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REGULATORY CAPITAL AND CREDIT RISK MODELS

On Assessment of Capital Adequacy in the Banking Sector

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Tiivistelmä

Tutkimuksen kohteena on luottolaitosten vakavaraisuussääntely yleisesti sekä erityisesti sen mahdollinen kehittäminen käyttämällä hyväksi luottoriskimalleja. Muutaman viimeksi kuluneen vuoden aikana eräät suurimmat kansainväliset liikepankit ovat kehittäneet luottoportfolioidensa luottoriskin mittaamiseen malleja, jota ovat pitkälti analogia markkinariskien mittaamiseen aikaisemmin kehitetyille Value-at-Risk –malleille. Viime vuoden aikana pankkien edustajat tekivät esityksiä voimassa olevien vakavaraisuussäännösten korvaamiseksi säännöksillä, jotka sallisivat pankkien käyttää puheena olevia malleja vakavaraisuuslaskennassa.

Tutkimuksen toisessa luvussa selvitetään voimassa olevan vakavaraisuussääntelyn sisältöä. Esitys perustuu Baselin pankkivalvontakomitean antamaan suositukseen vuodelta 1988 ja siihen myöhemmin tehtyihin muutoksiin. Voimassaoleva sääntely useimmissa kehittyneissä teollisuusmaissa Suomi mukaan lukien perustuu mainittuun suositukseen. Suosituksen säännökset kohdistuvat ennen muuta luottoriskeihin, mutta kuten tässä luvussa osoitetaan, ne ottavat puutteellisesti huomioon erot luotansaajien luottokelpoisuudessa. Lisäksi käytetyn laskentamenetelmän vuoksi luottosalkun hajauttaminen ei vaikuta vakavaraisuuslaskennan tulokseen. Näiden puutteiden mahdolliset vaikutukset pankkitoimintaan esitetään lyhyesti luvun lopussa.

Tutkimuksen kolmas luku käsittelee luottoriskin mallintamista. Esityksessä keskitytään niin sanottuihin *"bottom-up"* luottoriskimalleihin, koska niiden katsotaan toistaiseksi olevan menetelmiltään edistyneimpiä. Luottoriskin mallintamisen tavoitteena on estimoida luottosalkun luottotappioiden todennäköisyysjakauma, jotta tarvittava riskipääoma voitaisiin allokoida luottoriskin kattamiseksi. Luvussa esitetään luottoriskin mallintamisen vaiheet, erilaisissa malleissa eri vaiheissa tehdyt käsitteelliset valinnat ja mallintamisoletukset sekä parametrien estimointimenetelmät. Esitys perustuu pääosin Federal Reserve Boardin ja Baselin pankkivalvontakomitean julkaisemiin raportteihin pankeissa sisäisessä käytössä nykyisin olevista luottoriskimalleista.

Tutkimuksen neljännessä luvussa tarkastellaan nykyisiin malleihin liittyviä puutteita silmälläpitäen lähinnä niiden hyväksikäyttöä vakavaraisuussääntelyssä. Keskeisimmäksi puutteeksi osoittautuu asianmukaisen historiallisen datan puutteesta johtuvat yksinkertaistavat mallintamisoletukset eri malleissa. Niiden vaikutuksia mallintamisen tuloksiin ei toistaiseksi vielä tunneta, eikä erilaisten mallien tulosten testaamiseen ole vielä olemassa yleisesti hyväksyttyjä menetelmiä. Näyttääkin siltä, että vaikka angloamerikkalaiset pankkivalvojat ovat periaatteessa suhtautunut suopeasti luottoriskimalleihin, nykyisiin malleihin liittyy senlaatuisia puutteita, ettei sääntelyä voida lähiaikoina tältä osin kehittää.

Avainsanat
Luottoriski, vakavaraisuussääntely, luottoriskimallit

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INTRODUCTION

"A credit risk model cannot replace a banker's judgment. Models don't manage."

Tom de Swaan, a member of the managing board of ABN AMRO Bank and Chairman of the Basle Committee on Banking Supervision.

1 SUBJECT OF THE STUDY

This master thesis discusses the regulatory framework to access capital for credit institutions, especially commercial banks, and potential regulatory uses of banks' internal credit risk models. Credit risk associated with default is usually assumed as the most significant risk incurred by banks (see e.g., Anttila 1996, Bessis 1998, and Niemelä 1999). First, credit risk is paramount in terms of the importance of potential losses. Secondly, banks are themselves borrowers with high levels of leverage. Unexpected realization of credit risk have destabilized, de-capitalized and destroyed banks quickly. For that reason, regulatory capital standards are almost entirely addressed to credit risk. Consequently, the focus of this study is also on credit risk charges and credit risk modeling.

Before going on, it is worth clarifying why there is then any need to impose credit risk (or any other risk) capital charges on banking institutions, and not on other institutions? Simply because they are regarded as different i.e., banks collect deposits and play a key role in the payment system.

Deposits are usually insured, but still governments always act as a guarantor for credit institutions, and as a lender of last resort. Capital plays the role of a buffer against future, unanticipated losses, and in some sort participates in the privatization of the burden that would otherwise be born by the government in case of a bank failure. In addition, fixed-rate deposit insurance creates, by itself, the need for capital regulation because of the moral hazard and adverse selection problems that it may generate. Under current regime, insured banks have an incentive to take more risk, since fixed-rate deposit insurance is like a put option sold by government to banks at a fixed premium, independent of the riskiness of their assets. This option increases in value when the bank's assets become riskier. Furthermore, as deposits are insured, there is no incentive for depositors either to cautiously select their bank. Instead, depositors may be tempted to look for the highest deposit rate, without paying enough attention the banks' creditworthiness (See Anttila 1996; Niemelä 1999).

In the banking universe, credit risk arises from various types of instruments including loans and loan commitments, bonds, receivables, letters of credit as well as market-driven instruments (e.g. swaps and forwards). Credit risk is, loosely speaking, the risk that customers *default*, that is fail to comply with their obligations to service debt. The Dictionary of Financial Risk Management (G. Gastineau et al, NY: Frank J. Fabozzi Associates, 1996) defines more accurately the different components of credit risk as follows:

(1) Exposure to loss as a result of default on a swap, debt, or other counterparty instrument;
(2) Exposure to loss as a result of a decline in market value stemming from a credit downgrade of an issuer or counterparty;
(3) A component of return variability resulting from the possibility of an event of default; and

(4) A change in the market's perception of the probability of an event of default, which affected the spread between two rates or reference indexes.

Credit risk management has evolved dramatically over the past decade in response to a number of secular forces that have made its measurement and management more important than ever before. Altman and Saunders have listed these forces (1998):

(1) A worldwide structural increase in the number of bankruptcies
(2) A trend toward disintermediation by highest quality and largest borrowers
(3) More competitive margins on loans
(4) A declining value of real assets and (thus collateral) in many markets
(5) A dramatic growth of off-balance sheet instruments with inherent default risk exposure, including credit risk derivatives

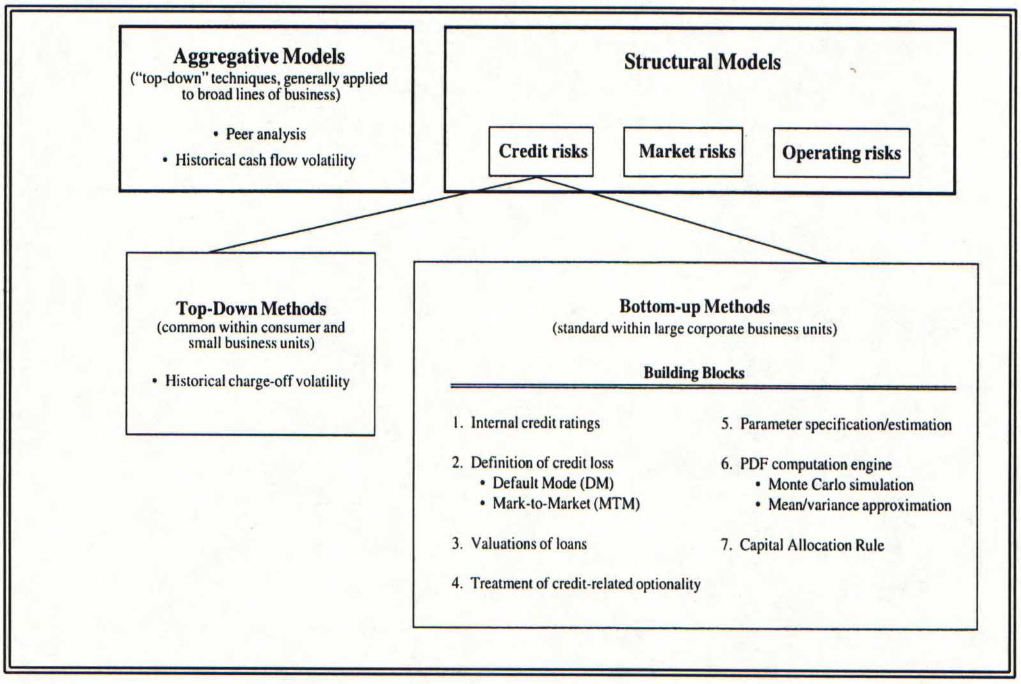
In response to these forces academics and practitioners alike have responded by:

(1) Developing new and more sophisticated credit-scoring/early-warning systems
(2) Moved away from only analyzing the credit risk of individual loans and securities towards developing measures of *credit concentration risk* (such as the measurement of *portfolio risk* of fixed income securities)
(3) Developing new models to price credit risk (such as the risk adjusted return on capital models) and
(4) Developing models to measure better the credit risk of off-balance sheet instruments.

Specifically, within the past two years, important advances have been made in modeling credit risk at portfolio level (2). Modeling methods build on the same statistical techniques employed by the Value-at-Risk (VaR) approach for measurement of market risks (see e.g. Jauri 1997). Large U.S. commercial banks, like Bank of America, Citibank and SBC

Warburg Dillon Read, have spent heavily to develop models for measuring the credit risk of their large and middle-market customers portfolios (see Figure 1 below). The modeling approach, referred as *"bottom-up"* approach by Jones & Mingo (1998), attempts to quantify credit risk at the level of each individual credit facility based on an explicit evaluation of the financial condition of the underlying customer and the structure of credit facility. Then, to measure credit risk for the portfolio as a whole, the risks of individual facilities are *aggregated*, taking into account diversification and correlation effects.

Figure 1: Overview of risk measurement systems



Source: Jones & Mingo (1998)

Banks use their internal credit risk models in estimating the *economical capital* needed to support their credit activities. Internal capital allocations are the basis for estimating the risk-adjusted profitability of various bank activities, which are used in evaluations of managerial performance and in determinations of managerial compensation (see e.g. Matten 1996). Of course, credit risk models and economic capital allocations have been incorporated into risk management processes, including risk-based pricing models, the setting of portfolio concentration and exposure limits, and a day-to-day credit risk management.

Besides strictly internal credit risk models, there are two banks that have gone public. Credit Suisse Financial Product has released CreditRisk+, which is free and can be downloaded from Internet. The model uses an analytic approach on the default rates associated with particular rating levels, the volatility of those default rates and a sector analysis. The basic mathematics it uses are similar to those used in insurance. The second well-known model is CreditMetrics data model published by J.P. Morgan & Co. The model uses Monte Carlo Simulation because its loss distributions are calculated from the probabilities of credit migration and the probabilities are correlated. Unlike CreditRisk+, CreditMetrics is only a methodology and dataset. Additional software is needed to run the model. Both the banks obviously assume that when their customers begin to use their models, demand for their credit derivative expertise will grow. Furthermore, there is a third well-known credit risk model, KMV developed by the consulting firm KMV Corporation and Stephen Kealhofer (see Kealhofer 1998).

The underlying idea of this thesis is based on the following two proposals: In March 1998, the Institute for International Finance Inc. (IIF) released a report recommending that regulatory capital framework should be refined to recognize banks' internal credit risk models for regulatory capital purposes. Similarly, the International Swaps and Derivatives Association

(Isda) published detailed proposals in March 1998 to try to persuade banking regulators to allow banks and other financial institutions to use credit risk models to calculate regulatory capital requirements. Both the organizations want a new regulatory framework which differentiates between high-quality and high-yield loan portfolios and which reward those banks which seek to manage their credit exposures with lower regulatory capital.

