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# Stress studies for permanent deformation calculations

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**ABSTRACT:** An analytical-mechanistic method for the calculation of permanent deformations in unbound pavement layers and subgrade has recently been developed in Technical Research Centre of Finland. The objective was to develop a relatively simple calculation method with a material model, which tie together permanent deformations to the most important effecting factors. The material model has been generated from the test results of accelerated pavement tests along with the complementary laboratory tests. This approach has created a new, important view to the research. The objective of this study is to compare the stress analysis done with the 3D and 2D modeling. The comparison of the 3D and 2D axisymmetric modeling showed that in the upper part of the pavement modeled with axisymmetric 2D overestimates stress responses. The stress analysis also proved that different non-linear elasto-plastic material models need separate material parameter C.

## 1 INTRODUCTION

Today need for evaluate of permanent deformations in pavement design is wide as well as global. The new procurement methods together with the functional requirements are underlining the demand for analytical and mechanical calculation methods for pavement rutting. The objective of the study was to compare stress responses of unbound pavement materials and subgrade analyzed with 2D and 3D models to give more confidence to the developed calculation method. Another objective was to study the material parameters of the calculation method. The calculation method has been derived from accelerated pavement tests (APT) made in Finland with a Heavy Vehicle Simulator (HVS) and from the laboratory test results of Finnish deformation project. The studied unbound materials include crushed rock, sandy gravel, crushed gravel and sand. So far, the tested pavement materials have been the most common granular, unbound materials.

The previous stress response studies (Korkiala-Tanttu & Laaksonen 2004) have proven the benefits of the implementation of the elasto-plastic material models in the calculation of permanent deformations.

The new calculation method is based on the elasto-plastic stress responses and corresponding shear strength capacities of the material. The shear strength approach is used because the rutting is supposed to be dominated by the shear strength ratio.

The stress analysis has included three different calculation cases: the most common 2D axisymmetric, 2D plane strain (long continuous line loading) and true

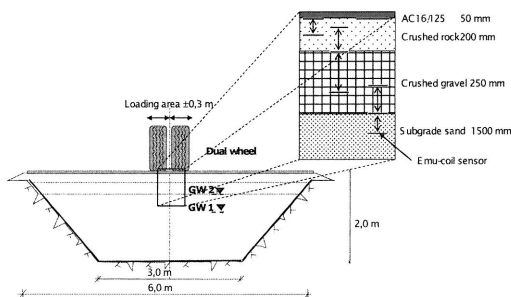


Figure 1. Cross section of the Spring-Overload test.

3D cases. All the calculations have been conducted with the Plaxis code; 2D cases with Plaxis version 8.6 and 3D cases with Plaxis 3D version 2. The HVS test setup for Spring-Overload test (SO) was chosen as a test structure (Fig. 1). The wheel load is a dual wheel type.

## 2 PREVIOUS STRESS RESPONSE COMPARISONS

The stress distribution studies of traffic load have shown that it is very important to calculate stresses in pavements with an elasto-plastic material model to avoid tensile stresses in unbound materials (Korkiala-Tanttu & Laaksonen 2004). The chosen material model drastically affects the stress distribution along with the permanent deformations, and also to some extent the resilient deformations.

Due to this, it is important to analyze stress responses for the permanent deformation calculations with a sophisticated model than a conventional linear elastic material model. By using a linear elastic material model there is a high likelihood that the calculations will generate tensile stresses in the unbound pavement layers. The phenomenon is emphasized in pavement structures which are thinly paved or totally unpaved. These tensions will cause unrealistic stress concentrations with misleading information about permanent deformation sensitivity. Thus, the needed variables of the calculation method (shear stress ratio and material parameters) can only be defined from the elasto-plastic modeling.

