V \mathbf{M}. \]  

Therefore, by applying (55), the magnetic moment can be expressed as

\[
m_x = V \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right) H_{xe} + V \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} - \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right) H_{ye},
\]

\[
m_y = V \left( -\frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right) H_{xe} + V \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right) H_{ye},
\]  

(58)

The averaged \( y_0 \)-component of the magnetic field, produced by the wire along the \( x_0 \)-axis, excites two non-zero components of the magnetic moment. As shown in (58), these components are orthogonal to the bias field. The excited magnetic moment induces electric current on the wires. The \( y_0 \)- and \( x_0 \)-components of the magnetic moment cause the electric current induction on the \( x_0 \)- and \( y_0 \)-directed wires, respectively. The electric currents at the center of the wires, due to the magnetic moment, can be written as

\[
I_{0x} = \frac{1}{Z_{in}} \int_{-l}^{+l} E_x \left( 1 - \frac{|x|}{l} \right) dx = \frac{\xi}{Z_{in}} m_y,
\]

\[
I_{0y} = \frac{1}{Z_{in}} \int_{-l}^{+l} E_y \left( 1 - \frac{|y|}{l} \right) dy = -\frac{\xi}{Z_{in}} m_x,
\]  

(59)

where \( \xi \) is an unknown coefficient. After determining all the polarizabilities, the Onsager-Casimir principle [41–43] will allow us to determine this unknown coefficient. \( E_x \) and \( E_y \) represent the tangential electric fields generated by the magnetic moment components to the wires.

The excited wire along the \( y_0 \)-axis produces \( x_0 \)-directed magnetic field. Similarly to the adjacent wire, the volume average of this component of the magnetic field at the ferrite sphere location can be written as

\[
H_{x-\text{average}} = -\frac{2a^2}{3V} I_{0y}.
\]  

(60)

Defining

\[
C_{xx} = C_{yy} \triangleq \frac{2a^2}{3} \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right),
\]

\[
C_{xy} = -C_{yx} \triangleq \frac{2a^2}{3} \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} - \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right),
\]  

(61)
and considering (54), (58), and (60), the magnetic moment components in terms of the electric currents at the center of the wires can be written as

\[
\begin{align*}
m_x &= C_{xy} I_{0x} - C_{xx} I_{0y}, \\
m_y &= C_{yy} I_{0x} - C_{yx} I_{0y}.
\end{align*}
\]

As a result, it can be stated that there is a cycle such that the electric currents excite the ferrite inclusion, and at the same time the magnetic moment excites the wires. This operation can be modeled by a block diagram. It can illustrate the relations between the currents and the magnetic moment components. Figure 5 shows the corresponding coupling block diagram. Also, the cycle can be modeled by the following equations

\[
\begin{align*}
I_{0x} &= \alpha_1 E + \alpha_2 m_y, \\
m_y &= \alpha_3 I_{0x} + \alpha_4 I_{0y}, \\
I_{0y} &= \alpha_5 m_x, \\
m_x &= \alpha_6 I_{0y} + \alpha_7 I_{0x},
\end{align*}
\]

where

\[
\begin{align*}
\alpha_1 &\approx \frac{l}{Z_{in}}, \quad \alpha_2 = -\alpha_5 = \frac{\xi}{Z_{in}}, \quad \alpha_3 = C_{yy}, \quad \alpha_4 = -C_{yx}, \quad \alpha_6 = -C_{xx}, \quad \alpha_7 = C_{xy}.
\end{align*}
\]

Knowing that the electric dipole moments and the electric currents at the center of the electrically small short-circuit wires are related to each other as [36]

\[
\begin{align*}
p_x &\approx \frac{4l}{j3\omega} I_{0x}, \\
p_y &\approx \frac{4l}{j3\omega} I_{0y},
\end{align*}
\]

and by solving (63), the electric and magneto-electric polarizabilities can be written as

\[
\begin{align*}
\alpha_{ee}^{co} &= \frac{\alpha_1 (1 - \alpha_5 \alpha_6)}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7} \times \frac{4l}{j3\omega}, \\
\alpha_{ee}^{cr} &= \frac{\alpha_1 \alpha_5 \alpha_7}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7} \times \frac{4l}{j3\omega}, \\
\alpha_{me}^{co} &= \frac{\alpha_1 \alpha_7}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}, \\
\alpha_{me}^{cr} &= \frac{\alpha_1 \alpha_3 (1 - \alpha_5 \alpha_6) + \alpha_1 \alpha_4 \alpha_5 \alpha_7}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}.
\end{align*}
\]
It is important to know that the incident electric field can be considered as an uniform external field for the ferrite sphere as a homogeneous dielectric sphere which has the relative permittivity $\varepsilon_r$. Therefore, an electric dipole moment is induced parallel to the incident field. The absolute value of the moment is given (in the quasi-static approximation) by [44]

$$p = 4\pi a^3 \varepsilon_0 \frac{\varepsilon_r - 1}{\varepsilon_r + 2} E,$$

where $\varepsilon_0$ is the permittivity of free space. Hence, there is an extra electric polarizability which should be added to the co-component of the electric polarizability in (66).

### 3.2.2 Magnetic and electro-magnetic polarizabilities

For achieving analytically the magnetic and electro-magnetic polarizabilities, it is required to have an incident electric field parallel to the magnetic bias field, so that
it is not able to excite the wires. The high-frequency incident magnetic field, for example in the \( x_0 \)-direction, with the peak value \( H \), excites the magnetic moment of the ferrite sphere. The excited magnetic moment induces electric current on the metal wires. Similarly to the previous part for deriving electric and magneto-electric polarizabilities, the coupling cycle is formed, because the induced electric currents excite the magnetic moment of the ferrite sphere. The following equations properly explain the cycle as

\[
I_{0x} = \alpha_2 m_y, \\
m_y = \alpha_3 I_{0x} + \alpha_4 I_{0y} + \alpha_9 H, \\
I_{0y} = \alpha_5 m_x, \\
m_x = \alpha_6 I_{0y} + \alpha_7 I_{0x} + \alpha_8 H,
\]

where

\[
\alpha_8 = 2\pi a C_{xx}, \quad \alpha_9 = 2\pi a C_{yx}.
\]

Solving (68) and using (65) give the magnetic and electro-magnetic polarizabilities as

\[
\alpha_{\text{co}}^\text{mm} = \frac{\alpha_8 - \alpha_2 \alpha_3 \alpha_8 + \alpha_2 \alpha_7 \alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}, \\
\alpha_{\text{cr}}^\text{mm} = \frac{\alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}, \\
\alpha_{\text{co}}^\text{em} = \frac{\alpha_2 \alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7} \times \frac{4l}{j \beta \omega}, \\
\alpha_{\text{cr}}^\text{em} = \frac{\alpha_5 \alpha_8 (1 - \alpha_2 \alpha_3) + \alpha_2 \alpha_5 \alpha_7 \alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7} \times \frac{4l}{j \beta \omega}.
\]

The corresponding coupling block diagram is shown in Figure 6.

