Magnetometry by a proximity Josephson junction interferometer

Robab Najafi Jabdaraghi

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Abstract

In quantum technology, several aspects of superconductivity such as proximity effect have studied to develop a wide range of attractive applications at sub-kelvin temperatures. A new type of interferometer based on the proximity effect, taking place around transparent interfaces between normal- and superconducting metals, the Superconducting Quantum Interference Proximity Transistor (SQUIPT), relies on the phase dependence of the density of states in the proximized weak link. The SQUIPT devices offer the possibility to realize sensitive low-dissipation magnetometers compared to conventional DC SQUIDS. In this thesis, we investigate the development of the sensitive SQUIPT magnetometers.

The first part of the thesis covers the characterization of non-hysteretic SQUIPTs with enhanced responsivity. In these structures, we demonstrate magnetic flux modulation of the device characteristics displaying no hysteresis at low temperatures by simply increasing the Josephson inductance of the weak link compared to self-inductance of the superconducting loop in the device. As a consequence, improvement in magnetic field responsivity is achievable.

We then turn to the implementation of SQUIPT devices based on different fabrication methods and superconductor materials for improving the device performance. In this aspect, we fabricate and characterize niobium-based SNS devices utilizing two separate lithography and deposition steps with strong Ar ion cleaning in between. We further investigate a prototype hybrid SQUIPT device based on an Nb-Cu-Nb SNS junction with a conventional Al probe in tunnel junction.

In the third part of the thesis, we present the flux noise characterization of a SQUIPT device using simultaneous measurement of DC transport properties and shot noise. To probe the noise, we use a cryogenic amplifier operating at frequencies in the range of a few MHz. We adapt this technique for flux noise measurements in SQUIPTs, adaptable also to low-temperature shot noise measurements of other nonlinear devices with high impedance. In order to investigate flux noise of SQUIPTs, we develop a model allowing one to optimize the figures of merit of the magnetometers such as the noise-equivalent flux.

Keywords Superconductivity, proximity effect, SNS weak link, tunnel junction, SQUID, SQUIPT

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*This work is dedicated to my beloved parents and family
for their love, endless support, encouragement and sacrifices*

Preface

The research presented in this thesis has been carried out in the PICO-group of Low Temperature Laboratory, Department of Applied Physics at Aalto University School of Science from May 2013 to October 2017. I started my initial research journey in Finland in 2012, which is when I joined the group of Doc. Sorin Paraoanu. Between the January 2012 and December 2012, it has been a privilege to work in Kvantti group to obtain guidance throughout my studies, working in the clean room of Nanotalo and especially introducing an opportunity to start at Low Temperature Laboratory, and I am grateful to Doc. Sorin Paraoanu for all the help and discussions before coming here and during 1 year staying in his group.

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I would like to thank the person who has mostly contributed to this work with the role of my thesis advisor, Dr. Joonas Peltonen. I am very grateful to him for the patient guidance, encouragement, and the advice, which he has provided throughout my time as I worked with him. Dr. Peltonen was the person who I could share freely all the ups and downs during the

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Since I joined Jukka's group at May 2013 I met supportive friends in his group. Especial thanks belong to Anna Feshchenko, who was not just my colleague and office mate but also one part of my family along our long journey in PICO. I should mention that Anna's supporting comes from the heart and she never stops that even after leaving the group, I always admire how kind and supportive she is. Many thanks to my office mates, with whom I shared daily life, different sort of cookies, chocolates and so on. I would like to express my sincere thanks to Dr. Jonne Koski, a brilliant office mate, who has always been ready to help with any problems that might come across in the lab, and office. Other members of my office mates should be appreciated: Elsa Mannila for sharing useful discussions and improving my Finnish language, Antti Jokiluoma, and Shilpi Singh. I would like to express the deepest appreciation to Bayan Karimi, of course not only my latest office mate but also my closest friend, who always believed in me. The most energetic and strong person I know, especially we could share our emotions, ups and downs with the same language.

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Since the start of my research journey in Finland, I have been very fortunate to receive scholarships from different sources. First, it was a partial grant from Iranian Ministry of Science, Research and Technology during the year 2012-2015. Then, in the year 2012, I got another partial grant from the Research Foundation of Helsinki University of Technology (Teknillisen korkeakoulun tukisäätiö). In May 2013, my research concerned to this research thesis started with part of the Academy of Finland Center of Excellence program. Scientific supports from the Aalto University at OtaNano- Micronova Nanofabrication center for providing the processing facilities are also appreciated: special thanks go to Antti Peltonen, Paula Kettula, Mika Koskenvuori.

The days would have passed far more slowly without the support of my friends, both in Finland and the other countries, whom I thank for providing such a rich source of conversation, education, and entertainment. Without their support, I could not overcome to my difficulties during this journey in Finland. Especial thanks belongs to my friend, Dr. Toroghi, who gave me useful advice to fight with all of my fears, and supported me to stay strong even when everything is falling apart. I am also heartily gratitude to my beloved friends in Finland, especially Dina Mosselhy, Jairan nafar Dastgerdi, Mona Mahboob Kanafi and her husband Armin, and Neda Mousakhani who gave me the family support here and with whom I have never felt nostalgic to my family. Many thanks to my other friends, Elnaz Abdollahi, Mehmet Yalcinkaya, Fereshteh Sohrabi, Sara Pourjamal, Sasha Hoshian, Farah Behnam, Bahareh Mehrabimatin, and Negin Karimi.

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Robab Najafi Jabdaraghi,

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Contents

Preface	1
Contents	5
List of Publications	7
Author's Contribution	9
1. Introduction	11
2. Superconductivity, tunnel junctions, and proximity effect	15
2.1 Superconductivity	15
2.1.1 Quasiparticle tunneling in hybrid junctions	16
2.1.2 Andreev reflection and superconducting proximity effect	18
2.2 SNS junction	19
2.2.1 Theoretical description	20
2.2.2 Long junction limit	22
2.2.3 Short junction limit	23
2.3 Basic principles of SQUIDs	25
2.3.1 Working principle	25
2.3.2 Hysteresis	26
2.3.3 Sensitivity	26
2.4 Superconducting quantum interference proximity transistor	27
2.4.1 DC transport in the SQUIPT device	28
2.4.2 I-V curve of a Josephson junction with small critical current	30
2.5 Current noise in tunnel junction devices	33
2.5.1 Quasiparticle current fluctuations in a general tunnel junction	33

2.5.2	Quasiparticle current fluctuations in a SQUIPT device	36
3.	Experimental methods	37
3.1	Device fabrication	37
3.1.1	One-step fabrication method	37
3.1.2	Two-step fabrication method	41
3.2	DC and noise measurement setup	44
3.2.1	Sample stage and cryostat wiring	46
3.2.2	HEMT-based cryogenic amplifier: Configuration and characterization	48
3.2.3	HEMT-based cryogenic amplifier for cross correlation technique	49
3.2.4	Model for evaluating shot noise in the SQUIPT device	50
4.	SQUIPT devices and their DC and noise characterization	53
4.1	DC transport of an Al-SQUIPT	53
4.2	Non-hysteretic SQUIPTs with enhanced responsivity	54
4.3	Low-temperature characterization of Nb-Cu-Nb weak links	57
4.4	Nb-SQUIPT	60
4.5	Noise performance of a superconducting magnetic flux sen- sor based on a proximity Josephson junction	63
4.5.1	Shot noise measurement of an Al-SQUIPT	65
4.5.2	Flux noise characterization	66
5.	Conclusions and outlook	71
	Bibliography	75
	Errata	85
	Publications	87

List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

I R. N. Jabdaraghi, M. Meschke, and J. P. Pekola. Non-hysteretic superconducting quantum interference proximity transistor with enhanced responsivity. *Applied Physics Letters* 104, 082601 (4 pages), February 2014.

II R. N. Jabdaraghi, J. T. Peltonen, O.-P. Saira, and J. P. Pekola. Low-temperature characterization of Nb-Cu-Nb weak links with Ar ion-cleaned interfaces. *Applied Physics Letters* 108, 042604 (5 pages), January 2016.

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IV R. N. Jabdaraghi, J. T. Peltonen, D. S. Golubev, and J. P. Pekola. Magnetometry with Low-Resistance Proximity Josephson Junction. *Journal of Low Temperature Physics*, DOI: 10.1007/s10909-018-1863-x, (10 pages), January 2018.

Author's Contribution

Publication I: “Non-hysteretic superconducting quantum interference proximity transistor with enhanced responsivity”

The author fabricated the devices, performed most of the measurements with the second author, analyzed the data, and wrote the first draft of the manuscript.

Publication II: “Low-temperature characterization of Nb-Cu-Nb weak links with Ar ion-cleaned interfaces”

The author fabricated the devices, took part in the experiments, participated in analyzing the data, and wrote the first draft of the manuscript.

Publication III: “Noise of a superconducting magnetic flux sensor based on a proximity Josephson junction”

The author fabricated the devices, implemented a design and built the cryogenic amplifiers used in the noise measurements, took part in the experiments, participated in analyzing the data, and wrote the first draft of the manuscript.

Publication IV: “Magnetometry with Low-Resistance Proximity Josephson Junction”

The author fabricated the devices, took part in the experiments, analyzed the data, and wrote the manuscript.

1. Introduction

We all know about magnets and magnetic fields. We are surrounded by various sources of magnetic fields such as those generated by the Earth, current flowing in a wire, TV, computers, power transmission lines, and interestingly by our own heart and brain. The magnitudes of the fields generated by the brain and heart are of the order of 10^{-13} tesla. Magnetic field detection, magnetometry, is of interest for various scientific purposes, navigation, medical science, biology, etc. In 1833 Carl Friedrich Gauss, head of the Geomagnetic Observatory in Göttingen, measured the Earth's magnetic field [1]. In the 19th century the Hall effect [2] was developed. One of the most significant developments in the condensed matter physics is the discovery of "superconductivity" [3], occurring in certain metals (superconductors) at low temperatures with several unique properties.

As a consequence of this fascinating discovery approximately 100 years ago, quantum technology has developed with several aspects of superconductivity, including for example Josephson effect and the proximity effect [4]. A major application of superconductivity is in superconducting magnets in medicine, with the development of magnetic resonance imaging (MRI). MRI is used to examine the soft tissues in human body, tumor screening [5], examining neurological functions, and revealing disorders in joints, muscles, heart and blood vessels. In quantum technology, superconducting quantum devices such as SQUIDs (superconducting quantum interference devices) [5, 6] are used as detectors to perform MRI. The SQUID measures extremely weak signals, such as subtle changes in human body's electromagnetic field. This device is based on a Josephson junction, consisting of two superconductors separated by an insulating layer through which electrons can tunnel quantum mechanically.

Besides the wide range of SQUID applications as a voltmeter, current amplifier, voltage standard [5, 7, 8], and in scanning microscopy (SSM)

[9, 10], one of the most important SQUID uses is in biomagnetism. It is the study of magnetic activity of the human organs such as brain (MEG), heart (MCG), lungs [11], and peripheral nerves [12]. An important element of mesoscopic electrical circuits are tunnel junctions, an electronic device consisting of two metallic electrodes separated by a very thin layer of insulating material. A wide range of attractive nanoscale applications, including quantum information [13], metrology [14, 15], quantum optics [16], and photon detection [17] has been reported in hybrid superconducting systems due to the electrical [18] and thermal [19] properties of these devices.

A novel application of these devices as an interferometer based on the proximity effect, Superconducting Quantum Interference Proximity Transistor (SQUIPT), has been investigated in 2010 [20]. It is a hybrid superconducting interferometer that exploits the phase dependence of the density of states (DoS) in the proximized weak link to achieve high detector sensitivity. Compared with a conventional DC SQUID, a SQUIPT can achieve significantly lower power dissipation [20]. Some features in this device, including simple DC readout scheme, both current and voltage biased measurements depending on the setup, and flexibility in the fabrication parameters makes it well-suited for nanoscale applications.

The SQUIPT has demonstrated in experiments high responsivity to magnetic flux [20–23] and theoretically predicted a low noise magnetometer [24]. A SQUIPT consists of a superconducting loop interrupted by a short normal-metal wire in direct metal-to-metal with it, while an additional superconducting probe electrode is tunnel-coupled to the normal region. In the fabrication process of these devices, a wide range of materials such as semiconductors [25, 26], carbon nanotubes [27–29], and graphene [30] can be used in the weak link instead of normal metals. The quality of the contact between the weak link material and the superconducting loop plays a key role in the device performance. The motivation to use higher temperature conventional superconductors such as Nb [31] and Vanadium [32, 33] is to increase the sensor sensitivity and range of operation temperatures compared to devices with an Al superconducting loop [20–23, 34–37].

A figure of merit of SQUID and SQUIPT sensors is the noise-equivalent flux (NEF) or flux sensitivity [38] which is obtained from the transfer function and the noise of the device. The limits to flux sensitivity of a SQUIPT have been considered theoretically in Ref. [24]. In the earliest

experimental realization [20], the NEF was limited by the preamplifier contribution to the noise, and estimated to be $\sim 20 \mu\Phi_0/\text{Hz}^{1/2}$. In a subsequent optimized device, $500 \text{ n}\Phi_0/\text{Hz}^{1/2}$ has been obtained at 240 mK in a low-frequency (sub-kHz) cross-correlation measurement, limited by the room-temperature amplifier noise [23]. Recently, $260 \text{ n}\Phi_0/\text{Hz}^{1/2}$ at 1 K was reported for a fully superconducting device [37]. In comparison, lower values of power have been obtained with state-of-the-art nanoSQUIDs [39, 40]. However, there is still a lot of room for optimizing the NEF value in SQUIPT by improving the magnetic flux sensitivity. Also observation of the non-bandwidth-limited intrinsic flux noise performance of these hybrid superconducting magnetometer devices, predicted to be determined by shot noise [41,42] in the current through the probe tunnel junction [24], has been experimentally probed only recently as part of this work.

Structure of the thesis

In this thesis, we summarize the development of the SQUIPT device towards sensitive magnetometry. We present different methods and materials for improving the device fabrication, and a flux noise characterization setup based on a cryogenic amplifier. For theoretical background, Chapter 2 discusses basic principles of superconductivity, tunnel junctions, and the proximity effect. This chapter summarizes the required theoretical details in tunnel junctions, including Normal-Insulator-Superconductor (NIS) and Superconductor-Normal Metal-Superconductor (SNS) structures. The short and long junction limits in the SNS weak link are presented in order to gain an understanding of the SQUIPT sensitivity to magnetic flux. The basics of the SQUID are presented in this chapter. They are useful for understanding the following section dealing with the main characteristics of the SQUIPT device. The DC transport of the SQUIPT is discussed theoretically. It will include I-V characteristics, current and voltage modulations, and the transfer function of the device. A brief theoretical calculation for SQUIPT with Nb superconducting loop and Al probe is presented, consequently. In the last section of this chapter, current fluctuation in tunnel junctions and the SQUIPT device will be theoretically addressed.

Chapter 3 presents practical methods, such as device fabrication processes and low temperature measurement techniques. The shot noise measurement scheme will be introduced in detail explaining all the com-

ponents using in the setup, such as resonant circuit on the sample stage, and the home-made HEMT-based cryogenic amplifier.

In Chapter 4, we discuss in detail the experimental results presented in Publications I-IV. The first section is focused on the non-hysteretic behavior of the SQUIPT structures, resulting in the enhanced responsivity. In Sec. 4. 2, we demonstrate experimentally the characterization of Nb-Cu-Nb weak links and a prototype Nb-SQUIPT at low temperatures. Finally, in Sec. 4. 3, shot noise measurements of an Al-SQUIPT and, consequently, flux noise of such a device is considered. In Chapter 5, the summary of this thesis gives a brief conclusion of the overall results obtained and their future prospects.

2. Superconductivity, tunnel junctions, and proximity effect

2.1 Superconductivity

In 1911, H. Kamerlingh Onnes observed the phenomenon of superconductivity [3, 43–45], noting that the electrical resistance of mercury dropped suddenly to zero when it was cooled below a certain temperature. This phenomenon is determining the main features of certain materials called superconductors below a material-specific critical temperature T_C . At low temperatures, electrons occupy all the states with energy below the Fermi energy $E < E_F$. According to Bardeen-Cooper-Schrieffer (BCS) theory [46, 47], due to the electron-phonon coupling, electrons with opposite momenta and spins can strongly correlate and form Cooper pairs. As a result, in the density of states there is the interval of energies $E_F \pm \Delta$ with no allowed states. Here $\Delta(T)$ is the temperature-dependent superconducting gap. The Cooper pairs are localized in the ground state of the superconductor and can be described by the single position-dependent superconducting wave function, which depends on the superconducting gap $\Delta(T)$. The magnitude of $\Delta(T)$ vanishes at $T = T_C$ where the transition between superconducting and normal states occurs. Based on the BCS theory, the attractive interaction between electrons and phonons is characterized by the coupling constant λ . The temperature dependence of the superconducting gap $\Delta(T)$ can be extracted from the equation [4]

$$\frac{1}{\lambda} = \int_{\Delta(T)}^{\hbar\omega_D} dE \frac{1}{\sqrt{E^2 - \Delta(T)^2}} \tanh\left(\frac{E}{2k_B T}\right), \quad (2.1)$$

where ω_D is the Debye frequency, and k_B is the Boltzmann constant. In the weak coupling limit $\lambda \ll 1$, the temperature dependence of the energy gap from Eq. (2.1) can be approximated by [48]

$$\frac{\Delta(T)}{\Delta_0} = \tanh[1.056(\tanh[\frac{\pi}{2}(1 - T/T_C)^{0.55}])^{0.5/0.55}], \quad (2.2)$$

Figure 2.1. (a) Calculated temperature dependence of the BCS superconducting energy gap. (b) Normalized BCS density of states DoS.

which depends only on T/T_C . As shown in Fig. 2.1 (a), at zero temperature, for $\lambda \ll 1$, one finds the constant $\Delta(T) = \Delta_0 = 1.76k_B T_C$ as the zero-temperature superconducting gap, and close to T_C the gap can be approximated by $\Delta(T)/\Delta_0 \simeq 1.746\sqrt{1 - T/T_C}$. In the ideal superconductor at zero temperature, the density of states (DoS) for single particles with energy E relative to E_F is obtained as

$$n_S(E) = n_0 \frac{|E|}{\sqrt{E^2 - \Delta^2}} \Theta(|E| - \Delta). \quad (2.3)$$

Here, n_0 is the density of states in the normal state at Fermi level E_F , and Θ is the Heaviside step function. Figure 2.1 (b) shows n_S as a function of energy E .