The development of the calculation method for unbound granular materials and subgrade is presented in a previous paper by Korkiala-Tanttu (2005). The developed method is based on the number of loadings, shear stress ratio and material properties. In this method the permanent deformations will be calculated from the stress responses determined with a finite element program with a separated permanent deformation model. The basic equation (1) of the permanent deformation in unbound material is a relatively simple hyperbolic function.

$$\varepsilon_p = C \cdot N^b \cdot \frac{R}{1 - R} \tag{1}$$

where  $\varepsilon_p$  = permanent vertical strain; C = permanent strain in the first loading cycle, a material state parameter; b = shear ratio parameter depending on the material; R = shear failure ratio =  $q/q_f$ , ( $q_f$  is defined in equation (2)); q = deviatoric stress, kPa.

### 3 MODELLING CASES

#### 3.1 Case 2D axisymmetric

Axisymmetric geometry was generated from the test structure (Fig. 1) so that the axisymmetric geometry had the same area as the plain strain geometry. Likewise the dual wheel loading area was changed to the corresponding pressure area. The radius of the bottom of the test structure was 2.4 meter and the radius of the loading area was 0.2 meter.

#### 3.2 Case 2D plane strain

For plain strain case the geometry corresponded to the actual cross section of the test structure (Fig. 1). The wheel load has been modeled as a continuous line loading. That is the reason why plain strain geometry is not widely applied to the pavement design. With infinite line loading, it is impossible to take into account the length (for heavy vehicles ~250 mm) of a wheel load. The plain strain modeling can only be used, when the shape of the geometry is studied.

Table 1. The modeling parameters for Mohr-Coulomb material of Spring-Overload test.

Material	Asphalt	Base course crushed rock	Subbase Crushed gravel	Subgrade Sand
Thickness, mm	50	200	250	1500
Modulus, MPa	5400	300-220-190	140-90	75
Poisson's ratio	0.3	0.35	0.35	0.35
Unit weight, kN/m <sup>3</sup>	24	21.2	22.0	18.0
Cohesion, kPa	–	30	20	8
Friction angle (°)	–	43	45	36
Dilation angle (°)	–	13	15	6
K <sub>0</sub>	1	0.32	0.30	0.42

#### 3.3 Case true 3D

For the simplicity the sloped walls of the SO test structure were not taken into account in the 3D calculations. Thus the geometry had the average width of 4.5 meter. The length of the geometry was 10 meter. The dual wheel load was generated in compliance with the true pressure measurements of HVS tests. Otherwise the material layers and material parameters were the same as in 2D cases.

### 4 STRESS RESPONSE COMPARISONS

#### 4.1 Used material models for stress calculations

Two different material models have been used for the axisymmetric and 3D calculations: linear elasto-plastic Mohr-Coulomb (MC) and non-linear elasto-plastic ‘Hardening Soil’ (HS). The hardening soil model (HS) is a non-linear elasto-plastic material model with Mohr-Coulomb failure criterion. A more detailed description of the material model is presented in Plaxis’s manual (Brinkgreve 2002). The plain strain case has been studied only with MC material model. The analysis is based on the calculated deviatoric and vertical stress components.

#### 4.2 Material parameters

The Table 1 presents the applied MC and Table 2 HS material parameters for in the modeling. It is important to note that the strength properties applied in the hardening soil model are higher than the normally applied static values. Several studies (Konrad & Juneau 2006, Hoff 1999, Courage 1999) have shown that the friction angle of well-compacted, partly saturated crushed rock in cyclic loading tests is typically between 50° to 60° and the apparent cohesion between 15 to 40 kPa.

Table 2. The modeling parameters for Hardening Soil material of Spring-Overload test (reference stress 100 kPa).

Material	Material model	DOC (%)/ w (%)	Friction angle°	Cohesion kN/m <sup>2</sup>	Unloading/reloading modulus, MPa	Compression modulus, MPa	Deviatoric modulus, MPa
Asphalt concrete	LE*	—	—	—	—	—	5400*
Crushed rock	HS†	95.8/4.6	55	20	750	173	250
Sandy gravel	HS†	98.1/7.3	58	20	900	201	330
Sand (dry)	HS†	101.4/9.9	40	15	420	110	120
Sand (moist)	HS†	—	36	8	420	95	100