### 3.2.3 Onsager-Casimir principle

Now, by applying the Onsager-Casimir principle [43]

\[
\bar{\alpha}_{\text{me}} (H_0) = -\bar{\alpha}_{\text{em}}^T (-H_0),
\]

it is possible to obtain the unknown coefficient \( \xi \). \( H_0 \) is the internal bias magnetic field and the superscript \( T \) indicates the transpose operation. By applying (71), and considering equations (66) and (70), after simple algebra the coefficient \( \xi \) can be calculated as

\[
\xi = \frac{-j \beta \omega \mu_0}{8\pi a}.
\]
3.2.4 Analytical results

The particle properties and dimensions are illustrated in Table 1. For having the resonance frequency at 10 GHz, the bias magnetic field should be 3570 Oe. The ferrite material is assumed to be lossy with a damping factor equal to 0.001. The length of each wire is 3 mm which is one tenth of the wavelength. Hence, the current distribution considered in (49) would be a valid model. Figure 7 shows the real and imaginary parts of the analytical polarizabilities of such Tellegen-omega particle.

<table>
<thead>
<tr>
<th>Ferrite Material</th>
<th>Relative permittivity</th>
<th>Saturation Magnetization</th>
<th>Applied Bias Field</th>
<th>Sphere Radius</th>
<th>Wires Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>YIG</td>
<td>15</td>
<td>1780 (gauss)</td>
<td>3570 (Oe)</td>
<td>0.5 (mm)</td>
<td>3 (mm)</td>
</tr>
</tbody>
</table>

Table 1: Tellegen-omega particle, properties and dimensions.
3.3 Simulations

To confirm the analytical results, we have achieved the polarizabilities also numerically. Simulations have been done by the Finite Element Method (FEM), applying Ansoft HFSS software. Simulations can be also done using the Method of Moment (MOM).

Briefly to state the method for obtaining the polarizabilities, we illuminate the object by two plane waves with the opposite propagation directions toward the particle. We extract the scattered fields in the far zone, and by analyzing them we get all the polarizabilities of the particle (see Appendix A). In fact, this method has been firstly proposed by Viktar Asadchy and Ihar Faniayeu both from Gomel State University, Belarus, for canonical chiral particle [45]. Recently, the method was generalized by Younes Ra’di and Viktar Asadchy at Aalto University to calculate polarizabilities for any electrically small particle. The corresponding paper is under preparation.

The analytical results are compared with the numerical results in Figure 8. As it is seen, the resonance frequency is approximately 10 GHz, and the simulated and analytical results are fairly well matched. The simulated electro-magnetic, and magneto-electric polarizabilities are shown in Figure 9. This figure shows that the considered particle possesses both Tellegen ($\alpha_{en}^{co} = \alpha_{me}^{co}$) and omega ($\alpha_{en}^{cr} = \alpha_{me}^{cr}$) characteristics.
Figure 7: Analytical polarizabilities of Tellegen-omega particle.
Figure 8: Comparison of simulated and analytical polarizabilities of Tellegen-omega particle.
Figure 9: Simulated electro-magnetic, and magneto-electric polarizabilities of Tellegen-omega particle.
4 Moving-Chiral Particle

This section investigates in detail the suggested artificial nonreciprocal bi-anisotropic particle which possesses moving and chiral electro-magnetic couplings. The geometry of such particle is illustrated in Figure 10. As it is seen, this particle consists of two metal wires placed on a ferrite inclusion which has nonreciprocal property.

In the first part 4.1, we derive analytically all the possible polarizabilities of the particle, and finally in the second part 4.2 we compare the analytical results with the numerical ones.

![Figure 10: Geometry of moving-chiral particle. The bias magnetic field is directed along the $z_0$-axis.](image)

4.1 Analytical Polarizabilities

To attain the polarizabilities theoretically, we illuminate the object by two electromagnetic plane waves propagating toward the particle. If the electric field is tangential to the plane of the wires (the $x_0y_0$-plane) and the magnetic field is parallel to the applied bias field, we can derive the electric and magneto-electric polarizabilities. This is because in this case only the incident electric field can excite the particle. On the other hand, if the electric field is along the $z_0$-axis and the magnetic field is in the $x_0y_0$-plane, the particle will be excited only by the incident magnetic field and the magnetic and electro-magnetic polarizabilities can be determined.

4.1.1 Electric and magneto-electric polarizabilities

An incident electric field in the $x_0$-direction can excite both metal wires, because the small part of the wire A and the large part of the wire B are parallel to the $x_0$-axis.
Assuming $l' \ll \lambda$, the small part of the wire A and the wire B have approximately uniform current distribution ($I_x$ and $I_y$, respectively). The large parts of the wires are supposed to be still much smaller than the wavelength. Hence, the wires have approximately the following current distributions:

\[ I_A = \begin{cases} 
I_x \left( 1 - \frac{y^2}{l'^2} \right) & \text{for } |y| > 0 \\
I_x & \text{for } |x| < l', 
\end{cases} \]  

(73)

\[ I_B = \begin{cases} 
I_y \left( 1 - \frac{x^2}{l'^2} \right) & \text{for } |x| > 0 \\
I_y & \text{for } |y| < l', 
\end{cases} \]

in which

\[ I_y = \frac{2 \tan \left( \frac{kl'}{2} \right)}{kZ_{in}} \approx \frac{l}{Z_{in}} E, \]  

(74)

\[ I_x = \frac{2l'}{Z_{in}} E. \]

Here, $E$ is the peak value of the incident electric field. $k$ represents the free-space wavenumber, and $Z_{in}$ is the impedance seen from the center of wire A and wire B. The constant currents $I_x$ and $I_y$ become secondary sources to produce magnetic field for exciting the ferrite sphere. The $y_0$-component of the magnetic field generated by $I_x$ and the $x_0$-component of the magnetic field generated by $I_y$ have the most principal role in ferrite sphere excitation. Similar to the theory for Tellegen-omega particle, because of existing non-uniform external magnetic field within the ferrite sphere, calculation of the average of the field over the volume of the sphere should be done. Using the Biot-Savart law for the small part of wire A (where the current distribution is approximately uniform), and taking the average of the field over the volume of the ferrite sphere gives

\[
H_{y\text{-average}} = \frac{1}{V} \int_V H_y \, dv \\
= \frac{I_x}{4\pi V} \int_0^{2\pi} \int_0^{\pi} \int_0^{-2a\sin\theta\sin\phi} f(r, \theta, \phi) \, dr \, d\theta \, d\phi = \frac{F}{V} I_x,
\]

(75)

\[
f(r, \theta, \phi) = \left( \frac{l' + r \cos \theta}{\sqrt{r^2 + l'^2 + 2rl' \cos \theta}} + \frac{l' - r \cos \theta}{\sqrt{r^2 + l'^2 - 2rl' \cos \theta}} \right) (-r \sin \phi).
\]

Calculation of the above integral is not straightforward. The value $F$ can be found numerically by, for instance, applying MATLAB simulator software. Similarly, the averaged $x_0$-component of the external magnetic field due to the constant current $I_y$ can be expressed as

\[
H_{x\text{-average}} = -\frac{F}{V} I_y,
\]

(76)
in which the sign ”−” implies that the produced magnetic field is opposite to the \( \mathbf{x}_0 \)-direction. From Section 3, we remember that equation (58) relates the magnetic moment to the external field as

\[
m_x = V \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right) H_{xe} + V \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} - \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right) H_{ye},
\]

\[
m_y = V \left( -\frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right) H_{xe} + V \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right) H_{ye}.
\]  

(77)

