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Performance Analysis of Relay Site Planning Over Composite Fading/Shadowing Channels With Cochannel Interference

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Abstract—The performance of relay deployments depends significantly on the capacity of the wireless relay link between a relay node (RN) and its serving base station (BS). Exploiting the deployment flexibility of RNs, relay site planning (RSP) can be utilized to overcome the limitations of the relay link. In particular, RSP is carried out by selecting RN deployment locations from a discrete set of alternatives considering the signal-to-interference-plus-noise ratio (SINR) on the relay link as the selection criterion. In this paper, we present an analytical framework for RSP that can be used for planning and dimensioning of two-hop cellular relay networks operating over composite fading/shadowing channels in the presence of cochannel interference. Nakagami-lognormal distribution is used to model the relay link, whereas the access link between a mobile terminal (MT) and its serving RN is modeled by Rician-lognormal distribution. As these composite models do not have closed-form distribution functions, we utilize mixture gamma (MG) distribution to accurately approximate them. Further, the total cochannel interference in the considered multicellular system is approximated using the moment-generating function (MGF) matching method. Accordingly, we present closed-form expressions for the distributions of relay-link SINR, link rates, and end-to-end rate. In addition, RSP is shown to effectively decrease the amount of fading (AoF) and, thus, mitigate the detrimental effects of composite fading/shadowing. Thorough results reveal significant performance improvements, which justify the use of RSP in cellular relay networks.

Index Terms—Cochannel interference, composite fading/shadowing, fourth-generation (4G) wireless networks, location selection, network planning, relay deployments.

I. INTRODUCTION

Relaying is considered an integral part of fourth-generation (4G) radio access networks, namely IEEE 802.16m and Third Generation Partnership Project (3GPP) Long-Term Evolution (LTE) Release 10 and beyond (LTE-Advanced). These two technologies fulfill the requirements of International Mobile Telecommunications Advanced (IMT-Advanced) specified by the International Telecommunication Union-Radiocommunication sector (ITU-R) for 4G networks [1] and, thus, have recently been accorded the official designation of IMT-Advanced.

The motive behind choosing relaying as an enhancement technology to current radio access networks has been well elaborated in the literature. According to previous technical studies, relay nodes (RNs) promise to increase the network capacity or, alternatively, extend the cell coverage area [2]–[8]. Relaying is also regarded as a cost-efficient technology. Installing RNs involves lower operational expenditure [9] and provides faster network upgrade when operators aim to improve the quality of service [10]. The cost efficiency of RNs is further investigated in [11] and [12].

A. Motivation for the Work

The performance of inband RNs was investigated in [6] from both coverage extension and system capacity perspectives. Inband RNs utilize the same frequency band on both the access link between an RN and a mobile terminal (MT) and the relay link between a base station (BS) and an RN and, hence, operate in half-duplex mode to avoid self-interference. Therein, the limitations of the relay link were highlighted, and it was shown that there is a potential for significant gain if these limitations are relaxed. One approach to address such a problem is characterized by relay site planning (RSP) techniques. In this context, the deployment flexibility of RNs can be exploited to enhance the system performance by means of improving the relay link through RSP. We emphasize that the discussion on RSP modeling has also been carried out in standardization [13], [14]. Consequently, a certain planning bonus has been added to the relay-link channel model in 3GPP evaluation guidelines [15].

Performance evaluation of RSP within the LTE-Advanced context was first given in [16]. The study investigated the effect of RSP on the relay-link signal-to-interference-plus-noise ratio (SINR) via system-level simulations considering shadowing only. Performing RSP, random deployment of RNs is eluded, and an RN site is chosen from a set of different possible locations to optimize the relay-link quality. Motivated by this study, a basic analytical model for RSP was deduced in [17].
where a single dominant interfering BS was considered. In addition to the simplified SINR model, the channel model in [17] considers lognormal shadowing on the relay link and Rayleigh fading on the access link. Nevertheless, composite fading/shadowing, where multipath fading is superimposed on shadowing, is frequently experienced particularly in scenarios with low or no mobility [18], [19]. In addition, given the full-frequency reuse in 4G cellular networks, cochannel interference is another vital factor to be taken into account for accurate performance analysis. However, difficulties arise when considering cochannel interference as interfering signals from different BSs are also subject to composite fading/shadowing and are correlated due to shadowing.

B. Contributions

In this paper, we build upon the concepts presented in [16] and [17] to deduce a comprehensive analytical model for the impact of RSP on the performance of relay deployments, which explains and justifies its gains on the relay link and on the end-to-end user rate. The main contributions of this paper are then summarized as follows.

• We deduce closed-form expressions for the SINR distribution of the relay link, the rate distributions of the relay and access links, and the end-to-end rate distributions. We also consider the impact of resource allocation and access-link limitations. Results show a clear gain on the relay link, which results in a higher achieved end-to-end rate when the system is limited by the relay link. The gains, however, reduce when the channel quality of the access link between the RN and the user deteriorates.

• A key performance measure in communications over fading channels is the amount of fading (AoF), which reflects the severity of the fading [19]. In this paper, we show how RSP can effectively decrease AoF on the relay link under various channel conditions in relay deployments.

Before deducing closed-form expressions, we carry out an extensive study on suitable composite fading/shadowing models. Contributions from this investigation are outlined as follows.

• We model the SNR on the relay and access links by Nakagami–lognormal and Rician–lognormal composite distributions, respectively, which are the two common models in the literature [18]–[21]. As these composite distributions do not have closed-form expressions, we utilize mixture gamma (MG) distribution [22] to accurately approximate them. Specifically, we adopt the Nakagami–lognormal model provided in [22] and derive the Rician–lognormal model in terms of MG distribution, and demonstrate their accuracy. We note that, in [22], it is shown that MG distribution can approximate Nakagami–lognormal more accurately than other existing models such as generalized-$K$ and $G$ distributions, without involving special functions. In addition, a key advantage of MG distribution is that it provides a unified framework, e.g., its cumulative distribution function (cdf), moment-generating function (MGF), and moments are readily available once the probability density function (pdf) is derived.

• The total cochannel interference on the relay link is characterized by a sum of Rayleigh–lognormal (also known as Suzuki) random variables (RVs) since RNs are not expected to have line-of-sight (LOS) links toward interfering BSs. Nevertheless, an exact closed-form expression for the sum of multiple Suzuki RVs is not available. Therefore, we adopt the MGF-matching method proposed in [23], where the sum of independent Suzuki and/or lognormal RVs is accurately represented by a single lognormal RV. Yet, in cellular networks, shadowing toward different BSs is typically correlated [24]. Thus, we also resolve this issue regarding the constraint on the independence of interferers. Moreover, the MGF-matching method requires an iterative solution of a system of nonlinear equations, where a properly selected initial value is essential. If the initial value is not feasible, iterations admit only slow convergence or become divergent [25]. In this paper, we tackle this challenge by using the so-called Wilkinson preconditioning, which we first proposed in [26]. In addition, we give guidelines on how to relax problems that may arise in the numerical solution when experienced low signal levels can necessitate accuracy that is too high in the computational environment.

• The SINR derivation becomes cumbersome when the effect of thermal noise is taken into account and the mean received power levels from different interferers are not equal (see [27] and [28] and references therein). The effect of the thermal noise, however, becomes significant in noise-limited scenarios such as suburb environments. Further, interferers imposed by different BSs may have different mean received power levels. Herein, we also derive generic SINR distributions covering such cases.

The remainder of this paper is organized as follows. Section II presents the channel models. In Section III, the modeling of RSP and adopted assumptions are summarized. In Section IV, the impact of RSP is analyzed along with SINR, AoF, and rate derivations. Performance results and evaluations are provided and discussed in Section V. Finally, Section VI concludes this paper.

II. Channel Models

Shadowing is usually modeled by a lognormal distribution with standard deviation $\sigma$ and mean $\mu$. The standard deviation $\sigma$ defines the severity of shadowing, where $\sigma = 0$ implies that shadowing is absent. The parameters of lognormal distribution are often given in decibels, and the mappings $\sigma = \lambda \sigma_{\text{dB}}$ and $\mu = \mu_{\text{dB}}$ with $\lambda = \ln(10)/10$ can be utilized for the conversion. In addition, the small-scale multipath fading is often characterized by Nakagami distribution with the fading parameter $(0.5 \leq n_{\text{CL}} \leq \infty)$ on a communication link (abbreviated by CL in this notation), Rician, or Rayleigh distribution. The fading parameter of Nakagami distribution inversely reflects the severity of the multipath fading, i.e., as $n_{\text{CL}} \to \infty$, the fading effect diminishes yielding a nonfading channel. Furthermore, Nakagami distribution yields Rayleigh distribution
when \( m_{\text{CL}} = 1 \) and can be used to approximate the Rician distribution when \( m_{\text{CL}} > 1 \) [18]. However, depending on the channel parameters, such an approximation can be inaccurate, particularly on the tails of cdf [18, Sec. 1.2.3], [29], which are important, e.g., for the analysis of outage probability.