2 PURPOSE AND SCOPE OF THE STUDY

The purpose of the study is two-fold. First, to provide insight into the current risk-based capital regulation and the current state-of-the-art in the design of credit risk models. Secondly, to investigate the key challenges involved in potential regulatory uses of banks' internal credit risk models. The literature analyzing regulatory capital standards e.g., in Finland, is by far sparse despite the obvious importance of the topic. A correct measurement of the risks of insolvency is extremely important to banks' managers, shareholders, and unsecured creditors, as well as to the insuring authorities.

In fulfillment of the above purpose, the rest of the study is organized as follows: Chapter 2 examines the current regulatory capital measurement. The chapter presents the main features of legislation suggested by the Basle Committee on Banking Supervision and implemented by banking regulators in several countries. In Finland, the Credit Institutions Act (Laki luottolaitostoiminnasta 1607/93; referred in the following as LIL) in force also follows the Basle Committee's guidelines. In addition, the inherent flaws of the current regime are presented briefly.

Chapter 3 gives an overview of the current credit risk modeling practices. As stated above, for the time being, the banking industry has made the greatest conceptual advances in "bottom-up" credit risk modeling and the chapter therefore focuses only on the "bottom-up" models. Information on current internal credit risk models is primarily obtained from the following two sources:

(1) Federal Reserve System Task Force on Internal Credit Risk Models, referred in the following as the "Fed Task Force", published a review of modeling practices at twelve large U.S. banking organization and two non-bank securities firms in May, 1998.

(2) The Models Task Force of the Basle Committee on Banking Supervision, referred in the following as the "BIS Task Force", published a survey of modeling practices at 20 banking institutions located in 10 countries in April, 1999.

Chapter 4 presents conclusions regarding the potential model-based approach to calculating credit risk capital requirements. The main body of the conclusions is based on the articles and papers published by the representatives of Federal Reserve Bank of New York and the report of the Models Task Force of the Basle Committee on Banking Supervision. Continental European regulators have been more skeptical about the prospects for credit risk models so far (Taylor 1998). Finally, the fourth chapter also provides a summary of this study.

Chapter 2

CREDIT RISK CAPITAL CHARGES IN THE BIS FRAMEWORK

3 BACKGROUND OF THE BIS FRAMEWORK

The underlying structure of all bank capital adequacy regulations throughout the G10, and some non-G10 countries including Finland, rely on principles which were laid out in the "International Convergence of Capital Measurement and Capital Standards" document, published in July 1988, and referred to in the following as the "Accord". The Accord was initially developed by the Basle Committee on Banking Supervision, and later endorsed by the central bank governors of the G10 countries. The regulatory guidelines established in the Accord are also known as the "BIS framework" since the Committee meets four time a year, usually in Basle, under the patronage of the Bank for International Settlements (BIS). When drafting the Accord, the Committee had two overriding objectives: first, "the new framework should serve to strengthen the soundness and stability of the international banking system"; and secondly, "the framework should be in fair and have a high degree on consistency in its application to banks in different countries with a view to diminishing an existing source of competitive inequality among international banks" (see the Accord 1988).

Considering the stability of international banking system, the Accord defined two minimum standards for meeting acceptable capital adequacy requirements: assets to capital multiple and a risk-based capital ratio. The first standard is an overall measure of the bank's capital adequacy. The second measure focuses on the *credit risk* associated with specific on- and off-balance sheet asset categories. This second measure is a solvency ratio, known as *the BIS ratio*. It is defined as a ratio of capital to risk-weighted on-balance sheet assets plus off-balance sheet exposures, where the weights are assigned on the basis of counterparty credit risk. At the time the Accord was drafted, the use of differential risk weights to distinguish among broad asset categories represented a truly innovative approach to formulating prudential regulations. The risk based rules also set the stage for the emergence of more general risk-based policies within the supervisory process. However, since 1988, banking and financial markets have changed considerably.

4 DIVISION BETWEEN BANKING AND TRADING BOOK

In April 1995, the Basle Committee issued a consultative proposal to amend to the initial Accord, known as "Amendment to the Capital Accord to Incorporate Market Risk". The final version of the proposal will be referred to in the followings as the "Market Risk Amendment". It requires banks to measure and hold capital to cover their exposure to *market risk* associated with debt and equity positions located in the trading book, and foreign exchange and commodity positions in the both the trading and banking books.

According to the Market Risk Amendment, market risk encompasses both "general market risk" and "specific market risk". General market risk refers to changes in market value of on-balance sheet assets and off-balance sheet instruments resulting from broad market movements, such as changes in the level of interest rates, equity prices, exchange rates, and commodity prices. Specific market risk refers to changes in the market value of individual positions due to factors other than broad market movements like liquidity, exceptional events, and credit quality.

The *trading book* is defined as the bank's proprietary positions in financial instruments, which are intentionally held for short term trading, and/or which are taken on by the bank with the intention making profit from short term changes in prices, rates and volatilities. All trading book positions must be marked-to-market or marked-to-model every day. For market risk purposes, a bank may include in its measure of general market risk certain non-trading instruments that it deliberately uses to hedge trading positions.

The initial Accord still applies to *non-trading items* both on- and off-balance sheet. Market risk must be measured for both on- and off-balance sheet *traded* instruments. However, on-balance sheet traded instruments are subject to market risk charges only, while off-balance sheet derivatives, like swaps and options are subject to both market risk and credit risk capital charges. Consequently, the bank's overall capital requirement is the sum of (see the Market Risk Amendment):

(1) *Credit risk capital charge*, which applies to all positions in the trading and banking book, but excluding debt and equity traded securities in the trading book, and all positions in commodities and foreign exchange; and
(2) *Market risk capital charge* for the instruments of the trading book on-, as well as off-balance sheet.

It is worth noting that market risk regulation in the Europe Union e.g., here in Finland (see LIL 5a §; RATAm 106.12), is primarily based on the Capital Adequacy Directive (CAD), adopted by the European Commission in 1993. The directive has been effective since January 1996, two years before the Market Risk Amendment applies. In many ways, the new BIS framework nevertheless follows the CAD rules. For instance, the "trading book" concept is very similar. Contrary to the CAD rules, the Market Risk Amendment however gives an important role to banks' internal VaR models. Banks have the choice between their own internal models and the standard model proposed by the Market Risk Amendment to determine market risk related regulatory capital. Instead, such internal models are not specifically provided for in the CAD (see Dale 1996; cf. RATAm 106.12 Appendix 3).

5 THE RISK-WEIGHTED AMOUNT USED TO COMPUTE THE BIS RATIO

In determining the BIS ratio, it is necessary to consider both the on-balance sheet items as well as specific off-balance sheet items. The BIS framework carries risk-weighted categories as to which all on-balance sheet items are allocated. These items are valued at their historical book values. Off-balance sheet items are first expressed as a credit equivalent and then are appropriately risk weighted by counterparty, see Section 6 below. The total risk-weighted amount is the sum of the two components: the risk-weighted assets for on-balance sheet instruments and the risk-weighted credit equivalent for off-balance sheet items. Table 1 gives the risk capital weights by asset categories, and Table 3, in Subsection 6.2, shows the weights which applies to credit equivalents for off-balance derivative positions by the type of counterparty (cf. LIL 76 §; RATAm 106.7).

Table 1: Risk capital weights by broad on-balance sheet asset category

Risk weights	Asset category
0 %	▪ Cash ▪ Claims on central governments and central banks denominated in national currency and funded in that currency ▪ Other claims on OECD central governments ▪ Claims collateralised by cash or OECD central government securities or guaranteed by OECD central governments.
20 %	▪ Claims on multilateral development banks and claims guaranteed by such bank ▪ Claims on banks in the OECD and loans guaranteed by OECD incorporated banks ▪ Claims on banks incorporated in countries outside the OECD (with a residual maturity of up to one year), and, claims guaranteed by the same banks ▪ Claims on non-domestic OECD public sector entities, and claims guaranteed by such entities ▪ Cash items in process on collection
50 %	▪ Loans fully secured by mortgage on residential property that is or will be occupied by the borrower or that is rented
100 %	▪ Claims on the private sector ▪ Claims on banks incorporated outside the OECD with a residual maturity of over one year ▪ Claims on commercial companies owned by the public sector ▪ Premises, plant and equipment and other fixed assets ▪ Real estate and other investments ▪ Capital instruments issued by other banks (unless deducted from capital) ▪ All other assets

The assignment of various groups of assets to particular risk-weight categories inevitable represent "rather broad, unspecific, regulatory judgement" (Norton 1994). Clearly, the BIS risk weights do not reflect some very obvious determinants of credit risk, such as differences in *credit quality* across commercial loans, *concentrations of risk* in a specific asset category or to particular obligator, industry, or region and *covariances* among values of financial instruments. Some empirical studies have analyzed the correspondence of risk weights with actual risk (see e.g., Avery & Berger 1991 and Cordell & King 1995). They indicated that the correspondence is indeed relatively loose.

6 CALCULATION OF THE CREDIT EQUIVALENT FOR OFF-BALANCE SHEET EXPOSURES

6.1 The Case of Non-derivative Exposures

Non-derivative off-balance sheet items are converted to credit equivalents by multiplying the nominal principal amounts by a credit conversion factor. Conversion factors depend on the nature of the instrument, as shown in Table 2.

Table 2: Credit conversion factors for non-derivative off-balance sheet exposures

Conversion factor	Off-balance sheet exposure
100 %	▪ Direct credit substitutes, e.g. general guarantees of indebtedness, bank acceptance guarantees and standby letter of credit serving as financial guarantees for loans and securities ▪ Sale and repurchase agreements and asset sales with recourse, where the credit risk remains with the bank ▪ Forward asset purchases, forward deposits and partly-paid shares and securities, which represent commitments with a certain drawdown
50 %	▪ Certain transaction –related contingent items ▪ Note issuance facilities and revolving underwriting facilities ▪ Other commitments with an original maturity of over one year
20 %	▪ Short-term, self-liquidating trade-related contingencies
0 %	▪ Other commitments with an original maturity of up to one year, or which could be unconditionally cancelled at any time

The resulting credit equivalents are weighted according to the type of the counterparty exactly as on-balance sheet instruments, see Table 1 above (cf. RATAm 106.7).

6.2 The Case of Derivative Positions

The Accord recognizes that the credit risk exposure of long dated financial derivatives fluctuates in value, and estimates this exposure both in terms of the current marked-to-market value, plus a simple measure of the projected future risk exposure. Calculation of the BIS risk-weighted amount for derivatives proceeds in two steps. The first step involves computing a credit equivalent amount, which is sum of the current replacement cost when it is positive (and zero otherwise), and an add-on amount that approximates future replacement cost. The second step involves computing a risk-weighted amount. It is simply derived by multiplying the credit equivalent amount by a counterparty risk-weighting factor given in Table 3 (cf. RATAm 106.7).

Table 3: Counterparty risk weighting factors for derivative off-balance sheet exposures

Risk weights	Type of Counterparty
0 %	OECD governments
20 %	OECD banks and public sector entities
50 %	Corporate and other counterparties

The current replacement value of a derivative is its marked-to-market value or liquidation value, only when it is positive. The add-on amount is computed by multiplying the notional amount of the transaction by the BIS required add-on factors, as shown in Table 4.

Table 4: Add on factors by the type of underlying and maturity

Residual Maturity	Interest Rate	Exchange Rate and Gold	Equity	Precious Metals	Other Commodities
One year or less	0 %	1 %	6 %	7 %	10 %
Over one year to five years	0.5 %	5 %	8 %	7 %	12 %
Over five years	1.5 %	7.5 %	10 %	8 %	15 %

In the table, interest rate contracts include single currency interest rate swaps, basis swaps, forward rate agreements and product with similar characteristics, interest rate futures and interest rate options purchased. Exchange rate contracts, in turn, include gold contracts, which are treated the same way as exchange rate contracts, cross-currency swaps, cross-currency interest rate swaps, outright forward foreign exchange contracts, and currency options purchased. Equity contracts based on individual stock as well as equity indices, precious metal contracts and contracts on other commodities include forward, swaps and purchased options.

6.3 Netting of Derivatives

In 1995 the initial Accord was modified to allow banks to reduce their credit equivalent when bilateral, legally enforceable netting agreements are in place (see the Treatment of the Credit Risk Associated with Certain Off-balance-sheet Items). The new BIS formula for add-on amounts is:

Equation 1

$$\text{add on amount} = \text{notional amount} \times \text{add on factor} \times (0.4 + 0.6 \times \text{NPR})$$

where add-on factors are the same as in Table 4. NPR denotes the net placement ratio, which is replacement cost when positive, divided by the gross replacement cost calculated as before, without taking account netting. That is, the sum of the positive replacement cost for the transactions covered by the netting agreement. Thus, the new formula either does not allow for complete offsetting even if netting agreement is in place. The above calculations are done by counterparty, and then the counterparty risk weight applies to derive the risk-weighted amount.