2.1.1 Quasiparticle tunneling in hybrid junctions

The hybrid Normal-Insulator-Superconductor (NIS) tunnel junction was first investigated by Giaever in 1960 [49]. It consists of two electrodes, one normal and one superconducting, separated by a thin layer of insulating material. The nonlinear temperature-dependent current-voltage characteristic of an NIS junction provides electronic refrigeration [50–52] and thermometry [53] to probe the electron temperature of the normal electrode at cryogenic temperatures [18, 54].

In an NIS junction, we approximate the weakly energy-dependent DoS

Figure 2.2. (a) Schematic illustration of electron tunneling in a voltage biased NIS junction. The Fermi levels in the two electrodes are indicated by dashed lines. By applying voltage V they are shifted relative to each other. (b) Calculated IV characteristics and (c) cooling power of an NIS junction at several values of temperatures between $T = 0.01 \text{ K}$ and $T = T_C$

in the normal metal N at energies close to E_F by a constant $n_N(E) = 1$. On the other hand, in a real-world BCS superconductor introduced by Dynes *et al* [55], the density of states is modeled by [18, 56]

$$n_S(E) = \text{Re}\left(\frac{E + i\gamma}{\sqrt{(E + i\gamma)^2 - \Delta^2}}\right). \quad (2.4)$$

Here, γ is a phenomenological parameter for describing the sub-gap current observed experimentally in NIS junctions [56, 57]. The energy level diagram for a voltage-biased NIS junction is shown in Fig. 2.2 (a). The current through the NIS junction can be written as

$$I_{NIS} = \frac{1}{eR_T} \int_{-\infty}^{+\infty} n_S(E) [f_N(E - eV, T_N) - f_S(E, T_S)] dE, \quad (2.5)$$

where $T_{N,S}$ and $f_{N,S}$ are the temperatures and the Fermi-Dirac distribution functions in the N and S electrodes, respectively. We define this result in more detail in Sec. (2.5.1). Furthermore, R_T is the normal state resistance of the junction. The current through the NIS junction is sensitive only to the normal metal temperature, and the S electrode temperature is negligible at $T_S \ll T_C/3$ [18]. To see this, Equation (2.5) can be rewritten as

$$I_{NIS} = \frac{1}{2eR_T} \int_{-\infty}^{+\infty} n_S(E) [f_N(E - eV) - f_S(E + eV)] dE. \quad (2.6)$$

At low temperatures $k_B T_N \ll \Delta$, the current at voltages $0 \ll V \ll \Delta/e$ can be approximated by [58]

$$I_{\text{NIS}} \sim \frac{\Delta}{2eR_T} \sqrt{\frac{2\pi k_B T_N}{\Delta}} e^{\left(\frac{eV-E}{k_B T_N}\right)}. \quad (2.7)$$

The calculated IV characteristic of a NIS junction at different values of electron temperatures T_N in N electrode are shown in Fig. 2.2 (b). The IV becomes smeared with increasing temperature. The mechanism of refrigeration in an NIS junction can be explained based on the hot quasi-particles tunneling through the insulating barrier. As displayed in Fig. 2.2 (a), in the NIS junction with applied bias voltage $eV \sim \Delta$, the most energetic particles can tunnel from the normal metal into the superconductor. The rate of the heat extracted from the N electrode of the junction is given by

$$P_N = \frac{1}{e^2 R_T} \int_{-\infty}^{+\infty} (E - eV) n_S(E) [f_N(E - eV, T_e) - f_S(E, T_e)] dE. \quad (2.8)$$

Figure 2.2 (c) illustrates this cooling power P_N for various electron temperatures. At bias voltages $V \leq \Delta/e$ it is positive and the maximum is achieved slightly below Δ/e . This range of bias voltage is allowing only the hot electrons to tunnel and results in electronic cooling of the normal metal [18, 50]. Furthermore, at high bias voltage $V > \Delta/e$, also quasiparticles below the Fermi level can tunnel, resulting in strong heating and negative cooling power.

2.1.2 Andreev reflection and superconducting proximity effect

Proximity effect [59–64] of the normal conductor is the occurrence of the superconducting-like properties when it is placed into good electrical contact with a non-superconducting material, for instance a normal metal (N). In the diffusive regime, one has $\xi_0 \gg l_{el}$ where $\xi_0 = (\hbar D / \Delta)^{1/2}$ is the so-called superconducting coherence length, and l_{el} is the elastic mean free path. The latter is related to the diffusion constant via $D = v_F l_{el} / 3$, where v_F is the Fermi velocity of electrons. As a consequence of proximity effect, the superconducting correlations can extend over a large length scale in the non-superconducting material and the density of states DoS in S region is modified [61, 65–69]. A simple geometry of a metallic N-S contact is illustrated in Fig 2.3 (a). Particularly, assuming the normal metal (L) and superconductor (L_S) lengths to be much larger than the coherence length, $L_S, L \gg \xi_0$, the DoS changes gradually from the BCS form deep inside the superconductor to the constant value $n_N(E)/n_0 = 1$ in the normal

metal. For a short normal metal in contact with a long superconductor, $L_S \gg L \sim \xi_0$, the DoS in the N side develops a minigap [67, 70, 71] around the Fermi energy. The magnitude of minigap is smaller than the superconducting gap and depends on the length of the normal metal. Increasing the length of the N part decreases the size of the minigap [65]. The important length and energy scales in the proximity effect are the length L of the normal metal and Thouless energy $E_{\text{Th}} = \hbar D/L^2$. In $\Delta \gg L$, magnitude of the minigap is of the order of E_{Th} as will be discussed below in Sec. (2.2.2). Andreev reflection (AR) [72] has a significant role to the proximity effect since it provides conversion of the dissipative electrical current in the normal metal into the dissipation-less supercurrent. The AR mechanism [72–74] is shown in Fig 2.3 (b): An incident electron, with energy E and spin $\uparrow$, can be transferred into the superconductor if another electron, with energy $-E$ and spin $\downarrow$, is also transferred through the interface thus crossing into the superconductor and forming a Cooper pair. As a result, in terms of single excitations, a hole with energy $-E$ and spin $\downarrow$ can be retro-reflected [72, 75] back into the normal metal. The AR process is significant when the barrier transparency is assumed to be high, with no oxide or tunnel layer. Simultaneously, close to N-S interface the leakage of the Cooper pairs weakens the superconductivity on the scale of ξ_0 from the interface on the S side. This phenomenon is called inverse proximity effect [64, 76]. The superconducting critical temperature in the superconducting metal is decreased close to the interface, and the energy gap is suppressed.

2.2 SNS junction

A Superconductor – Normal metal – Superconductor (SNS) weak link [4, 38] consists of a short normal conducting metal (N) embedded between two superconducting (S) electrodes. It can support a finite supercurrent due to proximity effect. As a consequence of the SNS junction properties, novel types of interferometers [20, 77, 78] can be introduced with a large number of foreseen applications including measurement of magnetic flux induced by atomic spins, single-photon detection and nanoelectrical measurements [79–81]. In this section, we discuss SNS junctions, focusing on two different parameter regimes, namely "short" and "long" junctions.

Figure 2.3. (a) Schematic geometry of an N-S contact. (b) Andreev reflection: An incident electron (solid blue circle) in the normal metal moves towards the superconductor producing a Cooper pair in the superconductor and a retro-reflected hole (open blue circle), moving into the normal conductor. Vertical arrows indicate the spin for each particle. (c) Schematic diagram of a 1D SNS junction, consisting of a normal conducting metal (N) embedded between two superconducting (S) electrodes. Due to proximity effect, superconductivity “leads” from each S electrode into the N part, and a finite supercurrent can flow through the weak link.

2.2.1 Theoretical description

The SNS circuit consists of a piece of normal metal in the diffusive limit in contact with two superconducting electrodes. In this structure the electron and hole quasiparticles of the normal metal are coupled by the pairing Hamiltonian and thus quantum state of quasiparticles are defined by Bogulubov-de Gennes equation [4]. The green function formalism does not require solutions of the Bogulubov-de Gennes equations. The three different types of Green functions, retarded ($\check{R}$), advanced ($\check{A}$) and Keldysh ($\check{K}$) [84] can be handled by a 4×4 matrix noted $\check{G}$, $\check{G} = \begin{pmatrix} \check{R} & \check{K} \\ 0 & \check{A} \end{pmatrix}$. In the dirty limit, the SNS structure is effectively described in terms of the quasi-1D spectral equations, satisfying the Usadel equation [61, 82, 83]

$$D\nabla(\check{G}\nabla\check{G}) = [-i(E + i\Gamma)\check{\tau}_z + \check{\Delta}, \check{G}]. \quad (2.9)$$

E is the energy, and Γ describes a small inelastic scattering rate. Here, $\check{\Delta} = \begin{pmatrix} \hat{\Delta} & 0 \\ 0 & \hat{\Delta} \end{pmatrix}$, $\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix}$, and $\check{\tau}_z = \begin{pmatrix} \hat{\tau}_z & 0 \\ 0 & \hat{\tau}_z \end{pmatrix}$,

where, Δ is the superconducting gap, and $\hat{\tau}_z$ is the third Pauli matrix.

The retarded Green function $\check{R}$ can be parametrized by a useful geomet-

rical description [83]

$$\hat{R} = \begin{pmatrix} \cosh(\theta) & \sinh(\theta)e^{i\chi} \\ -\sinh(\theta)e^{-i\chi} & -\cosh(\theta) \end{pmatrix}. \quad (2.10)$$

Here, θ and χ are in general complex scalar functions that depend on the energy and spatial coordinates. We assume that the width of the weak link is much smaller than its length L . This allows us to consider the wires as quasi-one dimensional (1D) structures as shown in Fig 2.3 (c). In this geometry, these two complex functions $\theta(x, E)$ and $\chi(x, E)$ depend on the position x and the energy E varying only in one dimension. The quasi-1D spectral equations can be simplified into [61, 85]

$$\hbar D \frac{\partial^2 \theta}{\partial x^2} = -2iE \sinh(\theta) + \hbar D/2 \left(\frac{\partial \chi}{\partial x} \right)^2 \sinh(2\theta) + 2i\Delta \cos(\varphi - \chi) \cosh(\theta), \quad (2.11)$$

$$\hbar D \frac{\partial j_E}{\partial x} = -2i\Delta \sin(\varphi - \chi) \sinh(\theta), \quad (2.12)$$

$$\text{and } j_E = -\sinh^2(\theta) \frac{\partial \chi}{\partial x}. \quad (2.13)$$

Here, φ is the superconducting order parameter phase difference between the two superconducting electrodes. The self-consistency equation for the energy gap Δ is assumed to have the position-dependent form

$$\Delta e^{i\varphi} = \frac{\lambda}{4} \int_{-\infty}^{+\infty} dE \text{Re}[\sinh \theta e^{i\chi}] \tanh\left(\frac{E}{2k_B T}\right), \quad (2.14)$$

where λ denotes the interaction constant, characterizing the strength of superconductivity [59, 61]. We can consider the boundary conditions in two reservoirs: In the S reservoir, the functions have the bulk values $\theta = \text{artanh}(\Delta/E)$ and $\chi = \varphi$, whereas these values in the N reservoir are $\theta = 0$ and χ not defined [86]. As a result of solving $\theta(x, E)$ and $\chi(x, E)$, the single-particle density of states at position x and energy E in the normal weak link is given by [61, 83]

$$n_N(x, E) = n_0 \text{Re}[\cosh(\theta(x, E))]. \quad (2.15)$$

The observable supercurrent is obtained from [83]

$$I_S(\varphi) = \frac{E_{\text{Th}}}{2eR_N} \int_{-\infty}^{+\infty} dE \text{Im}[j_E(E)] f_L(E). \quad (2.16)$$

Here, $R_N = L/(A\sigma)$ is the normal state resistance of the weak link with length L , normal state conductivity σ , and cross section A . The quantity $f_L(E) = \tanh(E/2k_B T)$ is related to the distribution function, assuming equilibrium at temperature T . Furthermore, the energy-dependent quantity $j_E(E)$ is the spectral supercurrent obtained from a numerical solution of the Usadel equations in the 1D SNS geometry, assuming perfectly

transparent interfaces [83, 87]. The magnitude of the maximum observable supercurrent, also known as the critical current I_C , can be found by maximizing Eq. (2.16) over the phase φ in the interval 0 to 2π .

2.2.2 Long junction limit

In the long junction limit, the weak link is much longer than the superconducting coherence length ξ_0 , $L/\xi_0 \gg 1$ (or equivalently $E_{\text{Th}} \ll \Delta$). Furthermore, $j_E(E)$ in Eq. (2.16) depends only weakly on the ratio Δ/E_{Th} [83]. In this limit, for the temperature dependence of the critical current I_C , we can find analytical estimates in certain temperature regimes, high and low temperature limits.

High-temperature regime: We first consider high temperatures $k_B T \gg E_{\text{Th}}$ (or equivalently $L \gg L_T = \sqrt{(\hbar D/k_B T)}$, where L_T is the characteristic thermal length in the diffusive limit). The mutual influence of the superconducting reservoirs is neglected and except in the vicinity of the N-S interfaces the Usadel equations are linearized for the N part. The critical current I_C can then be written as [87, 88]

$$\frac{eR_N I_C}{E_{\text{Th}}} = \frac{64\pi k_B T}{E_{\text{Th}}} \sum_{n=0}^{\infty} \left(\frac{2\omega_n}{E_{\text{Th}}} \right)^{1/2} \frac{\Delta^2 e^{-(\frac{2\omega_n}{E_{\text{Th}}})^{1/2}}}{[\omega_n + \Omega_n + \sqrt{2(\Omega_n^2 + \omega_n \Omega_n)}]^2}. \quad (2.17)$$

Here, $\omega_n = (2n+1)\pi k_B T$ are the Matsubara frequencies, and $\Omega_n = \sqrt{\Delta^2 + \omega_n^2}$. To highlight the dependence of I_C on the ratios $\frac{k_B T}{E_{\text{Th}}}$ and $\frac{\Delta}{E_{\text{Th}}}$, we rewrite Eq. (2.17) as

$$\frac{eR_N I_C}{E_{\text{Th}}} = \frac{64\pi k_B T}{E_{\text{Th}}} \sum_{n=0}^{\infty} \frac{(2x_n)^{1/2} e^{-(2x_n)^{1/2}}}{[x_n \frac{E_{\text{Th}}}{\Delta} + y_n + \sqrt{2(y_n^2 + x_n y_n \frac{E_{\text{Th}}}{\Delta})}]^2}, \quad (2.18)$$

with the dimensionless quantities $x_n = \frac{\omega_n}{E_{\text{Th}}} = (2n+1)\frac{\pi k_B T}{E_{\text{Th}}}$ and $y_n = \sqrt{1 + \frac{\omega_n^2}{\Delta^2}} = \sqrt{1 + (x_n \frac{E_{\text{Th}}}{\Delta})^2}$. In the very long junction limit $\frac{E_{\text{Th}}}{\Delta} \rightarrow 0$, and Eq. (2.18) can be further simplified to [87, 89]

$$\frac{eR_N I_C}{E_{\text{Th}}} = \frac{32}{3 + 2\sqrt{2}} \left(\frac{2\pi k_B T}{E_{\text{Th}}} \right)^{3/2} e^{-\sqrt{2\pi k_B T/E_{\text{Th}}}} \quad (2.19)$$

by keeping only the term with $n = 0$ in the sum. From Eq. (2.19), one obtains the temperature dependence of the critical current as $I_C(T) \propto T^{3/2} e^{-\sqrt{k_B T/E_{\text{Th}}}}$ [89].

Low-temperature regime: At lower temperature $k_B T \approx E_{\text{Th}}$, evaluation of I_C involves solving the Usadel equation at all energies. For $k_B T \ll E_{\text{Th}}$, the numerical solution can be approximated by [87]

$$I_C = a \left(\frac{E_{\text{Th}}}{eR_N} \right) [1 - b e^{(-\frac{a E_{\text{Th}}}{3.2 k_B T})}], \quad (2.20)$$

Figure 2.4. Calculated temperature dependence of the $eR_N I_C$ product, corresponding to various values of the ratio Δ/E_{Th} in the long junction limit.

where the constant coefficients a and b are 10.82 and 1.30, respectively. In the limit of zero temperature, the $eR_N I_C$ product is found to be proportional to E_{Th} , $I_C = 10.82 E_{Th} / eR_N$. Figure 2.4 shows the temperature dependence of $eR_N I_C$ for various values of the ratio Δ/E_{Th} .

2.2.3 Short junction limit

In the short junction limit $L/\xi_0 \ll 1$, and the superconducting order parameter is much smaller than the Thouless energy, $\Delta \ll E_{Th} = \hbar D/L^2$. In this regime, the Usadel equation can be analytically solved for the two functions, $\theta(E, x)$ and $\chi(E, x)$ [83]:

$$\theta(x) = \cosh\left(\frac{\sqrt{\alpha^2 + 1}}{\alpha} \cosh[j_E \alpha(x - x_0)]\right) \quad (2.21)$$

$$\chi(x) = \chi_0 - \arctan[\alpha \tanh(j_E \alpha(x - x_0))], \quad (2.22)$$

where α and x_0 are determined by the boundary conditions. We have $x_0 = 0$, and the values of θ and χ at $x = \pm L/2$ are given by [70]

$$\theta(E, x = \pm L/2) = \pi/2, \quad \chi(E, x = \pm L/2) = \pm \varphi/2. \quad (2.23)$$

Furthermore, we find

$$\alpha = \frac{\sqrt{E^2 - \Delta^2 \cos^2(\varphi/2)}}{\Delta \cos(\varphi/2)} \quad (2.24)$$

Figure 2.5. Calculated normalized DoS of the normal metal n_N in the SNS junction at different phases between the S electrodes. Inset: Minigap in units of the Thouless energy vs. the phase difference between the superconducting electrodes.

and

$$j_E = \frac{2\Delta \cos(\varphi/2)}{E^2 - \Delta^2 \cos^2(\varphi/2)} \cosh\left(\frac{E^2 - \Delta^2 \cos^2(\varphi/2)}{E^2 - \Delta^2}\right). \quad (2.25)$$

In the SNS junction, the density of states $n_N(x, E, \varphi)$ in the proximized normal metal is given by Eq. (2.15). Using Eq. (2.25) in Eq. (2.21), we find

$$\theta(x, E, \varphi) = \text{arcosh}(\gamma \cosh(2x \cosh(\beta))), \quad (2.26)$$

where

$$\gamma = \frac{\sqrt{\alpha^2 + 1}}{\alpha} \quad (2.27)$$

and

$$\beta = \arccos \frac{\sqrt{E^2 - \Delta^2(T) \cos^2(\varphi/2)}}{E^2 - \Delta^2}. \quad (2.28)$$

Figure 2.5 shows the DoS calculated for different phases between the S electrodes. In the short junction limit, the minigap opened in N part is $E_g = \Delta |\cos(\varphi/2)|$ whose amplitude depends on φ . For $\varphi = 0$, $E_g = \Delta$ and decreases by increasing φ and vanishes at $\varphi = \pi$, as shown in the inset of Fig. 2.5. Also in the one-dimensional long SNS structure, the most characteristic feature of the DoS is that the minigap E_g is periodically modulated as a function of the phase difference φ [70, 71]:

$$E_g/E_{\text{Th}} = \begin{cases} c_1(1 - c_2\varphi^2) & \varphi \ll \pi \\ c_3(\pi - \varphi) & \pi - \varphi \ll \pi, \end{cases} \quad (2.29)$$

with the numerical constants $c_1 = 3.122$, $c_2 = 0.0921$ and $c_3 = 2.467$. The magnitude of the minigap is maximized at $\varphi = 0$ and vanishes at $\varphi = \pi$.

