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# A new material model for permanent deformations in pavements

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**ABSTRACT:** Today's demand in pavement design for a calculation method to evaluate rutting (permanent deformations) is wide and global. The new procurement methods together with functional requirements are underlining this rutting evaluation demand in different pavement materials and layers. Permanent deformation research has become more general during the last decade. VTT (the Technical Research Centre of Finland) has researched many different aspects of permanent deformations with accelerated pavement tests. Accelerated pavement tests have been conducted with the Heavy Vehicle Simulator (HVS-Nordic). The HVS tests have been completed with a wide triaxial test programme in laboratory. The HVS tests have shown that it is difficult to predict in advance where in the pavement rutting happens: in the subgrade or in the structural layers.

A new material model based on the laboratory and HVS tests has been developed in VTT. The objective was to develop a material model for unbound materials, which is an analytical, nonlinear elasto-plastic model. The stress distribution studies of traffic load have shown that it was very important to calculate stresses in pavements with an elasto-plastic material model to avoid tensile stresses in unbound materials, especially when the asphalt layers were thin. The new material deformation model can take into account the number of the passes, the capacity of the material and its stress state. The deformations in each layer are calculated and then summed together to obtain the total rutting on the surface of the structure.

**KEY WORDS:** Rutting, permanent deformation material model, pavement design.

## 1 INTRODUCTION

VTT has focused its research during the last decade on developing an analytical and mechanistic calculation method for pavement design. An important part of this method has been permanent deformation (rutting) calculations. The development process started when the accelerated pavement testing facility HVS-Nordic was purchased in 1997. Several test series has been conducted since by both of the HVS's owner countries (Finland and Sweden).

HVS is a linear, mobile testing machine with full temperature control. The loading wheels are dual or single and the wheel load can vary from 20 to 110 kN. The test

constructions were constructed at VTT's waterproof test basins in Otaniemi. The test facility has been described in more detail in many other test reports and articles (e.g. Korkiala-Tanttu et al. 2003a).

The most important test series for this study have been 'Low volume road research', 'Spring – Overload' and 'Rehabilitated steep slope' tests. Each test series consisted of three different test structures. The objective of the 'Spring – overload' tests was to study the effect of an overload under spring conditions (Korkiala-Tanttu et al., 2003a). The objective of the 'Low volume' tests was to find out the effect of the steepness of side slope to the rutting (Korkiala-Tanttu et al., 2003b). The 'Rehabilitated steep slope' studied different rehabilitation methods of a rutted pavement (Korkiala-Tanttu et al., 2003c). The HVS tests have been completed with simultaneous laboratory tests, like cyclic triaxial tests and monotonic strength tests for the unbound materials. The laboratory tests are described in more detail in the 'Deformation' project's report (Laaksonen et al. 2004). The test data has been collected and analyzed to develop a material model for permanent deformations. The permanent deformation distribution of HVS tests has been analyzed in another article (Korkiala-Tanttu and Laaksonen 2003).

To determine why the biggest permanent deformations appeared in different layers, a wide stress distribution study was made (Korkiala-Tanttu and Laaksonen 2004). The study showed that permanent deformations are governed by both the stiffness and stress distribution of the materials. An important factor is to have the right kind of material model in the stress distribution calculation. The study included calculations with linear elastic, linear elastic with tension-cut-off and Mohr-Coulomb material models. The calculations showed that pure linear elastic material models produced far too high tensile stresses to the unbound material layers. Also Hoff et al. (Hoff et al. 1998) has detected serious shortcomings in pure linear elastic material models. The use of a non-linear material model, like Mohr-Coulomb, is even more vital for low-volume roads where the pavement layers are thin compared to the wheel pressure. The stresses in HVS test constructions for the studies have been calculated with the finite element code Plaxis.