\*linear elastic Young's modulus

†hardening soil

### 4.3 Shear stress ratio

According to Mohr-Coulomb's failure criteria, deviatoric stress in triaxial tests at failure  $q_f$  can be estimated with the help of equations 2–4. These equations are valid under the centre of the loading, where loading is axisymmetric and the angle of the major principal stress concurs with the vertical axis.

$$q_f = q_0 + M \cdot p' \quad (2)$$

$$M = \frac{6 \cdot \sin \phi}{3 - \sin \phi} \quad (3)$$

$$q_0 = \frac{c \cdot 6 \cdot \cos \phi}{3 - \sin \phi} \quad (4)$$

here  $q_f$  = deviatoric stress in failure;  $q_0$  = deviatoric stress, when  $p' = 0$ ;  $c$  = cohesion;  $M$  = slope of the failure line in  $p'$ - $q$  space in triaxial test;  $p'$  = hydrostatic pressure; and  $\phi$  = friction angle.

## 5 RESULTS

### 5.1 Stress responses – deviatoric stress

The calculated deviatoric stress responses with the 2D and 3D are compared with each other in Fig. 2. The stress components have been calculated under the centre line of the loading and the wheel load has been 50 kN.

### 5.2 Stress responses – vertical stress

The calculated vertical stress responses with the 2D and 3D are compared with each other in Fig. 3. The stress components have been calculated under the centre line of the loading and the wheel load has been 50 kN. Due to the stress sign rules of Plaxis compression stresses have negative values. Also the measured earth pressure at the top of the subgrade sand is presented in the Fig. 3.

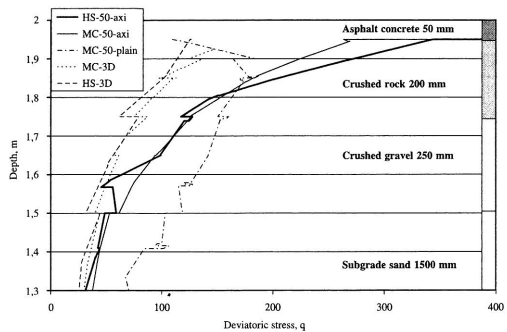


Figure 2. Comparison of the deviatoric stresses in the centre line of the loading (HS = Hardening soil, MC = Mohr-Coulomb) Spring-Overload test.

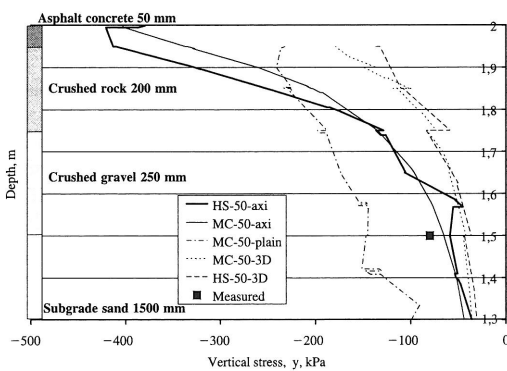


Figure 3. Comparison of the vertical stresses in the centre line of the loading Spring-Overload test.

### 5.3 Stress responses – shear stress ratio

The calculated shear stress ratios are compared with each other in Fig. 4. The shear stress ratio equation is only valid under the centre of the loading, where loading is axisymmetric and the angle of the major principal stress concurs with the vertical axis. Therefore, it can not be applied in the 3D results of two concurrent wheel loads, because the major principal

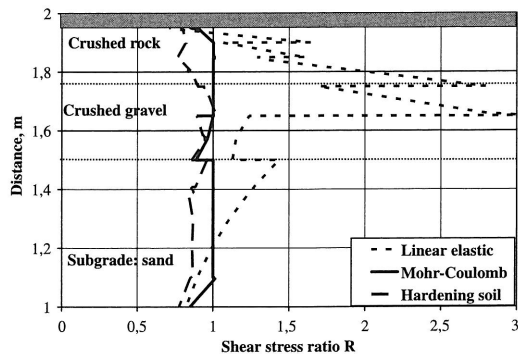


Figure 4. Comparison of the shear stress ratio in the centre line of the loading Spring-Overload test.

axis differs from the vertical axis even under the centre line of one wheel load.