Figure 2.6. (a) Schematic view of a DC SQUID, consisting of a superconducting loop interrupted by two parallel Josephson Junctions (JJs). (b) Current-voltage characteristics plotted for two extreme values of external magnetic flux $\Phi_a = 0$ and $\Phi_a = 0.5 \Phi_0$ (Adapted from Ref. [100]). (c) Voltage modulation with respect to magnetic flux for several values of constant bias currents (Adapted from Ref. [100]). (d) Flux noise of one Nb thin film SQUID presented in Ref. [150] at two different temperatures $T = 4.2 \text{ K}$ and $T = 13 \text{ mK}$.

2.3 Basic principles of SQUIDs

The superconducting quantum interference device (SQUID) [5, 6, 90] is a converter of magnetic flux into an electrical current or a voltage. At present, they are the most sensitive sensors for magnetic flux [6]. A SQUID consists of a superconducting loop interrupted by two or one parallel Josephson junctions, which are called DC SQUID [5–8] and RF SQUID [38, 91–93], respectively. The schematic view of a DC SQUID is shown in Fig. 2.6 (a).

2.3.1 Working principle

The SQUID operation is based on the physical phenomenon of the Josephson effect [4, 38, 94–96] and flux quantization [4] in a superconducting loop. The Josephson effect is the macroscopic quantum phenomenon which is appearing in a system of a so-called Josephson junction (JJ), consisting of two superconducting electrodes coupled by a weak link. The mathematical relationships for the current and voltage across the weak link

were predicted by Josephson in 1962 [97] and observed experimentally in 1963 [98]. In an SIS junction, a dissipationless supercurrent I_s can flow through the junction, depending on the phase difference $\varphi = \varphi_2 - \varphi_1$ across the tunnel barrier according to the DC Josephson relation

$$I_s = I_c \sin \varphi. \quad (2.30)$$

Based on the Ambegaokar-Baratoff formula [99] I_c , denoting the maximum supercurrent the junction can sustain, can be estimated from

$$I_c = \frac{\pi \Delta(T)}{2e R_n} \tanh\left(\frac{\Delta(T)}{2k_B T}\right), \quad (2.31)$$

valid in the tunnel junction limit. Here, R_n denotes the normal state resistance of the SIS tunnel junction. At $T \ll T_C$ Eq. (2.31) reduces to $I_c R_n = \pi \Delta_0 / 2e$.

2.3.2 Hysteresis

In the SQUID device, the total flux in the loop can be expressed as [6]

$$\Phi_T = \Phi_a - L_s I_c \sin\left(\frac{2\pi \Phi_T}{\Phi_0}\right), \quad (2.32)$$

where L_s and Φ_a are the self inductance of the loop and applied flux to the loop, respectively. The magnetic flux slope is defined by

$$\frac{d\Phi_T}{d\Phi_a} = \frac{1}{1 + \beta_L \cos(2\pi \Phi_T / \Phi_0)}, \quad (2.33)$$

with the dimensionless parameters parameter $\beta_L = 2\pi L_s I_c / \Phi_0$. For $\beta_L < 1$ or equivalently $L_s < \Phi_0 / 2\pi I_c$, the slope is positive and the plot Φ_T as a function of Φ_a is non-hysteretic. Here, L_J is the Josephson inductance of the tunnel junction in the SQUID device. On the other hand, for $\beta_L > 1$ the device makes transitions between two flux states and shows hysteretic behavior when Φ_a is varied. The hysteresis in the device appears when the Josephson inductance of the tunnel junction is smaller than the self inductance of the loop, $L_J < L_s$. In order to suppress the magnetic hysteresis in SQUID devices, increasing the Josephson inductance or reducing the loop inductance are necessary.

2.3.3 Sensitivity

The I-V characteristics of a typical SQUID device can be solved numerically using the differential equation presented in Refs. [100, 101]. A solution can be obtained from fixed Φ_a and β_L values while voltage-flux modulation $V(\Phi)$ can be achieved by varying the external magnetic flux Φ_a

and fixing β_L and bias current I_b . Figure 2.6 (b) displays IV characteristics for two values of applied magnetic flux, $\Phi_a = 0$ and $\Phi_a = 0.5 \Phi_0$ at $\beta_L = 1$. The DC SQUID can be operated as a flux-to-voltage transducer by current biasing it, leading to a magnetic flux voltage modulation $V(\Phi)$ as shown in Fig. 2.6 (c) for several values of bias currents I_b/I_0 and $\beta_L = 1$. Figures 2.6 (b) and (c) are adapted from Ref. [100]. A non-hysteretic SQUID can be employed straightforwardly as magnetic flux detector. At the optimal working point of the device, the value of $\partial V/\partial \Phi$ is maximum, $V_\Phi = (\partial V/\partial \Phi)_{\max}$. Another figure of merit of SQUIDS is given by the equivalent spectral density of flux noise $S_\Phi = S_V/V_\Phi^2$ or the rms flux noise $\sqrt{S_\Phi}$ with units $\Phi_0/\sqrt{\text{Hz}}$ [Fig. 2.6 (d)], where S_V is the power spectral density of voltage noise. The frequency dependence of the flux noise has two main parts: At low frequency $\sqrt{S_\Phi}$ is $1/f$ noise and at higher frequencies it becomes independent of f . This white noise is mainly due to Johnson-Nyquist noise associated with dissipative quasi particle currents in the JJs. A nanoscale SQUID "nanoSQUID" [39, 40, 102–107] offers significant reduced flux noise down to the level of a few tens of $\text{n}\Phi_0/\sqrt{\text{Hz}}$, strongly nanoSQUIDS are promising detectors for example for investigating localized magnetic signals produced by magnetic nanoparticles [107].

2.4 Superconducting quantum interference proximity transistor

As shown in experiments by Petrashov *et al.* [77, 78, 108], Andreev "mirrors" [72] with varying phase differences between superconductors provide a means to modulate the supercurrent through a short normal wire. Both hysteretic and non-hysteretic behavior of Andreev interferometers with three superconducting electrodes in voltage biased [109] and current biased regime [110] have been theoretically investigated. The superconducting quantum interference proximity transistor (SQUIPT) [20], is also an interferometer based on the proximity effect, relying on the modification of the density of states (DoS) [61, 65–67] in the proximized normal metal and on the opening of a minigap [61, 65, 70]. In the SNS junction, imposing a phase difference between the superconducting electrodes provides a means to modulate the density of states (DoS) in the normal metal (N) [77, 78, 108]. The operation of the SQUIPT is based on the magnetic field modulation of the DoS of the proximized normal metal N embedded in a superconducting loop. Some attractive features of SQUIPT are ultra-low dissipation, a simple DC read out scheme, and

Figure 2.7. Schematic view of a SQUIPT device, consisting of a superconducting S loop interrupted by an SNS weak link while the NIS junction is placed in the middle of it. The normal metal parameters are length (L), width (w_N) and thickness (d) of the wire. The width of the superconducting probe tunnel coupled to the middle of the weak link is denoted w_S . φ is the phase difference between two superconducting electrodes of the SNS junction and Φ is the magnetic flux applied through the superconducting loop.

flexibility in fabrication, materials and parameters. The schematic implementation of a SQUIPT device is illustrated in Fig. 2.7. It consists of a normal metal (N) embedded in clean contact with a superconducting loop (S), while a superconducting probe is tunnel coupled to the middle of the weak link. The geometry of the island is determined by the weak link width w_N , length L , and thickness d . As shown in Fig. 2.7, Φ_a is the external magnetic flux through the loop. By neglecting the loop inductance, we have $\Phi = (2\pi\varphi)\Phi_0$, where φ is the phase difference across the N wire, and $\Phi_0 = h/2e = 2.067 \times 10^{-15}$ Wb is the flux quantum. The loop geometry enables changing the phase difference across the normal metal/superconductor boundaries through the application of an external magnetic flux.

2.4.1 DC transport in the SQUIPT device

As shown in Fig. 2.5, the DoS in the normal metal, n_N , features an energy gap E_g whose magnitude can be controlled by the imposed order parameter phase difference. The magnitude of the minigap (E_g) in the normal metal is maximized at $\Phi = 0$ ($\varphi = 0$), where it equals the superconducting energy gap Δ_0 for a short junction. By increasing the magnetic flux through the loop, the minigap is gradually reduced and fully suppressed

Figure 2.8. (a) Calculated IV characteristics of the SQUIPT device at two values of magnetic flux, $\Phi = 0$ (blue) and $\Phi = \Phi_0$ (red) respectively. (b) Current vs voltage characteristics calculated for a few values of applied magnetic flux (Φ) between 0 and $0.5\Phi_0$ at $T = 0.1$ K, where Φ_0 is the flux quantum. For the bias current I_{bias} , $V(\Phi)$ is varied over the range $0 < \Phi < 0.5\Phi_0$.

at $\Phi = \Phi_0/2$ ($\varphi = \pi$). In particular, n_N is symmetric with respect to the energy. As a consequence of the variation of the DoS in the normal metal and the minigap, electron transport changes through the tunnel junction. The quasiparticle current, through the probe tunnel junction, biased at voltage V and located in the middle of the weak link ($x = 0$), can be written as [24, 96, 111]

$$I_{qp} = \frac{1}{ew_S R_T} \int_{-w_S/2}^{+w_S/2} \int_{-\infty}^{+\infty} n_N(x, E, \Phi) n_S(E - eV) [f_N(E - eV) - f_N(E)] dx dE, \quad (2.34)$$

where w_S is the width of the superconducting electrode of probe tunnel junction, and R_T is the normal state tunnel resistance of this NIS probe. n_N and n_S are the normalized densities of states in the normal metal and in the superconducting loop, respectively. As shown in Fig. 2.7, we consider the spatial coordinate $x \in [-L/2, L/2]$ along the weak link. In the simplest approximation of a narrow probe, in which n_N can be evaluated at single point ($x = 0$) along the tunnel junction overlap, I_{qp} is written as [24]

$$I_{qp} = \frac{1}{eR_T} \int_{-\infty}^{+\infty} n_N(E, \Phi) n_S(E - eV) [f_N(E - eV) - f_N(E)] dE. \quad (2.35)$$

Figure 2.8 (a) displays the current-voltage (IV) characteristics of the SQUIPT calculated for two extreme values of Φ at low temperature, $T = 0.1$ K. For $\Phi = 0$ the minigap in the N region is maximal, and the resulting current-voltage characteristic corresponds to a superconductor-insulator-superconductor (SIS) junction. In contrast, at $\Phi = 0.5\Phi_0$ the minigap vanishes and hence the IV curve turns to that of on a NIS junction. Figure 2.8 (b) shows a semi logarithmic scale the calculated low-temperature IV characteristics at different values of magnetic flux in the range of interest, $0 < \Phi < \Phi_0$. The SQUIPT behaves as a flux-to-voltage or flux-to-current transformer for which the responses $V(\Phi)$ and $I(\Phi)$ depend on the constant bias voltage I_{bias} and current V_{bias} , respectively. At a particular values of the the bias voltage or and current, the full extent of the modulation of $I(\Phi)$ or $V(\Phi)$ is observed as the flux varies over the interval $0 < \Phi < 0.5 \Phi_0$. The calculated current modulation of the device, $I(\Phi)$, is shown in Figs. 2.9 (a) and (b) for various bias voltages in two regimes, below and well above Δ_0/e , respectively. Furthermore, the calculated flux-to-current transfer function is illustrated in Fig. 2.10 for several values of bias voltages corresponding to those in Fig. 2.9 (a). The figure of merit of the voltage-biased SQUIPT interferometer is its noise equivalent flux (NEF) or flux sensitivity [38], defined as

$$\text{NEF} = S_{\text{IS}}^{1/2} / |\partial I / \partial \Phi|, \quad (2.36)$$

where S_{IS} is the spectral density of the current noise in the device.

2.4.2 I-V curve of a Josephson junction with small critical current

We consider a Josephson junction with a small critical current such that $\hbar I_C / 2e < k_B T$. In this case the I-V curve at low bias voltages and at low temperatures ($eV, k_B T < \Delta$) reads

$$I = \frac{V}{R_{\text{qp}}} + I_J(V). \quad (2.37)$$

Here R_{qp} is the quasiparticle resistance and we have also defined the contribution to the I-V curve coming from the strongly smeared Josephson current

$$I_J(V) = \frac{\pi \hbar I_C^2}{4e} [P(2eV) - P(-2eV)], \quad (2.38)$$

which is expressed in terms of the $P(E)$ -function [112, 113] defined as

$$P(E) = \int \frac{dt}{2\pi \hbar} e^{J(t) + iEt/\hbar}, \quad (2.39)$$

Figure 2.9. Calculated current modulation of the SQUIPT device as a function of the magnetic flux at different values of bias voltages in two regimes: (a) below the gap Δ_0/e and (b) at bias voltage between zero and $eV/\Delta_0 = 2.5$ well above gap Δ_0/e .

Figure 2.10. Calculated flux-to-current $\partial I / \partial \Phi$ transfer function vs. magnetic flux at several values of bias voltages in the sub-gap regime.

Figure 2.11. Calculated IV with Eq. (2.43) at four different values of magnetic flux between $\Phi = 0$ and $\Phi = 0.5 \Phi_0$. The parameters are: $R_{qp} = 30 \text{ M}\Omega$, $R_S = 8.6 \text{ k}\Omega$, $E_C = 22 \text{ }\mu\text{eV}$, $T = 60 \text{ mK}$. The values of the critical current, I_C , for different curves are: 0.65 nA , 0.57 nA , 0.41 nA , 0.1 nA .

where

$$J(t) = -\frac{4e^2}{\pi\hbar} \int_0^\infty d\omega \text{Re}[Z(\omega)] \left(\coth \frac{\hbar\omega}{2k_B T} \frac{1 - \cos \omega t}{\omega} + i \frac{\sin \omega t}{\omega} \right). \quad (2.40)$$

The impedance of the environment is unknown, but one can adopt a Lorentzian model

$$\text{Re}[Z(\omega)] = \frac{R_S}{1 + \omega^2 R_S^2 C^2}, \quad (2.41)$$

where C is the junction capacitance and R_S is the effective impedance of the environment. The critical current of the junction can be estimated via Ambegaokar-Baratoff formula [99]

$$I_C = \frac{\Delta_{\text{mg}}}{R_n} K \left(\sqrt{1 - \frac{\Delta_{\text{mg}}^2}{\Delta^2}} \right), \quad (2.42)$$

where Δ_{mg} is the minigap induced in the normal part by proximity effect, while Δ is the gap in the aluminum. Taking $\Delta_{\text{mg}} = 80 \text{ }\mu\text{eV}$, $\Delta = 200 \text{ }\mu\text{eV}$, and $R_n = 60 \text{ k}\Omega$ one finds $I_C = 3.15 \text{ nA}$ at $\Phi = 0$.

In the limit $k_B T \sim \hbar/2R_S C$ we can approximate the I-V curve as

$$I(V) = \frac{V}{R_{qp}} + \frac{\hbar I_C^2}{2eE_C} \int_0^\infty d\tau \exp \left[-\frac{8\pi k_B T}{gE_C} \left(\tau - \frac{\pi}{g} (1 - e^{-g\tau/\pi}) \right) \right] \sin \left[\frac{4\pi}{g} (1 - e^{-g\tau/\pi}) \right] \sin \left(\frac{2eV}{E_C} \tau \right). \quad (2.43)$$

Figure 2.12. Current noise of a NIS junction. (a) Normalized IV characteristics of a NIS tunnel junction (blue dashed line) together with the current noise S_I (red solid line) vs. bias voltage calculated at $T = 0.05$ K, with the superconducting Al gap $\Delta = 200 \mu\text{eV}$. Here, the dimensionless parameter γ is the ratio between NIS junction asymptotic resistance at high bias voltage and the sub-gap resistance, used in the modeling of a smeared BCS density of states $n_S(E) = |\text{Re}[(E/\Delta + i\gamma)/\sqrt{(E/\Delta + i\gamma)^2 - 1}]|$. The inset shows I and S_I as in (a) but plotted on a semilogarithmic scale. (b) Calculated current noise S_I at different temperatures between $T = 0.01$ K and $T = 1.3$ K.

Figure 2.11 shows the theoretical modeling for IV calculated at four different values of magnetic flux between $\Phi = 0$ and $\Phi = 0.5 \Phi_0$. From the calculation, the magnitudes of critical currents are obtained as 0.65 nA, 0.57 nA, 0.41 nA, 0.1 nA corresponding to different curves.

2.5 Current noise in tunnel junction devices

In this section, we provide the theoretical background which is used later in understanding the noise measurements. We first discuss current fluctuations in a general tunnel junction, focusing on the limiting cases of NIN and NIS junctions. Then we consider the noise spectrum in a SQUIPT device.