## **2 VTT PERMANENT DEFORMATION MODEL**

### **2.1 Background**

Many recently developed material models are based on the shakedown concept. The shakedown concept was developed to analyze the behavior of metal surfaces under repeated rolling loads (Johnson 1986). The material response is divided into four categories under repeated loading: purely elastic, elastic shakedown, plastic shakedown and incremental collapse (ratchetting). The unbound granular materials do not behave exactly in the same way as metals do. Thus, for example the Technical University of Dresden (Werkmeister et al. 2002) has further developed the shakedown concept for granular materials. In their approach the behavior of unbound materials can be divided into three ranges: plastic shakedown (range A), plastic creep (range B) and incremental collapse (range C).

The development of the VTT material model is based on the analogy of the material behavior under static and dynamic loading. The material model is founded on the theory of static loading, which is widened to the dynamic loading cases. The VTT material model includes implicitly the same kind of material behavior limits as the shakedown

concept. However, the applied terms are derived from geotechnical theories. The VTT material model is valid for all other unbound granular material behavior except for incremental collapse.

The VTT permanent deformation model consists of two different parts. In the following chapters each part is presented separately (2.2–2.3) and the final synthesis of the parts is presented in chapter 2.4.

## 2.2 Number of loading cycles

Many triaxial test studies have shown that the permanent deformation of unbound granular material depends on the number of passes, as given by Equation 1 (Sweere 1990) as a simple power function.

$$\varepsilon_p^1 = a \cdot N^b \quad (1)$$

where  $\varepsilon_p^1$  permanent axial strain  
 $a$  permanent axial strain in the first loading cycle  
 $b$  material parameter  
 $N$  number of load cycles.

All HVS tests together with triaxial tests have shown that Sweere's Formula (Equation 1) is valid for pavement materials (the fitted values in Figure 1) and for the total rutting depths in the surface of the pavement. Many permanent deformation models include the same kind of power function as Equation 1, i.e. Theyse's model (Theyse 1998), Zhang's and Mac Donald's energy – density model (Zhang and Macdonald 2000) and the model in the ARKPAVE FEM program (Qiu et al. 1999).

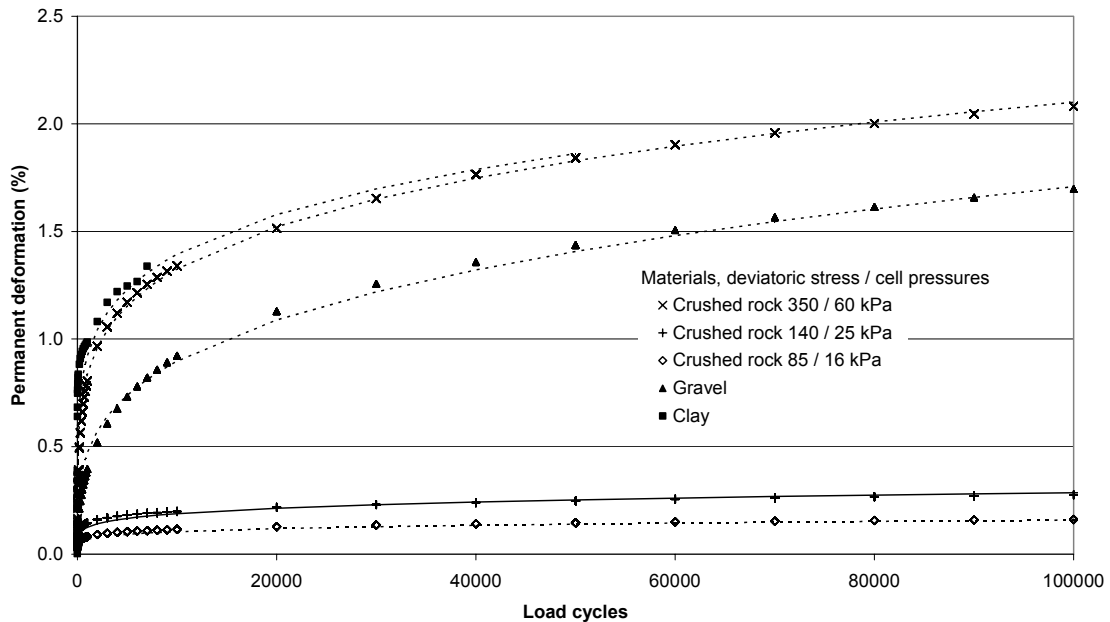


Figure 1: The measured and fitted permanent deformations in VTT's cyclic triaxial tests.

