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ROBUST MODEL PREDICTIVE CONTROL FOR CROSS-DIRECTIONAL PROCESSES

Tapio Mäenpää

TEKNILLINEN KORKEAKOULU
TEKNISKA HÖGSKOLAN
HELSINKI UNIVERSITY OF TECHNOLOGY
TECHNISCHE UNIVERSITÄT HELSINKI
UNIVERSITE DE TECHNOLOGIE D´HELSINKI

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Department of Automation and Systems Technology

Control Engineering Laboratory

Distribution:
Helsinki University of Technology
Control Engineering Laboratory
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E-mail: control.engineering@tkk.fi
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Opponent(s)	Professor Hannu Toivonen, Dr. John Shakespeare
Supervisor	Professor Heikki Koivo, As-74
(Instructor)	Dr. Kai Zenger, AS-74

Abstract

This study is concerned with the analysis and design of industrial robust model predictive control (MPC) for paper machine cross-directional (CD) processes, taking into account inherent uncertainty characteristics of the process. Paper machine CD control systems belong to the set of large scale, complex multivariable control systems, which are well known as ill-conditioned processes.

In this thesis, the concepts of structured model uncertainty and linear time-varying (LTV) systems with polytopic multi-models are exploited for describing uncertainty characteristics of the CD process. Complexity of the CD response system in terms of controller structure and inherent characteristics of the response model are evaluated based on the relative gain array (RGA) and the decentralized integral controllability (DIC). Results of the evaluation are utilized to select accurate CD response models for analysis of the primal robust MPC CD control strategy. An infinite horizon MPC problem with input and output constraints and polytopic system uncertainty is formulated as a convex optimization problem involving linear matrix inequalities (LMIs). The resulting time-varying state-feedback control law minimizes, at each time-step, an upper bound on the robust performance objective, subject to constraints.

Because the computation time of the primal robust MPC CD algorithm is too high for real time applications, a computationally efficient approach is adopted. The developed fast robust constrained state feedback MPC CD algorithm is based on a series of explicit control laws corresponding to a series of controlled invariant ellipsoids, calculated off-line, one within another in the state space. The concept of asymptotically stable invariant ellipsoid enables to provide robust stability without the demand of finding an optimum of the system at each sampling time. The advantage of this algorithm is that it gives off-line a set of stabilizing state feedback laws. Because no optimization is required, except a simple bisection search, the on-line computation time of the robust MPC CD algorithm is significantly reduced. The fast robust MPC CD algorithm is applied to the selected industrial large scale CD processes, and its efficiency is verified with numerous simulations and comparisons with the primal robust MPC CD algorithm. Simulations indicate that the loss of performance is minor and the benefit in on-line computation is over four orders of magnitude.

Keywords Paper machine, cross-directional (CD) control, robust model predictive control, linear matrix inequalities

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Robusti mallipredikitiivinen paperiradan poikkisuuntainen säätö

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Työn valvoja	Prof. Heikki Koivo, As-74
(Työn ohjaaja)	TkT Kai Zenger, AS-74

Tiivistelmä

Väitöskirjassa on tutkittu robustin mallipredikitiivisen säätömenetelmän soveltamista paperiradan poikkisuuntaiseen säätöön paperikoneella siten, että prosessin luonteenomaiset epävarmuustekijät tulevat huomioiduiksi. Prosessin epävarmuustekijät on esitetty strukturoidun epävarmuuden malliluokalla, jota käytetään muodostettaessa järjestelmän lineaarinen aikariippuva kuvaus. Väitöskirjassa prosessin vakiotilan vastemallien kompleksisuutta säätimen rakenteen ja mallien luonteenomaisten ominaisuuksien suhteen arvioidaan suhteellisen vahvistustaulukon menetelmällä ja hajautetun integroivan säädön soveltuvuudella muodostuneeseen probleemaan. Arvioinnin tulosten perusteella valitaan ne vastemallit, joihin säätöä sovelletaan. Tulo- ja lähtörajoitteinen äärettömään ennustehorisonttiin ulottuva polytooppisen epävarmuuden sisältävä mallipredikitiivinen säätöprobleema formuloidaan lineaaristen matriisiepäyhtälöiden avulla konveksiksi optimointitehtäväksi, jonka tuloksena saatava aikariippuva tilatakaisinkytkentään perustuva säätölaki minimoi jokaisella ajanhetkellä robustin kustannusfunktion ylärajan rajoitusten suhteen. Koska näin formuloidun säätöalgoritmin laskenta-aika kasvaa kuitenkin liian suureksi, on selkeä tarve nopealle säätöratkaisulle. Väitöskirjan nopea robusti mallipredikitiivinen säätöalgoritmi perustuu tilatakaisinkytkentään. Algoritmi muodostuu sarjasta eksplisiittisiä säätölakeja, jotka puolestaan vastaavat erillisesti määritettyä sarjaa invariantteja ellipsoideja tila-avaruudessa siten, että nämä sijaitsevat toinen toistensa sisällä. Asymptoottisesti stabiilit ellipsoidit mahdollistavat järjestelmän robustin stabiiliuden ilman, että järjestelmän absoluuttinen optimi löytyy jokaisella ajanhetkellä. Algoritmin etu on, että se mahdollistaa erillisesti määritettävän stabiloivan säätölain käytön. Koska algoritmi ei perustu optimointiin vaan suoraviivaiseen kahtiajakohakumenetelmään, nopean säätöalgoritmin laskenta-aika on lyhyt. Nopeaa säätöalgoritmia on sovellettu joukkoon realistisia paperiradan poikkisuuntaisia vastemalleja ja sen suorituskykyä on tutkittu lukuisin tietokonesimuloinnein ja vertaamalla sitä alkuperäiseen säätöalgoritmiin. Simulointikokeet osoittavat, että nopean algoritmin suorituskyky vastaa lähes alkuperäistä algoritmia, ja sen laskenta-aika täyttää teollisuusprosessin asettamat käytännön vaatimukset.

Asiasanat Robusti mallipredikitiivinen säätö, lineaariset matriisiepäyhtälöt, paperiradan poikkisuuntainen säätö

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This thesis is dedicated to the memory of my parents. They gave me the unforgettable lesson: In this life, one has to give up only once.

Helsinki, May 1, 2006 Tapio Mäenpää

Contents

List of abbreviations and symbols

The abbreviations and main symbols and notations used in the text are summarized below in alphabetical order.

Abbreviations

ARMA	Auto-Regressive Moving Average
CD	Cross Direction
DIC	Decentralized Integral Controllability
IC	Integral Controllability
LMI	Linear Matrix Inequality
LP	Linear Programming
LQ	Linear Quadratic
LTI	Linear Time Invariant
LTV	Linear Time Varying
LVDT	Linear Variable Displacement Transducer
MD	Machine Direction
MIC	Morari Index of Integral Controllability
MIMO	Multi Input Multi Output
MPC	Model Predictive Control
NI	Niederlinksi Index
PI	Proportional Integral
QP	Quadratic Programming
QPF	Quadratic Penalty Function
RGA	Relative Gain Array
SISO	Single Input Single Output
SM	Skogestad Morari Rule
SVD	Singular Value Decomposition

Symbols and notations

A	Coefficient matrix in state-space representation
A_o , A_1 , A_2,…	Coefficient matrices in state-space representation for polytopic systems
B	Coefficient matrix in state-space representation
$B_{o,}$ B_1, B_2, …	Coefficient matrices in state-space representation for polytopic systems
C	Coefficient matrix in state-space representation
$C(\mathrm{s})$	Laplace domain controller
Co	Convex hull
c	Element of Toeplitz matrix
$D(\mathrm{s})$	Laplace domain decoupling matrix
d_1, d_2, d_3	Step disturbances
$E(0)$	Steady-state interaction matrix
e	Profile deviation
F	Symmetric LMI matrix
$G(s)$	Laplace domain transfer function
$G_p(s)$	Laplace domain perturbed plant
$G(\mathrm{z})$	Discrete time transfer function
$G(0)$, G_o	Steady-state response matrix
$G^+(0)$	Steady-state response matrix with positive diagonal elements
$G_{diag}(0)$	Steady-state response matrix consisting only diagonal elements
$G^{n,n}$	Toeplitz symmetric response matrix
G_1 ,G_2 ,G_3, …	Steady-state CD response matrices
g	Static gain
$g_a(s)$	Laplace domain actuator transfer function
$g_d(s)$	Laplace domain time delay transfer function
i,j	Index, time index
J	Objective function
J_p	Objective function
J_∞	Objective function
$K(\mathrm{s})$	Laplace domain diagonal controller
K	Gain matrix
k	Time, discrete
k_a	Actuator gain
M	Total row size of the LMI system

m	Control horizon
N	Total number of scalar decision variables
n_s	Number of state variables
n_c	Number of manipulated variables
$O(\,.\,)$	Number of flops
P	Positive weight matrix
$P_o, P_\tau, P_{\tau 1}, \ldots$	Positive weight matrices for modified objective function
p	Prediction horizon
Q	Symmetric weight matrix
Q_1	Symmetric weight matrix
$Q(x)$	LMI matrix variable
R	Symmetric weight matrix
$R(x)$	LMI matrix variable
S	Size of an array
$S(x)$	LMI matrix variable
T	Toeplitz matrix
U	Output rotation matrix in the SVD decomposition
u	Vector of manipulated inputs
u_{ci}	Vector of desired inputs
$\hat{u}$	Vector of predicted inputs
$\tilde{u}$	Vector of shifted inputs
V	Input rotation matrix in the SVD decomposition
$V(x)$	Quadratic Lyapunov function
X	LMI variable
X	Block diagonal matrix
X_{rr}	LMI variable
x	Vector of state variables
x_1^{set}	Discretized one-dimensional subspace
$\hat{x}$	Vector of predicted states
$\tilde{x}$	Vector of shifted states
y	Vector of outputs
$\hat{y}$	Vector of predicted outputs
$\tilde{y}$	Vector of shifted outputs
Z_{rr}	LMI variable

w	Vector of augmented states
α_{i}	Scalar weight factor
τ	Time delay
τ_a	Time constant
$\Lambda(\mathrm{s})$	Relative Gain Array
Λ_{min}	Minimum RGA element value
Λ_{max}	Maximum RGA element value
λ	Relative gain, eigenvalue
σ	Singular value
$\overline{\sigma}$	Singular value maximum
$\underline{\sigma}$	Singular value minimum
γ	Condition number, LMI variable
ξ	Asymptotically stable invariant ellipsoid
ε	Tuning factor for gain
μ	Structured singular value
ρ	Spectral radius
Ω	Polytopic set
Ω_{sim}	Simulated polytopic set
Δ_{I}	Diagonal input uncertainty
$Re\{\lambda_i\}$	Real part of the eigenvalue
$\|\| . \|\|$	Euclidean vector norm , 2 -norm
$\| . \|$	Absolute value

Chapter 1

Introduction

Background

An increased emphasis on product quality, productivity, faster deliveries, and process optimization has been the driving force in paper industry behind the interest in automation and process controls. Quality and productivity are currently the key attributes that directly measure the effectiviness and cost of manufacturing processes. Productivity is closely linked to quality because there is an optimum rate of production when considering the principal manufacturing variables: raw material, process, and automation system used. Quality, on the other hand, is the judgment of the properties of the final output of the production process. Multible properties define the quality of the paper web. These include weight per unit area, moisture, thickness, gloss, physical properties and appearance. All of these properties can be affected by appropriate use of automation system and process controls. Variability of these properties in paper machine is controlled in both the machine direction (MD) and the cross-direction (CD) by quality controls. Today, improved control of paper production line by using efficient MD and CD quality controls can mean notable fiber savings, reduced quality variations, greater production rates, reduced sheet brakes, and significant energy savings (Fadum, 1989), (Wallace, 1986).

By applying current modern technologies in feedback control, along with developing new algorithms to handle open issues, a significant contribution can be made to quality improvements in papermaking. Current CD control methods for sheet processes do not take full advantage of modern control theory, and there is a clear interest in improving existing control strategies. Among other startegies, model predictive control (MPC) technology has been applied to a range of control applications in papermaking. The power of MPC, including its ability to handle constraints, makes this technology an attractive candidate for industrial implementation. However, specific characteristics of the sheet processes have limited an extensive use of MPC for CD control problems. Such characteristics include inaccurate spatial CD response models, large number of actuators and sensing locations, significant time delays, noisy sensors, actuator constraints, equipment faults, and uncertain disturbances. Numerical difficulties associated in handling large dimensional CD processes and uncertainty characteristics of the process have been the essential reasons for slow progress of MPC technology (Featherstone and Braatz, 2000). The main objective of this work is to attack the current inadequacies in papermaking and provide a robust MPC CD strategy, which takes into account the inaccuracies of the process model and thus enables implementation on real paper machines.

Modeling and control of CD processes

Modern CD controls are model-based controls. A model is created in order to describe how the process will respond to the control actions. However, it is impossible to generate an accurate process model because of lack of complete understanging of the underlying physical phenomena, inaccurate values for the physical parameters of the process, and several unknown disturbances. Therefore, in practical CD control application a linear, multivariable process model is used to describe cross-directional profile systems (Wilhelm and Fjeld, 1983), (Chen and Wilhelm, 1986). Actually this is an approximation of the real web forming process, which can be considered as a complex distributed parameter system (Duncan, 1989). In this kind of simplified approach, it is assumed that the response of actuators is separable, and it can be expressed as a product of a spatial (CD) response and a scalar dynamic term. Therefore, CD profile system may be presented as a *sampled data lumped parameter system*, which describes a finite number of actuators and a finite number of measuring points (Duncan ,1989), (Duncan *et al.,* 1996).

Usually the spatial CD response model is identified from input-output data by on-line or off-line identification methods. Several modern methods exist to accomplish this estimation (Wilhelm, 1982), (Wellstead *et al.*, 1998). Due to the inherent characteristics of the papermaking processes, the response model often has a structure of Toeplitz symmetric matrix, in which the same constant element is repeated along each diagonal (Laughlin *et al.,* 1993). Commonly CD process responds quite fast to the changes in the actuator settings. Therefore most of the system dynamics are attributed to the actuators, which are generally modeled as first-order-plus-dead-time (Laughlin *et al.,* 1993), (Kristinsson and Dumont, 1996). However, in practice the sampling rate for CD profiles is much slower the process dynamics, and therefore dynamics are ignored in most of the cases.

In practical CD applications several facts can cause imperfections to the CD process models. There will be uncertainty associated with the estimation procedure of the response shape. Although the shape of the response is not assumed to vary in the cross-direction, in practice the paper sheet may wander or shrink over time, leading to alignment error and mismatch between the true location of actuator's effect and its predicted location (Heaven *et al.*, 1993b). In a paper machine operating conditions may also vary in broad limits. In addition, the type of CD actuators affects on the form of model errors. For instance, mechanical coupling, clamping and backlash problems are common to all spindle actuators and electric motor applications (Cutshall, 1991). All characteristics of the uncertain CD process are connected to the response model. It is evident that response model uncertainty should be taken into account in the control strategy. In theory, model uncertainty can be either unstructured or structured (Skogestad and Postlethwaite, 1996). Both approaches have been used in CD control literature. Featherstone and Braatz (1998, 2000), Stewart *et al.* (2000) and Duncan (1994a) used unstructured model uncertainty, while Laughlin (1988) and Laughlin *et al.* (1993) applied a structured model uncertainty in their CD modeling work. In this thesis a separable CD response model approach is adopted and a linear, multivariable process model with first-order-plus-dead-time actuator dynamics is used. Structured model uncertainty, which takes into account the main characteristics of the process, is incorporated into the CD response model.

A wide variety of CD control strategies for papermaking processes appear in the literature. Two popular CD control schemes reported in the literature before late 1980s were linear-quadratic-optimal (LQ) and model inverse-based control (Boyle, 1977), (Chen *et al.*, 1986). Mostly steady state models were used. One weakness of these linear control approaches is that constraints can be satisfied only by sufficiently penalizing control actions in the objective function, producing rather sluggish control movements. Another concern in this method is closely related to numerical problems and ill-conditioned response models (Wilhelm and Fjeld, 1983). Following the works of Laughlin (1988) and Duncan (1989), who studied robustness of CD control systems, several methods have been suggested for designing robust CD controllers. However, many of these approaches utilize a special structure of the CD response model matrices in the controller design (Laughlin, 1988), (Laughlin *et al.*, 1993), (Stewart, 2000), which limits the applicability of these methods. Similarly, numerous model predictive control (MPC) approaches have been proposed for CD control problem (Campbell and Rawlings, 1998), (Backström *et al.*, 2000), (Dave *et al.*, 1997, 1999), (Barlett *et al.*, 2002). All these methods contain the basic features of MPC method, but thus do not explicitly address model inaccuracies. Therefore, for the time being robust constrained MPC has not been implemented in real time large scale CD processes because of heavy on-line computations needed. In this thesis, an efficient robust MPC strategy is proposed in order to solve this real time problem.

Analysis of CD model structure

A conventional approach to evaluate spatial CD response model is to study its condition number (Laughlin *et al*, 1993). Response models are commonly considered poorly conditioned and difficult to control if their condition numbers are large (Skogestad *et al.*, 1988). Laughlin *et al.* (1993) have shown that not only the process characteristics, but the size of the process, affect the value of the condition number, and that large dimensional matrices seem to be more poorly conditioned. This phenomenon has also been noticed in practice, because the majority of industrial CD processes are truly ill-conditioned (Featherstone and Braatz, 2000), (Laughlin *et al.,* 1993). However, the condition number is scaling dependent and it depends on the choices of units of the inputs and outputs. Therefore it should not be used as a measure of inherent ill-conditioning of the process. (Skogestad and Postlethwaite, 1996). In this thesis, we suggest that a better measure in this respect for the CD process is relative gain array (RGA) analysis, which is independent of scaling and takes into consideration the implicit characteristics of the CD process. Such characteristics contain especially the diagonal type input uncertainty of actuators, which is always present in a CD process. In addition, we suggest that RGA analysis combined with the concept of decentralized integral controllable (DIC) system (Skogestad and Morari, 1992) will provide more information about the behaviour of CD response model and controller design that pure condition number analysis. In this thesis, we will consider RGA and DIC as practical analysis and screening tools to select the controller structure for the process.

Robust model predictive CD control

Applications of model predictive control (MPC) have been increasingly used for paper machine control during last years. A notable advantage of MPC method is its ability to handle hard constraints on the actuators. However, its large scale nature and several model uncertainty characteristics have limited the availability of MPC for paper machine CD control. One approach to solve a large scale CD problem is to reduce the size of the model, and thus reduce the complexity of on-line computations. Haznedar and Arkun (2002) applied the principal component analysis method for model reduction and identification. VanAntwerb and Braatz (2000a) designed a fast MPC algorithm, which utilizes the iterated ellipsoid method, and is based on off-line singular value decomposition.

One of the main drawbacks of a standard MPC method is its inability to consider model uncertainties. However, robust MPC theory provides a method to take modeling errors into account in controller design. A min-max optimization approach with finite impulse response models has been an extensively used in robust MPC literature, e.g. (Campo and Morari, 1987), (Zafiriou, 1990), (Zheng and Morari, 1993). The same method has also been applied to discrete state-space models with polytopic model uncertainty (Lee and Yu, 1996). Recently, a robust MPC technique using linear matrix inequality (LMI) technique has been developed (Kothare *et al.*, 1996), (Lu and Arkun, 2000). In this method a robust infinite horizon MPC problem with constraints and model uncertainty can be reformulated to a convex optimization problem containing LMIs. An important advantage of this approach is that the stability of the robust MPC controller is guaranteed if the optimization problem is feasible.

The practicality of MPC has been limited by the difficulty to solve optimization problems in real time. To tackle these problems fast computation solutions have been introduced. Lee and Kouvaritakis (2000) presented a receding horizon dual-mode MPC algorithm, which uses a LP method to reduce computational complexity. Bemporad *et al*. (2002) suggested a state-feedback solution to finite and infinite horizon LQ control problems, which does not require time-consuming on-line QP solvers. VanAntwerb and Braatz (2000a) applied a fast MPC algorithm to CD control problem. They used truncated singular value method and iterated ellipsoid algorithm to minimize computing load. Wan and Kothare (2003) modified the robust MPC algorithm developed by Kothare *et al.* (1996), and introduced a concept of asymptotically stable invariant ellipsoid, which guarantees robust stability without the requirement of finding an optimum of the system. A significant benefit of this approach is that it gives off-line a set of stabilizing state feedback laws, and no optimization is required except a straightforward bisection search. The on-line computation time of robust MPC algorithm is thus considerably reduced. In this thesis, we will show that robust MPC methods developed by Kothare *et al.* (1996) and Wan and Kothare (2003) are efficiently applicable to the CD control problem, and they provide a new approach to this classical control problem.

Research objectives

This thesis focuses on ways to improve existing CD control methods to increase quality and productivity in papermaking. The objectives may be divided into the following main categories:

- To investigate ways to find out a practical method to evaluate the complexity of industrial CD response models

- To create a robust model predictive (MPC) CD control strategy, which will take into account uncertain characteristics of the process.

- To develop computationally efficient robust MPC CD algorithms, which may be implemented on real paper machines.

Layout of the Thesis

This thesis consists of seven chapters and it is organized as follows:

Chapter 1 is the introduction.

Chapter 2 presents first a paper machine cross-direction process and typical CD control systems. Basis weight, moisture and caliper controls are treated briefly. Then, characteristics of the CD process including scanning measurements, constrained actuators and complex high dimensional uncertain system are addressed. After this, a short review to the industrial CD control structure is presented.

Chapter 3 introduces cross-directional response models and clarifies how an approximated linear multivariable system description is derived. Model attributes and related process dynamics are also presented. Then, model uncertainties in terms of process characteristics, operating point and appearing actuator errors are introduced to derive the structure of the utilized uncertain response model. After that, an overview of reported control strategies is presented. The survey covers both the early history of CD control and modern model predictive and robust control approaches.

Response model analysis is the topic of Chapter 4. The relative gain array (RGA) method is introduced and its applicability to CD response analysis is evaluated in terms of diagonal type input uncertainty. Then, a concept of decentralized integral controllable (DIC) system model is presented and necessary conditions for DIC are defined to be used with steady state response models. Applicability of steady state inverse-based CD controller is studied, and RGA analysis is accomplished for a set of experimental or proposed CD response models. Observations and conclusions are given for demonstrating the usability of the proposed analysis.

Chapter 5 describes the robust model predictive CD control algorithm using linear matrix inequalities (LMI). First a brief description of linear matrix inequalities, linear time-varying (LTV) uncertain polytopic systems and basics of model predictive control is given. This supporting material will then be used to formulate a primary robust constrained MPC CD control problem with a state-feedback as a LMI problem. Then, several simulated examples based on industrial paper machine CD processes are given for illustrating the applicability of robust MPC approach. Selection of simulated response models is done based on RGA analysis.

Chapter 6 deals with an efficient robust LMI based MPC CD algorithm with guaranteed robust stability of the closed-loop system for polytopic model uncertainty description. First it is shown, that the algorithm provides the off-line a sequence of stabilizing state feedback laws, which consist in invariant ellipsoids one inside another in the state space. After this, closed loop performance is studied with simulations using large scale CD processes with realistic description of interaction matrices and general uncertainty structures. The performance of the fast robust MPC CD algorithm is compared both to the primary algorithm presented in Chapter 5 and to the standard, off-the-self MPC CD algorithm. Simulation results show that the fast robust MPC CD algorithm can reduce the on-line computation of robust constrained MPC considerably, with a little loss of performance.

Chapter 7 summarizes the main results of this study and makes some suggestions for further research.

Contributions

The main contributions of this thesis are:

- Complexity of cross-direction (CD) response model is evaluated based on relative gain array (RGA) analysis as distinct from conventional condition number approach.

- Relative gain array (RGA) analysis and the concept of decentralized integral controllable (DIC) system are introduced as a practical screening tool to select controller structure for CD processes.

- A linear time-varying (LTV) system with polytopic uncertainty structure is suggested in an unpresented way to describe uncertain CD processes.

- A concept of primal robust model predictive (MPC) CD algorithm is applied in a novel way and its applicability is proven with realistic industrial CD response model simulations.

- For the first time a computationally efficient robust MPC CD algorithm is presented and its efficiency is compared with the primal algorithm. Comparison reveals that the loss of performance is minimal and the advantage in on-line computation is over four orders of magnitude.

Chapter 2

Paper Machine Cross-Directional Processes

The paper machine, invented in the year 1799 by a French Nicolas-Louis Robert, has been constantly improved over the years. It has became wider and faster, and today modern paper machines make hundreds of tons of paper per day. Increased production rates and stricter quality, economic and environmental requirements, have set new demands to paper machine control. The complexity of the process, the scale of operation and production speed leave little room for error or mulfunction. Modern papermaking would not be possible without a great variety of technologies, and in particulat advanced process control and diagnostic. The following description of paper machine’s operations is very brief and simplified, a more detailed discussion of this complex process may be found e.g. in (Gullichsen and Paulapuro, 1999).

The first section of the paper machine is called the wet end, see Fig. 2.1. This is the area where the stock first comes into contact with the paper machine. In paper production fiber suspension consisting of wood fibers, additives and water, is mixed with white liquor and pumped into the headbox. The function of the headbox is to distribute the fiber suspension with approximately 0.2% - 1.0% consistency such that the fibers are evenly distributed over the width of the flow area. Several type of headboxes have been planned depending on the paper machine design: open headboxes, air-cushion headboxes, hydraulic headboxes and diffuser headboxes. After headbox the sheet-forming is taking place and the fiber suspension is fed onto the moving forming belt, which today may be rotating even 1800m/min. Depending on the machine lay-out various drainage units such as forming board, blades, suction boxes or rolls and hydrofoils are used to remove remarkable amount of water. Thus, when paper sheet leaves the wet end its consistency is approximately 20%.

The next phase of the paper machine is the press section where the sheet is both heated by steam boxes and more dewatered by mechanical compression in a nip formed by two rolls or a roll and a shoe. After press section, just before the dryer section, the solids content of the paper sheet has increased to 33% - 55% depending on the paper grade and press section design. The dryer section of the process starts with a series of very large-diameter, rotating steam heated cylinders. These cylinders remove most of the reamaining water. However, a small amount of water (5%-9%) remains in the paper even after the dryer section. Most drainage on a paper machine therefore occurs mechanically on the wire and press sections. After the dryer section, at the dry end, the paper is passed throught a series of calender rolls, where the paper sheet thickness and surface properties are being controlled. At the end of paper machine the sheet is wound up onto the reel.

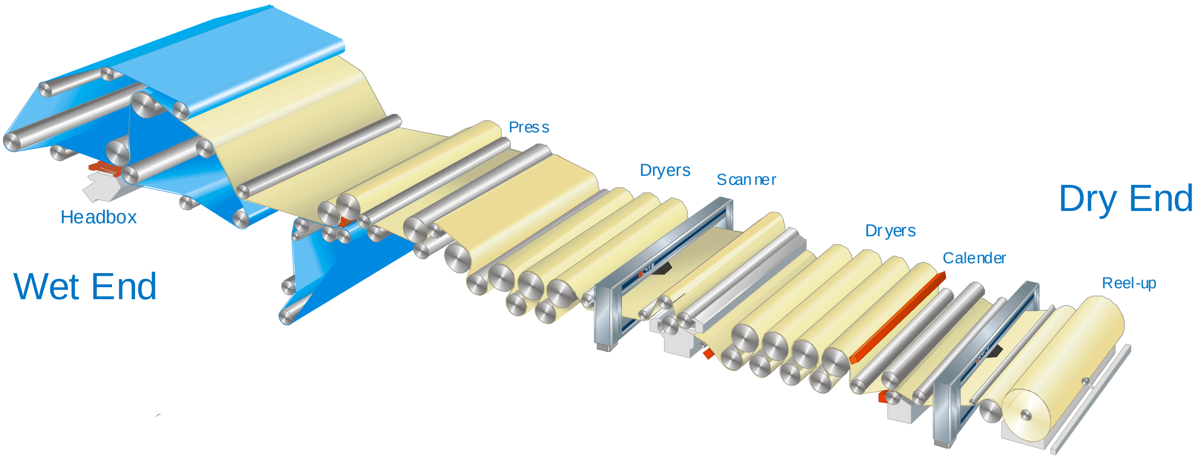

Figure 2.1: Wide view of modern paper machine (Courtesy of Metso Paper).

Depending on the paper machine design, after the dry end, there might nowadays exist a separate, off-machine paper finishing unit, consisting of coating machine or a soft calender. On the other hand, if the finishing units are included in the paper machine, an on-machine construction is considered. Off-machine coating machine is composed of coating stations, dryers and un-wind and wind-up units. Coating stations, either on-machine or off-machine, apply a thin layer of pigment-based coating colour to one or both surfaces of the sheet. The objective of coating is to improve appearance and printability of the paper.