The channel models to be derived pertain to a two-hop half-duplex decode-and-forward relay deployment where end-to-end performance is also degraded by interference on the relay link. Fig. 1 presents an exemplified schematic of the relay deployment, where an MT is connected to RN A on the access link. In the illustration, two neighboring BSs B and C interfere with the serving BS transmission on the relay link.

In the following, we first outline MG distribution and model the composite SNR distributions on the relay and access links in terms of MG distribution. The instantaneous SNR and the average SNR are denoted by \( \gamma \) and \( \bar{\gamma} \), respectively.

### A. MG Distribution

The pdf of the instantaneous SNR is approximated by MG distribution consisting of \( N \) Gamma components as [22]

\[
f_{\gamma}(x) = \sum_{i=1}^{N} w_i g_i(x) = \sum_{i=1}^{N} \alpha_i x^{\beta_i - 1} e^{-\zeta_i x}, \quad x \geq 0
\]

where \( w_i = \alpha_i \Gamma(\beta_i) \zeta_i^{-\beta_i} \) with \( \Gamma(\cdot) \) being the gamma function, \( g_i(x) = \zeta_i^{\beta_i} x^{\beta_i - 1} e^{-\zeta_i x} / \Gamma(\beta_i) \) is a standard Gamma distribution, and \( \alpha_i, \beta_i, \) and \( \zeta_i \) are the parameters of the \( i \)th Gamma component. Furthermore, \( \alpha_i = \theta_i / C \), where \( C = \sum_{i=1}^{N} \theta_i \Gamma(\beta_i) \zeta_i^{-\beta_i} \) is a normalization factor to ensure that \( \sum_{i=1}^{N} w_i = 1 \) as \( \int_0^\infty f_{\gamma}(x) = 1 \). Accordingly, \( \theta_i \) is also a parameter of the \( i \)th Gamma component. The number of components \( N \) determines the accuracy of the approximation and can be obtained by matching the first \( r \) moments of the approximation and the target distribution [22]. That is, we select \( N \) such that the approximated distribution and the exact distribution have the same nearest integer values for the first \( r = 3 \) moments.

Next, the cdf of the approximation is given as

\[
F_{\gamma}(x) = \sum_{i=1}^{N} \alpha_i e^{-\beta_i \gamma_i} e^{-\zeta_i x} \tag{2}
\]

where \( \gamma(a, b) \triangleq \int_0^b t^{a-1} e^{-t} dt \) is the lower incomplete gamma function [30, eq. (8.350.1)], which is available in mathematical software packages such as Mathematica and Matlab.

In addition, the \( r \)th moment of MG distribution of the instantaneous SNR is given as

\[
E(\gamma^r) = \sum_{i=1}^{N} \alpha_i \Gamma(\beta_i + r) \zeta_i^{-(\beta_i+r)} \tag{3}
\]

where \( E(\cdot) \) denotes the statistical expectation. The AoF can be then calculated from the first and the second moments of the SNR as [19]

\[
\text{AoF} = \frac{\text{var}(\gamma)}{[E(\gamma)]^2} = \frac{E(\gamma^2) - [E(\gamma)]^2}{[E(\gamma)]^2} = \frac{E(\gamma^2)}{[E(\gamma)]^2} - 1 \tag{4}
\]

The key advantage of the MG distribution is that, once the parameters of the \( i \)th gamma component, i.e., \( \theta_i, \beta_i, \) and \( \zeta_i \), are determined, the performance metrics are readily available [22] or can be easily derived.

### B. SNR Distribution on the Relay Link

The instantaneous SNR on the relay link is modeled by a gamma–lognormal distribution with pdf [19], [22], i.e.,

\[
f_{\gamma}(x) = \int_0^\infty \frac{x^{m_{\text{RL}}-1} e^{-\frac{m_{\text{RL}} x}{\bar{\gamma}}}}{\Gamma(m_{\text{RL}})} \frac{m_{\text{RL}} e^{-\frac{\ln x - \mu}{\sigma x}}}{\sqrt{2\pi}\sigma x} dy \tag{6}
\]

where \( m_{\text{RL}} \) is the fading parameter of Nakagami distribution on the relay link (abbreviated by RL in this notation). The approximation for the gamma–lognormal distribution in terms of MG distribution is derived using a series expansion based on Gauss–Hermite integration [22]. Then, the parameters of the \( i \)th gamma component are expressed as

\[
\theta_i = \left( \frac{m_{\text{RL}}}{\bar{\gamma}} \right)^{m_{RL}} w_i e^{-m_{\RL} \sqrt{2\pi} \sigma_{t_i} + \mu} \frac{\sqrt{\pi} \Gamma(m_{\text{RL}})}{\sqrt{\pi} \Gamma(\beta_i)} \tag{7}
\]

where \( t_i \) and \( w_i \) are the abscissas and weight factors of the \( N \)th order Hermite integration, respectively, and are tabulated for \( N \) up to 20 in [32, Tab. 25.10].

\(^2\)The Matlab implementation of this function, which is gammainc [31], deviates from the definition given here and thus should be modified as \( \Gamma(a) \) gammainc \((b, a)\).

\(^3\)The abscissas and weight factors can be also generated via various online tools such as in [33] for \( N \) up to 100.
The AoF for the SNR distribution on the relay link can be then easily obtained through (3), (4), and (7). A simplified expression of the AoF follows as [34]:

\[
\text{AoF} = \frac{\sqrt{\pi} (m_{RL} + 1)}{m_{RL}} \frac{\sum_{i=1}^{N} w_i e^{2/\zeta_{i}}}{\left(\sum_{i=1}^{N} w_i e^{2/\zeta_{i}}\right)^2} - 1. \tag{8}
\]

For certain ranges of parameters, namely \(m_{RL} = 1\) and \(\sigma_{DB} > 6\ dB\), or \(m_{RL} > 2\) and any \(\sigma_{DB}\), the Nakagami–lognormal distribution can be also approximated by a lognormal distribution [18]. The mean and the standard deviation of the approximating lognormal distribution are given by

\[
\begin{align*}
\mu_{DB}^{\text{approx}} &= \lambda^{-1} [\psi(m_{RL}) - \ln(m_{RL})] + \mu_{DB} \\
\sigma_{DB}^{\text{approx}} &= \sqrt{\lambda^{-2} \zeta(2, m_{RL}) + \sigma_{DB}^{2}} \tag{9}
\end{align*}
\]

where \(\psi(\cdot)\) and \(\zeta(\cdot, \cdot)\) are the Euler’s psi function and Riemann’s zeta function, respectively [30].

In Fig. 2, the cdfs for the exact distribution, MG, and lognormal approximations are plotted on a lognormal probability paper to compare the accuracy of the approximations. Note that, on the lognormal probability paper, the cdf of the lognormal distribution is a straight line. Two sets of channel parameters are considered. For the first set, i.e., \(m_{RL} = 1\) and \(\sigma_{DB} = 8\ dB\) (Rayleigh–lognormal), it can be seen that the MG distribution accurately follows the exact distribution; however, the approximating lognormal distribution deviates from the exact distribution, particularly on the tails of the cdf. For the second set of parameters, i.e., \(m_{RL} = 5.76\) and \(\sigma_{DB} = 6\ dB\), it is observed that both approximations yield high accuracy. The reason is that, for \(m_{RL} = 5.76\), the effect of shadowing is dominating, whereas the multipath fading is not severe; hence, the exact distribution can be well characterized by a lognormal distribution. For a comparison of the accuracy of the MG model with other approximations, see [22].

C. SNR Distribution on the Access Link

A relay cell is typically characterized by small coverage area due to lower transmit power levels relative to the BSs [6], [15]. Accordingly, we assume that a direct LOS component along with many weak non-LOS (NLOS) scatter components exist on the propagation paths between an RN and an MT on the access link. Furthermore, the LOS component may be partially or completely blocked by surrounding objects, e.g., trees and buildings, which implies random shadowing [19]. Therefore, we model the access link by Rician–lognormal distribution.