A similar rutting function was presented by Huurman (Huurman 1997). He treated the stress dependency of the parameters by binding them to the failure ratio of the major principal stress  $\sigma_1/\sigma_{1,f}$ . Usually Sweere's Formula (1) is valid for granular materials. Yet, some triaxial test have shown that even at a lower deviatoric stress states permanent deformations begin to accumulate after numerous loading cycles, i.e. (Kolisoja 1998) and (Werkmeister 2004). This kind of incremental collapse can not be described by means of Equation 1. Therefore Huurman also added a second term to Equation 1. His additional function resembles the creep curves for an asphalt mix (Francken et al. 1987).

Werkmeister (Werkmeister 2004) has developed Huurman's equation further, but instead of using failure ratio  $\sigma_1/\sigma_{1,f}$ , she has developed a stress dependency of parameters to principal stresses  $\sigma_1$  and  $\sigma_3$ . The model is called the DRESDEN-Model. The reason for the declining failure ratio in the DRESDEN-Model was that the definition of the failure parameters for unbound granular materials was troublesome.

### 2.3 Shear yielding

Permanent deformations mainly depend on the shear yielding of the material. In the VTT model the yielding and shear strains are described through the failure ratio  $R$ . This means that the deformations are larger when the failure ratio is close to failure. The failure ratio  $R$  in this case is defined as the ratio between deviatoric stress and deviatoric stress at failure ( $q/q_f$ ). The deviatoric stress is chosen because the deviatoric stress is supposed to be the most dominating stress component for the permanent stresses. Besides this, it is relatively easy to calculate from the normally used stress calculations. An analogical approach has been presented by Brown and Selig /Brown and Selig 1991/. Figure 2 shows how the vertical strains depend on the failure ratio  $R$ . It is notable that the vertical strains for different materials do not depend so much on the material, but on the failure ratio. The degree of compaction for the materials in Figure 2 varied from 95 % to 100 % and the water content from 4 % to 8 %.