2.5.1 Quasiparticle current fluctuations in a general tunnel junction

We consider two metallic electrodes, assuming right (R) and left (L) leads, separated by a thin insulating layer. For the current fluctuation in the

tunnel junction, the starting point is the tunnel Hamiltonian [4]

$$H = \sum_l E_l a_l^\dagger a_l + \sum_r E_r c_r^\dagger c_r + \sum_{l,r} (t_{lr} a_l^\dagger c_r + t_{lr}^* c_r^\dagger a_l), \quad (2.44)$$

where $H_L = \sum_l E_l a_l^\dagger a_l$ and $H_R = \sum_r E_r c_r^\dagger c_r$ are the Hamiltonians of the left and right lead, respectively, and $W = \sum_{l,r} (t_{lr} a_l^\dagger c_r + t_{lr}^* c_r^\dagger a_l)$ is the tunnel coupling. Here, $c_r^\dagger$ and c_r denote the fermionic quasiparticle creation and annihilation operators in the right lead and equivalently $a_l^\dagger$ and a_l in the left lead. E_l and E_r are the electron energies in the left and right leads, respectively. The number operator of electrons in the left and right leads reads $N_l = \sum_l a_l^\dagger a_l$ and $N_r = \sum_r c_r^\dagger c_r$, respectively. The current operator from right to the left lead is then $I = e\dot{N}_l$, which yields with proper commutation relations (for fermions)

$$I = e\dot{N}_l = \frac{ie}{\hbar} [W, N_l] = -\frac{ie}{\hbar} \sum_{l,r} (t_{lr} a_l^\dagger c_r - t_{lr}^* c_r^\dagger a_l). \quad (2.45)$$

The time dependence of the operators gives

$$I(t) = \frac{ie}{\hbar} \sum_{l,r} (t_{lr} a_l^\dagger c_r e^{i(E_l - E_r)t/\hbar} - t_{lr}^* c_r^\dagger a_l e^{-i(E_l - E_r)t/\hbar}). \quad (2.46)$$

We then obtain the correlator

$$\langle I(t)I(0) \rangle = \frac{e^2}{\hbar^2} \sum_{l,r} |t_{lr}|^2 (\langle a_l^\dagger a_l \rangle \langle c_r c_r^\dagger \rangle e^{i(E_l - E_r)t/\hbar} + \langle a_l a_l^\dagger \rangle \langle c_r^\dagger c_r \rangle e^{-i(E_l - E_r)t/\hbar}). \quad (2.47)$$

We transform the sums to integrals via the density of states (DoS), $n_{L,R}(0)$ of the left (right) lead at Fermi level in the normal state, and $n_{L,R}(E)$ the corresponding energy dependent normalized DoS. Furthermore, we assume that there is a bias voltage V across the junction. We also set $|t_{lr}|^2 = |t|^2$ (constant) as usual in the tunneling model. The averages read, for instance, $\langle a_l^\dagger a_l \rangle = 1 - \langle a_l a_l^\dagger \rangle = f_L(E_l - eV)$ and $\langle c_r c_r^\dagger \rangle = 1 - f_R(E_r)$. Then for the current through the tunnel junction, we can get

$$I_{\text{Tunnel}} = \frac{1}{R_T} \int dE_l dE_r n_L(E_l - eV) n_R(E_r) [f_L(E_l - eV) - f_R(E_r)], \quad (2.48)$$

where we have identified the inverse tunnel resistance

$$\frac{1}{R_T} = 2\pi e^2 |t|^2 n_L(0) n_R(0) / \hbar. \quad (2.49)$$

The corresponding noise spectrum reads

$$S_I(\omega) = \int dt e^{i\omega t} \langle I(t)I(0) \rangle = \frac{2\pi e^2}{\hbar} \sum_{l,r} |t_{lr}|^2 (\langle a_l^\dagger a_l \rangle \langle c_r c_r^\dagger \rangle \delta(E_l - E_r + \hbar\omega) + \langle a_l a_l^\dagger \rangle \langle c_r^\dagger c_r \rangle \delta(E_l - E_r - \hbar\omega)). \quad (2.50)$$

From Eq. (2.49), we have

$$S_I(\omega, V) = R_T^{-1} \int dE_L dE_r n_L(E_L - eV) n_R(E_r) \{ f_L(E_L - eV) [1 - f_R(E_r)] \delta(E_L - E_r + \hbar\omega) + [1 - f_L(E_L - eV)] f_R(E_r) \delta(E_L - E_r - \hbar\omega) \} \quad (2.51)$$

This yields then the general expression for the tunnel junction noise as

$$S_I(\omega, V) = R_T^{-1} \int dE n_L(E - eV) n_R(E) \{ f_L(E - eV) [1 - f_R(E + \hbar\omega)] + [1 - f_L(E - eV)] f_R(E - \hbar\omega) \}. \quad (2.52)$$

NIS junction: For an NIS junction, we set $n_L(E) = 1$ and $n_R(E) = n_S(E)$. Then for the current and current noise, we have

$$I_{\text{NIS}} = \frac{1}{R_T} \int_{-\infty}^{+\infty} dE n_S(E) [f_L(E - eV) - f_R(E)], \quad (2.53)$$

and

$$S_I(\omega, V) = R_T^{-1} \int dE n_S(E) \{ f_L(E - eV) [1 - f_R(E + \hbar\omega)] + [1 - f_L(E - eV)] f_R(E - \hbar\omega) \}, \quad (2.54)$$

respectively. For simplicity, let us consider equal temperatures and zero frequency, then

$$S_I(0, V) = R_T^{-1} \int dE n_S(E) \{ f(E - eV) [1 - f(E)] + [1 - f(E - eV)] f(E) \}. \quad (2.55)$$

Figure 2.12 displays the IV characteristics of such a NIS tunnel junction together with the current noise from Eq. (2.55) calculated at $T_e = 0.05$ K, assuming the superconducting Al gap of $\Delta = 200 \mu\text{eV}$.

NIN junction: For an NIN junction we set $n_N = n_S = 1$, resulting in

$$S_I(V) = 2R_T^{-1} \int dE \{ f(E - eV) [1 - f(E)] + [1 - f(E - eV)] f(E) \}. \quad (2.56)$$

Two basic cases are obtained from this expression then: (i) thermal equilibrium noise at $eV \ll k_B T$

$$S_I(0, 0) = R_T^{-1} \int dE \{ f(E - eV) [1 - f(E)] + [1 - f(E - eV)] f(E) \} = \frac{2eV}{R_T} \coth\left(\frac{eV}{2k_B T}\right), \quad (2.57)$$

and (ii) shot noise, $eV \gg k_B T$

$$S_I(0, V) \approx R_T^{-1} \left[\int_0^{eV} dE + \int_{-eV}^0 dE \right] = 2eV/R_T = 2e\langle I \rangle, \quad (2.58)$$

where $\langle I \rangle$ is the average current of the junction at this bias point.

Figure 2.13. (a) Calculated bias voltage dependence of the current noise S_I of a SQUIPT at different values of magnetic flux between 0 and $0.5\Phi_0$, assuming the short junction limit. (b) Calculated current noise as a function of magnetic flux at several values of the bias voltage.

2.5.2 Quasiparticle current fluctuations in a SQUIPT device

In a SQUIPT structure, taking into account the magnetic flux dependence of the density of states $n_N(E, \Phi)$ in the proximized normal metal, the current noise for a device with probe tunnel resistance R_T can be written

$$S_I(\omega, V, \Phi) = R_T^{-1} \int dE n_N(E, \Phi) n_S(E - eV) \{f(E - eV)[1 - f(E + \hbar\omega)] + [1 - f(E - eV)]f(E - \hbar\omega)\}. \quad (2.59)$$

Similar to Eq. (2.35), here we assume a narrow probe electrode and neglect the dependence of $n_N(E, \Phi)$ on the position along the N of the SNS junction [21, 24]. We further assume the low frequency limit $\hbar\omega \ll k_B T_e, eV, \Delta$, yielding

$$S_I(V, \Phi) = R_T^{-1} \int dE n_N(E, \Phi) n_S(E - eV) \{f(E - eV)[1 - f(E)] + [1 - f(E - eV)]f(E)\}. \quad (2.60)$$

Figure 2.13 (a) shows the the bias dependence of the noise at several values of magnetic flux between the two extreme flux values $\Phi = 0$ and $\Phi = 0.5\Phi_0$. In Fig. 2.13 (b) we show the flux dependence of the calculated current noise $S_I(\Phi)$, for several bias voltages V_b in the sub-gap regime.

3. Experimental methods

3.1 Device fabrication

The devices presented in this work have been fabricated by two main methods, using one step and two steps processes, respectively. In the device fabricated in Publication I (Al-SQUIPT), the one step method was employed, based on a single round of conventional electron beam lithography (EBL) and shadow deposition of thin films by a multi-angle evaporation technique [114, 115]. In Publication III (Al-SQUIPT), a Ge-based hard mask process has been applied for the device fabrication in one step. A different process, based on two separate rounds of EBL has been utilized to make the devices in Publications II and IV (Nb-SQUIPT). In this Chapter, these two fabrication methods are first summarized in details. Then we briefly discuss the experimental setups and techniques used in the low-temperature electrical measurements.

3.1.1 One-step fabrication method

The aluminum-based weak links and tunnel junctions with well-controlled interface transparencies are routinely fabricated by shadow evaporation through a suspended resist mask [115] that allows deposition of multiple metal films in a single vacuum cycle. In the following, we refer to this as a so-called one-step fabrication technique. This technique with a Ge-based hard mask is summarized in Fig. 3.1.

Wafer preparation: In this process, the samples were fabricated by electron-beam lithography (EBL) onto an Si wafer with typically 300 nm thick thermal SiO₂ oxide on top. In electron beam lithography, the suspended resist stack with large enough undercut profile consists of copolymer (P(MMA-MAA)-polymethelmetacrylate-methylmethacryllate) and poly-

Figure 3.1. Device fabrication process based on a Ge mask, using electron beam lithography and evaporation techniques. The process starts with the mask making and ends with metallic tunnel junction structure.

methyl methacrylate (PMMA). One first spin-coats a 900 nm thick layer of copolymer which is well baked to avoid cracking the Ge layer deposited in the following step. To achieve this layer thickness using 11% copolymer dissolved in ethyl lactate, a spinning speed of 4000-5000 rpm (depending on the desired thickness) for 1 min is used, followed by baking on a hot-plate at 180°C for ~ 30 min. Subsequently, a 22 nm thick film of Ge is deposited by electron beam evaporation. An evaporation rate of 0.2 Å/s was chosen to reduce the stress of the film coming from baking the top resist. This step is followed by spin coating a 50 nm thick PMMA layer as shown in Fig. 3.1(a). The PMMA resist (typically 2% PMMA dissolved in anisole) is spun for ~ 1 min with the speed of 2500 rpm and then baked on a hot plate at 160°C for ~ 1 min.

Mask preparation: The structures are patterned by electron beam lithography (Vistec, EBPG 5000+ES), for exposing a wide range of features varying wide nanowires from 20 nm up to ground planes and pads of several mm. The acceleration voltage used is 100 kV, which produces the electron scattering profile. The primary electrons are directed toward the wafer, then they are scattered in wafer and create secondary and back-scattering electrons. The top resist (PMMA) is mainly sensitive to the primary electrons while the copolymer is sensitive to the back-scattering

electrons emerging from the substrate. As a result, the polymer chains can break around the area where the beam is scanned. The exposed resist is dissolved by a suitable solvent, typically a solution of methylisobutylketone (MIBK) and isopropanol IPA (MIBK:IPA 1:3). After developing the structure, the chip is rinsed in pure IPA and blow-dried with nitrogen as shown in Fig. 3.1 (c).

Reactive ion etching (RIE) is a high resolution process for etching materials using reactive gas discharges. Different chemicals under specific pressure and interacted by an electromagnetic field can generate plasma, consisting of a wide variety of reactive ions and electrons. A DC bias is induced at the substrate by the free electrons which accelerates the ions towards the sample surface. The reactive ions with high energy react chemically with the chip, attack the wafer and remove the materials from it. The quality of the etching is determined by the etching parameters: DC bias voltage, RF power applied, gas pressure, flow rate and the etching time. Here, the etching process is done in two steps for the two different layers on the wafer (Ge and copolymer). The wet development is followed by Ge etching using carbon tetrafluoride CF_4 for ~ 140 s [Fig. 3.1 (d)] and then the second etching process is utilized for copolymer with O_2 for ~ 30 -60 min [Fig. 3.1 (e)]. Depending on the thickness of the copolymer, the second etching time is varies. The first very anisotropic etching developer removes the exposed Ge part of the mask, and the second step removes only the copolymer, resulting in the undercut profile shown in Fig. 3.1 (e). A large enough undercut enables the fabrication of structures by multi-angle evaporation. Between the CF_4 and O_2 etching steps, another optional wet development with MIBK:IPA 1:3 may be performed to assist in the undercut formation.

Shadow mask evaporation: The surface of the chip can be optionally further cleaned by argon plasma before metalization, especially if only a PMMA soft mask is used. In this step, the metals are deposited by electron-beam evaporation in high vacuum with pressure $\sim 10^{-7} - 10^{-8}$ mbar at different angles by tilting the chip in the vacuum chamber as illustrated in Fig. 3.1 (f) for the first and Fig. 3.1 (g) for the second evaporation. Between the deposition of the two films an *in situ* oxidation can introduce a tunnel barrier with specific parameters, determined by the oxidation pressure and time. In the devices presented in Publications I and III, Al is first deposited and subsequently oxidized for 1 min under pure oxygen at a pressure of 1 millibar to form the tunnel barrier of the normal

Figure 3.2. (a) Schematic view of the design and the actual structure presented in Publication I. The three angle evaporation consists of Al at 31° (black), Cu at -31° (dark grey), and Al at -10° (light grey). The black rectangle marks the area of the SEM image that is depicted in (c). (b) Sketch of the electrical setup of a SQUIPT attached to a voltage bias and current measurement, Φ represents the external magnetic flux through the Al superconducting loop. (c) SEM micrograph of the sample core illustrating a Cu wire embedded between two Al superconductors. The Al probe connected to the middle of the normal metal forms the normal metal insulator-superconductor (NIS) junction. (Adapted from Publication I)

metal-insulator-superconductor (NIS) probe. For the second deposition, approximately 15 nm of copper is evaporated to complete the NIS junction and to form the normal metal part of the SNS proximity weak link. Immediately after this, the superconducting Al loop with 120 nm thickness is deposited to form clean contacts to the copper island, which completes the structure. After the evaporation, the remaining metal-covered resists are dissolved in the lift-off process using acetone as the solvent. A typical final structure with tunnel junctions is shown in Fig. 3.1 (h).

Sample characterization (Scanning electron microscopy): In order to image the device, a scanning electron microscope (SEM) is used. This imaging method allows one to measure the dimensions of the tunnel junction overlap area and other critical structures. Figure 3.2 depicts the device presented in Publication I with the schematic view illustrating the shadow mask sequence and ending with the final device. Figure 3.2 (b) shows a scanning electron microscope (SEM) image of one

Figure 3.3. Two-step fabrication method: Making the Nb structure providing clean electrical contact between Nb and Cu, and depositing a probe tunnel junction.

of the fabricated Al-SQUIPT devices with an enlarged view of the zone around the weak link, with the attached tunnel probe in Fig. 3.2 (c). Here, L and d are the length and width of the normal metal, respectively, and w is the width of the Al-probe.

3.1.2 Two-step fabrication method

Concerning superconductors with higher T_C compared to Al, vanadium is one of the few that are easily suited for shadow evaporation [32, 116]. However, the evaporation of good quality Nb films requires significant attention due to the high melting temperature. Furthermore, some particular challenges such as a bilayer mask with a special thermostable polymer [117, 118], a fully inorganic mask [119, 120], or an evaporator with large target-to-sample distance [121] have to be used in order to gain high quality Nb films. Moreover, avoiding any unwanted normal metal structures, resulting from the multi-angle shadow evaporation, can be desirable in many detector applications such as sensitive hot-electron bolometers and calorimeters [122, 123]. A two-step process, an etching-based technique, can be useful for the difficult-to-evaporate materials such as Nb especially on the grounds of unrestricted geometry when combining with proximity weak links. In addition, in the structures combined with

Figure 3.4. (a) Effect of the etching time on the shape of the Nb-Cu-Nb structure. (b) A representative scanning electron micrograph of a junction with geometry A and B at left and right, respectively.

the tunnel junctions, it is advantageous to start with Nb deposition. In Publication II, a two-step process is used to realize high quality Nb-Cu-Nb weak links. Consequently, we introduce a prototype of an Nb-based SQUIPT which is explained in more details below.

Making Nb structures: The first step in this method is patterning the superconducting Nb structures in which the starting point is an oxidized four inch SiO_2 substrate with 200 nm sputter-deposited Nb. As shown in Fig. 3.3 (a), a positive tone AR-P 6200.13 resist is prepared with a spinning speed of 6000 rpm for 1 min, followed by baking at 150°C for 9 min. The first EBL [Fig. 3.3 (b)] is followed by development in AR600-546 for 3 min and 5 min reflow baking at 150°C to avoid abrupt edge profiles in the Nb structures [Fig. 3.3 (c)]. Reactive ion etching (RIE) with a mixture of SF_6 and Ar is then used to transfer the pattern into the Nb film as shown in Fig. 3.3 (d). The gas flows of 20 sccm and 10 sccm for SF_6 and Ar are employed, respectively, and the 100 W RF power resulted typically in DC self-bias of 300 – 320 V. To remove completely resists from the chip, we used AR300-76 remover at 80°C for 3 min. The shape of the Nb structure is strongly affected by the time as shown by the left and right panel of Fig. 3.4 (a), in which the etching time is changed from 160 s to 180 s to obtain minimum Nb electrode separation down to 700 nm.

Figure 3.5. a) Profilometer traces on top of the Nb and SiO₂ after Ar ion milling. (b) False color scanning micrograph of Nb-SQUIPT, together with the zoomed view of the center part.

Clean electric contact between Nb and Cu: The second step in this process provides the clean electric contact between Nb and Cu. Two different geometries (A,B) were chosen to study how to reduce controllably the separation between the Nb electrodes. This is illustrated in Fig. 3.4 (b). Thickness profile at the tip of the electrode is more gentle in geometry A, which may affect the contact quality and the physical extent of inverse proximity effect. Nb etching is followed by a second round of EBL using a conventional bilayer resist consisting of a 200 nm (600 nm) thick PMMA layer for geometry A (B), on top of a 900 nm layer of copolymer as shown in Fig. 3.3 (e).