Let \(\chi = S^2\) be the power envelope of the shadowed Rician fading channel. The pdf of \(\chi\) is given by [19, eq. 2.67]

\[
f_{\chi}(x) = \left(\frac{2b_0 m_{AL}}{2b_0 m_{AL} + \Omega}\right)^{m_{AL}} \frac{1}{2b_0} e^{-x/2b_0} \times \text{\_1\_F}_1\left(m_{AL}; 1; \frac{\Omega}{2b_0 (m_{AL} + \Omega)} x\right) \tag{10}\]

where \(\Omega\) is the average power of the LOS component, \(2b_0\) is the average power of the scatter component, and \(0 \leq m_{AL} \leq \infty\) describes the severity of shadowing on the access link (abbreviated by AL in this notation). For \(m_{AL} = 0\), the LOS component is completely obstructed (Rayleigh distribution), and for \(m_{AL} = \infty\), the LOS component is not obstructed (Rician distribution). Furthermore, \(\text{\_1\_F}_1(\cdot; \cdot; \cdot)\) is the confluent hypergeometric function [30, Sec. 9.2]. After applying the change of variable \(\gamma = A \chi\) with \(A = \sqrt{\pi/2b_0}\), the pdf of the composite SNR distribution attains the following form:

\[
f_{\gamma}(x) = \frac{1}{A} \left(\frac{2b_0 m_{AL}}{2b_0 m_{AL} + \Omega}\right)^{m_{AL}} \frac{1}{2b_0} e^{-x/A2b_0} \times \text{\_1\_F}_1\left(m_{AL}; 1; \frac{\Omega}{2b_0 (m_{AL} + \Omega)} x\right) \tag{11}\]

Using the series expansion of \(\text{\_1\_F}_1(\cdot; \cdot; \cdot)\) given by [30, eq. 9.210.1]

\[
\text{\_1\_F}_1(a; b; z) = \sum_{i=0}^{\infty} \frac{(a)_i}{(b)_i} \frac{z^i}{i!} = \sum_{i=1}^{\infty} \frac{(a)_{i-1}}{(b)_{i-1}} \frac{z^{i-1}}{(i - 1)!} \tag{12}\]

where \((\cdot)_i\) is the Pochhammer symbol, and substituting \((1)_1 = \Gamma(i)\), we can express (11) as

\[
f_{\gamma}(x) = \frac{1}{A} \left(\frac{2b_0 m_{AL}}{2b_0 m_{AL} + \Omega}\right)^{m_{AL}} \frac{1}{2b_0} e^{-x/A2b_0} \times \sum_{i=1}^{\infty} \frac{(m_{AL})_{i-1}}{\Gamma(i)} \left(\frac{\Omega}{2b_0 (m_{AL} + \Omega)}\right)^{i-1} \frac{x^{i-1}}{(i - 1)!} \tag{13}\]

Via a finite number of summands (cf. \(N\) in Section II-A) in (13), the pdf can be accurately approximated. Accordingly, we can approximate \(f_{\gamma}(x)\) in (13) using the MG distribution in (1). Substituting the Rician \(K\) factor \(K \equiv \Omega/2b_0\) and utilizing the
The AoF of the SNR distribution on the access link can be deduced using (3), (4), and (14). After some algebraic manipulations, we obtain the simplified AoF expression as follows:

$$\text{AoF} = \frac{1}{C} \left( \frac{m_{\text{AL}} + K}{m_{\text{AL}} + K} \right)^{m_{\text{AL}} - 1} K \times \sum_{i=1}^{N} \left( \frac{(m_{\text{AL}} - 1) i (i+2)}{(i+1)^2} \right)^{i} \left( \frac{K (1 + K)}{m_{\text{AL}} + K} \right)^{i} - 1 \quad (15)$$

where $C$ is the normalization factor, as discussed in Section II-A. It is worth noting that the AoF expressions both on relay link (8) and access link (15) are independent of the mean SNR $\bar{\gamma}$.

The derivation of the cdf from the pdf of the original model given in (10) requires complicated algebraic manipulations. Recently, in [21], a highly accurate closed-form expression for the power envelope $\chi$ has been deduced for any positive real fading parameter, herein $m_{\text{AL}}$. Again, considering the transformation $\chi \triangleq A\chi$, the cdf of the composite SNR distribution is obtained as

$$F_{\gamma}(x) = \frac{(1 + K) m_{\text{AL}} x}{(m_{\text{AL}} + K)^{m_{\text{AL}}}} \times \Phi_{2} \left( 1 - m_{\text{AL}}, m_{\text{AL}}, 2, -\frac{(1 + K) m_{\text{AL}} x}{\gamma} \right)$$

(16)

where $\Phi_{2}(\cdot,\cdot,\cdot,\cdot)$ is the bivariate confluent hypergeometric function of type 2 given by [30, eq. 29.6.2]\(^3\)

$$\Phi_{2}(\beta, \beta', \gamma, x, y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\beta)_{m}(\beta')_{n}}{(\gamma)_{m+n} m_{\text{AL}}} x^{m} y^{n}. \quad (17)$$

As it is seen through (16) and (17), the cdf expression includes a double infinite sum that can be approximated by a finite number of $m^{\ast} \times n^{\ast}$ summands. Moreover, the accuracy of this approximation is not proportional to the number of summands, and the number of summands should be carefully determined depending on the channel parameters to prevent numerical instabilities leading to $F_{\gamma}(x) > 1$. For further details regarding multivariate confluent hypergeometric series, see [35] and [36].

The cdfs of the approximations and the actual distribution are shown on a lognormal probability paper in Fig. 3, where

Fig. 3. CDFs of SNR on the access link for the exact distribution, MG, and $\Phi_{2}$- and $L_{\nu}^{\mp} (\cdot)$-based approximations. The average SNR is $\bar{\gamma} = 10$ dB, and the number of components in the MG model is $N = 3$ for frequent heavy shadowing and is $N = 19$ for infrequent light shadowing model, whereas $\Phi_{2}$-based approximation requires $m^{\ast} \times n^{\ast} = 32^2$ summands. The plots of the approximations and the exact distributions overlap.

The plots of approximations and actual distributions overlap. Two sets of channel parameters corresponding to different shadowing environments are considered, namely, frequent heavy shadowing implying a dense tree cover and infrequent light shadowing implying a sparse tree cover. These models, originally introduced in [37], are widely used in the literature [20], [38] and the references therein. Moreover, the channel parameters of the first and second environments are given as $m_{\text{AL}} = 0.739$ and $K = 0.007$ ($\sigma_{\text{DB}} = 3.5$ dB and $\mu_{\text{DB}} = -16.998$ dB), as well as $m_{\text{AL}} = 19.4$ and $K = 4.08$ ($\sigma_{\text{DB}} = 0.499$ dB and $\mu_{\text{DB}} = 0.499$ dB), respectively. We note that the mean and the standard deviation of the approximated lognormal shadowing can be obtained using the mappings provided in [20, eqs. 10, 11]. We further note that, due to numerical instability when using $\Phi_{2}$-based cdf expression (16) in infrequent light shadowing model, we utilize an approximate of $\Phi_{2}$ [21, eq. 4] based on Laguerre polynomials ($L_{\nu}^{\mp} (\cdot)$) [30, Sec. 8.97]. The cdf is then easily obtained by applying the transformation $\gamma \triangleq A\chi$ in [21, eq. 4]. Since $L_{\nu}^{\mp} (\cdot)$-based cdf expression is only valid for natural number $m_{\text{AL}}$, we approximate the channel model by setting $m_{\text{AL}} = 19$.

### III. System Model

#### A. Relay Site Planning Model

Cell planning and site selection tools are routinely used by operators to improve the system performance and to provide a satisfactory service with minimal deployment expenditure. In this context, the deployment flexibility of RNs can be exploited to enhance the system performance through RSP. The deployment flexibility stems in part from the wireless backhaul between RN and BS, RN’s compact physical characteristics, and low power consumption, which allows RNs to be mounted on structures such as lamp posts with power supply facilities, offering ample potential deployment sites. The deployment
flexibility of RNs can be exploited in choosing the location; hence, a random deployment of RNs is avoided.

An RN location is chosen from a set of possible locations. As exemplified in Fig. 1 for three RN candidate locations, RSP takes into account the channel properties at different locations and considers their links’ qualities toward the serving BS to optimize the relay-link quality. In addition, as also illustrated in Fig. 1, it is assumed that a single MT is served by an RN. In particular, we assume that there are $M$ potential locations for RN deployment in cell $k$, out of which we select the best location in terms of downlink SINR. In each location, RN is assumed to be served by a predefined BS solely. Then, the SINR in the selected location is of the following form:

$$\gamma_{m,k} = \max\{\gamma_{m,k} : m = 1, 2, \ldots, M\}$$  \hspace{1cm} (18)

where $\gamma_{m,k}$ is the SINR for the $m$th location in the $k$th cell.

