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QUANTIFYING MARKET RISK IN MONTE CARLO WAY: CASE SAMPO LIFE

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SUMMARY

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Objectives

The objectives of this thesis were to develop a Monte Carlo simulation and full valuation –based Value at Risk (VaR) system for Sampo Life, discuss all the necessary technical issues regarding the implementation and examine how to take full advantage of such a system in an asset management environment.

Data and research methodology

The VaR system was developed for Sampo Life, a major Finnish Life insurance company. The analysis considered about 50% of the whole portfolio and included equities, equity derivatives and bonds in 12 different currencies. To solve the dimensionality problem, equities were mapped with 4 different methods. For bonds, the whole cash flow matrix was used but individual zero-coupon yield curves were modeled with principal component analysis. Credit spreads for defaultable instruments were estimated using simple atheoretic approach. Several statistical test were utilized to test the statistical properties of financial time-series and conditional normality in particular. Two different variance/covariance estimators were also tested, namely moving averages (MA) and exponentially weighted moving averages (EWMA). To improve the convergence of the Monte Carlo simulation, a variance reduction technique called quadratic resampling was empoyed.

Results

The statistical test indicated that conditional normality is a reasonable assumption for time-series modeling, particularly with EWMA estimator which was favored also in the operational evaluation. The resulting VaR system performed very well for equities using industry index mapping for domestic and broad country indices for foreign assets. Equity mapping methods were considered highly sensitive to portfolio characteristics. For bonds, the credit spreads proved to be in need of more careful modeling whereas governmental instruments performed adequately. The developed VaR system was considered extremely useful basis for stress tests and for additional simulations concerning the efficiency of the asset allocation.

Key words

market risk, Value at Risk, Monte Carlo simulation, zero-coupon yield curve, principal component analysis, credit spread, bootstrapping, mapping, quadratic resampling

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Tavoitteet

Tutkielman tavoitteena oli kehittää Henki-Sammolle Monte Carlo –simulaatioon ja uudelleen arvottamiseen perustuva Value at Risk (VaR) –järjestelmä, käydä läpi tarpeelliset toteutukseen liittyvät tekniset yksityiskohdat sekä tarkastella toimivan järjestelmän täysimittaista hyödyntämistä omaisuudenhoito-ympäristössä.

Lähdeaineisto ja tutkimusmenetelmät

VaR –järjestelmä kehitettiin suurelle suomalaiselle henkivakuutusyhtiölle, Henki-Sammolle. Case-aineistona käytetyt sijoitukset muodostivat noin 50% koko yhtiön sijoitusomaisuudesta ja koostuivat osakkeista, osakejohdannaisista ja joukkovelkakirjoista 12:ssa eri valuutassa. Osakkeiden kohdalla dimensionaalisuusongelmaan haettiin ratkaisua neljällä eri menetelmällä kun taas joukkovelkakirjat purettiin yksittäisiksi kassavirroiksi ja keskityttiin mallintamaan korkokäyriä pääkomponenttianalyysillä. Konkurssiriskisten instrumenttien korkomarginaalit mallinnettiin yksinkertaisella menetelmällä. Taloudellisten aikasarjojen tilastolliset ominaisuudet testattiin usealla testillä kiinnittäen huomiota erityisesti ehdolliseen normaaliuteen. Myös kaksi varianssi/kovarianssi –estimaattoria, liukuva keskiarvo ja eksponentiaalisesti tasoitettu, testattiin. Monte Carlo –simulaation konvergenssia pyrittiin parantamaan kvadraattisella jälleenotannalla.

Tulokset

Tilastollisten testien perusteella ehdollinen normaalius on järkevä oletus aikasarjojen mallinnuksessa, erityisesti eksponentiaalisen tasoituksen estimaattorilla. Kehitetty VaR –järjestelmä toimi osakkeiden kohdalla parhaiten kun kotimaiset osakkeet mallinnettiin toimialaindekseillä ja ulkomaiset positiot maakohtaisilla osakeindekseillä. Osakkeiden mallinnuksen todettiin olevan erittäin herkkä sijoitusportfolion koostumukselle. Joukkovelkakirjojen kohdalla riskipreemioiden mallinnuksen todettiin olevan liian yksinkertainen, riskittömät valtion instrumentit puolestaan toimivat mallikelpoisesti. Kehitetty VaR –järjestelmä todettiin erinomaiseksi pohjaksi stressitesteille ja lisäsimulaatioille.

Avainsanat
markkinariski, Value at Risk, Monte Carlo –simulaatio, nollakuponkikäyrä, pääkomponenttianalyysi, korkomarginaali, bootstrapping, mapping

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LIST OF EQUITY MARKET ABBREVIATIONS:

Country/region	Index	Abbreviation
Japan	Nikkei 225	.N225
Japan	Nikkei 500	.N500
Japan	Topix	.TOPX
Sweden	Stockholm OMX	.OMX
Germany	Xetra DAX	.GDAXI
Germany	Neuer Markt	.NMDKX
Norway	OBX Oslo 25	.OBX
USA	S&P 500	.SPC
USA	Nasdaq Composite	.IXIC
USA	Dow Jones Industrial Average	.DJI
Austria	Austrian Traded Index	.AEX
Italy	MIB 30	.MIB30
Switzerland	Swiss Market	.SSMI
Denmark	KFX Constituents	.KFX
Spain	IBEX 35	.IBEX
UK	FTSE 100	.FTSE
France	CAC 40	.FCHI
Estonia	Tallinn SE General Index	.TALG
Latvia	Riga SE Index	.RICI
Lithuania	Vilnius SE Official Index	.LITIN
Finland	FOX Index	.FOX
Finland	HEX Portfolio Index Yield	.HPIY
Europe	Euro STOXX 50	.STOXX50E
Finland	HEX General Index	.HEX
Europe	Euro STOXX	.STOXXE
Finland	Banks & Finance	.HEB1
Finland	Insurance	.HEI1
Finland	Investment	.HEIN
Finland	Chemical	.HECH
Finland	Construction	.HEC
Finland	Energy	.HEE
Finland	Food industries	.HEFO
Finland	Forest industries	.HEF1
Finland	Metal & Engineering	.HEM1
Finland	Multibusiness Industries	.HEL1
Finland	Other Industries	.HET1
Finland	Media & Publishing	.HEME
Finland	Other Services	.HEO1
Finland	Tele & Electronics	.HETE
Finland	Trade Index	.HETD
Finland	Transport Index	.HETR

1 INTRODUCTION

1.1 NEED TO QUANTIFY RISK

Risk management means literally that we must manage risk, not to eliminate it. To be able to manage risk, we must know what are our exposures and what affects them. To aid the analysis, risk is usually divided into business, market, credit, liquidity, operational and legal risk. Of these, the market risk is certainly the easiest one to measure although it is not necessarily the most important source of risk. The focus of this thesis is on quantifying the market risk a financial institution faces. The case company, Sampo Life, is an institutional investor with a large, internationally diversified portfolio. As such, it is a good example of how difficult it actually is to measure risks and dependencies across asset classes or different instruments.

Looking back, the uncertainty within the financial markets has increased due to various reasons. In 1970's, the collapse of Bretton-Woods pegged exchange rate system led to highly fluctuating currencies whereas the prominent Black & Scholes equation enabled the amazing growth in derivatives trading. In 1980's, de-regulation and globalisation opened up unforeseen possibilities to diversify assets across the globe. In 1990's, internet and the speed of information changed the industry into truly 24-hour market. But as the possibilities have increased, so has the complexity of activities.

Fortunately, the preconditions for financial risk management are favorable nowadays. Some recent high profile financial disasters (e.g. Orange county, Barings, LTCM) have certainly made clear that it is worth to build up a robust risk management system. The simultaneous progress in financial theory and computer power have made it possible to actually calculate for example the market risk a financial institution faces.

1.2 VALUE AT RISK

Ever since JP Morgan launched their RiskMetrics risk management product to the public back in 1994, the pace at which the Value at Risk methodology has been adopted within the financial industry has been staggering. Value at Risk (VaR) is defined as *the maximum expected loss over a given time interval under normal market conditions at a given level of confidence* (Jorion, 1997, xiii). As a risk management technique, it differs from traditional risk measures which mostly tell us how sensitive our position is to some underlying risk factor. VaR tries to tell us, how much we might loose in monetary terms with given probability. An earlier attempt to quantify market risk in similar way was suggested by Leibowitz and Henriksson (1989) in the context of portfolio optimization. They called their approach as shortfall risk.

There exist three fundamental approaches to calculate VaR: analytical, historical simulation and Monte Carlo simulation. Suppose that the return of portfolio has a density function *f(r)* and we are dealing at a level of confidence of *1-c*. Then for a generic distribution, the probability to have a return less than r^* is (Jorion, 1996, 88):

$$P(r < r^*) = \int_{-100\%}^{r^*} f(r)dr = c \qquad (1)$$

Assuming normally distributed returns, it can be shown that our VaR becomes (Jorion, 1996, 91):

$$VaR = \alpha\sigma\sqrt{\Delta t}W_0 \qquad (2)$$

So, the VaR is simply a multiple of the standard deviation of the portfolio. α is an adjustment factor that is directly related to the confidence level. While very simple, the analytical method of calculating VaR applies only to well-diversified portfolios, which contain just a fraction of non-linear positions (e.g. options, bonds).

In historical simulation, realized returns are used to approximate the future return distributions. It avoids two most common difficulties present in other approaches: the estimation of volatilities and correlations and distributional assumptions. However, as the drawbacks are huge in terms of data requirements and implicit assumptions, this method is considered inappropriate for the purposes of this thesis.

The third method, Monte Carlo simulation, is the most powerful way to calculate VaR. Its strength comes from flexibility: it can handle any kind of positions, instruments and assumptions. This is also the pitfall as the inherent model risk in Monte Carlo methods is huge. Section 3.1 gives a more thorough discussion on issue.

In financial literature, VaR has provoked an extensive discussion both for and against. For an excellent general discussion on VaR, see Duffie and Pan (1997). Beder (1995) highlights the fact that different calculation principles produce totally different results. Marshall and Siegel (1997) extend the analysis by showing how the RiskMetrics based estimates vary as the complexity of the portfolio grows. Jorion (1997a) writes in defense of VaR by clarifying some common misconceptions about the methodology. After all, VaR figure is just an estimate of the true risk and approximating heavily non-linear positions linearly is clearly inappropriate. Recently there have been some interesting papers regarding the use of VaR for other purposes than just reporting the market risk. Kupiec (1998) develops stress testing using VaR as a starting point, Ahn *et al* (1999) show how to minimize VaR using options and Dowd (1999a, 1999b) employs VaR in the context of portfolio management and risk-return analysis.

Despite the criticism and misuse, the Value at Risk is currently the most powerful technique to estimate the market risk a financial institution faces. It can be applied to every asset class and every instrument. The real strength of properly designed VaR system is that it can actually measure the total portfolio risk by taking account dependencies over asset classes. It is a common and consistent measure of risk. Traditional measures (e.g. deltas, durations) may be adequate when considering the positions in isolation. But ending up with a reliable VaR estimate for a huge multi-currency, multi-asset portfolio is really a challenge in itself.

1.3 OBJECTIVES OF THIS THESIS

This thesis considers Value at Risk in the asset management framework, case company for the analysis being Sampo Life, a Finnish life insurance company.

Objectives of this thesis are:

1) Create a reliable VaR-system for Sampo Life based on Monte Carlo simulation with full valuation
2) Discuss the necessary and important technical aspects in creating such a system
3) Examine how to take full advantage of such a system within the asset management framework

The resulting VaR-system should fulfill certain features. Particularly, it should:

1) Be as simple as possible (in relative terms) but above all, reliable. This means an extensive backtesting needs to be employed;
2) Include all the assets susceptible to market risk;
3) Report not just the plain VaR figure but also other relevant information and sensitivity analysis;
4) Utilize the existing facilities to the fullest extent (market information sources, accounting systems etc.);
5) Run on a common desktop environment (that is, in Excel).

This list is utilized when making decisions on what methods to use to solve a particular technical problem.

1.4 STRUCTURE OF THE THESIS

Next, chapter 2 introduces the case company, Sampo Life. After a short company description, the main points of relevant legislation concerning the asset and risk management are laid out. Then are presented the company specific asset management principles and processes. The section, which describes the portfolio, explains also the selections on what to include in the VaR analysis. Section 2.5 reviews current risk management practices and evaluates their strengths and weaknesses. The last section is about what is the possible impact of building a VaR system into asset management environment and Sampo Life in particular.

Chapter three is all about methodologies. When building a Monte Carlo based VaR with full valuation, they are even more important than in delta-normal methods. This is due to huge model risk which is not always self-evident (e.g. relying on correlations to model dependencies). Each section in chapter 3 discusses all major alternatives and their strengths and weaknesses and explains why certain approach was selected to be the best in this context. Topics include basics of Monte Carlo simulation, dimensionality reduction techniques, price path generation, variance reduction techniques, parameter estimation, derivatives pricing, fixed income modeling and backtesting.

Chapter four lays out the empirical results of the developed VaR model. First, section 4.1 discusses simulation technical issues of convergence and variance reduction. The next two sections, 4.2 and 4.3, are intertwined as they consider the statistical properties of financial time-series and different variance-covariance estimators. Then, section 4.4 presents the results of different equity mapping techniques for various sub-samples of the considered equity portfolio. After that, the attention is turned to fixed income modeling in section 4.5, which consists of zero-coupon yield curve estimation, credit spread modeling and the efficiency of principal component analysis. Finally, the section 4.6 wraps up chapter and presents the backtesting results for the case portfolio.

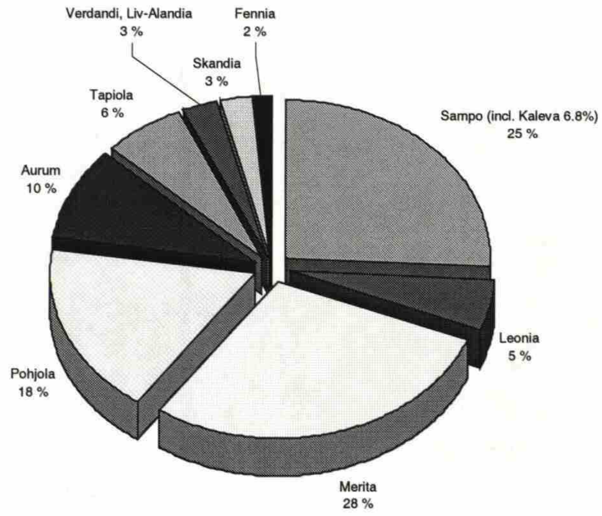
After empirical results, chapter five summarizes the key findings of the thesis, draws conclusions and suggests some ideas for further investigation.

2 CASE COMPANY, SAMPO LIFE

2.1 COMPANY IN BRIEF

Sampo Life is a Finnish life insurance company, which was formed in a merger between Nova Life Insurance Company Ltd. and Sampo Life Insurance Company Limited on 31st December 1998. Based on gross premiums written in 1999, the company was the second biggest player in the Finnish life insurance industry (Figure 1). Sampo Life's gross premiums written amounted to FIM 3 218 million and the value of investment portfolio stood at FIM 23 836 million in the end of 1999. At the moment company employs about 220 people and the headquarters are located in Helsinki at Boulevard 56, in the former Sinebrychoff's brewery. Sampo Life has no office network of its own, it rather utilizes the parent company's premises.

Figure 1: Market shares in the Finnish life insurance industry, 1999



However, the situation is about to change once again as the parent company Sampo Insurance Company plc and Leonia plc agreed in October 1999 on merger to form the first full-service financial services group in Finland. As a result, Sampo Life will merge with Leonia Life Insurance Company Ltd. in the end of 2000. The new Sampo Life becomes number one player in the market based on 1999 figures.

Within the whole insurance industry, life insurance business is the growth segment. This is because people are more and more interested on investment activities and interest earned from bank accounts are no longer tax free. Furthermore, for risk averse investor, the institutional feature of guaranteed interest rate is appealing combined with tax deduction possibilities. Based on this, the future prospects of Sampo Life seem to be favorable.

2.2 LEGISLATION

The main goal of legislation is to prevent insurance companies from ending up unable to fulfill their commitments. The main elements in the regulation of life insurance companies regarding their assets are solvency margin, solvency capital, the definition of coverage capital and the control of currency positions.

Solvency margin refers to the amount by which the company assets exceed its claims (Finnish Insurance Companies Act 11 §1). According to Instructions and regulations for domestic insurance companies prepared by the Ministry of Social Affairs and Health (MSAH), the minimum level for the required solvency margin is around 4% of the technical provisions (company's debt to its customers). This margin must be achieved in every disclosed financial statements. Failing to comply with the rule does not immediately result in a ban to conduct insurance activities. Instead of that, the company must prepare a restructuring plan and be subject to further scrutiny by MSAH.

Another measure used to control solvency is the solvency capital, which is the solvency margin plus equalization provision. Both these measures have been criticized for there is no strict mathematical reasoning behind them. These solvency measures were imported into the Finnish legislation from EU directives as a part of the harmonization process among EU membership countries. As such, both are quite vague and rarely limit the investment process unless the company is in dire straits.

Probably more important limitation from the viewpoint of asset management is how to cover the technical provisions. Technical provisions are company's debt to its clients. According to the principle of protection (Finnish Insurance Companies Act 12§), a (life) insurance company must be able to secure this debt momentarily. Company must maintain a coverage record, which lists the assets used to cover the technical provisions. The real problem is that not all assets may be used into this purpose. Statute given by MSAH divides the investment portfolio into asset classes. Each asset class may cover only a certain specified amount of technical provisions, for example bonds issued by the Finnish government or municipalities may cover 100%, whereas the shares of publicly listed companies may cover only 50% of the technical provisions. Also liquidity is taken into account as only 10% of technical provisions may be covered with illiquid assets.

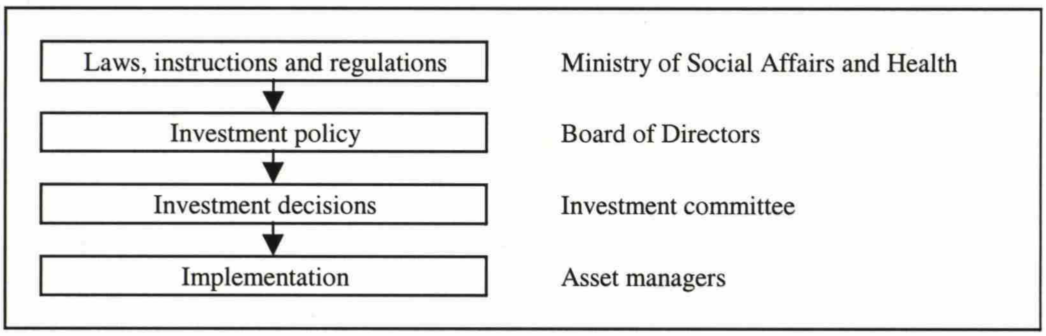
Currency issues are paid attention to in the legislation as well. The general principle is that technical provisions must be covered with assets in the very same currencies where the contractual claims have been made. Specifically, only 20% of the claims in certain currency may be covered with assets in other currencies. The emergence of the Euro area has reduced the effect of this rule but nevertheless, as the majority of the claims of Finnish life insurance companies are domestic, this rule must be taken into account. As USA and Japan are the worlds largest capital markets, the company may loose some important possibilities to improve returns and achieve diversification effects unless it has created sufficient surplus.

While above rules appear strict, once the company fulfills the required solvency levels, all assets not used to cover these requirements are not regulated at all. Currently, Sampo Life has a very strong financial position. Technical provisions stood at FIM 5 643 million in the last financial statements, which exceeds the required minimum level by a factor of seven. So, at the moment the legislation does not in practical terms affect Sampo Life's investment policy. From the risk management point of view, these regulations aim to guarantee the long-term survival of the company but the means are rather impractical. They for example ignore the portfolio theory by prohibiting the use of assets in other currencies to cover for technical provisions. At the same time, the legislation calls for companies to diversify their assets in order to reduce risk. This is somewhat inconsistent.

2.3 ASSET MANAGEMENT PRINCIPLES AND PROCESS

The basic framework for asset management in Sampo Life is laid out in Figure 2:

Figure 2: Asset management framework in Sampo Life



Laws, instructions and regulations were discussed in last section 2.2. At the company level, board of directors has the highest decision power. It decides on investment policy, risk capacity calculation principles, risk profile and asset allocation. As the board meets approximately only every six weeks, the decision power is further delegated to investment committee. It meets once a week to decide on tactical allocations and individual investments. These decisions are then carried out by asset managers on daily basis.