7 REGULATORY CAPITAL AND THE BIS RATIO

The main feature of the legislation suggested by the Basle committee is the minimum capital level of risk-based capital-to-asset ratio. As a protection against credit risk, banks are required to maintain a capital amount of at least 8% of the total risk weighted assets calculated as shown in the previous sections. However, capital, as defined by the Committee, is broader than equity capital. The BIS rules assign regulatory capital items to two tiers: core capital (Tier 1) and supplementary capital (Tier 2). The total capital of a bank comprises sum of core capital and supplementary capital, minus required deductions and limits as presented in Table 5 (see the Accord and cf. LIL 73 § and 75 §).

Table 5: Regulatory capital

+	**Tier 1**	▪ Fully paid common stock ▪ Non-cumulative preferred stock ▪ Disclosed reserves (reserves by appropriations of retained earnings, other surplus such as share premiums, retained profit, general and legal reserves)
-	**Deductions**	▪ Goodwill
+	**Tier 2**	▪ Undisclosed reserves ▪ Revaluation reserves ▪ General provisions and general loan loss reserves ▪ Hybrid debt capital instruments ▪ Subordinated term debt
-	**Deductions**	▪ Investments in unconsolidated subsidiaries engaged in banking and financial activities ▪ Banks' holdings of capital issued by other banks of deposit –taking institutions
-	**Limits**	▪ General loan loss reserves are limited to a maximum of 1.25 % of risk weighted assets ▪ Subordinated term debt instruments may be included within Tier 2 capital only to a maximum 50 % of the Tier 1 capital. ▪ Tier 2 capital *included in total capital* is limited in *amount to 100 % of Tier 1 capital*
=	**CAPITAL**	

As the BIS risk-based capital rules limit capital counted to Tier 2 capital to be no more than Tier 1 capital, the minimum capital levels can also be defined as:

Equation 2
$$\frac{\text{Tier 1 - Deductions}}{\text{Total Risk Weighted Assets}} > 4\%$$

Equation 3
$$\frac{\text{Tier 1 - Deductions + Tier 2 - Deductions - Limits}}{\text{Total Risk Weighted Assets}} > 8\%$$

At the time the Accord was drafted, little empirical work existed to support either the minimum capital levels or the risk weights included the BIS framework. In many respects, the proposals of the Basle Committee are however in accordance with the model presented by Kim and Santomero (1988). They used the mean-variance model and concluded that e.g. simpler financial leverage is ineffective means to limit the insolvency risk of banks. They argued that under such circumstances banks reshuffle assets toward riskier ones when a capital ratio is tightened. They suggested that risk-related capital regulation is potentially more effective, but they also noticed that the weights must be chosen optimally to be "theoretically correct". The Basle Committee justified its proposal on more pragmatic grounds:

(1) "It [the risk weight approach] provides a fairer basis for making international comparisons between banking systems whose structures may differ,

(2) It does not deter banks form holding liquid or other assets which carry low risk, and

(3) It allows off-balance-sheet exposures to be incorporated more easily into the measure."

Since 1988, some academic studies have evaluated the effectiveness of the BIS ratio's accuracy in classifying solvent and insolvent bank. The findings of e.g., Jones and King (1995) as well as Niemelä (1999) indicate that the risk-based capital ratio is actually a poor and unreliable indicator for troubled banks. First, credit risk is not the only determinant in failure prediction, and secondly, credit risk may be poorly measured by the BIS ratio.

8 FURTHER CONSIDERATIONS

It is said that the Accord was designed for "a stylized (or simplified) version of the banking industry at the end of the 80s" and tends to be somewhat "rigid in nature" (Swaan 1998 and Norton 1994). On the other hand, such elements have enabled the BIS framework to be widely applicable and that have contributed to greater harmonization. There is nevertheless a growing realization that the Accord, in spite of the above amendments, may be no longer up-to-date and needs to be modified. Moreover, the weaknesses of the current regime are not mere of academic or regulatory interest but have very real implications for banking businesses, as discussed briefly below.

Market participants argue that the current rules have a distortive effect on credit risk pricing, as the BIS risk weights do not fully reflect differences between degrees of default risk, different seniority of instruments or differences in the term of an exposure. Indeed, there seems to be some empirical evidence indicating that, at least to some extent, the relative regulatory capital cost of each type of instrument is influencing spreads and distorting the market pricing of credit risk (Isda 1998). The fact that riskier, higher-return business can be undertaken at the same "regulatory capital cost" also provides an incentive to lend to lower quality credits, as the relative return appears more favorable (see also Nishiguchi et al 1998).

As discussed above, the current regime ignores totally diversification effects. Therefore, market participants believe that the current capital ratios send an inaccurate and distorted signal to markets, causing perverse influences on a bank's relative funding costs, credit rating and share performance (Isda 1998). This is because regulatory capital ratios still play an important part in market perception of a bank relative to its peers; stock analysts, rating agencies, market commentators, investors and the press focus on the BIS ratio as a headline indicator of financial soundness.

Finally, it is highlighted how the current rules may have a distortive effect on risk management in general. Since they fail to recognize portfolio diversification and offsetting short credit risk positions, except in very limited circumstances, they inhibit credit risk hedging, through e.g., the use of credit derivatives, and provide no incentive to develop modeling techniques, which measure credit risk on a portfolio basis.

Chapter 3

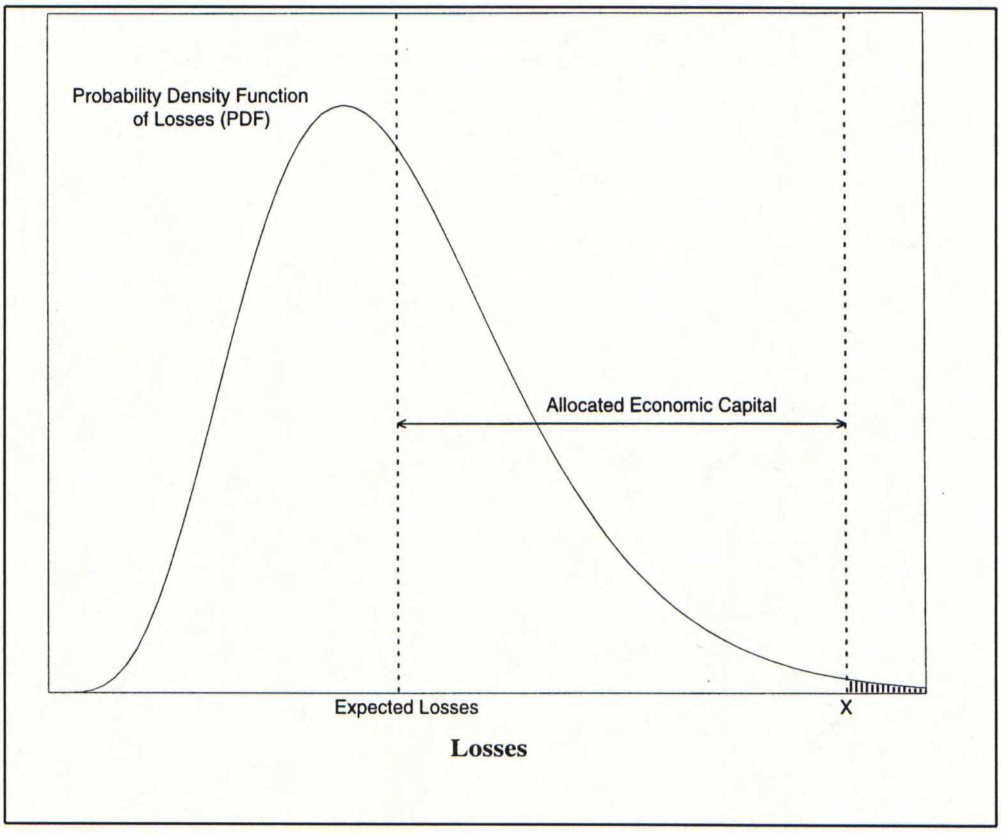
BOTTOM-UP CREDIT RISK MODELING

9 THE RELATIONSHIP BETWEEN PDF AND ALLOCATED ECONOMIC CAPITAL

Before discussing credit risk modeling techniques, it is useful to describe how in practice these models are used within banking institutions' internal capital allocation systems. Internal capital allocations against credit risk are based on a bank's estimate of the *probability density function* (PDF) for credit losses. Various credit risk modeling techniques do this by different means but the goal is the same; to estimate PDFs. A risky portfolio is, loosely speaking, one whose PDF has a relatively long, fat tail – where there is a significant likelihood that actual losses will be substantially higher than expected losses, shown as the left dotted line in Figure 2. In the figure, the probability of credit losses exceeding the level X is equal to the shaded area under the PDF to the right of X.

The estimated capital needed to support credit risk exposure is generally referred to as its "*economic capital*" for credit risk (Jones & Mingo 1998). The process for determining this amount is analogous to VaR methods used in allocating economic capital against market risks. Specifically, the economic capital for credit risk is determined in theory so that the probability of unexpected credit losses exhausting economic capital i.e., the probability of insolvency, is less than some "target insolvency rate". In practice, the target insolvency rate is usually chosen to be consistent with the bank's desired credit rating for its own liabilities (the Fed Task Force 1998; the BIS Task Force 1999). For example, if the desired rating is AA, the target insolvency rate might equal the historical default rate for AA-rated corporate bonds. According to a rule of thump used by practitioners, this rate is about three basis points (Gordy 1998). Consequently, such a bank ought to hold capital against credit loss equal to the 99.97^{th} percentile value on the cumulative distribution of portfolio losses. Capitalization sufficient to absorb up to the 99.50^{th} percentile value of losses in turn would be consistent with a BBB-rating (Gordy 1998).

Figure 2: Relationship between PDF and allocated economic capital



Within capital allocation systems, a critical distinction is made between *expected credit losses* and the *uncertainty of credit losses*. It is generally assumed that it is the role of reserving policies to cover expected credit losses (the Fed Task Force 1998; the BIS Task Force 1999), while it is role of equity capital to cover credit risk, or the uncertainty of credit losses. In Figure 2, for a target insolvency rate (the area shaded under the PDF to the right of X), the required economic capital equals the distance between the two dotted lines. The area under the PDF to the left of expected losses should be covered by the loan loss reserves.

As indicated, economic capital allocations for credit risk are based on two extremely critical inputs: the target insolvency rate and the estimated PDF for credit losses. Therefore, two banks with identical portfolios may have very different capital allocations for credit risk, owing to difference in their attitudes toward risk taking, as reflected in their target insolvency rates, or owing to differences in their methods for estimating PDFs as reflected in their credit risk models.

10 PLANNING HORIZON AND DEFINITION OF CREDIT LOSSES

Bottom-up credit risk modeling procedures are driven importantly by an underlying definition of credit losses and the "planning horizon" over which such losses are measured. The banks reviewed by the Fed/BIS Task Forces generally employ a *one-year planning horizon* and what Jones and Mingo (1998) refer to as either a *default-mode (DM) paradigm* or a *mark-to-market (MTM) paradigm* for defining credit losses.

For the time being, the DM paradigm appears to be the most common approach to defining credit losses, and e.g. CreditRisk+ also relies on this approach. It is sometimes called also a "two-state" approach because only two outcomes are relevant: non-default and default. If a loan does not default within the planning horizon, no credit loss is incurred; if the loan default, the credit loss equals the difference between the loan's book value and the present value its net recoveries. There are several possible definitions of default depending on the particular bank. A loan may be deemed to be in "default" if the loan is classified "substandard", if payments are past due, if the loan is placed on non-accrual status, or if recovery proceeding are initiated. Typically, default nevertheless arises if the obligor becomes unable to meet its payment obligations and the loan is placed non-accrual status (the Fed Task Force 1998). The DM paradigm can be thought of a representation of traditional "buy and hold" lending business of banks. Under this approach, secondary loan markets are regarded as not sufficiently developed to support a full mark-to-market approach to risk measurement.