It is worth noting that the location of the RN and the serving BS are to be decided on as part of the network planning phase. Operators usually perform coverage prediction simulations and carry out extensive drive tests, which could be used as input to identify the best location for an RN. More concretely, while using network planning tools, the additional complexity is imposed by the fact that a grid for potential RN locations should be created, and the corresponding losses from these potential locations toward the serving BS should be computed. Indeed, this is not a complex task once potential RN locations are known; however, the main practical complexity is due to the effort of pinpointing these potential locations. Better results can be attained as the number of potential locations increases. Nevertheless, the availability of the site locations is not necessarily always assured; verifying beforehand that all potential locations are available would increase the complexity substantially. To accomplish this task, for example, a list for the best potential locations can be made, and the availability of the site locations can be in turn checked based on the order in this list. We further note that network planning can become cumbersome if a constraint is set, which implies that RNs should cover the whole cell edge. On the other hand, in case of hot-spot coverage, a smaller number of RNs is required, which means less complexity in network planning.

B. Multicellular Network Model

1) Network Layout and RSP Location Trellises: The considered network is represented by a regular hexagonal layout with seven cells, where we seek a suitable location for a single RN in the $k$th cell assuming $M$ potential location candidates. Fig. 4 depicts the network layout along with two distinct RN location trellises that are utilized for RSP.

In Fig. 4, RN location trellis 1 models a practical scenario where the $M = 5$ candidate locations are localized in a target region. Moreover, $d_1$ denotes the distance between the serving BS and the midmost RN location. The outer candidate locations are at a distance of $d_2$ apart from the midmost candidate location (see also Fig. 1 for a similar scenario with $M = 3$ candidate RN locations). On the other hand, for RN location trellis 2, $M = 6$ candidate locations are selected such that each candidate location is at a distance of $d_1$ away from the serving BS, and due to the symmetry of the network layout, the set of distances to neighboring BSs is the same for all candidate locations. In particular, RN location trellis 2 will be utilized when determining AoF bounds in Section IV.

2) Path-Loss Model: Let us denote by $d_{m,k}$ the distance between the $m$th potential relay location and the $k$th BS, where $k = 0, 1, 2, \ldots, K$. Then, related path losses, including the shadowing, are given by

$$L_{m,k} = \alpha d_{m,k}^{\beta} / G$$ \hspace{1cm} (19)

where $\alpha$ and $\beta$ are a propagation constant and the path-loss exponent, respectively, and together, they define the distance-dependent path loss; and $G$ is dimensionless and reflects the impact of antenna gain, which is assumed to be the same for each BS. We note that isotropic antenna gain patterns are employed at BSs. Further, $\zeta_{m,k}$ is a zero-mean Gaussian RV that models the shadowing. Thus, in line with the empirical studies, $10^{\zeta_{m,k}/10}$ follows a lognormal distribution [18], [19]. In the following, we give assumptions that model shadowing in the considered multicellular environment.

(A1) At the $m$th RN location, the shadow fading variables $\zeta_{m,k}$ and $\zeta_{m,j}$ with respect to the $k$th and $j$th BSs are correlated according to the model of [24].

According to [24], RV $\zeta_{m,k}$, which models the log-normal shadowing, can be expressed as a sum of two components $\xi_m$ and $\eta_{m,k}$, where the former corresponds to the near field of the $m$th location and is the same for all BSs and the latter variable is a BS-dependent variable, which is independent from one BS to the other. Hence, we have

$$\zeta_{m,k} = \sqrt{\rho} \cdot \xi_m + \sqrt{1 - \rho^2} \cdot \eta_{m,k}$$ \hspace{1cm} (20)

where $\rho$ is the correlation coefficient related to any pair of BSs. According to [24], it is assumed that

$$E(\eta_{m,k} | \eta_{m,j}) = 0, \quad k \neq j$$

$$E(\zeta_{m,k}) = 0$$

$$\var(\xi_m) = \var(\eta_{m,k}) = \sigma_{dB}^2.$$  \hspace{1cm} (21)
With the given assumptions, we obtain
\[ E(\zeta_{m,k} \zeta_{m,j}) = \rho \sigma_{\text{dB}}^2, E\left((\zeta_{m,k})^2\right) = \sigma_{\text{dB}}^2, E(\zeta_{m,k}) = 0. \]  
(22)

Hence, the Gaussian-distributed shadow fading variables \( \zeta_{m,k} \) are zero mean and correlated. We note that, in 3GPP studies, the shadowing correlation coefficient of \( \rho = 0.5 \) between BSs is usually applied [15].

(A2) Variables \( \zeta_{m,k} \) corresponding to different RN locations are uncorrelated.

In accordance with the Gudmundson model [39], the correlation between shadowing samples at different locations in the \( k \)th cell is given by
\[ \rho(\zeta_{m,k}, \zeta_{n,k}) = e^{-\frac{|d_{m,n}|}{d_{\text{cor}}}} \]  
(23)

where \( \zeta_{m,k} \) and \( \zeta_{n,k} \) are the shadowing variables at locations \( m \) and \( n \), respectively; \( d_{m,n} \) is the distance between the two locations; and \( d_{\text{cor}} \) is the so-called decorrelation distance. The proposed value for \( d_{\text{cor}} \) in, e.g., [40], is 20 m. Then, shadow fading correlation between potential RN positions, e.g., adjacent lamp posts, with a mutual distance of around 50 m is small (\( \rho = 0.18 \)) and can be neglected in the closed-form analysis. Due to the low correlation in shadowing between candidate locations, the correlation between SINR values is also low and can be ignored.

3) SINR Formulation: Based on the aforementioned path-loss model, the received power from the \( k \)th BS at the \( m \)th location is of the following form:
\[ P_{\text{RX},m,k} = \frac{S_{m,k}^2 P_{\text{TX},k}}{L_{m,k}} \]  
(24)

where \( P_{\text{TX},k} \) is the transmission power of the \( k \)th BS, and \( S_{m,k}^2 \) is the power envelope of the multipath fading channel on the link between the \( k \)th BS and the \( m \)th location. Substituting (19) in this equation and reorganizing the terms, we obtain
\[ P_{\text{RX},m,k} = S_{m,k}^2 10^{-\zeta_{m,k}/10} P_{\text{TX},k} G_{\alpha}^{-1} d_{m,k}^{-\beta} \]  
(25)

where the term \( S_{m,k}^2 10^{-\zeta_{m,k}/10} \) defines the composite fading/shadowing channel, which is modeled by Nakagami–lognormal distribution, as discussed in Section II-B, and the rest of the terms contribute to the mean power. Accordingly, (25) can be reformulated by denoting \( X_{m,k} = -\zeta_{m,k} + 10 \log_{10}(P_{\text{TX},k} G_{\alpha}^{-1} d_{m,k}^{-\beta}) \). Then, we have
\[ P_{\text{RX},m,k} = S_{m,k}^2 10^{X_{m,k}/10} . \]  
(26)

Hence, \( X_{m,k} \) is a Gaussian RV with mean \( \mu_{X_{m,k}} \) and standard deviation \( \sigma_{X_{m,k}} \), i.e., \( X_{m,k} \sim N(\mu_{X_{m,k}}, \sigma_{X_{m,k}}^2) \), where
\[ \mu_{X_{m,k}} = 10 \log_{10}(P_{\text{TX},k} G_{\alpha}^{-1} d_{m,k}^{-\beta}) \]  
and \( \sigma_{X_{m,k}}^2 = \sigma_{\text{dB}}^2 \). We note that the assumption of the identical variances is reasonable as the standard deviation of lognormal shadowing is largely independent of the radio path length [18]. Moreover, this assumption is widely used in the literature and standardization bodies, e.g., in [15].

Consequently, assuming that the RN is connected to the \( k \)th BS and all \( K+1 \) BSs are active simultaneously, the SINR at the \( m \)th location is of the following form:
\[ \Upsilon_{m,k} = \frac{S_{m,k}^2 10^{X_{m,k}/10}}{P_N + \sum_{j \neq k} S_{m,j}^2 10^{X_{m,j}/10}} \]  
(27)

where \( P_N \) denotes the thermal noise. Furthermore, it is assumed that the interfering signals \( S_{m,j}^2 10^{X_{m,j}/10}, j \neq k \) are subject to Rayleigh–lognormal (Suzuki) composite fading/shadowing, i.e., \( m_{\text{RL}} = 1 \). It is worth noting that, in this formulation, the desired signal \( S_{m,k}^2 10^{X_{m,k}/10} \) and interfering signals are mutually dependent due to (A1).