As a long-term investor, Sampo Life aims at good long-term yield on a risk-adjusted basis. However, the industry specific feature of guaranteed interest rate, which is 3.5% p.a. at the moment, defines the lower limit that must be reached in the long run. In addition to that, the company policy aims at providing policyholders' with-profits savings, before charges and taxes, a total return that at least matches Finnish Treasury Bond yields. This reflects the Principle of Fairness stated in the Finnish Insurance Companies Act (13 §3). Of course, owners of the company want their share of the profits too and from the company point of view, part of the profits should be used to increase the solvency margin. As a result, the long-term level of required return on assets is around 8.0 - 8.5%.

In the short run, focus is given to:

1) Preventing losses on mark-to-market investments

2) Improving the yield level with realizations and better deals

3) Compensating decreasing yield levels with derivatives and trading

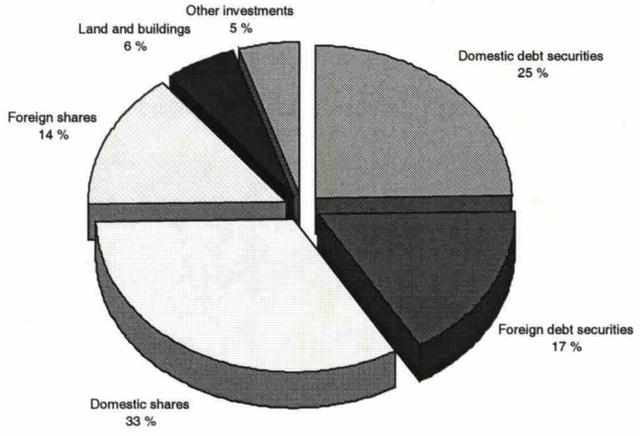
4) Improving the yield level of illiquid assets

There are certain limitations considering the value of trades investment committee is allowed to decide upon but as such they rarely affect the daily operations. Although short run objectives call for active portfolio management, active trading is not considered as the solution. Positions are rather taken with derivatives than by selling and buying a lot of underlying assets, unless the whole position in question is to be liquidated. As a long-term investor, the portfolio of Sampo Life includes a lot of valuation differences and it would be unreasonable to realize them in search of short-term profits.

2.4 PORTFOLIO

In the beginning of the analysis (1.1.1999) the market value of Sampo Life's portfolio stood at FIM 18 198 million. In the end of 1999 it grew up to FIM 23 836 million but part of the growth comes from written premiums which amounted to FIM 1 855 million (net of claims paid). So, the net growth was FIM 3 783. Portfolio is reasonably well diversified in a geographical sense although the bulk of assets are located in Finland (69%). Assets are nominated in 22 currencies in 27 different countries. Figure 3 describes how assets are allocated according to broad asset classes.

Figure 3: Sampo Life's investment portfolio 31.12.1999



As only market risk is considered, some unquoted portions of the portfolio are automatically left out from the analysis (lands and buildings, other investments). From the modeling point of view, the portfolio contains a wealth of different financial instruments: equities, equity options, warrants, equity index options and futures, commodity swap, money market instruments, straight bonds, floating rate notes (FRN), convertible bonds, forward rate agreements (FRA), index linked bonds, currency options, currency futures and forwards and deposits. Fortunately, most of the derivatives are plain vanilla, the only exotic ones being currency knock-in and knock-out options. This means that for the purposes of this thesis, all VaR calculations may be carried out with standard analytical formulas when valuing derivatives.

There were certain limitations for the VaR analysis regarding the portfolio. First, as the implementation requires a careful backtesting procedure to be carried out, the availability of data posed some difficulties. The starting point of the analysis was selected to be1.1.1999, as the company did not exist before that in the present form. The emergence of the Euro area complicated the analysis sizeably, particularly for interest rates. In the prevailing situation, the centralized interbank money marketd determine short rates whereas the long-rates still exhibit country-specific effects. Therefore, all modeled Euro countries had the same short rates beginning 1.1.1999. Before that, short-rates were taken from market specific interbank quotes. For currencies, it was easy to obtain artificial quotes against euro before 31.12.1998 because the conversion factors were known.

The biggest problem concerning the position data was that the company changed the accounting system it utilizes in the middle of 1999. As a result, data before that point was extremely difficult to obtain for the old system did not produce any decent reports. Furthermore, all derivatives were (some are still) handled separately in Excel files and position information had to be gathered manually. To keep the workload within reasonable limits, fixed income options, currency options and equity asset managers had to be removed from the analysis. Table 1 lists both the numbers of different instruments within the analysis as well as their market values and shares of total in the last two financial statements.

Table 1: Information on modeled portfolio

Number of instruments:	Bonds	Equities	Equity derivatives	Total portfolio
-Minimum	34	210	15	-
-Maximum	68	225	48	-
-Median	47	220	34	-
-Different	109	317	197	-
Market cap. 4.1.1999	5 256	3 825	-69	18 198
Share of total	63,1 %	69,4 %	100 %	49,5 %
Market cap. 31.12.1999	5 196	7 497	-51	23 836
Share of total	66,1 %	66,0 %	100 %	53,0 %

For equities, the biggest portions left out were foreign asset managers, mutual funds and private equities. Discarded bonds were mostly totally illuiqid domestic issues of corporates or municipalities. Fortunately it is possible to include the omitted and mark-to-market elements with little effort after the foundations of the model are well established and position information easily available from internal systems. Should all mark-to-market instruments be within the analysis, it would include about 90% of the equities and bonds.

2.5 CURRENT RISK MANAGEMENT PRACTISES

Current risk management considers mainly ensuring the long-term survival of the company. The board of directors has defined a risk profile for Sampo Life based on the current risk capacity. This measure is defined as the amount by which the solvency margin exceeds the required minimum level. For calculations, each asset class is given a variability parameter, which is based on long-run average historical volatility and estimations on future development. Parameters for different asset classes are: equities 25%, bonds 4-5% (average duration), real estates 8% and money markets 0%.

The total risk is calculated using a simple stress, which assumes that all markets plummet simultaneously an amount indicated by the variability parameter. So, to end up with a total risk figure, underlying market values of different asset classes are multiplied with corresponding variability terms. This total risk figure is then compared with the risk capacity to find out whether the level of market risk is acceptable. Total risk calculated may not exceed 75% of the risk capacity.

The risk profile definition includes maximum weights for each asset class. The purpose of these position limits is to guarantee optimum risk-return level in the long run. In the short run, investment committee is allowed to change the asset allocation in response to market movements. The positions are allowed to fluctuate even in excess of pre-specified maximum position limits according to fluctuation bands (+/- 1.5-15% depending on asset class).

To assess the risk factors of the portfolio more detailed, nominal amounts are used. For equity portfolio, alternatives are sector and geographical allocations taking account the derivatives position. Derivative positions are valued using delta-approximation and assuming perfect correlations across different positions. The risk of fixed income instruments is calculated using the traditional modified duration. To give richer view, the figures of bond portfolio are also given according to credit classes and country and sector allocations. The amount of credit risk is calculated using yield-to-maturity spreads and bond prices.

So, the quantitative risk management clearly needs to be improved. At the moment, the total portfolio risk is expressed only in terms of market values according to broad asset classes and in the form of extremely simple stress test. Also the calculation of derivatives risk is overly simplified.

2.6 SAMPO LIFE AND VAR

The investment committee is given authorities to adjust asset allocation in accordance with market fluctuations but it lacks the means to estimate the daily market risk across the whole portfolio. It is here, where a proper VaR system could impact the most. Although asset managers certainly have the feeling of the common volatility level in the market, it is prone to subjectivity and portfolio effects and derivative positions complicate things considerably. Of course, VaR figures could give valuable information for the board of directors as well as they are unlikely to watch the market as closely as asset managers do.

In general, VaR is most often discussed in the context of banks and brokers. Culp *et al* (1999, 1-2) argue VaR is not that well accepted in the institutional investment community as asset managers are typically in the business of taking risks. However, VaR is useful tool also for them as it may reveal whether the risks they are taking are those risks they want or need to be and think they are taking. Culp *et al* (1999, 14-22) consider four applications of VaR in the asset management environment: monitoring tool, what-if-modeling tool, risk targeting system and risk budgeting system. In addition to these, Monte Carlo simulation may provide interesting statistics as a by-product of VaR calculations. A prime example is the modeling of zero-coupon yield curve with principal components, which could also be used for hedging purposes (Litterman and Scheinkman, 1991). For Sampo Life, the aim is to develop a more realistic stress test procedure than prevailing system. Another topic is to examine the efficiency and risk/return structure of the asset allocation in the long-run. A good VaR system is sound basis for these applications.

Considering the size of the investment department, the main objectives for the VaR system were easy to define. It should be relatively simple, run in common desktop environment and report all additional information as well to be used in the asset management procedure. Portfolio was reviewed in section 2.4 and from that arises one important objective for the VaR system: assets must be mapped efficiently to reduce the dimensionality. In chapter 3, section 3.2 the issue is discussed in more detail and here we just need to conclude that with hundreds of different instruments, a straightforward implementation is right out of the question.

Reason to adopt Monte Carlo simulation with full valuation is based on the fact that the portfolio contains a lot of non-linear instruments. Although the VaR literature contains a wealth of papers, which deal with different approximation techniques for non-linearity, it is clear that full valuation is the most accurate method. As the duration of calculations is not the most important issue (within reasonable levels, of course), the full valuation is the method to choose.

As Value at Risk is defined as *the maximum expected loss over a given time interval under normal market conditions at a given level of confidence* (Jorion, 1997, xiii), the definition involves two arbitrarily chosen parameters: the time interval and the confidence level. When considering the target horizon, an institution need to assess the following (Dowd, 1998, 51): the liquidity of the markets in which it operates, the normal approximation, changes in the portfolio itself and the validation procedure. The liquidity factor might suggest a longer time horizon, especially when dealing with OTC or emerging markets. Time horizon should reflect the time it takes to orderly liquidate the whole position. Other factors suggest shorter horizons: normality works the better the shorter the time horizon, portfolio does not change too much and backtesting does not require too long history.

To be able to conduct an adequate backtesting and the use of normality assumption were seen as dominating factors here. Using time interval of one day, backtesting may be carried out using 355 observations and the zero-coupon yield curve may also be modeled with key-rates having normally distributed returns. As the bulk of the assets is invested in domestic instruments, the liquidity risk is a major concern but as such the liquidity factor was not modeled.

3 METHODOLOGIES

3.1 MONTE CARLO SIMULATION

Scientists working for the United States Government to develop the atom bomb invented Monte Carlo simulation back in the 1940's. Monte Carlo simulation inserts a probabilistic element into the non-probabilistic problem of estimating an integral. The dominant advantage of the method over classical numerical integration approaches is that the error convergence rate is independent of the dimension of the problem. As such, the method is perfect to proximate financial problems (and especially VaR related) where multidimensional integrals are frequent.

The basic process of Monte Carlo simulation for VaR analysis involves:

-Specifying initial positions and re-valuation formulas

-Identifying the key-factors and choosing appropriate stochastic processes for them

-Estimation of input parameters

-Generation of random numbers

-Price path creation and position re-valuation

The inherent problem with Monte Carlo simulation to proximate VaR is the huge model risk. With large multi-currency portfolio, one encounters practically every basic instrument and derivatives product there exists as is evident from the section 2.4. So, the modeling includes practically every issue that has been dealt with in the modern financial theory regarding financial markets. The issue is complicated by the fact that in many occasions, there is no right solution, only controversy and case examples. Therefore the results of Monte Carlo system must always be interpreted according to assumptions being made. But on the positive side, once the basic system is in place, it is very easy to experiment what the change in some assumption causes.

Furthermore, there exists no instrument that Monte Carlo VaR could not (at least in theory) handle.

There is also a related technique called Quasi-Monte Carlo simulation. In this technique, random numbers are not even pseudo-random but created to fill certain domain as efficiently as possible and therefore achieving fast convergence (section 3.6.3 gives more details about random numbers and section 3.7 discusses how to improve convergence). However, the gains from this method diminish as the dimensionality of the problem at hand increases. After some point, it is actually more efficient to use the normal Monte Carlo and pseudorandom numbers. For more on the subject, see Boyle, Broadie and Glasserman (1997, 1290-1302).

3.2 THE CURSE OF DIMENSIONALITY

The variance-covariance matrix, Σ, is the key player in Value at Risk (VaR) analysis if we consider either the delta-normal or structured Monte Carlo approach. Historical simulation method does not require this matrix, as it is already included in the historical, realized returns. To estimate the variance-covariance matrix, we need estimates both on the variances of the instruments we hold and covariances between them.

However, as the number of instruments we hold grows, the number of parameters needed to estimate Σ grows geometrically. If we hold n assets, we need to estimate n variances and $n(n-1)/2$ covariances, all in all $n(n+1)/2$ terms. In the case n=300, the number of terms needed to estimate would be 45 150. This results in enormous data requirements because we would like to have reliable estimates and at the same time ensure that the variance-covariance matrix is positively definite (Jauri, 1997, 192).

The positive definite property guarantees that the correlations between assets make sense as a whole and all variances are positive (Jauri, 1997, 192). There are two preconditions that must be met to ensure the positive definite property. First, the number of observations used to produce estimates must be at least equal to the number of variables, preferably much more (if moving averages are used). Second, none of the included time series may be linearly correlated with other series or group of series (multicollinearity precondition). In other words, each series must have some independent movement of its own. (Dowd, 1998, 77)

The analysis of the portfolio already made clear that achieving low dimensionality is one of the most important aspects in the design process. Next sections discuss all major techniques to achieve this goal.

3.2.1 Equity-specific methods

3.2.1.1 Representative mapping

RiskMetrics' representative mapping system for equities assumes that all stocks in any one country may be mapped to a broad stock index of the country in question. For this method to work, the portfolio in question should be reasonably well diversified because the systematic risk would then be the major source of the portfolio variance. If the portfolio contains positions which differ a lot from the index, then this method may produce inaccurate results. Dowd (1998, 82) describes a technique, which could be used if the case portfolio is imperfectly diversified.

Care must be exercised when selecting the relevant stock index. For example, here in Finland the problem with the representative approach does actually come from the fact that Nokia has so heavy weight in the HEX general index. If the portfolio is reasonably diversified, it most probably does not include over 60% of Nokia. That would be both poor risk and asset management. Using the HEX portfolio index is somewhat a remedy as it restricts the maximum weight of any one stock to 10%.

But there is an alternative way to handle the mapping procedure, at least within one country. Instead of a broad stock index we could map the portfolio using industry indices. In case the portfolio deviates a lot from the general index, this should be a major improvement. Of course, in a multicurrency situation it would be hard, if not impossible due to currency issues, to construct the needed global industry indices. But again, if there were hundreds of stocks, even the use of industry indices in each country would provide a major improvement.

3.2.1.2 Diagonal model

Jorion (1997, 158) describes a diagonal model originally proposed by Sharpe (1964) in the context of stock portfolios. The model is:

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i$$
$$E[\varepsilon_i] = 0,\ E[\varepsilon_i^2] = \sigma_{\varepsilon,i}^2,\ E[\varepsilon_i R_m] = 0,\ E[\varepsilon_i \varepsilon_j] = 0 \tag{3}$$

So, the equation is familiar from the CAPM analysis but here it is used just to simplify the variance-covariance matrix, not to estimate expected returns. The variance of an asset i can be decomposed:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon,i}^2 \tag{4}$$

The covariance between two assets is:

$$\sigma_{i,j}^2 = \beta_i \beta_j \sigma_m^2 \tag{5}$$

As a result, written in matrix notation, the variance-covariance matrix is:

$$\Sigma = \beta\beta^{T}\sigma_{m}^{2} + D_{\varepsilon} \tag{6}$$

where β is the beta vector and D_ε is a diagonal matrix consisting of residual variances. This is the diagonal model, which reduces the number of parameters needed to estimate quite dramatically. For 300 assets, a full variance-covariance matrix would require 45150 estimates but the diagonal model requires only 601 (the betas, residual variances and the market variance). But it requires regression run on each asset to estimate the residual variance. Furthermore, the actual data requirements do not change for we need all the same data as when using the full Σ.

3.2.1.3 Beta model

The beta model is a further simplification of the diagonal model. The variance of our portfolio according to the diagonal model is:

$$Var(R_p) = w^T \Sigma w = (w^T \beta\beta^T w)\sigma_m^2 + w^T D_\varepsilon w \tag{7}$$

The latter term consists of $w_i^2 \sigma_{\varepsilon,i}^2$ which will be very small when the portfolio is well diversified and has a lot of assets. So, the latter term converges to zero as the n (number of securities in the portfolio) increases. The variance of the portfolio therefore becomes:

$$Var\ (R_p) \rightarrow (w^T \beta\beta^T w)\sigma_m^2 \tag{8}$$

This equation is the beta model and it reflects dependence on only one factor, the market return. In case of 300 assets, it requires only 301 estimations (the betas and the market variance) which make it very attractive. Jorion (1997, 228) gives some numerical examples on the accuracy of both these models but he considers only three assets and therefore it is no surprise that the beta model underestimates the "true" (full variance-covariance matrix) VaR whereas the diagonal model performs very well.

The beta model offers (at least theoretically) a way to reduce the number of needed estimations considerably but it has some important shortcomings. First, in the case of a multicurrency stock portfolio, what is market return and how we should treat currencies? One way would be to consider all processes converted into a common currency and to use some worldwide stock market index. Second, the method as such considers only stocks. But if we have other instruments (bonds, options, commodities etc.) and we would like to calculate total portfolio VaR, the solution does not work. Third, we still need a lot of data for the estimation procedure although we do not have to estimate so much parameters (unless betas are acquired straight from other sources). For these reasons, the beta model might be most suitable for only domestic, pure, well-diversified stock portfolios.

3.2.1.4 Issues in equity-specific methods

As is evident from the discussion above, the portfolio structure is about to affect significantly the results of the VaR estimation. Johansson *et al* (1999, 106) draw similar conclusions as they apply twenty different VaR models based on three forecasting techniques to three equity portfolios of increasing degrees of diversification. To obtain accurate risk estimates, the characteristics of the portfolio must determine the technique applied. Chapter 4 gives empirical evidence regarding the case portfolio and different equity mapping techniques.

3.2.2 Mapping cash flows

When bonds and other instruments with known nominal cash flows are decomposed, result is a huge amount of individual cash flows in different currencies, maturities and risk classes. This is true even for a relative modest portfolio in terms of different assets. Instruments, that can be mapped to individual cash flows include following: floating rate notes (FRN), forward rate agreements (FRA), interest rate futures, interest rate swaps, structured notes, currency forwards, currency swaps etc. (see Dowd (1998, 80-87) or Longerstaye *et al* (1996, 107-117)).

As all these cash flows need to be discounted in full valuation Monte Carlo for each iteration, methods have been developed to map these cash flows in various ways to ease up things. Three common methods are described in Longerstaye *et al* (1996, 107-108): duration, principal and cash flow map. The duration map associates the market value of the bond at the time specified by its Macaulay duration whereas principal map allocates it to the maturity date. While very simple to implement, they are limited to only bonds and furthermore considered being too radical simplifications of reality.

The third alternative, which is also used by the RiskMetrics system, simplifies the structure by mapping individual cash flows into prespecified maturities (so-called RiskMetrics vertices, Longerstaye *et al*, 1996, 117). The procedure involves splitting all cash flows between the two closest RiskMetrics vertices. The procedure weights cash flows such that following conditions hold (Longerstaye *et al*, 1996, 188): market value, market risk and cash flow sign are preserved. The actual calculations required to achieve this are laid out in Longerstaye (1996, 119-120). The most serious problem with the algorithms is that as they include solving a quadratic equation, it is possible to have only imaginary roots.

Despite being rather complex, the RiskMetrics mapping procedure offers some insights on how to improve the VaR calculations. For example, suppose we have portfolio, which can be decomposed to 500 cash flows (in modeled portfolio, the number varied between 250 and 550). Now, if we run Monte Carlo simulation with 50 000 rounds, we have to re-value 25 million cash flows. That is no trivial task. If we had a RiskMetrics system for, say, 3 different currencies, we could map all our cash flows to a maximum of 3*14=42 vertices (RiskMetrics system provides 14 vertices for each country). Running 50 000 re-valuations require only 2.1 million calculations, which is only 8.4% of the original burden.

But the procedure is applicable only when considering cash flows which can be valued using the risk-free term structure. If portfolio contains corporate bonds or other instruments, which contain a credit spread over the risk-free term structure, the method fails. The credit spreads are discussed further in section 3.3, but as they play a significant role in the case portfolio, cash flows are not mapped in the RiskMetrics way.

3.2.3 Statistical methods to reduce dimensionality

It should be noted that with these statistical methods, data reduction is not in terms of how much data has to be collected, as all original variables are needed to form the inputs. Therefore they are no remedies for VaR analysis in terms of actual data requirements. The real benefits come from interpretation and further processing.