The MTM paradigm generalizes the DM approach by recognizing that the *economic value* of loan may decline even if the loan does not formally default. According to the Fed Task Force, few U.S. banks used the MTM framework (1998), but at the banks surveyed by the BIS Task Force, MTM-type models seem to be more common (1999). Furthermore, many practitioners believe the industry is likely to evolve from DM-based risk models for the banking book to the more general MTM-based models over the coming years. The earliest popular model J.P. Morgan's CreditMetrics can also be regarded as an MTM-model.

The MTM paradigm is "multi-state" in that "default" is only one of several possible credit ratings to which a loan could migrate. In effect, the credit portfolio is assumed to be marked to market or, more accurately, *"marked to model"* as further discussed below. For example, the value of a term loan typically would employ a discount cash flow methodology, where the credit spreads used in valuing the loan would reflect the market-determined term structure of credit risk spreads for loans of that grade. A credit loss under the MTM paradigm is defined as an unexpected reduction in the portfolio's value over the planning horizon due to deteriorations in credit ratings on the underlying loans or a widening of credit risk spreads in financial markets.

11 VALUATIONS OF LOANS AND RISK FACTORS

Under both the loss paradigms, the estimation of the current portfolio's PDF involves estimating (1) the portfolio's current value and (2) the probability distribution its future value at the end of the planning horizon. Consequently, model-builders are required to define how the *current and future values* of each credit instrument are determined at the beginning and end of the planning horizon. In addition, model-builders are required to specify the risk factors that determine each of the types of credit events leading to a *change* in the value of an instrument. In practice, these operational modeling details are however dependent on the specific concept of credit loss.

This section reviews the loan valuation processes and the specification of the risk factors under both the paradigms. To simplify the review, it is assumed that the bank's exposure level is known with certainty at the beginning of the planning horizon. Specifically, the credit portfolio is assumed to consist only of fixed-rate term loans and that each customer has only a single loan. For many types of credit instruments, such as lines of credit and derivatives, a bank's credit exposure over the planning horizon is *not* known with certainty. The treatment of such instruments is discussed briefly in Section 12.

11.1 Valuations within the DM Framework

The current and future values of loans in the DM paradigm are defined in a manner consistent with the underlying two-state notion of credit losses. Typically, for a simple term loan, the current value is measured as its *book value* at beginning of the planning horizon (see e.g. CreditRisk+: A Credit Risk Management Framework 1997, the Fed Task Force 1998, and the BIS Task Force 1999).

Under the DM paradigm, the future value of the loan depends only on whether or not the borrower defaults during the planning horizon. If the borrower does not default, the loan's future value is normally taken to be its *book amount* at the end of the planning horizon as well. Neither changes in credit risk spreads nor downgrades short of default affect the future values of non-defaulting loans within DM-type models. On the other hand, if the borrower defaults, the future value is usually measured as the loan's book value minus the present value its net recoveries, as further discussed in Subsection 11.3 below.

11.2 Discounted contractual cash flows approach

Within the MTM framework, the most common valuation approach appears to be discounted contractual cash flow methodology that is often also associated with J.P Morgan's CreditMetrics framework (see also the Fed Task Force 1998). To illustrate the methodology, suppose the credit portfolio consists N customers, where current credit rating of the customer i is g_i. The number of applied rating grades is denoted G, and grades 1 through G-1 are non-default states, and grade G stands for a default. The loan to the customer i has a contractual coupon payment of C_i per period until maturity in period M_i, at which point the final payment (principal plus coupon) equals the sum of C_i and P_i.

The current value of an individual loan to the customer *i* can be presented as the present discounted value of its contractual cash flows:

Equation 4

$$V_i = \frac{C_i}{[1 + r_1 + R_1(g_i)]} + \frac{C_i}{[1 + r_1 + R_1(g_i)][1 + r_2 + R_2(g_i)]} + \ldots + \frac{C_i + P_i}{\prod_{k=1}^{M_i} [1 + r_k + R_k(g_i)]}$$

The discount rate for period *k* equals the sum of the *forward risk-free rate* implied by the yield curve, denoted r_k; and for each credit rating grade, the *market-determined risk premium* for deflating period-*k* contractual cash flows of g_i-rated obligors, denoted $R_k(g_i)$.

Consistent with the determination of current values, the future values of a non-defaulting loan can be calculated as the present discounted value of its remaining contractual cash flows. Consequently, the future value of a non-defaulting loan to the customer *i* as of the end of planning horizon is given by:

Equation 5

$$V_i = \frac{C_i}{\left[1 + r_2 + \hat{R}_2(\hat{g}_i)\right]} + \frac{C_i}{\left[1 + r_2 + \hat{R}_2(\hat{g}_i)\right]\left[1 + r_3 + \hat{R}_3(\hat{g}_i)\right]} + \ldots + \frac{C_i + P_i}{\prod_{k=2}^{M_i - 1} \left[1 + r_k + \hat{R}_k(\hat{g}_i)\right]}$$

In the equitation, a hat (^) over a variable indicates that it is endogenous, i.e. dependent on other variables, and its value is taken as of the end of the planning period. Specifically, the discount rates can be different from those used in determining the value at the beginning of the planning horizon either because the loan's credit rating may have changed or because the term structure of credit spreads on loans of a given rating may have changed. In the equitation, $\hat{}R_k(\hat{}g_i)$ denotes the market-determined risk premium for obligators rated $\hat{}g_i$, where both the risk premium and the credit rating are *endogenous variables* measured as of the end of the planning horizon.

Although the discounted contractual cash flow approach is easily understood and implemented, it is not fully consistent with modern finance theory. Under the approach, loans to two identically rated borrowers receive the same discount factors, even if the two borrowers are not equally sensitive to the business cycle or other systematic factors. Accordingly, the discount factor for the customer *i* should include an idiosyncratic component, which affects only that individual customer. Within the current generation of credit risk models, this component is usually ignored meaning that credit risk spreads are assumed to depend only on the obligors credit rating.

11.3 Future Values of Defaulted Loans

The discounting of contractual cash flows is not appropriate for modeling the end-of-period values of defaulted loans. Instead, under both the loss paradigms, the *decline* in the economic value of a defaulted loan relative to its book value is typically determined as the book value, B_i times a *loss-rate-given-default* (LGD). That is, the future value of a defaulted loan V_i is given its recovery rate, equal to one minus the LGD:

Equation 6

$$V_i = B_i\left(1 - L\hat{G}D_i\right)$$

The sophistication of current modeling methods for LGDs seems to vary considerably across banks (see the Fed Task Force 1998 and the BIS Task Force 1999). LGDs may be treated as random variables whose values are uncertain as of beginning of the planning horizon. For example, LGDs may be assumed to equal the sum of a fixed average loss rate, L and a zero-mean random error term $\sim l_i$:

$$L\hat{G}D = L + \tilde{l}_i$$

A tilde ($\sim$) over variable indicate that it does not depend on other variables i.e., it is regarded as exogenous. Contrary to such models, in some models, LGDs are treated as deterministic and known in advance.

11.4 Credit Rating Migrations

11.4.1 Ratings Transition Matrix

Within the credit risk models reviewed by the Fed/BIS Task Forces, the likelihood of a credit facility migrating to another credit risk grade over the planning horizon is frequently represented through a "ratings transition matrix" similar to that J.P. Morgan's CreditMetrics matrix depicted in Table 6. Given the borrower's current credit rating, the probability of migrating to another grade is shown with intersecting cell. In Table 6, for example, the likelihood of a BBB-rated loan migrating to single-B within one year would be 0.32%. Since under the DM paradigm only rating migrations into the default state lead to changes in the values of loans, only the last column of matrix would be relevant representing the expected default frequency (EDF) of a particular grade.

Table 6: Sample Credit Transition Matrix

(Probability of migrating to another rating within one year, percent)

		AAA	AA	A	BBB	BB	B	CCC	Default
Current Credit Rating	AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
	AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
	A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
	BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
	BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
	B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
	CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60

Credit rating transition matrices are usually based on the historical migration frequencies of publicly rated corporate bonds, as in Table 6. However, probabilities in transition matrices are typically statistically "smoothed" in order to attenuate the effects of sampling variation in the actual migration patterns of corporate bonds (see CreditMetrics-Technical Document 1997). For modeling default/rating migrations, the banks reviewed by the Fed/BIS Task Forces have adopted either a reduced-form approach or a structural approach.

11.4.2 Structural Models

As exemplified by the CreditMetrics modeling framework, under the structural approach the model-builders typically posit some explicit microeconomic model of the process determining defaults and rating migrations of individual customers. To illustrate, for a given customer *i*, a rating migration from g_i to $\hat{}g_i$ may assumed to depend on the future realization of a customer-specific random variable, "a migration risk factor", denoted $\sim v_i$, representing the change in that borrower's financial condition over the planning horizon. Specifically, for a credit rating system with G grades:

Equation 7

$$\hat{g}_i = \begin{cases} 1 & \text{if} & & \tilde{v}_i & \leq & V_1(g_i) \\ 2 & \text{if} & V_1(g_i) & \leq \tilde{v}_i & \leq & V_2(g_i) \\ \vdots & & & & & \\ G-1 & \text{if} & V_{G-1}(g_i) & \leq \tilde{v}_i & \leq & V_G(g_i) \\ G & \text{otherwise} & & & & \end{cases}$$

where for a customer *i* having a credit rating of g_i, the $V_1(g_i),\ldots,V_G(g_i)$ denote the threshold levels of $\sim v_i$ that trigger rating downgrades or upgrades. Consequently, for a grade-4 facility a value of $\sim v_i$ less than or equal to $V_1(4)$ would imply a future credit rating of grade-1, a value greater than $V_1(4)$ but less than or equal $V_2(4)$ would imply a grade-2 and so forth. Of course, only migrations to "default" are relevant for DM-type models.

As within CreditMetrics, the change in the value of obligor's assets in relation to asset value thresholds V is often assumed to determine the change in its credit rating over the planning horizon (the BIS Task Force 1999). For example, given an obligor's current credit rating e.g., equivalent to BBB, an extremely large positive change to its net worth might correspond to an upgrade to AAA, while an extremely large negative realization might generate a downgrade to default. Mathematically, the asset threshold levels are appropriately scaled so that the probability of any borrower migrating to another grade agrees with the assumed rating transition matrix (see CreditMetrics-Technical Document 1997).

11.4.3 Reduced-Form models

In contrast to structural models, reduced-form models typically assume a particular functional relationship between borrowers' expected default rate/migration matrix and so-called *background factors*. These background factors may represent either (a) observable variables, such as indicators of macroeconomic activity, or (b) unobservable random risk factors. Under this approach, it is not necessary to explicitly estimate the means and variances of the underlying migration risk factors if a mean-variance methodology (see Subsection 14.1 below) is used to approximate the PDF. Rather, the model-builder must estimate each loan's probability of default.

An example of the reduced-form approach is the CreditRisk+ modeling framework where defaults are assumed to be driven entirely by a vector of K "risk factors" $x = (x_1, \ldots x_K)$. Conditional on x, defaults of individual obligors are assumed to be independently distributed Bernoulli draws. The conditional probability $p_i(x)$ of drawing a default for obligor i is a function of the rating grade g_i of obligor i, the realization of risk factors x, and the vector of "factor loadings" $(w_{i1}, \ldots, w_{iK})$ which measure the sensitivity of obligor i to each of the risk factors. CreditRisk+ specifies this function as:

Equation 8

$$p_i(x) = \bar{p}_{g_i} \sum_{k=1}^{K} x_k w_{ik}$$

where $\bar{p}_{gi}$ is the unconditional default probability for a grade-g obligor, and the x are positive-valued with mean one. The intuition behind this specification is that the risk factors x serve to "scale up" or "scale down" the unconditional $\bar{p}_{gi}$. A high draw x_k (over one) increases the probability of default for each obligor in proportion to the obligor's weight w_{ik} on that risk factor; a low draw of x_k (under one) scales down all default probabilities. The weights w_{ik} are required to sum to one for each obligor, which guarantees the $E[p_i(x)] = \bar{p}_{gi}$.

11.5 Changes in Credit Risk Spreads

Under the MTM paradigm, for purposes of modeling future values, changes in the risk-free yield curve are not treated as random credit events. Changes in the yield curve are normally set equal to the market expectations implied by the current risk-free term structure. Instead, it can be assumed that for a given credit rating g, changes in the credit risk spread for period k, are random:

Equation 9

$$\hat{R}_k(g) = R_k(g) + \tilde{z}_k(g) \text{ , for } k = 1, 2, \ldots, M$$

where M is the longest maturity of any loan and $\sim z_k(g)$ denotes a random risk factor. However, modeling changes in credit risk spreads appears to be in a very early stage of development, as discussed in Section 13.3 below.