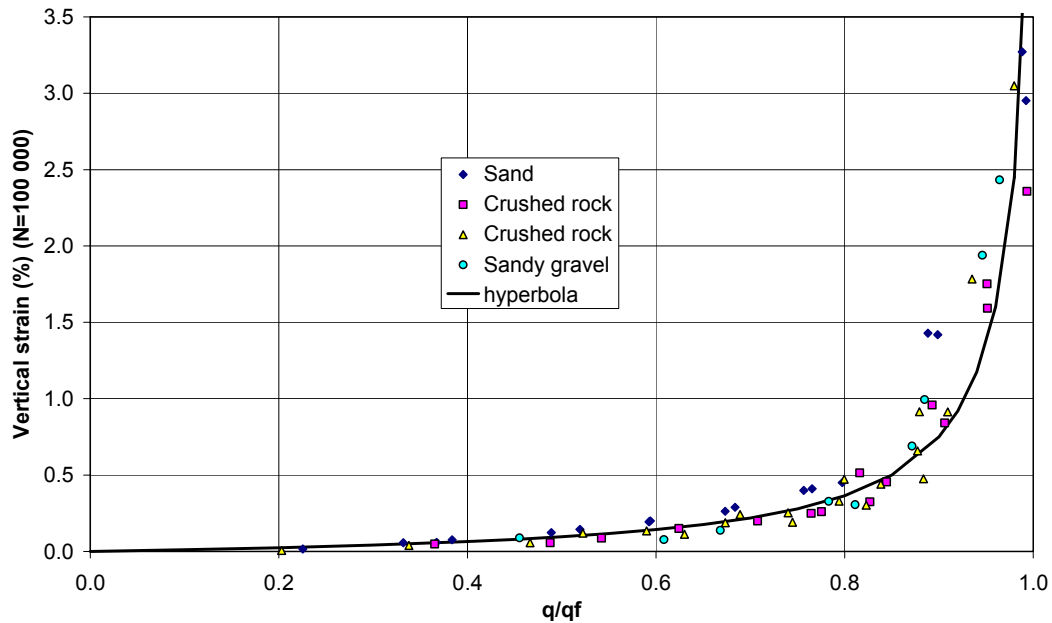


Figure 2: The failure ratios versus vertical permanent strains in VTT's cyclic triaxial tests.

Many different function types were attempted, but the hyperbolic function proved to work best. Hence, the hyperbolic constitutive equation of Kondner and Zelasko (Kondner and Zelasko, 1963) was chosen to describe the dependency of stresses and deformations even in cyclic tests (Equation 2). The shear strain  $\gamma$  can be replaced with permanent deformation  $\varepsilon_p$ . And if the shear ratio  $\tau/\sigma$  is chosen as the failure ratio  $R$  ( $q/q_f$ ), then the permanent deformation can be described by Equation 3. Parameter  $A$  is the maximum possible ratio for  $R$ , which theoretically is one. Because of the inaccuracies in calculation methods of the stress state and the definition method of the strength parameters, practical experiences have shown that it is better to use values between 1.02 and 1.05 for parameter  $A$ .

$$\frac{\tau}{\sigma} = \frac{A\gamma}{B + \gamma} \quad (2)$$

$$\text{if } \frac{\tau}{\sigma} \approx \frac{q}{q_f} = R \text{ and } \gamma \approx \varepsilon_p \Rightarrow \varepsilon_p = B \cdot \left( \frac{R}{A - R} \right) \quad (3)$$

where  $\tau$  shear stress  
 $\sigma$  normal stress  
 $B$  material parameter  
 $\gamma$  shear strain  
 $\varepsilon_p$  permanent strain  
 $q$  deviatoric stress, kPa  
 $R$  failure ratio ( $q/q_f$ )  
 $A$  the maximum value of the failure ratio  $R$ , theoretically  $A=1$ , in practical cases 1.02 to 1.05

$$q_f = q_0 + M \cdot p \quad (4)$$

$$M = \frac{6 \cdot \sin \phi}{3 - \sin \phi} \quad (5)$$

$$q_0 = \frac{c \cdot 6 \cdot \cos \phi}{3 - \sin \phi} \quad (6)$$

where  $q_f$  deviatoric stress in failure, kPa  
 $q_0$  deviatoric stress, when  $p' = 0$   
 $c$  cohesion, kPa  
 $M$  the slope of the failure line in  $p'$ - $q$  space (-)  
 $p'$  hydrostatic pressure, kPa  
 $\phi$  friction angle, °

Equation 3 expresses the acceleration of the growth of permanent strains when the failure envelope is approached. Figure 3 illustrates the situation in  $p'$ - $q$  space. To define

the deviatoric stress in failure, the strength properties of the materials should be measured with e.g. triaxial tests.