3.2.3.1 Principal components analysis

3.2.3.1.1 Definition

The basic idea in principal components analysis (PCA) is to form new variables (principal components) which are linear and uncorrelated (orthogonal) combinations of the original variables. The PCA requires either the correlation or the covariance matrix of the original variables as its input. Let the Σ be the covariance matrix for variables $\boldsymbol{X}=[X_1,..,X_p]$. The ith principal component is (Mustonen, 1995, 57):

$$Y_i = e_i^T \mathbf{X} = e_{1i} X_1 + ... + e_{pi} X_p;\ \mathrm{i} = 1,.., \mathrm{p} \tag{9}$$

where e_i are eigenvectors. Geometrically, these linear combinations represent the selection of a new coordinate system obtained by rotating the original system with original variables as coordinate axes (Johnson and Wichern, 1982,362). As a statistical method, PCA's objectives are data reduction and interpretation. If a substantial amount of the total variance in the data is accounted for by a few principal components, then we can use these for interpretational purposes or replace the original variables with them in further analysis.

3.2.3.1.2 Solution

The solution of PCA comes from the spectral decomposition of Σ (for exact mathematical derivation, see Mustonen (1995, 57-63):

$$\Sigma = E\Lambda E^T = \lambda_1 e^{(1)} e^{(1)} + \ldots + \lambda_p e^{(p)} e^{(p)}; \ \mathrm{i} = 1,.., \mathrm{p} \tag{10}$$

where Λ is a diagonal matrix of eigenvalues ($\lambda_1 \geq \ldots \geq \lambda_p \geq 0$) and $E=[e^{(1)},..,e^{(p)}]$ is an orthogonal matrix composed of eigenvectors $e^{(1)}, \ldots, e^{(p)}$. The first principal component is the one with the largest eigenvalue (λ_1, also the variance of the component) and its value is $Y_1 = e^{(1)T}\mathbf{X}$. Other principal components are formed analogously.

The foolproof way to actually calculate the eigenvectors and corresponding eigenvalues of symmetrical matrix (Σ is symmetrical) is to use the Jacobi method. The idea behind the Jacobi method is to systematically reduce the quantity (Golub and Van Loan, 1989, 445):

$$off(A) = \sqrt{\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij}^2} \tag{11}$$

i.e., to zero the off-diagonal elements (A denotes the matrix under scrutiny). This is done through a series of plane rotations of the form:

$$J(p,q,\theta)=\begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & & & & & \vdots \\ 0 & \cdots & c & \cdots & s & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -s & \cdots & c & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \tag{12}$$

where c and s are the cosine and sine of the rotation angle θ, so $c^2+s^2=1$. The basic procedure (Golub and Van Loan, 1989, 445) is (1) to choose an index pair (p,q) that satisfies $1\leq p<q\leq n$, (2) computing a cosine-sine pair (c,s) such that

$$\begin{bmatrix} b_{pp} & b_{pq} \\ b_{qp} & b_{qq} \end{bmatrix}=\begin{bmatrix} c & s \\ -s & c \end{bmatrix}^T \begin{bmatrix} a_{pp} & a_{pq} \\ a_{qp} & a_{qq} \end{bmatrix}\begin{bmatrix} c & s \\ -s & c \end{bmatrix} \tag{13}$$

is diagonal, and (3) overwriting A with $B=J^TAJ$. While Golub and Van Loan (1989, 444-459) lay out the mathematical foundations of the Jacobi method, a detailed application in C-language can be found in Press *et al* (1992, 463-469). It can be converted easily to run in the Excel's Visual Basic environment.

3.2.3.1.3 Issues in principal component analysis

As a dimensionality reducing technique, the method is efficient if the first few principal components account for a large portion of the total variation. This can be assessed by examining the eigenvalues: the larger the first couple of them, the better. The efficiency of the PCA really boils down to correlations: if variables are orthogonal in the first place (correlations are zero) then each principal component would account for the same amount of variance. With high correlations, the method is really efficient. (Sharma, 1997, 75-76).

3.2.3.2 Factor analysis

3.2.3.2.1 Definition

The factor analysis (FA) is similar to the PCA but the differences are not trivial. In the FA, the variation of variables is divided into common and specific variation. Furthermore, with an orthogonal rotation, the aim is to create a simple structure for resulting factors to aid the interpretation of the results. In the FA, the question is not about explaining the most of the total variation with new variables but rather finding a hidden and low-dimensional structure using the correlations between original variables. (Mustonen, 1995, 75)

When considering p variables and r factors ($r<p$), the equation of the FA is:

$$X_i = \mu_i + a_{i1}F_1 + ... + a_{ir}F_r + U_i;\ \mathrm{i} = 1,.., \mathrm{p} \tag{14}$$

Written in matrix notation, (10) becomes:

$$X = \mu + AF + U \tag{15}$$

where A is a $p\times r$ dimensioned factor matrix (factor loadings), F represents the r common factors and $U=[U_1,..,U_p]$ represents the specific factors. When the factors are uncorrelated, the covariance matrix of the original variables can be expressed as (Mustonen, 1995, 76):

$$\Sigma = AA^T + \Psi^2 \tag{16}$$

where Ψ^2 is a diagonal matrix and its elements are the variances of specific factors.

3.2.3.2.2 Solution

The simultaneous estimation of the factor loadings and the variances of specific factors is not a trivial task and it requires the use of some iterative procedure. Two common methods are the principal factor method (which is easily confused with the PCA) and the maximum likelihood method. The principal factor solution to factor analysis is a modification of PCA. As a result, the factor matrix A can be found by minimizing the following equation:

$$\left\| R - \Psi^2 - AA^T \right\|^2 \tag{17}$$

The method requires that we have some initial guess on specific variances (Ψ^2). Then the solution is found through an iterative process. First, we solve A using some assumption about Ψ^2 and then we recalculate Ψ^2 by: $\Psi^2=1-h^2$, where h^2 is vector of squared sums of factor loadings (communalities). Then we use this result as a new input and calculate A again. The process requires few rounds to converge to a solution (Mustonen, 1995, 78).

In maximum likelihood method, the original variables are assumed to be from multinormal distribution $N(\mu, \Sigma)$ where $\Sigma=AA^T+\Psi^2$. The following log-likelihood is then maximized:

$$\log \mathrm{L}(\mu,\Sigma) = -\tfrac{1}{2}\left[pn\log(2\pi) + n\log|\Sigma| + \mathrm{tr}\left(\Sigma^{-1}\mathrm{M}\right)\right] \tag{18}$$

Where n is the number of observations, p is the number of original variables and M is a moment matrix calculated from the sample (Mustonen, 1995, 79).

3.2.3.2.3 Factor scores

In the PCA, the values for principal components are obtained easily after the eigenvectors have been solved (equation (9)). The causality goes from original components to principal components and this is why the original variables are called formative indicators of the components. However, in the FA the variables are considered as functions of the hidden common factor(s) and the unique factors. Hence the variables are called reflective indicators. (Sharma, 1996, 128)

In VaR analysis, we are interested in the factor scores because we need them to estimate the variances and covariances with other variables in further analysis. The factor scores F cannot be directly solved from $X=\mu+AF+U$. The solution is to employ a sort of a regression to find estimates for F by $K(X-\mu)$. The equation for K is (Mustonen, 1995, 90):

$$K = (A^T \Psi^{-2} A + I)^{-1} A^T \Psi^{-2} \tag{19}$$

3.2.3.2.4 Issues in factor analysis

The factor analysis requires the most in terms of computing power. As the solution methods require optimization and sophisticated algorithms, the FA is clearly the Black-Box approach of the alternatives considered. The maximum likelihood method may suffer from the same drawback as the conventional variance and covariance forecasts: it gives equal weight to all observations. The normality assumption in maximum likelihood estimation is also a drawback but it should be noted that in general, maximum likelihood estimation may be carried out with any probability density function.

3.2.4 Selection criteria

Of the methods considered here, the representative mapping is by far the easiest method to implement and reduces also the actual data requirements by a sizeable amount. Should we add other asset classes into the analysis, this method has no problems to cope with the situation. In this thesis, all Finnish shares are mapped according to HEX industry indices. The empirical section provides evidence how misleading the use of HEX or HEX portfolio index might be. All the foreign equities are mapped to the relevant country index.

In VaR analysis, the statistical methods are found most useful when considering bonds. The yields are highly (although not perfectly) correlated (good for PCA) and highly homogenous (good for FA). Litterman and Scheinkman (1991) use utilize FA and conclude that movements in interest rates can modeled using just three factors: they interpret the factors as level, steepness and curvature (Litterman and Scheinkman, 1991, 57-58). Singh (1997) obtains similar results both for bonds and for some currencies. Bliss (1997b) points out that the statistical methods explain why single factor theoretical models for equity returns fail: equity returns are highly heterogeneous and contain a lot of individual variability. Jamsihidian and Zhu (1997) employ PCA in the context of scenario simulation and achieve quite remarkable results in terms of making the calculations more effective.

The selection criteria favor again the more simple approach, the PCA. FA is cumbersome to estimate, contains normality assumption and the underlying factor scores are more difficult to obtain. To illustrate the difference of these methods, both PCA and FA are used to estimate the Σ of the Finnish equity portfolio. Results of FA were obtained using a statistical add-in for Excel, XLSTAT 4.2.

3.3 ZERO-COUPON YIELD CURVE AND CREDIT SPREADS

3.3.1 Basic bond pricing equations

The estimation of a default-free zero-coupon yield is one of the major problems in finance. It stems from the fact that in practice we observe only prices for coupon bearing instruments although nowadays there exists also active markets for stripped instruments (namely in the USA). So, the zero-coupon yields must be derived from the prices of ordinary government bonds using some sophisticated method. The starting point is the simple bond pricing function (Bliss, 1997a, 5):

$$P_j = \sum_{i=1}^{m} C_{ij}\delta(t) = \sum_{i=1}^{m} C_{ij}e^{-y(t)t} \tag{20}$$

where P is the price for bond j, m is the number of cash flows, C is the ith cash flow, t is the time to ith cash flow, $\delta(t)$ is a discount function and $y(t)$ is the corresponding discount rate function. The discount function is usually transformed into a discount rate curve by $y(t)=-log[\delta(t)]/t$, hence the exponential. Unfortunately, many market frictions affect the observable prices (bid-ask spreads, tax-clientele effect, liquidity premia etc.) so the equation does not hold exactly. This leads to an inexact relation such as (Bliss, 1997a, 5):

$$P_j = f\lfloor C_{ij}, y(t)\rfloor + \varepsilon_j \tag{21}$$

where f captures all relevant aspects about bond pricing and $y(t)$ is fitted to minimize some function of the error term ε (sum of bond pricing errors). The selection of an appropriate weighing scheme for errors must be solved also. Bliss (1997a, 9) suggests using the inverse of the duration of the issue. After deciding whether to fit discount function or discount rates directly, the next step is to decide on the functional form to be used in the approximation. The most commonly used alternatives are cubic splines (McCulloch, 1975), exponential forms (Nelson and Siegel, 1987) and piecewise linear method (Fama and Bliss, 1987).

3.3.1.1 Splines

McCulloch (1975) suggested the use piecewise polynomial functions (splines) to approximate the discount function. Intuitively, a polynomial spline can be thought of as a number of separate polynomial functions, joined smoothly at a number of so-called knot points. Splines are flexible enough to model any reasonable-shaped discount function, in fact they can be too flexible (Seppälä and Viertiö, 1996, 14). As the number of knot points are increased, the quality of the fit increases. The stability of the curve decreases simultaneously, which may lead to unstable implied forward rates. Furthermore, there are no clear-cut solutions on how to select the appropriate number of knot points and how to locate them.

3.3.1.2 Nelson-Siegel exponential form

Nelson and Siegel (1987, 475) suggested a parsimonious exponential form to model the implied forward curve as:

$$f(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\tau}\right) + \beta_2\left[\left(\frac{t}{\tau}\right)\exp\left(-\frac{t}{\tau}\right)\right] \tag{22}$$

where $f(t)$ is the forward rate at maturity t and β_0, β_1, β_2 and τ are the parameters to be estimated. The estimation is done from the equation of the discount function:

$$\delta(t) = \exp\left\{-t\left[\beta_0 + (\beta_1 + \beta_2)\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)\frac{t}{\tau} - \beta_2 \exp\left(-\frac{t}{\tau}\right)\right]\right\} \tag{23}$$

This approach avoids the problem with knots inherent in spline-based models. The trade-off is the less flexible functional form and typically it fits the data less well. Literature contains a vast amount of extended specifications to increase the flexibility of this type of models.

3.3.1.3 Piecewise linear method

This is by far the easiest method to approximate the discount rates: it is an iterative procedure where rates are defined recursively from the shortest instrument onwards. Among practitioners it is called bootstrapping for obvious reasons. Fama and Bliss (1987) appear to be first to publish an implementation of this method, they use it to extract forward rates from U.S. Treasury bonds. The resulting zero-coupon yield curve from bootstrapping contains as many key-rates as the number of issues used in the extraction process. Between the key-maturities, the relevant discount rates are found using a linear estimate:

$$y(t) = y(t_0) + \left(y(t_1) - y(t_0)\right)\frac{t - t_0}{t_1 - t_0} \qquad t_o < t < t_1 \tag{24}$$

3.3.2 Zero-coupons and VaR calculations

Zero-coupon yields are an essential element of any VaR calculations as they are used to price bonds and different derivatives. The choice between different methods is not an obvious one. Once again, it is not quite clear which method produces the best results. We never observe any zero-coupon yield curve, so comparisons are necessarily indirect. Bliss (1997a) and Deacon and Derry (1994) provide some insights into this issue. Bliss (1997a, 25-28) observes that while the Fama-Bliss approach produces most accurate results, it may suffer from over-fitting. The Extended Nelson-Siegel and cubic spline performed comparably to each other. Deacon and Derry (1994) provide no statistical comparisons but they emphasize the importance of the implied forward rate curve and the need to take market specific frictions into account. All in all, as Bliss (1997a, 28) puts it, term structure estimation is an art. Theory offers no unconditionally superior method to modeling.

In the context of VaR, it is important to consider the shape of the implied forward rate curve. Otherwise the valuation of interest rate derivatives might produce awkward results. Mathematically, the discount function should be both positive and monotonically decreasing. The bootstrapping may not fulfill these preconditions should it suffer from overfitting. However, it has other attractive features when considering VaR calculations using Monte Carlo simulation and full valuation. For modeling purposes, we have to select some key-rates, which are then put through the PCA. When these principal components are simulated and sampled back to key-rates, the spline and Nelson-Siegel approach would require re-fitting for each iteration. Here bootstrapping and linear interpolation offer a major reduction in computational burden. The error weighing problem is also irrelevant as bootstrapping produces prices that equal the observed ones (Bliss, 1997a, 10). Simplicity is favored again and bootstrapping is utilized to derive default-free term structures for the purposes of this thesis.

3.3.3 Credit spreads

The risk of default complicates the pricing of corporate bonds considerably. In fixed income markets, credit spread refers to amount by which the corporate bond yield exceeds the yield of some suitable default-free bond (maturities are usually matched). These spreads convey information about the expectations on company's possible bankruptcy but the picture is blurred due to for example debt seniority. Unfortunately, this information is scarce (particularly in Europe and in Finland) as there exists only few liquid corporate bonds compared to amounts issued.

The modeling of credit spreads gets difficult when we abandon the market convention of yield-to-maturity –spreads and start to consider an entire spectrum of default-free rates. The method used in this thesis is again as simple as possible. Using observed corporate bond price and derived zero-coupon yield curve, credit spread is defined as a constant rate, which is added to the zero, yields when discounting bond cash flows.

It is constant over entire maturity but changes from day to day as interest rates evolve. The estimation requires the use of iterative procedure (e.g. Solver in Excel) similarly when calculating the normal yield-to-maturity. Approach is atheoretic but yields some interesting results as the empirical section later demonstrates.

In reality, the term structure of credit spreads is upward sloping, because subsequent coupon payments depend on each other. Thus, if the firm defaults the first coupon payment, all subsequent coupons are also defaulted on (Merton, 1974, 467). In literature, credit spreads are less discussed phenomenon as the topic depends also on the model used in bond pricing. Examples may be found in Merton (1974), Jarrow, Lando and Turnbull (1997) and Duffie and Singleton (1999). Litterman and Iben (1991) describe more practically oriented approach but the underlying assumptions are similar to more theoretical ones.

While all models mentioned above calculate reasonable credit spreads with different inputs and assumptions, they do not consider as such the variation of credit spreads over short intervals. The VaR estimates depend on the daily variation of credit spreads as they affect the pricing of corporate bonds. So, calibrating credit spreads using advanced model and assuming they remain constant is a faulty assumption as the empirical section demonstrates. Litterman and Iben (1991, 61) show how the generic credit spreads have evolved over time and conclude that even the best model can provide only a measure of credit risk implied by the market price. To model the change in corporate specific credit spreads over time, one would first need to extract historical implied spreads and examine what could be the key factors affecting them. The issue is very complex as it is about to depend on company, issue, industry and general macro events. Furthermore, in the case of sovereign bonds and bonds of unquoted firms, publicly available information is scarce.

All in all, credit spreads call for careful modeling. The simple and atheoretic approach employed here is used rather to highlight the importance of the issue than to solve it. Due to enormous complexity of explicit modeling of credit spreads, the issue is out of the scope of this thesis.

3.4 DERIVATIVES PRICING

Derivatives pose a significant problem for VaR calculations. Unless there exists a closed form solution, each re-valuation would require an additional simulation within simulation for derivatives position. Obviously, at some point the computational burden would become infeasible although this method would produce the most credible VaR figures. Remedy could be some sort of a grid approach (Alexander, 1998, 204), in which portfolio is revalued only at prespecified points. These grid points would be chosen to be representative of a wide but realistic set of underlying factor levels. The scenario simulation approach suggested by Jamshidian and Zhu (1997) is a specific application of this general idea.

In the case portfolio, derivatives constitute a sizeable (although time-varying) position. Almost all are standardized european options, futures or forwards. The only exceptions are currency derivatives, which include some knock-out options but as this asset class is not included in this analysis, this poses no problems. All european calls and puts are prized using the formulas laid out by Black and Scholes (1973, 644):

$$
\begin{aligned}
C &= SN(d_1) - e^{-rt}KN(d_2) \\
P &= e^{-rt}KN(-d_2) - SN(-d_1)
\end{aligned}
\tag{25}
$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

where $N(\cdot)$ is the cumulative density function normal, S is the spot price of the underlying, K is the exercise price, r is the risk free rate, t is the time to maturity in years and σ is the volatility of the underlying from now to maturity.

Forwards and futures are priced according to cash-and-carry principle. The pricing equation is (Jarrow and Turnbull, 1996, 46):

$$F_0 = S_0 * e^{t*r_t} \tag{26}$$

where F_0 is the futures/forwards price at time 0, S_0 is the spot price of the underlying, t is time to maturity in years and r_t is the risk-free rate for that period. This formulation ignores the expected dividend yield.

Unfortunately, it was quite impossible to obtain market data for options so the valuation was based totally on mark-to-model –approach. This should not affect the results too much for arbitrage reasons. The only deviation from reality is apparently the implied volatility, which may differ from the calculated estimate based on market data. The main point of the calculations was to give an approximation of the risk of the derivatives portfolio and to examine how it has affected the total equity risk.

3.5 THE ESTIMATION OF KEY PARAMETERS

Two type of estimators hold the key position in all VaR systems: the variability of financial time-series and their relations to each others. The variability is defined here in terms of variance and relationship structure in terms of covariances. Theory offers many alternatives to estimate the variance of financial time-series: random walk, long-term mean, moving average, exponential smoothing, regression models, the GARCH-family, stochastic variance and implied values from option prices. The issue is complicated by the fact that we must simultaneously model the entire multivariate distribution with covariances, which leaves us basically with three choices: moving averages, exponential smoothing and GARCH-models (Alexander and Leigh, 1997, 52).

3.5.1 Moving averages

The n-periodic moving average estimate (or sample variance) at time T assuming zero expected return is simply equally weighted average of past squared returns:

$$\hat{\sigma}_T^2 = \sum_{t=T-n}^{T-1} \tfrac{1}{n} r_t^2 \tag{27}$$

For the covariance, the equivalent estimate is:

$$\hat{\sigma}_{T:1,2} = \sum_{t=T-n}^{T-1} \tfrac{1}{n} r_{1t} r_{2t} \tag{28}$$

While very simple to estimate, this method has an obvious drawback: it gives an equal weight to all observations. For high-dimensional VaR-models, which require a long estimation period to achieve positive definite property, these estimates may become too insensitive. Furthermore, it assumes that the "true" underlying variance is constant, fails to allow for volatility clustering and ignores dynamic information from the temporal ordering of observations (Dowd, 1998, 94).