Next, for writing the relations more conveniently, we define the following coefficients

\[
C_{xx} = C_{yy} \triangleq F \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xx} + \frac{\beta}{\alpha^2 + \beta^2} \chi_{xy} \right),
\]

\[
C_{xy} = -C_{yx} \triangleq F \left( \frac{\alpha}{\alpha^2 + \beta^2} \chi_{xy} - \frac{\beta}{\alpha^2 + \beta^2} \chi_{xx} \right).
\]  

(78)

By applying (75), (76), and using (78), equation (77) reduces to

\[
m_x = C_{xy} I_x - C_{xx} I_y,
\]

\[
m_y = C_{yy} I_x - C_{yx} I_y,
\]  

(79)

which actually gives the magnetic moment in terms of the constant currents on the small parts of the wires A and B. It is important to consider that the excited magnetic moment induces electric current on the metal wires, because the \( y_0 \)-component of the magnetic moment produces an external \( \mathbf{x}_0 \)-directed electric field which is tangential to the small part of the wire A, and therefore can excite it. The same happens for the wire B due to the \( x_0 \)-component of the magnetic moment. The electric currents on the small parts of the wires, due to the magnetic moment, can be written as

\[
I_x = \frac{\xi}{Z_{in}} m_y,
\]

\[
I_y = -\frac{\xi}{Z_{in}} m_x,
\]  

(80)

where \( \xi \) is an unknown coefficient. Similarly to what we did for the Tellegen-omega particle, this coefficient will be found from the Onsager-Casimir principle.

Because the constant currents on the small parts of the wires A and B excite the ferrite inclusion, and simultaneously the magnetic moment induces electric current on the wires, a coupling cycle is created, which is illustrated by the block diagram in Figure 11.
Figure 11: Block diagram of coupling between the metal wires and the ferrite inclusion in moving-chiral particle in presence of an incident electric field in the $x_0$-direction.

The coupling cycle can be also expressed mathematically as the following relations:

\[
\begin{align*}
I_x &= \alpha'_8 E + \alpha_2 m_y, \\
m_y &= \alpha_3 I_x + \alpha_4 I_y, \\
I_y &= \alpha_5 m_x + \alpha_1 E, \\
m_x &= \alpha_6 I_y + \alpha_7 I_x,
\end{align*}
\]  (81)

where

\[
\begin{align*}
\alpha_1 &\approx \frac{l}{Z_{in}}, & \alpha_2 &= -\alpha_5 = \frac{\xi}{Z_{in}}, & \alpha_3 &= C_{yy}, \\
\alpha_4 &= -C'_{yx}, & \alpha_6 &= -C'_{xx}, & \alpha_7 &= C'_{xy}, & \alpha'_8 &= \frac{2l'}{Z_{in}}.
\end{align*}
\]  (82)

The electric dipole moments and the constant electric currents on the small parts
of the wires are related to each other as

\[ p_x \approx \frac{4l}{j3\omega} I_y + \frac{2l'}{j\omega} I_x, \]

\[ p_y \approx \frac{2l'}{j\omega} I_y - \frac{4l}{j3\omega} I_x. \]  

(83)

Using (81) and also considering (83), electric and magneto-electric polarizabilities can be written as

\[ \alpha_{co} = \frac{4l}{j3\omega} A_y + \frac{2l'}{j\omega} A_x, \]

\[ \alpha_{cr} = \frac{2l'}{j\omega} A_y - \frac{4l}{j3\omega} A_x, \]

\[ \alpha_{co} = \frac{\alpha_1 \alpha_6 (1 - \alpha_2 \alpha_3) + \alpha_1 \alpha_2 \alpha_4 \alpha_7 + \alpha_7 \alpha_8'}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}, \]

\[ \alpha_{me} = \frac{\alpha_3 \alpha_6' (1 - \alpha_5 \alpha_6) + \alpha_1 \alpha_4 + \alpha_4 \alpha_5 \alpha_7 \alpha_8'}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}. \]  

(84)

in which the coefficients \( A_x \) and \( A_y \) are

\[ A_x = \frac{\alpha_8' (1 - \alpha_5 \alpha_6) + \alpha_1 \alpha_2 \alpha_4}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}, \]

\[ A_y = \frac{\alpha_1 (1 - \alpha_2 \alpha_3) + \alpha_5 \alpha_7 \alpha_8'}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}. \]  

(85)

4.1.2 Magnetic and electro-magnetic polarizabilities

As mentioned in part 4.1, to derive the magnetic and electro-magnetic polarizabilities, an incident magnetic field should be assumed perpendicular to the direction of the applied bias field to be able to excite the ferrite inclusion. Most of the formulas described before can be used, and it is only needed to rewrite the relations between the constant currents and the magnetic moment as

\[ I_x = \alpha_2 m_y, \]

\[ m_y = \alpha_3 I_x + \alpha_4 I_y + \alpha_9 H, \]

\[ I_y = \alpha_5 m_x, \]

\[ m_x = \alpha_6 I_y + \alpha_7 I_x + \alpha_8 H. \]  

(86)
Here $H$ is the peak value of the high-frequency incident magnetic field. If we assume that this incident magnetic field has only an $x_0$-component, then

$$\alpha_8 = \frac{4\pi a^3}{3F} C_{xx}, \quad \alpha_9 = \frac{4\pi a^3}{3F} C_{yx}. \quad (87)$$

By applying (83) and (86), the magnetic and electro-magnetic polarizabilities can be expressed as

$$\alpha_{\text{cm}}^{\text{co}} = \frac{\alpha_8 (1 - \alpha_2 \alpha_3) + \alpha_2 \alpha_7 \alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7},$$

$$\alpha_{\text{cr}}^{\text{co}} = \frac{\alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}; \quad \alpha_{\text{em}}^{\text{co}} = \frac{4l}{j3\omega} B_y + \frac{2l'}{j\omega} B_x,$$

$$\alpha_{\text{em}}^{\text{cr}} = \frac{2l'}{j\omega} B_y - \frac{4l}{j3\omega} B_x,$$

where the coefficients $B_x$ and $B_y$ are given by

$$B_x = \frac{\alpha_2 \alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}, \quad (89)$$

$$B_y = \frac{\alpha_5 \alpha_8 (1 - \alpha_2 \alpha_3) + \alpha_2 \alpha_5 \alpha_7 \alpha_9}{1 - \alpha_2 \alpha_3 - \alpha_5 \alpha_6 + \alpha_2 \alpha_3 \alpha_5 \alpha_6 - \alpha_2 \alpha_4 \alpha_5 \alpha_7}.$$

As it is seen from (86), the coupling block diagram for this case is completely similar to what has been shown in Figure 6 for the Tellegen-omega particle.