The direction of sheet travel is known in the paper making industry as the machine-direction (MD), and the direction perpendicular to machine-direction is called cross-direction (CD). In paper production, the three most important quality properties of the sheet are: the sheet weight per unit area (g/m^2), the sheet moisture content (%) and the thickness or caliper of the sheet (μm). Regarding to paper finishing the coat weigh per unit area (g/m^2) is also a quality property worth mentioning. Quality properties are measured by a scanning sensor, installed typically at the end of the machine. The scanner traverses the moving sheet back and forth in the cross-direction (CD), measuring sheet quality properties even as fine as 10 mm wide. Because the scanning head traverses the moving sheet, it forms a diagonal path along the sheet, and thus the measurement profile comprises both MD and CD variations. In paper machine and also in paper finishing control applications these variations are commonly controlled separately. Thus, paper machines have two main control objectives. One is the maintenance of the average sheet property, which is referred to as the machine-direction (MD) control problem. The other is the maintenance of flat profiles across the machine web, referred to as the cross-directional (CD) control problem. We will not handle the extensive MD control problem in our work, we will focus on the CD control issues instead.

2.1 Paper machine CD control systems

The cross-direction (CD) control problem in paper manufacturing has been known since early 1950s when the problem of basis weight variation was officially noticed (Burkhard and Wrist, 1954). Actually in the 1960s the paper industry really discovered how bad the CD variation was when on-machine traversing weight and moisture gauges became commercially available (Beecher and Bareiss, 1970). The goal of quality control is to increase uniformity of manufactured product. However, this task is not necessarily the easiest in the paper manufacturing. It requires a great deal of process knowledge, consideration of suitable measurements and measurement processing, adequate actuators and suitable control algorithms. The objective of CD control is to minimize deviations of process variables from the setpoint across the width of the paper (Tong, 1975). Basis weight, moisture, caliper, and coat weight are examples of important sheet properties. Variations in these properties can result in paper that will not fulfill the quality requirements. On the other hand, successful CD control of these properties can mean significant reduction in raw material consumption and improvements in quality (Fadum, 1989). In each of these cases the variations are controlled adjusting the set points of an array of actuators. In most cases, paper machines have at least one actuator array for controlling each of the important sheet properties. Usually different sheet properties are controlled by separate and independent control systems. Current trend in the paper industry is to have larger and faster paper machines and the number of actuators in the array can vary from few up to more than hundred.

2.1.1 Basis weight control

Basis weight is an essential property of the paper sheet, which has also a signifigant influence to the other sheet properties. Depending on the paper machine desing basis weight values may vary widely from 35 g/m^2 for a light weigh paper grades even to 450 g/m^2 for board sheets (Gullichsen and Paulapuro, 1999). The function of CD basis weight actuators is to distribute the fibers evenly over the width of headbox. Two type of actuators are utilized for controlling CD basis weight: slice lip actuators and dilution actuators. Use of a slice lip actuators is a traditional way of controlling the CD profile of basis weight. The fiber suspension streams out from the headbox through a slice opening, which is as wide as the paper machine but only a couple of centimeters high. The amount of fiber suspension exiting the headbox is controlled by locally changing the height of the slice opening by deflecting the upper lip. The bending of the upper lip is controlled by an array of thermal, hydraulic or motorized actuators (Cutshall, 1991). The spacing of the actuators dependens on the installation, and may vary from 75 mm to 100 mm. The response of the basis weight profile to the slice lip actuators varies widely. For light grade papers the response width of a single actuator is quite narrow, covering only a couple of actuators. For heavy grades it may cover even a quarter of the entire width of the paper machine.

A modern method for basis weight control is consistency profiling or dilution control. In this method the basis weight profile is changed by locally altering the consentration of fibres in the headbox (Vyse *et al*., 1996). This is done by an array of dilution actuators distributed across the headbox. Dilution zones are commonly 40-60 mm wide. The consistency of the pulp stock is changed by diluting it with a flow of low consistency water as it enters to the headbox. An increase in the flow of a dilution actuator reduces the local consentration of fibres and thus locally reduces the basis weight. The advantage of dilution actuators, in comparison to the slice lip actuators, is a smaller actuator spacing, a narrower spatial response and much better bandwitdth for control of basis weight profiles. However, it requires a more accurate mapping information.

2.1.2 Moisture control

Moisture content of the paper is also a major quality property, which has an important influence on the paper strength. Normally moisture content varies from 5 % to 9 % of the total weight of the paper sheet. On the other hand, overdrying of the paper may reduce its strength and cause damages in the fibre structure. Analogous to CD basis weigh control, moisture CD profile control influneces several paper sheet properties like web structure, caliper and surface smoothness (Gullichsen and Paulapuro, 1999). It affects also the local CD dry content of the sheet. This may result in local changes in stress and strain and can create curl problems in the final product. In addition, an uncorrected moisture disturbance will appear in the caliper profile.

CD moisture profile control in the press section may use various kind of moisture actuators: zone controlled rolls to obtain a required line force profile to the press nip; steam boxes that heat the paper to improve the local mechanical dewatering; rewet shower actuators to prevent over drying of the sheet and to correct dry streaks. The use of segmented infrared heaters is also possible. On the other hand, infrared heaters are common in coating machines for CD moisture control. The actuator spacing of steam boxes, rewet showers and infrared heaters is usually between 75-150 mm, and for zone controlled rolls somewhat wider. Rewet showers and zone controlled rolls are fast actuators with a time constant less than 30 s, while steam boxes and infrared heaters are generally quite slow with a time constant of couple of minutes.

2.1.3 Caliper control

Usually calender stack is located at the dry end of the paper machine, or in the off-machine construction after it, as a separate unit. Calendering is papermaker's last chance to reduce caliper variations along the length and width of the finished paper sheet. A smoother sheet results in improved print quality, while more uniform caliper profiles improve the winding process and reduce sheet breaks in printing presses. Almost all paper and board is at least slightly hard nip calendered in order to control overall thickness and to even out cross-direction caliper non-uniformities. Calendering reduces paper caliper and roughness by pressing the sheet between two or more large cast-iron or soft-covered rolls. The high loads produced in a nip between two smooth calender rolls flatten high spots in the rough sheet by permanently deforming wood fibers on the surface of the sheet, thus reducing the roughness of the sheet.

Regardless of the calender construction, three main technologies for control of paper caliper are available; induction heating systems, confined air showers and zone controlled CD rolls. Induction heating and confined air showers externally heat the roll to increase the diameters which in turn increases the nip load. Zone controlled CD rolls vary the nip pressure by mechanically deflecting the roll shell. Introduction heating system heats directly the roll by applying an alternating magnetic field. On the other hand, magnetic field induces eddy currents into the roll surface, which in turn produce heating. System consists of a number of magnetic-induction coils shaped to conform closely to the contour of a roll. The coils are encapsulated in a flameproof and isolating resin compound. The center-to-center distance of the coils is typically 76.2 mm (Burma *et al*., 1996). A major disadvantage of induction heating system is a low speed of response and high energy consumption.

Air showers are based on a convection heating technology to transfer heat from a resistive heater to the roll surface. CD caliper control is done by changing the diameters of the roll through zone heating and cooling. Modern air showers use infrared units to heat constantly operating air jets that are directed at the roll surface. This kind of system can provide a narrow (38 mm) zone resolution of actuators, and with heaters at full power (79 kW/m), the air jet temperature rises to 450 °C (Impact Systems, 1994). However, this heating capability would not normally provide the CD control range or temperature increase required for soft nip calender applications. As a result, air shower applications are usually limited to hard nip stacks.

Nowadays, zone controlled CD rolls are commonly used as actuators for caliber control (Svenka and Minkenberg, 1995). The zone resolution of these actuators is at minimum 150-250 mm. Because of the stiffness of zone controlled roll shells, profile shape adjustment can be made over a bandwidth of 400-800 mm. An apparent advantage of the zone controlled CD rolls is that the control actions by hydraulic pressure can be accomplished in a few seconds, providing a fast speed of response.

2.1.4 Coat weight control

The coater is located at the dry end of the paper machine. In the off-machine construction it is a separate unit like calender stack. Paper is coated in order to improve its appearance and printability. Good printing characteristics demand a surface, which has an even ink absorbtion, adequate smoothness, good optical properties, and high surface strength. The coating result depends on the quality of the base paper, type of coating and the process used to apply the coating. Coating results a dense and homogenous surface that allows a high image definition. By coating the quality level of paper can be increased noticeably, and only coated paper can provide the high quality colour reproduction demanded by advertisers and customers. Various methods are used to apply and smooth the coating colour to final surfaces for different printing. However, the blade coating is the most important coating method for paper. In blade coating, the right amount of colour is metered onto the paper sheet surface, and the surface is levelled with a revising blade. Coat weight control is based on the surface roughness volume of the paper. This volume is controlled regulating the compression force on the coater blade (Luomi, 1991).

CD coat weight control is done by controlling an array of motor-driven spindles, which affect locally to the compression force of the blade. When a spindle is operated, a fast, local change in the displacement of the blade with respect to the backing roll is effected. This leads to a change in the blade load and, consequently, in the local coat weight level. Spacing of the coat weight actuators varies normally from 75 mm to 150 mm. In principle, CD coat weight system is similar to the CD basis weight control with slice lip actuators. However, certain process characteristics like wearing of the blade, critical dependence on the blade angle (Ellilä, 1994), less demanding mapping requirements and a shorter process delay make it distinct.

2.2 Process characteristics

The objective of CD control is to minimize deviations of process variables from the setpoints across the width of the paper sheet. Almost all cross-directional control systems consist of three basic components (Duncan, 1999), see Fig. 2.2. A sensing system which measures the variations across the full width of the web, an array of constrained actuators, and a control algorithm that uses the measurements from the sensing system to determine the inputs to the actuators. The control actions must always satisfy the constraint limits. When an actuator is manipulated sheet properties change for some distance on either side of the position of the actuator. These interactions are typically incorporated into the CD response model with a number of constant interaction parameters. The CD control problem is to calculate actuator actions based upon the measurement of the quality profile.

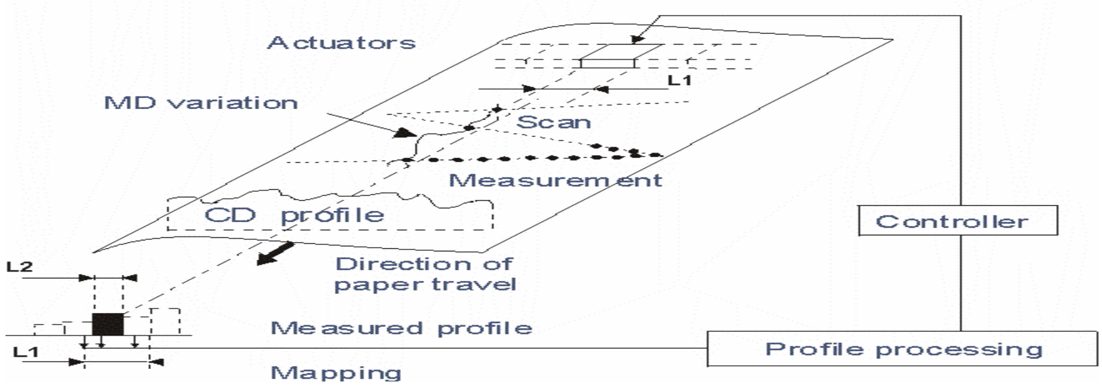

Figure 2.2: Cross-directional control problem.

Next we will describe some features of the CD process that make its control especially challenging.

2.2.1 Scanning measurements

Continuous on-line measurements are necessary for precise and reliable CD process control. However, existing scanning measurement systems can not measure the whole sheet; only a portion of the sheet is sampled and this is presumed to represent the entire product. Another difficulty is that some variations occur beyond the measurement capability of sensors. Because the gauge can not record wavelengths shorter than its frequency response limit, some faster variations can not be observed (Pfeifer, 1984). Next to measurement accuracy, the most critical factor is the processing of sensor data into profiles with high resolution. Today the total number of measurement points or data boxes in a measurement profile can easily be over one thousand depending on the width of the paper sheet. Perhaps the most important reason for having high profile resolution is to get a good mapping of the area of influence of a CD actuator (Dolphin,1988). Good CD control depends on knowing how actuator positions interact. In addition, high resolution profiles are necessary in identifying actuator response characteristics. A good determination of the proper response shape and size provides more confidence in the control strategy.

However, practical CD control systems have been restricted to controlling variations of the sheet based on the measurements which have been sampled in either the cross-directional (CD) or the machine-direction (MD) or both. When a sensor scans the sheet, it forms a diagonal path, measuring a new profile, which includes both MD and CD variations and a random component called residual (Beecher and Bareiss, 1970). One problem with the random component is that it can not be measured directly. It has to be estimated by subtracting the MD and CD from the total. Several modern methods have been presented to solve this MD/CD separation problem (Chen, 1992), (Duncan, 1994b). A conventional procedure to remove the MD variability from the measured profile is an exponential multiple-scan trending. In this method the removal of undesired MD variability from the profile is done by weighting the measurement at each cross direction position with the long-term historical value (Seborg *et al.,*1989). Another popular method is a moving-average filter, which averages a specified number of past data points by giving equal weight to each data point. The moving-average is usually less effective than the exponential filter, which gives more weight to the most recent data. To obtain a full profile information, use of an estimation algorithm, like a linear Kalman filter, is necessary to calculate missing measurements of the CD profile (Chen, 1988). However, even then the lack of profile information makes MD/CD separation difficult, even impossible for those MD and CD variations that have similar power spectrum. Also Dumont *et al.* (1993) and Wang *et al.* (1993a) used Kalman filter approach together with exponential forgetting and least squares algorithm to estimate basis weight and moisture content from scanned measurements. However, even if the MD or CD component is eliminated by a control system, the random component will not be reduced and it still remains in the process.

Another practical way to remove the MD variations from the CD profile is to scan more slowly to increase the averaging time for each cross-direction measurement zone and to reduce the amount of high-frequency MD variability. Therefore, in practice the result is often a compromise between the scan speed and the accuracy. Ideally the best scan is one that is instantaneous. This would eliminate all machine direction variation components. Such a technology is currently forthcoming and sensors, which simultaneously measure the CD profile have been introduced (ABB, 1998). However, also present measuring systems can be utilized in different modes. A sensor operating at fixed position can be used to estimate the MD component and herewith to compensate a scanning measurement to obtain a true CD measurement (Duncan, 1994b). Alternatively, a subscanning procedure may be used. This approach provides a way to scan some parts of the sheet more than once in each full traverse. Method gives a better MD resolution over part of the sheet, and provides estimation of the actual MD variation to frequencies higher than scanning frequency. More detailed discussion of this method and other cross-machine measurements may be found in Shakespeare (2001). However, for most of the existing measurement systems an appropriate scan speed must be defined to produce a good cross machine resolution. Thus, the traversing speed of existing measuring gauges varies typically from 250 mm to 500 mm per second, and therefore the scanning time in modern 10 meter wide paper machine may range between 20 and 50 seconds. Another issue related partly to the measurement system is the process delay, which is mainly determined by the distance between the actuators and scanning sensor, machine speed and the speed of traversing head. As a result, in modern paper machine it may vary from one scan to up to 6-8 scans, depending on the machine and scanning system design. In off-machine constructions the delay is significantly shorter.

2.2.2 Constrained actuators

In reality CD actuators are always constrained. There are upper and lower limits on the positions and often on movement of each actuator. The constraints in a real-time CD profile automatic control system include the available range of control adjustment, physical limits on the actuators, and possibly broken or manually controlled actuators. For instance in the case of headbox slice spindles to control the basis weight profile on a paper machine, there are high and low limits on the actuator positions as well as mechanical deformation or bending stress limit on the slice itself. Similar situation exists also in coating machines and calenders. Sometimes process as such dictates constraints because excessive control actions may compromise the integrity and strength of the sheet. Three typical sets of constraints may be presented as follows (Chen *et al.*,1986), (Chen and Wilhelm, 1986): (*i*) The available range of control adjustments u_i is constrained, that is, $u_{min} \leq u_i \leq u_{max,}$ for i = 1,…,n. These are known as min-max constraints. (*ii*) The difference between adjacent actuator positions u_i may be limited, that is, $|\delta u_i| = |u_{i+1} - u_i| \leq |\delta u|_{max,}$ for i = 1,…,n-1. These constraints are often called first order bending moment constraints. (*iii*) A difference between adjacent actuator positions u_i may be limited if control actions are made in opposite directions, that is, $|\delta^2 u_i| = |u_{i+2} - 2u_{i+1} + u_i| \leq |\delta^2 u|_{max}$, for i = 1,…,n-2. These bending stress constraints are also called second order bending moment constraints.

In CD control applications actuator constraints can be handled in different ways. Boyle (1977) and Chen and Wilhelm (1986) have shown that the optimal control subject to above shown constraints can be obtained with quadratic programming. However, this method expands the dimension of the problem considerably and is very sensitive to the condition of the process model. In linear CD control approach constraints may also be taken into account by defining additional vector inequality conditions and augmented performance index. An optimal solution of this new constrained optimization problem can be found by using quadratic programming (Wilhelm, 1986). A slightly more general technique reported by Braatz *et al.* (1992) is to include each constraint type with own weighting in the objective function. A solution is found by tuning the weightings to guarantee that the constraints are not violated.

Constraints can also be added explicitly to the linear quadratic control algorithm. Then the constrained control problem will be the unconstrained control problem plus the additional constraints (*i*) – (*iii*). Also this is a quadratic programming problem that must be solved at each time step for the optimal actuator movements (Braatz, 1993). A more convenient explicit approach is the model predictive control (MPC), which provides a way to explicitly handle constraints on the inputs and outputs of the system. However, heavy online computation load of MPC method in CD control applications has restricted direct use of this method (Featherstone and Braatz, 2000). We will discuss MPC method in more details later.

2.2.3 A complex high dimensional system

In theory, cross-directional control of paper machines is a distributed parameter control problem, which should be solved using partial differential equations (Duncan, 1989). However, in practical CD control applications this complex system is reduced to a lumped parameter system, which can easier be used for control purposes. A common assumption is that the CD process has a separable dynamical and spatial response, which simplifies the modeling problem distinctly (Duncan, 1989). We will discuss CD response modeling more in details in Chapter 3. However, in terms of spatial responses, CD process and its characteristics may vary significantly depending on the process conditions, operating point and produced grades. Spatial response to a single actuator can be as narrow as a few centimeters for the basis weight control by use of dilution actuators, or as wide as a couple of meters for the slice lip basis weight control on heavy grade papers. In addition, as pointed out earlier, some actuators may have a very fast, almost instantaneous response, while some others are extremely slow. Similarly, the process delay may change considerably at different production conditions.

Another problem is related to the high dimension of the CD control system (Featherstone and Braatz, 2000). A current trend in paper industry is to have smaller actuator spacing between adjacent actuators and adjacent sensing locations. Narrower process disturbances should be observed and eliminated (Wallace, 1986). Thus, existing industrial CD control systems may easily have over one hundred actuators in a single array, and over one thousand measurement values in a high resolution profile. The high dimension and increasing paper machine speeds put actual requirements on the time of on-line computation for the control algorithms. In addition, process models of large dimension may have poorly conditioned system matrices, which may lead to serious control problems (Skogestad *et al.*, 1988).

2.2.4 A poorly conditioned uncertain system

A traditional method to consider the characteristics of the CD response model is to examine its condition number (Laughlin *et al*., 1993), which describes the ratio between the largest and the smallest singular value of the response matrix. A matrix is said to be ill-conditioned if its condition number is large. In addition, the gain of the CD process response is dependent on the singular value directions of the process. Thus, a large condition number means that the system depends strongly on the input directions (Skogestad and Postlethwaite, 1996). In practice, majority of uncertain industrial CD processes are highly ill-conditioned (Featherstone and Braatz, 2000), (Laughlin *et al*., 1993), because their condition number values can reach into hundreds. This is a clear indication of expected critical control problems, such as sensitivity to the model uncertainty. In spite of its benefits, condition number is perhaps not the best possible index to measure inherent ill-conditioning of the process. We will propose in Chapter 4, that a better measure in this respect would be a relative gain array (RGA), (Skogestad and Postlethwaite, 1996). Another noteworthy matter is that the ill-conditioned characteristic of the response matrix affects the design of CD controllers. In several practical cases it excludes traditional CD control approaches and obliges to choose more advanced control methods. We will show in Chapter 4 that this is especially true in the case of decentralized integral CD controller.

Another important characteristic of practical CD control system is that a significant model uncertainty is related to it. Several facts can cause imperfections to the defined CD process models. There will be uncertainty associated with the identification procedure of the response models, which are commonly defined from uncertain data. Although the shape of the response is not assumed to vary in the cross-direction, in practice the paper sheet may wander or shrink over time, leading to the alignment and mapping error and mismatch between the true location of an actuator's effect and its predicted location (Shakespeare, 2001). In addition, a general assumption made when modeling CD process is that the response of the measured paper sheet properties to the actuators is described by a linear, time-invariant transfer matrix of low dynamic order (Duncan, 1989). However, this assumption is only an approximation of the complex CD process. In paper machine the operating conditions vary also between broad limits. In addition, the type of CD actuators and measurements affect the form of the model error. All the characteristics of the CD process are connected to the uncertain response model. We will return to these issues in Chapter 3.

2.2.5 Industrial paper machine CD control systems

Today industrial paper machine CD control applications are commonly implemented in a distributed control system (DCS), which have an open architecture and which can provide several benefits to the users (Ranta *et al*.,1992). Centralized operations, improved process performance through a higher level of control, cost-effective digitizing of data for use by information system, and distribution of risk into smaller modules with higher uptime and faster repairs are typical benefits. Of all these benefits, the most important ones are improved process control and improved process performance (Wallace *et al*., 1992).

A typical example of modern CD control application is a consistency profiling system for CD basis weight (Vyse *et al*., 1996). A block diagram, indicating the data flow in this control application, is presented in Fig. 2.3. A scanning sensor measures the sheet properties, producing a high-resolution raw measurement profile. After each scan, measured signal is dynamically filtered in order to separate machine-direction (MD) and cross-direction (CD) components of the profle. An antialiasing filter is used to remove uncontrollable variations, before the profile is mapped to the actuator zones by an adaptive alignment algorithm. Applied filter is a digital finite impulse response (FIR) filter (Powell *et al*., 1996). In this application the adaptive alignment algorithm can take care of nonlinear, asymmetric shrinkage profiles and uneven actuator spacing. The algorithm uses information from the scanner regarding the location of the sheet edges to update the mapping automatically. An on-line tool (AutoMap) is used to verify the alignment and determine the shrinkage profile based on automatic response tests.

Once the profile is aligned to the actuators, an adaptive estimation algorithm is used to extract the true CD profile from the measured profile. This algorithm adjusts the amount of averaging applied to each region of the sheet, based on the measurement and process noise. For example, if the edges of the sheet have more random process variability, the amount of averaging is automatically increased, rejecting the random variation. The multiple frequency actuator control (MFAC) block provides a means of coordinating multiple sets of actuators. MFAC uses digital spatial filters to split the control profile into frequency components, which can be in a best way controlled by each actuator set. The resulting control profile is compared to the target profile and the error from target is determined. The error is then passed through decoupling blocks to compensate for any spatial interaction across the web of the sheet. A grade-dependent, model based de-coupling is also possible.

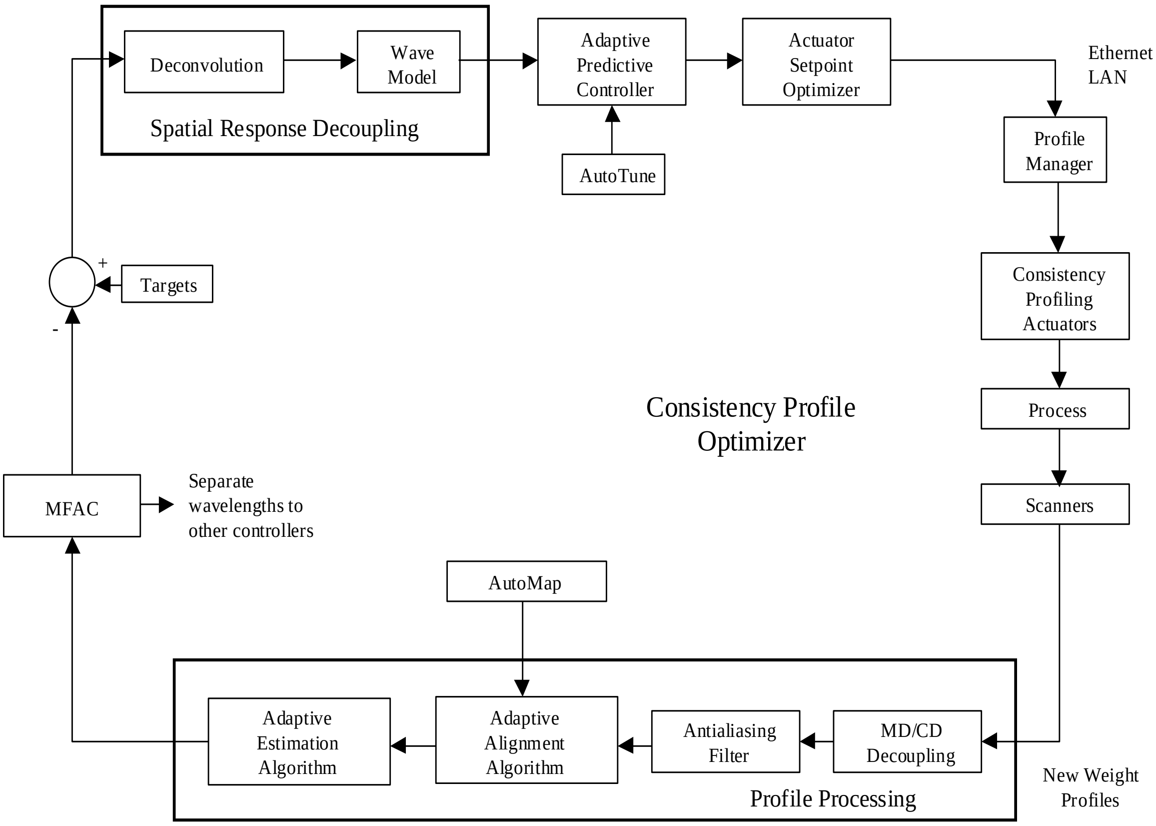

Figure 2.3: CD basis weight control strategy block diagram (Vyse *et al.*, 1996).

In this application an adaptive predictive controller determines the actuator positions or setpoint changes. Controller compensates for the delay between actions taken and the time it takes to see them at the scanner. A model of the process, obtained with an on-line tuning tool (AutoTune), determines the system delay time constant and gains. The controller adjusts automatically the closed-loop time constants, which are defined based on the nature of the profile variation. Once the changes to the actuator setpoints are defined, the overall setpoint average is adjusted. This can be used to ensure that the average of the profiling actuators is high enough to provide adequate control range in both directions. Actuator setpoints are sent to the I/O interface unit (Profile Manager) across an Ethernet local area network (LAN), and finally, the Profile Manager transmits new setpoints to the actuators.

Chapter 3

Modeling and Control of Cross-Directional Processes

Process models are used to describe different kinds of physical and chemical phenomena in the form of mathematical presentation. A mathematical model can be considered to be a group of equations, which represent the physical system and describe the connection between the system variables. Models in general can be classified in many ways. There are probabilistic and deterministic models, empirical and mechanistic, discrete and continuous, lumped and distributed. However, the main objective of mathematical modeling is to find a presentation that characterizes the real world phenomenon accurately enough. On the other hand, too complicated models should be avoided because the practical application of cumbersome models might prove to be troublesome. Models provide efficient, sometimes the only means of evaluation the results of alternative choices; for instance, a model is essential in cases where experimentation with the real world system is too expensive, dangerous, or even impossible (Denn, 1986).

The design of feedback controllers is always model-based in one form or another. A model is created in order to get a view how the process will respond to the control actions. Due to process complexity and absence of physical modeling, the CD process models are usually identified from measured input-output data (Chen, 1992). The spatial and dynamic models of the CD process are commonly assumed as a constant transfer matrix combined with linear low-order transfer function models (Duncan, 1989), (Laughlin *et al*.,1993). However, this assumption is only an approximation of the real industrial CD process, and in reality a high degree of uncertainty is related to the response models of actuators.

Another reason for the model mismatch is the process disturbances. In paper production many process disturbances can appear, which affect directly to the quality of CD profiles. These disturbances may be either steady-state or dynamically varying. Typical examples of CD disturbances are uneven flow of stock across the headbox, moisture streaks, sheet wraps on dryer cans and uneven nip pressure. A more thorough examination of process disturbances may be found in (Gullichsen and Paulapuro, 1999).