3.5.2 Exponential smoothing

The JP Morgan's RiskMetrics system utilizes an alternative, which should be an improvement over the sample variance: the exponentially weighted moving average (EWMA). Each observation is given a weight, which gradually decreases the more distant it is. The equations are (Longerstaye *et al*, 1996, 78, 83):

$$\hat{\sigma}_T^2 = (1-\lambda)\sum_{t=1}^{T} \lambda^{t-1} r_t^2 \qquad (29)$$

$$\hat{\sigma}_{T:1,2} = (1-\lambda)\sum_{t=1}^{T} \lambda^{t-1} r_{1t}^2 r_{2t}^2$$

The only parameter needed to estimate is λ, which makes this model quite attractive. Actually, the EWMA-estimator is a special case of an IGARCH model, with constant set to zero and the sum of coefficients set to 1. RiskMetrics utilizes constant l (0.94 for daily and 0.97 for monthly estimates) in all cases. For more on how the λ was set, see Longerstaye (1996, 97-100).

Alexander and Leigh (1997, 52) point out that EWMA method creates positive semi-definite Σ only if the same smoothing constant is applied to all series. So, the accuracy of the method cannot be improved by separate parameter estimation. They mention also that because the effective number of days (λ=0.94) used is only 74, models with more than 74 risk factors have Σ less than full rank. This should be considered as possible limitation for its use.

3.5.3 GARCH-family

This group of models takes a different approach to modeling time-series. The obscure term heteroscedasticity stands for changing variance, which refers to the empirical observation of volatility clustering in time series data. Engle (1982) was the first to propose this kind of model. His approach (ARCH) was later generalized by Bollerslev (1986) and ever since a growing number of variations has appeared in the literature. Here is presented only the GARCH(p,q) to illustrate the main properties of these models (Bollerslev, 1986, 309):

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{30}$$

The most common variation is the GARCH(1,1), e.g. only one lagged error square and autoregressive term. The model parameters need to fulfill certain conditions, namely $\omega>0$, $\alpha,\beta\geq 0$ and $\alpha+\beta\leq 0$. The size of these parameters determines the shape of the resulting volatility time series (Alexander,1998,137): large β coefficient indicates that shocks to conditional variance take a long time to die out (persistent volatility); large α mean that variance is quick to react to market movements ("spiky" volatility). The constant ω determines the long-term average level of volatility to which GARCH forecasts converge.

In a multidimensional framework, things start to get really difficult. Consider just a simple bivariate case of GARCH(1,1) (Alexander, 1998, 145):

$$\begin{aligned}
\sigma_{1,t}^2 &= \omega_1 + \alpha_1 \varepsilon_{1,t-1}^2 + \beta_1 \sigma_{1,t-1}^2 \\
\sigma_{2,t}^2 &= \omega_2 + \alpha_2 \varepsilon_{2,t-1}^2 + \beta_2 \sigma_{2,t-1}^2 \\
\sigma_{12,t} &= \omega_3 + \alpha_3 \varepsilon_{1,t-1} \varepsilon_{2,t-1} + \beta_3 \sigma_{12,t-1}
\end{aligned} \tag{31}$$

The estimation requires the use of maximum likelihood method in which an assumption of conditional normality is (usually) made. Here one must careful if another distribution assumption is made when modeling the stochastic process to create price paths. It would be clearly inconsistent first to produce variance estimates using another distributional assumption.

3.5.4 Selection criteria

The accuracy of different variance forecasts is extremely difficult to assess for we never actually observe the "true" variance. So, an indirect measure must be used for benchmarking but even the objective statistical evaluation procedures have produced conflicting results (see Alexander and Leigh (1997), Boudoukh et al (1997), Brailsford and Faff (1996), Dimson and Marsh (1990) and West and Cho (1995)). To summarize one should favor parsimonious models, avoid pre-test biases and base reasoning only on out-of-sample results (Dimson and Marsh, 1990).

In a multivariate environment, we must also ensure that our estimated variance-covariance matrix is positive definite. Otherwise portfolio variance might be negative. This calls for consistency in the estimation methods used: if we use moving average method to estimate variances, we must use it to estimate also covariances. This undermines especially the usefulness of GARCH-models, for it is notoriously difficult to estimate a multivariate model as the number of parameters needed to estimate grows exponentially with the number of covariances. Alexander (1998, 147-148) proposes an orthogonal GARCH-model in which the risk variables are first made orthogonal by PCA. Then, GARCH-model is applied to produce variance estimates for these new factors.

Again, simplicity is the rule and therefore moving average and EWMA-method are both assessed in empirical part. True variance for a given day is defined as squared return and two criteria are used: mean squared error (MSE) as a standard statistical measure and Kupiec likelihood ratio described in section 3.8 functions as an operational evaluation tool.

3.6 PRICE PATH GENERATION

3.6.1 Stochastic processes

For Monte Carlo simulation, the selected stochastic process determines the behavior of the simulated variables. For equities, the *de facto* method to simulate price paths is the geometric Brownian motion (GBM) described by the stochastic differential equation:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dz \tag{32}$$

where S is the underlying, μ is the drift, σ is the volatility and $z \sim N(0,\sqrt{t})$ is a standard Wiener process. Assuming log-normal returns, the discretized equation becomes:

$$S_t = S_0 \exp\left[\left(\mu - \tfrac{1}{2}\sigma^2\right)\Delta t + \sigma \Delta z\right] \tag{33}$$

The use of logarithmic returns instead of arithmetic ones eliminates one type of a model risk. In GBM with arithmetic returns, there is positive probability for returns in excess of –100%, which would mean negative prices.

For interest rates, the GBM is applicable only for really short-run simulations such as daily ones. In the longer run, interest rates tend to mean revert and this is not captured by the standard GBM. There exists a vast amount of different alternatives to model the mean reversion property of interest rates (see Alexander, 1998, 173-176). The issue with all mean reverting models is how to estimate the needed parameters to achieve the mean reverting property. As the focus in this thesis is on modeling the daily changes, the GBM with logarithmic returns is used also for interest rates.

3.6.2 Some pitfalls in time-series modeling

3.6.2.1 Normality assumption

Mandelbrot (1963) and Fama (1965) were the first ones to describe the common phenomenon about statistical properties of financial time-series: normality assumption does not seem to hold. Compared to normal, the empirical return distributions are peaked and have fat-tails. Statistically speaking, the return series have excess kurtosis and skewness. This is extremely bad news for VaR analysis as the interest is particularly on tail returns. Taking account the heteroscedastic variance, we must speak of conditional normality. It means that standardized distribution rather than the observed returns are assumed to be normal. So, hereinafter testing normality refers to testing conditional normality.

There exists many ways to test the normality assumption in univariate environment, here are used χ^2-test, parametric Kolmogorov-Smirnov –test and a quantile-to-quantile test. χ^2-test is based on frequency distribution of sample returns. These empirical observations are compared with theoretical ones to form the test statistic, which is χ^2–distributed. To test the normality assumption, we must first estimate the sample mean and variance. Then the sample is divided into suitable intervals and empirical observations are recorded along with ones based on normal distribution. Test statistic is formed as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(f_i - e_i)^2}{e_i} \sim \chi^2(n) \tag{34}$$

where f_i is the empirical frequency, e_i the corresponding theoretical frequency, k is the number of intervals and $n=k-r-1$ where r is the number of parameters estimated. The null hypothesis H_0: $F(x)=F_0(x)$ is rejected if the test statistic exceeds $\chi^2_\alpha(n)$.

The Kolmogorov-Smirnov –test is based on comparing the sample cumulative distribution function with the theoretical one. Two-sided test statistic is formed as:

$$D = \max_x |F_0(x) - S(x)| \tag{35}$$

where F_0 is the theoretical cumulative distribution and S is the empirical one. The resulting test statistic is compared to the critical value, which must be looked from the table. For large sample size, the critical value at α=5% for two sided test is approximately $1.36/\sqrt{n}$. The null hypothesis H_0: $F(x)=F_0(x)$ is rejected if the test statistic exceeds the critical value.

One useful method to illustrate the distributional properties of time-series is the quantile-to-quantile (Q-Q)–plot. Along the X-axis are empirical observations in ascending order and along the Y-axis are the theoretical counterparts. As the empirical observations are estimates of the f fractile of the theoretical distribution, these implied f values are found by the following transformation (Fama, 1965, 52):

$$f = \frac{(i - \frac{1}{2})}{n} \tag{36}$$

These probability levels are then transformed to actual theoretical values using the inverse of the cumulative distribution function. Visually, the observations follow theoretical distributions if the plotted observations form a 45° straight line. More preferably, the linearity of the Q-Q plot can be assessed by calculating the correlation coefficient between the sample and theoretical quantiles. The null hypothesis H_0: $F(x)=F_0(x)$ is rejected if the sample correlation coefficient does not exceed the critical value of 0.998 (depends on the sample size) at confidence level α=5% (Sharma, 1996, 377, 466).

Despite the problematic empirical evidence, the normality assumption is particularly useful when modeling multivariate distributions. The problem is not how to create uncorrelated random numbers with certain distributional properties but how to apply the desired correlation structure into them. There exists tests to examine the multivariate normality property but they rely on homoscedastic Σ. This is problematic, for multivariate normality property is not guaranteed even if all marginal distributions satisfy univariate normality (Mustonen, 1995, 28). In addition to that, all possible linear combinations of underlying variables must be normally distributed. As suitable test does not exist, the multivariate normality is assumed to hold here.

3.6.2.2 Autocorrelations

The independence of successive returns on individual time-series must be tested also. In the long run, cyclical fluctuations in economic activity may cause time-series to have some predictable components. In the short-run, the problem is apparently almost non-existent as the common observation for return series is that they follow random

$$r_k = \frac{\frac{1}{T-k}\sum_{t=1}^{T-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\frac{1}{T}\sum_{t=1}^{T}(x_t - \bar{x})^2} \quad k = 0, \pm 1, \pm 2, .. \pm (T-1) \tag{37}$$

walk (Fama, 1965, 80). For return series $x_1 \ldots x_T$ the *k*th autocorrelation is defined as:

To test whether the sample is autocorrelated, a Ljung-Box test is employed. Test statistic and hypothesis are:

$$Q = T(T+2)\sum_{k=1}^{s}\frac{r_k^2}{T-k} \sim \chi_s^2 \tag{38}$$
$$H_0 : \rho(k) = 0$$

where s is fixed integer that denotes the number of autocorrelations tested. The practical maximum for s is around $T/4$ but here the interest is especially on few first as the simulation is done only one-day forward.

3.6.2.3 Expected return

One common assumption is to assume zero expected return when modeling the short-run price movements. This is totally reasonable when returns are measured using the first log-property. When past returns are calculated using first log property, the mean return over any time-period is simply:

$$\mu(t_0, t_1) = \log\left(\frac{p_1}{p_0}\right) \tag{39}$$

This is due to basic calculation rules concerning logarithms. As expected return depends only on the time window used (first and last observation), it would be rather unreasonable to model expected return this way. When modeling short-run (e.g. daily) returns, this is also well in line with the common assumption about asset prices following random walk. Jorion (1995, 510) finds that the zero expected return – assumption is unlikely to affect the estimation of variance. However, when building longer simulations, the expected return has very big impact on the outcome and neglecting it would be clearly a mistake. In the short-run, volatility is the dominating factor.

3.6.2.4 Market data

From the practical point of view, time-series modeling suffers from two problems: missing or unavailable data and unsimultaneous observations. Data availability is rarely a problem but the quality of the data may be substandard. Usually the data for given day simply do not exist due to lack of trades, but some times the reason is poor databases. There exists several statistical techniques to fill these holes (see Beder *et al*, 1999, 296-301) but in this thesis, holes were filled with previous price observation. In the Finnish equity market, the lack of daily trading is very frequent and when trades occur, the price level may change substantially. Also corporate bonds are subject to this problem. In fact, some papers had to be removed from the analysis due to data problems.

The time-zone issue complicates the estimation of covariances if the observations are recorded at very different times. In real (up and running) systems, it is easy to record observations at prespecified time but obtaining historical data to verify the accuracy of certain procedure is the problem. In this thesis, the only possibility was to use closing prices so the results must interpreted accordingly.

3.6.3 Generation of random numbers

Ensuring randomness in Monte-Carlo simulation is crucial to achieve unbiased results. Random numbers may be classified into three categories based on their "randomness":

1) True random numbers based on some physical process
2) Pseudorandom numbers generated by some deterministic algorithm
3) Quasi random sequences which are designed for example to fill certain domain

The generation of random numbers from deterministic grounds is as much art as science. A good pseudorandom number generator (PRNG) should have at least following features:

1) Generated numbers should be "random enough" and have good distributional properties
2) Numbers should be uncorrelated
3) Long cycle (e.q. a lot of different random numbers)
4) Repeatable for testing purposes
5) Fast creation

For most people, the only familiar random number generator is the one found in Microsoft Excel. It may be adequate for some small scale testing but serious simulation is right out of the question. Therefore this study utilizes a powerful PRNG called Mersenne Twister (Matsumoto and Nishimura, 1998) which has a cycle of 2^{19937}-1. In contrast, the cycle of Excel's PRNG is only 2^{15} or 32 768 (Excel 97) different random numbers. Furthermore, the Mersenne Twister is able to create a 623-dimensioned equidistribution, which makes it suitable also for large scale VaR-systems. Creators consider these properties to be the best among all generators ever implemented. The source code for different platforms may be found on the following website: http://www.math.keio.ac.jp/~matumoto/emt.html

At the first stage, the PRNG's produce uniformly (*U~(0,1)*) distributed numbers which have to be converted according to stochastic process used. In this study, the process is the standard geometric Brownian motion which requires normally distributed (*N~(0,1)*) numbers. The most popular and easiest way to convert *U~(0,1)* to *N~(0,1)* is the Box-Muller algorithm which converts a pair U_1,U_2 to (Alexander, 1988, 177):

$$\begin{aligned} Z_1 &= \sqrt{-2\log U_1}\cos 2\pi U_2 \\ Z_2 &= \sqrt{-2\log U_1}\sin 2\pi U_2 \end{aligned} \tag{40}$$

which are N~(0,1). However, the resulting Z_1,Z_2 are not genuinely independent (Alexander, 1998, 177) which calls for more sophisticated solutions. In this study, the transformation is handled by the inverse transformation method. Let Φ be the standard normal cumulative distribution and Φ^{-1} its inverse. Since Φ takes values between 0 and 1, Φ^{-1} is a function on the unit interval (0,1); and if U is uniform on the unit interval, then $\Phi^{-1}(U)$ has the standard normal distribution (Alexander, 1998, 178). Unfortunately, there exists no closed form solution to Φ^{-1} but it can approximated accurately by rational functions. This study utilizes an approximation algorithm for the standard normal cumulative distribution proposed by Moro (1995).

3.6.4 Creating correlated random numbers

So far the created random numbers are independent in all dimensions (IID, indepent, identically distributed). Next step is to modify these numbers to include the estimated variance-covariance structure. Methods to achieve this include Cholesky decomposition, spectral decomposition, Gram-Schmidt factorization and singular value decomposition (Mustonen, 1995, 193). As the Σ should be positively definite (otherwise it is not valid), the Cholesky decomposition is the most practical alternative to implement. For a $n \times n$ matrix, the definition is:

$$A = LL^T \tag{41}$$

where L is either lower- or upper triangular matrix. Golub and Van Loan (1989, 139-149) give various algorithms to calculate A, but the general solution is (Rogers *et al*, 1997, 96):

$$l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2} \qquad i = 1,..,n \tag{42}$$

$$l_{ji} = \frac{1}{l_{ii}}\left(a_{ij} - \sum_{k=1}^{i-1} l_{ik} l_{jk}\right) \qquad j = i+1, i+2,..,n$$

Once the L is solved, it is used to multiply the random number vector Z to apply the structure of A.

The spectral composition involves calculating the eigenvalues and eigenvectors of Σ, which is a much more complicated procedure. However, as the zero-coupon yield curves are modeled using the principal component analysis (see section 3.2.3.1), this technique is therefore also employed. Equation (10) defines the spectral decomposition and when we set $X \sim N(0,\Sigma)$ and $Z \sim (0,I)$, the X are achieved by (Jauri, 1997, 281):

$$X = E\sqrt{\Lambda}Z \tag{43}$$

where E is a matrix of orthogonal eigenvectors, Λ is a diagonal matrix of eigenvalues and Z is a vector of random numbers.

3.7 VARIANCE REDUCTION TECHNIQUES

The problem with Monte-Carlo simulation is the slow convergence. Although the strong law of the large numbers guarantees that eventually the simulated values reach their "true" levels, the computational burden might be excessive. The standard deviation of the estimation error falls as $1/\sqrt{n}$, where n is the sample size (Rogers and Talay, 1995, 22). So, to halve the statistical error we need to quadruple the sample size. Methods developed to reduce the sample variance include antithetic variables, control variates, stratified sampling, importance sampling and moment matching to name a few. Here attention is paid only to the moment matching technique. For a thorough discussion on other techniques, see Boyle, Broadie and Glasserman (1997, 1270-1290).

Moment matching techniques adjust the created random numbers in a way that the empirical sample exactly fits the input data. One specific approach, which is utilized in this study, is the quadratic resampling proposed by Barraquand (1995). It adjusts the empirical first and second-order moments (mean and covariances) in the following way: let Σ denote the given covariance structure, μ denotes the given mean vector, Z are the created random numbers, S is the sample covariance matrix and E(Z) is the sample mean. Modified random numbers Y are achieved by applying the modification (Barraquand, 1995, 1887):

$$Y = H[Z - E(Z)] + \mu \tag{44}$$
$$H = \sqrt{\Sigma}\sqrt{S}^{-1}$$

As with all variance reduction techniques, which modify the created random numbers, the efficiency of this method boils down to how the final measured variable behaves. The quadratic resampling is supposedly several times more efficient than the brute force alternative (depending on the problem at hand) but it has some drawbacks too (Boyle, Broadie and Glasserman, 1997, 1277).

3.8 BACKTESTING

For a casual observer, VaR may seem as a bulletproof answer to quantitative risk management when employed carefully. However, it is crucial to understand the fact that we never actually observe any realized VaR value after the event: the model forecasts an unobservable variable with unobservable variables (variances, covariances and zero-coupon yields). Furthermore, the model itself is subject to many errors (e.g. sampling errors, data problems, inappropriate models, poor assumptions, plain human error…) so the verification is not a trivial task (Dowd, 1998, 55).

The most obvious way to assess system performance is to investigate how often losses are realized in excess of the estimated VaR. If our confidence level were set to 95%, we would expect excess losses to occur 5% of the time. The practical issue is how to tell if the realized frequency of such excessive losses is sufficiently different from the predicted frequency to be statistically significant.

Kupiec (1995) proposes a test according to which the probability of observing N failures in a sample of size T is governed by a binomial process of $(1-p)^{T-N}p^{N}$. According to Kupiec, the most appropriate test of the null hypothesis that $p=p^*$ (observed probability equals the one implied by the confidence level) is a likelihood ratio (LR) test given by:

$$LR = -2\ln\left[(1-p^*)^{T-N}p^{*N}\right] + 2\ln\left[\left(1-\tfrac{N}{T}\right)^{T-N}\left(\tfrac{N}{T}\right)^{N}\right] \qquad (45)$$

This test statistic is distributed as a chi-squared with one degree of freedom under the null hypothesis. The problem is the low power: it requires relatively long comparison sample period. Otherwise test has difficulty discriminating among flawed and correct models.

4 EMPIRICAL RESULTS

This chapter presents the empirical results. First, attention is paid to convergence and variance reduction. Then come few statistical tests on time-series properties and after that, comparison of two variance and covariance estimators. Results for different equity mapping methods are followed by fixed income modeling section. Finally, backtesting results for the whole system complete the chapter. Results were calculated on 5 Pentium III workstations with 500 MHz CPU and 256 Mb memory. It took approximately 9 minutes to calculate a single VaR estimate with 50 000 simulation rounds using one workstation. So, the total machine time for backtesting totaled to over 50 hours. The code needed to produce the estimates took about twenty pages.

4.1 CONVERGENCE AND VARIANCE REDUCTION

When presenting the VaR figures, the obvious question is how reliable are they? Leaving the model risk aside, the question with Monte Carlo simulation is how many simulation rounds are needed to achieve acceptable results. What is acceptable, must be defined in forehand and partly it depends on the problem at hand. Recent developments in computer power have diminished the importance of this question but as always, there exists some tradeoff between speed and accuracy. For this, the accuracy of the simulated VaR figures is assessed as in Boyle, Broadie and Glasserman (1997). The importance of reporting estimation error is discussed also in Jorion (1996).

The calculation of accuracy is fairly simple: we just calculate the same VaR figure with different random numbers to obtain a sample of estimates. Here the sample size was selected to be 500, to keep the computational burden at reasonable limits. As the calculations took 100 machine hours, a random date was selected for estimation purposes. Sample average, standard deviation, maximum and minimum were calculated for 1 000-50 000 rounds in 5 000 steps. Results for equity portfolio are presented in Appendix A both in monetary figures and as percentages of the underlying portfolio. Results for other portfolio combinations were derived also but the general analysis is just the same.