### 4.1.3 Onsager-Casimir principle

The coefficients $\alpha_2$ and $\alpha_5$, which show the effect of the magnetic moment on the metal wires, are not known. As it was mentioned before, the Onsager-Casimir principle

$$\bar{\alpha}_{\text{me}} (H_0) = -\bar{\alpha}_{\text{em}}^T (-H_0), \quad (90)$$

gives the opportunity to determine these coefficients. Here $H_0$ denotes the internal bias field of the ferrite inclusion, and the superscript $T$ denotes the transpose operation. Thus by applying (90) and after some algebraic manipulations we can get

$$\alpha_2 = -\alpha_5 = \mu_0 \frac{\alpha_1 \alpha_4 + \alpha_3 \alpha_8'}{\left(\frac{4l}{j3\omega}\right) \alpha_9 - \left(\frac{2l'}{j\omega}\right) \alpha_8}. \quad \left(91\right)$$
4.1.4 Analytical results

We have assumed that the long parts of the wires ($l$) are large compared to $l'$, but small compared to the wavelength. Hence, we need to decrease the resonance frequency used in the example of a Tellegen-omega particle (10 GHz) to be able to increase the value of $l$, such that the wavelength is still larger than the size of particle. Therefore, 2.5 GHz can be a proper choice with the wavelength 120 mm. The ferrite material does not change (the same relative permittivity, the saturation magnetization and damping factor). For having the resonance at 2.5 GHz, the applied bias field is chosen to be 892.5 Oe. We assume $l' = 0.6$ mm, and $l = 6.9$ mm. Figure 12 shows the real and imaginary parts of the analytical electric and the magnetic polarizabilities. Subsequently, the analytical electro-magnetic and the magneto-electric polarizabilities are shown in Figure 13. As it is seen in the figures, the polarizabilities $\alpha_{em}^{co}$ and $\alpha_{me}^{co}$ or the polarizabilities $\alpha_{em}^{cr}$ and $\alpha_{me}^{cr}$ are not exactly opposite to each other in the whole frequency range from 2.45 GHz to 2.55 GHz.

For a pure moving-chiral particle, we should have

\[
\begin{align*}
\alpha_{em}^{co} &= -\alpha_{me}^{co}, \\
\alpha_{em}^{cr} &= -\alpha_{me}^{cr},
\end{align*}
\]

(92)

in which the upper formula indicates the chiral property, and the next one is the condition for having the "moving" property. For the structure of our particle, there are also couplings of the other two types: Tellegen and omega, in addition to the moving and chiral ones. The fundamental reason for existing Tellegen and omega couplings is the central parts of the wires placed on the ferrite inclusion. Therefore mixing all four couplings causes that we can not have a pure moving-chiral particle.
Figure 12: Analytical electric and magnetic polarizabilities of moving-chiral particle.
Figure 13: Analytical electro-magnetic and magneto-electric polarizabilities of moving-chiral particle.
4.2 Simulations

The used method for extracting the numerical polarizabilities is the same as for the Tellegen-omega particle. By analyzing the scattered fields in the far zone, we are able to achieve the all polarizabilities (see Appendix A). The analytical results for the particle described in part 4.1.4 are compared with the simulated ones in Figure 14 and Figure 15. Figure 14 shows the electric and the magnetic polarizabilities, and Figure 15 shows the electro-magnetic and the magneto-electric polarizabilities.

As it is clear in the figures, there is a very small difference in the resonance frequency between the analytical and the simulated results. That can be because of inaccuracy in calculating the applied bias field for having the resonance at 2.5 GHz. However, it is thought that the main source of that shift is in inaccuracies of determining the reactive part of the input impedances of wire elements.
The analytical polarizabilities: $\alpha_{e\text{e}}^{co}$, $\alpha_{ee}^{cr}$, $\alpha_{m\text{m}}^{co}$, and $\alpha_{m\text{m}}^{cr}$ are nicely matched with the simulated results. For electro-magnetic and magneto-electric polarizabilities, although the analytical and simulated results are not well matched, some similarities can be seen between them.

Concentrating only on the simulated results for electro-magnetic and magneto-electric polarizabilites, it is found that the relation to the moving ($\alpha_{e\text{m}}^{cr} = -\alpha_{m\text{e}}^{cr}$) and the chiral ($\alpha_{e\text{m}}^{co} = -\alpha_{m\text{e}}^{co}$) characteristics is not satisfied, and the particle is not purely moving-chiral. In part 4.1.4, we concluded the same thing that the electro-magnetic and magneto-electric polarizabilities are not completely opposite to each other ($\alpha_{e\text{m}}^{cr} \neq -\alpha_{m\text{e}}^{cr}$, and $\alpha_{e\text{m}}^{co} \neq -\alpha_{m\text{e}}^{co}$), but the situation for the numerical case is worse.

We think in addition to this fact that such a particle has the all four features: chiral, Tellegen, omega, and moving couplings simultaneously, the long arms of the wires are very close to the ferrite inclusion and they can create parasitic effects on

Figure 15: Comparison of analytical, and simulated (electro-magnetic and magneto-electric) polarizabilities of the moving-chiral particle with $l = 6.9 \text{ mm}$ and $l' = 0.6 \text{ mm}$.
Figure 16: Comparison of analytical, and simulated (electro-magnetic and magneto-electric) polarizabilities of the moving-chiral particle with $l = 9$ mm and $l' = 1.5$ mm.

The ferrite sphere (due to strong non-uniformity of the fields). In theory we have not taken into account these parasitic effects, and therefore the numerical and the theoretical results have so much difference. Hence, we should decrease in some way the impacts of the long arms of the wires, and definitely one solution is moving them away from the ferrite sphere by increasing the length of the central part of the wires. So we choose $l' = 1.5$ mm. As mentioned before, $l$ is assumed to be much larger than $l'$. To realize this, we suppose that $l$ is six times larger than $l'$, which gives $l = 9$ mm. Because the size of the particle should be much smaller than the wavelength, we replace 2.5 GHz with 2 GHz as the resonance frequency. The corresponding applied bias field will be 714 Oe.

Figure 16 indicates the real and imaginary parts of the electro-magnetic and the magneto-electric polarizabilities obtained analytically and numerically. As it is seen, now there is fairly good agreement between the analytical and simulated results. Still because of inaccuracy, a little difference exists in the resonance frequencies.
Consequently for achieving an approximate moving-chiral particle, it is required to optimize the shape of the primary particle and the length of its wires in order to decrease the parasitic effects and also the Tellegen and omega couplings.

We have simulated many structures, and here we just mention our final design which is relatively close to the pure moving-chiral particle. Compared to the initial particle ($l = 6.9 \text{ mm}$ and $l' = 0.6 \text{ mm}$), we increased the length of the small parts of the wires and instead of using straight long arms at the end of small parts, we applied slant arms. The new particle is shown in Figure 17. Because $l \gg l'$, and the particle size should be smaller than the wavelength, we choose 2 GHz as the resonance frequency. The electro-magnetic and magneto-electric polarizabilities of this particle are shown in Figure 18. As it is seen, at the resonance frequency $\alpha_{\text{em}}^{\text{cr}} \approx -\alpha_{\text{me}}^{\text{cr}}$ and $\alpha_{\text{em}}^{\text{co}} \approx -\alpha_{\text{me}}^{\text{co}}$. Although some Tellegen and omega couplings or small parasitic effects still remain, the particle is fairly purely moving-chiral particle.

Figure 17: Modified moving-chiral particle.
Figure 18: Simulated electro-magnetic and magneto-electric polarizabilities for improved moving-chiral particle.
5 Experimental Work

The goal in this section is to introduce a proper measurement technique to determine the magneto-electric polarizabilities (Tellegen, chiral, omega and moving parameters) of the studied Tellegen-omega and moving-chiral particles. One simple and practical technique which can be used in every radio engineering laboratory is the waveguide measurement method. In this technique, by placing the particle inside a waveguide and measuring the reflection and transmission coefficients ($S_{11}$ and $S_{21}$), we can extract the polarizabilities of the particle.