The crucial step in this fabrication is to create a transparent contact between Nb and Cu. This is done by exposing the chip to *in situ* Ar ion etching, in the same vacuum cycle immediately prior to the Cu deposition. The surface profile of the chip after Ar ion etching is measured by a profilometer. Based on profilometer traces typically 10 – 22 nm of Nb is removed in this cleaning step for 10 – 20 min as illustrated in Fig. 3.5 (a). To complete the SNS and NIS junctions, before the 60 nm thick Cu evaporation [Fig. 3.5 (g)], the 20 nm thick Al electrode of the NIS junction is deposited and subjected to *in situ* oxidation to create the AlO_x tunnel barrier [Fig. 3.5 (f)]. The typical low-temperature values of the Cu sheet

Figure 3.6. (a) Simple sketch of the DC measurement setup, consisting of bias voltage V applied to SQUIPT device with typical divider. The DC current I is measured by using preamplifier at room temperature. (b) The setup for applying the magnetic flux Φ through the loop. (c) The schematic layout of the simultaneous DC and noise measurement system. The green part of the layout indicates the setup for obtaining the noise signals.

resistance R_Ω is estimated to be in the range $0.4 - 0.5\text{ }\Omega$. The Ar ion flux hits the sample perpendicular to the substrate, i.e. at the same angle as the deposited Cu. A typical Nb- SQUIPT interferometer is shown in Fig. 3.5(b), consisting of a suitable shadow mask and Ar etching of Nb contacts in a fashion identical to the Nb-Cu-Nb weak links.

3.2 DC and noise measurement setup

Figure 3.6 (a) shows the simplified sketch of the DC measurement setup attached to the SQUIPT device. To apply a DC bias voltage V to the SQUIPT tunnel probe electrode, we typically use Agilent 33220 A function/arbitrary waveform generators or Stanford Research Systems (SRS) SIM928 floating isolated voltage sources. To obtain the small bias voltage across the sample in the range of mV and below, room-temperature voltage dividers are used, formed by resistive networks with typical division $1/1000$. The average current I is measured by using room temperature transimpedance preamplifiers (Femto LCA series). Magnetic flux Φ through the sample loop is applied by a superconducting coil in helium

Figure 3.7. Typical sample and the noise measurement setup. (a) False color SEM image of the SQUIPT device prepared with Ge-based process, together with a zoomed-in view of the Cu wire (green) embedded in the superconducting Al loop (brown). The Al tunnel probe (blue) contacts the middle of the proximity SNS junction. (b) Schematic view of the simultaneous DC and noise measurement system in the dilution refrigerator.

bath, wound around the inner vacuum can (IVC) of the cryostat. This coil is biased by a room temperature voltage source through a suitable (typically $1 \text{ k}\Omega - 10 \text{ k}\Omega$) bias resistor as illustrated in Fig. 3.6 (b).

So far, a successful technique for measuring the shot noise [41, 42] of a high-impedance semiconducting samples relies on a cryogenic amplifier based on a high electron mobility transistor (HEMT) and an RLC circuit with high frequency of a few MHz [124, 126–128]. This technique has been widely used in various systems, including quantum point contacts (QPCs) [129, 130] and quantum dots (QDs) [131, 132]. The schematic layout of the noise measurement system is shown in the green part of Fig. 3.6 (c). The DC current is injected to the sample by applying DC bias voltage with typical $1/1000$ divider and resistance R_I . In order to avoid the capacities loss, the LC circuit is formed by inserting inductor L with the cable capacitance C_{coax} . The injector side of the sample is grounded for the high frequency ($\sim \text{MHz}$) through the C_c capacitance, whereas the detector side of the sample is attached to the LC resonator. The AC voltage fluctuations caused in the sample is applied to the gate of the HEMT-based cryogenic amplifier and then is amplified by that at 4.2 K . Consequently,

Figure 3.8. Sample stage for the simultaneous DC and noise measurements. (a) The bottom side and (b) top cap of the sample holder. (c) Top view of the sample holder printed circuit board (PCB). The inductors of the LC resonant circuit, capacitors, and resistors are soldered onto the sample holder.

the noise signal is amplified by the secondary amplifier at room temperature. The resultant signal is captured by the low pass (LP) filtering and digitizer, and it transformed to spectral density data.

In our work, the simultaneous DC and noise measurements are performed in plastic $^3\text{He}/^4\text{He}$ dilution refrigerator reaching a base temperature of ~ 50 mK [133, 134] as shown in Fig. 3.7 (b). As the main elements, our home made double-HEMT cryogenic amplifier [128] and the inductors of the LC resonant circuit are placed in the liquid helium bath and on the sample holder at base temperature, respectively. In the following, these two main parts of the setup are introduced in more details. Simultaneously with the DC measurement of the average current I , current noise through the SQUIPT is probed by the HEMT amplifier via the capacitor C_c . The cryogenic HEMT amplifier is placed in the liquid helium bath at 4.2 K and a DC supply voltage V_{SD} is applied to the amplifier by a room temperature voltage source.

3.2.1 Sample stage and cryostat wiring

As illustrated in Fig. 3.8, the sample holder has 12 DC and 6 RF pads which connect to the DC and RF measurement lines in the cryostat. The

Figure 3.9. (a) Schematic diagram of the cryogenic amplifier together with the circuit of passive components including surface mount metal-film resistors and laminated ceramic capacitors. (b) Inside view of the amplifier with the pair of Avago ATF-34143 HEMT transistors.

electrical connection between the sample holder and cryostat is done by a multi-pin connector and SMA connectors, Fig. 3.8 (a). As sketched in the diagram of the measurement setup at $T = 60 \, \text{mK}$ [Fig. 3.7 (b)], all the elements such as inductors, capacitors and resistors are soldered on the stage. On the sample holder PCB, the DC lines utilized for the noise measurements contain a surface mount series resistor of value $R_1 = 330 \, \Omega$, combined with a capacitance $C_1 = 22 \, \text{nF}$ to form a low-pass RC filter as shown in the top panel of Fig. 3.8 (c).

The DC and RF lines continue with Thermocoax coaxial cable up to the 1 K plate in the cryostat. From this part, first resistive manganin twisted pairs are used to connect the main liquid helium bath at 4.2 K and then the twisted pairs continue to a room temperature connector box with BNC connectors. Samples can be diced to small pieces to be glued to the center of the sample holder. The fabricated SQUIPT device [Fig. 3.7 (a)] is electrically connected by ultrasonic bonding with Al wire (Delvotek 5332, deep-access wedge-wedge bonder). This makes the electrical contact between the bonding pads on the chip and the copper contacts on the sample stage PCB board. We formed the home made LC resonator on the sample holder as shown in Fig. 3.8 (c). It is working at frequencies of the

order of the resonance at $f_0 = 1/(2\pi\sqrt{L'C_{\text{coax}}}) \approx 4.2$ MHz, formed by the inductance $L' = (L^{-1} + L_1^{-1})^{-1} \approx 16.5$ μH (due to the coils $L = L_1$) and the capacitance $C_{\text{coax}} \approx 92$ pF. This capacitance is mainly due to the coupling between the sample and the HEMT amplifier via ~ 50 cm coaxial cable. In the setup, the capacitors $C_c = C_1$ can be considered as electrical shorts at frequencies $f \sim f_0$. In Fig. 3.8 (c), the phenomenological resistor $R \sim 50$ k Ω denotes the parasitic losses in the circuit, mainly the inductors L and L_1 . It accounts for the losses in the circuit when the differential resistance of the sample $R_S(V, \Phi) = dV/dI \sim R$. The noise signal is amplified first by the HEMT amplifier at 4.2 K. It is further enhanced by another stage (SRS SR445A) at room temperature, followed by low pass (LP) filtering by a commercial 5 MHz low pass filter to avoid aliasing. The amplified voltage signal is finally captured by a 16-bit digitizer running continuously at 50 MSamples/s, converted into spectral density of voltage noise by windowing and Fast Fourier Transform of blocks with typically 2^{15} samples [124]. A desired number of spectra are averaged to improve the signal-to-noise ratio.

3.2.2 HEMT-based cryogenic amplifier: Configuration and characterization

Our cryogenic amplifier was made on a Printed-Circuit Board (PCB) of copper material. It consists of the passive, surface mount metal film resistors and laminated ceramic capacitors. The only active element of the amplifier is a high-electron-mobility-transistor (HEMT: ATF-34143). The schematic circuit of the complete amplifier is shown in Fig. 3.9 (a) together with the PCB layout (Cadsoft EAGLE). The PCB board is placed in a brass box with the outer size 34 mm $\times$ 34 mm [see Figure 3.9 (b)]. In order to reduce the $1/f$ noise of the amplifier, we prepared a variant of the amplifier with two HEMTs in parallel [128]. One part of the amplifier development was to build a so called dip-stick that was immersed into a bath of cryogenic liquid. With the help of it, we could directly characterize the amplifiers in liquid nitrogen ($T = 77$ K) and helium ($T = 4.2$ K) temperatures. Figure 3.10 (a) illustrates the bottom and top part of the dip-stick attached to the amplifier by three coaxial lines. The process was started by measuring the room and low temperature characteristics of the HEMT amplifier. The typical source-drain current I_{SD} [Fig. 3.10 (b)] and gain [Fig. 3.10 (d)] as a function of the supply voltage V_{SD} is shown at two different temperatures $T = 4.2$ K (blue solid line) and $T = 300$ K (red solid

Figure 3.10. (a) Dip-stick configuration for the test measurement of the cryogenic amplifier. (b) Source-drain current I_{SD} and (c), gain as a function of the supply voltage V_{SD} at two different temperatures $T = 4.2$ K (blue solid line) and $T = 300$ K (red solid line) at 3 MHz. By decreasing the temperature, I_{SD} decreases while the gain increases. (d) Frequency dependence of the gain at $V_{SD} = 2$ V at two different temperatures.

line) at 3 MHz. When lowering the temperature, I_{SD} decreases while the gain increases. Furthermore, the gain is almost constant in the saturation limit of the V_{SD} , $V_{SD} > 1.2$ V. For the noise measurement setup, we prepared the LC resonance circuit with the resonant frequency close to 4 MHz. Figure 3.10 (c) shows the frequency dependence of the gain at $V_{SD} = 2$ V, remaining flat (~ 3.4 at $T = 4.2$ K) in the frequency range of interest. For the equivalent input voltage noise of the amplifier, we found $V_N \approx 0.9$ nV/Hz $^{1/2}$ at temperature $T = 4.2$ K, as shown in Fig. 3.11. The inset of Fig. 3.11 illustrates the schematic setup for the measurement of the equivalent input voltage noise of the amplifier.

3.2.3 HEMT-based cryogenic amplifier for cross correlation technique

In order to establish a higher resolution noise measurement setup with a cross-correlation technique [124, 125] in a $^3\text{He}/^4\text{He}$ dilution refrigerator, two amplifiers can be used to avoid the background noise level. In this section, we only discuss the characterization of the cryogenic amplifiers using the setup. Figure 3.12 (a) shows the circuit schematic of each

Figure 3.11. The equivalent input voltage noise of the amplifier at $T = 4.2$ K. Inset: The simple measurement setup for the equivalent input voltage noise of the amplifier at $T = 4.2$ K

amplifier and splitter together with the passive components. The cryoamplifier placed at 4.2 K contains the only active element, a high-electron-mobility-transistor (HEMT: ATF-34143). The splitter board at room temperature separates two output signals, in the low and high frequency regimes. The outer sizes of the amplifier and splitter are $2\text{ cm} \times 2.5\text{ cm}$ and $4\text{ cm} \times 4\text{ cm}$, respectively, as shown in Fig. 3.12 (b). With the help of a dipstick, we can characterize the two separate amplifiers at $T > 4.2$ K. Figure 3.13 (a) illustrates the simple setup for each amplifier at room temperature. The splitter is connected to amplifier with Channel 1 via RF connector and Channel 2 is biased with DC supply voltage V_{SD} , whereas the output signal can be characterized at Channel 3. Figure 3.13 (b) shows the typical source-drain current I_{SD} as a function of the supply voltage V_{SD} for two separate amplifiers in liquid nitrogen ($T = 77$ K) and room ($T = 300$ K) temperatures. Similar to the cryoamplifier introduced in Sec. 3.2.2, we characterized the gain of the two amplifiers as a function of frequency [Fig. 3.13 (c)] and supply voltage V_{SD} [Fig. 3.13 (d)]. From Figs. 3.13 (b), (c), and (d), we can realize that the behavior of each amplifier is almost equal.

3.2.4 Model for evaluating shot noise in the SQUIPT device

In order to calculate the flux noise of the SQUIPT, we can consider the noise measurement setup depicted in Fig. 3.14 in more details writing

Figure 3.12. (a) Schematic diagram of the amplifier and splitter together with the passive components for cross correlation technique. (b) Photograph of a cryoamplifier and splitter boards.

down. The spectral density of the total voltage fluctuations at the amplifier input yields

$$S_V(\omega) = |Z_{\text{eff}}(\omega)|^2 S_I + S_{V_A}. \quad (3.1)$$

Here $Z_{\text{eff}}(\omega)$ is the parallel impedance of the sample and the RLC circuit

$$1/Z_{\text{eff}}(\omega) = 1/R_S + 1/Z_R(\omega). \quad (3.2)$$

S_I and S_{V_A} are the total current noise and the amplifier input noise, respectively. The total current noise S_I can be separated into different components

$$S_I = S_{I_S} + S_{I_R} + S_{I_A}, \quad (3.3)$$

where S_{I_S} , S_{I_R} , and S_{I_A} are the total sample noise, the RLC current noise, and the current spectral density of the amplifier, respectively. The total sample noise can be written as $\delta I_S = \delta I_{\text{shot}} + (\partial I / \partial \Phi) \delta \Phi$, corresponding to $S_{I_S} = S_{I_{\text{shot}}} + (\partial I / \partial \Phi)^2 S_\Phi$. Here $\partial I / \partial \Phi$ indicates the responsivity of the SQUIPT and S_Φ is the flux noise.

Figure 3.13. (a) Simple test setup for cross correlation amplifiers at room temperature. (b) I_{SD} of two amplifiers as a function of the supply voltage V_{SD} at two different temperatures $T = 77$ K and $T = 300$ K. (c) Frequency dependence of the gain at $V_{SD} = 4$ V at two different temperatures for two amplifiers. (d) V_{SD} dependence of the gain at two different temperatures for two separate amplifiers at 3 MHz frequency.

Figure 3.14. Circuit model for evaluating the shot noise in SQUIPT.

4. SQUIPT devices and their DC and noise characterization

In this Chapter, the summary of the results in Publications I-IV is presented. First, we introduce the characterization of an Al-SQUIPT with non-hysteretic properties and enhanced responsivity reported in Publication I. In Secs. 4.3-4.4, measurements of SNS weak links based on Nb and Cu from Publications II and IV presented, and consequently an SNS junction in an Nb-SQUIPT device is investigated. In the last part, Sec. 4.5, shot noise measurements of SQUIPT device from Publication III are described in more detail.

4.1 DC transport of an Al-SQUIPT

The devices presented in Publication I are SQUIPTs based on an Al superconducting loop. The magnitude of the supercurrent peak geometrical I_S of the NIS junction depends on its resistance R and the geometry of the SNS weak link [?]. Table I shows different Al-SQUIPTs measured in Publication I together with the values of the parameters including the tunnel junction width w . In the SNS weak link, the significant parameter is the length of the wire L which indicates the strength of the proximity effect. For strong proximity effect in the normal metal, L should be of the order of the superconducting coherence length ξ_0 , $L = \gamma\xi_0$ where $\xi_0 = (\hbar D/\Delta_1)^{1/2}$ and Δ_1 is the superconducting gap of the aluminum leads. For all the samples in Table I, we set $D = 0.01 \text{ m}^2\text{s}^{-1}$ and $\Delta_1 = 220 \text{ } \mu\text{eV}$ and the magnitude of γ is then obtained to be $\sim 1.37, 1.45, 1.5$ and 1.62 for samples A to F. The current-voltage (IV) characteristics of sample A is plotted in Fig. 4.1 (a) at $T = 50 \text{ mK}$ with maximum supercurrent I_S of approximately 50 pA at $\Phi = 0$ [see Fig. 4.1 (c)].

As we discussed in Sec. 2.2.3, in the diffusive regime of a SNS junction [135, 136], the magnitude of the minigap Δ_2 in the normal metal is

Table 4.1. Parameters of different samples measured at $T_{\text{bath}} = 50$ mK. Here d , L , w , are the width and length of the copper island, and the width of the probe respectively. Al superconducting loop and Cu are 100 nm and $a = 20$ nm thick respectively. The resistance of the NIS junction, R , was measured at low temperature and the maximum supercurrent of the probe is given by I_S . The maximum current and voltage responsivity as a function of magnetic flux are shown as $|\partial I/\partial \Phi|_{\text{max}}$ and $|\partial V/\partial \Phi|_{\text{max}}$. (Adapted from Publication I)

sample	L	d	w	R	I_S	$ \frac{\partial I}{\partial \Phi} _{\text{max}}$	$ \frac{\partial V}{\partial \Phi} _{\text{max}}$
	(nm)	(nm)	(nm)	(k Ω)	(pA)	($\frac{nA}{\Phi_0}$)	($\frac{mV}{\Phi_0}$)
A	237	45	70	104	48	23	1.7
B	250	50	80	243	7	8.4	1.5
C	250	55	80	178	5	6.5	0.47
D	250	65	80	145	12	7	0.45
E	275	66	107	137	10	4	0.55
F	280	50	90	188	9	2	1

minimized for $\varphi = \pi$ ($\Phi = \Phi_0/2$) and maximized for $\varphi = 0$ ($\Phi = 0$) [67, 70]. The IV of sample A is measured at different values of the magnetic flux between 0 and $0.5 \Phi_0$ [see Fig. 4.1 (b)]. For the estimation of the energy gap and minigap at different temperatures, we consider the measured differential conductance as a function of the bias voltage. As shown in Fig. 4.1 (d), arrows indicate the positions of $\Delta_1 - \Delta_2$ (brown), $\Delta_1 + \Delta_2$ (blue) and $2\Delta_1$ (green). The minigap increases the conductance at the bias voltage ~ 0.16 mV, when the edge of the minigap and the superconducting Al gap are aligned at $V = (\Delta_1 - \Delta_2)/e$. The superconducting gap edge faces the minigap edge at $V = (\Delta_1 + \Delta_2)/e \sim 0.37$ mV, where the largest peak appears in the differential conductance vs. voltage (G-V) curve [21]. The magnitude of the minigap in the devices listed in Table I reaches a value of approximately 0.6–0.7 of the full superconducting gap Δ_1 .