The general result is straightforward: quadratic resampling improves the quality of the VaR estimate in terms of reduced variation. The sample standard deviation using quadratic resampling is roughly 70% of the base case all the time. This is in line with the results presented in Boyle, Broadie and Glasserman (1997, 1279): the improvement factor with moment matching is essentially constant as sample size increases. The reduced standard deviation results in improved confidence interval (calculated assuming normality) and smaller difference of sample maximum and minimum and hence gives more accurate VaR figure. For there was no significant cost in terms of increased computing time to employ quadratic resampling, the method is utilized to derive the VaR figures for the purposes of this thesis.

What comes to the number of simulation rounds, even quite a modest amount (e.g. 5 000 rounds) gives decent indicator of the risk level. However, Appendix A is a bit misleading in a sense that it gives the results of sample averages. In daily operations, it is possible to leave the system to calculate VaR estimates overnight but for backtesting purposes, it is just not possible to spend 100 machine hours per day to calculate 355 VaR estimates. Looking at the difference between sample maximum and minimum, even 50 000 simulation rounds with quadratic resampling produce difference of approximately 700 000 Euros (or 0.054%). As the backtesting was conducted based on percentages, VaR estimates were rounded up to two decimals (e.g. -2.45%). This was deemed adequate because even the realized portfolio returns may contain errors due to poor market data. In the real application, even 100 000 simulation rounds are definitely not too much. Percentages were used because all portfolios have grown in size due to cash inflow and therefore the development of risk level in monetary terms over time is blurred by this effect.

4.2 STATISTICS OF FINANCIAL TIME-SERIES

The autocorrelation issue is discussed first, for it may affect the normality tests in a serious way. All the results from statistical tests are presented in the Appendix B. The first column gives the value Ljung-Box Q and if the test statistic exceeds the critical value of approximately 3.8415 (α=5%) the time series in question is deemed to be autocorrelated. For the majority of time-series, the Ljung-Box test poses no problems. However, some markets seem suffer from serious problems, namely the emerging markets, the entire Swedish fixed income market and the European money market. Using more strict confidence level (α=1%), the other notable exceptions are Norwegian krone and two Finnish industry indices.

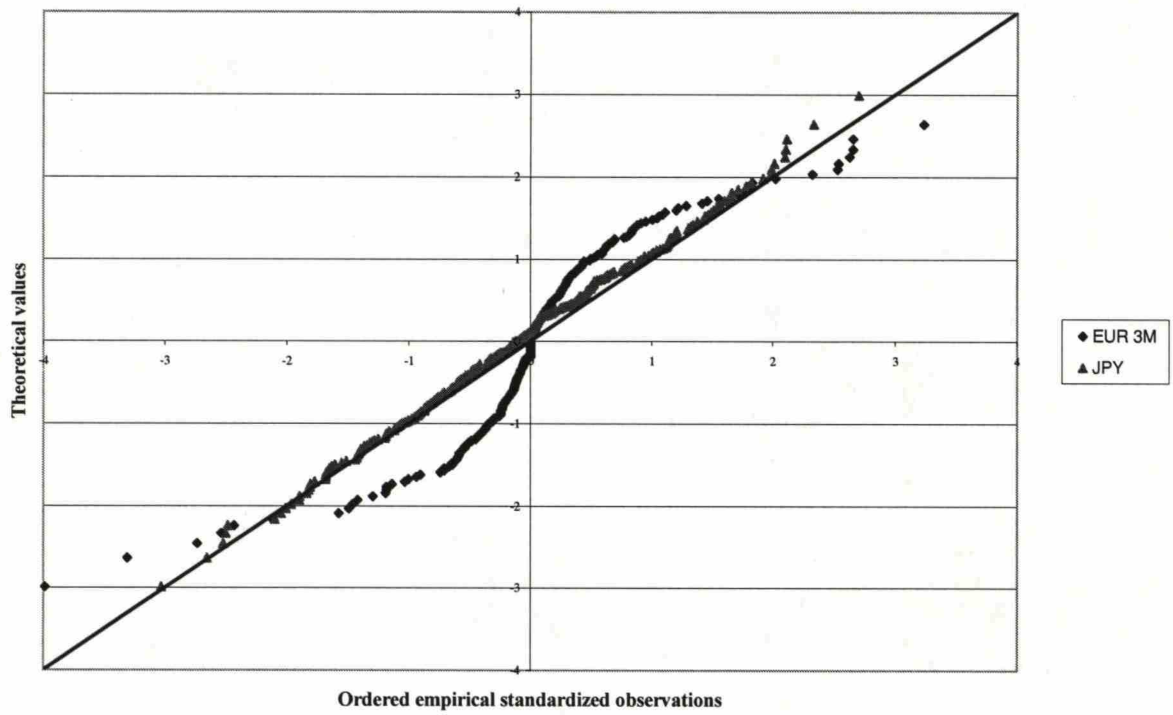
While the Ljung-Box test shows that the autocorrelations for some series are statistically significant, the question is are they big enough to cause major problems in modeling. Fitting a simple one-lag autoregressive model to suspicious series reveals that for the most part, the coefficients are too small in magnitude (-0.13 - 0.17) to be of major concern. Only Lithuanian general index, HEX forest index and one year European money market rate pose coefficients in excess of 0.2. Basically, one should first clear the time-series from serial correlation before any parameter estimations and simulations to assure sound results. In the light of these small and from the case portfolio point of view negligible (only very small positions) autocorrelation coefficients, all the time-series were used on as is basis.

Next the attention is paid to normality tests. The return observations were standardized using both an EWMA- and MA -estimate for variance and assuming zero expected return. Variance estimators were calculated using 120 days of price change information, which included also the final value to be standardized. For more on variance estimators in the VaR context, see section 4.3 below. As such, all subsequent results depend partly on the quality of the variance estimator. When calculating the theoretical quantiles, the assumptions were a variance of 1 and zero expected return. As the MA-estimate produced significantly worse results than EWMA, only EWMA results are reported in Appendix B. Corresponding MA results are mentioned in the analysis below.

The columns 2-5 give information on the statistics of the standardized observations. As can be seen, the zero expected return assumption seem to hold quite well although almost all series pose slightly positive figures whereas the variance is generally less than unity (for MA, variance tended to exceed unity). All series exhibit excess skewness and kurtosis, which indicates departure from normality. Average kurtosis across all series for MA estimates was ten times bigger than for EWMA method (4.08 versus 0.41).

For the purpose of χ^2- and Kolmogorov-Smirnov –test, the observations were ordered into 23 quantiles. The χ^2-test was not able to distinguish any departure from normality for EWMA method but for MA, 32 series indicated non-normality. The test was very sensitive to the selection of quantiles and was considered almost useless. The Kolmogorov-Smirnov –test proved to be more consistent. For EWMA method, it indicated non-normality for 20 series and for MA, 36 series. Common features for these rejected series were either large excess skewness, kurtosis or variance much smaller than unity. The most stringent test proved to be the correlation test based on quantile-to-quantile –plots. This is no wonder as one observation forms one quantile. To illustrate different shapes of these plots, Figure 4 gives an example of European 3-month money market rate and Japanese yen using the EWMA method.

Figure 4: Quantile-to-quantile –plot: EUR 3M and JPY



Should the time-series be log-normally distributed, the plotted values should trace the 45° straight line, which is drawn to aid in the analysis. When the tails of empirical distribution are longer than normality would suggest, the graphs in general take the shape of elongated S with the curvature at the top and bottom varying directly in relation to the size of so-called fat-tails. The EUR 3M is a prime example of non-normality whereas the JPY behaves very nicely according to theoretical distribution.

As the VaR calculations involve precisely estimating the tail behavior of time-series, the results of Q-to-Q –test are troubling news. But again, using other distributional assumptions in multivariate environment is extremely difficult also. Fortunately, the confidence level of the calculated VaR figure may be a partial cure here. The question is, at what level do fat-tails start to dominate standard normal distribution. For this, the Q-to-Q –test was modified to calculate the correlation coefficient only for those observations that should be within the theoretical limits of –1.644 and 1.644. Whereas the basic Q-to-Q –test rejects 66 series out of 90 for the EWMA method and all for MA, the modified test rejects normality assumption only for 38 EWMA series and 67 MA series.

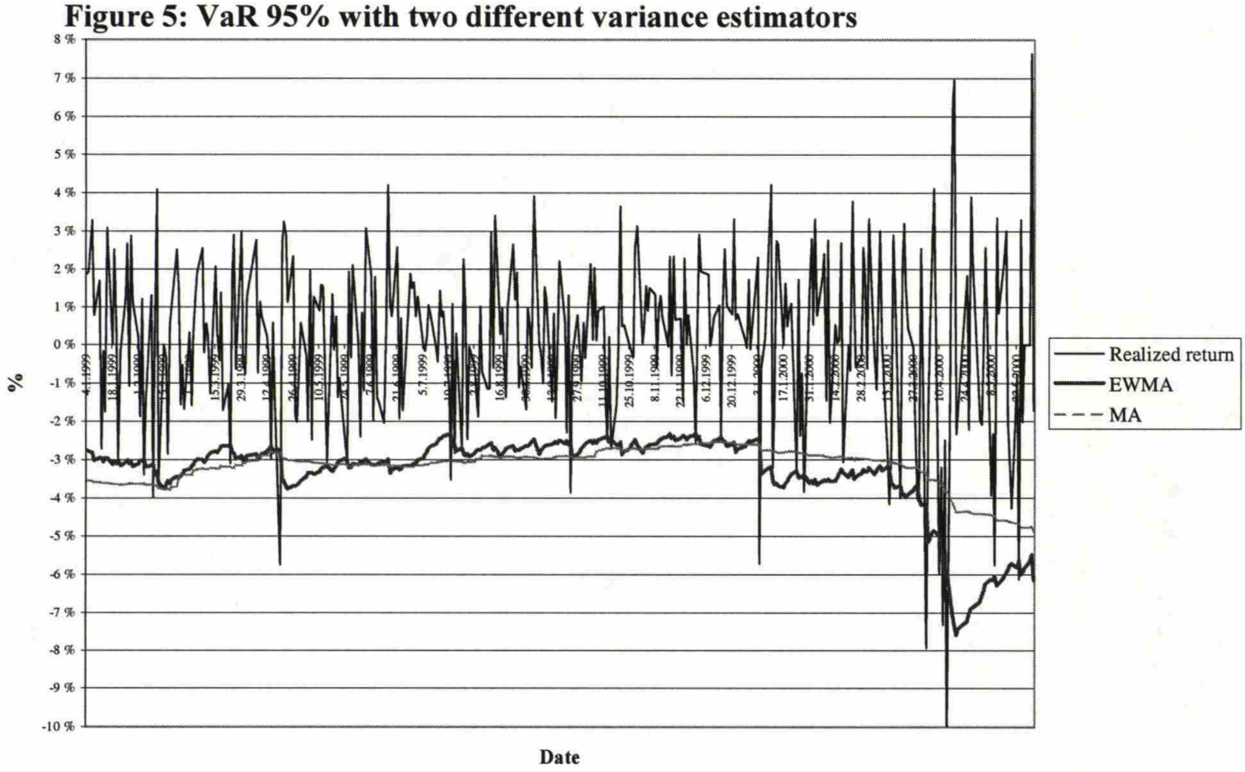
All in all, the normality assumption appears to hold much better in the case of EWMA than MA estimates. In the basic Q-Q –test, EWMA method produced larger correlation coefficients in 97% of cases and 85% in the modified version. The modified Q-Q –test suggests that the conditional normality assumption is reasonable although notable exceptions do occur for some series.

4.3 VARIANCE AND COVARIANCE ESTIMATES

Although the conditional normality tests seem to favor EWMA methodology over MA, results need to be confirmed also for the case of predicting variances. Again, variance forecasts were calculated using 120 days of past return information. The comparison results are given in Appendix C. According to the conventional statistical measure, RMSE, the EWMA-estimator is better 57 times out of 90 compared to 33 for moving average. But Alexander and Leigh (1997, 55) criticize this measure because it focuses on the accuracy of the center of the predicted returns distribution. On the other hand, VaR estimates need accuracy in the tails of the distribution and therefore some sort of an operational benchmark is needed. Alexander and Leigh (1997, 57) use the evaluation procedure given by the Bank for International Settlements (BIS) but for purposes of this thesis, it is considered unsuitable. Instead, the Kupiec likelihood-ratio approach is utilized and results are reported at the usual confidence level of α=5%.

The results were calculated for VaR 95% because at 99% level the distributional assumptions could affect the results significantly. Furthermore, as the sample size is only 355 days, the confidence interval of Kupiec LR for VaR 99% would be <8. So, it would be unable to distinguish between overly conservative and appropriate models. At VaR 95%, the confidence interval for excess returns is $10<n<27$. According to this operational benchmark, EWMA-estimator clearly outperforms the moving average method. EWMA-estimate fails to comply only in 5 cases whereas moving average produces 14 failures. The results are subject to sample size issue as the power of test statistic is generally poor. Figure 5 gives a graphical example of differences between the two competing estimators.

Figure 5: VaR 95% with two different variance estimators



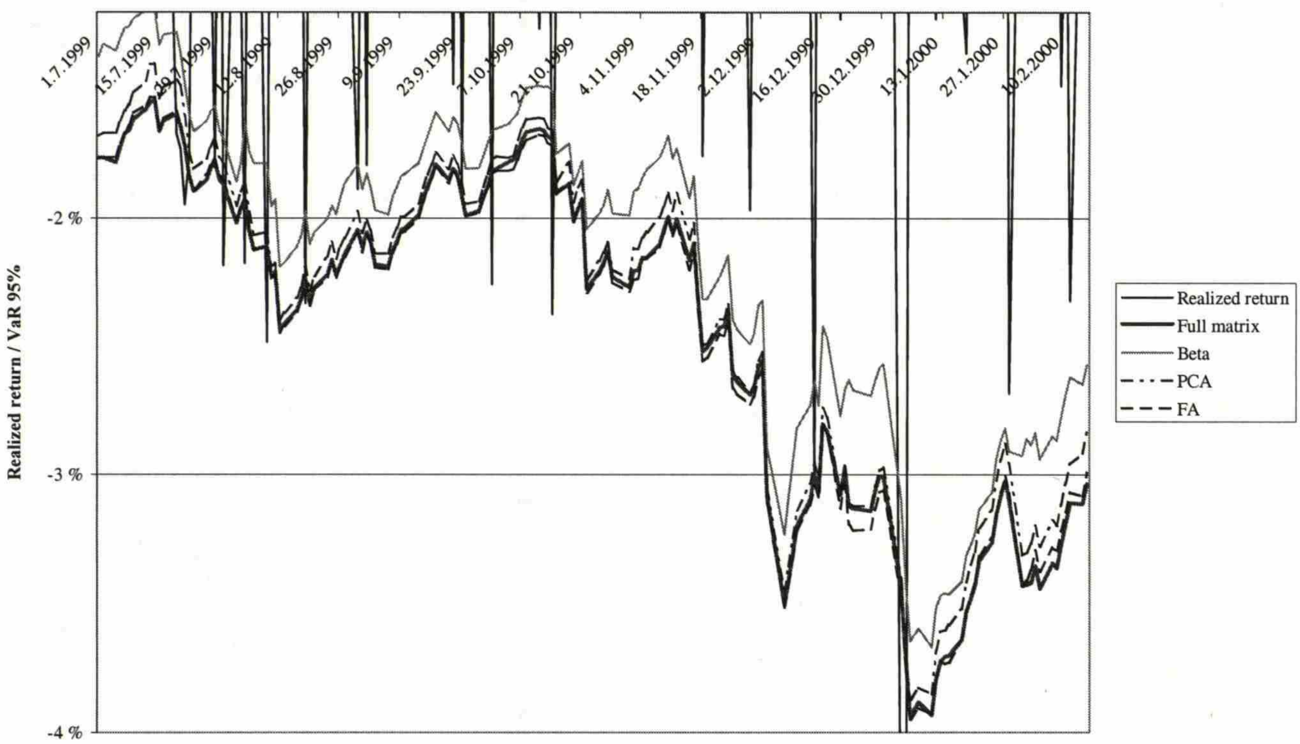
4.4 EQUITY MAPPING

The results in this section are for time period of 1.7.1999 – 15.2.2000 (159 days). They served as preliminary stage in building the VaR system for Sampo Life and were presented more detailed in a working paper for the course Advanced Risk Management at the Helsinki School of Economics and Business Administration. The analysis includes only the Finnish portion of the portfolio but as such includes 69-80% of the total equity portfolio under consideration. The portfolio included from 79 to 84 different companies within the time period, while the total number of different companies amounted to 92.

Results are presented for VaR level of 95% and the benchmark is the value calculated using the full variance-covariance matrix. At this stage, the analysis included no simulation but the use of equation (2) to calculate the VaR number. This way, the results are free from any random number sampling errors and give the most accurate information on the efficiency of different methods to recreate the original Σ. PCA and FA were modeled with 16 components to facilitate comparison with the RiskMetrics mapping using industry indices (HEX has 16 industry indices). Furthermore, with 16 components both methods achieved satisfactory explanatory levels

Results for the more statistically orientated methods are laid out first but a comment on beta estimation deserves attention. The normal OLS-regression suffers from similar problems as moving averages variance and covariance estimators. As evident from the section 4.2, they are too insensitive to new information. Therefore the betas are estimated using EWMA variance and covariance forecasts. So, Figure 6 gives the results for the beta method, PCA and FA along with the full Σ.

Figure 6: Equity VaR 95% with statistical mapping methods



Results are well in line with the expectations. Both purely statistical methods are able to recreate the original Σ very accurately, although the FA does marginally better job. Beta method clearly looses too much information. Even if the sample size is too small for any serious statistical conclusions, the Kupiec LR test anyway indicates that beta method provides inaccurate VaR forecasts at 95% level. The results of the test are presented in Table 2. Also PCA method is rejected but with only one outlier. Interestingly, FA performs better than the original Σ.

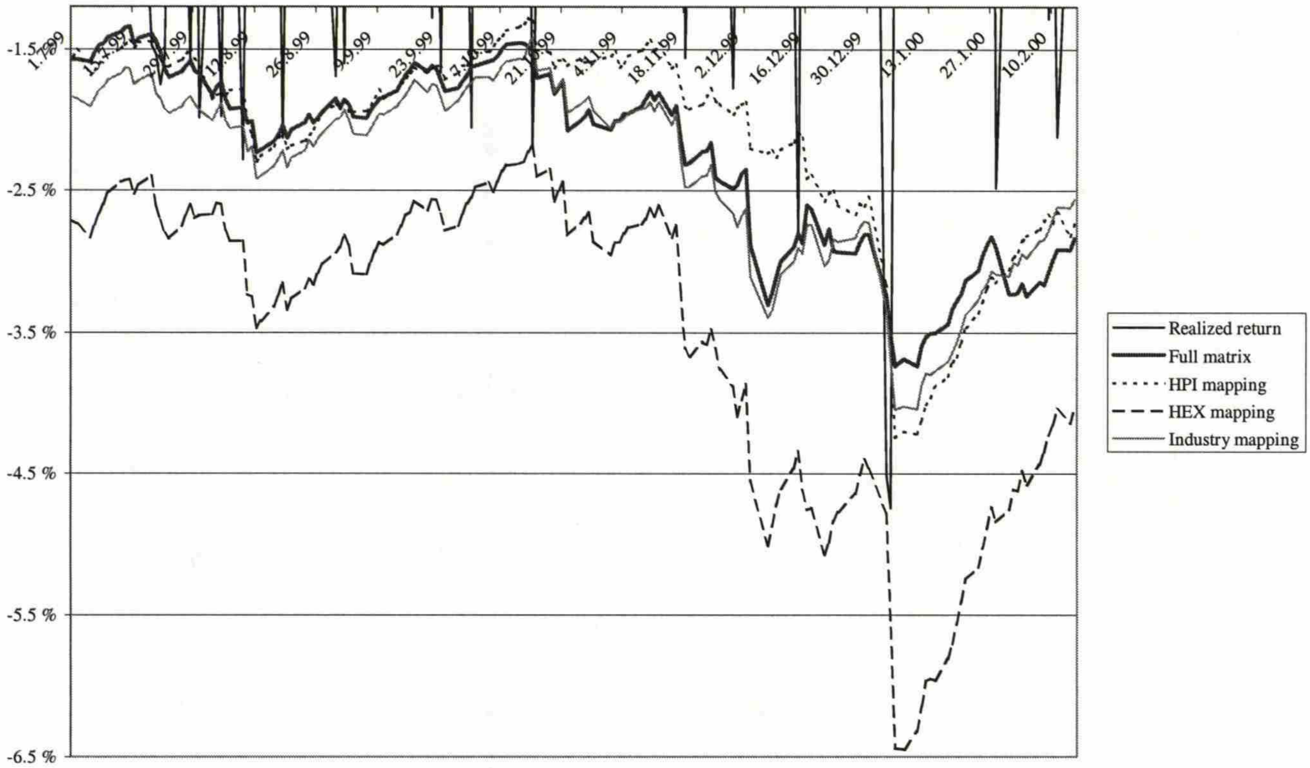
Table 2: Kupiec likelihood ratio test for equity mapping methods

Method	Full matrix	Industry mapping	HEX mapping	HPI mapping	Beta method	PCA	FA
Sample size	159	159	159	159	159	159	159
Probability	5 %	5 %	5 %	5 %	5 %	5 %	5 %
Expected losses	7.95	7.95	7.95	7.95	7.95	7.95	7.95
Excessive losses	13	7	0	14	15	14	12
Kupiec LR, α=5%	Confidence intervals for LR (n=159):			**3<LR (VaR 95%)<14**			
LR (VaR 95%)	2.86	0.12	-	3.99	5.28	3.99	1.89
χ^2	9.10 %	72.44 %	-	**4.58 %**	**2.16 %**	**4.58 %**	16.91 %
Decision rule	Accept H_0	Accept H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Accept H_0

The statistical methods as such succeed very well in recreating the original Σ but relatively large number of factors are required. As Bliss (1997b, 19) points out, the relatively moderate ability of a few linear factors to explain stock returns underscores the poor performance of specific stock return models. This is starkly in contrast with results below in section 4.5.3 for fixed income market where systematic movements cover for the most part of the variation.