In this chapter we will first define a cross-directional control problem and present a category of CD response models, which will be used throughout the thesis. Model uncertainties in the CD process are discussed in terms of operating point, actuators and structure of model errors. After that a brief survey to the early history of CD control and reported CD control strategies is presented. Linear quadratic, model predictive and robust CD controls are reviewed.

3.1 Cross-directional response models

All sheet-forming processes can be considered as distributed parameter systems, where the output variables change over time and place. A mathematical solution of this kind of problem requires a solution of a partial differential equation (Duncan, 1989). If the parameters of the distributed process are assumed to be constant and if the process is assumed to behave linearly within a reasonable operating region, the process may be characterized by a linear partial differential equation with constant parameters over its domain of definition. However, even in this case the general solution may be very complex for control purposes. One problem with this method is that a partial differential equation formulation of the response has to be acquired from the physics of the process and in practice there are always uncertainties associated with this formulation. The response is usually different for different operating points of the process during grade change. In addition, the partial differential equations tend to be very complex, and demanding computation can be required to produce the desired control actions (Duncan *et al*.,1996).

3.1.1 An approximated model

In practical CD control applications the distributed parameter response model is often reduced to an approximated model by using an *interaction matrix based approach*. In this approach, it is assumed that the response of the actuators is separable, in the sense that the response can be expressed as the product of a fixed spatial (CD) response and a scalar dynamic term. (Duncan, 1989), (Duncan *et al*.,1996). This separation of the actuator response means that when a change is applied to the set point of an actuator, the CD shape of the response remains fixed and the actuator dynamics simply change the amplitude of this CD shape. Commonly it is also assumed that the response of the full array of actuators can be presented as the product of a scalar dynamic term and the steady-state spatial responses of each of the actuators. This description of the system is infinite-dimensional, but by expressing the spatial response in terms of a basis function expansion and truncating the higher order terms, a finite-dimensional description of the system is obtained. The simplest method of reducing the problem to finite dimensions is to divide the sheet into a number of zones with one zone for each actuator, and then to average the measured response over each zone. It is convenient to assume that the actuators are equally spaced and that they all have the same shape of response and identical dynamics (Duncan, 1989), (Wilhelm and Fjeld, 1983). This assumption reflects the arrangement of most practical cross-directional control systems, which consist of an array of equally spaced, identical actuators (Chen and Wilhelm, 1986). The only exception from this assumption occurs when some of the actuators, usually those at the edges of the web, have a different shape of response due to the constraints of the process (Kniivilä, 2003), (Chen and Wilhelm, 1986).

Sensor measurements are taken after some form of processing and they are located at certain distance down the machine-direction from the actuation. The normal mode of operation in most practical CD control systems is to take a measurement of the web and then to use this profile to calculate the new set points for the actuators. The actuator set points remain fixed until the next profile measurement is taken and new set points are calculated. This kind of approach means that continuous variations in the web profile are sampled in two directions and operation produces responses, which are discontinuous in the time domain, but continuous in the spatial domain. Therefore the original distributed parameter system model may be reduced to a system model which is discrete in both the cross-directional (CD) spatial domain and the time (MD) domain (Duncan, 1989), (Duncan *et al*.,1996). This kind of system can be represented as a *sampled data lumped parameter system,* which characterizes the response between a finite number of actuators and a finite number of measuring points. If it is assumed that there are m measuring points and n actuators, then the response of the actuator array is given by a discrete-time model (Duncan, 1989), (Duncan and Bryant, 1997), (Heath, 1996).

$$y(z) = G(z)u(z), \qquad G(z) = g_a(z)G_o \tag{3.1}$$

where $y(z) \in R^m$ is the vector of measurements in z-domain, $u(z) \in R^n$ is the vector of actuator set points in z-domain, $g_a(z)$ is the scalar function describing the dynamics of each of the actuators and $G_o \in \mathrm{R}^{mxn}$ is a constant steady-state impulse response matrix whose ij^{th} element contains the response of the i^{th} actuator. This equation describes a linear multivariable system. Similar presentation can be derived for a continuous time model, which is commonly used in the literature (Wilhelm and Fjeld, 1983), (Chen and Wilhelm, 1986), (Gräser and Neddermeyer, 1986). Process disturbances like MD variations, CD variations and gauge noise may be connected to the model as uncorrelated white noise (Duncan, 1994b), (Duncan and Corscadden, 1996). It should be noted that the assumption about each of the actuators having identical dynamics allows to define $g_a(z)$ as a scalar, so that the impulse matrix contains only real values. In practice, it is likely that this assumption is not necessarily true because individual actuators age and deteriorate at different rates. In the impulse response matrix G_o, the partial g_{ij} represents the static gain between the control adjustment at the i^{th} CD actuator and the profile change at the corresponding CD location. The partials, g_{ij} for $j \neq i$, are the spatial coupling gains between the control action at the j^{th} actuator and the profile change at the i^{th} CD position (Chen and Wilhelm, 1986). Since the shape of the response of each of the actuators is assumed to be the same, the shape of each of the columns is the same, except that the center of each response is shifted.

3.1.2 Model characteristics

The impulse matrix G_o, also known as a response model, interaction matrix or process transfer function (Chen and Wilhelm, 1986), (Featherstone and Braatz, 2000) is in practice seldom known beforehand. Therefore the partials must be estimated with on-line or off-line identification methods. There are many ways to accomplish this estimation (Chen *et al.*, 1986), (Wilhelm, 1982), (Corscadden and Duncan, 1996), (Wang *et al.*, 1993a), (Featherstone and Braatz, 1995), (Wellstead *et al.*, 1998). Basically it involves perturbing several actuators and using least-squares estimation or correlation studies to find the steady state gain from the actuators to the measurements over an area of the measured profile corresponding to the perturbed actuators. One convenient way to determine the response of actuators is to observe the effect of a bump test as outlined in Fig. 3.1.

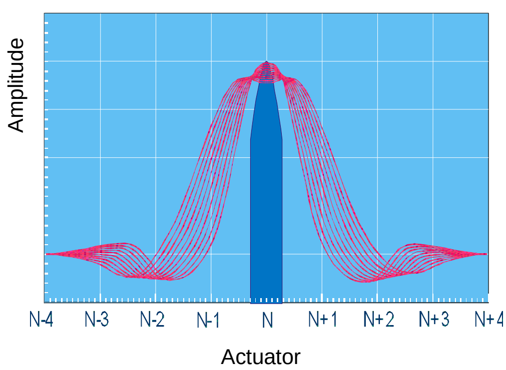

Figure 3.1: A scene of uncertain response of an actuator

A step is applied to the setpoint of a single actuator, while CD system is running in open loop so that the setpoints of all other actuators remain fixed. For a single actuator, several estimations might have to be run, one for each measurement, and then from those estimations the response shape can be obtained. The experiments are then repeated for several actuators across the machine. The response shapes from those several actuators are generally averaged to get a single response shape to represent the response to any actuator across the machine.

The complexity and the difficulty of the CD profile control are related to the nonzero spatial coupling partials (Chen and Wilhelm, 1986). The spatial coupling for each actuator usually spreads over a certain width in the cross-machine direction and has negligible effects on the rest of the profile. Thus, the process model G_o is a band-diagonal matrix as shown in equation (3.2). The partials beyond the certain width from the diagonal are all zero. G_o as its name suggests, represents the spatial influence of each actuator on the system outputs. This kind of model follows from the assumption that changes observed downstream from one actuator caused by adjustment at nearest neighboring actuator is independent of position across the machine. This model structure is the most often found in the literature (Laughling *et al.*,1993).

$$G_o = \begin{pmatrix} g_1 & g_2 & \cdots & g_m & 0 & \cdots & \cdots & 0 \\ g_2 & g_1 & g_2 & \cdots & g_m & \cdots & \cdots & \cdots \\ \cdots & g_2 & g_1 & g_2 & \cdots & \cdots & \cdots & \cdots \\ g_m & \cdots & g_2 & \cdots & \cdots & \cdots & g_m & 0 \\ 0 & g_m & \cdots & \cdots & \cdots & g_2 & \cdots & g_m \\ \cdots & \cdots & \cdots & \cdots & g_2 & g_1 & g_2 & \cdots \\ \cdots & \cdots & \cdots & g_m & \cdots & g_2 & g_1 & g_2 \\ 0 & \cdots & \cdots & 0 & g_m & \cdots & g_2 & g_1 \end{pmatrix}_{nxn} \tag{3.2}$$

One of the major problems with band-diagonal matrices is that they can easily approach singularity - their condition number can grow infinity (Wilhelm and Fjeld, 1983), (Laughling *et al.*,1993). Processes with high condition number can be difficult to control, especially in the presence of uncertainty in the interaction matrix. Because of the uncertainty, the direction of a large input signals may be much larger than expected from the model. In the limit as the G_o matrix becomes singular, the plant is uncontrollable, as the actuator settings can not be determined based on the measurements (Skogestad *et al.,* 1988).

3.1.3 A square response model

Square CD models have been used commonly in the CD control literature (Laughling *et al.*,1993), (Stewart, 2000), (Stewart *et al.*,2003), (Hovd and Skogestad, 1994), (Hovd *et al.*, 1997). Also in this work a square response model G_o is considered. Use of square response model can be argued for a concept of controllable subspace. Duncan (1989) has shown that the space of completely controllable profiles is equal to the dimension of actuator responses. All uncontrollable high frequency components of profile measurements should be lowpass filtered before sampling, so that the profile signal could be sampled at intervals equal to the distance between actuators. A more profound theory behind this statement is based on the concept of almost spatially-invariant processes (Duncan, 1989), (Duncan and Bryant, 1997). If the number of actuators is assumed to be large, the output profile of the CD process can be obtained using convolution of the input profile and response of a single actuator (Duncan and Bryant, 1997). Further, the output profile can be transferred from the spatial domain into the spatial frequency domain by the Fourier transform, and the spatial bandwidth of the whole array of actuators depends on the spatial frequency response of a single actuator. As a result, spatial frequency approach provides a method to separate the controllable and uncontrollable components of the CD disturbances in the spatial frequency domain.

In several practical CD control applications a square response model is very common (Chen and Wilhelm, 1986). Although a high-resolution profile with hundreds of measurement points is available for control, in practice this profile is anyway mapped down to the actuator resolution by using constant mapping matrix (Stewart *et al*., 2000), (Heaven *et al.*,1993a). Thus the number of actuators is equal to the profile measurements and a square model is applied. However, there are two main problems associated with this matrix form that are also typical for a non-square G_o matrix. The most serious problem is the mapping, that is, determining which actuator belongs to which measurements data boxes of the profile (Carey *et al*., 1975), (McFarlin, 1983), (Amyot and Nuyan, 1989). Another problem is how to model the edges (Kniivilä, 2003). If the edges are not accounted for, the model might show some edge effect, that are not present in the real process. The edges of the interaction matrix might have to be identified separately otherwise the model might create some phantom ripples at the ends of the profile.

Due to the symmetric inherent in the physical architecture of the web forming processes, the interaction matrix G_o usually has the structure of a Toeplitz symmetric matrix (Laughling *et al*.,1993), in which the same constant element is repeated along each diagonal of the matrix. However, centrosymmetric and circulant symmetric interaction matrices have also been reported (Laughling *et al*.,1993), (Stewart, 2000).

The matrix T is said to be a Toeplitz matrix if the scalars (c_{-p+1}, …, c_o, …, c_{p-1}) are such that the $(i,j)^{th}$ element of matrix T is c_{j-i}. The matrix shown in (3.3) can be expressed as T = Toeplitz (c_{-p+1}, …, c_o, …, c_{p-1}). In general, the Toeplitz matrix is not a symmetric matrix. However, in a special case when $c_{-i} = c_i$, it is a symmetric matrix (Gray, 2002).

$$T = \begin{pmatrix} c_0 & c_1 & c_2 & . & . & . & . & c_{p-1} \\ c_{-1} & c_0 & c_1 & . & . & . & . & c_{p-2} \\ c_{-2} & c_{-1} & c_0 & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & c_0 & c_1 & c_2 \\ . & . & . & . & . & c_{-1} & c_0 & c_1 \\ c_{-p+1} & c_{-p+2} & . & . & . & c_{-2} & c_{-1} & c_0 \end{pmatrix} \tag{3.3}$$

For example, the constant square band-diagonal interaction matrix $G^{n,n}$ with *m* independent spatial coupling partials in equation (3.2) can be expressed as a Toeplitz symmetric matrix: $G^{n,n}$ = $\text{Toeplitz}_n(0, ... ,0, g_1, g_2, \dots , g_m,0, \dots ,0)$. This matrix structure will occur repeatedly throughout the text, and therefore is abbreviated using notation adopted from the software literature (Matlab, 1996).

$$G^{n,n} = \text{toeplitz}_n\{g_1, \dots , g_m,0, \dots ,0\} \tag{3.4}$$

3.1.4 Process dynamics

The process dynamics in (3.1) is attributed to the actuators $g_a(z)$, which are usually represented as low order, stable and minimum phase except for the transport delay. A continuous time model of the actuator dynamics is generally presented with a first-order-plus-dead-time model, which is defined in terms of gain, time-delay and a single negative pole (Laughling *et al.*,1993), (Stewart, 2000), (Bergh and MacGregor, 1987).

$$g_a(s) = \frac{k_a e^{-\tau s}}{\tau_a s + 1}; \quad \tau > 0, \ \tau_a > 0 \tag{3.5}$$

The discrete time model of the actuators has a similar form of a first order transfer function with a delay (Kristinsson and Dumont, 1996), (Backström *et al.*, 2000), (Haznedar and Arkun, 2002), (Rigopoulos, 1999).

$$g_a(z) = \frac{z^{-d}}{1 - az^{-1}} \tag{3.6}$$

In regard to the physics of the paper production process, the delay is usually significant since there is distance between the location of the actuators and the array of sensors. Typically the delay d, presented now in terms of sampling rate, is in the range $0 \le d \le 4$ and the pole $0 \le a \le 1$. Equations (3.5) and (3.6) give the transfer function representation of the dynamics of a single actuator. However, since all actuators in a single bank of actuators are assumed to be identical, the same transfer function is used for each actuator. In many cases it has been assumed that the spatial responses of the actuators do not have any dynamic response (Chen and Wilhelm, 1986). This allows the use of spatial responses, which are independent of time. If the lengths of the transient of the dynamic response of the spatial responses are less than the sampling rate, then the measurements do not observe the dynamics, which mean that the assumption is valid, when the sampled model is used. If the length of the transients is longer than the sampling rate, then it is necessary to take the dynamics into account.

3.2 Model uncertainties in the CD process

In practical applications several facts can cause imperfections to the CD process models. There will be uncertainty associated to the estimation procedure, and it is likely that the shape of the response will change during processing, particularly when changing to a different grade of material. A standard procedure is to make a series of step response or bump tests with a set of selected actuators and record the observed responses to the quality variables. Tests can be made manually or automatically. The position of the CD actuator is changed in order to get change in a corresponding measured profile (Heaven *et al.*, 1993b). However, there is no guarantee that the responses of the unmeasured actuators will be the same as those that are tested. Especially at the sheet edges the CD response shape may be very different (Kniivilä, 2003).

An especially critical factor to the success for a CD control is the spatial relationship between the measurements taken at the scanner and the actuators used to control the profile. Due to shrinkage, sheet wander, and the physical distance between actuators and sensors, this relationship is surprisingly difficult to determine with the required precision. The center of the produced CD response in the profile is the mapping position of the actuator. A major difficulty with implementing a CD control system is determining the CD location of the effect of an actuator movement (Carey *et al.*,1975). (McFarlin, 1983), (Amyot and Nuyan, 1989). Using the results from bump tests on a few actuators is a relatively crude method to determine the mapping of the whole array of actuators. The shape of the response is assumed not to vary in the cross-direction. However, in practice the paper sheet may wander or shrink over time, leading to alignment error and mismatch between the true location of an actuator's effect and its predicted location. Trim squirt measurement error, edge measurement error, mapping error, and error from web wandering during one scan will affect to the accuracy of mapping, and in the worst case the total error can be centimeters.

On the other hand the MD-CD cross coupling is another source of model error (Tong, 1975), (Chen, 1992). The scanning sensor is traversing back and forth across the moving paper tracing a diagonal path on the sheet. Therefore, in addition to the usual measurement noise, the sensor will alias certain MD variations into the CD profiles which are generally not identifiable and controllable (Shakespeare, 2001). The amplitude of the response may also vary. In the case of basis weight variables like total dilution ratio, consistencies and profile dry weight average affect to the amplitude of the response. The procedure is very much operating point and grade dependent and usually the step response is performed to different grades in order to calculate relative gains to different reference grades.

3.2.1 Role of operating point

Typically a paper machine, calender and coating machine are designed to produce a very wide range of products. The heaviest grade is often twice or more heavier than the lighter grades of paper. In paper machine the operating conditions, such as machine speed, slice opening and headbox consistency, may range between broad limits (Gullichsen and Paulapuro, 1999). Actually the most distinct characteristic is the varying profile response to the slice screws on headbox. The profile response shape and response width of the slice screw is highly dependent on the machine speed and the dewatering process on the wire. Edge profile problems are also very common for almost all paper machines. For heavier weight paper machines, the edge profile problems are significantly worse. On the other hand, at soft calender process conditions like linear load, roll temperature and machine speed with soft cover material properties and used actuators define the shape, width and amplitude of the CD response (Vyse *et al.*,1993), (Svenka and Minkenberg, 1995), (Gullichsen and Paulapuro, 1999). In coating machine the blade angle, blade load, properties of the coating colour and machine speed affect properties of the CD response (Ellilä, 1994).

All the characteristics of the process are connected to the process response model, which quantifies the change in the CD profile to the change in actuator position. Process dynamics, rise time and transport delay change over time, with changing process conditions. Because the grade range can be large, unique process response models must be determined for each grade and operating point. Therefore if for a specific grade the operating conditions of the machine change, the process model should also change or to be updated. Several methods to accomplish this identification of the response model have been presented in the literature (Wilhelm, 1982), (Chen, 1988), (Wang *et al.*,1993b), (Wellstead *et al.*, 1998). In practical paper machine control applications grade and operating point specific process model and controller parameters are pre-tuned and stored in a recipe data base, from where they are obtained with an on-line tool (Vyse *et al.*, 1996) or with a grade change application software. A detailed description of the grade change on a paper machine can be found in Viitamäki (2004).

3.2.2 Model errors caused by actuators

A wide selection of actuators is available for CD control purposes depending on the object of the controlled process. A traditional way of performing CD basis weight control is to change the position of the slice lip of the headbox in the cross machine direction. Several methods exist: motors, thermal and hydraulic actuators and motorized robots (Cutshall, 1991). In modern dilution headboxes the basis weight control is done by using dilution valves and white water injection to change the composition of material (Vyse *et al.*,1996). Use of dilution to control the dry weight profile does not guarantee good fiber orientation profile. Thermal deformations and minor defects can vary with operating conditions. Jet misalignment leads to fiber orientation problems especially at low rush to drag values. In coating machine CD coat weight control is done by adjusting rods similar to slice screws in paper machine headboxes to modify coating blade pressure at intervals across the web (Ellilä, 1994), (Hoeke, 1990), (Sollinger, 1990). With super and soft calenders for the control of paper CD thickness three main technologies are available: induction heating systems, confined air showers, and hydraulic zone-controlled CD rolls. For the control of paper smoothness and gloss, steam shower technology has established its position as a leading method. Modern steam profilers use electromechanical actuators with synchronous motors and feedback signals for accurate positioning. However, the type of the CD actuators affects on the possible forms of the model error. Inaccurate calibration of actuator electronics and position feedback sensors like LVDT sensors are well known sources of the errors. Mechanical coupling, clamping and backlash problems are common to all spindle actuators and electric motor applications (Cutshall, 1991). An insufficient cooling of electronics at high temperatures can cause a drifting problem of measurement values and other serious malfunctions. Unstable zero points with constant calibration procedures are common to many hydraulic systems. In practice, a possible actuator failure should also be taken into account. In industrial process an uneven wear or tear is common and can easily lead to the actuator nonlinearities.

3.2.3 Appearance of the model errors

We may conclude that in practice a notable uncertainty is related to the CD response models and it should be taken into account in the control strategy. Generally model uncertainty is presented by describing the process model as a set of plants G_p, given by a nominal model G and a set of norm bounded perturbations Δ. Model uncertainty can be either unstructured or structured (Skogestad and Postlethwaite, 1996). Unstructured uncertainty is defined as a *full* complex perturbation Δ, while structured uncertainty usually means that the perturbations can be arranged into block diagonal form $\Delta = diag(\Delta_i)$. In the literature usually six major types of multivariable uncertainty descriptions are presented: additive, multiplicative input, multiplicative output, inverse additive, inverse multiplicative input and inverse multiplicative output (Zhou *et al.,*1996). Additive and multiplicative output uncertainties are the most commonly used to represent unmodeled process dynamics. Additive uncertainty, which typically represents unmodeled process dynamics is presented as a *full* matrix, whereas multiplicative uncertainties are presented as being either *full* or diagonal. Diagonal uncertainty can be represented as having diagonal elements that are independent scalars or repeated scalars.

Featherstone and Braatz (1998), (2000) used an additive unstructured model uncertainty based on uncertain pseudo-singular values in their CD controller study. Stewart *et al.* (2000) and Duncan (1994a) utilized a similar uncertainty structure to describe a paper machine plant-model mismatch. Stewart *et al.* (2000) applied this approach also to two industrial CD process cases. Laughlin (1988) and Laughlin *et al.* (1993) applied a structured model uncertainty description in their CD modeling work. The authors addressed the parameter uncertainty for both dynamic response and spatial response. Gorinevsky and Stein (2001) used similarly a structured uncertainty representation for a process model, which was described by rational transfer functions of spatial and dynamic variables.

Gorinevsky and Heaven (1997) studied also the model uncertainty structure in terms of typical CD response shapes by defining an empirical expression for a continuous response shape. Their expression describes well the majority of practically encountered CD responses. The parameterized structure models the effect of an actuator on the continuous paper profile. In their presentation four scalar parameters are used to describe the shape of the response. These CD model parameters are: the gain parameter, the attenuation parameter, the width parameter and the divergence parameter for describing the spatial response shape. The influence of these parameters is depicted in Fig. 3.2, which shows the nominal steady-state responses to a single actuator and the responses with parameter uncertainties in the spatial domain. In Fig. 3a-d, only one parameter has uncertainty. The effect of the gain parameter uncertainty, which affects directly the amplitude of the response, is shown in Fig. 3a. The attenuation parameter, as shown in Fig. 3b, changes the size of the negative lobes of the response. For large parameter values these lobes are not observable, for smaller ones, they are more profound. The width parameter, as it’s name expresses, affects the width of the response, especially at the bottom lobes of the response, as shown in Fig. 3c. On the other hand, the divergence parameter, as presented in Fig. 3d, defines the presence of two maxima. This kind of behaviour is especially typical for heavy grade papers.

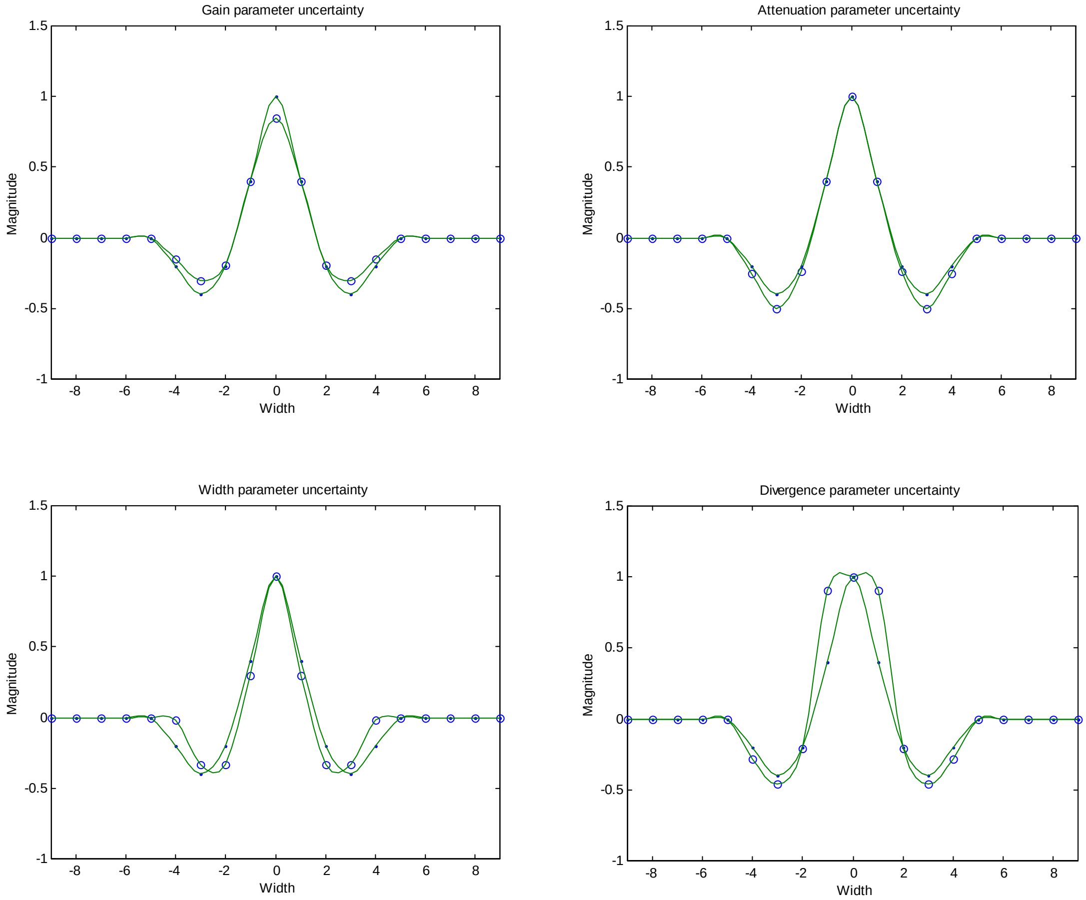

Figure 3.2: Influence of the uncertainty parameters on the shape of response. The nominal spatial response (solid + dotted) and its corresponding one with uncertainty (solid + circle): (a) the gain parameter uncertainty; (b) the attenuation parameter uncertainty; (c) the width parameter uncertainty and (d) the divergence parameter uncertainty.

We will use this presentation to illustrate the uncertainty structure of our response model. If we consider the row structure of the CD actuator system it is clear that in reality each manipulated CD actuator is a source of the input uncertainty. As shown by Duncan (1994a) an independent diagonal uncertainty description would be more adequate for characterizing inaccuracies in the CD actuator models because each actuator is expected to have a slightly different dynamic response. On the other hand, a repeated diagonal uncertainty description may be appropriate for modeling inaccuracies in the sensor model, because the sensor is usually of the traversing type and the same sensor is used for all measurements. Due to the physical structure of the CD control system, we may say that, diagonal input and output uncertainties are always present in the real CD control system. Diagonal input uncertainty represents inaccuracies associated with the actuators and diagonal output uncertainty represents inaccuracies associated with the measurement devices.

We assume that the true CD process response belongs to a set of possible response models (Laughlin *et al.*, 1993), described in continuous form by

$$G^{n,n}(s) = g(s)G^{n,n} \tag{3.7}$$

where the elements g_i of the constant interaction matrix $G^{n,n}$ are constrained as in $g_i \in [g_{i\min}, g_{i\max}]$, and *g(s)* is defined by the first order actuator dynamics $g_a(s)$ and time-delay $g_d(s)$

$$g(s) = g_a(s)g_d(s) = \frac{k_a e^{-\tau s}}{\tau_a s + 1} \tag{3.8}$$

$$k_a \in [k_{a\min}, k_{a\max}], \tau_a \in [\tau_{a\min}, \tau_{a\max}], \tau \in [\tau_{\min}, \tau_{\max}]$$

Each real parameter in the actuator dynamics in (3.8) is permitted to vary between the specified upper and lower bounds independent of the other real parameters. Uncertainty in actuator scalar dynamics is described by the bounds for k_a and τ_a; time-delay uncertainty by the bounds for τ. This kind of description of the CD model uncertainty covers the structured model uncertainty and especially block diagonal input and output uncertainties. Therefore the previously mentioned uncertainties related to identification method, operating point and actuators are included into the approach. Structured model uncertainty is also called parametric uncertainty, because the structure of the model is known, but some of the parameters are uncertain. Alternative approach for parametric uncertainty is to assume a probabilistic distribution of the parameters, and apply the average response. However, this kind of stochastic uncertainty is difficult to analyze (Skogestad and Postlethwaite, 1996). Another well applicable approach for parametric uncertainty is the multi-model approach in which a finite set of alternative models is considered. In this approach the performance is measured in terms of the worst-case or some average of these models' responses. We will discover that the multi-model approach is applicable to describe a set of uncertain CD response models.