In the first part 5.1, it is explained how we can apply the waveguide method to measure the co-component of the magneto-electric polarizability (Tellegen or chirality parameter). Subsequently, in the second part 5.2, we state how we can measure the cross component of the magneto-electric polarizability (omega or moving parameter) using the waveguide measurement technique.

5.1 Measuring Tellegen or Chiral Parameter

In Tellegen or chiral coupling, the incident field and the induced dipole moment are in the same direction. Therefore we should measure the reflected or transmitted electric field which is along the same direction as the incident magnetic field. This will be completely impossible in a rectangular waveguide where only the fundamental mode $TE_{10}$ propagates. One solution is using a square cross section waveguide which has two orthogonal degenerate propagating modes $TE_{10}$ and $TE_{01}$.

We assume that the particle is placed at the center of the waveguide cross section and also in the middle of the waveguide, as shown in Figure 19.

Figure 19: Geometry of Tellegen-omega (or similarly moving-chiral) particle inside a square cross section waveguide. The bias magnetic field is directed along the $x_0$-axis.
As it is seen, we excite the square waveguide with a TE$_{10}$ mode which is supposed to have an amplitude equal to $A$. The unit electric and magnetic field vectors for the TE$_{10}$ mode can be written as [40]

\[
\mathbf{e}_{10} = -Z \frac{j \beta_w}{K_{\text{cut-off}}} \sin \frac{\pi x}{b} \mathbf{y}_0,
\]

(93)

\[
\mathbf{h}_{10} = \frac{j \beta_w}{K_{\text{cut-off}}} \sin \frac{\pi x}{b} \mathbf{x}_0 + \cos \frac{\pi x}{b} \mathbf{z}_0.
\]

where $b$ is the height and width of the waveguide, $K_{\text{cut-off}}$ is the cut-off wave number, $\beta_w$ is the phase constant, and $Z$ represents the characteristic impedance for the TE$_{10}$ mode. Hence, we have

\[
K_{\text{cut-off}} = \frac{\pi}{b},
\]

(94)

\[
\beta_w = \left( k^2 - K_{\text{cut-off}}^2 \right)^{0.5},
\]

(94)

\[
Z = \frac{k}{\beta_w} \eta.
\]

As above, $k$ and $\eta$ are the free-space wave number and the intrinsic impedance of free space, respectively.

For the present geometry of the particle (see Figure 19), if we consider equation (93), we will find out that the waveguide excitation by a TE$_{10}$ mode causes that only the wire which is parallel to the applied electric field (Ae$_{10}^+$) is excited and the incident magnetic field cannot excite the particle at all. The applied bias field is directed along the $x_0$-axis. Now, the particle will be polarized and the reflected and transmitted waves are excited for both TE$_{10}$ mode and the cross-polarized degenerated TE$_{01}$ mode. The amplitudes of the reflected (or transmitted) TE$_{10}$ and TE$_{01}$ waves are denoted as $B$ and $C$, respectively. The unit electric and magnetic field vectors for TE$_{01}$ mode are given by [40]

\[
\mathbf{e}_{01} = Z \frac{j \beta_w}{K_{\text{cut-off}}} \sin \frac{\pi x}{b} \mathbf{x}_0,
\]

(95)

\[
\mathbf{h}_{01} = -\frac{j \beta_w}{K_{\text{cut-off}}} \sin \frac{\pi x}{b} \mathbf{y}_0 + \cos \frac{\pi x}{b} \mathbf{z}_0.
\]

The cross-polarized amplitude $C$ can be found as [33]

\[
C = -\frac{j \omega \mathbf{e}_{01} \cdot \mathbf{p}}{P_{01}} + \frac{j \omega \mathbf{h}_{01} \cdot \mathbf{m}}{P_{01}},
\]

(96)

where

\[
P_{01} = 2 \int_S (\mathbf{e}_{01} \times \mathbf{h}_{01}) \cdot dS,
\]

(97)
and \( S \) is the cross sectional area of the waveguide. The normal vector \( \mathbf{S} \) is directed along the \( z_0 \)-axis. For the TE\(_{01} \) mode with the fields given by equation (95), we have

\[
P_{01} = 2 \int_0^b \int_0^b Z \left( \frac{\beta_w}{K_{\text{cut-off}}} \right)^2 \sin^2 \left( \frac{\pi x}{b} \right) \, dx \, dy
\]

\[
= Z \left( \frac{b \beta_w}{K_{\text{cut-off}}} \right)^2.
\]

(98)

The electric and magnetic dipole moments of the particle are induced in both \( y_0 \)- and the \( z_0 \)-directions. In other words,

\[
P = p_y \mathbf{y}_0 + p_z \mathbf{z}_0,
\]

\[
m = m_y \mathbf{y}_0 + m_z \mathbf{z}_0.
\]

(99)

Because the particle is positioned at the center of the waveguide, the \( z_0 \)-component of the unit magnetic field \( \mathbf{h}_{01} \) is zero and only its \( y_0 \)-component is important. Also, because the unit electric field \( \mathbf{e}_{01} \) has only one component (in the \( x_0 \)-direction), the scalar product of the unit electric field and the electric dipole moment is zero. Hence, by using (95) and (99), equation (96) reduces to

\[
C = \frac{\omega \beta_w}{P_{01} K_{\text{cut-off}}} m_y.
\]

(100)

The induced magnetic moment can be written as

\[
m_y = \alpha_{\text{me}}^{yy} E_{\text{local}},
\]

(101)

in which \( E_{\text{local}} \) is called the local field and it is the summation of the incident field and the fundamental mode fields produced by the particle. In other words,

\[
E_{\text{local}} = A e_{10}^+ + B e_{10}^- = -Z \frac{j \beta_w}{K_{\text{cut-off}}} (A + B) \mathbf{y}_0.
\]

(102)

In fact, the higher-order modes are also induced by the particle. They exist near the particle and contribute to the local field. However, because they are reactive fields, their effect is only a small resonance frequency shift in the measured results. Therefore, in (102) we neglect them. By using (101) and (102), and substituting the value of \( P_{01} \), equation (100) can be rewritten as

\[
C = -j \frac{\omega}{b^2} (A + B) \alpha_{\text{me}}^{yy}.
\]

(103)

Finally, the magneto-electric polarizability can be written as

\[
\alpha_{\text{me}}^{yy} = j \frac{b^2}{\omega (A + B)} = j \frac{b^2}{A} \frac{C}{1 + \frac{B}{A}} = j \frac{b^2}{\omega (1 + \Gamma)}.
\]

(104)
in which \( \Gamma \) and \( \tau \) are the reflection and transmission coefficients, respectively. Therefore, by measuring the scattering parameters \( (S_{11} \text{ and } S_{21}) \) at the input and output ports of the square waveguide, we are able to extract the Tellegen parameter for a Tellegen-omega particle, or the chirality parameter for a moving-chiral particle. If the particle has both the Tellegen and chiral effects, we can separate them by using this fact that the Tellegen parameter changes sign when the bias field is reversed, while the chirality parameter is independent of this.

5.2 Measuring Omega or Moving Parameter

For particles with omega or moving couplings the situation is different as compared to Tellegen or chiral couplings. The direction of the dipole moment has a 90-degree difference with the incident field direction. Therefore it appears that using a proper rectangular waveguide which operates at the fundamental mode TE\(_{10}\), we can measure the cross component of the magneto-electric polarizability.