4.2 Non-hysteretic SQUIPTs with enhanced responsivity

Similar to SQUIDS, hysteresis in the SQUIPT devices appears when the self-inductance of the superconducting loop L_s well exceeds the Josephson inductance of the weak link $L_J = \Phi_0/(2\pi I_C)$, where I_C is the critical current of the SNS junction [16]. A detailed view of the sample presented in Publication I is displayed in Fig. 3.2, where the geometry of the island is determined by weak-link width d , length L , and thickness a . The usability of the SQUIPTs is limited by the hysteresis. In the SQUIPT device

Figure 4.1. (a) I-V characteristics of sample A measured at $T_{\text{bath}} = 50$ mK. (b) An enlarged view of current-voltage curve at several values of magnetic flux Φ between 0 and $0.5\Phi_0$. (c) The magnitude of supercurrent appearing around zero bias voltage in device A is $I_S = 48$ pA. (d) Measured differential conductance vs voltage bias at three different bath temperatures for sample E. Arrows indicate the positions of $\Delta_1 - \Delta_2$ (brown), $\Delta_1 + \Delta_2$ (blue) and $2\Delta_1$ (green). (Adapted from Publication I)

presented by Meschke *et al.* [21], hysteresis appears towards low temperatures $T < 300$ mK where the self-inductance of the superconducting ring is about $L_S = 8$ pH and the SNS junction Josephson inductance of the order of 1 pH. For structure in the first version of the SQUIPT [21], the dimensions were $d = 200$ nm, $L = 300$ nm and $a = 20$ nm respectively. Figure 4.2 (a) illustrates an example of the hysteresis at low temperature, corresponding to a typical SQUIPT at three values of bias voltages. The suppression of the hysteresis is possible by increasing the temperature or by reducing the self inductance of the loop compared to Josephson inductance [83, 135]. In Fig. 4.2 (b), for the same sample we obtain the suppression of hysteresis by increasing the temperature from 55 mK to 399 mK at $V = 0.3$ mV. In an ideal SNS junction, the critical current I_C is proportional to the normal state resistance of the weak link $R_N = \rho l / A$. Here, $\rho = 1/(\nu_F e^2 D)$ is the island resistivity, A is the island cross section, ν_F is the density of states at the Fermi level in N and D denotes again the diffusion coefficient of the normal metal [135, 137, 138]. In order to suppress hysteresis by design of the SQUIPT device, we shrink the cross

Figure 4.2. (a) Current through one of the measured SQUIPT devices fabricated by Ge-process at three different bias voltages as a function of magnetic flux through the ring at the base temperature $T = 55$ mK, resulting in hysteretic behavior. (b) Current modulation of the same sample measured at different values of bath temperatures at $V = 0.3$ mV. Suppression of hysteresis is possible by increasing the temperature.

section A of the weak-link leading to increased normal-state resistance and Josephson inductance.

Measured $I(\Phi)$ and $V(\Phi)$ modulations are illustrated at different values of bias voltage and current at the base temperature $T_{\text{bath}} = 50$ mK in Figs. 4.3 (a) and (c), together with detailed views in Fig. 4.3 (b) and (d), respectively. From this figure, it is obvious that the hysteresis is absent at low temperatures in contrast to previous work [21]. In the devices presented in Publication I, the ratio between the magnitudes of Josephson inductance L_J and the geometric self inductance of the loop L_s is $L_J/L_s \sim 1.5$, due to an increase of L_J by a factor of five in comparison to earlier work.

To characterize the flux sensitivity of the SQUIPT device, we evaluated its flux-to-current transfer function $\partial I / \partial \Phi$. In Figure 4.4, the blue curve corresponds to the transfer function at $V = 0.251$ mV for which we achieve the maximum device responsivity $|\partial I / \partial \Phi|_{\text{max}} \cong 23$ nA/ Φ_0 at $T_{\text{bath}} = 50$ mK. This value is about one order of magnitude higher than the responsivity reported in previous work [21]. In a subsequent optimized device [23], high flux-to-current (exceeding $|\partial I / \partial \Phi|_{\text{max}} \cong 100$ nA/ Φ_0) transfer func-

Figure 4.3. (a) Current modulation $I(\Phi)$ of the NIS junction of sample A at different values of bias voltage applied to it at the base temperature $T_{\text{bath}} = 50$ mK. (b) A zoomed view for several values of V in the range $-340\mu\text{V} < V < 298\mu\text{V}$. (c) Measured flux-to-voltage $V(\Phi)$ curves at different magnitudes of current through the junction of sample B and (d) a zoomed image at some values of current in the range 0.18 - 1.8 nA. The curves are not symmetric around $\Phi = 0$ due to a constant offset flux. Both $I(\Phi)$ and $V(\Phi)$ modulations are non-hysteretic. This happens as the Josephson inductance L_J exceeds the self-inductance of the superconducting ring. (Adapted from Publication I)

tion value has been obtained at sub-Kelvin temperatures. The typical room temperature low noise current pre-amplifiers have a noise level of $5 \text{ fA}/\sqrt{\text{Hz}}$. Based on the responsivity above, the flux resolution is then $\sim 0.2 \times 10^{-6} \Phi_0/\sqrt{\text{Hz}}$. The flux-to-voltage transfer function yields a maximum responsivity $|\partial V/\partial \Phi|_{\text{max}} \cong 1.7 \text{ mV}/\Phi_0$ at base temperature. In this Al-SQUIPT device, we increased the Josephson inductance by factor of five as compared to the earlier work. Further enhancement of the magnetic flux responsivity is feasible by increasing the superconducting loop area [139], using advanced lithography in further reducing cross section and thickness of the weak link, and low temperature deposition [140,141].

4.3 Low-temperature characterization of Nb-Cu-Nb weak links

In this part, we present the investigation of niobium-based SNS weak links, formed by the two-step fabrication process that relies on Ar ion-etching. We consider both triangular (geometry A) and rectangular (ge-

Figure 4.4. Current responsivity $\partial I / \partial \Phi$ characteristics as a function of the external magnetic flux through the superconducting loop of sample A at $T_{\text{bath}} = 50 \text{ mK}$ at four different values of applied bias voltage close to the optimum working point. (Adapted from Publication I)

ometry B) shaped S electrode terminations as shown in Fig. 3.4 (b). Better minimal S electrode separation L_1 is achieved in geometry B needed to gain optimal sensitivity in sensor applications. In Table 4.1, the parameters of the measured SNS junctions with two different geometries are listed showing the minimum Nb electrode separation L_1 in the SNS junction, the full length of Cu wire L_2 , and the minimum width of the copper island w . The main panel in Fig. 4.5 indicates the current-voltage characteristics of sample B2 measured in a four-probe configuration at $T = 80 \text{ mK}$. At zero bias voltage, it features a supercurrent branch with a sudden switch to a resistive branch with constant resistance, at the so-called switching current I_{sw} . The IV characteristic in SNS junctions is hysteretic originating mainly from self-heating in the finite-voltage state- [118, 142]. Upon decreasing the current, the voltage jumps back to zero only at a retrapping current I_r significantly smaller than I_{sw} .

In sample B2 we observe hysteretic behavior at $T < 0.65 \text{ K}$ from which we deduce a switching current I_{sw} and retrapping current I_r of about $42 \mu\text{A}$ and $9 \mu\text{A}$, respectively. The samples in Table 4.2 are in the long junction limit where $L \gg \xi_0$ ($\xi_0 = (\hbar D / \Delta)^{1/2}$) or equivalently $\Delta \gg E_{\text{Th}}$, $\Delta \approx 1.2 \text{ meV}$ is the superconducting Nb energy gap. The top inset of Fig. 4.5 shows IV characteristics of sample A3 measured at several bath temperatures. The temperature dependence of the switching and retrapping currents, I_{sw} and I_r , is displayed in the bottom inset of Fig. 4.5 for

Table 4.2. Parameters of the measured samples. L_1 is the minimum Nb electrode separation in the SNS junction, L_2 the full length of the Cu wire, and w the minimum width of the copper island (see Fig. 3.4). E_{Th} (and hence L) and α are obtained from a comparison of the temperature-dependent switching current measurements to the theoretical model. R_{N} is the measured low-temperature normal state resistance of the wire, and $I_{\text{sw}}^{\text{max}}$ denotes the maximum observed switching current. (Adapted from Publication II)

sample	L_1	L_2	w	L	α	R_{N}	E_{Th}	$I_{\text{sw}}^{\text{max}}$	$\frac{eR_{\text{N}}I_{\text{sw}}^{\text{max}}}{E_{\text{Th}}}$
	(μm)	(μm)	(μm)	(μm)		(Ω)	(μeV)	(μA)	
A1	0.29	2	0.55	1.09	0.52	0.83	5.5	33	5
A2	0.36	2	0.55	1.18	0.67	0.83	4.7	33	5.9
A3	0.48	2.15	0.54	1.18	0.48	0.57	4.8	30	3.6
A4	0.49	2.15	0.54	1.21	0.5	0.59	4.5	30	4
A5	0.84	2.5	0.85	1.80	0.49	0.5	2	10	2.5
B1	0.32	2	0.15	1.09	0.61	0.98	7.7	42	5.3
B2	0.38	2	0.15	1.09	0.57	0.89	7.3	41	5
B3	0.40	2.12	0.15	0.96	0.5	2.31	7	14	4.6
B4	0.39	2.17	0.14	0.91	0.47	2.42	8	14	4.3
B5	0.40	2.12	0.15	0.85	0.53	2.51	9	18	5

sample B2 on a linear scale. Furthermore, Fig. 4.6 summarizes the measured switching currents I_{sw} vs the bath temperature T for four of the samples from Table 4.2 together with the calculated model according to Eq. (2.16). In the model, we used the Thouless energy E_{Th} and a dimensionless reduction parameter α as adjustable parameters. The reduction parameter α can contribute to suppression of the observed switching current I_{sw} below the true critical current. The values of these parameters are listed in Table 4.2 for various measured samples. We can observe the value of α for each sample with assuming partially transparent interfaces [83, 143]. The total resistance of the normal metal (Cu) is $R_{\text{N}} \approx R_{\text{L}} + 2R_{\text{I}}$, where R_{L} is the N wire resistance and R_{I} is the resistance of the contact. Based on the measured R_{N} for each sample and estimated value of Cu wire resistance, we find $\alpha \approx 0.5$ for our shadow-evaporated Al-Cu weak links.

From Fig. 4.6 it is clear that there is no qualitative difference in the behavior of the switching currents between two geometries A and B, while the hysteresis properties vary to some extent. If we consider the normal metal wire with length L' , width w , and thickness t , its resistance is given by $R_{\text{N}} = \rho L' / (tw)$. Based on the measured resistance of the normal

Figure 4.5. Main panel: current–voltage characteristics of sample B2 measured at $T = 80$ mK. Arrows indicate the switching I_{sw} and retrapping current I_r . The top inset shows measured IV characteristics of sample A3 at different values of temperatures. The measured switching current I_{sw} and the theoretical model (solid line) as a function of temperature for sample B2 are shown in bottom inset. The star symbols show the rerapping current I_r , which is almost temperature-independent in the hysteretic regime.

metals, the Cu sheet resistance ρ/t at low temperatures is in the range of the $0.4 - 0.5 \, \Omega$, and consequently, the estimated diffusion constant is $D \approx 0.01 \, \text{m}^2\text{s}^{-1}$. As shown in Table 4.2, the effective SNS junction length L is derived from the fitting parameter E_{Th} and diffusion constant of the normal metal D . The magnitudes of L in Table 4.2 correspond to the expected values, $L_1 < L < L_2$, due to the inverse proximity effect where the Cu island overlaps the Nb electrodes.

4.4 Nb-SQUIPT

The performance of the SQUIPT device is optimized in the short junction limit where the proximity effect in the weak link is maximized [67,83]. As a consequence of the strong proximity effect, the magnitude of the minigap is enhanced [24]. By replacing the superconductor with higher T_C than that of Al, such as Nb, for the superconducting loop in the structure, the magnetic field modulation of the minigap can be increased and higher sensitivity of the SQUIPTs can be achieved. Here we introduce

Figure 4.6. Temperature dependence of the measured I_{sw} of samples A1, A2, B1, and B2 together with the basic theoretical model based on Usadel equations [Eq. (2.16)] indicated by solid lines. The vertical arrows identify the samples and indicate the temperature below which the hysteresis in the IV curve appears for each of them.

the first realization of a SQUIPT based on a Nb superconducting loop (Nb-SQUIPT). It consists of a Nb-Cu-Nb link embedded in a Nb superconducting loop, placed into an external magnetic field. The magnetic field dependence of the DoS can be probed by a weakly coupled Al tunnel electrode in contact with the Cu wire.

To realize such a device, we use the two-step fabrication technique presented in Sec. 3.1.2. We first prepare a suitable shadow mask and Ar etch the Nb contacts, then without breaking the vacuum immediately before the Cu deposition, a 20 nm thick Al electrode is evaporated and subjected to *in situ* oxidation to create the NIS tunnel junction. The SEM image of the initial Nb-SQUIPT is shown in Fig. 3.5 (b). This device was fabricated based on an A-type SNS junction. The IV characteristics of the device measured at two different magnetic fields is displayed in Fig. 4.7 (a). Furthermore, the inset demonstrates the extent of the flux modulation of the IV curve close to zero bias voltage at two different temperatures $T = 80$ mK (brown) and 190 mK (green), respectively. Almost full magnetic flux modulation is observed due to increased SNS weak link inductance compared to the Nb loop inductance [23] which is obvious at $T = 190$ mK. We further achieved a maximum supercurrent I_s of 3.6 nA, due to the larger

Figure 4.7. (a) IV characteristics of a Nb-SQUIPT based on an A-type SNS junction measured at $T = 80$ mK, for two values of magnetic flux through the loop: $\Phi = 0$ (red) and $\Phi = 0.5\Phi_0$ (blue). The inset shows the flux modulation of the IV curve around zero bias voltage at $T = 190$ mK (green solid) and $T = 80$ mK (brown dashed). (b) Current modulation $I(\Phi)$ at $T = 80$ mK for several values of the bias voltage between 250 and 320 μ V. (c) Temperature dependence of the maximum flux-to-current transfer function $|\partial I/\partial \Phi|_{\max}$ on the supercurrent branch.

junction size and relatively low tunnel resistance $R_T \approx 3$ k Ω . To estimate the flux sensitivity of the device, we first measured current modulation $I(\Phi)$ at several values of the bias voltage V and then characterized the flux-to-current transfer function $\partial I/\partial \Phi$ by numerical differentiation as shown in Fig. 4.7 (b). We consider the maximum sensitivity $|\partial I/\partial \Phi|_{\max}$ in two regimes, the quasiparticle current (above gap) and supercurrent (subgap) branch. At higher bias voltages corresponding to the onset of the quasiparticle current, we find $|\partial I/\partial \Phi|_{\max} \approx 40$ nA/ Φ_0 . The overall maximum $|\partial I/\partial \Phi|_{\max} \approx 50$ nA/ Φ_0 , at the base temperature, is interestingly reached in the supercurrent branch at $V \approx 32$ μ V. In Fig. 4.7 (c), the maximum sensitivity as a function of temperature is shown in the low-bias regime, decreasing monotonously as T increases.

For comparison, the measured flux-to-current transfer function is shown in Fig. 4.8 (a) at two different bias voltages, in the supercurrent regime at $V = 32$ μ V and in the quasiparticle branch at $V = 307$ μ V. The magnitudes of maximum current responsivity are $|\partial I/\partial \Phi|_{\max} \approx 35$ nA/ Φ_0 and $|\partial I/\partial \Phi|_{\max} \approx 50$ nA/ Φ_0 at $V = 307$ μ V and $V = 32$ μ V, respectively. We,

Figure 4.8. (a) Flux-to-current transfer function measured at two different bias voltages, in the supercurrent regime at $V = 32 \mu\text{V}$ and in the quasiparticle branch at $V = 307 \mu\text{V}$. Maximum flux-to-current transfer function $|\partial I / \partial \Phi|_{\text{max}}$ with respect to the bias voltage, measured on the (b) quasiparticle and (c) supercurrent branch at $T = 78 \text{ mK}$.

furthermore, investigate the bias voltage dependence of the maximum sensitivity at the base temperature $T = 78 \text{ mK}$. The maximum flux-to-current transfer function $|\partial I / \partial \Phi|_{\text{max}}$ at two different regimes of bias voltages, quasiparticle and supercurrent, are illustrated in Fig. 4.8 (b) and Fig. 4.8 (c), respectively. Figure 4.8 (c) indicates that the maximum responsivity is obtained at $|V| = 30 \mu\text{V}$, corresponding to $|\partial I / \partial \Phi|_{\text{max}} \approx 55 \text{ nA} / \Phi_0$. The magnitude of power dissipation in this device is then achieved to be below 10^{-13} W , improving the power dissipation by up to two orders of magnitude compared to our conventional device based on an Al-Cu-Al SNS junction and an Al tunnel probe (Al-SQUIPT) [22].