3.3 Reported CD control strategies

Control systems for regulation of cross-machine (CD) paper sheet properties have been on-line in the paper mills for over two decades. Several control schemes and algorithms are applied and reported in the literature. According to Dumont (1986) the pulp and paper industry is, in general, conservatively using well established control methods. Therefore, it is not surprising that very traditional controllers, like PI controllers with output de-coupling features, are still extensively used for CD control purposes in the paper industry Heaven *et al.,* (1993a). In this section we will discuss the main CD control methods presented in the literature. The survey does not cover the whole topic but will give an overview to the matter.

3.3.1 Early history of CD control

CD control first became possible with the invention of sensors to measure the paper across the sheet. Burkhard and Wrist (1954) were the first to classify basis weight variations into three components. Initial efforts at reducing variations concentrated on the control of MD variations, mainly because CD sensors were not available and more computer power was required for control of cross-directional variations. In 1967 Åström published one of the early papers on computer control of paper machines (Åström, 1967). This paper concerned the machine direction control of basis weight and a minimum variance controller was designed by use of linear stochastic control theory. Two years later Dahlin (1969) presented an algorithm based on exponential filtering to extract the MD and CD components from raw data and gave a description of the sensors used for basis weight and moisture measurements.

A simple model structure of CD response (3.1) suggests that the system could easily de-coupled by introducing an inverse of G in the controller definition. Carey and co-workers (Carey *et al.*,1975) utilized this observation and reported a reduction in fibre usage following installation of a CD control system, which used the interaction matrix approach. Two years later Boyle (1977) showed that the interaction matrix approach could result in extensive bending or displacement of the slice lip and that the solution may well be infeasible. To avoid the bending problems two methods were introduced to limit bending and displacement of the lip. The first method, quadratic programming, minimized variations subject to hard limits on the actuator movements. However, it was found to be computationally intensive (Boyle, 1978). The second method, minimization of quadratic objective function, penalized actuators moves or bending of the slice. Boyle (1977) described a quadratic programming formulation for simulation and observed that computational considerations limit the applicability of the method to only a few actuators. In Wilkinson and Hering (1983), a linear quadratic optimal design using on line identification of the interaction matrix was used in industrial settings. After these work several applications based on the quadratic programming have been introduced. Due to popularity of linear CD control we will study it more in details.

3.3.2 Linear CD control

A linear-quadratic (LQ) optimization represents a very traditional mathematical programming manner to solve a multivariable constrained steady-state optimization problem, and minimizing or maximizing a quadratic performance index yields a straightforward linear control law. The basic control objective in design methods based on quadratic cost functions is to force the CD profile to some desired shape. An obvious approach is to try to choose the actuator setting to minimize the mean square deviation from the desired profile. Therefore, the performance index is directly proportional to the variance of the profile deviation (Wilhelm and Fjeld, 1983), (Chen and Wilhelm, 1986). If the process model is accurate, the optimal control corrects totally the profile deviation. However, an accurate model is normally not available and in practice the control actions are always limited by the constraints.

Constraints in steady-state CD control problem can be taken into account by defining additional vector inequality conditions and augmented performance index. An optimal solution of this new constrained optimization problem can be found by using a quadratic programming (QP). Chen and Wilhelm (1986) presented that the Kuhn-Tucker optimality conditions can be used to convert the problem into a linear complementary problem, which can be solved with a complementary pivoting algorithm. An optimal solution of the problem can be obtained with a finite number of pivots. However, the pivoting calculation is very sensitive to modeling errors if the process model *G* is ill-conditioned. Another way to solve the constrained optimization problem is to use quadratic penalty function (QPF) approach, which provides a sub-optimal solution (Chen and Wilhelm, 1986). Also in this method the original performance index is modified and a constrained optimization problem is converted to non-constrained minimization of modified performance index with a proper choice of defined penalties. The choice of penalties is done iteratively and an appropriate searching procedure with constraint-checking logic and repeated calculation of control is used. The achieved value for performance index is nearly minimal subject to the constraints. Chen *et al.* (1986) extended studies of quadratic programming (QP) and quadratic penalty function (QPF) methods. Even if QP gives an optimal solution, the time and memory requirements of this approach prohibit it’s use in real-time control, and therefore the authors turned to QPF.

Also Wilhelm and Fjeld (1983) reviewed some control algorithms for CD control in their study. They formulated control methods for both the inverse interaction matrix approach and for quadratic optimal design. They noted that strong coupling in the interaction matrix could result in the matrix being singular and therefore not invertible. They suggested that strong coupling simply represented a poor choice of actuator spacing. The problem could be avoided by controlling groups of actuators together or computing the control based on a wide spacing of actuators and then using interpolation to calculate the settings of the remaining actuators. Bergh and MacGregor (1987) used LQG (Linear Quadratic Gaussian) control theory to jointly control MD and CD moisture variations using spatially distributed actuators. They modeled the disturbances for a web forming process as a multivariate time series. Similarly LQG control with recursive identification to sequentially adjust the slice lip actuators with a traversing robot is presented by Halouskova *et al.* (1993).

3.3.3 Model predictive CD control

Model predictive control (MPC) is currently the most widely implemented advanced process control technology in process industry (Qin and Badgwell, 1997). While model predictive control is very popular in process industry it is less common in paper machine CD or other web forming process control. The reason for this is related to the large scale nature and uncertainty characteristics of CD processes. However, the main reasons for MPC favor in process industry is that it can easily handle multi-input multi-output systems and it provides a way to explicitly handle constraints on inputs and possibly outputs of the system. In MPC the control objective is optimized on-line subject to constraints and a linear or quadratic optimization is solved at each sampling instance. MPC strategies have been formulated using finite pulse or step response models, as well state space models (Maciejowski, 2001). Morari and Lee (1999) give a recent overview of the state-of-the-art in the field as well the issues that are still open. Characteristics of MPC will be discussed further in Chapters 5 and 6.

A limitation of MPC has traditionally been due to the enormous online computation load and its sensitivity to model uncertainty (Featherstone and Braatz, 2000). MPC CD optimization problems can be very large – over 200 variables and constraints for steady-state control of medium size web process. Solving an MPC CD problem is similar to solving a linear (LP) or quadratic program (QP) of size mn, where m is the control horizon and n is the number of decision variables. Typically n is the same as the number of actuators. The fastest QP algorithm (Nesterov and Nemirovskii, 1994) requires $O((mn)^3)$ flops to solve an MPC problem. A lot of efforts have been directed towards reducing the solution time for these large optimization problems and advances in computing power coupled with theoretical work in model reduction and robustness are beginning to make MPC techniques possible. One way to solve large scale CD problems is to reduce the size of the model. A general way to implement a model reduction is to use different kinds of basis functions to approximate the output and input profiles. Halouskova *et al*. (1993) used first order spline functions to describe the web profiles. Kristinsson and Dumont (1996) and Heath (1996) used Gram polynomials or discrete Chebyshev coefficients for model reduction. This method provides a convenient way to avoid inverses of large interaction matrices and reduces greatly computational requirements. Duncan and Bryant (1997) showed that the web profile of finite width can be separated into controllable and uncontrollable parts. Fourier transform is used to represent the controllable subspace in terms of spatial frequencies.

The other way to overcome the large scale characteristic of CD process is to use either quadratic programming (QP) or linear programming (LP) solvers. Campbell and Rawlings (1998) reduced the computational time of the QP problem by utilizing the characteristic of state-space matrices. Barlett *et al.* (2002) developed a fast QP solver, which uses the Schur complement algorithm. Backström *et al.* (2000) reported the use of MPC for CD control on a linerboard machine to control both CD weight and CD moisture. Their approach uses a fast QP solver and utilizes the sparse structure of CD process models. Achieved control performance and computational time of the MPC optimization were satisfactory. Dave *et al.* (1997), (1999) developed a linear programming MPC algorithm which may be used for large scale CD control problems in real time. Their algorithm is based on the fast computing of an approximate control action that is close to optimality. Once an approximate solution is obtained, it is used to initialize the basis of the original problem to obtain an exact solution.

3.3.4 Robust CD control

Robust control is a new modern sophisticated control method, which provides a way to incorporate model inaccuracies to the control strategy (Morari and Zafiriou, 1989). Robust control approach can be used in CD controller design by utilizing the special structure of CD response model matrices. Because most of the CD models are Toeplitz symmetric matrices, this special model structure can be used to reduce the computational expense associated with robust control. In addition, circulant matrix theory can been utilized to develop methods for designing conservative robust multivariable controllers based on the design of only one single loop (SISO) controller. Laughlin presented this kind of approach already in 1988. He suggested a MIMO (multiple input multiple output) robust control scheme based on internal model control (Laughlin, 1988). He was able to guarantee both robust stability and robust performance by assuming knowledge of the parameter uncertainty. A major drawback with this scheme was that the gain matrix describing the relationship between actuators and sensors had to be positive definite. Furthermore he did not take into account the constraints of the slice lip, so the control action from this scheme could not be realized. Laughlin *et al.* (1993) utilized circulant matrix theory to develop conservative robust multivariable controllers based on the design of only one single loop controller. Circulant symmetric, Toeplitz symmetric and centrosymmetric models were covered by the theory. However, the controllers were restricted to be either decentralized or decentralized controllers in series with a constant decoupler matrix. Another method presented also in (Laughlin *et al.* 1993) was a model-inverse-based control. However, this technique requires a well-conditioned interaction matrix, which is often impossible to guarantee in practice (Laughlin *et al.* 1993), (Heath, 1996).

One way to handle numerical difficulties related to the inverse of interaction matrix G**,** is to use a singular value decomposition (SVD) of matrix G, see Appendix 1. For plants where the singular vector directionality is independent of frequency, singular value decomposition method can be used to decouple a system into nominally independent subsystems of lower dimensions (Hovd, 1992), (Hovd *et al.*, 1997). Because the CD response model (3.1) is expressed as a product of spatial response G_o and a dynamic term $g_a(z)$, it is clearly suitable for singular value decomposition. Duncan developed a robust controller design algorithm for sheet process with arbitrary interactions across the machine based on this approach (Duncan, 1989), (Duncan, 1994a). Sufficient conditions for robust performance with multiplicative input and output uncertainties were derived in terms of satisfying robust performance for SISO subsystems. The controller satisfied Nyquist stability requirements for all control modes based on limits on maximum and minimum singular values. Braatz and VanAntwerp (1996) and Featherstone and Braatz (1997), (1998) proposed a modification of the singular value decomposition called pseudo-SVD, where the elements of diagonal matrix, referred to as pseudo singular values, are transfer functions and they have a sign. In (Braatz and VanAntwerp, 1996) a reduced order pseudo-SVD controller was suggested. Controller is based on the certainty of each pseudo singular value. Robustness of the controller is ensured by controlling only those directions whose corresponding pseudo-singular values' signs are known with sufficient certainty.

A robust loop shaping CD controller has been proposed Stewart (2000) and Stewart *et al.* (1999a), (1999b), (2003). The CD process is modeled as a linear, quadratic, circulant symmetric system with a norm bounded additive unstructured uncertainty. This method is related to the SVD approach and it is based on the eigenvalue-eigenvector decomposition of a symmetric matrix. The method is efficient when G is circulant, since any circulant matrix has a complete set of independent eigenvectors. In addition, every circulant matrix of the same order can be diagonalized by the same eigenvector matrix, namely the Fourier matrix (Davis, 1979). When the circulant matrix is symmetric, the analysis can be simplified further, because the associated eigenvector matrix can be chosen to be a real Fourier matrix. This is based on the fact that the eigenvalues of the circulant matrix appear in pairs and the corresponding eigenvectors can be chosen to be real (Stewart, 2000).

3.3.5 Two-dimensional CD control

Heath (1992) and Wellstead and Heath (1992) proposed to handle paper properties as a two-dimensional dynamic system with the cross and machine direction parts as two mutually dependent dynamic processes. In this approach, the response of both the actuators and disturbances are presented in terms of rational two-dimensional polynomials. The quality variations are modeled using two-dimensional ARMA models. The proper selection of the order of the two-dimensional models is included into the approach and similarly a recursive updating of the ARMA parameters. The advantage of this method is that it provides a convenient way of formulating the response of the system, without assumption of general separability. Present theory of two-dimensional systems covers methods for estimating responses (Heath, 1992), designing controllers (Heath, 1992), (Zarrop and Troyas, 1994), and determining optimal measurement (Gacon and Zarrop, 1996). However, as pointed out by Duncan (1999), controllers based on two-dimensional approach have not been implemented, because in practice the assumption of separability has to taken into account because of the sluggishness of the system.

Chapter 4

Use of RGA and DIC in Analysing Cross-Directional Control Systems

Since Bristol (1966) first proposed the relative gain array (RGA) as a means to choose pairing between inputs and outputs for decentralized control, the technique has gained considerable practical utilization. It is one of the most widely used techniques in the design of control systems for multivariable plants. Relative gain analysis is based on a relative gain array, which is a matrix of interaction measures for all possible single-input single-output (SISO) pairings of the variables considered. Thus RGA indicates the preferable variable pairings in decentralized control systems based on interaction considerations. Because only steady-state gain matrix is required, RGA analysis is usable for many practical process applications.

Originally, RGA was defined only at steady-state, but it is straightforward to include also dynamics. Further developments have shown that the RGA is much more than a simple measure of interactions. The RGA provides information about fundamental properties such as closed-loop stability, controllability and robustness with respect to modeling errors and input uncertainty. However, a frequency-dependent RGA can provide more detailed information than the steady state, especially regarding control performance. Main advantage of the RGA approach is that it is easy to use and only the process gains, which can be determined from a *steady-state response model G(0)*, are required. In addition it is scaling independent. Despite of its considerable popularity, the RGA has some shortcomings. It does not take into account the dynamics of the system and control loops may interact both in the steady state and dynamically, even if the RGA suggests that there is little interaction (Friedley, 1984). To deal with the deficiencies of the RGA, a number of complementing measures and procedures have been proposed. The block relative gain (BRG) and Niederlinski index (NI) are two measures of similar simplicity as the RGA. BRG is used to analyze the feasibility of a block-decentralized control structure and NI gives, among other things, a necessary condition for closed-loop stability of a block decentralized control system. However, we will use it with RGA to evaluate the applicability of integral control to the decentralized CD control problem.

Decentralized integral controllability (DIC) is a *plant dependent* characteristic, which defines the applicability of a diagonal integral control (Skogestad and Morari, 1992). The concept DIC is based on the demand that if a negative feedback is used under integral control, the sign of the plant gain must be known. Typically DIC analysis of the steady-state systems G(0) has been used, like RGA analysis, to choose pairing between inputs and outputs for decentralized control. However, DIC approach has some special features, which can be utilized to *classify* CD control problems.

In this chapter *steady-state based* RGA and DIC approaches are used to analyze the behavior of cross-directional (CD) control system in terms of *complexity of the response model G(0)* and *applicability of the controller structure*. As far as the author knows this kind of an analysis has not been applied to the CD control problems earlier. Almost all practical CD control algorithms are based on *steady-state response models G(0)*, which provides a good basis for the RGA and DIC analysis. The approach in this chapter is as follows. First basic definitions and characteristics of the methods are presented. Then, the applicability of methods for CD control problem is evaluated and a brief review to diagonal CD controller and general decentralized integral controllable (DIC) plant is done. Analyses are especially focused on the models with diagonal type of input uncertainty. After that, some ideas of applicability of the inverse-based steady-state controller for the CD control problem in terms of RGA analysis is presented. Finally, CD response complexity analysis based on RGA and DIC is done for some typical CD models and general observations regarding RGA and CD controller structure are presented.

4.1 Definitions and basic properties

4.1.1 Condition number and singular value decomposition

The matrix G is said to be ill-conditioned if its condition number is large. The condition number of a matrix is defined as the ratio between the maximum and minimum singular values, see Appendix A.

$$\gamma(G) \equiv \frac{\bar{\sigma}(G)}{\underline{\sigma}(G)} \tag{4.1}$$

Therefore a system is ill-conditioned if some combinations of the inputs have a strong effect on the outputs, whereas other combinations have a weak effect on the outputs. For a non-singular (square) matrix $\underline{\sigma}(G) = 1/\bar{\sigma}(G^{-1})$, so $\gamma(G) = \bar{\sigma}(G)\bar{\sigma}(G^{-1})$. A large condition number means that the system depends strongly on the input direction and it may cause serious control problems (Skogestad and Postlethwaite, 1996). Large condition number may be caused by a small value of minimum singular value, which should always be avoided. On the other hand, large condition number may express that the plant has large relative gain array (RGA)- elements or that the system is sensitive to full-block input uncertainty.

Because the condition number represents the maximum amount by which any relative uncertainty in G will be amplified and transmitted to the output, it is clearly a measure of error sensitivity. Similarly it is also a ratio between the gains in the strong and weak directions, which can be defined by using singular value decomposition. On the other hand, the minimum singular value of the system $\underline{\sigma}(G)$, calculated as a function of frequency, is a useful measure for evaluating the feasibility of achieving acceptable control. On the other hand, if the inputs and outputs have been scaled properly, then with a manipulated input of unit magnitude we can achieve an output magnitude of at least $\underline{\sigma}(G)$ in any output direction. Therefore the minimum singular value $\underline{\sigma}(G)$ indicates to which value we are able to track the reference changes without reaching input constraints (Skogestad and Postlethwaite, 1996). Generally $\underline{\sigma}(G)$ is required to be as large as possible.

The condition number is scaling dependent, i.e. dependent on the choices of units of the inputs and outputs. Therefore it should not be used as a measure of the inherent ill-conditioning of a process. A better measure in this respect is the relative gain array (RGA), which is independent of scaling (Skogestad and Postlethwaite, 1996). Another important issue related to the condition number is that it is a function of the system dimension. When the dimension of the matrix G increases, a system with originally a relatively low condition number may become singular, i.e. $\gamma(G) \rightarrow \infty$ because the model may be singular (Laughlin *et al.,* 1993).

The concept of process directionality is generally analyzed by using singular value decomposition (SVD), (Skogestad and Postlethwaite, 1996), see in details Appendix A. SVD analysis shows that the largest gain for any input direction is equal to the maximum singular value $\overline{\sigma}(G)$. SVD analysis shows also that the input singular vector relating to the maximum singular value $\overline{\sigma}(G)$ corresponds to the input direction with largest amplification, and the output singular vector relating to the $\overline{\sigma}(G)$ corresponds to the output direction in which the inputs are most effective. Similarly, least important, weak or low-gain directions of input singular vector and output singular vector are associated with $\underline{\sigma}(G)$. In addition, the high-gain direction is orthogonal to the low-gain gain direction.

4.1.2 Relative gain array

We will consider a linear *nxn* system described by the model

$$y(s) = G(s)u(s) \tag{4.2}$$

where $G(s)$ is stable and strictly proper matrix and the steady-state gain matrix $G = G(0)$ is nonsingular. The open loop gain from input u_j to output y_i is $g_{ij}(s)$ when all other outputs y are uncontrolled. Writing equation (4.2) as

$$u(s) = G^{-1}(s)y(s) \tag{4.3}$$

it can be seen that the gain from u_j to y_i is $1/[G^{-1}(s)]_{ji}$ when all other y's are perfectly controlled. The relative gain is the ratio of these *open-loop* and *closed-loop* gains. Thus the relative gains, the RGA matrix, can be computed using formula

$$\Lambda(s) = G(s)\otimes(G(s)^{-1})^T \tag{4.4}$$

where the $\otimes$ symbol denotes element by element multiplication (Hadamard or Schur product). A steady state RGA is obtained when the transfer functions are evaluated at $s = 0$. In this work only the steady-state RGA is considered.

Equation (4.4) implies that the open-loop gain G_{ij} between y_i and u_j will change by the factor λ_{ij}^{-1} when the other control loops are closed. This means that variable pairings corresponding to positive relative gains as close to unity as possible should be preferred. Negative relative gains or relative gains much larger than unity should be avoided and large negative gains are especially undesirable. A more detailed description of RGA, its interpretations and properties is presented in Appendix A.

4.1.3 Decentralized integral controllability

A fundamental control system design requirement is that a negative feedback is needed to guarantee stability under integral control. If the sign of the system model gain between a specific system model input and output changes as the other loops are closed, integral control is not possible. Therefore the sign of the system plant gain must be known in advance. One aspect, which is related to the diagonal CD controller is the concept of decentralized integral controllability, see Fig. 4.1.

Decentralized Integral Controllability. A plant *G* (corresponding to a given pairing) is defined to be decentralized integral controllable (DIC), if it is possible to design a diagonal controller which (*i*) has integral action, (*ii*) yields stable individual loops, (*iii*) is such that the system remains stable when all loops are closed simultaneously and (*iv*) has the property that each loop gain may be detuned independently by a factor ε_i ($0 \leq \varepsilon_i \leq 1$) without affecting the closed loop stability (Skogestad and Morari, 1992), (Morari and Zafiriou, 1989).

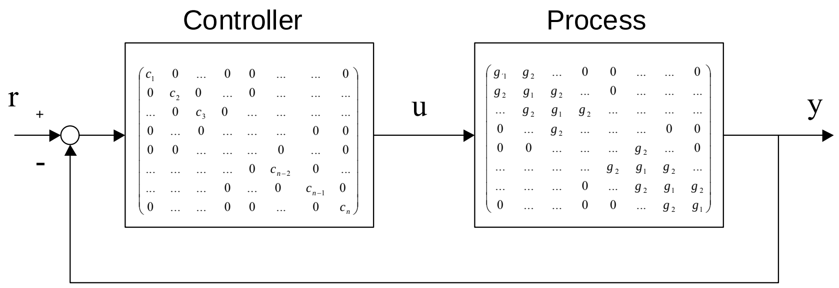

Figure 4.1: Decentralized CD control structure.

DIC implies that any of the control loops can be detuned or taken out of service without introducing any instability to the rest of the system. In terms of CD control applications this kind of requirement is very justified and reasonable, because commonly some actuators, especially those at the edges, are tuned separately. An important characteristic of DIC is that it *depends only on the plant*, i.e. it is independent of the choice of the controller. Therefore, we may say that it is an inherent property of the plant. A reason for this is that we are permitting each loop gain to be reduced independently, which is the same as allowing any ratio between elements in the controller. Thus, all potential diagonal controllers (at least at steady state) are considered (Skogestad and Morari, 1992).

4.2 RGA and DIC as analysis tools for CD control problem

As was shown in Chapter 3.1 we are considering cross-directional (CD) response models, which are presented as a linear multivariable system

$$y(s) = g_a(s)G(0)u(s) \tag{4.5}$$

where $y(s) \in R^n$ is the vector of measurements, $u(s) \in R^n$ is the vector of actuator set points, $g_a(s)$ is a scalar function describing dynamics of the actuators, and $G(0) \in R^{nxn}$ is a constant steady-state response matrix. This presentation is directly applicable to the RGA and DIC analyses, because we are interested in events, which are observed in the steady-state. However, we will not use RGA and DIC to choose pairing between inputs and outputs, instead are we interested in applying them for the *complexity analysis of CD response models* and *applicability of the controller structure*.

4.2.1 Role of model uncertainty

RGA is perhaps best known as a way to choose pairing between inputs and outputs for decentralized control. However, it has also an important application as an indicator of sensitivity to uncertainty. Let us consider the effect of uncertainties to the behavior of the CD control system. Skogestad and Morari (1987) have considered how especially a diagonal type input uncertainty affects the multivariable control system. We will utilize a similar approach and assume that each manipulated input is a source of the input uncertainty. Let $u_{ci}(s)$ represent the desired value of the i^{th} manipulated input as computed by the controller, and let Δ_i denote the relative uncertainty associated with this input. The input in the vector form is $u(s) = u_c(s)(I + \Delta_I)$, where $\Delta_I = \text{diag}\{\Delta_i\}$ depicts the diagonal input uncertainty. The perturbed plant is

$$G_p(s) = G(s)(I + \Delta_I) \qquad \Delta_I = \text{diag}\{\Delta_i\} \tag{4.6}$$

Let us consider the diagonal uncertainty in terms of loop transfer matrix as presented in Fig. 4.2.

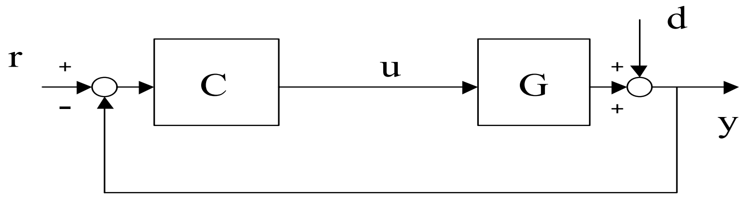

Figure 4.2: Closed loop control

The loop transfer matrix, $G_p(s)C(s)$, is related to the performance of control loop because of the identity $y(s) = (I+G_p(s)C(s))^{-1}d(s)$. $G_p(s)C(s)$ may be written in terms of the nominal $G(s)C(s)$ and error terms $C^{-1}(s)\Delta_I C(s)$ or $G(s)\Delta_I G^{-1}(s)$, as follows

$$G_p(s)C(s) = G(s)C(s)(I + C^{-1}(s)\Delta_I C(s)) \tag{4.7}$$

$$G_p(s)C(s) = (I + G(s)\Delta_I G^{-1}(s))G(s)C(s) \tag{4.8}$$

For multivariable systems the effect of the input uncertainty on $G_p(s)C(s)$ may be amplified significantly as shown by Skogestad and Morari (1987). For nxn plants the diagonal elements of the error terms $C^{-1}(s)\Delta_I C(s)$ and $G(s)\Delta_I G^{-1}(s)$ may be written as

$$(C^{-1}(s)\Delta_I C(s))_{ii} = \sum_{j=1}^{n} \lambda_{ji}(C)\Delta_j \qquad (G(s)\Delta_I G^{-1}(s))_{ii} = \sum_{j=1}^{n} \lambda_{ij}(G)\Delta_j \tag{4.9}$$

where $\lambda_{ij} = \lambda_{ij}(G)$ denotes the RGA elements of the plant. Controllers and systems with large RGA elements will indicate large elements in the matrices $C^{-1}(s)\Delta_I C(s)$ and $G(s)\Delta_I G^{-1}(s)$. Large elements in either of these matrices will lead to large elements in $G_p(s)C(s)$ and therefore unfavorable performance when input uncertainty is related. Equations (4.9) can be used as an analysis tool to evaluate the feasibility of the CD control structure. For example, the worst case combination of the input uncertainty can be found from the RGA. For an inverse-based controller the error term $G(s)\Delta_I G^{-1}(s)$ is directly related to the change in G(s)C(s). This kind of approach can be used in the simulation studies of the controller structure to evaluate its applicability for the problem. The approach contains characteristics of the RGA and it takes into account diagonal input uncertainty.

Chen and Seborg (2002) have also studied the influence of the process model uncertainty on the RGA analysis. They have shown that analytical worst-case bounds for RGA uncertainty can be derived in terms of a prescribed degree of uncertainty in square nominal steady-state gain matrix. Their approach covers also correlated uncertainties in elements of the gain matrix. In authors' study also a method for statistical uncertainty bounds is presented. Worst-case bounds can be used to analyze the maximum allowed degree of uncertainty in the model that will not influence to the controller pairing. Especially in the case of steady-state models it gives information about how much the operating conditions can vary before the chosen controller structure becomes inactive. This guarantees the robustness of the control structure for the whole uncertainty range of the plant model.

4.2.2 Diagonal CD controller

As it was pointed out in Chapter 3 a diagonal, fully decentralized controller has commonly been applied to the CD control problem (Heaven et al., 1993a), (Laughlin *et al.,* 1993). We will study now this controller structure more closely. Generally we can say that a diagonal controller has one special property, namely $\lambda_{ii}(C) = 1$, therefore the error term (4.9) will get a form $C^{-1}(s)\Delta_I C(s) = \Delta_I$. In this special case, the input uncertainty has only a light influence to the response. This justifies the use of diagonal controller in many practical CD control cases.

However, a diagonal controller is able to produce only a limited correction for the directionality of the system and $\gamma(GC)$ may be rather large. Therefore the response is dependent on the direction of the disturbance d on the output. Diagonal controllers do not usually correct for directionality of the system. Generally $\gamma(GC)$ is large when $\Lambda(G)$ has large elements. The model G with large RGA values always has large $\gamma(GC)$ values, and therefore a diagonal controller will produce a poor control performance. This is especially true when set-point changes are considered. However, as shown by Skogestand and Morari (1987), there is one special case when a diagonal controller can produce an acceptable performance for an ill-conditioned system with large condition number $\gamma(G)$. Namely when the system is naturally decoupled at the input, i.e. when the input singular vectors are equal to identity vector (V = I), see SVD description in Appendix A. Then the RGA elements $\Lambda(G)$ of the system are always less than 1 in magnitude. We may conclude that a diagonal, fully decentralized CD controller is insensitive or even robust with respect to input uncertainty, but it will be unable to compensate for strong couplings, as expressed by the large RGA elements, and therefore it will, even nominally, yield a poor performance. Therefore, it is not a surprise that the robust decentralized CD control approaches presented in the literature (Laughlin, 1988), (Laughlin *et al.,* 1993), (Duncan, 1994a), have confirmed this statement.