## 6 DISCUSSION

### 6.1 Deviatoric stresses

The stress comparisons clearly show that the deviatoric stresses with HS material model give smaller deviatoric stresses in both 2D and 3D cases. This is quite natural, because the HS model has a non-linear, hyperbolic material model for the elastic deformations. The calculated deviatoric stresses in 3D for MC and HS models were close to each other. The average relative difference between 3D MC and HS calculated stresses was 11% and it varied between -1 to 21%. For 2D the difference was smaller: in average MC defined stresses were about 4% bigger than HS defined.

At the depth of 500 mm the deviatoric stresses for 3D and 2D axisymmetric cases approached each other and the differences were less than 10 kPa. The differences between plain strain and axisymmetric modeling were the largest in the lower part of the structure. These results are all expected. In the plain strain case the deviatoric stresses decreased surprisingly slowly downwards. Thus, also deformations can easily be overestimated if plain strain modeling is used. After all, the plain strain modeling can only be recommended to be used when the shape of the pavement is analyzed (like shoulder and side slope steepness).

### 6.2 Vertical stress

The vertical stress component shows the same phenomenon as deviatoric stress comparison: stresses calculated with MC and HS models are relatively close to each other. The 2D and 3D stresses separate from each other in the upper part of the pavement (to the depth of about 500 mm). The 3D stress calculation is supposed to give more reliable results, because the load

distribution can be modeled more correctly. It is also probable that with the single wheel load the difference between 3D and 2D is smaller in the upper part of the pavement.

Again the plain strain case's vertical stresses decrease very slowly downwards.

### 6.3 Shear stress ratio

If the HS and MC stress responses were reasonably close to each other, the shear stress ratios separated more clearly. The stress calculations for the permanent deformations should be done with HS model. If a suitable non-linear elasto-plastic model is not available, it is also possible to use MC type model. In that case the denominator in equation 1 should be of form  $(X \cdot R)$ , where  $X$  is about 1.05. This correction must be done, because otherwise the deformations become infinite.

### 6.4 Material parameters

The material parameter  $C$  for the permanent deformation method has been defined from the triaxial laboratory tests. Its values have also been fitted to match stress responses calculated with MC model. The problem with parameter definition was that the amount of full-scale tests was only two. Because in the axisymmetric 2D case the HS stress responses and especially shear strength ratio  $R$  are smaller than MC's, the parameter  $C$  needs redefining. Otherwise the method will give far too low deformations. Table 3 presents the material parameters used in the permanent deformation method for the MC model and Table 4 for HS model. The material parameter  $C$  has been estimated to have about 2...4 times larger values for the HS model than for the MC model. The value of parameter  $C$  has been defined from the Finnish accelerated pavement tests (Korkiala-Tanttu et al. 2003a & 2003b).

### 6.5 Permanent deformations

The permanent deformations have been calculated from the stress responses of 2D axisymmetric modeling case and they have been compared with the measured permanent strains for each layer. The permanent strains have been measured with Emu-Coils (Korkiala-Tanttu et al. 2003b). The 2D axisymmetric stress responses were chosen because shear stress ratio  $R$  can not be determined from the 3D results (see chapter 5.2). Both MC and HS models were applied to evaluate the differences between them. Fig. 5 illustrates the calculation results of the SO structure and the loading of 50 kN and Fig. 6 the loading of 70 kN.

The calculation approaches mostly underestimate the permanent strains especially for the high load of

Table 3. The parameters for permanent strain calculations for Mohr-Coulomb (MC) model.

Material	Parameter d	Parameter c	C (%)	DOC (%)	w (%)
HVS: Sand	0.16	0.21	0.0038 ( $\pm 0.001$ )	95	8
HVS: Sandy gravel	0.18	0.15	0.0049 ( $\pm 0.003$ )	97	5...7
HVS: Sandy gravel	0.18	0.15	0.0021 ( $\pm 0.001$ )	100	5...7
HVS: Crushed rock	0.18	0.05	0.012 ( $\pm 0.004$ )	97	4...5

Table 4. The parameters for permanent strain calculations for Hardening Soil (HS) model.