Suppose that the particle is positioned inside and at the center of a rectangular waveguide which possesses the width \( b \) and the height \( d \) as indicated in Figure 20. The amplitudes of the incident field and the reflected (and transmitted) wave are denoted as \( A \) and \( B \), respectively.

![Figure 20: Geometry of Tellegen-omega (or similarly moving-chiral) particle inside a rectangular waveguide. The bias magnetic field is directed along the \(-z_0\)-axis.](image)

To remember, the unit field vectors can be expressed as

\[
\begin{align*}
\mathbf{e}_{10} &= -Z \frac{j \beta_w}{K_{\text{cut-off}}} \sin \frac{\pi x}{b} \mathbf{y}_0, \\
\mathbf{h}_{10} &= \frac{j \beta_w}{K_{\text{cut-off}}} \sin \frac{\pi x}{b} \mathbf{x}_0 + \cos \frac{\pi x}{b} \mathbf{z}_0.
\end{align*}
\]
As it is clear, both the incident electric field (in the $y_0$-direction) and the incident magnetic field (in the $x_0$-direction) can excite the particle. The amplitude of the reflected wave can be calculated as

$$B = -\frac{j\omega e_{10} \cdot p}{P_{10}} + \frac{j\omega h_{10} \cdot m}{P_{10}},$$

(106)

where the normalization constant $P_{10}$ is

$$P_{10} = 2 \int_S (e_{10} \times h_{10}) \cdot dS = Z \left( \frac{\beta_w}{K_{\text{cut-off}}} \right)^2 bd.$$  

(107)

Taking into account that the electric and magnetic dipole moments have two components, in the $x_0$- and $y_0$-directions, by applying equation (105) we can rewrite the amplitude $B$ as

$$B = -Z \frac{\omega \beta_w}{P_{10}K_{\text{cut-off}}} p_y + \frac{\omega \beta_w}{P_{10}K_{\text{cut-off}}} m_x.$$  

(108)

The relation between the induced moments and the local fields is given by

$$p_y = \alpha_{ee}^{yy} E_{\text{local}} + \alpha_{em}^{yx} H_{\text{local}},$$

$$m_x = \alpha_{me}^{xy} E_{\text{local}} + \alpha_{mm}^{xx} H_{\text{local}},$$

(109)

in which the local fields are

$$E_{\text{local}} = A e_{10}^+ + B e_{10}^- = -Z \frac{j\beta_w}{K_{\text{cut-off}}} (A + B) y_0,$$

$$H_{\text{local}} = A h_{10}^+ + B h_{10}^- = \frac{j\beta_w}{K_{\text{cut-off}}} (A - B) x_0.$$  

(110)

From the last three equations written above, we can prove that

$$\Gamma = \frac{j\omega}{P_{10}} \left( \frac{Z \beta_w}{K_{\text{cut-off}}} \right)^2 (1 + \Gamma) \alpha_{ee}^{yy} - \frac{j\omega}{P_{10}} \left( \frac{\beta_w}{K_{\text{cut-off}}} \right)^2 (1 - \Gamma) \alpha_{mm}^{xx},$$

$$= \frac{j\omega}{P_{10}} Z \left( \frac{\beta_w}{K_{\text{cut-off}}} \right)^2 (-\alpha_{me}^{xy} - \Gamma \alpha_{me}^{xy} - \alpha_{em}^{yx} + \Gamma \alpha_{em}^{yx}),$$

(111)

where the reflection coefficient $\Gamma$ is the ratio between the amplitudes $B$ and $A$ ($\Gamma = B/A$). By substituting the value of $P_{10}$, we can simplify equation (111) as

$$G = \frac{j\omega}{bd} \left[ -\alpha_{me}^{xy} - \alpha_{em}^{yx} - \Gamma (\alpha_{me}^{xy} - \alpha_{em}^{yx}) \right],$$

(112)
where
\[ G = \Gamma - j \frac{Z\omega}{bd} \left( 1 + \Gamma \right) \alpha_{\text{ee}}^{xy} - j \frac{\omega}{Zbd} \left( 1 - \Gamma \right) \alpha_{\text{mm}}^{xx}. \] (113)

We can consider two cases. For a pure Tellegen-omega particle, we can measure the cross component of the magneto-electric polarizability (the omega parameter) as
\[ \alpha_{\text{me}}^{xy} = \frac{jGb \omega}{2\omega \Gamma}, \] (114)
and for a pure moving-chiral particle, we can measure the moving parameter as
\[ \alpha_{\text{me}}^{xy} = \frac{jGb \omega}{2\omega}. \] (115)

If the particle inside the waveguide has both the omega and moving couplings, we can determine the omega and moving parameters by doing another measurement such that the direction of the bias magnetic field is inverted (here \(+z_0\)-axis). Firstly, we assume that the direction of the bias field is still the same and it has not changed. We can decompose the magneto-electric and electro-magnetic polarizabilities \(\alpha_{\text{me}}^{xy}\) and \(\alpha_{\text{em}}^{xy}\) as
\[ \alpha_{\text{me}}^{xy} = \alpha_{\text{me}}^{xy}(\text{moving}) + \Gamma \alpha_{\text{me}}^{xy}(\text{omega}), \] (116)
\[ \alpha_{\text{em}}^{xy} = \alpha_{\text{em}}^{xy}(\text{moving}) + \Gamma \alpha_{\text{em}}^{xy}(\text{omega}), \]
where
\[ \alpha_{\text{me}}^{xy}(\text{moving}) = \alpha_{\text{em}}^{xy}(\text{moving}), \]
\[ \alpha_{\text{me}}^{xy}(\text{omega}) = -\alpha_{\text{em}}^{xy}(\text{omega}). \] (117)

Therefore, by substituting (116) into (112), and by considering (117), we can rewrite equation (112) as
\[ G = \frac{-2j\omega}{bd} \left[ \alpha_{\text{me}}^{xy}(\text{moving}) + \Gamma \alpha_{\text{me}}^{xy}(\text{omega}) \right]. \] (118)

Now, at the second measurement, we invert the direction of the bias field. If the direction of the bias field is inverted, the reciprocal omega parameter does not change while the nonreciprocal moving parameter changes sign. Thus, equation (118) can be expressed as
\[ G' = \frac{-2j\omega}{bd} \left[ -\alpha_{\text{me}}^{xy}(\text{moving}) + \Gamma \alpha_{\text{me}}^{xy}(\text{omega}) \right]. \] (119)

Therefore, we have two equations ((118) and (119)) and two unknowns. We can simply calculate the omega and moving parameters as
\[ \alpha_{\text{me}}^{xy}(\text{moving}) = \frac{jbd (G' - G\Gamma)}{2\omega (\Gamma + \Gamma')}, \]
\[ \alpha_{\text{me}}^{xy}(\text{omega}) = \frac{jbd (G + G')}{2\omega (\Gamma + \Gamma')}. \] (120)
As it is seen from the above equations, the coefficients $G$ and $G'$ depend on the co-components of the electric and magnetic polarizabilities. Therefore, it is required to firstly find a way to measure these two polarizabilities.