4.2.3 Necessary conditions for DIC

Definition of decentralized integral controllability (DIC) was presented earlier in section 4.1, next we will study some other important properties of DIC. Skogestad and Morari (1992) have derived *necessary* conditions for DIC to avoid input-output pairing where the plant gain may change sign. In terms of CD response and controller analysis, these conditions are extremely useful, because violating such a condition means that DIC is not possible for corresponding CD response model.

This follows because we are not trying to influence the choice of input-output pairings, which are already fixed by the design of actuator spacing and measurement mapping. Thus, *necessary* DIC conditions indicate if a decentralized integral controller can be applied to the chosen CD response model.

All *necessary* conditions, presented here as rules for DIC, are based on the fact that a negative feedback is required to guarantee stability under integral control. The proof of these conditions can be found from Skogestad and Morari (1992).

RGA- rule

Assume that C(s) is a diagonal controller and G(s)C(s) is proper, then

$$\mathrm{RGA}_{ii}(G(0)) < 0 \text{ for some } i \Rightarrow \text{not DIC} \tag{4.10}$$

Here $\mathrm{RGA}_{ii}(G)$ denotes the i'th diagonal element of the RGA of $G(0)$. If the sign of this gain changes as we change or close other loops, then we are not able to apply negative feedback in all cases, and the plant is not DIC.

Niederlinski Index (NI) – rule

$$\frac{\det(G(0))}{\prod_{i=1}^{n} g_{ii}} < 0 \;\Rightarrow \text{not DIC} \tag{4.11}$$

where $\prod_{i=1}^{n} g_{ii}$ is the product of the diagonal elements of $G(0)$. We should not use decentralized control on pairings which have the sign of the plant (given in terms of its determinant) different from the product of the plant gains for the loops. This is a condition for avoiding the use of positive feedback.

Morari Index of Integral Controllability (MIC) -rule

$$\min_i \mathrm{Re}\{\lambda_i\,(G^{+}(0))\} < 0 \;\Rightarrow \text{not DIC} \tag{4.12}$$

Here $G^{+}(0)$ denotes a plant steady-state gain matrix with the signs adjusted so that all diagonal elements have positive signs and $Re\{\lambda_i\}$ is the real part of eigenvalues. MIC -rule advises us to eliminate pairing with negative MIC index.

Skogestad- Morari (i) (SM(i)) –rule

$$Re\ \{\lambda(E(0))\} < -1 \Rightarrow \text{not DIC} \tag{4.13}$$

where $E(0) = (G(0) - G(0)_{diag})\, G_{diag}^{-1}(0)$ is an interaction matrix. Here $G(0)_{diag}$ denotes plant steady-state matrix consisting of only diagonal elements. SM(i) -rule advises us to eliminate pairing with negative values less than -1.

Skogestad- Morari (ii) (SM(ii)) -rule

$$Re\ \{\lambda(G^{+}(0)K)\} < 0 \ \Rightarrow \text{not DIC} \tag{4.14}$$

SM(ii) -rule advises us to eliminate pairing for which there exists a controller K(s) (diagonal matrix with positive entries) which yields negative values.

Skogestad and Morari (1992) have shown that NI -rule is redundant, because MIC -rule implies NI as a special case. However, none of rules RGA, MIC or SM(i) is mutually redundant, and all of them are applicable. MIC - and SM(i) -rules are special cases of SM(ii). They can be derived from SM(*ii)* -rule by choosing K(s) equal to identity matrix I and $G_{diag}^{+}(0)^{-1}$ respectively. SM(*ii)* -rule is difficult to test, and because it requires specifying a controller, it is not very useful for practical purposes. However, all above presented *necessary* conditions have to be met for a plant to be truly DIC. This means that a plant that does not pass these tests is not DIC, but there may be other plants that pass the tests, but still turn not to be DIC.

Skogestad and Morari (1992) have also derived *sufficient* and *necessary and sufficient* conditions for DIC. A *sufficient* condition is presented in terms of the structured singular value (Doyle, 1982) μ of E(0) as follows:

μ(E(0)) –rule

$$\mu(E(0)) < 1 \Rightarrow \text{DIC} \tag{4.15}$$

where interaction matrix E(0) is as defined earlier. $\mu(E(0))$ -rule can not be used to eliminate variable pairings, but it indicates that DIC is satisfied for a particular pairing. If the condition $\mu(E(0)) <1$ is satisfied, then the controllers for each loop may be designed independently.

A necessary and sufficient condition for DIC is presented in terms of finding a diagonal controller matrix $K(s)$ with real, positive (nonzero) entries. A system is DIC if the following condition is satisfied:

$$\min_K \min_i \mathrm{Re}\{\lambda_i(G^+(0)K)\} > 0 \Leftrightarrow \mathrm{DIC} \quad (4.16)$$

However, as pointed out by the authors, this condition is difficult to test, and it is not very useful for practical analysis. Authors have developed a numerical optimization method to test this condition.

Another common issue, related also to the diagonal controller, is the concept of *Integral Controllability* (IC) of the system. Definition of IC is very similar to DIC (Grosdidier *et al.,* 1985).

Integral Controllability. A system (plant and controller) is integral controllable (IC) if (*i*) the controller has integral action, (*ii*) the overall system is stable, and (*iii*) all controller gains may be reduced by the same factor ε ($0 < \varepsilon < 1$) without introducing instability.

We can observe that for IC all the gains are reduced by the same degree, while for DIC each loop may be operated separately. Thus IC is a property that depends on both the *plant and controller C*. As pointed out by Skogestad and Morari (1992), if a certain decentralized controller satisfies IC, it does not mean that the plant with these input-output pairings will satisfy DIC. However, the reverse, with any decentralized controller with positive loop gains, is true that DIC $\Rightarrow$ IC.

Skogestad and Morari (1992) have derived a *sufficient* condition for IC, which is based on the calculation of spectral radius of the interaction matrix $\rho(E(0))$.

$$\rho(E(0)) < 1 \Rightarrow \mathrm{IC} \quad (4.17)$$

For CD response and controller structure analysis IC is not as convenient as DIC, but in control literature it has been used quite commonly. Yu and Luyben (1987) combined IC with RGA analysis and defined a perturbation equation, which gives an upper limit on the maximum change in a single process gain element that will guarantee integral controllability. They specified this property as an integral robustness array (IRA), which defines a quantitative measure of system's robustness to integral controllability. They emphasized that IRA criterion is applicable to any controller with integral action including multi-loop SISO controllers, Internal Model Control, Dynamic Matrix Control, etc. Also Grosdidier *et al.* (1985) studied the problem of integral controllability. They derived a general result for integral controllability in terms of RGA and model errors. They formulated an inequality equation, which guarantees the closed-loop stability of the system under integral control.

4.2.4 Applicability of inverse-based controller for the steady-state CD control problems

As previously mentioned an inverse-based controller has been one of the typical controller structures for steady-state CD control problem (Carey *et al.,* 1975), (Boyle, 1977), (Boyle, 1978). We will study now more closely the applicability of this controller for the CD control problem in terms of RGA. An inverse-based controller $C(s) = G^{-1}(s)K(s)$, where $K(s)$ is a diagonal controller, is often a desirable solution for many process control purposes. One special case of the inverse-based controller is a decoupler (Skogestad and Morari, 1987), Skogestad and Postlethwaite, 1996). An ideal dynamic decoupler is $D(s) = G^{-1}(s)G_{diag}(s)$, where $G_{diag}(s)$ denotes the matrix consisting of diagonal elements in $G(s)$. The basic idea of using a decoupler is that a decoupler takes care of the multivariable aspects. Tuning of the control system is then reduced to a series of single-loop problems. Let the diagonal matrix $K(s)$ denote these single loop controllers. The overall controller $C(s)$ including the decoupler is $C(s) = D(s)K(s)$. A constant steady-state decoupling is obtained with $D = G(0)^{-1}$. With $C(s) = G^{-1}(s)K(s)$, it can be shown that $\Lambda(C) = \Lambda(G^{-1}\mathrm{K}) = \Lambda(G^{-1}) = \Lambda(G^{\mathrm{T}})$. Thus, if the elements of $\Lambda(G)$ are large, so will be the elements of $\Lambda(C)$ and high sensitivity to input uncertainty is expected. The physical reason for the problems with the inverse-based controller is that the controller tries to apply large input signals in certain directions to match weak directions in the plant. The input uncertainty changes these directions and ruins the design match. We see from equation

$$G_p(s)C(s) = K(s)(\mathrm{I} + G(s)\Delta_I G^{-1}(s)) = K(s)(\mathrm{I} + C^{-1}(s)\Delta_I C(s)) \tag{4.18}$$

that large elements in $G(s)\Delta_I G^{-1}(s)$ indicate that loop transfer matrix $G_p(s)C(s)$ differs from the nominal one $G(s)C(s) = K(s)$. Poor response, instability and serious robustness problems may be expected if $\Delta_I \neq 0$. In this case $G(s)C(s) = K(s)$ has no directionality that could guarantee $G_p(s)C(s)$ remain small. We may say that the inverse-based CD controller, that corrects the interactions of the plant, may yield excellent nominal performance, but will be very sensitive to input uncertainty, and will not yield robust performance. Skogestad and Morari (1987) have shown that the most important reason for the robustness problems encountered with decouplers is probably not decoupler errors but the previously mentioned input uncertainty. Any controller of the form $C(s) = G^{-1}(s)K(s)$ is sensitive to input uncertainty if the plant has large RGA elements. Decouplers are generally of this form and they should therefore not be used for CD response systems with large RGA elements.

Let us study next more closely the *steady-state* linear quadratic (LQ) optimization method mentioned in Chapter 3. The basic control objective in CD control methods based on quadratic cost function is to force the profile to some desired shape. An obvious approach is to try to choose the actuator setting to minimize the mean square deviation from the desired profile. Usually weighting factors are included to allow increased significance on some areas of the profile (Wilhelm and Fjeld, 1983). Thus the minimized cost function is:

$$J = 1/2\ (y - y_r)^T Q\ (y - y_r) = 1/2\ e^T Q e \tag{4.19}$$

where:

$e = y - y_r$ is the deviation of y from the desired profile y_r

y_r is often flat, typically all elements are equal to zero

Q is a diagonal matrix of weighting factors

This performance index is directly proportional to the variance of the profile deviation if the weighting matrix Q is the identity matrix. The optimal solution of this unconstrained steady-state minimization problem is (Wilhelm, 1986), (Wilhelm and Fjeld, 1983):

$$u = -(G^T(0)QG(0))^{-1}G^T(0)Q(y_o - y_r) + u_o \tag{4.20}$$

where y_o and u_o represent the profile and the control setting respectively before the control adjustment. If the process model is accurate, the optimal control corrects totally the profile deviation y_o-y_r. If we assume that the response matrix $G(0)$ is square and $Q = q\mathrm{I}$ then we get that u $= -G^{-1}(0)(y_o - y_r) + u_o$. This solution is based on the fact that for a square matrix $G(0)$ its pseudoinverse is identical to $G^{-1}(0)$. This is a clear decoupling solution to the steady-state LQ problem. Decoupling solution can be interpreted as a special case of the general steady-state LQ optimal control solution. A practical LQ solution for CD control problem follows very often this kind of approach (Wilhelm and Fjeld, 1986), (Gräser and Neddermeyer, 1986), (Wilhelm, 1986). Almost constantly the response matrix $G(0)$ is assumed to be square which leads to the decoupling solution and actually to the inverse-based controller $C(s) = G^{-1}(0)K(s)$. It is evident that this kind of CD controller can not be the best possible solution if input uncertainty is expected.

4.3 CD response model analysis

4.3.1 Studied response models

We will apply next RGA and DIC analysis for some typical CD response models presented in the literature. Laughlin *et al.* (1993) and Featherstone and Braatz (2000) have studied a set of experimental or proposed CD basis weight response models in their work. We will use these CD response models as general examples in our study. Response models are presented in Table 4.1 using the Toeplitz notation (3.4) and adjusted from narrower to the wider. We will use a notation G_k, (k = 1,…,12) to indicate these models. Selected models represent nicely typical CD basis weight responses for newsprint, kraft sack paper and paper board. However, similar CD responses can be found widely in the literature. For example response models like G_1 and G_2 are typical for air showers and rewetting actuators, induction coils (Vyse *et al.,* 1993) and deflection compensated rolls (Svenka and Minkenberg, 1996). Similarly, response models like G_3 and G_6 are typical for modern consistency profiling systems.

Studied models have been normalized so that the response magnitude at position g_1 is equal to 1.0. Values of the interaction parameters may vary, but the bell shape of the response is very characteristic. Negative CD response elements can be found in many of the models reflecting the observations that efforts to increase the basis weight downstream from one actuator position may actually decrease it on either side of that position. Strong interaction leads to large positive and negative off-diagonal elements in the G matrix.

TABLE 4.1: Reported Toeplitz CD response models (Laughlin *et al.*, 1993), (Featherstone and Braatz, 2000).

Model	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	1.0	0.2								
2	1.0	0.4								
3	1.0	0.1	-0.3							
4	1.0	0.45	-0.55							
5	1.0	0.5	-0.5							
6	1.0	-0.15	0.03	-0.01						
7	1.0	0.4	-0.5	0.05						
8	1.0	0.2	-0.1	-0.1						
9	1.0	0.4	-0.2	-0.4	-0.2					
10	1.0	1.3	0.8	-0.6	-0.3	0.0	-0.1			
11	1.0	1.2	0.6	-0.4	-0.9	-0.2	-0.2			
12	1.0	0.9	0.7	0.8	1.0	0.6	-0.5	-0.4	-0.2	-0.2

Graphical illustration of responses G_2, G_7, G_{10} and G_{12} is presented in Chapter 5. It is surprising that a graphical plot describes the behavior of the CD response very poorly. Usually the width and the shape are depicted. Some indication numbers can be calculated based on the width or area of the response, but they are more related to the mapping and tuning of the CD controller than for the analysis of the system. We will consider a response model of size $G^{86,86}$ that will represent a typical CD response rather well. The number of actuators (86) may be considered normal for web widths between 6 to 10 meters depending on the spacing of the actuators, typically from 75 mm to 150 mm.

4.3.2 Some observations of the condition number and singular values

In Table 4.2 a condition number $\gamma(G)$, a minimum and maximum singular value $\sigma(G)$ and both minimum and maximum element value Λ of RGA of the studied response models are presented.

TABLE 4.2: Condition number $\gamma(G)$, min-max singular values $\sigma(G)$, min-max RGA element values Λ, and results of RGA-, MIC-, SM(i)-, DIC-, $\mu(E(0))$- and IC- rules of studied response models $G^{86,86}$.

Model	$\gamma(G)$	$\underline{\sigma}(G)$	$\overline{\sigma}(G)$	Λ_{min}	Λ_{max}	RGA	MIC	SM(i)	DIC	$\mu(E(0))$	IC
1	2.33	0.600	1.399	-0.045	1.091	Yes	Yes	Yes	Yes	Yes	Yes
2	8.97	0.201	1.799	-0.333	1.666	Yes	Yes	Yes	Yes	Yes	Yes
3	7.96	0.202	1.607	-0.165	1.406	Yes	Yes	Yes	Yes	Yes	Yes
4	90.36	0.024	2.189	-0.571	0.549	No	No	No	No	No	No
5	132.55	0.016	2.123	-0.460	1.741	Yes	No	No	No	No	No
6	1.86	0.740	1.379	-0.023	1.046	Yes	Yes	Yes	Yes	Yes	Yes
7	93.26	0.022	2.029	-0.237	1.451	Yes	No	No	No	No	No
8	2.71	0.552	1.498	-0.054	1.145	Yes	Yes	Yes	Yes	Yes	Yes
9	13.28	0.201	2.671	-0.407	2.326	Yes	Yes	Yes	Yes	No	No
10	315.50	0.015	4.608	-2.706	2.996	Yes	No	No	No	No	No
11	346.69	0.016	5.728	-3.141	2.853	No	No	No	No	No	No
12	631.44	0.011	7.051	-2.865	4.577	No	No	No	No	No	No

As can be seen the condition number $\gamma(G)$ alternates very widely from 1.86 to 631.44. Generally large condition number means that the plant depends strongly on the input direction and system is sensitive to unstructured input uncertainties. From the control point of view high condition number of the CD response model means that a strong control action is required to dampen disturbances entering the process in the direction of the left singular vector corresponding to the minimum singular value $\underline{\sigma}(G)$. Strong control action taken in the wrong direction can lead to instability or poor performance. Therefore it is difficult to design an acceptable controller based on CD response models with high condition number, when input uncertainty is also present. On the other hand, the minimum singular value $\underline{\sigma}(G)$ indicates also to which value we are able to track the reference changes without reaching input constraints. Therefore from the control point of view response models like G_4, G_5, G_7, G_{10}, G_{11} and G_{12} are undesired because they have large condition numbers and relatively small minimum singular values.

Especially models G_5, G_{10}, G_{11} and G_{12} indicate how a large condition number may be caused by a small value of $\underline{\sigma}$(G) that is generally regarded undesirable. Generally negative values of the CD response are not a problem. This can be seen from models G_3, G_6 and G_8. On the other hand, very small changes in the element values can produce a significant alteration to the condition number. Response models G_4 and G_5 indicate how an incremental change (Δ=0.05) of the element value may increase the condition number unexpectedly. As can be seen the maximums of singular value $\overline{\sigma}(G)$ are almost the same in magnitude but the minimums differ sufficiently to affect the condition number. This example indicates how sensitive the response model can be to the modeling errors and how important the robustness and RGA study of the model actually is. It also shows that purely a visual observation of responses based on graphical plotting will never tell or even hint at the latent control problem. A wide response, as in models G_{10}, G_{11} and G_{12}, indicates strong interactions between adjacent profile zones. Typically this leads to a very high condition number and obvious control problems. For instance, response model G_{12} was originally presented by Karlsson *et al.* (1985) who found out that wider responses imply inherently limited possibilities for CD basis weight control on the board machines.

4.3.3 Some observations of the RGA

RGA analysis was done for all studied response examples. Calculations were done using Matlab (1996) software. In Appendix B 11*x*16 first elements of studied $RGA^{86,86}$ matrices are presented. We observe that all in Appendix A mentioned RGA alternatives are represented: from negative values to quite high RGA element values. Therefore some control loops interact with other loops, some closed-loop systems are very sensitive to parameter changes, some inverse response events are expected and some control loops depend entirely on the other control loops.

As can be seen CD response models G_1, G_2, G_3, G_6 and G_8 represent rather simple and acceptable cases. All condition numbers are less than 10, RGA matrices are diagonal dominant with positive relative gains quite close to unity. Especially models G_1 and G_6 seem to be very appropriate for decentralized control. Although in the model G_6 the width of the response is four elements and some of these elements are even negative, the RGA calculation implies that this kind of response model would inherently fulfill almost optimal variable pairing. Because the maximum values of RGA gains in our case are close to unity and all off-diagonal elements are rather insignificant, it is obvious that for this kind of CD response model even a traditional decentralized PI controller would give an acceptable control result. Therefore it is not surprising that decentralized PI controllers have been used for some CD applications rather successfully.

Strong interactions between adjacent zones in response models result in higher RGA values as can be seen by comparing model G_1 to G_2 and models G_6 and G_8 to G_7. However, this result can not be generalized and RGA values of response model G_4 are a practical indication of that. We can observe that RGA elements of model G_4 are less than 1 in magnitude. Therefore, G_4 represents a naturally decoupled system with a rather large condition number, and as pointed out by Skogestad and Morari (1987), a diagonal controller would produce a reasonable performance in this special case. However, as we can see the controller can not be based on integral control. We will discuss controller selection more in the next section.

When the response is wide enough and the interactions are dominating RGA elements will achieve relative high values. Studied model G_{12} represents the most complex response example with large RGA element values. On the other hand, from models G_5, G_7, G_{10}, G_{11} and G_{12} we can see that the large negative RGA values are attached to the cases with high condition numbers and small minimum of singular values. We may decide that large RGA elements of these response models indicate fundamental control problems because only very small uncertainties in the elements are allowed without impaired performance of the control loop (4.8). In these cases a special attention should be focused on the selection of the controller structure.

4.3.4 Some observations of the controller structure and DIC applicability

Complexity of the CD response models in terms of controller structure was evaluated by using DIC analysis. Results of the individual RGA-, MIC- and SM(*i*)- rules, equations (4.10), (4.12) and (4.13) respectively, are presented in Table 4.2. Defined results are combined to form an estimate of those CD response models, which are DIC. This total result is marked with a column notation DIC. NI- rule (4.11) was excluded because it is redundant, and calculation of SM(*ii*)- rule (4.14) was omitted impractical. First of all, we may notice that condition number is a good suggestive index to indicate control difficulties of the system, but it does not advise in the selection of the control structure. Secondly, we can observe that RGA- rule alone produces an incomplete DIC estimate for the CD response models. If we compare results of RGA -rule to the combined DIC, we can discover that RGA- rule concludes three times to the false outcome, while MIC- and SM(i)- rules seem to be more accurate.

However, one important practical advantage of RGA- rule is that it is very simple to compute. On the other hand, MIC- and SM(i)- rules require only a calculation of eigenvalues of the steady-state matrices, which is similarly a straightforward operation. DIC analysis, based on the *necessary* conditions, excluding SM(ii)- rule, produces an estimate that CD response models G_4, G_5, G_7, G_{10}, G_{11} and G_{12} are not decentralized integral controllable. However, consideration of *sufficient* condition (4.15) for DIC corrects this estimate slightly. Computation of structured singular value μ of interaction matrix $E(0)$ for all CD response models adds also G_9 to this set of not DIC models. However, definition of structured singular value μ requires special numerical algorithms (Chiang and Safonov, 1992), which might limit the usability of this method. For DIC analysis, in terms of CD response complexity, condition (4.15) is very powerful because it indicates directly if DIC is satisfied. We may conclude that DIC analysis and the *sufficient* condition (4.15) provide us enough information to evaluate complexity of the CD response model in terms of controller structure.

On the other hand, if we accept that all gains of the integral controller may be changed by the same degree, and apply the IC- rule (4.17) for analysis, we can observe that it produces, in our cases, precisely the same results as DIC condition (4.15). In addition, calculation of spectral radius of steady-state interaction matrix $E(0)$ is much more easier than using the structured singular value. Integral controllability (IC) of the CD systems can also be studied by using methods presented by Grosdidier *et al.* (1985) and Yu and Luyben (1987). Their approaches provide tools to calculate estimates for the maximum degree of uncertainty in the CD response model, i.e. they show how much variation in the response model parameters is tolerated before the system becomes uncontrollable in terms of IC. Also Laughlin *et el*. (1993) concluded, that integral controller is applicable only to such CD response models, which have all eigenvalues in the right half plane.

4.3.5. Conclusions of the CD response model analysis

Commonly RGA and DIC analyses are used to choose pairing between inputs and outputs for decentralized control. We have shown that they can be applied efficiently for *complexity analysis* *of the CD response models G(0)* and *applicability of the controller structure*. RGA analysis together with singular values, condition number and DIC analysis is a suitable tool to study the properties of the CD process. The main reason for this is that almost all practical CD control applications are based on *steady-state response models G(0).* We can say that in general a CD process model with large RGA elements is difficult to control. However, if the RGA values of the model are large, but there exists a controller with small RGA elements, which produces an appropriate response for all major disturbances in favorable directions, the statement is not absolutely valid. This implies that the CD process model is not ill-conditioned for the predictable disturbances. Similarly, we can say that the CD process model with a large condition number is not necessarily always difficult to control. Because it is the RGA, rather than the condition number, which defines the sensitivity of the CD process to the diagonal input uncertainty. Similarly, the worst case combination of the input uncertainty can easily be found with the RGA analysis.

RGA analysis with DIC study reveals directly when a *diagonal integral controller is unsuitable* for CD purposes. RGA analysis gives a clear indicator of sensitivity to model uncertainty, especially when uncertainty on each manipulated input is considered. It gives also information about applicability of the inverse-based controller for CD control problem. CD control system is always sensitive to input uncertainty if an inverse-based CD controller is used for process models with large RGA elements. On the other hand, we know that the decentralized CD control system is insensitive to input uncertainty if a diagonal controller is used, but its closed-loop performance will be poor. Perhaps RGA is not in general an absolute indicator of the system's sensitivity to the input uncertainty, but for *practical CD controller design purposes* it is a very adequate method. Especially CD control applications which are based on the steady-state linear quadratic (LQ) optimization should be analyzed carefully, because in practice they often reduce to the inverse-based controllers, which have been shown to be sensitive to the input uncertainty.

For evaluation of the CD control structure DIC analysis appears to be quite suitable. Especially, its ability to consider and *classify inherent properties* of the CD response models G(0) makes it very useful. Although it advises only on the applicability of decentralized integral controller, it provides an unambiguous and transparent indication of system complexity. DIC analysis can be utilized to classify CD response models in different categories based on sophistication. Together with RGA analysis it can be used as a *practical screening method* to select controller structure for the CD processes. It shows clearly when a traditional CD control approach is inappropriate and a more advanced control technology is required.

RGA and DIC analyses can also be used as a designing tool to evaluate the applicability of CD actuator structure. Based on RGA and DIC analyses we know what kind of CD response is desirable, robust and easy to control. The next task is just to develop that kind of actuator or machine part that fulfils these requirements. In practice, several methods exist to accomplish this task. Realization of desirable response characteristics may be done based on mathematical modeling (Duncan *et al.,* 2000) and simulation (Tarvainen and Rouhiainen, 2001), experimental trials on pilot machine (Ellilä, 1994) or even on production machine. This knowledge can be combined with the modern machine design to generate required implementation (Tarvainen and Rouhiainen, 2001).

Chapter 5

Robust Constrained Model Predictive CD Control using Linear Matrix Inequalities

Model Predictive Control, also known as Moving Horizon Control (MHC) or Receding Horizon Control (RHC) is a popular technique used in industrial process control. MPC solves an on-line optimization problem at each step to compute an optimal control profile over finite horizon of future time. Typically a sequence of predicted control moves will be calculated, but only the first one is implemented. At the next sampling time, the optimization problem is solved again with new measurements, and the control input is updated. This control technique is very popular since it is possible to handle constraints on input and output signals during the design and implementation of the controller (Morari and Lee, 1997), (Maciejowski, 2001). However, one of the main drawbacks of standard MPC is the difficulty to incorporate plant model uncertainties explicitly. The reason is that MPC is in principle a computational approach, and an analytic expression for the controller is generally not available. This limits further the study of closed-loop stability which is based on such information. Standard MPC schemes virtually have no guaranteed robustness because they use nominal models and perform finite horizon optimization. Therefore in the presence of model mismatch, this type of algorithm can behave poorly.

Robust MPC is an MPC theory that increases the effectiveness of the control actions when modeling errors are present by explicitly taking in to account the modeling errors in the controller design procedure. Instead of using one process model in predicting the system behavior as in MPC, robust MPC forecasts system behavior for every model in the uncertainty set. The optimal control actions are defined by a min-max optimization that minimizes the deviations of the forecasted behavior from the desired behavior for the model with the largest deviation. Campo and Morari

(1987), Zafiriou (1990) and Zheng and Morari (1993) proposed using min-max optimization in MPC with finite impulse response models. Lee and Yu (1996) proposed using min-max optimization on discrete state-space models with polytopic model uncertainty. Generally the on-line min-max optimization is computationally very demanding.

In the design of MPC, robust stability is an important issue. The aim is to design a controller that is stable independent of the operating conditions, which usually affect to the process model. For the nominal case, where only the most probable model is handled by the controller, several methods to obtain a stable MPC are available. A popular approach to obtain a stable MPC is based on an infinite output horizon (Rawlings and Muske, 1993). For stable systems, the infinite horizon open loop cost can be expressed as a finite cost with the inclusion of a terminal state penalty, which is computed by solving a Lyapunov equation. The extension of this approach to the robust multi-model MPC was proposed by Badgwell (1997), with the inclusion of contracting constraints for the costs associated with the possible plants. In that method, it was assumed that, for the computed input sequence, the state goes to zero at infinite time for all possible plant models. Morari and Zafiriou (1989) discussed how to improve the robust stability of MPC in framework of internal model control (IMC) by tuning IMC filters. By modifying the optimization problem to "min-max" problem, Campo and Morari (1987) studied "worst-case" performance over all model uncertainties and formulated the constrained robust stabilization problem as a linear programming (LP) problem. Zafiriou (1990) and his coworkers (Zafiriou and Marchal, 1991) analyzed the effect of incorporating constraints to the model predictive controller using the contraction mapping principle, and developed some necessary/sufficient conditions for robust stability of MPC. As pointed out in (Zafiriou, 1990), the existence of hard constraints can largely deteriorate the performance and stability property, which is even worse in the case of model-plant mismatch.