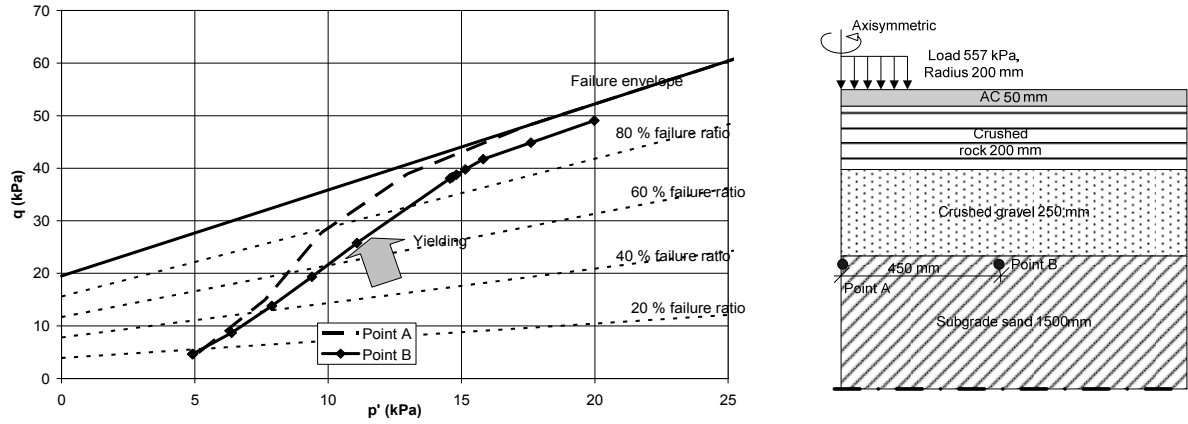


Figure 3: The contours of failure ratios for sand and two different stress paths in  $p'$ - $q$  space.

## 2.4 The material model

The permanent deformation in the first loading cycle (a in Equation 1) can be described with the aid of shear strain (Equation 3). When the permanent shear strain component is combined together with the cyclic loading function, a new permanent material model, which calculates the vertical permanent strain, is introduced as Equation 7. The definition of the parameters b and D is presented in chapters 3.1 and 3.2. The research is continuing and a detailed description about the definition of the parameters b and C will be published in the future.

$$\varepsilon_p = C \cdot N^b \cdot \frac{R}{1 - R} \quad (7)$$

where  $\varepsilon_p$  permanent vertical strain  
C material parameter.

The vertical deformations in each layer are calculated with Equation 7 and then changed to vertical compression, which are summed together to obtain the total rutting in the surface of the structure.

## 3 THE MODEL PARAMETERS

### 3.1 Material parameter C

The material parameter C describes the amount of permanent deformation in different materials. The value of parameter C is stress dependent and thus values of parameter C can not be directly compared with each other. The value of the parameter C depends on various factors. The most important factors are the material, its degree of compaction

(DOC) and water content. Figure 4 illustrates the values of parameter C for sandy gravel in different HVS tests (SE06 = Swedish test number 06 and SO = Spring Overload test in Finland), VTT's triaxial tests and Werkmeister's triaxial tests (Werkmeister 2004).