IV. ANALYSIS OF RELAY SITE PLANNING

A. Derivation of the Relay-Link SINR

To derive an analytically tractable SINR expression, we need the expression for the thermal noise plus total cochannel interference. However, an exact closed-form expression for the distribution of the sum of multiple lognormal and/or Suzuki RVs is not available. In [23], the sum of a mixture of independent Suzuki and lognormal RVs is accurately approximated by a lognormal RV utilizing MGF-matching method. In this regard, the analytical modeling of the relay-link SINR inherits two main difficulties. The first difficulty arises due to mutual dependence of the desired and interfering signals due to shadowing, and the second difficulty is due to constant thermal noise term \( P_N \). To overcome these difficulties, we reformulate the SINR expression provided in Section III-B3.

Substituting (20) in (27), we obtain
\[ \Upsilon_{m,k} = \frac{S_{m,k}^2 10^{\sqrt{\pi} \mu_{X_{m,k}} + \sqrt{\pi} \sigma_{X_{m,k}}}}{P_N 10^{-\sqrt{\pi} \mu_{X_{m,k}} + \sqrt{\pi} \sigma_{X_{m,k}}} + \sum_{j \neq k} S_{m,j}^2 10^{\sqrt{\pi} \mu_{X_{m,j}} + \sqrt{\pi} \sigma_{X_{m,j}}} / 10} \]  
(28)

where \( \mu_{X_{m,k}} \) and \( \mu_{X_{m,j}} \) are the means of \( X_{m,k} \) and \( X_{m,j} \), respectively, following the discussion on (26). Dividing the numerator and denominator by the common shadowing term \( 10^{\sqrt{\pi} \mu_{X_{m,k}} / 10} \) yields
\[ \Upsilon_{m,k} = \frac{S_{m,k}^2 10^{\sqrt{\pi} \mu_{X_{m,k}} + \mu_{X_{m,j}}}}{P_N 10^{-\sqrt{\pi} \mu_{X_{m,k}} / 10} + \sum_{j \neq k} S_{m,j}^2 10^{\sqrt{\pi} \mu_{X_{m,j}} + \mu_{X_{m,j}}} / 10} \cdot \]  
(29)

This form is particularly beneficial since the RVs in this reformulated SINR expression are mutually independent. Moreover, the newly introduced RV \( P_N 10^{-\sqrt{\pi} \mu_{X_{m,k}} / 10} \) follows a lognormal distribution with mean \( 10 \log_{10}(P_N) \) and standard deviation \( \sqrt{\pi} \cdot \sigma_{X_{m,k}} \). Since the sum in the denominator of (29) consists of a multiple independent Suzuki RVs and a lognormal RV, it can be well approximated by a new lognormal RV as
\[ I = P_N 10^{-\sqrt{\pi} \mu_{X_{m,k}} / 10} + \sum_{j \neq k} S_{m,j}^2 10^{\sqrt{\pi} \mu_{X_{m,j}} + \mu_{X_{m,j}}} / 10 \]  
= \[ I_0 + \sum_{j \neq k} I_{m,j} \approx 10^{0.1Z} \equiv \hat{I} \]  
(30)

where \( Z \sim N(\mu_Z, \sigma_Z^2) \). The aim of the approximation is then to determine \( \mu_Z \) and \( \sigma_Z \), both given in decibels.
1) MGF-Matching Method: An accurate lognormal approximation is obtained by matching the MGF of the sum $I$ with that of the approximating RV $\tilde{I}$ [23]. The MGF-matching method provides parametric flexibility, as compared with well-known Wilkinson method (also known as Fenton–Wilkinson), and it is shown to be more accurate than the Wilkinson method [23]. The benefit of the Wilkinson method [41] is that it provides a simple closed-form solution for the mean and variance of the approximating lognormal RV. Yet, the accuracy of the method in approximating $\mu_Z$ and $\sigma_Z$ degrades when the spread of the mean values decreases or the spread of the standard deviations of the summands increases [42]. It is also reported that this accuracy is inversely proportional to the number of the summands and that the degradation of the accuracy becomes considerable for $\sigma_{\text{dB}} > 4 \text{dB}$ [18]. Therefore, we rather utilize the Wilkinson method as a preconditioning approach to the MGF-matching method. This will be discussed in the following.

Since there is no general closed-form expression available for the lognormal MGF, it is given by a series expansion based on Gauss–Hermite integration as [23]

$$\hat{\Psi}_f(s; \mu_Z, \sigma_Z) = \sum_{i=1}^{N} \frac{w_i}{\sqrt{\pi}} \exp \left(-s \times \exp \left(\frac{\sqrt{2} \sigma_Z t_i + \mu_Z}{\lambda^{-1}}\right)\right), s \in \mathbb{R} \ (31)$$

where $t_i$ and $w_i$ are abscissas and weight factors for the $N$th-order Hermite integration, respectively, as discussed earlier. Furthermore, the MGF of the Suzuki RVs $\Psi_{I_{m,j}}$ can be easily obtained through (5) and (7) by setting $m_{\text{RL}} = 1$. We note that, when using (7) for MGF calculation, $\gamma$ is set to 1, as it is already included in the term $\mu_{X_{m,j}}$. Since lognormal RV $I_0$ and Suzuki RVs $I_{m,j}$, $j \neq k$ in (30) are mutually independent, the resultant MGF of the sum is the product of the MGFs of the summands [43], which is given by

$$\Psi_{\{I_0 + \sum_{j \neq k} I_{m,j}\}}(s) = \hat{\Psi}_{I_0}(s; \mu_{I_0}, \sqrt{\rho} \cdot \sigma_{\text{dB}}) \prod_{j \neq k} \Psi_{I_{m,j}}(s; \mu_{X_{m,j}}, \sqrt{1 - \rho} \cdot \sigma_{\text{dB}}) \ (32)$$

where $\mu_{I_0} = 10 \log_{10}(P_N)$. Consequently, the MGF of $\tilde{I}$ is matched with that of the sum at two different positive values of $s$, i.e., $s_1$ and $s_2$, which yields the system of two independent nonlinear equations as

$$\sum_{i=1}^{N} \frac{w_i}{\sqrt{\pi}} \exp \left(-s_k \times \exp \left(\frac{\sqrt{2} \sigma_Z t_i + \mu_Z}{\lambda^{-1}}\right)\right) = \Psi_{\{I_0 + \sum_{j \neq k} I_{m,j}\}}(s_k) \text{ for } k = 1 \text{ and } 2 \ (33)$$

where the choice of $s_1$ and $s_2$ depends on the portion of the lognormal distribution that needs to be modeled accurately. In particular, larger values for $s_1$ and $s_2$, e.g., $(s_1, s_2) = (0.2, 1)$, yield accurate matching for the head portion of the cdf (small values of the sum power) and smaller values for $s_1$ and $s_2$, e.g., $(s_1, s_2) = (0.0125, 0.0625)$, yield accurate matching for the tail portion of the cdf (large values of the sum power) [23]. The nonlinear equations of (33) should be solved numerically to calculate $\mu_Z$ and $\sigma_Z$.

2) Numerical Solution and Wilkinson Preconditioning: The system of the nonlinear equations given in (33) can be numerically solved, e.g., using Matlab function $\text{fsolve}$ [44] and Mathematica function $\text{FindRoot}$ [45]. Both of the functions require a starting point, i.e., an initial guess to be able to initiate the solution process.