However, all methods that use company specific observations are undermined by the lack of daily price observations, which is very usual phenomenon in the Finnish market. It is here where the RiskMetrics mapping is potentially very useful. Equity indices certainly have observations for each day but the difficulty lies in selecting the right index as Table 2 demonstrates. The case portfolio differs too much from the two major Finnish indices, HEX general and HEX portfolio index, to be reliably mapped to them. HEX general is overly conservative due to Nokia's heavyweight whereas HPI and its 10% weight limit prove to be too restrictive. The solution is to employ more detailed mapping with 16 HEX industry indices. As can be seen above in the Table 2 and below in the Figure 7, industry mapping is able to catch variation even better (according to LR test) than the full Σ.

Figure 7: Equity VaR 95% with RiskMetrics mapping



So, to achieve good results with RiskMetrics mapping is a question of finding a suitable set of indices, which capture the key drivers of the equity portfolio risk. Table 3 below lists the set of country indices, which were used to find a suitable set to model the internationally diversified portion of the equity portfolio. These results were derived for the whole time period under consideration, 1.1.1999 – 31.5.2000 (355 days).

Table 3: RiskMetrics mapping for foreign equities

5 %	probability, p*		17.75	expected failures	
355	sample size, T				
Country	**Index**	**Failures**	**LR (VaR 95%)**	χ^2	**Decision rule**
Japan	.N225	28	5.34	2.08 %	Reject
Japan	.N500	28	5.34	2.08 %	Reject
Japan	.TOPX	29	6.35	1.17 %	Reject
Germany	.GDAXI	11	3.11	7.79 %	Accept
Germany	.NMDKX	4	16.13	0.01 %	Reject
USA	.SPC	28	5.34	2.08 %	Reject
USA	.IXIC	6	10.89	0.10 %	Reject
USA	.DJI	33	11.13	0.09 %	Reject
Sweden	.OMX	15	0.47	49.19 %	Accept
Norway	.OBX	47	35.65	0.00 %	Reject
Netherlands	.AEX	34	12.49	0.04 %	Reject
Italy	.MIB30	16	0.19	66.49 %	Accept
Switzerland	.SSMI	23	1.50	22.05 %	Accept
Denmark	.KFX	24	2.10	14.76 %	Accept
Spain	.IBEX	38	18.59	0.00 %	Reject
UK	.FTSE	29	6.35	1.17 %	Reject
France	.FCHI	21	0.59	44.12 %	Accept
Estonia	.TALG	20	0.29	59.09 %	Accept
Latvia	.RICI	25	2.78	9.54 %	Accept
Lithuania	.LITIN	23	1.50	22.05 %	Accept
Confidence intervals for LR (n=355):				**10<LR (VaR 95%)<27**	

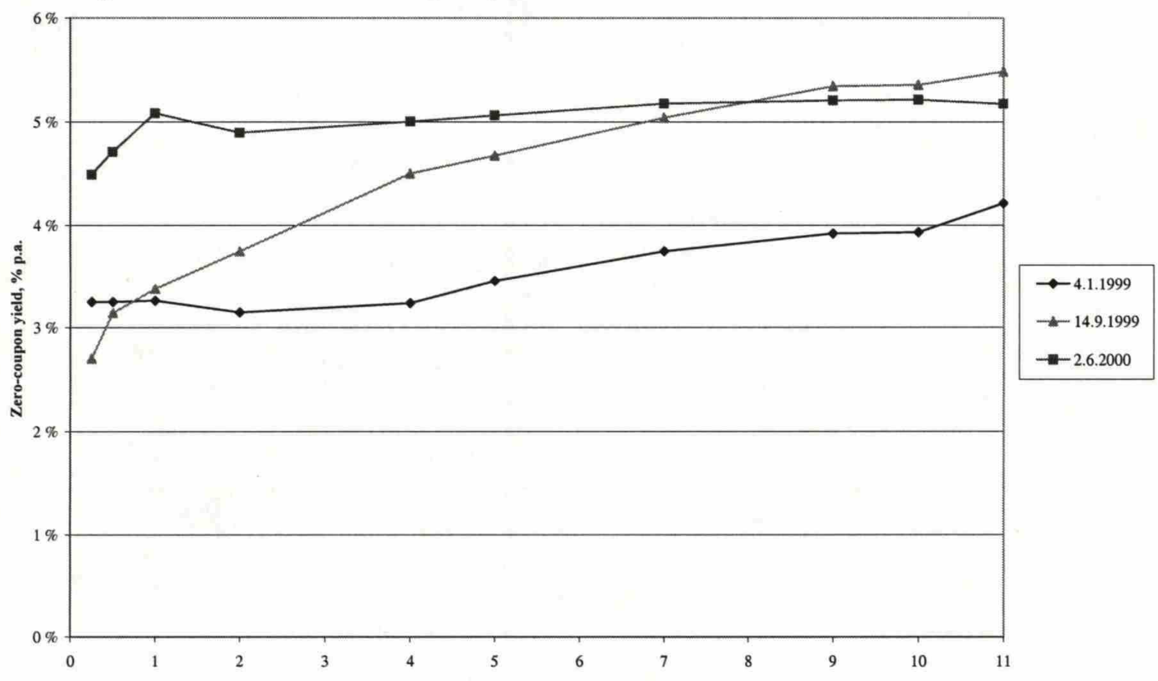
As such the results appear discouraging but the section 4.5, which presents the whole picture, shows how individual deficiencies may cancel each other out. So, while at the micro level results are poor, the aggregate figures are reliable. The reason for these discouraging figures is simply that the geographical portfolio composition is significantly different from the broad country indices. For the case portfolio, the major source of equity risk lies in Finland and the implementation of industry index mapping is really the key to success here.

4.5 FIXED INCOME MODELING

4.5.1 Zero-coupon yield curve

Estimated zero-coupon yield curves were all well behaving and contained no suspicious observations. To illustrative how the term structures have evolved over time, Figure 8 presents the Finnish zero-coupon yield curve for three dates. Surface presentation would have been better but due to Excel's insufficient capabilities the resulting graphs were inadequate. The Figure 8 demonstrates how the simple bootstrapping method is able to capture quite well the different shapes of the entire term structure.

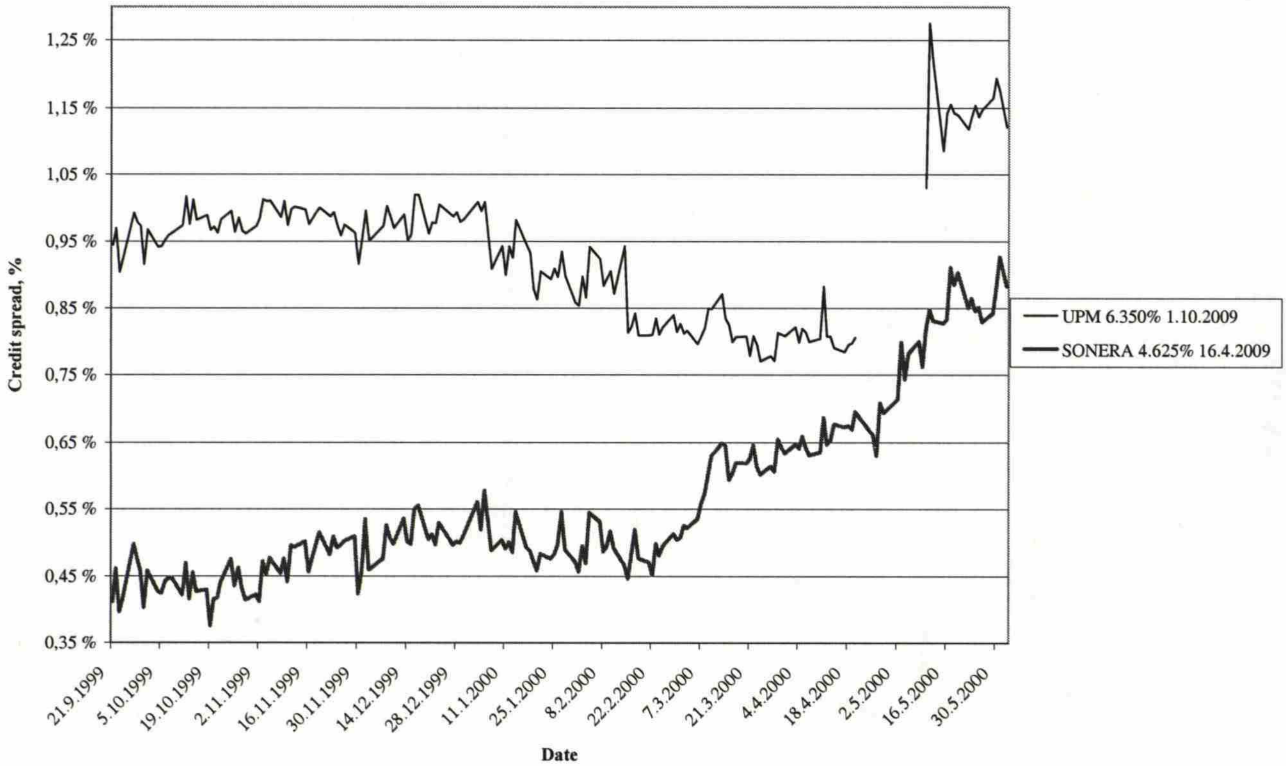
Figure 8: Finnish zero-coupon yield curve for three dates



4.5.2 Credit spreads

Although the approach in this thesis to modeling credit spreads is atheoretic, they nevertheless contain information about the riskiness of the issuing companies. As a whole, the credit spreads have widened within the sample period. This, combined with the overall rising trend in riskless rates, has made the credit market quite a poor investment. To illustrate the time-varying nature of credit spreads, two issues are examined in greater detail. First, Sonera's issue maturing at 16th April 2009 with an annual coupon of 4.625%. The loan was issued on 30th March 1999 and the amount outstanding is EUR 300 million. Second, UPM's issue maturing at 1st October 2009 with an annual coupon of 6.350%. The issue date was 21st September 1999 and the amount outstanding is EUR 250 million. Figure 9 depicts the development of the issue specific credit spreads for the time period 21.9.1999-31.5.2000 against the Finnish riskless term structure.

Figure 9: Estimated credit spreads for Sonera and UPM



The difference of credit spreads between Sonera's and UPM's loan is due to credit ratings. While Sonera's rating is A+, UPM has only BBB+ according to Standards & Poor. For Sonera, the development of credit spread over time reflects the market situation where mobile operators are considered being riskier than before. This is due to auctions being held for third generation mobile phone licenses, which are regarded as overly expensive. The English auction was held in spring this year and Sonera's credit spread started to increase simultaneously. At the same time, stock market turbulence reached high levels and Finnish government sold 1.5% of its Sonera stake on 7th March 2000 (12 million shares). After that, the share price has more than halved which partly indicates overvaluation and partly increased riskiness of the company.

The case of UPM reflects very clearly, how company specific issues affect the risk levels. Earlier in 1999, UPM had made a bid to buy a US forest company Champion with cash and stocks. Markets reacted negatively as the premium per share was considered too high. This led to deterioration of both UPM's share price and the bid for Champion as it was partly paid with UPM's stocks. Then on 25th of April the major US player International Paper (IP) made a competing offer which significantly exceeded UPM's bid. As it took couple of days for UPM to reconsider their bid, there were no deals for the bond. Clearly, the risk level was about to change. After UPM did raise its offer, the credit spread soared for the company would have been overly indebted had it won the bid. Companies made bids for few rounds and finally IP stood out as a winner. The credit spread for UPM did not return to previous levels as company made clear, it was searching new potential buy-out candidates.

These two examples highlight how even these atheoretic credit spreads convey information about the company specific risk and how they evolve over time. As the section 4.6 makes it clear, some form of modeling is needed to capture this change in risk as it affects corporate bond prices in serious way. For UPM's bond, the rise of credit spread from 80 basis points to 128 basis points meant a drop of over 4% in the clean bond price. At the same time, the clean price for the Finnish government bond maturing at 15th October 2010 rose 0.27%.

4.5.3 Principal component analysis

Next we assess the efficiency of the PCA in explaining the interest rate behavior. Based on results of Litterman and Scheinkman (1991) and portfolio characteristics, three principal components were used to model interest rates in five countries: Finland, Sweden, France, Germany and USA. Table 4 summarizes the findings for the sample period of 1.1.1999 – 31.5.2000 (number of day=355).

Table 4: The efficiency of principal component analysis with 3 components

	Market				
Explanatory power	Finland	Sweden	France	Germany	USA
-Average	94.25 %	97.37 %	90.19 %	95.04 %	92.41 %
-Maximum	98.24 %	99.24 %	97.11 %	98.36 %	95.72 %
-Minimum	86.39 %	95.22 %	83.56 %	89.56 %	86.79 %
Standard deviation	2.37 %	0.99 %	3.07 %	1.97 %	1.68 %
Standard error	0.13%	0.05%	0.16%	0.10%	0.09%
Average first component	73.57%	79.97%	60.18%	68.14%	66.69%
Number of key rates	10	15	9	9	8

Results are clearly in line with findings of Litterman and Scheinkman (1991, 58) and Bliss (1997b, 20-21) despite the small differences in the modeling approach. Litterman and Scheinkman (1991, 57) fit the three component model using factor analysis to weekly excess returns (they use the generic overnight repo rate as true risk-free rate) on US Treasury bond market with eight key rates while Bliss (1997, 18) examines monthly yields with ten key rates. The results presented in this thesis exhibit a bit lower explanatory power compared with earlier findings but this may very well be due to use of daily returns. Daily yield changes are likely to contain more idiosyncratic movements than returns over longer time span.

Nevertheless, three principal components excel in explaining the entire term structure. The first component, which Litterman and Scheinkman (1991, 57) name as level factor, is able to explain the bulk of the variation although generally less than in earlier findings. According to Bliss (1997, 18), this explains why the traditional measure of interest rate risk, the Macaulay duration, is so successful. But the existence of three components also indicates why it is an incomplete indicator of risk: parallel movements are only part of the story. To better illustrate the time varying pattern of principal components, Figure 10 depicts the results for the Finnish market, which is the main source of interest rate risk in the case portfolio.

Figure 10: Principal components of the Finnish zero-coupon yield curve

So, it is quite clear that three principal components seem to do fine in explaining the variation of entire term structure of default-free zero-coupon yields. The real test is how the method succeeds in the context of VaR estimation.

4.6 BACKTESTING

This section finally presents the results for the developed system as a whole. The total portfolio was first divided according to asset classes, into equities and fixed income. These classes were further assigned to sub-classes, domestic and foreign risk components for equities and government and corporate risk for fixed income securities. Equity derivatives were considered in the context of the whole equity portfolio to examine their effect on equity risk. Table 5 gives the backtesting results for Kupiec likelihood ratio. VaR estimates were calculated both for 95% and 99% confidence level although VaR 99% results are more difficult to verify. The confidence level for likelihood ratio test is the common α=5%.

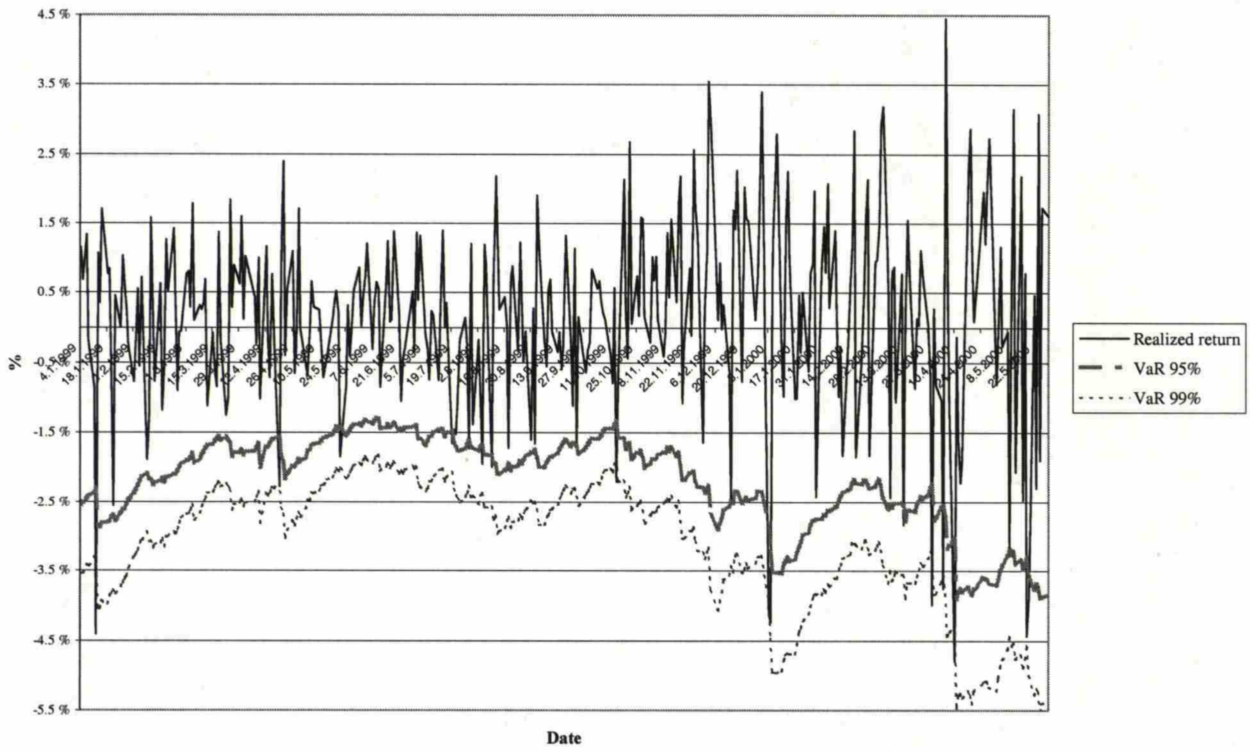
Table 5: Backtesting results for the whole system

	Total portfolio	Equity portfolio	FIM equities	Ex-FIM equities	Equity derivatives	Bonds	Govt. bonds	Corp. bonds
Sample size (T)	355	355	355	355	355	355	355	355
Probability	5 %	5 %	5 %	5 %	5 %	5 %	5 %	5 %
Expected losses	17.75	17.75	17.75	17.75	17.75	17.75	17.75	17.75
Excessive losses	19	17	15	24	19	33	21	37
Probability	1 %	1 %	1 %	1 %	1 %	1 %	1 %	1 %
Expected losses	3.55	3.55	3.55	3.55	3.55	3.55	3.55	3.55
Excessive losses (n)	6	7	6	7	11	12	6	25
Kupiec LR, α=5%	Confidence intervals for excessive losses (T=355):					10 < n < 27	and	n < 8
LR (VaR 95%)	0.09	0.03	0.47	2.10	0.09	11.13	0.59	16.98
χ^2	76.33 %	85.41 %	49.19 %	14.76 %	76.33 %	0.09 %	44.12 %	0.00 %
Decision rule	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Reject H_0	Accept H_0	Reject H_0
LR (VaR 99%)	1.41	2.64	1.41	2.64	10.14	12.54	1.41	56.03
χ^2	23.43 %	10.42 %	23.43 %	10.42 %	0.15 %	0.04 %	23.43 %	0.00 %
Decision rule	Accept H_0	Accept H_0	Accept H_0	Accept H_0	Reject H_0	Reject H_0	Accept H_0	Reject H_0

Looking at the total portfolio, the system appears to produce acceptable results both for VaR 95% and VaR 99%. At the VaR 95% level, the number of realized losses in excess of VaR estimate is very close to the predicted value. However, looking at the broad asset class level, VaR estimates for fixed income instruments seem to underestimate the true risk level. Equities perform very nicely as a whole and also both sub-classes are within the limits of the confidence interval. The reason for the system to produce acceptable results for the whole portfolio despite the fixed income failure, is that in monetary terms the equity risk dominates the fixed income risk significantly. In Euros, the equity VaR 95% is on the average eight times bigger than corresponding bond estimate, fluctuating from 3 to 17 times. This highlights the fact that nominal values (or market values) tell little about the risk levels of different asset classes.