11.6 Summary of Loan Valuations and Risk Factors

The preceding presentation has emphasized three types of "credit events" that can lead to a change in the value of a loan. The credit loss for a single term loan reflects the combined influence of the risk factors, which correspond to (1) the random variables affecting rating migrations (only migrations to "default" are relevant for DM-type models); (2) the random variables affecting the loan's LGD; and within MTM-type models, (3) the random variable affecting credit risk spreads.

12 TREATMENT OF CREDIT RELATED OPTIONALITY

12.1 Lines of Credit

In contrast to simple term loans or bonds, for many instruments a bank's credit exposure is not fixed advance, but rather depends on future (random) events. One important example of such *exposure risk* or, in other words, "credit-related optionality" is a committed line of credit, where the borrower is allowed to draw on those lines whenever he wants to, depending on his needs and subject to a pre-defined credit limit. Optionality reflects the fact that drawdown rates tend to increase as a borrower's credit quality deteriorates.

Within current credit risk models, the credit-related optionality associated with a committed line of credit usually is represented by treating the drawdown rate as a known function of the customer's end-of-period credit rating (The Fed Task Force 1998 and the BIS Task Force 1999). The method can be illustrated considering a one-year line of credit that is completely undrawn at the beginning of the planning horizon. Conditional on the customer's credit grade at the end of the planning horizon, the assumed end-of-period drawdown rate would be based on the average historical drawdown experience of customers having that future grade. The future value of the line would then be calculated as if the line were a loan equal to the assumed conditional drawdown.

Within the DM-type models, a simpler approach is often employed. The undrawn credit facility is converted into a "loan equivalent exposure" (LEE) to make it comparable to a term loan (The Fed Task Force 1998). The loan equivalent exposures are assumed independent of customer's credit quality. Ideally, the loan equivalent exposure should be calculated as the expected drawdown under the line in the event the customer were to become insolvent by the end of period. Within the CreditRisk+ modeling framework, it is suggested that a committed line of credit is usually drawn down totally prior to default and the exposure at risk should therefore be assumed to be full nominal amount (CreditRisk+: A Credit Risk Management Framework 1997).

12.2 Derivatives Exposures

Credit-related optionality also arises with products outside traditional lending operations. In derivative transactions, such as interest rate and currency swaps, equity derivatives, FX derivatives, etc., the source of uncertainty is not the customers' behavior but lies in market movements.

That is, counterparty exposure changes randomly over the life the contract, reflecting changes in the amount by which the bank is "in the money".

The current exposure of various derivative positions can be assessed by identifying the market-to-market values of positions and applying the above modeling techniques to assess the probability of loss, but assessing *potential future exposure* introduces further complications. Essentially, this is because future exposure will vary as changes in relevant market rates e.g., interest rates, levels of equity indices etc., effect the value of portfolio of deals with any individual counterparty. Moreover, future exposure is likely to correlate with counterparty default likelihood, and exposure will be affected by the extent to which legally enforceable netting arrangements are in place. Finally, exposure anyhow changes over time as deals mature and roll off.

Ideal counterparty risk modeling would involve establishing a distribution of possible losses that not only looks at default and recovery rates of individual names and combinations of names, but also at the volatility of the size of the underlying derivative exposure. To treat derivative products in full detail, it would be necessary to develop an integrated model of credit and market risk. Such integrated models are still evolving. For example, CreditMetrics specifies only a procedure for calculating plain-vanilla interest rate swap values and treats those instruments consistent with way bonds and loans are treated. In addition, credit derivatives can be incorporated in the CreditMetrics modeling framework (see CreditMetrics-Monitor 1998). Neither CreditRisk+ nor CreditMetrics can handle non-linear derivative products as options and cross-currency swaps.

So far credit-related optionality arising from derivative transactions is generally incorporated into credit risk models by associating with each derivative instrument a non-random loan equivalent exposure (LEE), which equals the instrument's current market-to-market value plus an add-on future exposure. To set these add-ons, banks employ standard tables that approximate the volatility of different market factors which drive their derivative portfolios (Isda 1998). This approach seems to be quite similar to the current Basle add-ons approach, except that more sophisticated systems have many more market factor assessed than is the case under the Basle add-on matrix (Isda 1998). Instead of the standard tables, at some banks reviewed by the Fed/BIS Task Forces derivative contracts' LEEs are calculated as some variant of the amount by which the bank is expected to be "in the money" on that contract over the planning horizon, based on simulations using the bank's trading account VaR models.

13 PARAMETER SPECIFICATION AND ESTIMATION

The most challenging aspect of the credit risk modeling is the calibration of model parameters. In principle, to measure credit risk for a portfolio as a whole, each risk factor's joint probability distribution with all other risk factors should be specified. Reflecting the longer term nature of credit cycles, even in the best of circumstances where parameter stability can be assumed, many years of data, spanning multiple credit cycles, would be needed to estimate joint default/rating migration probabilities, correlations, and other key parameters with adequate precision (Altman–Saunders 1998).

Unfortunately, at the banks reviewed by the Fed/BIS Task Forces, data on historical loan performance have been warehoused only since the implementation of their economic capital allocation systems, within the last few years at best. Owing to such data limitations, the current generation of model specification practices tends to involve several crucial simplifying assumptions and considerable judgment, as further discussed herein.

13.1 Correlation among different types of risk factors

From the classic portfolio theory, the overall uncertainty around a portfolio's rate of return depends on its systematic risk. Similarly, in the context of long-term credit portfolios, uncertainty around *expected credit losses* depends on co-movements in loan values arising from their dependence on common influences. Under the MTM paradigm, the systematic risk may reflect four types of correlations among risk factors that potentially could contribute to co-movements in loan valuations:

(1) Correlations between risk factors affecting credit migrations, especially those corresponding to borrowers operating in related markets, like the same geographic region or industrial sector;
(2) Correlations between risk factors determining LGDs;
(3) Correlations between risk factors driving changes in the term structures of credit risk spreads; and
(4) Cross-correlations among the risk factors affecting rating migrations, LGDs and credit spreads.

Of course, only three types of correlations are relevant within DM-type models: correlations between borrower defaults, correlations between LGDs, and cross-correlations among defaults and LGDs.

Although critically important, at least in theory, correlations among random variables are difficult to estimate reliably with relatively short historical sample periods. Therefore, the builders of existing credit risk models have imposed pretty restrictive assumptions on the correlations among the risk factors. Essentially, nearly all current models assume zero correlation between risk factors of *different* type. The risk factors affecting changes in credit ratings are assumed to be independent of those affecting changes in credit risk spreads, which are assumed to be independent of those affecting LGDs. Model-builders typically focus on specifying the probability distribution for each type of risk factor *separately* from the others. In fact, according to the Fed/BIS Task Forces' findings, within virtually all credit risk models the only correlation effects considered at present are the correlations between default/credit migrations of different customers.

13.2 Risk Factors Affecting LGDs

The availability of historical loss data dictates the degree of complexity and choice of methodology in modeling LGDs. In practice, the loss in the event of default is dependent on numerous factors such as the type of default, the seniority class and collateral status of the debt, and the context at the time of default. As expected, historical recovery rate statistics indicate that there is indeed significant variation in the level of loss, given the default of an obligator, as illustrated in Table 7. To make this estimation problem manageable, within the current generation of credit risk models, LGDs are typically assumed to depend only on a limited set of variables characterizing the structure of a particular credit facility. These variables may include the type of credit product, its seniority, collateral and country of originator.

Table 7: Recovery rates by seniority class

Seniority class and security	Mean (% of face value)	Standard Deviation (%)
Senior secured bank loans	71.18	21.09
Senior secured public debt	63.45	26.21
Senior unsecured public debt	47.54	26.29
Senior subordinated public debt	38.28	24.74
Subordinated public debt	28.29	20.09
Junior Subordinated public debt	14.66	8,67

Source: CSFP (1997)

As referred in Subsection 11.3 above, in some models LGDs are treated as deterministic, while in other models they may be treated as random. In the latter case, the probability distribution for each LGD may be assumed to take a specific parametric form, such as that of a beta distribution within the CreditMetrics modeling framework. Generally, models assume LGDs (after controlling for seniority, collateral, etc.) to be *independently* and *identically distributed* over time and across borrowers, and hence no systematic risk due to LGD volatility. Furthermore, zero correlation among the LGDs is often assumed even across obligations of the same borrower.

According to the surveys of the Fed/BIS Task Forces, outside the consumer and small business lending areas, an individual bank's historical data generally can provide very little information with which to estimate LGDs. Unfortunately, especially for middle-market and large corporate loans, the number of defaulted loans with in an individual bank's historical database is typically too small to allow the probability distribution of LGDs for any particular type of loan to be estimated adequately. Consequently, for a given set of facility characteristics, the underlying parameters of the probability distribution for LGDs are typically inferred judgmentally by pooling information from several sources. Along with their own databases on historical loan losses, the banks reviewed by the Task Forces pooled data e.g., from trade association reports and publicly available regulatory reports; consultants' proprietary databases on the LGDs of their clients; and publicly available rating agency studies on the historical LGDs of corporate bonds.

13.3 Risk Factors Affecting Changes in Credit Risk Spreads

Due to a lack of extensive databases on secondary market yields, this area is still in an early stage of development. It appears to be difficult to obtain reliable credit spread data, even for more developed bond markets. Credit spreads between the yield of an obligation and that of a risk-free bond do not typically correct for differences in liquidity (the BIS Task Force 1999). In fact, some studies have begun to question the efficiency of bond markets, and hence the utility of estimates of default probabilities based on the term structure of credit spreads as well.

Owing to the data limitations, virtually all current applications of MTM-type models treat the term structure of the credit spreads *fixed* and *known* over the planning horizon. Within the current version of CreditMetrics, for example, the risk factors affecting changes in credit risk spreads actually are set to zero for purposes of modeling future values. When appropriate historical data is available, non-parametric approaches may however be used to estimate the joint probability distribution of future changes in credit risk spreads. The Fed Task Force identified (1998) one such procedure that involves constructing, for each credit rating grade, a database of historical term structures of credit risk spreads. The joint probability distribution of future spreads is then estimated using a with-in sample Monte Carlo simulation procedure.

13.4 Risk Factors Affecting Changes in Credit Ratings

As referred above, the current generation of credit risk models generally relate the process determining the customer defaults or rating migration to two types of parameters: (a) for each customer, the expected default frequency (DM-type models) or rating matrix (MTM-type models), and (b) across customers, the correlation among defaults and rating migrations.

Two types of procedures are generally used to estimating these parameters: actuarial-based methods and equity-based methods (see the Fed Task Force 1998 and the BIS Task Force 1999).

13.4.1 Estimation of EDF/Rating Transition Matrices

Actuarial-based methods are used to calibrate expected default frequencies (EDF) or rating transition matrices in both structural and reduced-from models. The basic approach involves using historical data on the default rates of borrowers to predict EDFs and rating migrations for customers having similar characteristics.

As presented in Section 11.4.2, in the case of structural credit risk models, EDFs/rating transition matrices are not actual parameters used in specifying models. The actual parameters represent the means and variances associated with the underlying migration risk factors. They together with the thresholds define upgrades and downgrades. However, in general, there is a one-to-one mapping between the two sets of parameters and, in practice, actuarial-based methods calibrate the latter by "reverse-engineering" them from the former. Likewise, in reduced-from models, the underlying model parameters are typically calibrated to be consistent with estimated EDF/transition matrices for individual assets or pools of assets.

One actuarial approach utilizes formal credit scoring models to predict EDFs/ rating transition matrices. While some of the banks reviewed by the Fed/BIS Task Forces have developed their own in-house credit scoring models for their corporate customers, others purchase credit scores from external vendors. Owing to a lack of historical data on loan performance, it is often assumed that credit transition probabilities among the underlying risk factors for corporate loans are identical to those for similarly rated corporate bonds, as in Table 6 above. Techniques for estimating borrower default models as such seem to be well researched within the economic literature, but data availability tends to be the critical limiting factor.

A second actuarial approach involves grouping borrowers into discrete "risk segments" based on observable characteristics. Within any risk segment, all obligors, and the stochastic properties of their underlying migration factors, are assumed to be statistically identical, and hence all customers in the same risk segment would assumed to have the same EDF/transition matrix. For large corporate customers, it is possible define risk segments on the basis of factors such as the borrower's internal credit rating, size, country and industrial sector. Given the assumption that all borrowers within a segment have the same EDF/rating transition matrix, models typically attempt to estimate parameters from average historical rating migration data of borrowers in that segment. In practice, data availability severely limits the length of time over which such an average can be calculated.