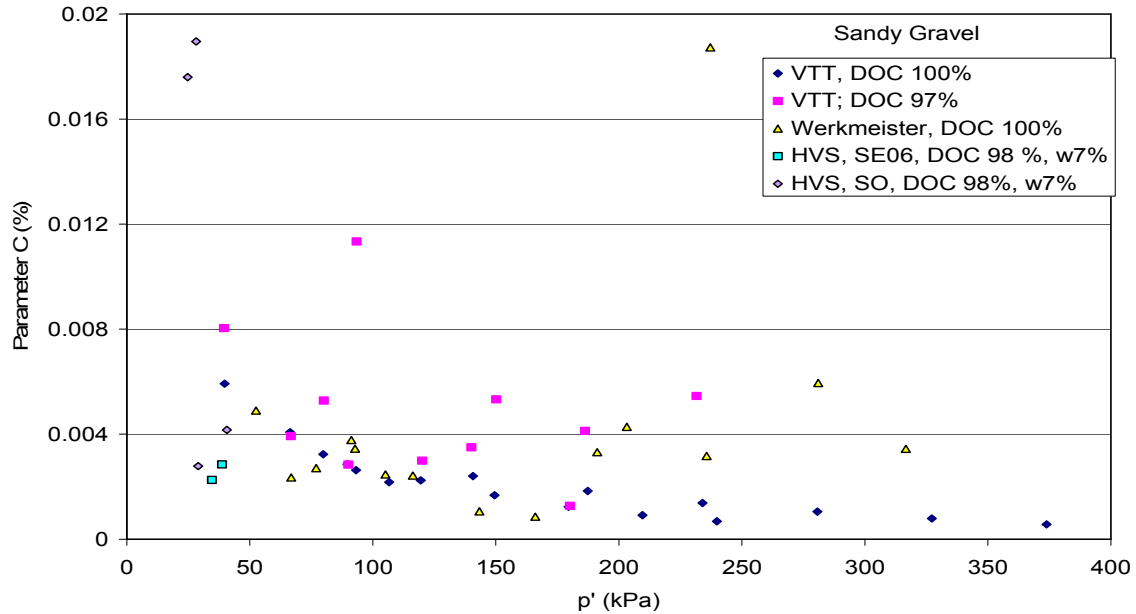


Figure 4: The parameter C for sandy gravel in HVS and triaxial tests.

The degree of compaction and the water content have changed from test to test. In the HVS tests the value of parameter C has varied a lot. One reason for this is that the loading situation and water contents in the HVS test are not as well defined in the in-situ conditions as in the laboratory. Another reason is the inaccuracy of the Emu-Coil measurements in HVS tests.

On the bases of the test results, parameter C shows a clear dependency on the degree of compaction and water content. For sandy gravel the test results also show a slight dependency on the hydrostatic stress. The proposed values for parameter C for sand, sandy gravel and crushed rock materials are presented in Table 1. The maximum values should be used for materials that are saturated or when the degree of compaction (DOC) is low.

Table 1: The values of parameter C (%) for different materials.

Material	Parameter C (%)
Sand (DOC 95 %, w 8 %)	0.0038 ( $\pm 0.001$ )
Sandy Gravel (DOC 97 %, w 5...7 %)	0.0049 ( $\pm 0.003$ )
Sandy Gravel (DOC 100 %, w 5...7 %)	0.0021 ( $\pm 0.001$ )
Crushed rock (DOC 97 %, w 4...5 %)	0.012 ( $\pm 0.004$ )



It is interesting to notice that the value of parameter C for crushed rock is bigger than for sand. This is due to the hyperbola function. If the deviatoric and hydrostatic stresses are the same, the value of the hyperbola can have much bigger values for sand, whose strength properties are not as high as for crushed rock. And because of this, the permanent deformations will be bigger for sand than for crushed rock in the same stress state. Besides this, parameter b depends on the strength properties and emphasizes the effect of the shearing to the permanent deformations.

### 3.2 Parameter b

The value of the parameter b (Equation 7) has been calculated from the laboratory tests and deformation measurements of HVS tests. This parameter b gives the damping shape of the permanent deformation curve. If b is 1, the permanent deformations are linearly dependent on the amount of load repetitions. If b is small (near 0), the permanent deformations are only slightly dependent on the amount of load repetitions. The in-situ and laboratory parameters have been compared with each other in Table 2. With most materials the in-situ values of parameter b seem to be bigger than laboratory values. This means that in-situ the deformations do not damp as easily as they do in laboratory conditions. The main reason for this is that in the in-situ conditions the loading includes the rotation of principal stresses, which laboratory testing does not include. The material model is sensitive to the choice of parameter b. The field values of b have been calculated from the Emu-coil measurements of the HVS tests.

*Table 2: Parameter b in HVS test materials, determined from laboratory tests and in-situ (Emu-coil) measurements.*

Material / parameter b	Crushed rock, $\sigma_3$ 60 kPa	Crushed rock, $\sigma_3$ 25 kPa	Crushed rock, $\sigma_3$ 16 kPa	Crushed gravel	Clay	Sandy gravel	Sand
In laboratory	0.20	0.18	0.18	0.28	0.18	0.13–0.48	0.23–0.39
In situ	0.28–0.4	0.3–0.38	0.4	0.38	0.18	0.25–0.45	0.25–0.38

The parameter b depends on many factors, the most important of which are the stress state and failure ratio. The basic geotechnical hypothesis is that deformations grow quickly when the stress state approaches failure. Equation 8 suggests the value of parameter b as a simple linear function of stress failure ratio  $q/q_f$ . The degree of compaction (DOC) and water content also affect the value of parameter b. If the DOC increased, parameter b will reduce and vica versa. If the water content increases, one can suppose that parameter b also increases. The test data is insufficient to show these dependencies. In Table 3 some values for the parameters c and d are suggested.

$$b = d \cdot \left( \frac{q}{q_f} \right) + c \quad (8)$$

where c and d are material parameters.