An issue that may arise in the numerical solution of (33) is the calculation of the right side of the equation that includes nested exponentials, as given in (31). Given the thermal noise and mean signal levels typical to mobile cellular systems, such a calculation can require to cope with very small numbers and, hence, high accuracy, which may not be available in the computational environment. That is, for example, in Matlab, to limit the requirement on the accuracy to 15 significant digits, it is necessary to satisfy $\kappa > -36$ in $\exp[-\exp(\kappa)]$ for negative $\kappa$. The issue due to nested exponentials can be easily handled by means of a deterministic scaling parameter $\varepsilon$, which effectively shifts the power levels toward the desired range so that the required accuracy is kept low. We rewrite (30) by multiplying both sides of the approximation by $\varepsilon$ as

$$I^* = \varepsilon \left(I_0 + \sum_{j \neq k} I_{m,j}\right) \approx \varepsilon 10^{0.1Z} = 10^{0.1Z^*} = \tilde{I} \ (34)$$

where $\varepsilon$ can be selected as the median of the power means such that $\varepsilon = \text{median}(E(10 \log_{10}(I_j))); E(10 \log_{10}(I_{m,j})), j \neq k)$. It can be noticed that multiplication by $\varepsilon$ implies only a shift in the mean power. For instance, it immediately follows that $Z^*$ in (34) has a Gaussian distribution with mean $\mu_{Z^*} = \mu_Z + 10 \log_{10}(\varepsilon)$ and standard deviation $\sigma_{Z^*} = \sigma_Z$. Accordingly, $\mu_Z$ and $\sigma_Z$ can be easily obtained. Then, the starting point required for the numerical solution denoted as $(\mu_o, \sigma_o)$ is obtained via Wilkinson preconditioning.

Although the Wilkinson method itself may not provide the required accuracy, it provides an appropriate starting point for the MGF-matching method. In [26], we have shown that, via Wilkinson preconditioning, the convergence of the numerical solution can be guaranteed, and the efficiency in terms of convergence rate and the total number of function evaluations in the numerical computations can be significantly enhanced compared with some conventional approaches studied therein. The following adapted approach is utilized herein. First, each Suzuki RV $\varepsilon I_{m,j}$, $j \neq k$ is approximated by a new lognormal RV $I_{m,j}$, $j \neq k$, using (9) for $m_{\text{RL}} = 1$. Then, the Wilkinson method is applied, where the first two moments of the lognormal sum $I^*$ are matched with those of the approximating lognormal RV $\tilde{I}$. That is, we require

$$E(I^*) = E(\tilde{I}^*) \quad \text{and} \quad E\left[(I^*)^2\right] = E\left[(\tilde{I}^*)^2\right] \ (35)$$

where $E(\tilde{I}^*)$ and $E\left[(\tilde{I}^*)^2\right]$ are evaluated by using the initial assumption that $I^*$ is a lognormal RV. Let $\Xi_l := \ln(\varepsilon I_0), \ l = 1$.

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4In [23], it is stated that $\text{NSolve}$ in Mathematica can be used to solve this system of nonlinear equations. However, $\text{NSolve}$ solves a system of polynomial equations numerically; thus, $\text{FindRoot}$ can indeed be used to solve the systems that involve more complicated functions [46], as in our case.
and $\Xi_l := \ln(\hat{f}_{m,j})$, $j \neq k$, $l = 2, \ldots, K + 1$ with the mean values $\mu_{\Xi_l}$ and standard deviations $\sigma_{\Xi_l}$. Following the derivation detailed in [42] for the original Wilkinson method with the mutual independence assumption, the simple closed-form expressions for the starting point follow as

$$\mu_o = \lambda^{-1} \left(2 \ln(u_1) - \frac{1}{2} \ln(u_2)\right)$$  \hspace{1cm} \text{(36)}$$

$$\sigma_o = \sqrt{\lambda^{-2} \left(2 \ln(u_2) - 2 \ln(u_1)\right)}$$  \hspace{1cm} \text{(37)}$$

where $u_1$ and $u_2$ are given as

$$u_1 = \sum_{l=1}^{K+1} \exp \left(\mu_{\Xi_l} + \frac{1}{2} \sigma_{\Xi_l}^2\right)$$  \hspace{1cm} \text{(38)}$$

$$u_2 = \sum_{l=1}^{K+1} \exp \left(2 \mu_{\Xi_l} + 2 \sigma_{\Xi_l}^2\right)$$

$$+ 2 \sum_{l=1}^{K} \sum_{n=l+1}^{K+1} \exp(\mu_{\Xi_l} + \mu_{\Xi_n}) \exp \left(\frac{1}{2} \left(\sigma_{\Xi_l}^2 + \sigma_{\Xi_n}^2\right)\right).$$  \hspace{1cm} \text{(39)}$$

Consequently, after having determined the approximating log-normal RV $10^{\hat{Y}_{m,k}}$, the approximated SINR can be formulated by following the expression in (29) as

$$\hat{Y}_{m,k} = S_{m,k}^2 10^{\left[\sqrt{1 - \rho} \sigma_{\mu_{X_{m,k}}} - \mu_{X_{m,k}} + \mu_{X_{m,k}}\right]/10} = S_{m,k}^2 10^{\Delta_{m,k}/10}$$  \hspace{1cm} \text{(40)}$$

where $\Delta_{m,k}$ is a Gaussian RV with mean $\mu_{X_{m,k}} - \mu_{X}$ and standard deviation $\sqrt{(1 - \rho) \sigma_{\mu_{X_{m,k}}}^2 + \sigma_{\mu_{X}}^2}$. Accordingly, the SINR distribution on the relay link follows a Nakagami–lognormal composite distribution, which is characterized by (1)–(5), where the parameter expressions are provided by (7) in which $\bar{\gamma}$ is set to $1$. The accuracy of this SINR approximation will be presented in Section V.

3) Impact of RSP on Final SINR Distribution: According to (A2) and considered RN location trellises in Section III-B1, we have $E(\hat{Y}_{m,k} \cdot R_{m,k}, \Delta_{m,k}) = 0$, if $m \neq n$; thus, variables $\{\hat{Y}_{m,k} : m \neq n\}$ are independent. This stems from the model given in (A2) that the correlation between variables $\zeta_{m,k} : m \neq n$ can be ignored because uncorrelated shadowing is assumed among the different candidate RN locations according to [39]. Note here that, in [24], the losses toward two different BSs are assumed to be jointly Gaussian.

Based on the preceding discussion (see Section III-A) when RSP is carried out in the $k$th cell over $M$ candidate locations, the cdf of the SINR attains the following form:

$$F_{\hat{m},k}(\hat{Y}) = \prod_{m=1}^{M} F_{m,k}(\hat{Y})$$  \hspace{1cm} \text{(41)}$$

where $F_{m,k}(\hat{Y})$ is given by (2) following the discussion after (40). The pdf is then obtained by taking the derivative of (41) and reorganizing the terms, which yield

$$f_{\hat{m},k}(\hat{Y}) = \left(\prod_{m=1}^{M} f_{m,k}(\hat{Y})\right) \left(\sum_{m=1}^{M} f_{m,k}(\hat{Y}) \frac{f_{m,k}(\hat{Y})}{F_{m,k}(\hat{Y})}\right)$$  \hspace{1cm} \text{(42)}$$

where $f_{m,k}(\hat{Y})$ is given by (1) following the discussion after (40). We note that these expressions cover the general case, e.g., RN location trellises 1, where the SINR distribution may not be the same at different locations due to different mean received power, i.e., $\mu_{X_{m,k}}$ and $\mu_{X_{m,j}}$. Nevertheless, in the case where SINR distributions are the same at different locations, e.g., RN location trellises 2, these equations reduce to

$$F_{\hat{m},k}(\hat{Y}) = \left[F_{m,k}(\hat{Y})\right]^M$$  \hspace{1cm} \text{(43)}$$

and the pdf is given by

$$f_{\hat{m},k}(\hat{Y}) = M \left[F_{m,k}(\hat{Y})\right]^{M-1} f_{m,k}(\hat{Y}).$$  \hspace{1cm} \text{(44)}$$

B. Impact of RSP on Relay Link AoF

Diversity combining techniques, e.g., maximal-ratio combining and selection combining (SC), are routinely used to mitigate the effects of multipath fading and thus to enhance the overall received SNR [19]. In particular, exploiting different diversity branches, e.g., multiple-receiver antennas, these techniques aim at avoiding the deleterious effect of fading. The effectiveness of the diversity combining techniques is also shown for radio channels being subject to composite fading/shadowing [47].

Among various widely used diversity combining techniques, SC is relatively less complicated since only one of the diversity branches is processed. Namely, the branch with the highest SNR is selected by the combiner [19]. Accordingly, recalling the RSP model in Section III-A, we can characterize a simple analogy between SC and RSP. Specifically, in RSP, the RN location having the highest SINR is selected, which is analogous to the diversity branch with the highest SINR in SC. Furthermore, the number of RN locations considered in RSP corresponds to the number of diversity branches in SC.