Equity-based method, most often associated with the option theoretic model of Merton, is used exclusively for estimating the EDFs and the credit migrations of large and middle-market business customers within structural models. This technique uses publicly available information on a firm's liabilities, the historical and current market value of its equity and the historical volatility of its equity to estimate the level, rate of change and volatility of the economic value of the firm's assets. In addition, for analytical convenience, customers' asset values are usually assumed to be jointly normally distributed. Thereby, under the assumption that the default occurs when the value of a firm's assets falls below its liabilities, expected default probabilities can be inferred from the option models. Alternatively, using a approach pioneered by Kealhofer (see e.g. 1998) and KMV Corporation, it is possible to calculate the number of standard deviation the current value is away form the default threshold, termed the "distance to default". Given a firm's estimated distance to default its EDF is calculated as the historical default frequency for firms having that same distance to default, derived from a proprietary KMV database on the historical default experience of publicly rated businesses.

13.4.2 Interdependence between Defaults/Rating Migrations

Within both structural and reduced-form models, the interdependence between defaults and/or rating migrations is a key determinant of a portfolio's PDF. Structural models parameterize this interdependence in terms of the correlations among customers' migration risk factors, i.e. usually customers' asset values or net worth positions. In the context of reduced-form models, the interdependence between customers' defaults and/or rating migrations in turn reflects the assumed or estimated process relating observable and unobservable background factors to EDFs or credit rating transition matrices. To extent that two obligors are sensitive to the same set of background factors, their default/migration probabilities will move together. The effects of interdependence may be modeled at the level of either individual credit exposures or pools of relatively homogenous exposures.

An actuarial method here is an extension of the above risk segmentation approach to estimating EDFs/transition matrices, and is used in calibrating correlation parameters in both structural and reduced-form models. Within each risk segment, borrowers are assumed to be statistically identical. Given the EDF for a particular risk segment, mathematically there is a one-to-one relationship between the variance of the risk segment's default rate and the correlation of the risk factors associated with the loans in that risk segment. An estimate of the default correlation among the loans is often reverse-engineered from an estimate of the historical variability of the risk segment's aggregate default rate. This procedure involves two stages. In the first stage, the means, variances and covariances of aggregate default rates are used to estimate default or migration correlations between loans of various types. In the second stage, correlations between migration factors are inferred from the default correlations generated in the first step (see CreditMetrics – Technical Document 1997). A broadly similar reverse engineering method can be used to infer risk factor correlations between borrowers in different risk segments from the historical covariance between the annual aggregate default rates for those risk segments. The relationship between default correlations and migration risk factors is further discussed e.g. by Chunsheng Zhou (1997).

Under the equity-based method, used solely within structural models, it is usually assumed that underlying migration risk factors for each borrower equals the underlying value of the firm's asset, as noted earlier. Consequently, in principle, an estimate of the correlation can be calculated directly from estimates of firms' historical asset values. In practice, however, some vendors have observed such estimates tend to be quite unstable. To mitigate this problem, KMV econometrically averages that asset value correlations across the customers within various risk segments, defined in terms of the borrower's industry and country, any possible other characteristics (see Kealhofer, 1998).

14 ECONOMIC CAPITAL ESTIMATION

14.1 PDF Computation

Once the parameters of the credit risk model have been specified, the portfolio's probability density function generally is computed by one of two methods: Monte Carlo simulation or approximations using a mean-variance methodology.

The Monte Carlo techniques employed in credit risk modeling are essentially identical to those used within VaR models in the trading book (see Jauri 1997). A Monte Carlo simulation procedure typically involves the following steps in the context of credit risk modeling (see, e.g. Nishiguchi et al 1998 and CreditMetrics - Technical Document 1997). A simulation is employed to create scenarios corresponding to a possible "state of the world" at the end of the planning horizon. The "state of world" is just the credit rating of each of the obligors in the portfolio. For each scenario, the credit portfolio is revalued to reflect the new credit ratings (assuming that the risk factors affecting in credit spreads are set zero). If loss-rates-given-default (LGDs) are not treated as deterministic, each default scenario furthermore requires an independently generated LGD. Given the large number of possible future portfolio values generated in these steps, the simulation technique results in an estimated PDF whose shape is consistent with the parameters of the underlying credit risk model.

The mean-variance methodology aims only to generate the first two moments of the distribution, i.e. its mean and standard deviation. The general shape of the PDF remains implicit in the model. Often the PDF is assumed to take the shape of a *beta* or, in some instances, even *normal distribution* having a mean and standard deviation identical to the estimated mean and standard deviation of the portfolio's credit losses. However, neither above distribution may be strictly consistent with the other assumptions and parameters of the model. Observed credit losses are markedly non-normal. They are typically skewed toward large losses, and leptokurtic meaning that, for a given mean and standard deviation, the probability of large losses occurring may be substantially greater than would be the case if the distribution were normal.

The mean-variance approximation is used primarily within the context of the DM paradigm. A portfolio's expected loss μ over the planning horizon equals the expected losses for the individual credit facilities:

Equation 10

$$\mu = \sum_{i=1}^{N} EDF_i LEE_i \overline{LGD}_i$$

where for the credit facility i, LGD_i is the expected loss-rate-given-default, EDF_i is expected default frequency, and LEE_i is the loan equivalent exposure. Recall that for plain-vanilla term loan, the loan equivalent exposure equals the amount of the loan. Other credit facilities, like lines of credit and derivatives, are converted into loan equivalent exposures as expressed in Section 12.

The portfolio's standard deviation of credit losses σ can be composed into the contribution from each of the individual credit facilities:

Equation 11

$$\sigma = \sum_{i=1}^{N} \sigma_i \rho_i$$

where σ_i denotes the stand-alone standard deviation of credit losses for the credit facility *i*, and ρ_i denotes the correlation between credit losses on the facility *i* and those on the overall portfolio. Furthermore, under the assumptions that (1) each facility's exposure is known with certainty, (2) the random risk factors affecting customer defaults and LGDs are independent of one another, and (3) LGDs are independent across borrowers, the stand-alone standard deviation of credit losses for the facility *i* can be expressed as:

Equation 12

$$\sigma_i = LEE_i \sqrt{EDF_i(1 - EDF_i)\overline{LGD}_i^2 + EDF_i VOL_i^2}$$

where *VOL* denotes the standard deviation of the facility's LGD. These equations provide a convenient way of summarizing the overall portfolio's credit risk within the DM framework in terms of each instrument's EDF, ρ, LGD, VOL and LEE.

The Fed Task Force reported that relatively few U.S banks used Monte Carlo methods to estimate PDFs (1998). For purposes of analytical simplicity and computational speed, the vast majority used mean-variance approximation. Given the number of sources of variability and the number of positions to be estimated, simulation processes can indeed be computationally very burdensome. However, recent advances in computing capabilities have made it more feasible to estimate PDFs using the Monte Carlo simulation methods. In fact, many of the banks reviewed the BIS Task Force already used Monte Carlo simulation to characterize the full distribution of portfolio losses at least for some sub-portfolios (1999).

14.2 Capital Allocation Rules

Finally, once the PDF for portfolio credit risk has been estimated, the bank must specify a particular rule for determining how much economic capital should be held against credit risk. As referred in Section 9, at most banking institutions this "allocation rule" is expressed as the capital necessary to achieve some target insolvency rate over the planning horizon. Among the banks sampled by the Fed Task Force, the most widely used target rate was around three basis points, or equivalently, the 99.97th percentile value on the cumulative distribution of portfolio losses (1998). At the banking institutions reviewed by the BIS Task Force target rates were slightly lower falling in the range of 99-99.98th percentile values, with the majority converting in the middle (1999).

In cases where the portfolio's PDF is estimated directly via Monte Carlo simulation, the economic capital allocation against credit risk can be computed directly from the estimated PDF. For banks using mean-variance approximation methods, economic capital is calculated as some *multiple* of portfolios estimated *standard deviation* of credit losses, where multiple is chosen to be consistent with the target insolvency rate and the assumed shape of the PDF. The Fed Task Force found that these multiples vary in the range of 3-7 depending on the insolvency rate and on whether the "actual" PDF is assumed to be beta- or normal-shaped. Consequently, final economic capital allocations differ considerably across banks that use mean-variance approximation methods.

Chapter 4

POTENTIAL MODELS-BASED APPROACH TO CREDIT RISK CAPITAL CHARGES

15 REGULATORY CONSIDERATIONS

Of course, the Basle Committee as well as several banking regulators have recognized the weaknesses of current capital regime referred in Chapter 2. That is (1) the denominator of the risk-based capital ratio, total risk-weighted assets, may not be accurate measure of total credit risk and, more importantly, the measurement of total risk-weighted assets ignores critical differences in credit risk among financial instruments as well as differences across banks in hedging and portfolio diversification, and (2) the regulatory measure of "capital" may not represent a bank's true capacity to absorb either expected or unexpected losses. For instance, banks' loan loss reserves often tend to exceed credit losses during good times but to understate expected credit losses during times of stress.

Furthermore, the anomalies of the current regime have created substantial opportunities for "regulatory capital arbitrage" that are rendering the formal risk-based capital ratios increasingly less meaningful. The Task Force realized that, for example through securitization and other financial innovations, many large U.S. banks have lowered their risk-based capital requirements substantially without reducing materially their overall credit risk exposure (1998). In fact, the implementation of Market Risk Amendment have created additional arbitrage opportunity by affording certain credit risk positions much lower risk-based capital requirement when held in the trading book rather than in the banking book. The basic arbitrage techniques involve re-engineering financial contracts to convert a bank's on-balance sheet credit risk into a nearly equivalent off-balance sheet exposures having a lower capital requirement, or removing from the banking book financial instruments for which the 8% capital standard is too high, relative to underlying economic risks, while retaining instruments for which the standard is too low.

Considering the flaws of the current regime, there may be need to begin developing the next generation of credit risk capital standards, at least for the largest, most sophisticated banks, before the current framework is completely outmoded (see e.g., the Fed Task Force 1998; Jones & Mingo 1998; de Swaan 1998). It is nevertheless infeasible to establish internationally applicable risk weightings that accurately reflect banks' risks at all times and under all conditions. For that reason, "internal models" approaches to prudential regulation may be the only long-term solution on the horizon (Jones & Mingo 1998). Especially the representatives of the Federal Reserve Board and the Basle Committee have examined whether banks' current internal credit risk models could also be used for regulatory capital purposes. However, there are still serious obstacles on this road when viewed from a regulatory perspective.

16 COMPARISON OF CALCULATION METHODOLOGIES

To illustrate the potential models-based approach to setting credit risk charges, this section reviews testing results published by Isda in March 1998. Isda tested the relative performance of the current BIS rules and three credit risk models. Models included in the straightforward test were:

CreditMetrics, using a one year planning horizon and 99 percentile of the loss distribution; CreditRisk+, using the same parameters; and a proprietary credit risk model of an ISDA member (SBC Warburg Dillon, no further information on the model available in public), using the same parameters.

To assess the different calculation methods, three different portfolios, each with a total exposure of USD 66.8 billion, were composed as follows:

- *Portfolio A:* A high credit quality diversified (500 name) portfolio
- *Portfolio B:* A low credit quality diversified (500 name) portfolio
- *Portfolio C:* A high credit quality concentrated (100 name) portfolio

The portfolios consisted solely of one-year corporate term loans and loan amounts were equal size for all names. Credit ratings, default rates and default rate volatilities were provided for the test (from 1.1.1981 to 31.12.1996 U.S. data, from Standard & Poors). The test shows both 0% and, maybe more realistically, 50% recovery rates for the portfolios. Zero recovery uncertainty was assumed in both the cases. Furthermore, the tests shows the performance of the credit risk models assessed with in terms of modeled correlations (or the application of default volatilities under CreditRisk+) but also with correlation set to zero. The test results are shown in Figure 3:

Figure 3: Comparison of capital calculation Methodologies

Correlation Assessed, zero recovery (millions USD)

1	CAPITAL CHARGE		
	Portfolio A	Portfolio B	Portfolio C
BIS	5,304	5,304	5,304
CreditMetrics	2,264	11,346	2,941
CreditRisk+	1,638	10,000	2,547
SBC	1,373	9,654	2,366

Correlation Assessed, 50 % recovery, zero recovery uncertainty (millions USD)

2	CAPITAL CHARGE		
	Portfolio A	Portfolio B	Portfolio C
BIS	5,304	5,304	5,304
CreditMetrics	1,132	5,718	1,471
CreditRisk+	819	5,000	1,287
SBC	686	4,827	1,183

Zero Correlation, zero recovery (millions USD)

3	CAPITAL CHARGE		
	Portfolio A	Portfolio B	Portfolio C
BIS	5,304	5,304	5,304
CreditMetrics	777	2,047	2,020
CreditRisk+	789	1,907	1,967

The application of the current BIS rules means that credit risk capital charges are exactly equal for all three test portfolios regardless of credit quality and portfolio concentration. As all of the loan exposures are assumed to be to corporates, all of them would receive a 100% risk weighting. Consequently, the credit risk capital charge is 8%×66,3 billion = 5,304 millions.