*Table 3: Parameters c and d for different materials, DOC and water content.*

<b>Material</b>	<b>Parameter d</b>	<b>Parameter c</b>	<b>DOC (%)</b>	<b>w (%)</b>
Sand	0.16	0.21	95	8
Sandy gravel	0.18	0.15	97	6
Crushed rock	0.18	0.05	97	4

## **4 DISCUSSION**

To be able to calculate permanent deformations in pavement, the stress distribution in the pavement should be calculated in a reliable way. A very important factor is to have the right kind of material model in the stress calculations. Pure linear elastic material models have proven to produce far too high of tensile stresses to the unbound material layers. Thus, more complex material models, like Mohr-Coulomb or a non-linear elasto-plastic model together with a finite element program are recommended for the stress calculation (Korkiala-Tanttu & Laaksonen 2004). In most cases, when there is no real three dimensionality the best way to simulate wheel loading is to use an axisymmetric geometry. In the near future the wheel loadings can hopefully be modeled in a more realistic way through three dimensional modeling.

The VTT deformation model is founded on the theory of static loading, which is widened to the dynamic loading cases. The developed material model is relatively simple, but still it succeeds to bind permanent deformations to the most important factors. The calculation of stress state needs three parameters to be defined: friction angle, cohesion and resilient modulus. For materials not including the basic materials – sand, sandy gravel and typical Finnish crushed rock – also parameters C, c and d should be defined in laboratory tests. In the parameter definition the degree of the compaction and water content can be slightly varied from those presented in the tables. In other cases caution should be followed.

The VTT permanent deformation model is quite sensitive to the changes in parameter b. Also definition of the material strength parameters is important, because the failure ratio has a great effect on the total deformations. In the deformation calculation, typical parameter values should be used. Because of the high compaction, the friction angle and cohesion can have quite high values in the pavement layers.

The permanent deformation model has been developed from the in-situ accelerated pavement tests. The model has to be verified with other accelerated pavement tests. Besides this, more knowledge about the model and its limitations is needed. The deformation model also has to be verified for other kinds of materials, like cohesive, bound base and recycling materials. Theoretically the material model is valid when there is no incremental collapse.

The research revealed how important it is that the development of the permanent deformation material model is based on both accelerated loading and triaxial tests. The stress state and distribution in the full scale tests differ very much from the laboratory conditions. If only laboratory tests are applied the risk to accentuate an insubstantial factor is big. It was also obvious that many compromises had to be made when a simple material model was to be achieved.

The parameters of the model need more research to find out their dependency on i.e. the water content and degree of compaction. More materials should also be studied to define their material parameters. Because the main objective is to achieve a working tool to estimate rutting in the field, the material model will be verified with other accelerated pavement tests. The material model and stress calculation method also need a wider system to evaluate all other factors that affect the rutting phenomenon, like lateral wander of the loading, deformations in asphalt and wearing of the studded tire. The VTT material model and its materials need more research, but even in this format it can be applied for estimating the rut depth of traditional pavement structures.

## 5 CONCLUSIONS

A new material model for the calculation of vertical permanent strains has been developed in VTT. The model is founded on the theory of static loading, which is widened to the dynamic loading cases. The objective was to develop a relatively simple model, which binds permanent deformations to the most important governing factors. The calculation of stress state, when a more complex model is used, needs three parameters to be defined: friction angle, cohesion and resilient modulus. The deformation calculations need three parameters  $C$ ,  $c$  and  $d$ . The material model is sensitive to the changes in material strength parameters as well as to parameter  $b$ . The permanent deformation model has been developed from in-situ test results. It needs verification with other in-situ tests, other unbound materials and more knowledge of its limitations.

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