Material	Parameter d	Parameter c	C (%)	DOC (%)	w (%)
HVS: Sand	0.16	0.21	0.016 ( $\pm 0.004$ )	95	8
HVS: Sand	0.16	0.21	0.035 ( $\pm 0.01$ )	95	saturated
HVS: Sandy gravel	0.18	0.15	0.02 ( $\pm 0.01$ )	97	5...7
HVS: Sandy gravel	0.18	0.15	0.008 ( $\pm 0.003$ )	100	5...7
HVS: Crushed rock	0.18	0.05	0.048 ( $\pm 0.016$ )	97	4...5

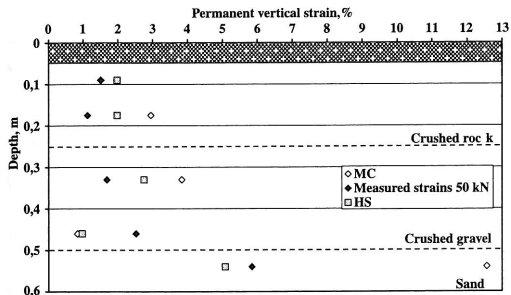


Figure 5. Comparison of the vertical strains in the centre line of SO structure with the 50 kN loading.

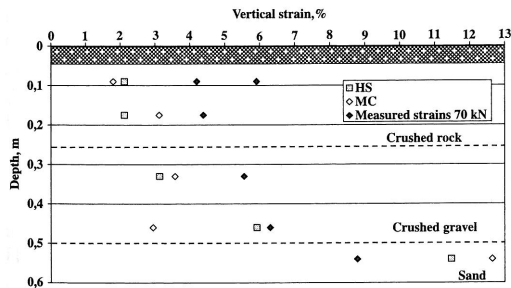


Figure 6. Comparison of the vertical strains in the centre line of SO structure with the 70 kN loading.

70 kN. Yet, for the subgrade sand the MC model gives the highest values for both load levels. In general the underestimation is much bigger for the MC model than for the HS model. The problem with the high load level is that when the maximum shear strength ratio 1 for

MC model has been reached the permanent deformations will be the dependent on the load level. Another reason for the underestimation is that the method does not take into account the rotation of the principal axis. The studies of Kim & Tutumluer (2006) have been proven that the rotation of the principal axis has a significant effect on the permanent deformations.

The vertical strains are calculated from layer to layer, multiplied with the thickness of the layer and then they are summed up to get the total rut depth.

The measured error of Emu-Coil pairs according to Janoo et al. studies (1999) was within  $\pm 1$  mm, which is also at the threshold limit of the ability to detect permanent deformations. This error corresponds to the %-unit error of  $\pm 0.5\%$  to  $1.25\%$  depending on the distance of the coils.

## 7 CONCLUSIONS

From the stress analysis it can be concluded that:

- 2D axisymmetric modeling gives quite reasonable stress distributions in the lower part of the pavement structure,
- in the upper part 2D axisymmetric stresses overestimate the stress state especially for the dual wheel load,
- the differences between HS and MC analyzed stresses in 3D case is small,
- 2D plain strain modeling can be used for the scaling of different pavement geometries, but it is not recommend to be used in the deformation calculations because it overestimates greatly the stress state in the lower part of the pavement,

– 3D stress response can not be applied to the developed calculation method, because the maximum deviatoric stress calculation method is not valid in real 3D conditions.

The recommendation is that the stress responses for permanent deformation calculation should be done using 2D axisymmetric calculations. Preferably HS approach should be used. If MC is used, the denominator in equation 1 should be of form  $(X-R)$ , where  $X$  is about 1.05. This correction has to be done, because otherwise the deformations will be infinite. Also the material parameter  $C$  should be chosen according to the used material model.

The permanent deformation method gives tolerable results for the normal load levels. For the high load levels it will probably underestimate the strains. The method suits also to the comparison of different pavement structures and their rutting sensitivity.

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