The importance of portfolio diversification effects is illustrated in Figure 3 by contrasting the results for portfolios A and C. In all cases, the modeled charge differentiate between the two portfolios, recognizing that the relative concentration of portfolio C makes it riskier even though overall credit quality is the same between these portfolios. All three models reflect this impact in roughly same manner.

The credit risk models also differentiated charges in the light of credit quality. This can been seen contrasting the credit risk capital charges for portfolios A and B. Portfolio concentration is equal, but credit quality is lower in the latter portfolio. This is reflected by the significantly higher capital charges produced by all three models.

In relation to the current standardized rules, the modeling results adjusted for 50 percent recovery seem to be very close in the case of the low credit quality diversified portfolio C. The models-based approach would strongly reward the higher credit quality portfolios A and C and also further rewards the greater diversification of portfolio A relative C. As regards potential regulatory capital requirements, these modeling results may, however, be misleading because of the low 99.00^{th} percentile value. Recall that the most widely used target rate appears to be around three basis points i.e., the 99.97^{th} percentile value on the cumulative distribution of portfolio losses.

Furthermore, these testing results also indicate a challenge involved in credit risk modeling at least to some extent. That is, the sensitivity of the modeling results to default correlation analysis. In Figure 3, the strong impact of correlation on the test results can been seen by contrasting the results for diversified portfolios A and B in Table 1 and Table 3. Given the empirical difficulty of estimating default correlations with precision, regulators have been concerned for the sensitivity of modeling results.

17 ROBUSTNESS OF CREDIT RISK MODELS

17.1 Choice of Planning Horizon and Loss Paradigm

As stated earlier, banks typically employ a one-year planning horizon for purposes of credit risk modeling. The reasons put forward for this choice seem to favor computational convenience rather than model optimization. In support of the one-year horizon, practitioners suggests this interval represents a period over which in the *normal course of business* either (a) capital can be raised to fully offset portfolio credit losses, or (b) risk mitigating actions, such as loan sales or the purchase of credit protection, can be taken to eliminate the possibility of further credit losses. Perhaps the most important consideration however is that the estimations of key model parameters is viewed as infeasible for planning horizons beyond one year due to the lack of historical data.

According to regulators, the choice of planning horizon should be viewed outside the normal course of business (the Fed Task Force 1998). Regulators tend to view capital adequacy within the context of a bank under stress attempting to unload the credit risk of a significant portfolio of deteriorating credits. That is, fluctuations in economic activity and in credit losses tend to be positively serially correlated from one year to the next, implying that a bank's capital buffer may be called upon to absorb significant credit losses extending beyond a single year. Indeed, two or more years are typically required to resolve asset-quality problem at troubled banks, as the experiences of Finnish or Japanese banking sector, for instance, has shown. The markets for secondary loan trading and credit derivatives appear to be expanding and are becoming more liquid at least in some countries, but they have not yet been tested by any large bank under severe stress.

Besides the planning horizon, loss paradigm is inevitably critical decision variable in the current credit risk modeling processes. While various justification may be put forward in support of the DM paradigm versus the MTM paradigm, the determination of model superiority tends to be influenced by the fit between model output and model application. For example, a bank that utilizes credit risk models for performance measurement purposes associated with the buy-and-hold portfolio might reasonably opt for a much simpler DM-type model. In contrast, certain pricing decision for a portfolio of more liquid credits may require a loss measurement definition that incorporates potential shift in credit spreads.

From a regulatory perspective, materiality of loss paradigm is not yet clear. However, some regulators have been especially skeptical of the ability of DM-type models to capture the effects of potentially adverse credit events (see the Fed Task Force 1998). Due to the paradigm's "two-state" nature, DM-type models may be particularly sensitive to the choice of a one-year planning horizon. For example, with respect to a three-year term loan, the one-year horizon means that more than two-thirds of the credit risk is potentially ignored. In order to reduce this bias of the DM approach, banks apply sometimes various ad hoc adjustments, such as making a loan's estimated default frequency an increasing function of its maturity (see the Fed Task Force 1998 and the BIS Task Force 1999). In practice, adjustments of this sort may however lead to internal inconsistencies with in the modeling procures. For example, multi-year EDFs may be used in combination with loss correlations calculated on the basis of one year time horizon. It is therefore difficult to assess these adjustments' overall impact and effectiveness.

17.2 Treatment Credit-related Optionality

For many types of credit instruments, a bank's exposure is not known with certainty, but rather may depend on the occurrence of future random events. Modeling methods for dealing with such credit-related optionality are however still evolving as noted in Section 12. Both the Fed/BIS Task Forces observed great diversity in current modeling practices. The diversity leads to very large differences across banks in credit estimates for similar instruments. For example, with regard to virtually identical lines of credit, estimates of stand-alone credit risk can differ as much as a ten-fold. The Fed Task Force (1998) realized that these differences sometimes reflect modeling assumptions that are even fundamentally inconsistent with a bank's own views regarding the nature of the underlying business. Considering committed lines of credit, the common assumption that future drawdown rates are independent of future changes in the customer's credit quality, despite clear evidence to the contrary, may lead systematic underestimates of the loan equivalent exposures (LEE) for lines of credit.

The treatment of optionality arising from derivatives transactions appears to be even more complex topic. Most modeling methods for calculating loan equivalent exposures for derivative contracts rely on the assumption that any unexpected future change in the bank's exposure with respect to a given derivative contract is independent of both (a) changes in all other derivative contracts and (b) changes in the credit quality of the bank's counterparty. Both these assumptions may bias the overall output of credit risk models. First, counterparty credit risk exposures may actually be positively correlated across contracts. Secondly, in certain derivative transactions, the extent to which a bank is "in the money" may be negatively correlated with changes in the credit quality of its counterparty. Satisfactory treatment of these issues would require a much closer integration of internal market risk and credit risk measurement systems. Given current technologies, it nevertheless seems to be very difficult to conduct simultaneous Monte Carlo simulations of both the credit risk model and the bank's VaR market risk models, which could be used to simulate random changes in the derivative contract's mark-to-market value over its lifetime (the BIS Task Force 1999).

17.3 Analytical Soundness of Modeling Practices

Under both the DM and MTM framework, estimates of portfolio credit risk are driven largely by assumptions and parameter estimates regarding the joint probability distribution of the relevant risk factors. As noted earlier, available data on the historical performance of different types of loans generally do not span sufficiently long time period to enable precise estimation of the distributions. For that reason, parameter values are often established through a judgmental process involving critical assumptions and considerable uncertainty. As presented in Section 13, these include the followings:

(1) Joint normality or other parametric assumptions on the probability distributions of the risk factors determining credit ratings migrations
(2) In the case of mean-variance DM models, the assumption that portfolio credit losses have a beta or normal probability distribution
(3) Independence between risk factors affecting changes in credit ratings, changes in LGDs, and credit risk spreads
(4) Independence of LGDs across borrowers; and
(5) Stability of model parameters,

In reviewing the assumptions, regulators have highlighted that estimation of the extreme tail percentile values of a credit portfolio's PDF is likely to be highly sensitive to variations in key parameters, such as correlations. For the time being, in practice there is generally little analysis supporting these assumptions. Nor is it standard practice to conduct sensitivity testing of models' vulnerability to key parameters or assumptions (The Fed Task Force 1998 and the BIS Task Force 1999). In addition, adequate accounting for the uncertainty in modeling parameters would significantly increase measured credit risk (see e.g. Duffey 1996). The following subsections review key consideration presented from a regulatory perspective.

17.3.1 Treatment of LGDs

As noted in the preceding sections, current methods for estimating LGDs vary considerably in terms of sophistication. In some models, LGDs may be treated as deterministic and known with certainty while in other they may be treated as random. Furthermore, while some banks appear to rely on almost exclusively on LGDs parameters set intuitively, other banks with access to large amounts of historical data may rely heavily on objective empirical analysis (The Fed Task Force 1998 and the BIS Task Force 1999).

For portfolios characterized by distributions of exposure size that are highly skewed, the assumption that LGDs are known with certainty may tend to bias downwards the estimated tail to the PDFs credit losses. The simulation results of e.g. Michael Gordy's comparative study illustrate how percentile values on loss distributions are directly proportional to LGDs (1998). For example, if LGDs are assumed to be fixed proportions of $\lambda=0.3$ of book values, holding fixed other modeling parameters and the target insolvency rate, the required capital given a loss rate of e.g. $\lambda=0.45$ would simply be 1.5 times the required capital given $\lambda=0.3$. Consequently, this kind of credit risk models incorporates only default risk, and thus additional capital should be held for recovery uncertainty.

The reliability of pooled LGD data is of course also a key consideration, as it will affect accuracy of estimation results. First, owing to data limitations, sample periods for estimating tend to be relatively short. Secondly, the BIS Task Force realized that in setting parameters for corporate customers located outside the U.S., some banks appear to rely entirely on historical loss studies for publicly rated U.S. corporate bonds. Extrapolating U.S. data to other countries may be especially problematic due to differences in bankruptcy laws (Shirreff 1998 and The BIS Task Force 1999).

Moreover, regulators seem to concern for the common assumption that LGDs between borrowers are mutually independent (the Fed Task Force 1998). It is argued that this assumption may represent a serious shortcoming when the bank has significant industry concentration of credits e.g., commercial real estate loans within the same geographical region.

17.3.2 Estimation methods for PDFs/Rating Transition matrices

Subsection 13.4 presented the two methods that are generally used for mapping observable data historically into EDFs/rating transition matrices. Actuarial-based parameter estimates are inherently backward-looking, while in theory, the equity-based approaches are forward-looking at least to some extent. However, many of the assumptions underlying the equity-based models may be stylistic (the BIS Task Force 1999). These include the assumption that: (a) all equity price movements reflect changes in the underlying economic values of firms, rather than any change in the market price of equity risk, and (b) equity prices fully reflect all available information. This efficient market assumption is often viewed as extremely questionable. For that reason, regulators have noted that the relative accuracy of actuarial-based versus equity-based methods may be ultimately an empirical issue (the BIS Task Force 1999).

As said earlier, data availability generally dictates the empirical methods used to estimate EDFs/transition matrices. Banks typically have comparatively little useful default/migration data internally. Therefore, the banks reviewed by the Fed/BIS Task Forces attempt to estimate EDFs/transition matrices using historical performance studies on corporate bonds published by the rating agencies and other researchers. These studies often report historical default and rating experience, by rating category, over time spans covering 20 or more years. However, when using such supplemental information, the model-builder must usually determine judgmentally whether the geographical and industry composition of published data is appropriate to the characteristics of the loan portfolio being modeled; if not the model-builder often attempt to make data more comparable through the processes that seem to involve a high degree of subjectivity (see the BIS Task Force 1999).

Furthermore, owing to the lack of sufficient data, current models are not designed to capture business cycle effects, such as tendency for credit ratings to improve or deteriorate more during the cyclical upturns or downturns. In this sense, virtually all current bottom-up credit risk models can be termed "unconditional" (the Fed Task Force 1998). They reflect relatively limited borrower- or facility-specific information. A potential implication of using "unconditional" transition probabilities is that estimates of expected losses and credit risk could be biased downward during the early stages of recession, and biased upwards during the early stages of recoveries. An example of a "conditional" credit risk model is McKinsey & Co's CreditPortfolioView (see Wilson 1997 I and II). Within its modeling framework, rating transition matrices are functionally related to the state of the economy. On the other hand, conditional techniques may also have drawbacks. For example, a conditional model may underestimate losses just as the credit cycle enters a downturn and overestimate losses just as the cycle bottoms out. Consequently, regulators' key consideration seems to be the fact that business cycles effects raise the possibility that parameter estimates may be subject to considerable uncertainty.