Recently, the robust MPC technique using linear matrix inequality (LMI) technique has been developed by Kothare *et al.* (1996). In this method, the authors formulate an infinite horizon MPC problem with input and output constraints and plant uncertainty as a convex optimization problem involving LMIs. The LMI formulation is suitable to deal with uncertain systems and input – output constraints. Using the "worst-case" performance index with respect to plant perturbations over a moving infinite prediction horizon, they considered the state-feedback robust MPC problem for affine uncertain linear systems. The synthesis problem was formulated as an on-line optimization problem, subject to input and output constraints. Sufficient solvability conditions in terms of LMI optimization were also derived. In this method linear matrix inequalities (LMI) are actually used to solve a state feedback control problem $u(\mathrm{t}) = Kx(\mathrm{t})$ for the gain K that minimizes the model in the polytope with the largest deviation from the origin. The LMI -based optimization is done on-line to determine the gain K at every sampling time. A nice property of their MPC scheme is that the stability of robust model predictive controller is guaranteed if the optimization problem is feasible. We will study this approach more in details in Chapter 5.3. Lu and Arkun (2000) extended this technique to polytopic linear parameter-varying systems for the scheduling MPC problem.

In this chapter we will examine the robust infinite horizon model preditive control problem. First we present some background material such as linear matrix inequalities, model of systems with special type of uncertainties and basics of model predictive control (MPC). We will use this supplementary material to formulate a robust constrained model predictive CD control problem with a state-feedback as a linear matrix inequality problem. Uncertain discrete-time linear time-varying CD systems with time-delays are also considered. After that performance of the robust MPC algorithm for cross-directional (CD) control problem is investigated by performing a number of simulations. Studied CD response models represent a realistic description of the interactions across the paper machine. Selection of simulated CD response models is done based on the RGA and DIC analyses. But next we will discuss about linear matrix inequalities and a special type of systems called polytopic uncertain systems.

5.1 Linear matrix inequalities

A wide variety of problems in system and control theory can be reduced to a standard convex optimization problems that involve linear matrix inequalities or LMIs. The form of an LMI can be very general; linear inequalities, quadratic inequalities, matrix norm inequalities, and various constraints from control theory such as Lyapunov and Riccati inequalities. Also optimal LQG control and H_∞ control problems can be formulated by using LMIs. Morover, multiple LMIs can always be written as a single LMI of larger dimension. Further applications of LMIs arise in estimation, identification, optimal design, structural design and matrix scaling (Boyd *et al.*,1994a). The main strength of LMI formulation is the ability to combine various design constraints and objectives in a numerically effective way. For a few special cases there are analytical solutions to LMI optimization problems, but usually they can be solved numerically very efficiently. Actually the growing popularity of LMI optimization for control purposes can be counted on the recent developments in the interior point-point methods for LMI optimization.

Next we will give a brief description of linear matrix inequalities and some optimization problems based on LMIs. Especially, we summarize a number of terms and results which will be used in the rest of the thesis. A detailed discussussion of the extensive literature on LMIs is beyond the scope of this thesis and can be found in the book of Boyd *et al.*(1994a).

5.1.1 Definition and properties of linear matrix inequalities

Here we define the LMI and some of its basic properties (VanAntwerp and Braatz, 2000b).

A linear matrix inequality or LMI has the form

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0 \tag{5.1}$$

where $x \in R^{m}$, $x = [x_1 ,\ldots, x_m]$ are the variables, and symmetric matrices $F_i = F_i^T \in R^{nxn}$ are given. $F(x) > 0$ means that $F(x)$ is positive definite, that is

$$z^T F(x) z > 0,\ \forall\ z \neq 0,\ z \in R^n \tag{5.2}$$

Linear matrix inequality is a *constraint* on the variable x. Thus, $F(x)$ is an affine function of the elements of x. The LMI (5.1) is equivalent to n polynomial inequalities because a matrix is positive definite if and only if each leading principal minor of $F(x)$ is nonnegative.

An important property of LMIs is that the set $\{x|F(x) > 0\}$ is convex, that is, the LMI forms a convex constraint on x. Equation (5.1) is a strict LMI. Requiring that $F(x)$ be positive semidefinite is referred to as a nonstrict LMI. The strict LMI is feasible if the set $\{x|F(x) > 0\}$ is nonempty.

Multiple LMIs can be expressed as the single LMI. Consider a set defined by p LMIs:

$$F^1(x)>0;\ F^2(x)>0;\ \ldots\ ;\ F^p(x)>0$$

Then the equivalent single LMI is given by

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i = diag\{F^1(x), F^2(x), \ldots, F^p(x)\} > 0 \tag{5.3}$$

where

$$F_i = \mathrm{diag}\{(F_i^1, F_i^2, \ldots ,F_i^p\},\ \forall i = 0, \ldots, m$$

and $\mathrm{diag}\{X_1, X_2, \ldots , X_p\}$ is a block diagonal matrix with blocks $X_1, X_2, \ldots , X_p$. This statement can be proved based on the knowledge that the eigenvalues of a block diagonal matrix are equal to the union of the eigenvalues of the blocks, or from the definition of positive definiteness (VanAntwerp and Braatz, 2000b). Therefore we will make no distiction between a set of LMIs and a single LMI.

Convex nonlinear inequalities are converted to LMI form using *Schur complements*. Let $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ depend affinely on x. Then the LMI

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0 \tag{5.4}$$

is equivalent to the matrix inequalities

$$\begin{aligned} &R(x) > 0,\ Q(x) - S(x)R(x)^{-1}S(x)^T > 0 \\ &Q(x) > 0,\ R(x) - S(x)^T Q(x)^{-1}S(x) > 0 \end{aligned} \tag{5.5}$$

The proof of the Schur complement lemma is based on the straightforward elementary calculus. See in details (VanAntwerp and Braatz, 2000b).

Several traditional control problems can be formulated to LMIs. Typical examples are eigenvalue problems, singular value problems and Lyapunov stability of linear time invariant (LTI) and time varying (LTV) systems (Boyd *et al.*,1994a). An important LMI-based problem to this work is that of *minimizing a linear objective subject to LMI constraints*:

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && F(x) > 0 \end{aligned} \tag{5.6}$$

where F is a symmetric matrix that depends affinely on the optimization variable x, and c is a real vector of required size. This is a convex nonsmooth optimization problem (Boyd *et al.,* 1994a).

5.1.2 Solving LMI-based problems

A large number of control problems are reducible to LMI-based convex optimization problems. Over the past few years efficient algorithms for numerically solving these optimization problems have been developed (Boyd *et al.*, 1994b). The most important practical implementation is that these algorithms can rapidly compute the global optimum, with non-heuristic stopping criteria, and prove that the optimum really has been obtained to within some prespecified accuracy. In general, LMI problems can be solved in polynomial-time, which means that they have low computational complexity. In these problems the exact time required to solve the problem is bounded by a single function, which is polynomial in the amount of data needed to define the problem. Computational LMI problems which are solvable in polynomial-time include both the linear and convex quadratic optimization problems. On the other hand, computational problems which are defined as NP-hard cannot be computed in polynomial-time in the worst case, because in these cases the computation grows exponentially with the problem size.

The simplest algorithm for solving convex LMI problems is the *ellipsoid algorithm* which has a polynomial-time complexity. The algorithm works well for smaller problems but can be rather slow when the size of the problem is large. The basic idea of the algorithm is as follows. In the first step an ellipsoid that is guaranteed to contain an optimal point is calculated. Next a cutting plane that passes throught the centre of this ellipsoid is computed. Boyd *et al.* (1994a) have shown an analytical formulation to define a cutting plane for each of the standard LMI problems. Next a bisected half-ellipsoid contains an optimal point and a new ellipsoid of minimum volume that contains this bisected half-ellipsoid is defined. The procedure is repeated until the algorithm converges to the optimal solution

In practice a more efficient method for solving LMI problems is the *interior-point algorithm.* This method is based on the early work of Nesterov and Nemirovsky in 1988 (Boyd *et al.,*1994a, 1994b). The development of interior point methods has meant that many problems in system and control theory for which no *analytical solution* has been found, can today be solved by reducing them to LMI problems. The key element in this method is the knowledge of a barrier function with a certain property called *self-concordance*. Linear matrix inequalities represent a class of convex constraints for which easily computable self-concordant barrier functions are known. Interior-point algorithm proceeds as follows: In the first step a specific logarithmic barrier function which is convex within the feasible set and becomes infinite outside it is defined based on the system constraints. Then the primal objective function is augmented to contain this barrier function. Next the original constrained optimization problem is replaced with an unconstrained optimization problem. Optimization is solved applying Newton's method with appropriate step length. An analytic center of the LMI is defined to represent the point which minimizes the unconstrained optimiztion problem. A scalar parameter in the objective to the unconstrained optimization problem is repeated until the analytic center of the LMI is optimal for the original optimization problem (Boyd *et al.,*1994a).

5.1.3. Software for solving LMI problems

Several research groups have made numerical software packages available for solving LMI problems. Nowadays optimization problems with convex objective functions and LMI constraints can be solved efficiently with powerful off-the-shelf software algorithms. Gahinet and Nemirovskii developed a software package called LMI-Lab (Gahinet and Nemirovski, 1993), which is based on the Nemirovskii's projective interior-point algorithm (Nesterov and Nemirovsky, 1994), (Nemirovsky and Gahinet, 1994). LMI-Lab allows the user to describe an LMI problem in a high-level symbolic form. Matlab's LMI Control Toolbox (Gahinet *et al.*, 1996) is based on the same algorithm, and offers a graphical user interface and extensive support for control applications. The control applications are built around the LMI-Lab and allow for the treatment of various robust control problems. Toolbox includes also tools for classical Riccati-based H_∞ control (Gahinet *et al.*, 1994). VanDenberghe and Boyd (1994) wrote the code SP which is based on a primal-dual potential reduction method for semidefinite programming with Nesterov and Todd scaling. The code is written in C with calls to BLAS and LAPACK library programs and SP includes an interface to Matlab. Software packages SDPSOL (Wu and Boyd, 1996) and LMITOOL (Elghanoui *et al.*, 1995) offer user-friendly interfaces to SP that simplify the specifications of semidefinite programming problems where the variables have a matrix structure. SDPSOL is a parser solver that calls SP code. SDPSOL can run without Matlab and it enables the user to specify the problem in a high level leanguage. The Induced-Norm Control Toolbox (Beran, 1995) is a Matlab toolbox for robust and optimal control. It is in turn based on LMITOOL.

5.2 Polytopic systems

In this section we describe a model of systems with uncertainties called *polytopic systems* which are commonly used in many modeling and estimation approaches. In general, polytopic systems form a special class of Linear Fractional Representation (LFR) of the uncertain systems. For these systems, a lot of research has been done on analysis and synthesis using quadratic Lyapunov functions (Boyd *et al.*,1994a). Polytopic systems can be presented as a linear time-varying (LTV) system

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= Cx(k) \\ [A(k)\ \ B(k)] &\in \Omega \end{aligned} \tag{5.7}$$

where $u(k) \in R^{n_u}$ is the control input, $x(k) \in R^{n_x}$ is the state of the plant and $y(k) \in R^{n_y}$ is the plant output, R is a set of real numbers and Ω is some prespecified set.

For polytopic systems, the set Ω is a polytope

$$\Omega = \mathrm{Co}\{[A_1\ B_1],[A_2\ B_2],\ldots,[A_L\ B_L]\} \tag{5.8}$$

where Co refers to convex hull. If $[A\ B] \in \Omega$, then for some nonnegative $\lambda_1, \lambda_2, \ldots, \lambda_L$ summing to one will give

$$[A\ B] = \sum_{i=1}^{L} \lambda_i \left[A_i\, B_i\right]$$

When L = 1 a linear time-invariant (LTI) system without plant-model mismatch will occur. Graphical presentation of polytopic uncertainty is shown in Figure 5.1.

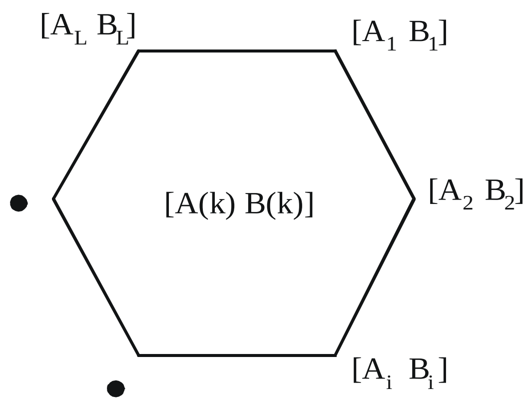

Figure 5.1: Polytopic uncertainty.

Polytopic multi-model systems can be derived as follows (Boyd *et al.*, 1994a). Suppose we have a real system that is rather well modeled as a linear system. We collect several sets of input-output measurements at different operating points and at different times, so that the measurement data characterizes comprehensivly all plant variations that can be expected. For each data set we develop a linear system model of the plant which will always contain the same accessible state vector. We suggest to model the system as a polytopic system (5.7) with the vertices (5.8) given by the measured or estimated linear system models. Thus, we model the plant as a time-varying linear system, with system matrices that are allowed to alternate among all of the multi-models we defined. Alternatively, a nonlinear system can be approximated by a polytopic uncertain linear time-varying system using a global linearization approach presented by Liu (1968).

The development of polytopic multi-models is in principle similar to the traditional parametric identification techniques such as least squares, maximum likelihood and instrumental variable methods, which utilize a pseudo-random binary sequence (PRBS) technique to excite the process in determining the process dynamics, unknown parameters and imposing interactions. Also these approaches provide a method to define e.g. a CD response model based on the input-output data from the varying process (Heaven *et al.*, 1993b). Similarly, parametric identification methods enable the evaluation of parameter validity range in terms of statistical confidence.

Other type of uncertainties or perturbations can also be connected to the linear system models. Kothare *et al.* (1996) have shown how a structured uncertainty in a feedback loop with linear time-invariant systems can be developed. This method provides a way to handle repeated scalar block or a full block uncertainty models. However, later we will see that the polytopic uncertain systems can very conveniently be applied to the CD processes.

5.3 Model predictive control

Model predictive control (MPC) techniques are widely used in industrial process control. Its general structure is shown in Fig. 5.2. At each sampling time, MPC solves a trajectory optimization problem, typically a linear or quadratic program to compute optimal control inputs over a fixed future time horizon, using a plant model to predict future plant outputs. All MPC systems rely on the idea of generating values for process inputs as solutions of the on-line optimization problem. The underlying principle of MPC is that a model can be used to predict the effects of past and future inputs on the future system outputs. Although more than one optimal control input is generally calculated, only the first one is implemented. At the next sampling time, the optimization problem is reformulated with the horizon shifted forward by one time step and solved utilizing the new measurement information obtained from the system. Thus, the process measurements provide the feedback element in the MPC structure. The main advantage of MPC is its ability to explicitly handle constraints on both the input and output variables. However, due to the on-line optimization involved, the application of MPC is restricted to slow processes, which allow the on-line computation to be completed between two sampling instances. MPC is probably the only methodology currently available, which can explicitly handle constraints on the manipulated and output variables systematically during the design and implementation of the controller (Maciejowski, 2001).

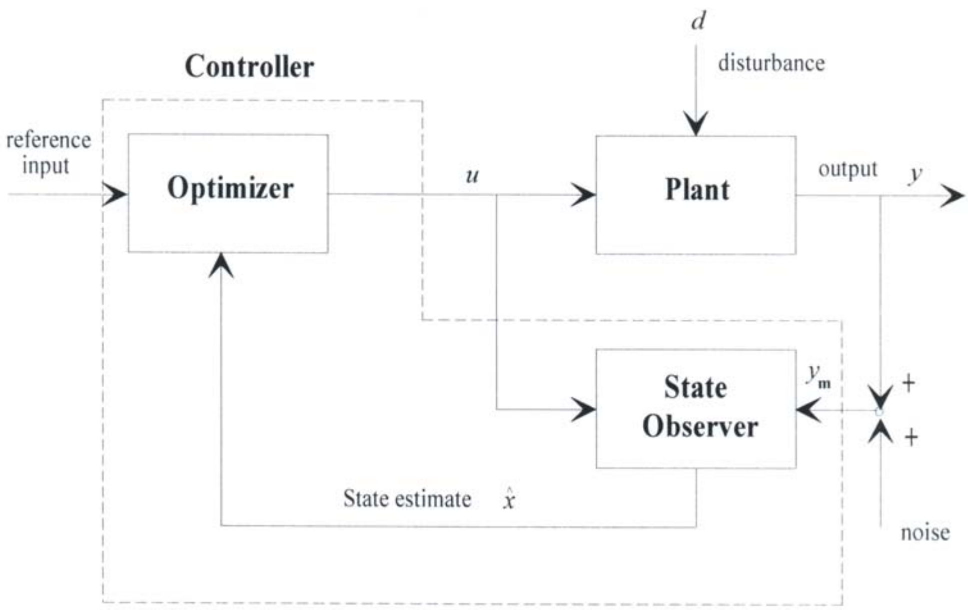

Figure 5.2: General structure of MPC.

Model predictive control is an open-loop control design procedure where at each sampling time *k*, plant measurement y_m of the output *y* is obtained, as shown in Fig. 5.2.

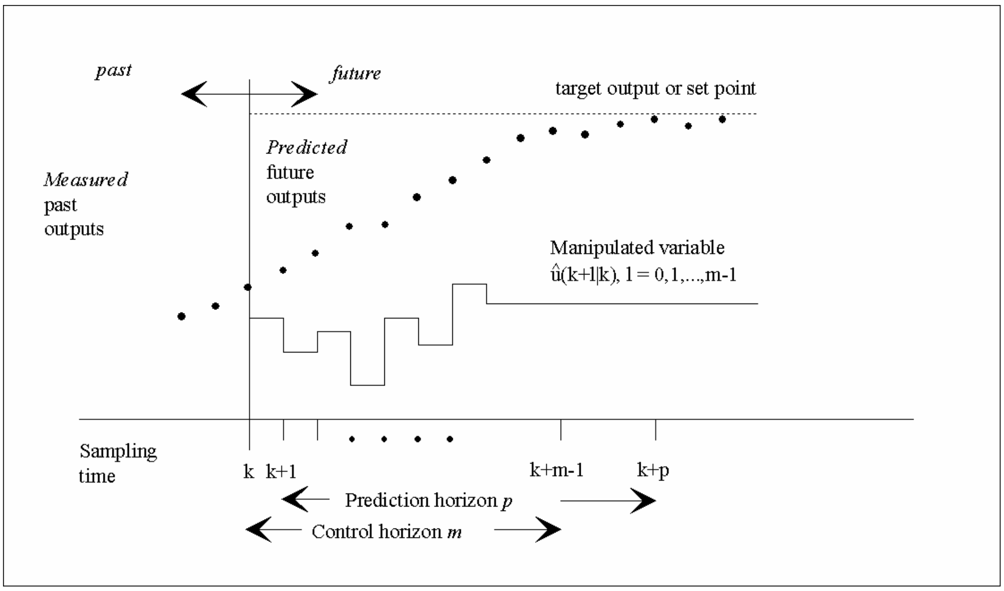

Figure 5.3: Basic principle of MPC.

This measurement and knowledge of the plant input *u(k)* at the current sampling time are used by the observer to define an estimate $\hat{x}(k)$ of the plant state *x*. This state estimate $\hat{x}(k)$ and a model of the plant are used to predict future states and outputs $\hat{x}(k+i|k)$, $\hat{y}(k+i|k)$, i = 1,…, p of the system over the future time *prediction* horizon p, see Fig. 5.3. At the same time the manipulated input $\hat{u}(k+i|k)$, i = 0,1,…, m-1 is changed over the future time *control* horizon m.

MPC optimizer computes a sequence of m control moves $\hat{u}(k+i|k)$, i = 0,1,…,m-1 such that the predicted output follows the specified target or set-point as defined. Inequality constraints on the inputs and outputs are taken into account in the optimization. The sequence of control moves are computed by minimizing an objective function $J_p(k)$ over the prediction horizon p as follows

$$\min_{\hat{u}(k+i|k),i=0,1,\dots,m-1} J_p(k) \tag{5.9}$$

subject to constraints on the control input $\hat{u}(k+i|k)$, i = 0,1,…,m-1 and possibly also on the state $\hat{x}(k+i|k)$ and the output $\hat{y}(k+i|k)$,i=0,1,…,p. Here

$\hat{x}(k+i|k)$, $\hat{y}(k+i|k)$	:	state and output respectively, at time $k+i$, predicted based on the measurements at time k; $x(k|k)$ and $y(k|k)$ refer respectively to state and output measured at time k.
$\hat{u}(k+i|k)$	:	control move at time $k+i$, computed by the optimization problem at time k; $u(k|k)$ is the control move to be implemented at time k.
p	:	prediction horizon
m	:	control horizon

It is assumed that the control action is not changed after time $k+m$-1, i.e., $\hat{u}(k+i|k) = \hat{u}(k+i-1|k)$, $i \geq m$. Similarly for state regulation problems, $\hat{u}(k+i|k) = 0$, $i \geq m$. Thus, although more than one optimal control input is calculated, only the first computed control action $u(k|k)$ is implemented and the rest of the control sequence is discarded. At the next sampling time k+1, new measurents $\hat{y}(k+1)$ are received from the plant and a new estimate $\hat{x}(k+1)$ of the plant state $\hat{x}(k+i|k)$ is received from the observer. Next, predictions of the plant state and output $\hat{x}(k+1+i|k+1)$, $\hat{y}(k+1+i|k)$, i = 1,…, p can be defined over a shifted prediction horizon from k+1+1 to k+1+p, and the optimization is resolved again using these predictions to recompute m optimal control moves $\hat{u}(k+1+i|k+1)$, i = 0,1,…, m-1. Therefore, both the control horizon m and the prediction horizon p move ahead by one step as time moves ahead by one step. The advantage of using new measurements at each time step is to diminish the significance of unmeasured disturbances and model inaccuracies in the MPC structure (Maciejowski, 2001).

In this study we assume that an exact measurement of the state is available at each sampling time k, i.e., $x(k|k) = x(k)$. We will also assume that the specified objective function $J_p(k)$ is quadratic

$$J_p(k) = \sum_{i=0}^{p} \left(\hat{x}(k+i \mid k)^T Q_1 \hat{x}(k+i \mid k) + \hat{u}(k+i \mid k)^T R \hat{u}(k+i \mid k) \right) \tag{5.10}$$

where $Q_1 > 0$ and $R > 0$ are symmetric weighting matrices. We will adopt the approach of Kothare *et al.* (1996) and consider an infinite horizon MPC technique ($p = \infty$). The main benefit to use infinite horizon approach is that it guarantees better nominal stability than finite horizon control laws and it does not require any parameter tuning for stability. When the control horizon and the prediction horizon both approach infinity, and when there are no constraints, we obtain a standard Linear Quadratic Regulator (LQR) problem.

In this study the input and output constraints are defined as component-wise peak bounds:

$$\left| \hat{u}_j(k+i \mid k) \right| \le u_{j,\max},\ k,i \ \ge\ 0, \qquad j = 1,2,\ldots, n_u \tag{5.11}$$

$$\left| \hat{y}_j(k+i \mid k) \right| \le y_{j,\max},\ k \ge\ 0,\ i \ge\ 1 \qquad j = 1,2,\ldots, n_y \tag{5.12}$$

Notice that because the current output cannot be affected by the current or future control actions, the output constraints are defined strictly over the future horizon (i.e., $i \ge 1$) and not at the current time (*i.e,* $i = 0$). In addition, constraints on the input are usually hard constraints, because they are physical limitations of the process equipments like actuators. On the other hand, constraints on the output are often performance specifications which can be softened by allowing them to make y_{max} as small as possible, subject to the input constraints.

5.4 Robust constrained model predictive CD control algorithm using linear matrix inequalities

In this section, we consider a robust infinite horizon MPC problem. First we discuss the minimization of a worst-case objective function with input and output constraints. A linear objective minimization problem is solved and a required state feedback matrix is defined. Then a system with delays are considered. We will see that the derived robust MPC algorithm is applicable to the cross-directional (CD) control problem.

We assume that the system is described by the equation (5.7) with the uncertainty set Ω. Similar to the approach from linear robust control, the nominal objective function in equation (5.10) is replaced by the minimization of a robust performance objective. For notational clarity, we have omitted the indication of predictions, because they are clearly shown by time indexes. At each sampling time k minimization of the worst case infinite horizon quadratic objective function is performed (Kothare *et al.*, 1996).

$$\min_{u(k+i|k),i=0,1,\ldots,m} \; \max_{[A(k+i)B(k+i)]\in\Omega, i\geq 0} J_\infty(k) \tag{5.13}$$

where $$J_\infty(k) = \sum_{i=0}^{\infty}\left(x(k+i\,|\,k)^T Q_1 x(k+i\,|\,k) + u(k+i\,|\,k)^T R u(k+i\,|\,k)\right).$$

With $Q_1 > 0$, $R > 0$, subject to polytopic uncertainty set Ω in (5.8) and component-wise peak input - output constraints

$$\left|u_j(k+i\,|\,k)\right| \leq u_{j,\max},\ k,i \geq 0, \qquad j = 1,2,\ldots, n_u \tag{5.14}$$

$$\left|y_j(k+i\,|\,k)\right| \leq y_{j,\max},\ k \geq 0,\ i \geq 1 \qquad j = 1,2,\ldots, n_y \tag{5.15}$$

Euclidean norm type of constraints are also handled in Kothare *et al.* (1996). The problem as stated above is a typical "min-max" problem. We address this problem by first deriving an upper bound on the robust performance objective $J_\infty(k)$. The maximization is done over the set Ω thus that the time-varying plant $[A(k+i)\ B(k+i)] \in \Omega$, $i \geq 0$ will give the worst-case value of $J_\infty(k)$ among all plants in the set Ω. This upper bound value of $J_\infty(k)$, over present and future control actions $u(k+i|k)$, $i=0,1,\ldots,m$, is then minimized with a constant feedback control law $u(k+i|k) = Kx(k+i|k)$, $i \geq 0$.

In equation (5.13) the state feedback law $u(k+i|k) = Kx(k+i|k)$ is used at each sampling time to minimize the worst case value of $J_\infty(k)$. Next we derive an upper bound on $J_\infty(k)$. At sampling time k, define a quadratic function $V(x) = x^T P x$, $P > 0$. For any $[A(k+i)\ B(k+i)] \in \Omega$, $i \geq 0$, suppose $V(x)$ satisfies the following robust stability constraint:

$$V(x(k+i+1|k) - V(x(k+i|k) \leq -(x(k+i|k)^T Q_1 x(k+i|k) + u(k+i|k)^T R u(k+i|k)) \tag{5.16}$$

Summing up the above inequality from $i = 0$ to ∞ and requiring $x(\infty|k) = 0$ or $V(x(\infty|k)) = 0$, we get

$$\max_{[A(k+i)B(k+i)]\in\Omega, i\geq 0} J_{\infty}(k) \leq V(x(k|k)) \leq \gamma \tag{5.17}$$

Equations (5.16) and (5.17) give an upper bound on $J_{\infty}(k)$, which is defined as γ. The condition $V(x(k|k)) \leq \gamma$ in (5.17) can be expressed equivalently as LMIs

$$\begin{bmatrix} 1 & x(k|k)^T \\ x(k|k) & Q \end{bmatrix} \geq 0 \text{, } Q > 0 \tag{5.18}$$

where $Q = \gamma P(k)^{-1}$. The robust stability constraint (5.16) for system (5.7) is satisfied if for each vertex of Ω.

$$\begin{bmatrix} Q & QA_j^T + Y^T B_j^T & QQ_1^{\frac{1}{2}} & Y^T R^{\frac{1}{2}} \\ A_j Q + B_j Y & Q & 0 & 0 \\ Q_1^{\frac{1}{2}} Q & 0 & \gamma I & 0 \\ R^{\frac{1}{2}} Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \; j = 1,2,\ldots,L \tag{5.19}$$

where, $Q = \gamma P(k)^{-1}$ and $K(k)$ is expressed by $K = YQ^{-1}$. Thus we want to minimize the upper bound of the equation (5.17)

$$\min_{\gamma,Q,Y} \gamma \tag{5.20}$$

subject to (5.18) and (5.19)

This defines our *unconstrained* linear objective minimization problem. For notational convenience, we have excluded the time index k in the above optimization. Actually, the variables should be expressed by Q_k, F_k, Y_k etc., to indicate that they are computed at time k.

The corresponding *robust stability* and *feasibility* for the system (5.7) can be found in Kothare *et al.* (1996) and is omitted here for brevity. For nominal case, (L=1), it can be shown that a standard discrete-time Linear Quadratic Regulator (LQR) solution is obtained.