For equities, it is also interesting to note that despite the rather problematic results for international equity mapping with country indices presented in section 4.4, the international equity portfolio as a whole behaves rather well. Individual errors cancel each other out as the international portfolio is rather well diversified. But it would be clearly misleading to report country specific VaR estimates based on this system. Of course, the estimates for domestic equities are robust due to more detailed implementation. They dominate the risk of equity portfolio as well because the majority of assets are Finnish (for the modeled portion of the equity portfolio, the portion is 74%). On average, the foreign equity VaR 95% is 25% of the domestic equity VaR, which is very close to the portion based on market values. However, the relation fluctuates very much as the currency risk affects the situation, too. The evolution of equity risk within the sample period is illustrated in Figure 11 below. The time varying nature of variance is evident: the recent turbulence in the stock market has raised the risk level of the equity portfolio significantly.

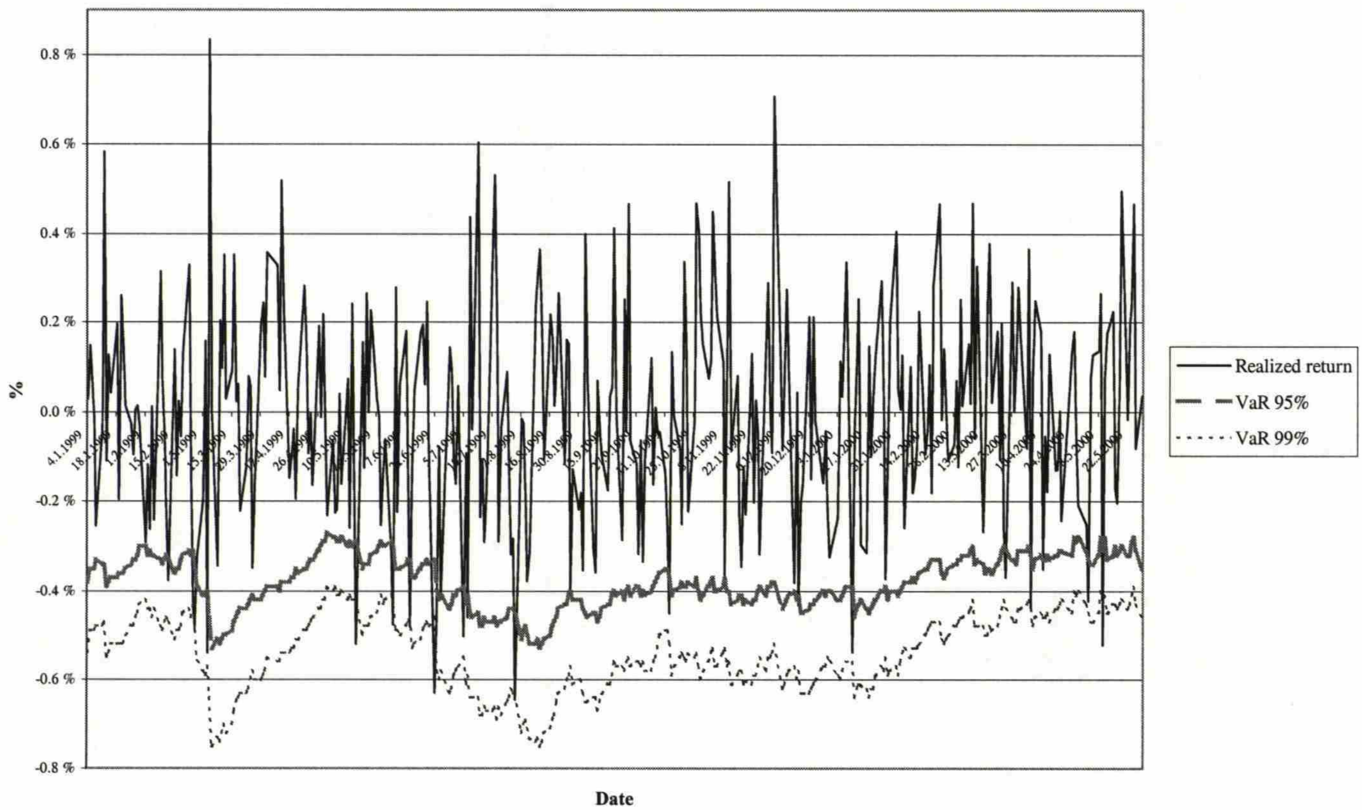
Figure 11: VaR estimates for equity portfolio



As the equity derivatives were solely marked to model, the results are not too surprising. Practically, they measure how the volatility estimate and the stochastic process used for the underlying instrument perform. As can be seen, the VaR 95% level is reasonable for reporting purposes but VaR 99% is way out of the confidence limits. From the viewpoint of asset management, more interesting information is how the derivatives have affected the equity risk. In Sampo Life, derivatives are mainly used for hedging purposes but quite often options are written in search of extra returns, too. Looking at the whole sample, the use of derivatives did not result in a major change of risk profile. At times, they actually increased the risk level quite sizeably.

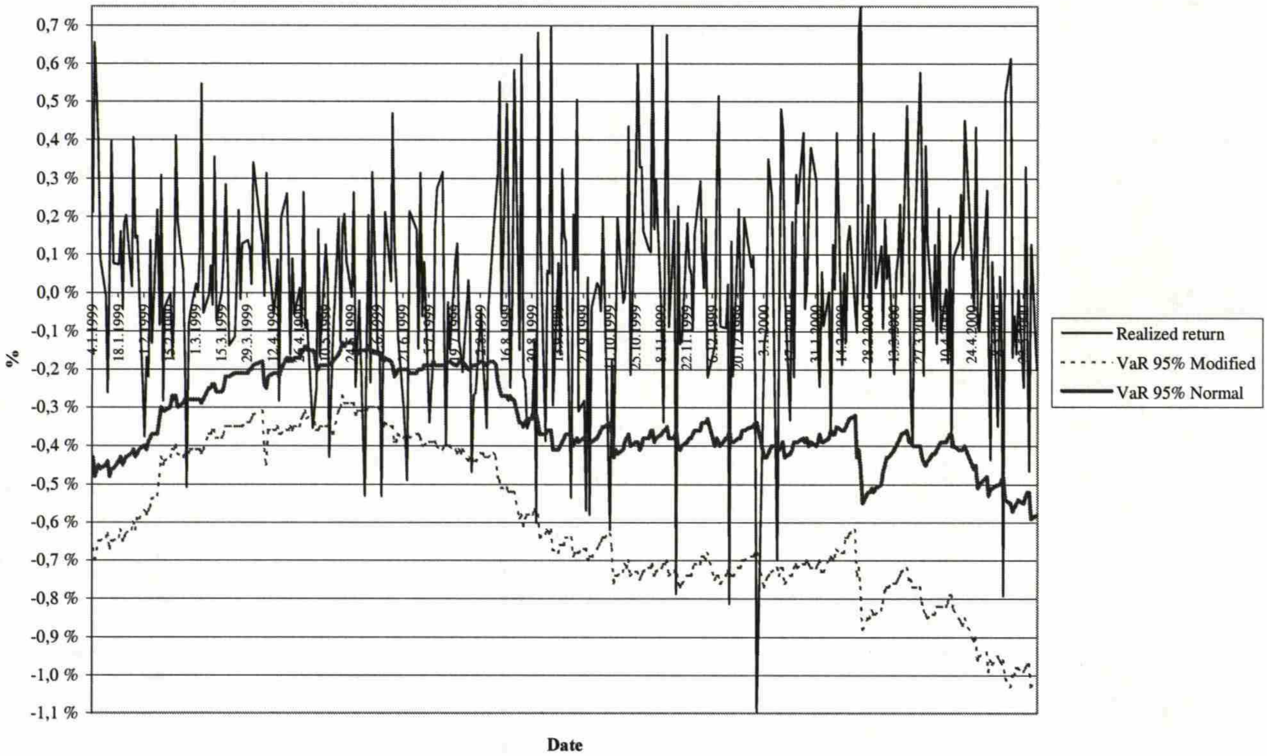
Turning the attention back to the fixed income results, it is clear that credit spread need to modeled more carefully. Theoretical models do not necessarily perform any better than the simple atheoretic approach used in this thesis if they are only used to calibrate the credit spreads based on observed prices. Of course, also the zero-coupon yield curve should be modeled with different techniques to isolate possible errors in that procedure. But as the Figure 12 below and Table 5 above demonstrate, the bootstrapping method produces accurate results for default-free instruments. As such, it does perfectly well for VaR calculations of default-free instruments. Unfortunately, the risk inherent in credit spreads dominates the systematic risk for the case portfolio and therefore VaR for fixed income instruments is reliable only government bonds.

Figure 12: VaR estimates for government bond portfolio



To quickly test, what could be the effect of modeling credit spreads, an *ad hoc* procedure was used. When calculating the VaR estimates for corporate bonds, each credit spread was multiplied by factor of 1.10. This is of course totally unrealistic procedure but the aim was to test how an artificial "confidence interval" would affect the results. Figure 13 lays out the results.

Figure 13: Credit spread modeling



As can be seen, the systematic risk is unable to fully explain the variation in corporate bond returns. The widening of credit spreads within the sample period affects the artificially created "confidence interval" as it tends to grow over time. Clearly, credit spread modeling should take into account the time-varying nature of this risk measure. Otherwise VaR system is not able to produce decent estimates on market risk of credit risky portfolio.

5 SUMMARY AND CONCLUSIONS

This thesis has developed for Sampo Life a VaR system, which is based on Monte Carlo simulation and full valuation. As the financial literature demonstrates, the impact of VaR is not just in calculating the daily estimate on market risk. It serves as a basis for several applications which include stress testing, monitoring, risk-return analysis, hedging, asset allocation etc. Many authors argue that perhaps the biggest impact of VaR is it imposes a structured methodology for critically thinking about risk. Thus the process of getting to VaR may be as important as the number itself. For Sampo Life, future plans include developing a more realistic stress test and examining the efficiency and riskiness of the asset allocation in the long run.

In an asset management environment like Sampo Life where the business is risk taking on behalf of the customers, VaR may reveal whether the risks taken are those risks that are wanted or needed or even thought to be taken. Looking at the descriptive statistics of the case portfolio, it is evident that current risk calculation practices are inadequate. The biggest advantage of VaR figure as a market risk measure is, that it is common and consistent, takes account the portfolio effects and hence enables comparison of totally different positions. However, as the critics point out, the results may be highly dependent on the system employed. Therefore we must always interpret the VaR figures according to assumptions made in their calculations.

Looking at the empirical results, the importance of adequate backtesting cannot be over-emphasized. One should always experiment with financial time-series to find out how certain assumptions affect the outcomes. The usual assumptions about independent and log-normally distributed returns do not hold strictly but the whole VaR ideology is not about getting the most precise figures but getting an estimate of the risk. As the VaR calculations include so many unobservable elements (variances, covariances, zero-coupon yields, credit spreads and so forth), it is a bit surprising that in the end it works so well.

For equities, the characteristics of the case portfolio must determine the technique and model applied. The beta method is reasonable only for highly diversified portfolio whereas the purely statistical techniques are a bit too cumbersome due to highly idiosyncratic stock prices and extensive data requirements. On the other hand, RiskMetrics mapping method is sensitive to the selection of the index used in mapping. Using industry indices proved to be the best solution, which offered accurate results very easily. As equity derivatives were mark-to-model, the only meaningful result was to assess their impact on equity risk.

The starting point of fixed income modeling is the derivation of default-free term structure of zero-coupon yields from coupon bearing instruments. Again even the simplest model, bootstrapping, proved to be efficient way of extracting the unobservable elements from the prices of government bonds. Since the movements in bond prices are highly systematic, the principal component analysis with just three components was sufficient to model the entire zero-coupon yield curve. Resulting VaR figures for government bonds were within the acceptable limits according to the likelihood ratio test and visual examination.

However, the presence of default risk in corporate bonds complicates things considerably. The modeling of credit spreads to take account for this extra risk was atheoretic and simple: a constant rate over the entire term structure. Even this simple procedure was able to highlight how the company or industry specific issues affect the implied riskiness. Due to time-varying nature of credit spreads, backtesting results were disappointing as they also appeared to dominate the risk of whole bond portfolio. The need for a more careful modeling was proved with a simple *ad hoc* test, which essentially placed a small confidence interval for all credit spreads.

Reviewing the objectives set in the beginning, the created system is reliable for equities and governmental securities. As such it is able to cover roughly 75% of the whole portfolio although the analysis here modeled it only partly. The relatively simple approach to modeling proved to be effective in calculating aggregate VaR figures. Furthermore, it forms a sound basis for additional applications such as stress testing.

REFERENCES

Ahn, Dong-Hyun, Boudoukh, Jacob, Richardson, Matthew and Whitelaw, Robert F. 1999. "Optimal Risk Management Using Options". The Journal of Finance 54:1 359-375.

Alexander, Carol (ed.) 1998. Risk Management and Analysis. Vol. 1: *Measuring and Modeling Financial Risk.* John Wiley & Sons Ltd., West Sussex.

Alexander, C.O. and Leigh, C.T. 1997. "On the Covariance Matrices Used in Value at Risk Models". Journal of Derivatives 4:3 50-62

Asetus ensivakuutusliikettä harjoittavan vakuutusyhtiön vastuuvelan katteesta, N:o 461/1995.

Beder, Tanya Styblo 1995. "VAR: Seductive but Dangerous". Financial Analysts Journal 51:5 12-24.

Beder, Tanya, Minnich, Michael, Shen, Hubert and Stanton, Jodi 1999. "Vignettes on VAR". The Journal of Financial Engineering 7:3 289-309.

Black, Fisher and Scholes, Myron 1973. "The Pricing of Options and Corporate Liabilities". Journal of Political Economy 81:3 637-654.

Bliss, Robert R. 1997a. "Testing Term Structure Estimation Methods". Advances in Futures and Options Research 9 197-231.

Bliss, Robert R. 1997b. "Movements in the Term Structure of Interest Rates". Federal Reserve Bank of Atlanta Economic Review 97:4 16-33.

Bollerslev, Tim 1986. "Generalized Autoregressive Conditional Heteroscedasticity". Journal of Econometrics 31 307-327.

Boudoukh, Jacob, Richardson, Matthew and Whitelaw, Robert F. 1997. "Investigation of a Class of Volatility Estimators". The Journal of Derivatives 4:3 63-71.

Boyle, Phelim, Broadie, Mark and Glasserman, Paul 1997. "Monte Carlo Methods for Security Pricing". Journal of Economic Dynamics and Control 21:8-9 1267-1321.

Brailsford, Timothy J. and Faff, Robert W. 1996. "An evaluation of Volatility Forecasting Techniques". Journal of Banking and Finance 20:3 419-438.

Culp, Christopher L., Mensink, Ron and Neves, Andrea M.P. 1999. "Value at Risk for Asset Managers". Forthcoming in Derivatives Quarterly 5:2.

Deacon, Mark and Derry Andrew 1994. "Estimating the Term Structure of Interest Rates". Bank of England Working Paper No 24.

Dimson, Elroy and Marsh, Paul 1990. "Volatility Forecasting without Data-Snooping". Journal of Banking and Finance 14:2 399-421

Dowd, Kevin 1998. Beyond Value at Risk: A New Science of Risk Management. John Wiley & Sons Ltd., West Sussex.

Dowd, Kevin 1999a. "A Value at Risk Approach to Risk-Return Analysis". The Journal of Portfolio Management 25:4 60-67.

Dowd, Kevin 1999b. "Financial Risk Management". Financial Analysts Journal July-August 65-71.

Duffie, Darrell and Pan, Jun 1997. "An Overview of Value at Risk". The Journal of Derivatives 4:3 7-49.

Duffie, Darrel and Singleton, Kenneth J. 1999. "Modeling Term Structures of Defaultable Bonds". The Review of Financial Studies 12:4 687-720.

Engle, Robert F. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation". Econometrica 50:4 987-1007.

Fama, Eugene F. and Bliss, Robert R. 1987. "The Information in Long-Maturity Forward Rates". The American Economic Review 77:4 680-692.

Golub, Gene H. and Van Loan, Charles F. 1989. Matrix Computations, Second Edition. The John Hopkins University Press, Baltimore.

Jamshidian, Farshid and Zhu, Yu 1997. "Scenario Simulation: Theory and methodology". Finance and Stochastics 1:1 43-67.

Jarrow, Robert A. and Turnbull, Stuart M. 1996. Derivative Securities. South-Western College Publishing, Cincinnati, Ohio.

Jarrow, Robert A., Lando, David and Turnbull, Stuart M. 1997. "A Markov Model for the Term Structure of Credit Risk Spreads". The Review of Financial Studies 10:2 481-523.

Jauri, Osmo 1997. Riskienhallinta uudesta näkökulmasta. Kauppakaari Oy, Helsinki.

Johanssen, Frederik, Seiler, Michael J. and Tjarnberg, Mikael 1999. "Measuring Downside Portfolio Risk". The Journal of Portfolio Management 26:1 96-107.

Johnson, Richard A. and Wichern, Dean W. 1982. Applied Multivariate Statistical Analysis. Prentice Hall, Inc., New Jersey.

Jorion, Philippe 1995. "Predicting Volatility in the Foreign Exchange Market". The Journal of Finance 50:2 507-528.

Jorion, Philippe 1996. "Risk2: Measuring the Risk in Value at Risk". Financial Analysts Journal 6:52 47-56.

Jorion, Philippe 1997a. "In Defense of VAR". Derivatives Strategy

Jorion, Philippe 1997b. Value at Risk: The New Benchmark for Controlling Market Risk. Irwin Professional Publishing, Chicago.

Kupiec, Paul H. 1995. "Techniques for Verifying the Accuracy of Risk Management Models". The Journal of Derivatives 3 73-84.

Kupiec, Paul H. 1998. "Stress Testing in a Value at Risk Framework". The Journal of Derivatives 6:1 7-24.

Laki vakuutusyhtiölain muuttamisesta, N:o 752/1993

Leibowitz, Martin L. and Henriksson, Roy D. 1989. "Portfolio Optimization with Shortfall Constraints: A Confidence-Limit Approach to Managing Downside Risk". Financial Analysts Journal March-April 34-41.

Litterman, Robert and Scheinkman, José 1991. "Common Factors Affecting Bond Returns". The Journal of Fixed Income June 54-61.

Litterman, Robert and Iben, Thomas 1991. "Corporate Bond Valuation and the Term Structure of Credit Spreads". The Journal of Portfolio Management 17:3 52-64.

Longerstaye, Jacques *et al* 1996. RiskMetrics™-Tehcnical Document. Fourth Edition. Morgan Guaranty Trust Company, New York.

Marshall, Chris and Siegel, Michael 1997. "Value at Risk: Implementing a Risk Measurement Standard". The Journal of Derivatives 4:3 91-111.

Matsumoto, Makoto and Nishimura, Takuji 1998. Mersenne Twister: A 623-dimensionally equidistributed uniform pseudorandom number generator. Department of Mathematics, Keio University, Tokyo.

McCulloch, J.H. 1975. "The Tax-Adjusted Yield Curve". The Journal of Finance 30 811-830.

Merton, Robert C. 1974. "On the Pricing of Corporate Debt: the Risk Structure of Interest Rates". The Journal of Finance 29:2 449-470.

Moro, B. 1995. "The Full Monte". RISK 8 57-58.

Mustonen, Seppo 1995. Tilastolliset monimuuttujamenetelmät. Helsingin Yliopisto, Tilastotieteen laitos, Helsinki.

Nelson, C.R. and Siegel, A.F. 1987. “Parsimonious Modeling of Yield Curves”. Journal of Business 60 473-489.

Press, William H., Teukolsky, Saul A., Vetterling, William T. and Flannery, Brian P. 1992. Numerical Recipes in C: The Art of Scientific Computing. Cambridge University Press, Cambridge.

Rogers, L.C.G and Talay D. (ed.) 1997. Numerical Methods in Finance. Cambridge University Press, Cambridge.

Sampo Life, Investment plan 2000.

Seppälä, Juha and Viertiö, Petri 1996. “The Term Structure of Interest Rates: Estimation and Interpretation”. Bank of Finland Discussion Papers 19/96.

Sharma, Subhash 1996. Applied Multivariate Techniques. John Wiley & Sons, Inc., New York.

Sharpe, W. 1964. “Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk”. The Journal of Finance 19: 425-442.