A key performance measure in analysis of RSP is then its impact on the resultant AoF on the relay link. The AoF after RSP can be evaluated\(^6\) using the definition (4) along with the pdfs given in (42) for RN location trellis 1 and (44) for RN location trellis 2. In addition, the gain achieved by SC decreases in the case of average SNR imbalance among diversity branches [19]. Hence, to find out the maximum achievable gains by RSP in terms of decrease in AoF on the relay link, i.e., lower bounds on the resultant AoF, we utilize RN location trellis 2, where each RN location is subject to the same SINR distribution implying equal average SINRs.

C. Link Rate Distributions

In the $k$th cell and at the $m$th RN location, the relay-link rate $R_{r,m,k}$ is given in terms of the relay-link SINR as follows:

$$R_{r,m,k} = \delta_r \cdot A_r \cdot \log_2(1 + B_r \cdot \hat{Y}_{m,k})$$  \hspace{1cm} \text{(45)}$$

where $A_r$ and $B_r$ are the bandwidth and SINR efficiency factors, respectively, and $\delta_r$ is the overhead scaling factor that accounts for, e.g., LTE overhead through reference symbols

\(^6\) Matlab is utilized in AoF evaluations after RSP.
and control signaling. These factors are commonly used to fit the system rate with the set of adaptive modulation and coding (AMC) curves obtained via system-level simulations. Herein, we have used the contribution of [48] to enable the AMC functionality in the system. The single root of (45) is then formulated as
\[
\tilde{r}_{m,k} = \left(2^{\frac{R_{r,m,k}}{(\delta_r A_r)} - 1}\right) / B_r = g^{-1}(R_{r,m,k}).
\] (46)

The cdf of the relay-link rate follows as in [43]:
\[
F_{r,m,k}(R_{r,m,k}) = F_{m,k}(g^{-1}(R_{r,m,k})).
\] (47)

Similarly, using (41) or (43), depending on the employed RN location trellis, we obtain the relay-link cdf in case of performing RSP as
\[
F_{r,m,k}(R_{r,m,k}) = F_{m,k}(g^{-1}(R_{r,m,k})).\] (48)

On the other hand, the access-link instantaneous rate \(R_a\) is of the following form:
\[
R_a = \delta_a \cdot A_a \cdot \log_2(1 + B_a \cdot \gamma_a) = h(\gamma_a)
\] (49)
where \(\gamma_a\) is the instantaneous SNR on the access link, and the parameters \(A_a\), \(B_a\), and \(\delta_a\) may differ from \(A_r\), \(B_r\), and \(\delta_r\); e.g., the antenna configurations on the relay and access links are different, and relay-specific control signaling is taken into account.

The cdf of the access-link rate now follows from (2) based on (14) and (49), i.e.,
\[
F_a(R_a) = F_{\gamma_a}(h^{-1}(R_a)).
\] (50)

D. End-to-End Rate Distributions

For the relay deployment, the end-to-end rate is given in terms of the rate on the two hops. As we consider half-duplex decode-and-forward relays, transmissions from the BS to the RN and from the RN to the MT are scheduled on different time slots. This is aligned with 3GPP and IEEE 802.16m specifications, where transmissions on the access and relay links are time-division multiplexed on a single-carrier frequency [15], [49].

Fig. 5 exemplifies a resource-allocation scheme on the access and relay links. Time resources allocated for the relay-link communication constitute \(\tau_r\) of the total system resources. Similarly, access-link communication is scheduled on \(\tau_a\) of the total available resources, where resource normalization is given as \(\tau_r + \tau_a = 1\).

Subsequently, the end-to-end rate experienced by a single user served by the RN in the \(k\)th cell and the \(m\)th location is defined as the minimum of the user rate achieved on the relay and access links, i.e.,
\[
R_{e,m,k} = \min(\tau_r \cdot R_{r,m,k}, \tau_a \cdot A_a)
\] (51)
where rates on the relay and access links are scaled by the portion of resources allocated to each. The achievable rates \(R_{r,m,k}\) and \(R_a\) are given as in (45) and (49), respectively, and they are independent RVs. Thus, the cdf of the end-to-end rate is formulated as
\[
F_{e,m,k}(r) = F_{r,m,k}(r/\tau_r) + F_a(r/\tau_a).
\] (52)

When RSP is performed, the rate distribution \(F_{e,m,k}\) is formulated similarly by
\[
F_{e,m,k}(r) = F_{r,m,k}(r/\tau_r) + F_a(r/\tau_a).
\] (53)

The end-to-end rate (51) is maximized when the rates on the relay and access links are equal. Then, the optimal resource allocation on the access link and the achieved maximum end-to-end rate are given by
\[
\tau_a^{opt} = \frac{R_{r,m,k}}{R_{r,m,k} + R_a}
\] (54)
\[
F_{e,m,k}^{max} = \frac{R_{r,m,k}R_a}{R_{r,m,k} + R_a}.
\] (55)

However, in practice, due to resource-allocation granularity in time, \(\tau_r\) or \(\tau_a\) takes discrete values, e.g., in LTE [15], from the set of \(\{0.1, 0.2, \ldots, 0.9\}\).

V. PERFORMANCE EVALUATION

Here, we evaluate the effect of RSP on the relay-link quality and on the end-to-end performance. In addition, we demonstrate the impact of RSP on the resultant AoF on the relay link. All analytical results have been validated via simulations (denoted by Empirical in the figures) through which the accuracy of the deduced formulations is also presented. The simulations are conducted using Matlab as the computational environment. Specifically, 5 × 10^4-sample Monte Carlo simulations are carried out to ensure reliable statistics. Moreover, the simulation models follow the 3GPP guidelines given in [15]. It should be stressed that, although spatial shadowing correlation between two candidate RN locations (23) is ignored in analytical formulations (see (A2) in Section III-B), this correlation is taken into account in the simulations. In addition, the considered cellular network layout, which is depicted in Fig. 4, consists of \(K + 1 = 7\) cells, out of which six neighboring cells cause cochannel interference with the relay-link reception in the midmost cell. Furthermore, we focus on coverage-oriented planning, i.e., RNs are positioned at the cell edge where users experience high interference and/or severe propagation losses.
toward the serving BS. We note that, in 3GPP and IEEE, the main interest is on the coverage extension by relays [49].

In Table I, the system parameters that are used for analytical and simulation results are provided in accordance with [15]. The tabulated values of the parameters \((s_1, s_2)\) of (33) for the MGF-matching method yield a tradeoff performance between head and tail portions of the cdf, noting that higher accuracy at either portion can be attained by using different values. Moreover, RN location trellis 2 is analyzed only for the resultant AoF on the relay link, whereas RN location trellis 1 is employed for other results.

We utilize distribution functions to illustrate the performance because they include information about the whole statistics. Following the convention of IMT-Advanced standardizations (see, e.g., [15]), cdfs are used for system performance evaluations, and numerical results are usually documented for the 50%-ile and 5%-ile cdf levels to indicate the rate gains for in-cell and cell-edge users, respectively.

### A. Relay-Link SINR Distribution

The impact of RSP on the relay-link SINR distribution is illustrated by cdf plots in Fig. 6 for different numbers of RN candidate locations along with two sets of channel parameters. These sets of channel parameters are

1. \((m_{RL}; \sigma_{dB}) = (1; 8)\), which corresponds to a scenario with severe fading;
2. \((m_{RL}; \sigma_{dB}) = (5.76; 6)\), which corresponds to a scenario with comparatively light fading.

First, it is observed that the cdf plots pertaining to analytical results (solid and dashed lines) are in good agreement with the simulation results (circular markers). Second, it is noticed that RSP provides SINR gain particularly at lower cdf percentiles in both scenarios. The gain through RSP is more prominent for the first scenario where severe fading is experienced, which highlights the impact of RSP in alleviating the effects of severe fading. For instance, considering the first scenario, RSP with \(M = 5\) RN candidate locations achieves a 17-dB SINR gain at the 5%-ile cdf level. On the other hand, in the case of the scenario with comparatively light fading, RSP with \(M = 5\) RN candidate locations yields an 11-dB SINR gain at the 5%-ile cdf level. It can be also inferred that, as the number of RN candidate locations in RSP increases, the deviation of the SINR cdf plots reduces, implying a decrease in the AoF. In what follows, we demonstrate this impact of RSP on the resultant AoF on the relay link.