The limits on the control signal can be incorporated into the robust MPC algorithm as sufficient LMI constraints. At sampling time k, consider the component-wise peak constraint (5.14). The constraint is imposed on the present and the entire horizon of future manipulated variables if there exists a symmetric matrix X such that the linear matrix inequality holds

$$\begin{bmatrix} X & Y \\ Y^T & Q \end{bmatrix} \geq 0 \quad \text{with } X_{rr} \leq u_{r,\max}^2 \ r = 1,2, \ldots, n_u \tag{5.21}$$

This is an LMI in X,Y and Q. Inequality represents sufficient LMI constraints which guarantee that the specified constraints (5.14) on the manipulated variables are satisfied.

Similarly, output constraints can be defined as LMIs. At sampling time k, consider the component-wise peak constraint (5.15). This is the worst-case constraint over the set Ω and it is imposed over the future prediction horizon ($i \geq 1$). The output constraints are satisfied if there exists a symmetric matrix Z such that for each vertex of Ω the LMI

$$\begin{bmatrix} Z & C(A_j Q + B_j Y) \\ (A_j Q + B_j Y)^T C^T & Q \end{bmatrix} \geq 0,\ j = 1,2,\ldots,L \tag{5.22}$$

with $Z_{rr} \leq \ y_{r,\max}^2$, r = 1,2, …, n_y holds. Condition (5.22) represents a set of LMIs in Z,Y and $Q > 0$ for the polytopic uncertainty model.

We can now define the robust constrained MPC algorithm which will be the basis for our primal robust MPC CD algorithm.

Algorithm 5.1 Constrained linear objective minimization problem (Kothare *et al.*, 1996)

For the system (5.7), at sampling time k, the state feedback matrix K(k) in the control law u(k+i|k) = Kx(k+i|k), i ≥ 0, which mimimizes the upper bound γ on the worst case MPC objective function $J_\infty(k)$ *, is given by K(k) =* YQ^{-1} *where Q > 0 and Y are obtained from the solution (if it exists) of the following linear objective minimization problem:*

$$\min_{\gamma, Q, X, Y, Z} \gamma$$

subject to (5.18), (5.19), (5.21) and (5.22)

This MPC algorithm, if initially feasible, robustly asymptotically stabilizes the closed-loop system.

Thus, the goal of the primal robust MPC CD algorithm is at each sampling time k, define a constant state-feedback control law $u(k+i|k) = Kx(k+i|k)$ to minimize the upper bound $V(x(k|k))$. Only the first computed input $u(k|k) = Kx(k|k)$ is implemented. At the next sampling time, the state $x(k+1)$ is measured and the optimization is repeated to recompute K. The robust MPC CD controller stabilizes all matrices within the matrix polytope Ω.

The derived robust constrained MPC algorithm can be extended in several ways. Kothare *et al.* (1996) have shown how optimal tracking problems, constant set-poit tracking and disturbance rejection problems can be treated with the robust MPC algorithm. They have also proved how process delays can be incorporated into the algorithm. We will study next this subject.

5.4.1 Systems with delays

Delays can be taken into account in the robust MPC algorithm as follows. Consider the following uncertain discrete-time linear time-varying system with delays:

$$\begin{aligned} & x(k+1) = A_o(k)x(k) + \sum_{i=1}^{m} A_i(k)x(k-\tau_i) + B(k)u(k-\tau) \\ & y(k) = Cx(k) \\ \text{with}\quad & [A_o(k)\ A_1(k)\ \ldots\ A_m(k)\ B(k)] \in \Omega \end{aligned} \tag{5.23}$$

We assume that the discrete delays τ_i can be organized $0 < \tau < \tau_1 < \ldots < \tau_m$. At sampling time $k \geq \tau$, we would like to formulate a state-feedback control law $u(k+i-\tau|k) = Kx(k+i-\tau|k)$, $i \geq 0$, and minimize the modified infinite horizon robust objective function

$$\max_{[A(k+i)B(k+i)]\in\Omega, i\geq 0} J_\infty(k) = \sum_{i=0}^{\infty} \left(x(k+i|k)^T Q_1 x(k+i|k) + u(k+i-\tau|k)^T R u(k+i-\tau|k) \right) \tag{5.24}$$

subject to input and output constraints. An augmented state is defined as

$$w(k) = [x(k)^T\ x(k-1)^T\ \ldots\ x(k-\tau)^T\ \ldots\ x(k-\tau_1)^T\ \ldots\ x(k-\tau_m)^T]^T \tag{5.25}$$

If the augmented state is assumed to be measurable at each time $k \geq \tau$, an upper bound on the robust objective function (5.24) can be derived. The constrained minimization problem of the upper bound with the state-feedback control law $u(k+i-\tau|k) = Kx(k+i-\tau|k)$, $k \geq \tau$, $i \geq 0$, can be formulated as the linear objective minimization in Algorithm 5.1. These details can be derived straightforwardly. As pointed out by authors (Kothare *et al.*,1996) the specified choice of the function $V(w(k))$ which satisfy the condition (5.16) is

$$V(w(k)) = x(k)^T P_o x(k) + \sum_{i=1}^{\tau} x(k-i)^T P_\tau x(k-i) + \sum_{i=\tau+1}^{\tau_1} x(k-i)^T P_{\tau_1} x(k-i) + \ldots + \sum_{i=\tau_{m-1}+1}^{\tau_m} x(k)^T P_{\tau_m} x(k)$$

$$= w(k)^T \mathrm{P} w(k) \qquad (5.26)$$

where P is properly defined in terms of P_o, P_τ , $P_{\tau 1}$, ... $P_{\tau m}$. This kind of modified choice of $V(w(k))$ is closely related to the Modified Lyapunov-Krasovskii (MLK) functional (Feron *et al.,*1992).

5.4.2 Disturbance rejection

The presence of disturbances is one of the main reasons for using control, because in all practical applications some disturbances enter to the system. It is customary to distinguish among different types of disturbances, such as load disturbances, measurement errors and parameter variations. In process control load disturbances are typically quality variations which vary slowly or periodically. Four different types of disturbances – impulse, step, ramp and sinusoid – are commonly used in analyzing control systems. However, we will use only impulse and step disturbances in our CD study. The impulse is a simple idealization of sudden disturbance of short duration. It may represent load disturbances as well measurement errors. On the other hand, the step disturbances are typically used to represent load disturbances or offsets in a measurement.

Kothare *et al*. (1996) have shown how disturbance rejection problems can be treated with the robust MPC algorithm. Impulse disturbances can directly be handled with the robust MPC algorithm. Regarding load disturbances, one traditional way to approach this problem is to estimate them and use feed-forward from the estimated disturbance. In classical design, steady-state errors are eliminated introducing integrators if an unknown constant additive disturbance is acting at the output of the plant, see Fig. 4.2. One way to introduce an integrator is to define a new state that integrates the error between plant output and measured output y_m. However, in our case the augmentation would increase the dimension of the system and lead to serious computation problems. Thus it is an inadvisable approach. Another way to eliminate steady-state output errors or load disturbances, without use of integral action, is based on the same method used for offset-free tracking of constant set-points (Kothare *et al*. 1996). Basically this method utilizes the fact that currently estimated disturbance will persist at the same level also into the future and it will continue to act throughout the whole prediction horizon. As pointed out by Maciejowski (2002) this approach is commonly used in several commercial MPC software applications.

Let us consider (5.7), which we will now assume to represent an uncertain linear time-invariant system i.e., $[A\ B] \in \Omega$ are constant unknown matrices. Suppose that the system output y is required to eliminate the constant load disturbance y_d or in the case of set-point tracking to track the target vector y_d, by moving the system to the set-point x_s, u_s where

$$x_s = Ax_s + Bu_s \; , \; y_d = Cx_s \tag{5.27}$$

We assume that x_s, u_s, y_d are feasible, i.e., they satisfy the imposed constraints. The choice of $J_\infty(k)$ for the robust load disturbance rejection or set-point tracking objective in the optimization (5.13) is the following:

$$J_\infty(k) = \sum_{i=0}^{\infty}\left((Cx(k+i\,|\,k) - Cx_s)^T Q_1 (Cx(k+i\,|\,k) - Cx_s) + (u(k+i\,|\,k) - u_s)^T R(u(k+i\,|\,k) - u_s)\right) \tag{5.28}$$

where $Q_1 > 0$, $R > 0$.

As discussed in (Kothare *et al*. 1996), we can define a shifted state $\tilde{x} = x(k) - x_s$, a shifted input $\tilde{u} = u(k) - u_s$ and a shifted output $\tilde{y} = y(k) - y_d$ to reduce the problem to the standard form as shown earlier in this chapter. Component-wise peak bounds on the control signal u can be translated to constraints on $\tilde{u}$ as follows:

$$\left|u_j\right| \le \left|u_{j,\max}\right| \;\Leftrightarrow\; \left|\tilde{u}_j + u_{s,j}\right| \le u_{j,\max} \;\Leftrightarrow\; -u_{j,max} - u_{s,j} \le \tilde{u}_j \le u_{j,max} - u_{s,j} \tag{5.29}$$

Constraints on the deviation $y(k)$ from the value y_d can be incorporated in a similar way. We will use this approach to eliminate step disturbances at the output of the robust MPC CD system and refer it to the *modification* of primal robust MPC CD algorithm.

5.5 Simulation of the primal robust MPC CD algorithm

At present simulation is a widely applied method in pulp and paper industry, and applications vary from process control and production planning to economic evaluation. Simulation has been commonly used in troubleshooting, production and energy control and operator training (Woodward *et al.*, 1988). Modeling and simulation are traditionally used in process studies and optimization and in defining and testing the process control strategies. By off-line simulation, alternative operation strategies, sensitivity studies and optimization methods can easily be tested without disturbing the actual process. However, this kind of approach requires a clear knowledge of the process behaviour and relatively accurate process models.

The performance of the implementation of the proposed primal robust MPC algorithm for cross-directional control problem was investigated by performing a number of simulations. The results of these inverstigations are presented in this section. A LMI Control Toolbox (Gahinet *et al.*,1996) software in Matlab 5.2 environment was used in the simulations to compute the solution of the linear objective minimization problem.

5.5.1 Simulated CD response model

In our simulation study we will apply a CD response model to design a robust MPC CD controller for the system with interactions. The interaction matrix is a Toeplitz symmetric matrix $G^{n,n}$ = toeplitz$_n\{g_1, g_2, \ldots, g_m, 0, \ldots, 0\}$ with uncertainty bounds for each element. Uncertainty in the CD interaction matrix $G^{n,n}$ is expressed by $g_i \in [g_{imin}, g_{imax}]$ in the CD response model (3.1). This kind of correlated coefficient uncertainty in $G^{n,n}$ indicates observed deviations in experimentally measured CD response interactions (Laughlin *et al.,* 1993). In our study we use uncertainty bound values which represent 25 % errors from the nominal values of g in the response Table 4.1 presented in Chapter 4. Elements of the response model vary randomly between specified bounds $g_i \in [g_{imin}, g_{imax}]$. The uncertainty bounds are equal for all elements, excluding the diagonal elements that represent the center of the response and stay unchanged. Of course element specific bounds can be defined, but they increase the number of vertexes of Ω and affect to the equations (5.19) and (5.22).

Due to clarity of description we will use the continuous time model of first-order actuator dynamics (3.5) with uncertain model parameters and a constant time-delay in the simulation:

$$g_a(s) = \frac{k_a e^{-\tau s}}{\tau_a s + 1} \tag{5.30}$$

$$k_a \in [0.9, 1.1]; \quad \tau_a \in [6.0, 10.0]; \quad \tau \in Z$$

The time-delay τ is assumed to be an integer number of sample times $\{\tau \in Z\}$. The gain k_a and the time constant τ_a are allowed to vary randomly and independently between the specified upper and lower bounds. In the simulation a *process generator* describing the uncertain CD process produces the randomly varying elements of the response model $G^{n,n}$ and actuator parameters.

We assume that the number of actuators is 20 and the actuator spacing is 150 mm, thus the width of the sheet is roughly 3000 mm. If the traversing speed of the measuring head is 500 mm per second, a scanning time of 6 sec for the profile measurement is achieved. This is our sampling time for the simulation. Thus we assume that the sampling rate is less or equal than the lengths of the transient of the dynamic response of spatial responses, and therefore the measurements will observe the dynamics. Tustin's method is used to accomplish the transformation from *s*-domain to *z*-domain. Simulated discrete-time response model (3.1) will be of the form

$$y(z) = G(z)u(z)\ , \qquad G(z) = g_a(z)G_o$$

where the elements of the transfer matrix $G(z)$ are given by

$$G_i(z) = C[zI - A_i\ ]^{-1}B_i\ z^{-\tau}\ ,\ i = 1,2,\ldots,L \tag{5.31}$$

and (A_i, B_i, C) are defined in (5.7) and (5.8). In addition a steady-state assumption (Rawlings and Chien, 1996) is presumed, see also Chapter 3.1. We assume that the discrete-time the CD response model is defined from specified input/output data set at different operating points and it will take into account the uncertainty bounds. From each data set a discrete-time linear state-space model is defined and prespecified set of CD models is expressed as a disctrete-time time-varying linear system as presented in equation (5.7). The operating point dependent set Ω is a polytope as depicted in Fig. 5.1. Because we have three pairs of varying parameters: $g_i \in [g_{imin}, g_{imax}]$, $k_a \in [k_{amin}, k_{amax}]$ and $\tau_a \in [\tau_{amin}, \tau_{amax}]$, thus $L = 6$ and the multi-model polytope will be $\Omega_{sim} = \mathrm{Co}\{[A_1\ B_1],[A_2\ B_2],\ldots,[A_6\ B_6]\}$. An example of the discrete-time response model is presented in Appendix C.

We assume that the *state vector* $x(k)$ i.e mapped CD profile of the uncertain system will allways be the same and *measured* at each sampling time k. Therefore the state observer in Fig.5.2 is just a calculation block in terms of general structure of the MPC. This assumption simplifies our approach considerably. We want to define a constant state-feedback CD control law $u(k+i|k) = Kx(k+i|k)$ to minimize the upper bound $V(x(k|k))$ in equation (5.17) and simultaneously take into account constraints (5.14) and (5.15). This requirement is expressed equivalently as equation (5.20) and LMI equations (5.18, 5.19, 5.21, 5.22). These equations, the discrete-time linear polytopic system model (5.7) and polytope Ω_{sim} define our *primal robust MPC* CD simulation problem. Regarding rejection of step disturbances, modification of primal robust MPC CD algorithm is used as defined in Chapter 5.4.2.

5.5.2 Studied CD response models

We will apply robust MPC control approach for selected CD response models presented earlier in the Chapter 4.3. For the simulations response models have been normalized such that the magnitude of the response at the center position is one. In the simulations a quadratic response model of size $G^{20,20}$ is considered. The size of model is selected based on the reasonable computation time. This issue is discussed more in details later. However, the selected size can be seen to bring out the basic behavior of the CD response.

We will use four different kinds of disturbances in the simulation either to characterice impulse or step disturbances. This approach is partly similar to Laughlin *et al.* (1993) apart from we will use impulse disturbances and also a typical unit disturbance d_4 to characterise clearly load disturbances. Disturbance like d_2 enters with a large component in the direction of the vector corresponding to the minimum singular value and is therefore difficult to reject. Disturbances d_1 and d_3 enter the system in different, more favorable directions and are therefore easier to handle.

$$\begin{aligned} d_1 &= [1.0,0.8,0.4,-0.4,-0.8,-1.0,-0.8,-0.4,0.4,0.8,1.0,0.8,0.4,-0.4,-0.8,-1.0,-0.8,-0.4,0.4,0.8] \\ d_2 &= [1.0,-1.0,1.0,-1.0,1.0,-1.0,1.0,-1.0,1.0,-1.0,1.0,-1.0,1.0,-1.0,1.0,-1.0,1,-1.0,01.0,-1.0] \\ d_3 &= [1.0,0.9,0.7,0.4,0.0,-1.0,1.0,-1.0,-0.7,-.3,0.3,0.7,1.0,-1.0,1.0,0.0,-0.4,-0.7,-0.9,-1.0] \\ d_4 &= [0.0,0.0,0.0,0.0,0.0,0.0,1.0,0.0,0.0,0.0,0.0,0.0,0.0,-1.0,0.0,0.0,0.0,0.0,0.0,0.0] \end{aligned} \tag{5.32}$$

The behavior of the DC responses is analyzed by using the RGA and DIC analysis methods presented in Chapter 4. In Table 5.1 the condition number $\gamma(G)$, the minimum and maximum singular value, and both the minimum and maximum element value Λ of RGA of studied CD response models $G^{20,20}$ are presented. Similarly results of the DIC rules are shown.

TABLE 5.1: Condition number $\gamma(G)$, min-max singular values $\sigma(G)$, min-max RGA element values Λ, and results of RGA-, MIC-, SM(i)-, DIC-, $\mu(E(0))$- and IC- rules of studied response models $G^{20,20}$.

Model	$\gamma(G)$	$\underline{\sigma}(G)$	$\overline{\sigma}(G)$	Λ_{min}	Λ_{max}	RGA	MIC	SM(i)	DIC	$\mu(E(0))$	IC
1	2.31	0.604	1.395	-0.045	1.091	Yes	Yes	Yes	Yes	Yes	Yes
2	8.57	0.208	1.791	-0.333	1.666	Yes	Yes	Yes	Yes	Yes	Yes
3	6.98	0.227	1.586	-0.165	1.405	Yes	Yes	Yes	Yes	Yes	Yes
4	11.38	0.189	2.153	-0.046	0.798	No	No	No	No	No	No
5	15.82	0.132	2.089	-0.099	1.052	Yes	No	No	No	No	No
6	1.85	0.743	1.372	-0.023	1.046	Yes	Yes	Yes	Yes	Yes	Yes
7	33.27	0.060	1.995	-1.105	0.668	No	No	No	No	No	No
8	2.64	0.560	1.479	-0.054	1.145	Yes	Yes	Yes	Yes	Yes	Yes
9	12.01	0.213	2.563	-0.407	2.308	Yes	Yes	Yes	Yes	No	No
10	23.43	0.191	4.464	-0.662	0.623	No	No	No	No	No	No
11	145.40	0.036	5.329	-3.529	4.464	No	No	No	No	No	No
12	464.37	0.014	6.676	-9.271	7.182	No	No	No	No	No	No

As can be seen the condition number $\gamma(G)$ alternates widely from 1.84 to 464.37. By comparing these values to Table 4.2 we may observe that the condition number is really dependent on the dimension of the system. Based on the condition number we may say that, from control point of view, responses like G_4, G_5, G_7, G_9, G_{10}, G_{11} and G_{12} are undesired, because they have rather large condition numbers and relatively small minimum singular values. Especially models G_7, G_{11} and G_{12} indicate how the large condition number may be caused by a small value of $\underline{\sigma}(G)$. On the other hand, RGA analysis indicates that CD response models G_1, G_2, G_3, G_6 and G_8 represent less complex cases, and like in the case of model size $G^{86,86}$, also now RGA matrices are diagonal dominant with positive gains rather close to unity. Similarly, all these CD response models are decentralized controllable. In regard to the response model G_4 the same phenomenon that was observed already in Chapter 4 can be verified, i.e., G_4 represents a naturally decoupled system. Similarly, response models G_7, G_{10},G_{11} and G_{12} represent the most complex CD cases, and it is not a surprise, that traditional decentralized integral controllers should not be applied to these CD response models. Therefore, a more sophisticated controller is required.

From Table 5.1we may conclude that those CD response models, which are not decentralized integral controllable, and which also have large RGA element values, and a high condition number are primary candidates for our simulation study. Therefore, we will use response models G_2, G_7, G_{10} and G_{12} in the simulation, and as we can see only G_2 is DIC. Graphical illustration of these responses is presented in Fig. 5.4.

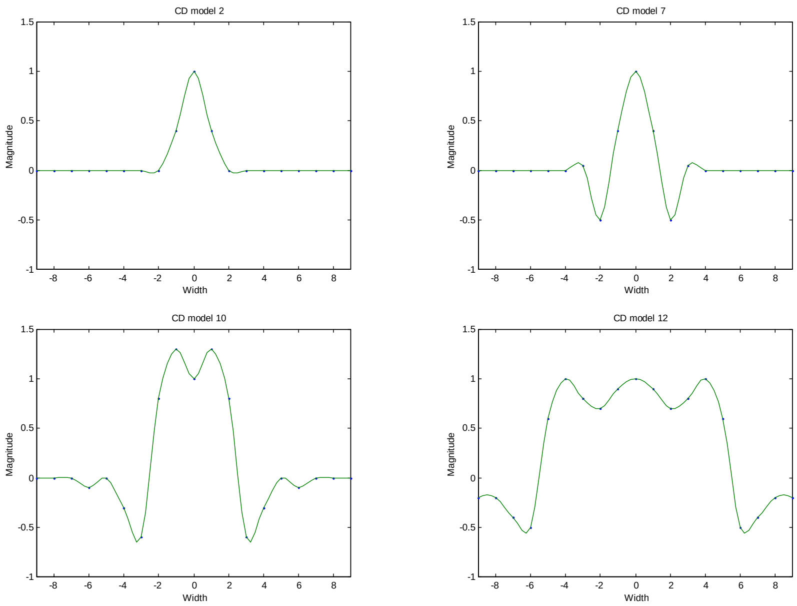

Figure 5.4: Toeplitz symmetric CD response models G_2, G_7, G_{10} and G_{12} from Table 4.1.

Studied response G_2 represents a CD model with strong decouplings (Wilhelm and Fjeld, 1983), while the model G_7 (Wilkinson and Hering, 1983) is a typical example of CD response of the basis weight to the slice lip actuator. On the other hand, models G_{10} and G_{12} describe examples of much wider and complex CD responses from the sack paper and board machines (Karslsson *et al.*, 1985).

5.5.3 Simulation of CD models without delays

In the first simulations we assume that the delay is not present ($\tau = 0$). Responses of the closed-loop system to the disturbances are shown in Figs. 5.5, 5.6. and 5.7. All CD response models were analyzed and similarly all disturbance options were studied. However, only the results of the selected CD response models, based on the RGA and DIC response analyses, are presented. In Fig. 5.5 the response and control actions of the closed-loop system to the impulse disturbance d_2 is shown.

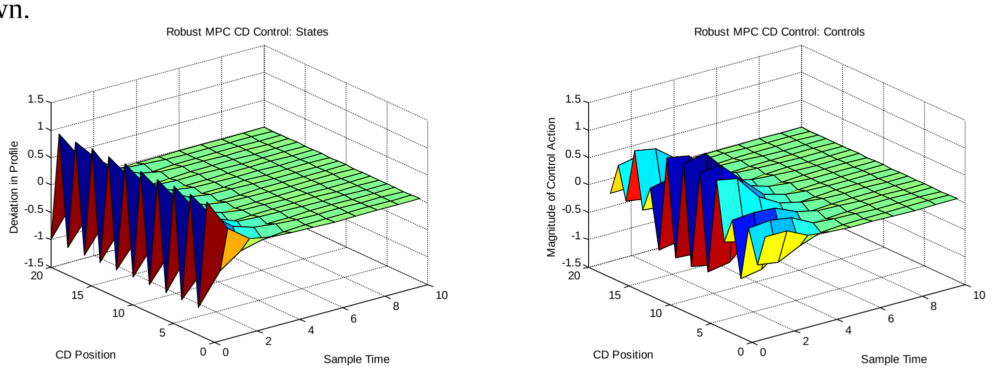

Figure 5.5: Response and control actions of the closed-loop system to the impulse disturbance d_2. CD model G_{12}.

CD response model G_{12} was used because it represents the most demanding and complex case from the control point of view. Large condition number (464.37) and unfavorable negative RGA elements indicate major control problems. Simulation parameters were $R = 0.000001I$ and $Q = I$ and all control actions $|u(k)| \leq 1_{max}$ and outputs $|y(k)| \leq 1_{max}$ were limited to the same value. We see the robust MPC CD controller rejects impulse disturbances effectively and stabilizes the closed loop system quite well in spite of the complexity of the model.

In Fig. 5.6 the response and control actions of the closed-loop system to the step disturbance d_2 is shown. Now the robust MPC CD controller is formulated as presented in Chapter 5.4.2. As expected the step disturbance d_2 is difficult to handle and the number of iteration loops increses easily over forty. Anyhow, the MPC CD controller operates effectively within defined bounds of constraints.

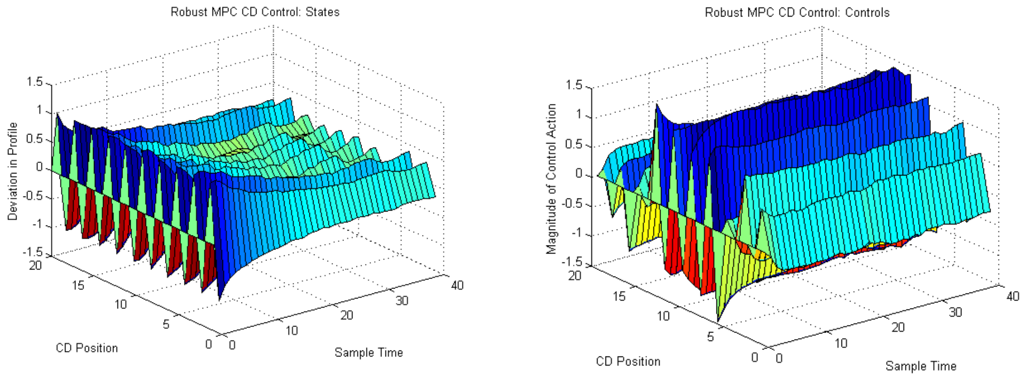

Figure 5.6: Response and control actions of the closed-loop system to the step disturbance d_2. CD model G_{12}.

It is clear that behavior of the closed-loop response is dependent on the selected CD model and type of disturbances. More complex and more demanding CD responses require aggressive control actions and more time for controller to stabilize the system. When the complexity of the CD model decreased it is easier for the controller to perform the control task. This can be seen from Fig. 5.7 where the response of the closed-loop system is presented for CD models G_7 and G_2, which have quite moderate condition numbers and acceptable RGA element values.

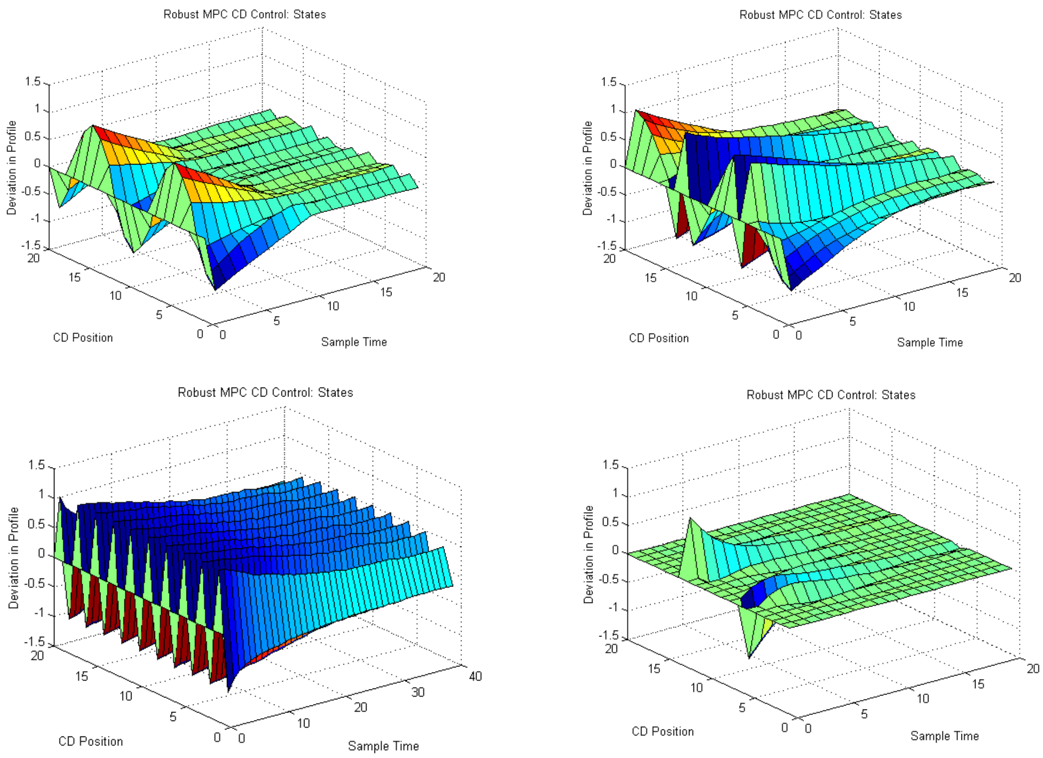

Figure 5.7: Responses of the closed-loop system. Above CD model G_7 with step disturbances d_1 and d_3. Below CD model G_2 with step disturbances d_2 and d_4.