Sosiaali- ja terveysministeriön määräys- ja ohjekokoelma kotimaisille vakuutusyhtiöille. Dnro 58/021/97

Vakuutusyhtiölaki, N:o 1062./1979

West, Kenneth D. and Cho, Dongchul 1995. ”The Predictive Ability of Several Models of Exchange Rate Volatility”. Journal of Econometrics 69:2 367-391.

LECTURE NOTES:

Simulation Methods in Finance 27D051. Helsinki School of Economics and Business Administration, Department of Economics.

Modelling of Dependencies in Economics 36C120. Helsinki School of Economics and Business Administration, Department of Economics.

APPENDIX A: CONVERGENCE OF VAR 95% FOR THE EQUITY PORTFOLIO

No resampling					Confidence interval					
Rounds	Average	Average-%	Standard dev.	St. dev. %	Lower bound	Upper bound	Lower %	Upper %	Max - Min	Max - Min %
1,000	-28,000,078	-2.180 %	1,076,306	0.084 %	-28,094,648	-27,905,509	-2.187 %	-2.172 %	5,784,045	0.450 %
5,000	-28,033,616	-2.182 %	501,014	0.039 %	-28,077,638	-27,989,595	-2.186 %	-2.179 %	3,025,859	0.236 %
10,000	-28,022,508	-2.181 %	337,432	0.026 %	-28,052,157	-27,992,860	-2.184 %	-2.179 %	1,842,465	0.143 %
15,000	-28,029,760	-2.182 %	274,274	0.021 %	-28,053,859	-28,005,661	-2.184 %	-2.180 %	1,487,346	0.116 %
20,000	-28,022,004	-2.181 %	233,231	0.018 %	-28,042,497	-28,001,511	-2.183 %	-2.180 %	1,224,669	0.095 %
25,000	-28,030,899	-2.182 %	215,967	0.017 %	-28,049,875	-28,011,923	-2.183 %	-2.180 %	1,206,713	0.094 %
30,000	-28,036,479	-2.182 %	204,769	0.016 %	-28,054,471	-28,018,487	-2.184 %	-2.181 %	1,230,138	0.096 %
35,000	-28,035,499	-2.182 %	182,521	0.014 %	-28,051,536	-28,019,462	-2.184 %	-2.181 %	1,137,861	0.089 %
40,000	-28,038,789	-2.183 %	173,798	0.014 %	-28,054,060	-28,023,519	-2.184 %	-2.181 %	1,063,425	0.083 %
45,000	-28,042,165	-2.183 %	165,156	0.013 %	-28,056,676	-28,027,654	-2.184 %	-2.182 %	1,066,847	0.083 %
50,000	-28,042,063	-2.183 %	154,187	0.012 %	-28,055,610	-28,028,515	-2.184 %	-2.182 %	1,027,325	0.080 %

Quadratic resampling					Confidence interval					
Rounds	Average	Average-%	Standard dev.	St. dev. %	Lower bound	Upper bound	Lower %	Upper %	Max - Min	Max - Min %
1,000	-27,940,800	-2.175 %	769,178	0.060 %	-28,008,384	-27,873,217	-2.180 %	-2.170 %	4,381,528	0.341 %
5,000	-28,034,457	-2.182 %	340,121	0.026 %	-28,064,342	-28,004,573	-2.185 %	-2.180 %	1,960,888	0.153 %
10,000	-28,017,544	-2.181 %	249,757	0.019 %	-28,039,489	-27,995,599	-2.183 %	-2.179 %	1,496,579	0.116 %
15,000	-28,049,273	-2.183 %	218,799	0.017 %	-28,068,497	-28,030,048	-2.185 %	-2.182 %	1,353,559	0.105 %
20,000	-28,036,016	-2.182 %	183,796	0.014 %	-28,052,166	-28,019,867	-2.184 %	-2.181 %	1,205,097	0.094 %
25,000	-28,038,483	-2.182 %	157,217	0.012 %	-28,052,297	-28,024,669	-2.184 %	-2.181 %	867,776	0.068 %
30,000	-28,038,181	-2.182 %	153,009	0.012 %	-28,051,625	-28,024,737	-2.184 %	-2.181 %	916,442	0.071 %
35,000	-28,037,485	-2.182 %	144,610	0.011 %	-28,050,191	-28,024,779	-2.183 %	-2.181 %	775,926	0.060 %
40,000	-28,035,645	-2.182 %	129,321	0.010 %	-28,047,007	-28,024,282	-2.183 %	-2.181 %	843,850	0.066 %
45,000	-28,033,651	-2.182 %	119,065	0.009 %	-28,044,112	-28,023,189	-2.183 %	-2.181 %	666,700	0.052 %
50,000	-28,035,834	-2.182 %	114,167	0.009 %	-28,045,865	-28,025,802	-2.183 %	-2.182 %	690,019	0.054 %

APPENDIX B: STATISTICS OF FINANCIAL TIME-SERIES

	Ljung-Box Q	Skewness	Kurtosis	Average	Variance	χ^2	Kolmogorov-Smirnov	Basic Q-to-Q	Modified Q-to-Q
.N225	3.206	-0.119	0.456	0.048	0.932	74	0.060	**0.9964**	**0.9966**
.N500	**9.707**	-0.279	0.293	0.108	0.976	59	**0.078**	**0.9955**	**0.9973**
.TOPX	0.315	-0.251	0.433	0.085	0.949	48	**0.077**	**0.9955**	**0.9961**
.OMX	1.183	-0.089	-0.442	0.128	0.949	50	0.069	**0.9966**	**0.9973**
.GDAXI	0.906	-0.143	-0.146	0.070	0.928	55	0.063	**0.9969**	**0.9964**
.NMDKX	**8.864**	-0.244	-0.012	0.085	0.985	56	0.072	**0.9957**	**0.9949**
.OBX	2.231	-0.077	0.427	0.085	0.881	60	0.072	**0.9961**	**0.9976**
.SPC	0.000	-0.156	-0.081	0.023	0.962	55	0.032	**0.9980**	0.9991
.IXIC	0.039	-0.333	-0.396	0.097	1.009	64	**0.080**	**0.9933**	**0.9961**
.DJI	0.279	-0.065	-0.114	0.024	0.967	50	0.030	**0.9978**	0.9981
.AEX	**4.293**	-0.252	-0.280	0.051	0.913	53	0.044	**0.9957**	0.9985
.MIB30	0.097	-0.095	-0.424	0.046	0.954	55	0.035	**0.9967**	0.9989
.SSMI	2.034	-0.150	-0.041	0.017	0.895	52	0.030	0.9982	0.9991
.KFX	3.325	-0.223	-0.296	0.070	0.967	49	0.047	**0.9966**	0.9986
.IBEX	**5.327**	-0.142	-0.212	0.016	0.947	50	0.024	0.9981	0.9990
.FTSE	2.597	-0.146	-0.358	0.020	0.958	54	0.024	**0.9962**	0.9990
.FCHI	**5.099**	-0.200	-0.255	0.104	0.950	47	0.063	**0.9976**	0.9984
.TALG	**14.381**	0.003	1.613	0.031	0.833	69	0.066	**0.9881**	**0.9946**
.RICI	**13.626**	0.197	-0.080	-0.013	0.902	68	0.044	**0.9974**	**0.9971**
.LITIN	**29.605**	0.157	0.950	0.019	0.828	63	0.058	**0.9927**	**0.9946**
.FOX	1.442	-0.204	-0.310	0.124	0.979	47	0.063	**0.9973**	0.9988
.HPIY	3.153	-0.248	-0.118	0.122	0.972	57	**0.080**	**0.9967**	**0.9977**
.STOXX50E	**5.505**	-0.205	-0.238	0.095	0.951	53	**0.075**	**0.9970**	**0.9968**
.HEX	2.114	-0.224	-0.272	0.144	1.000	58	**0.075**	**0.9967**	0.9989
.STOXXE	3.552	-0.330	0.006	0.093	0.942	63	**0.080**	**0.9953**	**0.9955**
.HEB1	0.148	0.121	0.314	0.035	0.911	54	0.052	**0.9965**	**0.9954**
.HEI1	2.480	-0.171	1.245	0.054	0.884	65	0.060	**0.9930**	0.9984
.HEIN	0.036	0.372	0.546	-0.055	0.886	47	0.058	**0.9949**	0.9981
.HECH	1.589	0.030	0.951	0.039	0.904	53	0.061	**0.9946**	0.9980
.HEC	0.945	0.236	0.290	0.066	0.874	57	0.058	**0.9961**	**0.9978**
.HEE	1.080	0.377	0.175	-0.050	0.903	56	0.069	**0.9946**	**0.9977**
.HEFO	0.983	0.197	2.058	-0.044	0.819	63	**0.080**	**0.9870**	**0.9965**
.HEF1	**18.762**	0.010	0.301	0.044	0.936	62	0.046	**0.9968**	**0.9967**
.HEM1	**5.637**	0.249	0.415	0.024	0.895	55	0.043	**0.9965**	**0.9971**
.HEL1	0.213	0.275	0.977	0.033	0.945	70	0.049	**0.9933**	0.9980
.HET1	0.034	-0.176	-0.076	0.048	0.936	49	0.033	0.9980	0.9987
.HEME	0.565	0.293	0.477	-0.022	0.948	52	0.038	**0.9958**	0.9991
.HEO1	**8.303**	0.072	0.115	0.008	0.944	56	0.038	**0.9970**	**0.9968**
.HETE	1.954	-0.229	-0.322	0.145	0.991	58	**0.089**	**0.9963**	0.9986
.HETD	**3.908**	0.036	0.085	-0.044	0.921	51	0.041	**0.9979**	**0.9979**
.HETR	0.412	0.091	0.430	-0.072	0.898	54	**0.077**	**0.9979**	0.9987
SEK	0.513	0.092	0.179	-0.084	0.918	46	0.047	0.9987	0.9996
NOK	**8.038**	0.536	0.446	-0.053	0.860	57	**0.077**	**0.9913**	**0.9942**
GBP	0.868	0.230	0.270	-0.088	0.961	52	0.069	**0.9973**	0.9980
JPY	2.254	0.034	-0.041	-0.101	0.948	40	0.055	0.9990	0.9987
CHF	2.606	-0.294	1.026	-0.025	0.887	56	0.052	**0.9930**	0.9981
USD	0.545	0.164	0.075	-0.106	0.980	57	0.060	**0.9977**	0.9984

Boldface indicates the rejection of null hypothesis

APPENDIX B: CONTINUED

	Ljung-Box Q	Skewness	Kurtosis	Average	Variance	χ^2	Kolmogorov-Smirnov	Basic Q-to-Q	Modified Q-to-Q
FIM 2Y	0.4335	-0.123	0.074	0.097	0.954	45	0.049	0.9985	0.9992
FIM 3Y	0.9487	-0.018	-0.204	0.099	0.929	41	0.044	0.9991	0.9991
FIM 4Y	0.0962	-0.066	-0.131	0.075	0.949	49	0.047	0.9985	0.9987
FIM 5Y	0.0040	-0.091	-0.211	0.075	0.929	48	0.052	0.9984	0.9987
FIM 8Y	0.2304	-0.085	-0.014	0.071	0.938	55	0.049	0.9983	0.9991
FIM 10Y	0.0543	-0.084	-0.186	0.068	0.930	50	0.035	0.9987	0.9996
FIM 11Y	**3.9543**	0.025	0.622	0.057	0.929	58	0.066	**0.9956**	**0.9976**
SEK 3M	**6.9282**	0.158	2.166	-0.018	0.852	109	**0.106**	**0.9799**	**0.9882**
SEK 6M	**4.9400**	0.048	0.999	0.019	0.911	64	0.055	**0.9922**	**0.9950**
SEK 1Y	3.0482	0.128	1.041	0.054	0.877	81	**0.083**	**0.9917**	**0.9941**
SEK 2Y	**7.0143**	-0.135	0.318	0.065	0.915	54	0.058	**0.9976**	0.9984
SEK 3Y	**11.3715**	-0.140	0.379	0.070	0.915	54	0.049	**0.9977**	0.9987
SEK 4Y	**12.9333**	-0.180	0.144	0.070	0.926	53	0.047	**0.9980**	0.9983
SEK 5Y	**11.3981**	-0.148	0.108	0.062	0.939	48	0.052	**0.9979**	0.9987
SEK 6Y	**9.1156**	-0.141	0.019	0.056	0.948	53	0.041	0.9982	0.9993
SEK 7Y	**7.3867**	-0.101	-0.107	0.053	0.959	45	0.030	0.9982	0.9994
SEK 8Y	**8.4653**	-0.155	-0.135	0.049	0.968	49	0.035	**0.9978**	0.9991
SEK 9Y	**8.2925**	-0.140	-0.168	0.044	0.973	50	0.038	**0.9978**	0.9992
SEK 10Y	**8.2991**	-0.156	-0.077	0.046	0.973	59	0.033	**0.9976**	0.9991
SEK 15Y	**9.9237**	-0.111	-0.202	0.032	0.971	45	0.027	0.9984	0.9993
FRF 3Y	0.1348	0.275	0.530	0.083	0.926	61	0.052	**0.9957**	**0.9967**
FRF 6Y	0.9592	-0.016	0.264	0.069	0.924	57	0.052	**0.9979**	0.9987
FRF 9Y	0.0836	0.049	-0.002	0.074	0.926	58	0.055	0.9981	**0.9972**
FRF 15Y	0.0029	0.115	-0.003	0.061	0.922	59	0.049	0.9982	0.9985
FRF 20Y	0.0395	0.048	0.042	0.042	0.924	48	0.032	0.9991	0.9997
FRF 30Y	**6.1794**	0.058	-0.200	0.035	0.933	47	0.024	0.9989	0.9994
EUR 3M	**6.6072**	0.237	7.824	0.073	0.585	354	**0.218**	**0.9080**	**0.9703**
EUR 6M	**29.0712**	0.671	4.222	0.165	0.732	234	**0.187**	**0.9427**	0.9562
EUR 1Y	**20.0526**	0.503	2.447	0.156	0.806	168	**0.142**	**0.9675**	0.9732
DEM 3Y	1.3239	0.168	-0.057	0.081	0.945	49	0.035	0.9982	0.9982
DEM 6Y	0.1824	0.054	0.150	0.077	0.914	51	0.046	0.9981	**0.9977**
DEM 7Y	0.2663	0.147	0.014	0.071	0.913	54	0.041	**0.9977**	**0.9975**
DEM 8Y	0.1223	0.141	0.092	0.072	0.933	51	0.041	0.9987	0.9986
DEM 20Y	0.7149	0.236	-0.100	0.043	0.939	57	0.032	**0.9962**	**0.9978**
DEM 30Y	0.0431	0.018	-0.219	0.029	0.942	53	0.030	0.9983	0.9988
USD 3M	0.0326	0.036	0.141	0.068	0.916	67	0.046	**0.9979**	**0.9971**
USD 6M	0.0697	0.330	1.036	0.123	0.849	79	**0.092**	**0.9889**	**0.9929**
USD 1Y	0.8502	0.540	1.377	0.106	0.804	92	**0.097**	**0.9871**	**0.9932**
USD 2Y	0.0958	-0.046	0.117	0.117	0.847	73	**0.108**	0.9985	0.9980
USD 5Y	1.2919	-0.046	0.014	0.089	0.888	52	**0.080**	**0.9978**	**0.9968**
USD 8Y	2.6950	0.149	-0.001	0.069	0.896	50	0.049	0.9985	0.9988
USD 10Y	0.2420	0.194	0.147	0.083	0.938	43	0.047	0.9981	0.9991
USD 30Y	0.3988	-0.157	-0.166	0.043	0.965	53	0.041	**0.9971**	**0.9972**

Boldface indicates the rejection of null hypothesis

APPENDIX C: COMPARISON OF VARIANCE ESTIMATORS

	Excess returns		RMSE*1000	
Time-series	EWMA	MA	EWMA	MA
.N225	16	14	0.4028	**0.4067**
.N500	22	19	**0.5015**	0.5024
.TOPX	22	17	**0.3642**	0.3633
.OMX	14	17	**0.3835**	0.4233
.GDAXI	22	19	0.3627	**0.3774**
.NMDKX	18	**27**	**1.2371**	1.2782
.OBX	16	14	0.3530	**0.3876**
.SPC	21	17	**0.3054**	0.3053
.IXIC	24	21	**0.8952**	0.9363
.DJI	21	19	**0.2833**	0.2855
.AEX	17	18	0.2936	**0.3137**
.MIB30	21	23	0.3454	**0.3659**
.SSMI	18	16	0.2198	**0.2506**
.KFX	22	20	**0.2239**	0.2399
.IBEX	21	18	0.3942	**0.4115**
.FTSE	21	20	0.2070	**0.2110**
.FCHI	20	17	0.2897	**0.2997**
.TALG	16	12	0.9138	**0.9324**
.RICI	15	12	0.2221	**0.2746**
.LITIN	16	11	0.2689	**0.3123**
.FOX	19	18	**0.6633**	0.6862
.HPIY	19	20	**0.4293**	0.4428
.STOXX50E	20	20	0.3211	**0.3313**
.HEX	19	18	**1.1319**	1.1695
.STOXXE	21	18	0.3370	**0.3472**
.HEB1	18	11	**0.8255**	0.8335
.HEI1	16	11	3.0191	**2.9798**
.HEIN	16	13	0.2960	**0.2933**
.HECH	16	13	**0.5206**	0.5220
.HEC	15	**9**	**0.3436**	0.3605
.HEE	14	14	**0.4897**	0.5081
.HEFO	14	**8**	2.3317	**2.2995**
.HEF1	15	16	1.0157	**1.0156**
.HEM1	13	**9**	**0.3713**	0.3728
.HEL1	16	19	**1.4184**	1.4225
.HET1	18	16	**0.2777**	0.2827
.HEME	18	15	**0.9700**	0.9838
.HEO1	19	15	**1.8360**	1.8608
.HETE	21	18	**1.6614**	1.7175
.HETD	19	21	**0.2587**	0.2616
.HETR	16	13	**0.3430**	0.3450
SEK	19	13	0.0305	**0.0340**
NOK	12	**9**	**0.0265**	0.0305
GBP	21	18	**0.0584**	0.0592
JPY	24	18	0.1606	**0.1598**
CHF	19	16	**0.0093**	0.0098
USD	26	**27**	0.0709	**0.0699**

	Excess returns		RMSE*1000	
Time-series	EWMA	MA	EWMA	MA
FIM 2Y	15	16	**0.2226**	0.2239
FIM 3Y	15	16	**0.2022**	0.2023
FIM 4Y	16	16	**0.2271**	0.2313
FIM 5Y	21	20	**0.1898**	0.1913
FIM 8Y	16	17	**0.1678**	0.1689
FIM 10Y	14	15	**0.1591**	0.1613
FIM 11Y	20	21	0.3114	**0.3101**
SEK 3M	17	12	0.5188	**0.5151**
SEK 6M	17	13	**0.1794**	0.1817
SEK 1Y	16	13	**0.2456**	0.2475
SEK 2Y	16	18	**0.2602**	0.2615
SEK 3Y	15	14	**0.2446**	0.2456
SEK 4Y	16	13	**0.2254**	0.2259
SEK 5Y	16	14	**0.2204**	0.2205
SEK 6Y	16	16	**0.1959**	0.1973
SEK 7Y	15	16	**0.1934**	0.1946
SEK 8Y	18	17	**0.1915**	0.1931
SEK 9Y	18	18	**0.1853**	0.1865
SEK 10Y	17	18	**0.2107**	0.2121
SEK 15Y	19	19	0.1665	**0.1661**
FRF 3Y	14	12	**0.4352**	0.4352
FRF 6Y	14	13	0.2983	**0.2980**
FRF 9Y	14	13	0.1759	**0.1737**
FRF 15Y	12	13	0.2088	**0.2063**
FRF 20Y	17	11	0.1407	**0.1394**
FRF 30Y	19	17	**0.1540**	0.1543
EUR 3M	**7**	**4**	1.0462	**1.0362**
EUR 6M	**9**	**9**	0.3163	**0.3125**
EUR 1Y	**8**	**9**	0.2708	**0.2673**
DEM 3Y	14	15	**0.2853**	0.2868
DEM 6Y	15	15	**0.2546**	0.2552
DEM 7Y	15	16	**0.2015**	0.2016
DEM 8Y	16	17	**0.1929**	0.1931
DEM 20Y	17	19	**0.1460**	0.1469
DEM 30Y	13	18	**0.1370**	0.1377
USD 3M	15	**10**	**0.1599**	0.1894
USD 6M	**7**	**6**	**0.1421**	0.1672
USD 1Y	**7**	**7**	**0.2493**	0.2497
USD 2Y	12	**7**	**0.1276**	0.1402
USD 5Y	15	11	**0.1800**	0.1853
USD 8Y	12	**8**	**0.2125**	0.2145
USD 10Y	13	11	0.3721	**0.3707**
USD 30Y	21	21	0.1509	**0.1496**

For excess returns, boldface indicates rejection (α=5%)
For RMSE, boldface indicates better estimator