4.5 Noise performance of a superconducting magnetic flux sensor based on a proximity Josephson junction

In this section, we move on to present measurements of current noise of a SQUIPT device fabricated with the Ge process discussed in Sec. 2.1.1. In this device, we used Al as the superconducting loop material. The simultaneous DC and noise measurement setup are installed in a $^3\text{He} / ^4\text{He}$

Figure 4.9. DC transport measurements. (a) IV characteristics at $T = 60$ mK, measured at two values of magnetic flux $\Phi = 0$ (red solid line) and $\Phi = 0.5\Phi_0$ (blue solid line), respectively. (b) Flux modulation of the IV curve (solid lines) around zero bias voltage at four values of magnetic flux between $\Phi = 0$ and $\Phi = 0.5\Phi_0$, together with the theoretical model at each flux (dotted lines). (c) Current modulation $I(\Phi)$ at various fixed bias voltages $V \sim \Delta/e$, and (d), in the sub-gap region close to zero bias voltage. Here the bias voltages are indicated in microvolts. (e) Measured flux-to-voltage curves $V(\Phi)$ at several values of the bias current through the device. (f) Current responsivity $\partial I/\partial \Phi$ and (g), voltage responsivity $\partial V/\partial \Phi$ as functions of the magnetic flux at the optimum bias points, $V = 0.249$ mV and $I = 4.2$ nA, respectively. (Adapted from Publication III)

dilution refrigerator with base temperature close to 60 mK as discussed in more detail in Sec. 2.2. A home made cryogenic HEMT-based amplifier was placed in the 4.2 K bath in the fridge, and a proper sample stage was made as explained in Sec. 2.2.1. DC transport characterization of the device is presented in Fig. 4.9, starting with the IV characteristics [Fig. 4.9 (a)] and proceeding with flux responsivity by measuring current $I(\Phi)$ [Fig. 4.9 (c)] and voltage $V(\Phi)$ [Fig. 4.9 (e)] modulations. Figure 4.9 (b) shows the measured (solid lines) flux modulation of the IV curve around zero bias voltage at four values of magnetic flux between $\Phi = 0$ and $\Phi = 0.5\Phi_0$, together with the theoretical model at each flux (dotted lines) presented at Sec. 2.4.2.

4.5.1 Shot noise measurement of an AI-SQUIPT

In order to estimate current noise of the device, we can start from Eq. (3.1). Introducing the effective resistance $R_{\text{eff}} = (R_S^{-1} + R^{-1})^{-1}$ with using $\omega_0 = 1/\sqrt{LC}$ as the resonance frequency, we can derive frequency dependence of power spectral density of the voltage noise $S_V(f)$ from Eq. (3.1) as

$$S_V(f) = S_{V_A} + \frac{R_{\text{eff}}^2 S_I}{1 + (f^2 - f_0^2)^2 / (f \Delta f)^2}. \quad (4.1)$$

The $S_V(f)$ is centered around resonance frequency f_0 . Figure 4.10 (a) shows examples of the measured $S_V(f)$ together with the theoretical model obtained from Eq. (4.1). The measured blue dots in Fig. 4.10 (a) were achieved at base temperature $T = 60$ mK and fixed magnetic flux close to $\Phi = 0$ through the loop at the few indicated values of bias voltage V across the device. For tunnel junction devices such as the SQUIPT, we expect the spectral density of the current shot noise to follow $S_{I_S} = 2e|I|$ [42], we therefore investigate the dependence of S_V and hence S_I on V , Φ , and T . We make theoretical fits to using the model S_{V_A} , f_0 , the peak height $P_0 = R_{\text{eff}}^2 S_I$, and the peak width $\Delta f = 2\pi L' f_0^2 / R_{\text{eff}}$ as adjustable parameters. Here, Δf gives directly the full width at half maximum (FWHM) of the peak in S_V in the limit $f_0 \gg \Delta f$. Furthermore, the background level $S_{V_A} \approx 3 \times 10^{-18}$ V²/Hz due to the amplifier voltage noise, and the resonance frequency $f_0 \approx 4.18$ MHz can be kept fixed, whereas the peak height and width depend systematically on V , Φ , and T .

As a result of the fitting procedure, we can extract the value of the peak height P_0 which is shown in Fig. 4.10 (b) as a function of bias voltage at several values of magnetic flux, corresponding to a few equally spaced flux values between $\Phi = 0$ and $\Phi = 0.5 \Phi_0$. The vertical arrows indicate the bias voltages at $\Phi = 0$ for the spectra displayed in panel (a). In addition, Fig. 4.10 (c) demonstrates the extracted R_{eff} . The bias dependence of R_{eff} resembles the differential resistance of the SQUIPT device, due to the fact that R_{eff} is a parallel combination of R_S and the constant parasitic resistance R . Based on Eq. (4.1), we can extract the total current noise S_I as a function of the bias voltage as shown in Fig. 4.10 (d). In this figure, the two curves correspond to measurements at bath temperature $T = 4.2$ K with the SQUIPT fully in the normal state (top), and at the base temperature $T = 60$ mK (bottom) at a constant magnetic flux close to $\Phi = 0$. To calibrate the total gain of the setup, we can use the measured data at 4.2 K, to achieve $S_I = 2e|I|$ at large voltages. In the normal state

Figure 4.10. Noise measurements. (a) Power spectral density of the measured voltage noise at $\Phi = 0$ for the indicated values of the bias voltage V (blue dots), plotted on a semilogarithmic scale. The solid lines are fits to Eq. (4.1). (b) Bias dependence of the peak height $R_{\text{eff}}^2 S_I$ and (c) R_{eff} , extracted from fits to Eq. (4.1), for a few equally spaced values of magnetic flux between $\Phi = 0$ (red solid line) and $\Phi = 0.5\Phi_0$ (blue solid line). (d) Total current noise S_I vs. the DC bias, measured at $T = 4.2$ K with the junction in the normal state, and at $T = 60$ mK in the superconducting state, together with the theoretical predictions (see text for details). The dashed horizontal line indicates the background noise level, independent of V and Φ .

at $T = 4.2$ K, the noise spectrum is given by [42]

$$S_{\text{IS}} = (2eV/R_T) \coth(eV/2k_B T), \quad (4.2)$$

shown by the pink solid line in Fig. 4.10 (d). The red solid line in Fig. 4.10 (d) is obtained from the theoretical calculation presented in Sec. 2.5.2. The noise is dominated by the shot noise of the SQUIPT tunnel junction. With increasing V the noise increases as $S_{\text{IS}} \approx 2e|I(\Phi)|$. In Fig. 4.10 (d), at base temperature the expected $S_{\text{IR}} \approx 0.7 \times 10^{-29} \text{ A}^2/\text{Hz}$ is much smaller than the background term $S_{\text{IA}} \approx 2.3 \times 10^{-27} \text{ A}^2/\text{Hz} \approx (48 \text{ fA})^2/\text{Hz}$.

4.5.2 Flux noise characterization

Figure 4.11 (a) illustrates the bias dependence of the current noise measured at two extreme flux values $\Phi = 0$ and $\Phi = 0.5\Phi_0$, reflecting the shot noise in the average current $I(V, \Phi)$. To compare with the IV characteristics of the SQUIPT device, the $S_{\text{IS}} \approx 2e|I|$ noise spectra are included

Figure 4.11. (a) Voltage dependence of the current noise at two extreme magnetic flux values $\Phi = 0$ and $\Phi = 0.5\Phi_0$. (b) IV characteristics of the SQUIPT (red and blue dots) compared to the measured current noise S_{IS} (red and blue solid lines) vs. bias voltage, at the two extreme flux values, $\Phi = 0$ and $\Phi = 0.5\Phi_0$.

in the Fig. 4.11 (b). In the S_{IS} curve at $\Phi = 0$, we attribute the apparent excess noise around zero bias (at the gap edge) to an uncertainty in the fitting to extract the exact value of R_{eff} when the peak is at its narrowest (lowest height). It originates from the residual interfering peaks in the background noise of S_V , present for example at $f \approx 4.02 \text{ MHz}$ in Fig. 4.10 (a).

The flux sensitivity can be estimated from simultaneous measurements of the current $I(\Phi)$ and noise $S_{IS}(\Phi)$ modulations in same setup at several values of the bias voltage V . Figures 4.12 (a) and (b) show examples of $I(\Phi)$ and $S_{IS}(\Phi)$, respectively, at some values of bias voltage around the onset of the quasiparticle current. There is reasonable qualitative agreement between $I(\Phi)$ and $S_{IS}(\Phi)$, $S_{IS} \propto |I(\Phi)|$. The transfer function is obtained by numerical differentiation of $I(\Phi)$ which is shown in Figure 4.12 (c) for different values of bias voltages. Combining S_{IS} and $\partial I/\partial \Phi$, we obtain the NEF plots as displayed in Fig. 4.12 (d). The bias voltage $V = 0.24 \text{ mV}$ results in the lowest NEF (green), whereas $V = 0.29 \text{ mV}$ gives the highest $|\partial I/\partial \Phi|$. The minimum NEF $\approx 4 \mu\Phi_0/\text{Hz}^{1/2}$ (green solid / dotted line) is achieved at $V = 0.24 \text{ mV}$, $\Phi \approx 0.4 \Phi_0$. The obtained flux sensitivity for the device in Publication III is not impres-

Figure 4.12. Characterization of flux noise performance of the voltage-biased SQUIPT device. (a) DC current and (b), current noise as a function of magnetic flux at several values of bias voltage, measured simultaneously in the same setup at 60 mK. (c) Responsivity $\partial I/\partial \Phi$ at several biases. (d) Flux sensitivity at the 2nd and 3rd lowest bias voltages $V = 0.24$ mV and 0.29 mV in panels (a)–(c). The dotted lines use S_{IS} obtained by direct fitting of the measured S_V spectra, whereas the solid lines assume full shot noise $S_{IS} = 2e|I(\Phi)|$ with $I(\Phi)$ from the DC measurement. Each color in panels (a)–(d) corresponds to a specific bias voltage. (Adapted from Publication III)

sive compared to nanoSQUIDs. Significant improvements to our initial demonstration of the noise performance in the MHz-range are expected to result from optimizing the geometry and materials of the SNS junction and the consequently enhanced responsivity [22–24]. This is possible, for instance, by replacing the interferometer loop with a larger-gap superconductor [31] and shortening the normal metal weak link [22,23]. We expect the transfer function to be enhanced by a few orders up to $\mu\text{A}/\Phi_0$ [23] under voltage bias, and flux noise in the $n\Phi_0/\sqrt{\text{Hz}}$ range [20, 24]. For the Nb-SQUIPT [31], interestingly the optimum working point is in the supercurrent branch. For these devices, we expect flux noise in the range of $50 n\Phi_0/\sqrt{\text{Hz}}$ to be achievable. Furthermore, higher-bandwidth readout of a SQUIPT, fast magnetometry, is possible by embedding the device in a lumped element or coplanar waveguide resonator with high resonance frequency, similar to fast NIS tunnel junction thermometry [144–146]. This is similar to work on quantum-limited dispersive SQUID magne-

tometry with conventional Al tunnel junctions [147] or nanobridge weak links [148], with flux noise down close to $20 \text{ n}\Phi_0/\text{Hz}^{1/2}$ and bandwidth of the order of 10 MHz.

5. Conclusions and outlook

In this thesis, we have investigated at low temperatures various SQUIPT devices based on different fabrication methods and two different superconducting loop materials. In earlier work, the first realization of a SQUIPT- [20] featured a normal metal weak link of length $L \approx 1.5 \mu\text{m}$, resulting in an SNS junction in the long diffusive limit with only modest minigap modulation. A subsequent SQUIPT device [21] increased the responsivity by reducing the copper wire length in the SNS contact into the intermediate junction regime. However, the usability of this device suffered from hysteresis appearing at low temperatures. The hysteresis is emerging from the self-induced magnetic field caused by the high critical current in the weak link.

In this work, in Publication I we have demonstrated experimentally the suppression of magnetic hysteresis in SQUIPTs by tuning the Josephson inductance of the weak link. This device features reduced tunnel junction area and smaller weak link cross section. As a consequence, we achieved almost an order of magnitude improved magnetic flux responsivity of the device in contrast to earlier work [21]. This achievement allows to design ultra-sensitive SQUIPTs in the field of nanoscale magnetometers at low temperatures. The devices presented in Publication I were implemented with a fabrication protocol based on conventional aluminum-copper (Al-Cu) technology. Later, further improvements to achieve a highly sensitive SQUIPT have been reported [23]. With this improved version of an Al-SQUIPT, magnetic flux resolution as low as $\text{NEF} \approx 0.5 \mu\Phi_0/\text{Hz}^{1/2}$ was obtained at sub-kelvin temperatures [23]. The performance of these devices can be improved by optimizing the SQUIPT parameters, such as using a superconducting loop material with a larger energy gap [31, 32]. The flux sensitivity was predicted to reach below $\text{NEF} \approx 40 \text{ n}\Phi_0/\text{Hz}^{1/2}$ in an optimized device using a short copper wire ($L \approx 150 \text{ nm}$) and Nb in the

superconducting loop.

In the experiments presented in Publications II and IV, we characterized Nb-Cu-Nb weak links through low-temperature switching current measurements and tunnel spectroscopy. We investigated the fabrication of SNS devices in two separate lithography and deposition steps, combined with strong argon ion cleaning before the normal metal deposition. This technique enables more flexibility in the choice of materials and pattern design. The Nb-Cu-Nb junctions studied in these publications are in the long junction limit. Furthermore, an initial SQUIPT device based on Cu-Nb technology (Nb-SQUIPT) is introduced, achieving the maximum sensitivity on the supercurrent branch of the order of that of an optimized Al-SQUIPT [23].

The most significant improvements to the performance of the prototype Nb-SQUIPT are expected to be the result from shortening the effective length L of the SNS weak link closer to the short junction limit. We estimate that with somewhat thinner Nb electrodes, appropriate values for L_1 (the distance between superconducting electrodes) are achievable down to slightly below 100 nm. For optimized parameters, transfer functions up to a few mV/Φ_0 under current bias and intrinsic flux noise in the range of $\sim n\Phi_0/\sqrt{\text{Hz}}$ are predicted [20, 24]. The flux sensitivity of these Nb-SQUIPTs can be further improved with narrower Al probe junctions, leading to less spatial averaging of the N DoS. In the fabrication technique presented in Publications II and IV, we observe a notable broadening of the geometry after *in situ* etching. This needs to be addressed in future work.

Since the first realization of the SQUIPT, their direct noise measurement and then flux noise characterization has remained challenging. Publication III presents the DC characterization of an Al-SQUIPT sensor based on Al and Cu, together with a detailed characterization of the current noise and flux sensitivity of the device at 4 MHz frequency. The flux sensitivity is the figure of merit of magnetic flux sensors such as SQUIDS. It is not straightforward to quantify for SQUIPTs due to their large impedance. The noise measurement setup developed as part of this thesis is suited for shot noise measurements of not only SQUIPTs but also other nonlinear devices.

The performance of the setup can be improved by cross-correlation techniques using two cryogenic amplifiers, and by employing a lower-noise room temperature amplifier. The flux noise of SQUIPT devices can be sig-

nificantly improved by optimizing the dimensions of the SNS weak link. Similar to fast thermometry [144–146] by NIS tunnel junctions and fast magnetometry with SQUIDs [147, 148], fast SQUIPT magnetometry can be realized by embedding the device in a lumped element or coplanar waveguide resonator. The readout frequency can be increased into the range of several hundred MHz or several GHz in the future.

Bibliography

- [1] C. F. Gauss, *The intensity of the Earth's magnetic force reduced to absolute measurement*, <http://21sci-tech.com/translations/gaussMagnetic.pdf>, (1832), Retrieved 2017-10-09.
- [2] E. Hall, American Journal of Mathematics 2 (3): 287 (1879), JSTOR 2369245.
- [3] H. K. Onnes, Leiden Comm. 120b, 122b, 124c, (1911).
- [4] M. Tinkham, *Introduction to Superconductivity*, 2nd ed. (McGraw-Hill, 1996).
- [5] J. Clarke and A. I. Braginski, *The SQUID handbook: Applications of SQUIDs and SQUID systems*, Volume II (Wiley-VCH, Weinheim, 2006).
- [6] J. Clarke and A. I. Braginski, *The SQUID handbook: Applications of SQUIDs and SQUID systems*, Volume I (Wiley-VCH, Wienheim, 2004).
- [7] R. K. Fagaly, Rev. Sci. Instrum. **77**, 101101 (2006) .
- [8] P. Seidel (Ed.), *Applied Superconductivity: Handbook on Devices and Applications* (Wiley, Weinheim, 2015).
- [9] J. R. Kirtley and J. P. Wikswo, Annu. Rev. Mater. Sci. **29**, 117 (1999).
- [10] S. M. Frolov, M. J. A. Stoutimore, T. A. Crane, D. J. Van Harlingen, V. A. Oboznov, V. V. Ryazanov, A. Ruosi, C. Granata, and M. Russo, Nat. Phys. **4**, 1315 (2008).
- [11] D. N. Paulson, R. M. Toussaint, and R. L Fagaly, IEEE Trans. Magn. **23**, 1315 (1987).

- [12] B. M. Mackert, *Magnetoneurography* **115**, 2667 (2004).
- [13] J. Clarke and F. K. Wilhelm, *Nature* **453**, 1031 (2008).
- [14] P. Solinas, S. Gasparinetti, D. Golubev, and F. Giazotto, *Sci. Rep.* **5**, 12260 (2015).
- [15] J. P. Pekola, J. J. Vartiainen, M. Möttönen, O-P. Saira, M. Meschke, and D. V. Averin, *Nat. Phys.* **4**, 120 (2008).
- [16] J. Q. You and F. Nori, *Nature* **474**, 589 (2011).
- [17] J. Govenius, R. E. Lake, K. Y. Tan, and M. Möttönen, *Phys. Rev. Lett.* **117**, 030802 (2016).
- [18] F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, *Rev. Mod. Phys.* **78**, 217 (2006).
- [19] F. Giazotto and M. J. Martínez-Pérez, *Nature* **492**, 401 (2012).
- [20] F. Giazotto, J. T. Peltonen, M. Meschke, and J. P. Pekola, *Nat. Phys.* **6**, 254 (2010).
- [21] M. Meschke, J. T. Peltonen, J. P. Pekola, and F. Giazotto, *Phys. Rev. B* **84**, 214514 (2011).
- [22] R. N. Jabdaraghi, M. Meschke, and J. P. Pekola, *Appl. Phys. Lett.* **104**, 082601 (2014).
- [23] A. Ronzani, C. Altimiras, and F. Giazotto, *Phys. Rev. Appl.* **2**, 024005 (2014).
- [24] F. Giazotto and F. Taddei, *Phys. Rev. B* **84**, 214502 (2011).
- [25] Y.-J. Doh, J. A. van Dam, A. L. Roest, E. P. A. M. Bakkers, L. P. Kouwenhoven, S. D. Franceschi, *Science* **309**, 272 (2005).
- [26] F. Giazotto, P. Pingue, F. Beltram, M. Lazzarino, D. Orani, S. Rubini, and A. Franciosi, *Phys. Rev. Lett.* **87**, 216808 (2001).
- [27] P. Jarillo-Herrero, J. A. van Dam, and L. P. Kouwenhoven, *Nature* **439**, 953 (2006).
- [28] A. F. Morpurgo, J. Kong, C. M. Marcus, and H. Dai, *Science* **286**, 263 (1999).
- [29] J.-P. Cleuziou, W. Wernsdorfer, V. Bouchiat, T. Ondarcuhu, and M. Monthieux, *Nat. Nanotech.* **1**, 53 (2006).