### B. Resultant AoF on the Relay Link

To gain more insight into the impact of RSP, we have plotted in Fig. 7 the AoF values on the relay link as a function of the shadowing standard deviation. The solid lines are obtained by using RN location trellis 1 based on (4) when \(M = 5\) and based on (8) following the discussion after (40) when \(M = 1\) (no RSP). The dashed lines illustrate the lower bounds for AoF, which are obtained by using RN location trellis 2, based on (4) and (44). For \(m_{RL} = 5.76\), the multipath fading is not severe; hence, shadowing dominates.
the effectiveness of RSP in mitigating the deleterious impact of shadowing on the relay link. The lower bounds for AoF in Fig. 7 are obtained by using RN location trellis 2. It is shown of shadowing on the relay link. The lower bounds for AoF in the effectiveness of RSP in mitigating the deleterious impact between the resultant AoF values of RN location trellis 1 and (53) when \( M = 5 \). On the relay link, we have \((m_{RL}; \sigma_{RL}) = (5.76; 6)\). On the access link, (a) frequent heavy shadowing environment with an average access-link SNR of 10 dB and (b) infrequent light shadowing environment with average access-link SNR of 20 dB are considered.

![Fig. 8. Relay-link, access-link, and end-to-end rate cdfs which are, respectively, based on (47), (50), and (52) when \( M = 1 \) (no RSP) and (48), (50), and (53) when \( M = 5 \). On the relay link, we have \((m_{RL}; \sigma_{RL}) = (5.76; 6)\). On the access link, (a) frequent heavy shadowing environment with an average access-link SNR of 10 dB and (b) infrequent light shadowing environment with average access-link SNR of 20 dB are considered.](image)

**Fig. 8.** Relay-link, access-link, and end-to-end rate cdfs which are, respectively, based on (47), (50), and (52) when \( M = 1 \) (no RSP) and (48), (50), and (53) when \( M = 5 \). On the relay link, we have \((m_{RL}; \sigma_{RL}) = (5.76; 6)\). On the access link, (a) frequent heavy shadowing environment with an average access-link SNR of 10 dB and (b) infrequent light shadowing environment with average access-link SNR of 20 dB are considered.

C. Link and End-to-End Rates

Fig. 8 presents the cdfs of the access, relay, and the end-to-end link rates for \( M = 5 \) and \( M = 1 \) (no RSP). Two cases are considered for the channel conditions on the access link.

1) Frequent heavy shadowing with an average access-link SNR of \( \gamma_a = 10 \) dB reflects relatively moderate channel conditions, as shown in Fig. 8(a).

2) Infrequent light shadowing with an average access-link SNR of \( \gamma_a = 20 \) dB corresponds to good channel conditions, as shown in Fig. 8(b).

In both cases, we have \((m_{RL}; \sigma_{RL}) = (5.76; 6)\), and the resource-allocation parameters are set as \( r_a = \tau_a = 0.5 \). We note that this resource allocation may not be optimum. Optimum resource allocation will be addressed in Section V-D.

D. Resource Allocation

Optimum resource allocation should take into account the qualities of both relay and access links and balance the achieved rates on them. We consider two performance measures, namely the 5%-ile and 50%-ile end-to-end rate cdfs, and investigate the gains achieved using different resource allocations. We recall that the 5%-ile rate cdf level reflects the cell coverage performance, whereas, the 50%-ile level indicates the median user performance within the cell.

Fig. 9 presents the achieved 5%-ile versus 50%-ile end-to-end rates for different resource-allocation combinations on the access and relay links, based on (52) when \( M = 1 \) (no RSP) and on (53) when \( M > 1 \) (with RSP). The parameter \( \tau_r \) ranges from 0.1 to 0.9 with a step size of 0.1 (each mark indicates a different \( \tau_r \)). The arrows on the curves indicate the direction of increase in \( \tau_r \). Dashed and solid curves correspond to relatively moderate and good channel conditions on the access link, respectively. The 5%-ile rate cdf level reflects the cell coverage performance, whereas the 50%-ile rate cdf level indicates the median user performance.

Fig. 8 reveals that RSP results in significant rate gain on the relay link. The achievable end-to-end rate, however, depends on the capacities of both the relay and access links. Under moderate access-link channel conditions, the maximum achievable gain in the end-to-end rate through RSP is limited by the capacity of the access link; the cdf plots of end-to-end and access link rates almost overlap. This is due to the relatively low access-link rate, which renders it a bottleneck; hence, improving the relay link further does not translate into any end-to-end rate gain. In this case, RSP yields 24% and 33% end-to-end rate gains at 5%-ile and 50%-ile cdf levels, respectively. On the other hand, under good access-link channel conditions, the maximum achievable gain in the end-to-end rate through RSP is, by contrast, limited by the capacity of the relay link. In such a case, RSP gains are more prominent. When performing RSP, a clear gain in end-to-end rate is observed, and 280% and 72% rate gains are seen at 5%-ile and 50%-ile cdf levels, respectively.
50%-ile rate cdf level) or to decide on tradeoff between both criteria. We further note that the increase in the relay-link resource allocation is indicated by the direction of the arrows on the curves.

The two cases of channel conditions on the access link analyzed in Section IV-C are also applied in Fig. 9, where dashed and solid lines correspond to relatively moderate and good channel conditions, respectively. In both cases of dashed and solid lines, the relay-link channel parameters are $(m_{RL}; \sigma_{dB}) = (5.76; 6)$. The results depicted in Fig. 9 point out the importance of the resource allocation for end-to-end rates. Moreover, the 50%-ile and 5%-ile end-to-end rate targets may lead to different optimum resource allocations. This is particularly observable under moderate access-link conditions (dashed lines), e.g., when performing RSP with $M = 5$, the 50%-ile end-to-end rate is maximized by setting $\tau_R = 0.4$, whereas 5%-ile end-to-end rate achieves the maximum when $\tau_R = 0.2$. In addition, an optimum value of $\tau_R = 0.2$ implies that the access-link quality clearly lags behind that of the relay link. In what follows, the gains are determined relative to no RSP with optimum resource allocations maximizing either 5%-ile or 50%-ile end-to-end rates.

In the case of moderate access-link channel conditions (dashed lines), performing RSP with $M = 5$ provides clear gains. For example, the maximum gain of 89% is achieved at 5%-ile end-to-end rate cdf level when $\tau_R = 0.2$. Moreover, it can be seen that, due to improvement in relay-link quality through RSP, the optimum gains are achieved with reduced resource shares on the relay link. It is also worth noting that, when even with only one extra RN candidate location is available, i.e., $M = 2$, significant gains can be achieved.

If access-link channel conditions are good (solid lines), the relay-link quality lags behind the access-link quality, e.g., for $M = 1$ (no RSP), the best 5%-ile end-to-end rate is achieved with $\tau_R = 0.8$. Nevertheless, with RSP, fewer resources are needed on the relay link when maximizing 5%-ile and 50%-ile end-to-end rates. This indicates that the relay-link limitations can be eased by RSP yielding better overall end-to-end rates. As an example, in case of performing RSP with $M = 5$, the maximum gain reads as 177% at 5%-ile end-to-end rate cdf level. Moreover, similar to the previous case, performing RSP, even with $M = 2$, provides significant gains.

VI. CONCLUSION

In this paper, we have investigated RSP as a technique to enhance the wireless relay-link performance of RNs by exploiting their deployment flexibility. In this manner, an RN deployment location is selected from a discrete set of alternatives such that the SINR on the relay link is maximized. The performance of RSP has been analyzed assuming composite fading/shadowing channels. The impact of cochannel interference on the relay-link quality is taken into account within the framework of multicellular wireless networks, where the desired and interfering signals are correlated due to shadowing. In particular, tractable closed-form approximations for SINR, relay-link and access-link rates, and the end-to-end rate are obtained. Specifically, MG distribution has been employed to accurately approximate the Nakagami–lognormal and Ricean–lognormal composite distributions on the relay link and the access link, respectively. Moreover, the MGF-matching method has been used to approximate cochannel interference where the total cochannel interference is characterized by a sum of correlated Rayleigh–lognormal RVs.

Through numerical simulations, it is shown that deduced expressions achieve high accuracy. In addition, AoF is utilized as the performance measure to illustrate the impact of RSP on reducing the severity of fading. Comprehensive results show that performing RSP provides significant gains on the relay-link SINR. It is also shown that not only does RSP increase average SINR, it also substantially decreases the resultant AoF on the relay link, particularly boosting the low SINR regime. Achieved gains on the relay link are shown to translate into clear improvements in end-to-end rates unless the access link is a bottleneck. Moreover, the importance of balancing resource allocation to realize such gains is illustrated. Concretely, it is shown that the 50%-ile and 5%-ile end-to-end rate targets may lead to different optimum resource allocations. In addition, it is illustrated that with RSP, fewer resources are needed on the relay link when achieving the optimum end-to-end rates.

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