17.3.3 Default Correlation Analysis

According to the BIS Task Force, the assumptions and approximation used in estimating default correlations also highlight various conceptual and empirical questions, including (a) whether the choice or risk factor distribution function, e.g. normality or gamma, makes a material difference to model output, (b) whether the technical approximations have a material impact, and (c) whether the default correlations generated by the different models are within the same range, result in a correct correlation structure, and are stable over the planning horizon. As regards the correlation analysis, the discussion in Subsection 11.4 may suggest that the structural and reduced-form approaches are based on inevitably inconsistent views of the world.

Recent studies however argue that there may be good agreement between, for example, the CreditMetrics and CreditRisk+ PDFs. To illustrate, Michael B. Gordy has run a set of simulations within CreditRisk+ and the restricted "two-state" form of CreditMetrics, denoted as CM2T in Table 8 (1998). Both the models were calibrated to yield the same unconditional expected default rate for an obligor of a given rating grade, and the same default correlation between any two obligors with in a single rating grade. The test portfolio consisted of 5000 obligors, and portfolio concentration across rating grades was calibrated using data from internal Federal Reserve Board surveys of large U.S banking institutions. In all the simulations, it was assumed that LGD is a fixed proportion 0.3 of the loan book value. The total portfolio value outstanding is immaterial here because losses were calculated as a percentage of the total portfolio book value. A summary of the simulation results is given in Table 8.

Table 8: Effects of increased default volatilities

	CM2T		CR+	
	σ	2σ	σ	2σ
Mean	0.481	0.480	0.480	0.480
Std Dev	0.319	0.590	0.325	0.610
Skewness	1.696	3.221	1.844	3.860
Kurtosis	8.173	20.278	8.228	25.182
0.9500	1.089	1.597	1.120	1.648
0.9900	1.578	2.854	1.628	3.130
0.9950	1.795	3.467	1.847	3.818
0.9997	2.714	6.204	2.736	6.772

Source: Gordy (1998)

where σ denotes the normalized volatility of default probabilities in the portfolio. Note that skewness is a measure of the symmetry of the loss distribution, and kurtosis is a measure of the relative thickness of the tails of the distribution. For the portfolio credit risk models, high kurtosis indicates relatively high probability of very large credit losses.

In Table 8, the expected loss under either model is 48 basis points of the portfolio book value. The standard deviation of loss in turn is roughly 32 basis points. Capital requirements implied by the simulations seem relative low. That is, a bank ought to hold only about three percent capital against credit risk in order to maintain a AA-rating standard (equal to the 99.97^{th} percentile value).

More problematically, both the credit risk models however appears to be sensitive to the volatility of default probabilities, or equivalently to the default correlations in the portfolio. When the normalized volatility of default probabilities was doubled in the above test, extreme tail percentile values increases substantially. Specifically, the 99.97^{th} percentile values more than doubled under both the models. In addition, as the volatility of default probabilities increases, the loss distributions become increasingly kurtotic. Since capital decisions depend on extreme tail percentile values of the loss distribution, this sensitivity issue has been of primary concern among regulators (Shirreff 1998, the Fed Task Force 1998, and the BIS Task Forces 1999).

18 MODEL VALIDATION

18.1 Backtesting

Given the difficulties associated with calibrating credit risk models, regulators attention has focused on the need for effective model validation procedures. Unfortunately, the same data problems that make it exceedingly difficult to calibrate these models also make it difficult to validate the models.

The task of estimating the extreme tail of the PDF is comparable to predicting the frequency at which credit losses in any year will exceed many multiples of a normal year's losses. The only entirely objective method for evaluating the statistical accuracy of a credit risk model is to compare the model's *ex ante* estimates of PDFs against *ex post* realization of actual credit losses. Specifically, only the realization of more frequent, extreme credit losses relative to the modeling prediction can provide purely statistical basis for concluding a model is deficient. Hence, backtesting is almost certain to be very problematic in practice, owing to insufficient data for out-of-sample testing. Consequently, the banks reviewed by the Fed/BIS Task Forces generally didn't conduct statistical backtesting on their estimated PDFs (1998). At the sampled banks, the backtesting was limited to comparison of expected and actual credit losses or default rates. However, such tests do not address the accuracy of the model's predictions of credit risk, against which economic capital is allocated.

Instead of the backtesting, credit risk models are often validated indirectly; through various market-based "reality" checks (Jones & Mingo 1998). For instance, peer-group analysis may be used to measure the reasonableness of credit risk models and internal capital allocation processes. Another market-based method involves comparing the break-even spreads implied by the bank's internal pricing models with actual credit spreads on corporate bonds or syndicated loans having a particular credit rating, e.g. AA. An actual credit spread well below or above the bank's break-even spread might be interpreted as evidence that the model's capital allocation for AA-rated credit is too high or low. From a supervisory perspective, it is said that the use of market-based validation methods can raise serious concerns regarding the comparability and consistency of a risk model over time (the Fed Task Force 1998).

18.2 Stress Testing

In principle, stress testing could at least partially compensate for the data limitations, estimation problems, and short-comings available in backtesting methods for credit risk models. Stress tests circumvent these difficulties by specifying, albeit arbitrarily, e.g. particular economic scenarios against which the bank's capital adequacy may be judged without regard to the probability of that event actually occurring. In general, the types of credit risk stress tests that may be employed are similar to those used for VaR market risk models (see Jauri 1997).

Specifically, in the credit risk context, stress testing should involve assessing the impact of extreme "fat-tail" credit events on the current portfolio of credit exposures. A significant downward shift in credit ratings may be assumed across a portfolio, or recovery rates may be arbitrarily adjusted downward by a significant amount, and changes to corralations may be assessed. In terms of assessing credit spread risk, shocks to credit spreads can be also undertaken. These changes may be applied individually or in combination, for individual borrowers, portfolios of borrowers or across the whole credit portfolio.

As is the case with the market risk models, historical scenario stress testing may also be considered, as experience about past credit crisis with particular sectors or about losses at extreme points of the credit cycle can be assessed. Furthermore, the reasonableness of capital allocation levels needs to be examined assessing the particular scenarios (an individual event or combination of events) that would be especially damaging to the current portfolio. Although several stress testing protocols might be developed for internal credit risk models, the Fed/BIS Task Forces appear to be unaware of the reviewed banks were actively pursuing this approach.

19 CONCLUDING REMARKS

Credit risk modeling builds on the same statistical techniques employed by the VaR approach for the calculation of market risk. Hence, market participants' proposals recommending that the ten-years-old BIS framework covering credit risk should be updated to reflect advances in credit risk modeling are inevitably motivated by the Market Risk Amendment that incorporated banks' internal VaR models into the determination of capital requirements for market risk. Accordingly, it is logical to conclude this study by summarizing the key differences in credit risk versus market risk modeling.

Underpinning all credit risk models are what is termed the probability density function of credit losses (PDF), but a consensus within the banking industry about a "standard" shape of the PDF has yet to emerge. This stands in contrast with market risk VaR models, where the normal distribution is frequently used as a standard (Jauri 1997). Observed portfolio credit loss distributions are typically skewed toward large losses meaning that, for a given mean and standard deviation, the probability of large losses occurring is greater than would be the case if the distribution were normal. Due to the long-tailed nature of the distributions of credit risk models, the range of required capital corresponding to choice of a target loss rate within the 99.00-99.98 interval tend to be considerably wider than the range corresponding to the 95.00-99.00 interval used in market risk models.

One reason why no industry standard portfolio credit loss PDF has emerged may be that the modeling of losses from individual credit exposures as such is more difficult than is the case for market risk, and a wide range of simplifying assumptions is therefore made. Individual losses might be assumed to be binary, or else to follow one of a range of continuos distributions. The portfolio PDF that result from aggregating individual exposure losses *may* depend strongly upon these assumptions and further the assumptions made in estimating credit correlations.

Neither practitioners nor regulators are however fully aware of the effects of different modeling assumptions on estimates of the extreme tails of the loss distribution. Hence, it is unclear whether the high target credit loss percentiles used in the measurement of credit risk, and the resulting estimates of economic capital, can be estimated with an acceptable degree of precision for regulatory purposes. Due to the shape of distributional tails, alternative modeling assumptions that appear reasonable may imply large differences in estimates of very high percentiles regardless of e.g. PDF computation method. Altogether, it is so far inappropriate to develop qualitative and quantitative standards to ensure that the quality of modeling outputs is comparable across banking institutions.

The difficulties in the modeling of credit losses are foremost due to data limitations. Contrary to market-driven instruments, most credit instruments are not *marked to market*. For this reason, the predictive nature of a credit risk model does not derive from a statistical projection of future prices based on comprehensive records of historical prices. In addition, the scarcity of the data required to estimate model parameters also arises from the infrequent nature of default events and the longer-term horizon used in measuring credit risk. Consequently, in specifying model parameters, credit risk models require the use of simplifying assumptions and proxy data, which in turn highlights the importance of model validation processes to ensure that capital requirements generated using credit risk models will in practice provide an adequately large capital buffer.

For market risk VaR models, the Basle Committee has outlined a particular supervisory framework for backtesting and the supervisory interpretation of testing results (see 1996b). The methodology applied to backtesting market risk VaR models is not easily transferable to credit risk models due to the data constrains. Where market risk models typically employ a horizon of a few days, the Committee's backtesting framework requires the use of minimum of 250 trading days of forecasts and realized losses for backtesting purposes. A similar standard for credit risk models would inevitably require an impractical number of years of data, spanning multiple credit cycles. The longer holding period, associated with the higher loss percentiles used in credit risk models, presents problems to model-builders in assessing the accuracy of their models. At present, there actually seems to be no commonly accepted framework for periodically verifying the accuracy of credit risk models.

In many respects, the issues presented above are however qualitatively similar to concerns that raised in the course of developing the internal models approach to regulatory capital charges for market risk (Dale 1996). Within the context of credit risk modeling, these problems are however much more severe. The size of the banking book and the length of its relevant planning horizon are many times larger than those of the trading book. Therefore, errors in measuring credit risks for the banking book are more likely to affect the assessments of the bank's *overall financial soundness*. In addition, the banking book does not benefit from high liquidity and a daily mark-to-market process that, in the context of the trading book, may provide substantial safeguards against significant losses accumulating unnoticed.

20 SUMMARY

The regulatory framework to access capital for credit institutions throughout many countries relies on principles laid out by the Basle Committee on Banking Supervision in the International Convergence of Capital Measurement and Capital Standards" document. Although this ten-years-old BIS framework has been developed and improved continuously, banking institutions as well as regulators are aware of the inherent flaws of the framework. Perhaps the most important of them is the fact that the BIS risk weights do not reflect some very obvious determinants of portfolio credit risk, such as differences in credit quality of corporate obligators or concentrations of risk in a specific asset category or to particular obligor, industry, or region.

After drafting the Accord, a number of the large international banks have developed sophisticated systems in an attempt to model credit risk at portfolio level. The most important advances have been made in the context of so-called bottom-up credit risk models. Banks use such models primarily in estimating the economic capital needed to support their credit activities similarly as VaR methods are used in allocating economic capital against market risks. Capital allocations are the basis for internal risk management processes, including risk-based pricing models, and the setting of portfolio concentration or exposure limits.

Lately, several market participants have suggested that there is a need to ensure greater convergence between the regulatory capital regime and best practice in internal credit risk management. Forcing banks to maintain a flawed standardized credit risk capital calculation methodology in addition their own more sophisticated internal risk management systems has been regarded as a significant and unnecessary diversion of resources.

This study highlights, however, that any consensus has not yet been reached on the best practice in credit risk modeling. Instead, the current generation of bottom-up credit risk models includes a range of practices in the conceptual approaches to modeling. To summarize, these different conceptual choices include:

(1) Different approaches to the measurement of credit loss (the DM and MTM paradigms
(2) Different methodologies for measurement loss given default
(3) Unconditional and conditional models
(4) Different approaches to aggregation of credit risk
(5) Different techniques for measuring the interdependence of factors that contribute to credit losses.

Neither the materiality of these choices on models' accuracy nor their impacts on the size of required economic capital are well understood so far. However, before a portfolio modeling approach could be used in formal process of setting capital regulatory capital requirements for credit risk, regulators would have to be confident not only that models are being used to actively manage credit risk, but also that they are conceptually sound, empirically validated, and produce capital requirements that are comparable across banking institutions. This study shows that at this time, significant obstacles, concerning especially data availability and weaknesses in model validation, still need to be cleared before these objectives can be met.

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