5.2.1 Measuring the co-component of the electric polarizability

If we position the particle inside the rectangular waveguide so that the incident magnetic field cannot excite the particle, and the incident electric field can induce the electric current on the metal wires, we are able to measure the electric polarizability $\alpha_{ee}^{co}$. The geometry of the particle to fulfill this goal is shown in Figure 21. The particle is at the center of the waveguide.

![Figure 21: Geometry of Tellegen-omega (or similarly moving-chiral) particle inside a rectangular waveguide. The bias magnetic field is directed along the $x_0$-axis.](image)

As before, the amplitude of the reflected wave can be calculated as

$$B = -\frac{j\omega e_{10}}{P_{10}} \cdot \mathbf{p} + \frac{j\omega h_{10}}{P_{10}} \cdot \mathbf{m} = -Z \frac{\omega \beta_w}{P_{10} K_{cut-off}} p_y. \quad (121)$$

Because the unit magnetic field $\mathbf{h}_{10}$ and the magnetic moment $\mathbf{m}$ are orthogonal to each other, their inner product will be zero. Also, only the $y_0$-component of the electric dipole moment is significant because of the electric field direction. We know that

$$p_y = \alpha_{ee}^{yy} E_{local} = -Z \frac{j\beta_w}{K_{cut-off}} (A + B) \alpha_{ee}^{yy}. \quad (122)$$
Hence, by applying (107) and after some algebraic manipulations, we can get the co-component of the electric polarizability as

$$\alpha_{ce}^{yy} = -j\frac{bd}{A} \frac{B}{\omega} \frac{1}{1 + \frac{B}{A}} = -j\frac{bd}{Z\omega} \frac{\Gamma}{1 + \Gamma}.$$  (123)

### 5.2.2 Measuring the co-component of the magnetic polarizability

To measure the polarizability $\alpha_{mm}^{co}$, the particle inside the rectangular waveguide must not be excited by the incident electric field. To realize this, the particle should be positioned at the center of the waveguide as shown in Figure 22. As it is clear, electric field of the fundamental mode cannot excite the metal wires. We assume that the electric dipole moment induced by the incident electric field in the ferrite sphere (which can be considered as a dielectric sphere) is very small.

The reflected wave amplitude $B$ is given by

$$B = -j\frac{\omega e_{10}}{P_{10}} \cdot p + j\frac{\omega h_{10} \cdot m}{P_{10}} = \frac{\omega \beta_w}{P_{10} K_{cut-off}} m_x.$$  (124)

The unit electric field and the induced electric dipole moment are perpendicular to each other, so their scalar product equals zero. Also the unit magnetic field is in the $x_0$-direction, therefore only the $x_0$-component of the magnetic moment is important. This component is equal to

$$m_x = \alpha_{mm}^{xx} H_{local} = j\frac{\beta_w}{K_{cut-off}} (A - B) \alpha_{mm}^{xx}.$$  (125)
By using the last two equations and substituting the value of $P_{10}$, we can find the co-component of the magnetic polarizability as

$$\alpha_{mm}^{xx} = -j \frac{Zbd}{\omega} \frac{B}{1 + B} = -j \frac{Zbd}{\omega} \frac{\Gamma}{1 - \Gamma}. \quad (126)$$
6 Conclusion and Future Work

We applied the antenna theory concepts (for electrically small short-circuit wire antennas) and also the knowledge about electromagnetic properties of ferrite materials to derive analytically the electric, magnetic, electro-magnetic and magneto-electric polarizabilities of two nonreciprocal bi-anisotropic particles, named Tellegen-omega, and moving-chiral. By illuminating with an electromagnetic plane wave such that only the incident electric field can excite the particle, we could get the electric and magneto-electric polarizabilities. For deriving the magnetic and electro-magnetic polarizabilities, we illuminated by a plane wave such that only the incident magnetic field interacts with the particle.

Subsequently, we compared the analytical results with the simulated ones. For the Tellegen-omega particle, the analytical polarizabilities showed very good agreement with the numerical polarizabilities. For the moving-chiral particle, initially the analytical results did not agree well with the simulated results. The reason was assumed to be in parasitic effects which had not been taken into account in deriving the theoretical polarizabilities. However, after optimization of the particle, we observed that the analytical and simulated results were almost well-matched. Comparison of the electro-magnetic and magneto-electric polarizabilities for the particle expected to be purely moving-chiral, indicated that the particle had also some Tellegen and omega couplings. Hence, we optimized the particle to decrease the Tellegen and omega effects, and we could achieve approximately a pure moving-chiral particle.

It is needed to confirm the analytical and numerical polarizabilities also experimentally. In Section 5, by applying the waveguide measurement technique and placing the particle inside the waveguide in different orientations, we introduced proper measurement setups to allow us to extract the co- and cross components of the magneto-electric polarizability (Tellegen, chiral, omega, and moving parameters). For measuring the Tellegen or chirality parameter, we used a square cross section waveguide. On the other hand, a rectangular waveguide was used to get the omega or moving parameter.

Due to the shortage of time and also the lack of facilities (such as a permanent magnet for providing the bias magnetic field), we could not do the measurements and this will be the next step to be done in the future. Also, optimizing and improving the performance of the suggested moving-chiral particle still requires research work. In the future, it is necessary to find a pure moving particle.

Finally, the plan is to use these nonreciprocal particles in microwave devices, such as nonreciprocal perfect absorbers or thin-sheet isolators.
References


Appendix A: Formulas for Extracting the Polarizabilities from Numerical Data

This method has been firstly proposed by Viktar Asadchy and Ihar Faniayeu both from Gomel State University, Belarus, for canonical chiral particle [45]. Recently, the method was generalized by Younes Ra’di and Viktar Asadchy at Aalto University to calculate polarizabilities for any electrically small particle. The corresponding paper is under preparation.

We assume an incident plane wave which propagates in the \(+z_0\)-direction, as shown in Figure A1(a). We extract the scattered field \(E_{\text{scat}}\) at two points (1 and 2) in the far zone. Also, we consider another incident plane wave such that its propagation is in the opposite direction of the propagation of the first incident wave, as shown in Figure A1(b). In this case, the scattered field at the points 1 and 2 is denoted as \(E_{\text{scat}}^*\). In Cartesian components,

\[
E_{\text{scat}} = E_{x-\text{scat}}x_0 + E_{y-\text{scat}}y_0 + E_{z-\text{scat}}z_0, \tag{A1}
\]

\[
E_{\text{scat}}^* = E_{x-\text{scat}}^*x_0 + E_{y-\text{scat}}^*y_0 + E_{z-\text{scat}}^*z_0.
\]

The particle is positioned at the origin of the coordinate system, and the bias magnetic field is in the \(z_0\)-direction.

Far away from the particle as an electrically small scatterer, the scattered (radiated) fields are found to be [38]

\[
rE_{\text{scat}} = \frac{k^2}{4\pi\varepsilon_0} \left[ (n \times p) \times n - \frac{n \times m}{\mu_0 c} \right],
\]

\[
rE_{\text{scat}}^* = \frac{k^2}{4\pi\varepsilon_0} \left[ (n \times p^*) \times n - \frac{n \times m^*}{\mu_0 c} \right], \tag{A2}
\]

in which \(k\) is the wavenumber of free space, \(c\) is the speed of light, and \(\varepsilon_0\) and \(\mu_0\) are the permittivity and permeability of free space, respectively. Also, \(n\) is the unit vector in the direction of observation, and \(p\) (\(p^*\)) and \(m\) (\(m^*\)) represent the electric and magnetic moments of the particle, respectively.