- [30] H. B. Heersche, P. Jarillo-Herrero, J. B. Oostinga, L. M. K. Vander-sypen, and A. F. Morpurgo, *Nature* **446**, 56 (2007).
- [31] R. N. Jabdaraghi, J. T. Peltonen, O-P. Saira, and J. P. Pekola, *Appl. Phys. Lett.* **108**, 042604 (2016).
- [32] A. Ronzani, M. Baillergeau, C. Altimiras, and F. Giazotto, *Appl. Phys. Lett.* **103**, 052603 (2013).
- [33] N. Ligato, G. Marchegiani, P. Virtanen, E. Strambini, and F. Giazotto, *Sci. Rep.* **7**, 8810 (2017).
- [34] S. D'Ambrosio, M. Meissner, C. Blanc, A. Ronzani, and F. Giazotto, *Appl. Phys. Lett.* **107**, 113110 (2015).
- [35] E. Enrico, E. Strambini, and F. Giazotto, *arXiv:1609.08525* (2016).
- [36] R. N. Jabdaraghi, D. S. Golubev, J. P. Pekola, and J. T. Peltonen, *Sci. Rep.* **7**, 8011 (2017).
- [37] A. Ronzani, S. D'Ambrosio, P. Virtanen, F. Giazotto, and C. Altimiras, *arXiv:1611.06263* (2016).
- [38] K. K. Likharev, *Dynamics of Josephson Junctions and Circuits* (Gordon Breach, New York 1986).
- [39] M. Schmelz, V. Zakosarenko, T. Schönau, S. Anders, S. Linzen, R. Stolz, and H-G. Meyer, *Supercond. Sci. Technol.* **30**, 014001 (2017).
- [40] D. Vasyukov, Y. Anahory, L. Embon, D. Halbertal, J. Cuppens, L. Neeman, A. Finkler, Y. Segev, Y. Myasoedov, M. L. Rappaport, M. E. Huber, and E. A. Zeldov, *Nat. Nanotech.* **8**, 639 (2013).
- [41] M. Reznikov, M. Heiblum, H. Shtrikman, and D. Mahalu, *Phys. Rev. Lett.* **75**, 3340 (1995).
- [42] Ya. M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 15 (2000).
- [43] H. K. Onnes, *Proceedings of the Koninklijke Nederlandse Akademie Van Wetenschappen*, 100 (3-4): 153, 1911.
- [44] H. K. Onnes, *Proceedings of the Koninklijke Nederlandse Akademie Van Wetenschappen*, 100(3-4): 158, 1911.
- [45] H. K. Onnes, *Proceedings of the Koninklijke Nederlandse Akademie Van Wetenschappen*, 100(3-4): 156, 1911.

- [46] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
- [47] L. N. Cooper, Phys. Rev. **104**, 1189 (1956).
- [48] T. Faivre, *Low dissipation Thermometry using superconducting tunnel junctions*, Ph.D. thesis, Aalto University (2015).
- [49] I. Giaever, Phys. Rev. Lett. **5**, 147 (1960).
- [50] M. Nahum, T. M. Eiles, and J. M. Martinis, Appl. Phys. Lett. **65**, 3123 (1994).
- [51] M. M. Leivo, J. P. Pekola, and D. V. Averin, Appl. Phys. Lett. **68**, 1996 (1996).
- [52] A. M. Clark, A. Williams, S. T. Ruggiero, M. L. van den Berg, and J. N. Ullom, Appl. Phys. Lett. **84**, 625 (2004).
- [53] J. M. Rowell and D. C. Tsui, Phys. Rev. B **14**, 2456 (1976).
- [54] J. T. Muhonen, M. Meschke, and J. P. Pekola, Rep. Prog. Phys. **75**, 046501 (2012).
- [55] R. C. Dynes, V. Narayanamurti, and J. P. Garno, Phys. Rev. Lett. **41**, 1509 (1978).
- [56] J. P. Pekola, T. T. Heikkilä, A. M. Savin, J. T. Flyktman, F. Giazotto, and F. W. J. Hekking, Phys. Rev. Lett. **92**, 056804 (2004).
- [57] J. P. Pekola, V. F. Maisi, S. Kafanov, N. Chekurov, A. Kemppinen, Yu. A. Pashkin, O.-P. Saira, M. Möttönen, and J. S. Tsai, Phys. Rev. Lett. **105**, 026803 (2010).
- [58] L. Solymar, *Superconductive tunneling and applications* (Wiley, New York 1972).
- [59] P. G. de Gennes, Rev. Mod. Phys. **36**, 225 (1964).
- [60] P. G. de Gennes, *Superconductivity of metals and alloys* (W. A. Benjamin, 1966).
- [61] W. Belzig, F. K. Wilhelm, C. Bruder, G. Schön, and A. D. Zaikin, Superlatt. Microstruct. **25**, 1251 (1999).
- [62] C. J. Lambert and R. Raimondi, J. of Phys. Cond. Mat. **10**, 901 (1998).
- [63] B. Pannetier and H. Courtois, J. Low Temp. Phys. **118**, 599 (2000).

- [64] A. I. Buzdin, *Rev. Mod. Phys.* **77**, 935 (2005).
- [65] W. Belzig, C. Bruder, and G. Schön, *Phys. Rev. B* **54**, 9443 (1996).
- [66] S. Guéron, H. Pothier, N. O. Birge, D. Esteve, and M. H. Devoret, *Phys. Rev. Lett.* **77**, 3025 (1996).
- [67] H. le Sueur, P. Joyez, H. Pothier, C. Urbina, and D. Esteve, *Phys. Rev. Lett.* **100**, 197002 (2008).
- [68] N. Moussy, H. Courtois, and B. Pannetier, *Europhys. Lett.* **55**, 861 (2001).
- [69] M. Vinet, C. Chapelier, and F. Lefloch, *Phys. Rev. B* **63**, 165420 (2001).
- [70] F. Zhou, P. Charlat, B. Spivak, and B. Pannetier, *J. Low Temp. Phys.* **110**, 841 (1998).
- [71] D. A. Ivanov, R. von Roten, and G. Blatter, *Phys. Rev. B* **66**, 052507 (2002).
- [72] A. F. Andreev, *Zh. Eksp. Teor. Fiz.* **46**, 1823 (1964) [*Sov. Phys. JETP* **19**, 1228].
- [73] P. de Gennes and D. Saint-James, *Phys. Lett.* **4**, 151 (1963).
- [74] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, *Phys. Rev. B* **25**, 4515 (1982).
- [75] P. A. M. Benistant, H. van Kempen and P. Wyder, *Phys. Rev. Lett.* **51**, 817 (1983).
- [76] M. A. Sillanpää, T. T. Heikkilä, R. K. Lindell, and P. J. Hakonen, *Europhys. Lett.* **56**, 590 (2001).
- [77] V. T. Petrashov, V. N. Antonov, P. Delsing, and T. Claeson, *JETP Lett.* **60**, 606 (1994).
- [78] V. T. Petrashov, V. N. Antonov, P. Delsing, and T. Claeson, *Phys. Rev. Lett.* **74**, 5268 (1995).
- [79] C. P. Foley and H. Hilgenkamp, *Supercond. Sci. Technol.* **22**, 1 (2009).
- [80] L. Hao, J. C. Macfarlane, S. K. H. Lam, C. P. Foley, P. Josephs-Franks, and J. C. Gallop, *IEEE Trans. Appl. Supercond.* **15**, 514 (2005).

- [81] L. Hao, J. C. Macfarlane, J. C. Gallop, D. Cox, P. Joseph-Franks, D. Hutson, and J. Chen, IEEE Trans. Instrum. Meas. **56**, 392 (2007).
- [82] K. D. Usadel, Phys. Rev. Lett. **25**, 507 (1970).
- [83] T. T. Heikkilä, J. Särkkilä, and F. K. Wilhelm, Phys. Rev. B **66**, 184513 (2002).
- [84] J. Rammer and H. Smith, Rev. Mod. Phys. **58**, 323 (1986).
- [85] P. Virtanen and T. Heikkilä, Appl. Phys. A **89**, 625 (2007).
- [86] J. T. Peltonen, *Fluctuations, relaxation and proximity effect in superconducting circuits*, Ph.D. thesis, Aalto University (2011).
- [87] P. Dubos, H. Courtois, B. Pannetier, F. K. Wilhelm, A. D. Zaikin, and G. Schön, Phys. Rev. B **63**, 64502 (2001).
- [88] H. Courtois, Ph. Gandit, D. Mailly, and B. Pannetier, Phys. Rev. Lett. **76**, 130 (1996).
- [89] F. Wilhelm, A. Zaikin, and G. Schön, J. Low Temp. Phys. **106**, 305 (1997).
- [90] R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercereau, Phys. Rev. Lett. **12**, 159 (1964).
- [91] J. D. Jackel and R. A. Buhrman, J. Low Temp. Phys. **19**, 201 (1975).
- [92] G. J. Ehnholm, J. Low Temp. Phys. **29**, 1 (1977).
- [93] T. RyhDnen, H. Seppä, R. Ilmoniemä, and J. Knuutila, J. Low Temp. Phys. **76**, 287 (1989).
- [94] K. K. Likharev, Rev. Mod. Phys. **15**, 101 (1979).
- [95] T. V. Duzer and C. W. Turner, *Principles of Superconductive Devices and Circuits*, 2nd ed. (Prentice-Hall, 1999).
- [96] A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect*, 1st ed. (Wiley, 1982).
- [97] B. D. Josephson, Phys. Lett. **1**, 251 (1962).
- [98] P. W. Anderson and J. M. Rowell, Phys. Rev. Lett. **10**, 230 (1963).
- [99] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963).
- [100] C. Granata and A. Vettoliere, Physics Reports, 614 (2016).

- [101] A. M. Chang, H. D. Hallen, L. Harriott, H. F. Hess, H. L. Kao, J. Kwo, R. E. Miller, R. Wolfe, and J. van der Ziel, *Appl. Phys. Lett.* **61**, 1974 (1992).
- [102] A. Finkler, Y. Segev, Y. Myasoedov, M. L. Rappaport, L. Ne'eman, D. Vasyukov, E. Zeldov, M. E. Huber, J. Martin, and A. Yacoby, *Nano Lett.* **10**, 1046 (2010).
- [103] L. Hao, J. C. Macfarlane, J. C. Gallop, D. Cox, J. Beyer, D. Drung, and T. Schurig, *Appl. Phys. Lett.* **92**, 192507 (2008).
- [104] A. G. P. Troeman, H. Derking, B. Borger, J. Pleikies, D. Veldhuis, and H. Hilgenkamp, *Nano Lett.* **7**, 2152 (2007).
- [105] M. J. Martínez-Pérez and D. Koelle, *Phys. Sci. Rev.* **2**, 20178001 (2017).
- [106] M. Schmelz, Y. Matsui, R. Stolz, V. Zakosarenko, T. Schönau, S. Anders, S. Linzen, H. Itozaki, and H.-G. Meyer, *Supercond. Sci. Technol.* **28**, 015004 (2015).
- [107] M. J. Martínez-Pérez, B. Müller, D. Schwebius, D. Korinski, R. Kleiner, J. Sése, and D. Koelle, *Supercond. Sci. Technol.* **30**, 024003 (2017).
- [108] W. Belzig, R. Shaikhaidarov, V. V. Petrashov, and Yu. V. Nazarov, *Phys. Rev. B* **66**, 220505 (2002).
- [109] A. V. Galaktionov, A. D. Zaikin, and L. S. Kuzmin, *Phys. Rev. B* **85**, 224523 (2012).
- [110] A. V. Galaktionov, and A. D. Zaikin, *Phys. Rev. B* **88**, 104513 (2013).
- [111] E. L. Wolf, *Principles of electron tunneling spectroscopy*, (Oxford University Press, New York, 1985).
- [112] G.-L. Ingold and Y. V. Nazarov, in *Single Charge Tunneling*, Vol. 294 of NATO ASI Series B, eds. H. Grabert and M. Devoret (Plenum Press, 1992), pp. 21–107.
- [113] H. Grabert and H. Horner, *Z. Phys. B* **85**, 317 (1991).
- [114] C. Enss, *Cryogenic Particle Detection*, (Springer, Germany, 2005).
- [115] G. J. Dolan, *Appl. Phys. Lett.* **31**, 337 (1977).

- [116] C. Pascual Garcia and F. Giazotto, *Appl. Phys. Lett.* **94**, 132508 (2009).
- [117] P. Dubos, P. Charlat, Th. Crozes, P. Paniez, and B. Pannetier, *J. Vac. Sci. Technol. B* **18**, 122 (2000).
- [118] F. Chiodi, M. Aprili, and B. Reulet, *Phys. Rev. Lett.* **103**, 177002 (2009).
- [119] T. Hoss, C. Strunk, and C. Schönenberger, *Microel. Eng.* **46**, 149 (1999).
- [120] S. Samaddar, D. van Zanten, A. Fay, B. Sacépé, H. Courtois, and C. B. Winkelmann, *Nanotechnology* **24**, 375304 (2013).
- [121] N. Kim, K. Hansen, J. Toppari, T. Suppala, and J. Pekola, *J. Vac. Sci. Technol. B* **20**, 1071 (2002).
- [122] J. Govenius, R. E. Lake, K. Y. Tan, V. Pietilä, J. K. Julin, I. J. Maasilta, P. Virtanen, and M. Möttönen, *Phys. Rev. B* **90**, 064505 (2014).
- [123] S. Gasparinetti, K. L. Viisanen, O.-P. Saira, T. Faivre, M. Arzeo, M. Meschke, and J. P. Pekola, *Phys. Rev. Appl.* **3**, 014007 (2015).
- [124] L. DiCarlo, Y. Zhang, D. T. McClure, C. M. Marcus, L. N. Pfeiffer, and K. W. West, *Rev. Sci. Instrum.* **77**, 073906 (2006).
- [125] A. Kumar, L. Saminadayar, D. C. Glatthli, Y. Jin, and B. Etienne, *Phys. Rev. Lett.* **76**, 2778 (1996).
- [126] M. Hashisaka, S. Nakamura, Y. Yamauchi, S. Kasai, K. Kobayashi, and T. Ono, *Phys. Stat. Sol. (c)* **5**, 128 (2008).
- [127] M. Hashisaka, Y. Yamauchi, S. Nakamura, S. Kasai, K. Kobayashi, and T. Ono, *J. Phys.: Conf. Ser.* **109**, 012013 (2008).
- [128] T. Arakawa, Y. Nishihara, M. Maeda, S. Norimoto, and K. Kobayashi, *Appl. Phys. Lett.* **103**, 172104 (2013).
- [129] Y. Nishihara, S. Nakamura, K. Kobayashi, T. Ono, M. Kohda, and J. Nitta, *Appl. Phys. Lett.* **100**, 203111 (2012).
- [130] S. Nakamura, M. Hashisaka, Y. Yamauchi, S. Kasai, T. Ono, and K. Kobayashi, *Phys. Rev. B* **79**, 201308(R) (2009).

- [131] Y. Yamauchi, K. Sekiguchi, K. Chida, T. Arakawa, S. Nakamura, K. Kobayashi, T. Ono, T. Fujii, and R. Sakano, *Phys. Rev. Lett.* **106**, 176601 (2011).
- [132] Y. Okazaki, S. Sasaki, and K. Muraki, *Phys. Rev. B* **87**, 041302(R) (2013).
- [133] F. Pobell, *Matter and Methods at Low Temperatures*, 3rd ed. (Springer, 2007).
- [134] J. Pekola and J. Kauppinen, *Cryogenics* **34**, 843 (1994).
- [135] P. Dubos, H. Courtois, B. Pannetier, F. K. Wilhelm, A. D. Zaikin, and G. Schön, *Phys. Rev. B* **63**, 064502 (2001).
- [136] J. J. A. Baselmans, A. F. Morpurgo, B. J. van Wees, and T. M. Klapwijk, *Nature*. **397**, 43 (1999).
- [137] F. Giazotto, T. T. Heikkilä, G. Pepe, P. Heliöstö, A. Luukanen, and J. P. Pekola, *Appl. Phys. Lett.* **92**, 162507 (2008).
- [138] Y. Blum, A. Tsukernik, M. Karpovski, and A. Palevski, *Phys. Rev. B* **70**, 214501 (2004).
- [139] J. Gallop, *Supercond. Sci. Technol.* **16**, 1575 (2003).
- [140] M. Zgirski, K. P. Riikonen, V. Touboltsev, and K. Arutyunov, *Nano Lett.* **5**, 1029 (2005).
- [141] X. S. Wu, P. W. Adams, Y. Yang and R. L. McCarley, *Phys. Rev. Lett.* **96**, 127002 (2006).
- [142] H. Courtois, M. Meschke, J. T. Peltonen, and J. P. Pekola, *Phys. Rev. Lett.* **101**, 067002 (2008).
- [143] J. C. Hammer, J. C. Cuevas, F. S. Bergeret, and W. Belzig, *Phys. Rev. B* **76**, 064514 (2007).
- [144] D. S. Schmidt, C. S. Yung, and A. N. Cleland, *Appl. Phys. Lett.* **83**, 1002 (2003).
- [145] S. Gasparinetti, K. L. Viisanen, O.-P. Saira, T. Faivre, M. Arzeo, M. Meschke, and J. P. Pekola, *Phys. Rev. Appl.* **3**, 014007 (2015).
- [146] O.-P. Saira, M. Zgirski, K. L. Viisanen, D. S. Golubev, and J. P. Pekola, *Phys. Rev. Appl.* **6**, 024005 (2016).

- [147] M. Hatridge, R. Vijay, D. H. Slichter, J. Clarke, and I. Siddiqi, Phys. Rev. B **83**, 134501 (2011).
- [148] E. M. Levenson-Falk, N. Antler, and I. Siddiqi, Supercond. Sci. Technol. **29**, 113003 (2016).
- [149] R. Kleiner, D. Koelle, F. Ludwig, and J. Clarke, Proc. IEEE **92**, 1534 (2004).
- [150] M. J. Martínez-Pérez, J. Sesé, F. Luis, D. Drung, and T. Schurig, Rev. Sci. Instr. **81**, 016108 (2010).

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