At point 1 (see Figure A1), the unit vector \(n = z_0\). If we simplify equation (A2), the components of the scattered fields at point 1 can be written as

\[
rE_{x-\text{scat}1} = \frac{k^2}{4\pi\varepsilon_0} \left( p_x + \frac{m_y}{\mu_0 c} \right), \quad rE_{y-\text{scat}1} = \frac{k^2}{4\pi\varepsilon_0} \left( p_y - \frac{m_x}{\mu_0 c} \right),
\]

\[
rE_{x-\text{scat}1}^* = \frac{k^2}{4\pi\varepsilon_0} \left( p_x^* + \frac{m_y^*}{\mu_0 c} \right), \quad rE_{y-\text{scat}1}^* = \frac{k^2}{4\pi\varepsilon_0} \left( p_y^* - \frac{m_x^*}{\mu_0 c} \right). \tag{A3}
\]
Figure A1: Geometry of Tellegen-omega (or similarly moving-chiral) particle in presence of the incident field. The bias magnetic field is directed along the $z_0$-axis.

On the other hand, for point 2, the unit vector $\mathbf{n} = -\mathbf{z_0}$. Hence, the components of the scattered fields can be derived as

$$
\begin{align*}
  rE_{x - \text{scat}2} &= \frac{k^2}{4\pi\varepsilon_0} \left( p_x - \frac{m_y}{\mu_0 c} \right), \\
  rE_{y - \text{scat}2} &= \frac{k^2}{4\pi\varepsilon_0} \left( p_y + \frac{m_x}{\mu_0 c} \right), \\
  rE_{x* - \text{scat}2} &= \frac{k^2}{4\pi\varepsilon_0} \left( p_x^* - \frac{m_y^*}{\mu_0 c} \right), \\
  rE_{y* - \text{scat}2} &= \frac{k^2}{4\pi\varepsilon_0} \left( p_y^* + \frac{m_x^*}{\mu_0 c} \right).
\end{align*}
$$

(A4)

We can express the components of the electric and magnetic moments with respect to the incident field. As seen in Figure A1(a), when the propagation of the
incident wave is in the \(+z_0\)-direction, the electric and magnetic fields are in the \(+x_0\)- and \(+y_0\)-directions, respectively. Therefore,

\[
\begin{align*}
    p_x &= \alpha_{ee}^{xx} E_{\text{inc}} + \alpha_{em}^{yy} H_{\text{inc}}, \\
    p_y &= \alpha_{ee}^{yx} E_{\text{inc}} + \alpha_{em}^{yx} H_{\text{inc}}, \\
    m_x &= \alpha_{me}^{xx} E_{\text{inc}} + \alpha_{mm}^{xy} H_{\text{inc}}, \\
    m_y &= \alpha_{me}^{yx} E_{\text{inc}} + \alpha_{mm}^{yy} H_{\text{inc}},
\end{align*}
\]

(A5)

where \(E_{\text{inc}}\) is fixed to be 1 volt per meter in the simulator, and we know that \(H_{\text{inc}} = (1/\eta)E_{\text{inc}}\). \(\eta\) is the wave impedance of free space. For the incident wave which propagates in the \(-z_0\)-direction, the magnetic field changes direction to the opposite, as shown in Figure A1(b). Therefore,

\[
\begin{align*}
    p_x^* &= \alpha_{ee}^{xx} E_{\text{inc}} - \alpha_{em}^{yx} H_{\text{inc}}, \\
    p_y^* &= \alpha_{ee}^{yx} E_{\text{inc}} - \alpha_{em}^{yy} H_{\text{inc}}, \\
    m_x^* &= \alpha_{me}^{xx} E_{\text{inc}} - \alpha_{mm}^{xy} H_{\text{inc}}, \\
    m_y^* &= \alpha_{me}^{yx} E_{\text{inc}} - \alpha_{mm}^{yy} H_{\text{inc}}.
\end{align*}
\]

(A6)

Equations (A5) and (A6) result in

\[
\begin{align*}
    p_x + p_x^* &= 2\alpha_{ee}^{xx} E_{\text{inc}}, & p_x - p_x^* &= 2\alpha_{em}^{yx} H_{\text{inc}}, \\
    p_y + p_y^* &= 2\alpha_{ee}^{yx} E_{\text{inc}}, & p_y - p_y^* &= 2\alpha_{em}^{yy} H_{\text{inc}}, \\
    m_x + m_x^* &= 2\alpha_{me}^{xx} E_{\text{inc}}, & m_x - m_x^* &= 2\alpha_{mm}^{xy} H_{\text{inc}}, \\
    m_y + m_y^* &= 2\alpha_{me}^{yx} E_{\text{inc}}, & m_y - m_y^* &= 2\alpha_{mm}^{yy} H_{\text{inc}}.
\end{align*}
\]

(A7)

By using (A7), and by applying (A3) and (A4), the polarizabilities of the particle can be derived as

\[
\begin{align*}
    \alpha_{ee}^{co} &= -\frac{\varepsilon_0 C^2}{4\pi f^2} \left[ r E_{x-\text{sact}} + r E_{x-\text{sact2}} + r E_{x-\text{sact1}}^* + r E_{x-\text{sact2}}^* \right], \\
    \alpha_{ee}^{cr} &= -\frac{\varepsilon_0 C^2}{4\pi f^2} \left[ r E_{y-\text{sact}} - r E_{y-\text{sact2}} + r E_{y-\text{sact1}}^* - r E_{y-\text{sact2}}^* \right], \\
    \alpha_{me}^{co} &= \frac{C}{4\pi f^2} \left[ r E_{y-\text{sact1}} - r E_{y-\text{sact2}} + r E_{y-\text{sact1}}^* - r E_{y-\text{sact2}}^* \right], \\
    \alpha_{me}^{cr} &= \frac{C}{4\pi f^2} \left[ r E_{x-\text{sact1}} - r E_{x-\text{sact2}} + r E_{x-\text{sact1}}^* - r E_{x-\text{sact2}}^* \right].
\end{align*}
\]

(A8) (A9) (A10) (A11)
\begin{align}
\alpha_{em}^{co} &= -\frac{c}{4\pi f^2} \left[ rE_{y-sact1} + rE_{y-sact2} - rE_{y-sact1}^* - rE_{y-sact2}^* \right], \quad (A12) \\
\alpha_{em}^{cr} &= -\frac{c}{4\pi f^2} \left[ rE_{x-sact1} + rE_{x-sact2} - rE_{x-sact1}^* - rE_{x-sact2}^* \right], \quad (A13) \\
\alpha_{mm}^{co} &= -\frac{\mu_0 c^2}{4\pi f^2} \left[ rE_{x-sact1} - rE_{x-sact2} - rE_{x-sact1}^* + rE_{x-sact2}^* \right], \quad (A14) \\
\alpha_{mm}^{cr} &= +\frac{\mu_0 c^2}{4\pi f^2} \left[ rE_{y-sact1} - rE_{y-sact2} - rE_{y-sact1}^* + rE_{y-sact2}^* \right], \quad (A15) \\
\end{align}

where \( f \) denotes the frequency.