Four different kinds of step disturbances were used. Simulation parameters were the same as before. It is obvious that the controller can perform the control task faster when the complexity of CD model is not high. However, the type of the disturbance affects clearly to the performance of the controller. Disturbance d_2 is really difficult to reject, which can be observed from the response of model G_2.

The weight factors Q and R in the cost function J affect to the behaviour of the CD control algorithm as expected; weighting either states or controls. During the simulation following values were used $Q = I$ and R varied in the range from $0.01I$ to $0.000001I$. The increase of value R restricts the use of large control actions which affects to the gain K. Behaviour of the gain matrix K is presented in Fig. 5.8. As can be seen the matrix is diagonal if the interactions between adjacent zones are small as in the CD response model G_2. When the complexity of the response model increases (model G_{12}) widens the gain matrix K in a similar way.

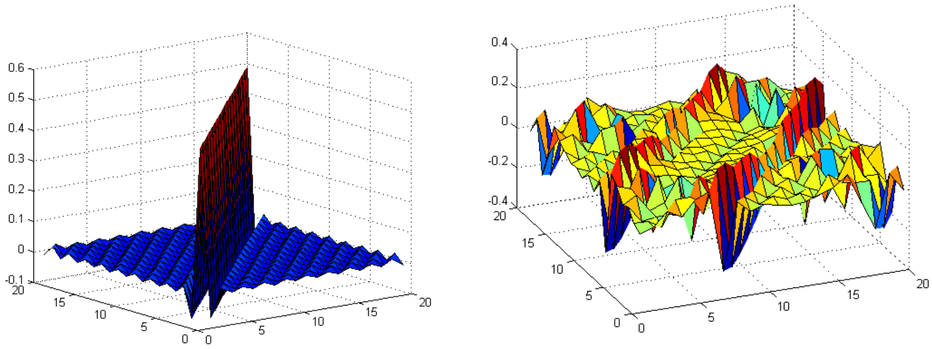

Figure 5.8: Gain matrix K for CD response models G_2 and G_{12}.

In Fig. 5.9 the norm of K as function of iterations for the robust MPC CD controller is shown. As can be noticed, to meet the constraint $|u(k)| = |Kx(k)|$ for small k, K must be small because $x(k)$ is large for small k.

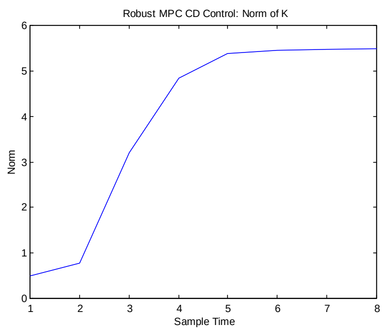

Figure 5.9: Norm of the feedback matrix K as a function of iterations. CD response model G_{12}.

But as $x(k)$ approaches zero, K can be made larger while still meeting the input constraint. This favourable use of the control constraint is possible only if K is recomputed at each time k, as in the robust MPC CD controller.

Next we will study how a *nominal* MPC CD controller behaves. Previously we have assumed that the multi-model polytope will have equal importance all over the uncertainty set Ω. Let us assume that our confidence of CD model is based on the most reliable observations of a single model and the rest of set is ignored. Let us define $L = 1$, which corresponds to the case when there is no plant-model mismatch and the model polytope is reduced to $\Omega_{\text{sim}} = \text{Co}\{[A_1\ B_1]\}$. As mentioned earlier for the nominal case a standard discrete-time Linear Quadratic Regulator (LQR) solution is obtained. This solution is based on the minimization of *nominal* performance objective function like (5.10).

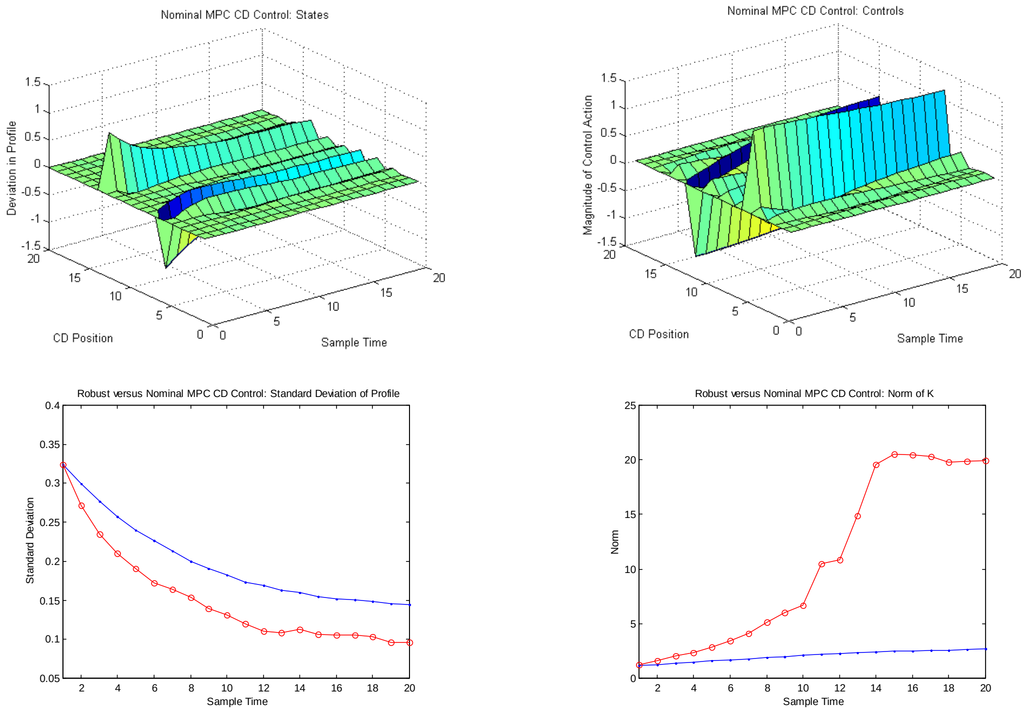

Figure 5.10: Above the response and control actions of the closed-loop system to step disturbance d_4. Below the development of standard deviation of robust (solid + circle) versus nominal (solid + dotted) MPC CD control and development of norm of the feedback matrix K as a function of iterations. CD model G_2.

Let us choose the CD response model G_2 which represents an easy case from control point of view. All the simulation parameters are now the same as previously. Nominal CD model G_2 is as defined in Table 4.1 and the actuator parameters (5.30) are constants with nominal values: $k_a = 1$, $\tau_a = 8$. In Fig. 5.10 the response and control actions of the closed-loop system to the step disturbance d_4 is shown. Similarly the development of standard deviation of robust versus nominal MPC CD control and development of norm of the feedback matrix K is shown. Robust MPC CD control and model G_2 with step disturbance d_4 was shown earlier in Fig. 5.7. We can see that the response of nominal MPC CD controller is about three times slower than the response of robust MPC CD controller. This slowness can be understood if we consider the development of norm of the feedback matrix K. In both cases the receding horizon controller recomputes the feedback matrix K at each time k, but only in the case of robust MPC controller it is done based on minimization of real *robust* performance objective (5.13). The maximization of the "min-max" problem (5.13) is done over the multi-model polytope $\Omega_{sim} = \text{Co}\{[A_1\ B_1],[A_2\ B_2],\ldots,[A_6\ B_6]\}$ and it corresponds to choosing that time-varying plant which would lead to the largest value of $J_\infty(k)$ among all plants in Ω_{sim}. While this worst-case value is minimized over present and future control moves with a constant state feedback control law $|u(k)| = |Kx(k)|$, the feedback matrix K is allowed to enlarge optimally within bounds of constraints, as $x(k)$ approaches zero.

Studied impulse response models converge from initial values in 4 - 6 iteration loops to the acceptable accuracy tolerances. However, in the case of step disturbances the convergence speed was much slower because of active contraints. The type of disturbances influenced crucially the convergence speed of the closed loop response. Likewise complexity of the response model affected the computation time but its maximum result was only about 15 to 20 per cent. Times required to compute the closed loop responses were about 120-180 seconds per sample, on a 3 GHz Pentium 4 PC with 2 GB of RAM using Matlab 5.2 code. For step disturbances values were higher than for simple impulse disturbances. However, no attempt was made to optimize the computation time, and therefore the times required for the computation of the closed-loop responses only indicate the status of the present LMI solvers. In the simulation most of the time was required to solve the LMI optimization at each sampling time. Constraints and initial values affect to the converge and computation time, but the influence is minimal.

5.5.4 Simulation of CD models with delays

Next we will study the case where the time-delays are taken in to account in the MPC CD algorithm. In our simulation we assume that the time-delay τ is an integer number of sample times and $\tau = 2$. The state space presentation (5.23) becomes

$$x(k+1) = A_o(k)x(k) + B(k)u(k\text{-}2)$$
$$y(k) = Cx(k)$$
$$[A_o(k)\ B(k)] \in \Omega \tag{5.33}$$

The state-feedback control law is $u(k+i\text{-}2|k) = Kx(k+i\text{-}2|k)$, $k \geq 2$, $i \geq 0$ and the modified infinite horizon robust performance objective is

$$\max_{[A(k+i)B(k+i)]\in\Omega, i\geq 0} J_p(k) = \sum_{i=0}^{\infty}\left(x(k+i\,|\,k)^T Q_1 x(k+i\,|\,k) + u(k+i-2\,|\,k)^T Ru(k+i-2\,|\,k)\right) \tag{5.34}$$

As in the standard non-delayed system, only the first computed input $u(k\text{-}2|k) = Kx(k\text{-}2|k)$, is implemented. At the next sampling time, the state $x(k+1)$ is measured and the optimization is repeated to recompute K. The augmented state vector (5.25) is expressed as $w(k) = [x(k)^T\ x(k\text{-}1)^T\ x(k\text{-}2)^T]^T$ and the function $V(w(k))$ in the equation (5.26) will be $V(w(k)) = x(k)^T P_o x(k) + x(k\text{-}1)^T P_\tau x(k\text{-}1) + x(k\text{-}2)^T P_\tau x(k\text{-}2)$. Augmentation increases the computational dimension of the LMI optimization problem from original 20 to 60. Naturally it has also an effect to the computation time.

In Fig. 5.11 the response and control actions of the delayed closed-loop system to the impulse disturbance d_1 is shown. Simulated CD response model was G_7, and the simulation parameters and input-output constraints were the same as previously with the model size 20. Because the profile measurements are not available before time $k \geq \tau$, the process model is not used to predict the future outputs of the system before that time.

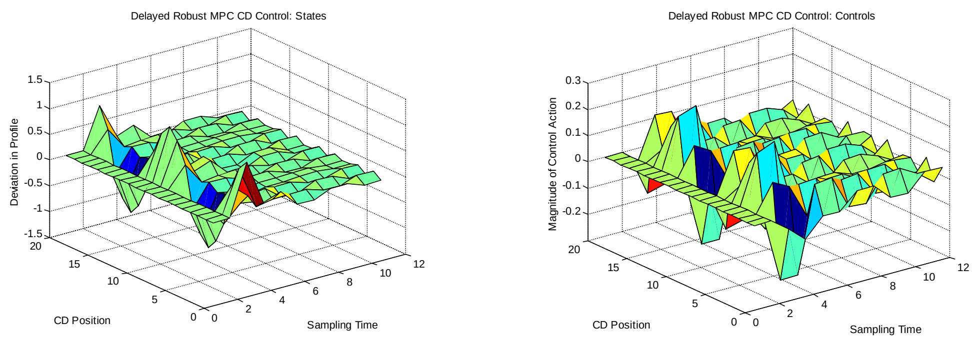

Figure 5.11: Response and control actions of the delayed closed-loop system to the impulse disturbance d_1. CD model G_7

Studied delayed model converges from initial impulse response values in 5 - 8 iterations to the acceptable accuracy tolerances. However, the time required to compute the closed loop response was long, 25.20 hours per sample. Especially this simulation showed how critically the size of the response model affected to the computation time.

5.5.5. Conclusions of the CD simulations

Several simulation examples have been presented to demonstrate the performance of the new robust MPC CD controller. Robust MPC CD algorithm was applied to a simulated paper machine, which has a realistic description of interactions across the machine. Simulated CD response models were selected based on the RGA and DIC analysis shown earlier in the Chapter 4 and some of the chosen models represented extremely complex cases with large RGA element values and high condition numbers.

Simulation results show that the control strategy yields acceptable steady-state performance when remarkable polytopic model uncertainty is present. Study of the nominal MPC CD controller revealed that this kind of controller has significantly slower closed-loop performance than robust MPC CD controller, which allows the optimal use of feedback matrix *K*. This kind of sluggish behaviour is a result of using nearly a static state-feedback controller. In addition, the basic primal robust MPC CD controller is able to handle sudden disturbances like impulses inherently, but load disturbances like steps require additional modifications of the algorithm. This kind of extension of algorithm increases the computation time, but only slightly when the number of states is low. However, both algorithms acted consistently and effectively and we may state that they are suitable for complex and demanding CD response models, which are not controllable by using traditional decentralized CD controllers based on integral control.

On the other hand, simulations revealed clearly the complexity of the large scale robust CD control problem and also the limitation of the existing off-the-shelf LMI optimization software (Gahinet *et al.*,1996). The importance of the size and dimension of the response model can be noticed by considering the used LMI Matlab algorithm. The fastest interior point algorithms show $O(MN^3)$ growth in computation (Gahinet *et al.*,1996) where M is the total row size of the LMI system and N is the total number of scalar decision variables. Because M is proportional to L, which is the number of vertices of uncertain model and $N \approx n_s^2/2 + n_s n_c$, where n_s is the number of state variables and n_c the number of manipulated variables, it can be seen that the size of the response model defines clearly the computation time.

In the simulations this appeared especially when the delayed model was studied. However, the maximum number of state variables ($n_s = 60$) did not cause any memory problems on a 3 GHz Pentium 4 PC with 2 GB of RAM. Concerning computation, no action was made to optimize the computation time, and in all simulations default values for the LMI functions and control parameters of the LMI optimization algorithm were used. Feasibility of the solution to the given system of LMIs was computed during each simulation loop, and in the LMI optimization also a QR factorization was allowed to solve the least-squares problem, see in details Gahinet *et al.* (1996). However, LMI optimization algorithm did not have to use it, and computation was based on the default Cholesky factorization.

Although the achieved computation times of the robust MPC CD algorithm are too high for real time applications, the method itself seems to be very developable. In the next chapter we will present such a solution for the robust CD control problem that the computation drawbacks are solved and the developed robust MPC CD algorithm is applicable to the real time CD control problem.

Chapter 6

An Efficient Robust MPC Algorithm for CD Processes

One of the main drawbacks of MPC method is the high on-line computation requirement which limits the applicability of MPC to reduced scale processes with slow dynamics. To overcome these problems many researchers have started to develop fast computational solutions to the optimization problems related to MPC. Lee and Kouvaritakis (2000) introduced a receding horizon dual-mode algorithm for constrained systems with polyhedral model uncertainty. Computational complexity was reduced through the use of LP method. For constrained MPC, Bemporad *et al*. (2002) presented the explicit state-feedback solution to finite and infinite horizon LQ optimal control problem. A piece-wise linear and continuous solution is provided and on-line QP solvers are no more required which decreases considerably the computational complexity.

On the other hand, regarding the control of cross-machine (CD) paper sheet properties several control schemes and algorithms have been applied and reported in the literature. However, concerning the MPC method the large scale nature and several model uncertainty characteristics of the CD process have limited the availability of MPC in paper machine CD control. As pointed out earlier one way to solve the large scale problem of CD process is to reduce the size of the model and herewith intensify the on-line computation. Haznedar and Arkun (2002) used principal component analysis method for model reduction and identification. VanAntwerb and Braatz (2000a) designed a fast MPC algorithm for CD control problem to avoid exciting uncontrollable plant directions. They applied truncated singular value method to maintain robustly controllable components in the admissible set of models. They used an iterated ellipsoid algorithm, which is based on an off-line singular value decomposition of the model. In this method an ellipsoid is applied to approximate the polytopic input constraint set in an off-line calculation and the size of ellipsoid is optimized on-line during calculation.

Regarding robust control, after Laughlin (1988) and Duncan (1989) who in late 1980s studied the robustness of CD control systems, several methods have been suggested for designing robust CD controllers. Similarly, as pointed out earlier, numerous model predictive control (MPC) approaches have been proposed that contain actuator limitations but do not explicitely address model inaccuracies. For the time being, the robust constrained MPC has not been implementable in real-time large scale CD processes because of the heavy on-line computation.

In this chapter, an efficient LMI based robust model predictive controller is proposed in order to control the CD processes. First the proposed robust MPC strategy, which incorporates directly actuator limitations and model uncertanties into the control algorithm, is introduced. We will see that the control algorithm guarantees the robust stability of the closed-loop system for polytopic uncertainty descriptions, and it provides off-line a sequence of stabilizing state feedback laws, which consist in invariant ellipsoids one inside another in the state space. After this, the control algorithms closed-loop performance is studied with simulations by using a large scale CD processes with realistic description of interaction matrices and general uncertainty structures. Similarly a simulation comparison with the standard off-the-self MPC CD algorithm is performed. Simulation results will show that the performance of the robust MPC CD algorithm is efficient and algorithm is applicable to the complex CD response models.

6.1 Fast robust constrained MPC CD algorithm

In this section we present a fast robust constrained MPC algorithm based on the concept of the *asymptotically stable invariant ellipsoid*. Presentation follows closely the structure shown by Wan and Kothare (2003) and Wan (2003). Developed algorithm will be used later for polytopic CD response systems to illustrate efficiency of the approach.

Definition (Wan and Kothare, 2003)

Given a discrete dynamical system $x(k+1) = f(x(k))$, a subset $\xi = \{x \in R^{n_x} \mid x^T Q^{-1} x \leq 1\}$ of the state space R^{n_x} is said to be an asymptotically stable invariant ellipsoid, if it has the property that, whenever $x(k_1) \in \xi$, then $x(k) \in \xi$ for all times $k \geq k_1$ and $x(k) \to 0$ as $k \to \infty$.

If we consider a closed-loop system consisting of (5.7) and a state feedback controller $u(k) = YQ^{-1}x(k)$, where Y and Q^{-1} are achieved by applying the robust constrained MPC algorithm of *the linear objective minimization problem* from Chapter 5 to a system state x_0. Then the subset ξ =

{$x \in R^{n_x}$ |$x^T Q^{-1} x \leq 1$} of the state space R^{n_x} is an asymptotically stable invariant ellipsoid. Within an asymptotically stable invariant ellipsoid ξ ={$x \in R^{n_x}$ |$x^T Q^{-1} x \leq 1$}, we define the distance between the state *x* and the origin as a weighted norm $\|x\|_{Q^{-1}} \equiv \sqrt{x^T Q^{-1} x}$. For an input constrained system we know that when the state is approaching the origin, less constraints on the choice of the feedback matrix will be focused (Kothare *et al*., 1996). If we apply *the linear objective minimization problem* from Chapter 5 to a state far from the origin, the resulting asymptotically stable invariant ellipsoid has a more constrained feedback matrix. However, we do not have to keep this feedback matrix constant while the state is converging to the origin (Kothare *et al*., 1996). It is possible to formulate inside the ellipsoid another asymptotically stable invariant ellipsoid, which is based on a new state closer to the origin. Therefore, by repeating this procedure, we have more possibilities to construct acceptable feedback matrices based on the distance between the state and the origin.

Algorithm 6.1 Off-line robust constrained MPC algorithm (Wan and Kothare, 2003)

Consider an uncertain system (5.7) subject to input and output constraints (5.14) and (5.15). Off-line, given an initial feasible state x_1, process a sequence of minimizers, γ_i, Q_i , X_i , Y_i and Z_i (i = 1,…,N) as follows.
Set i : = 1.

1. *Compute the minimizer γ_i, Q_i , X_i , Y_i , Z_i at x_i as defined in the linear objective minimization problem in Chapter 5.4 with additional constraint $Q_{i-1} > Q_i$ (ignored at i = 1), store Q_i^{-1} , K_i (=$Y_i Q_i^{-1}$), X_i , Y_i in a data base*
2. *If i < N, choose a state x_{i+1} satisfying $\|x_{i+1}\|^2_{Q_i^{-1}} < 1$.*

Set i : = i +1, return to step 1

On-line given an initial state x(0) satisfying $\|x(0)\|^2_{Q_i^{-1}} \leq 1$, let the sate x(k) at time k. Execute a bisection search over Q_i^{-1} in the data base to find the largest index i or equivalently the smallest ellipsoid ξ_i ={$x \in R^{n_x}$ |$x^T Q_i^{-1} x \leq 1$} such that $\|x(k)\|^2_{Q_i^{-1}} \leq 1$. Apply the control law u(k) = $K_i x(k)$.

Wan and Kothare (2003) have shown that the first step in the off-line processing algorithm is always feasible for *i* > 1, if it is assumed that it is feasible for *i* = 1. Likewise, they have shown that for a given dynamical system (5.7) and the initial state $x(0)$ satisfying $\|x(0)\|^2_{Q_i^{-1}} \leq 1$, the off-line processing MPC algorithm robustly asymptotically stabilizes the closed-loop system. As pointed out previously, the optimal robust MPC law and the equivalent asymptotically stable invariant ellipsoid are dependent on the state. Although the control law can be applied to all states within the ellipsoid, it is not necessarily optimal. However, it provides a *stable suboptimal* solution and reduces substantially the on-line computation time.

The sequence of state feedback matrices, generated in the off-line algorithm, is constant between two adjacent asymptotically stable invariant ellipsoids, and discontinuous on the boundary of each ellipsoid. However, this can be overcome and a continuous feedback matrix over the state space can be constructed by utilizing the following algorithm.

Algorithm 6.2 Design of continuous feedback matrix (Wan and Kothare, 2003)

Consider the data base generated by the off-line part of Algorithm 6.1. If for each x_i ($i = 1,\ldots,N-1$)

$$Q_i^{-1} - (A_j + B_j K_{i+1})^T Q_i^{-1}(A_j + B_j K_{i+1}) > 0 \;, \; j = 1,\ldots,L \tag{6.1}$$

is satisfied, then on-line, given an initial state $x(0)$ satisfying $\|x(0)\|^2_{Q_i^{-1}} \leq 1$ and the current state $x(k)$ at time k, perform a bisection search over Q_i^{-1} in the data base to find the largest index i or equivalently the smallest ellipsoid ξ_i such that $\|x(k)\|^2_{Q_i^{-1}} \leq 1$.

If $i \neq N$, solve $x(k)^T(\alpha_i Q_i^{-1} + (1 - \alpha_i)Q_{i+1}^{-1})x(k) = 1$ for α_i and apply the control law $u(k) = (\alpha_i K_i + (1 - \alpha_i)K_{i+1})x(k)$.

If $i = N$, apply $u(k) = K_N x(k)$.

For a given dynamical LTV system (5.7) and the initial state $x(0)$ satisfying $\|x(0)\|^2_{Q_i^{-1}} \leq 1$, the off-line processing robust constrained MPC algorithm robustly asymptotically stabilizes the closed-loop system, and similarly the feedback matrix K implemented in the off-line processing algorithm is a continuous function of the state x. Both above presented algorithms represent a general approach to construct a Lyapunov function for uncertain and constrained systems. Even if this function is not necessarily continuous on the boundary of each asymptotically stable invariant ellipsoid, it is monotonically decreasing within the smallest ellipsoid and within each ring region between two adjacent ellipsoids. This guarantees the stability of the closed-loop system (Wan and Kothare, 2003).

In both *Algorithms 6.1* and *6.2*, the choice of the state x_{i+1}, which satisfy $\|x_{i+1}\|^2_{Q_i^{-1}} < 1$ is arbitrary. Therefore, Wan and Kothare (2003) suggest to choose an arbitrary one-dimensional subspace $\Im = \{\alpha x^{max} \,|\, 1 \geq \alpha > 0, \alpha \in R, x^{max} \in R^{n_x}\}$, where the chosen state x^{max} should be far enough from the origin and still preserve the feasibility of the problem. This can be accomplished by discretizing the chosen set and constructing a set of discrete points, $\Im^d = \{\alpha_i x^{max} \,|\, 1 \geq \alpha_1 > \ldots > \alpha_N, \alpha_i \in R, x^{max} \in R^{n_x}\}$. Since the asymptotically stable invariant ellipsoid constructed for each discrete point actually passes through that point, $\|\alpha_{i+1} x^{max}\|^2_{Q_i^{-1}} < \|\alpha_i x^{max}\|^2_{Q_i^{-1}} = 1$ is satisfied. To cover a sufficient dimension of the state space a logarithmic scale discretization of the one-dimensional subspace is suggested. This enables the reduction of the number of discretization points and keeps the size of the data base reasonable. In addition, it is always possible to find a new feasible set of minimizers in *Algorithm 6.2* by readjusting the discretization.

If the LMI minimization problem is solved by using an interior point method, a set of strictly convex unconstrained minimization problems are used to solve a convex constrained minimization problem. In addition, for a strictly convex unconstrained minimization problem is characteristic that the objective function and all the minimizers are unique. Thus, the optimal solutions for the optimizations in the off-line part of *Algorithm 6.1* are assumed to be unique. Therefore, it is always possible to find the feasible set of minimizers for *Algorithm 6.2* if the discretization is done tight enough. Actually, condition (6.1) becomes trivial if x_{i+1} is chosen to be sufficiently close to x_i.

For the fast robust MPC algorithm the on-line calculation time is defined by the bisection search from the data base in which the Q_i^{-1} matrixes are stored. An array of S stored Q_i^{-1} requires $\log_2 S$ searches, because the discretization of the subspace is based the logarithmic scale. The matrix-vector multiplication in one search obeys square-law $O(n_s^2)$ in the number of flops, with n_s the number of state variables. Therefore the total number of flops required to calculate a control action is $O(n_s^2 \log_2 S)$, (Wan and Kothare, 2003). This value can be compared with the fastest interior point algorithm presented by Gahinet *et al.* (1996) which has $O(M\{n_s^2/2 + n_s n_c\}^3)$ growth in computation. M is the total row size of the LMI system and the expression in the brackets represents the total number of scalar decision variables. M is proportional to the number of vertices of the uncertain model and n_c represents the number of manipulated variables. It is evident that the bisection search method of the off-line approach can reduce signifigantly the on-line computation of the robust MPC algorithm. In the next section we will show by CD process simulations how notable this can be.

6.2 Simulation of the fast robust MPC CD algorithm

In this section, the proposed fast robust MPC control algorithm is applied to the sheet forming processes and its closed loop performance is studied by performing a number of simulations. Also in these simulations A LMI Control Toolbox software (Gahinet *et al.*, 1996) in Matlab 5.2 environment is used to compute the solution of the control problem.

6.2.1 Simulated CD response models

In the simulation we will apply the fast robust MPC control algorithm for the same CD response models as presented in Chapter 4. Our interaction matrix is a Toeplitz symmetric matrix $G^{n,n}$ = toeplitz$_n\{g_1, g_2, \ldots, g_m, 0, \ldots, 0\}$ with uncertainty bounds for each element $g_i \in [g_{i\min}, g_{i\max}]$. We will use three complex CD response models from the Table 4.1, and four step disturbances for the simulation, see equation (5.32). Simulated CD models are as follows:

$$G_7^{n,n} = \text{toeplitz}_n\{1.0,\ 0.4,\ -0.5,\ 0.05,\ 0,\ \ldots,0\}$$

$$G_{10}^{n,n} = \text{toeplitz}_n\{1.0,\ 1.3,\ 0.8,\ -0.6,-0.3,0.0,-0.1,\ 0,\ \ldots,0\}$$

$$G_{12}^{n,n} = \text{toeplitz}_n\{1.0,\ 0.9,\ 0.7,\ 0.8,1.0,0.6,-0.5,-0.4,-0.2,-0.2,\ 0,\ \ldots,0\}$$

Like in the previous simulations in Chapter 5, the amplitude of the uncertainty is 25 % from the nominal value of each element g_i. However, as before all diagonal elements remain unchangeable. First-order actuator dynamics with uncertain process model parameters is used, see equation (5.30). Now the time delay is excluded (τ = 0), but the limits of the parameters are as previously. The continuous CD response model is transformed to the discrete-time form (5.31) based on input-output data set, as shown in Chapter 5. The achieved set of CD models is expressed as a disctrete-time time-varying linear system (5.7). Model uncertainties are incorporated by utilizing the polytopic uncertainty description (5.8) and taking into account the varying parameter limits: $g_i \in [g_{i\min}, g_{i\max}]$, $k_a \in [k_{a\min}, k_{a\max}]$ and $\tau_a \in [\tau_{a\min}, \tau_{a\max}]$. The structure of the polytope will be $\Omega_{sim} = \text{Co}\{[A_1\ B_1],[A_2\ B_2],\ldots,[A_6\ B_6]\}$. The robust performance objective function J_∞(k), subject to input $|u(k+i|k)| \leq 1$, $i \geq 0$ and output constraints $|y(k+i|k)| \leq 1$, $i \geq 1$ is as defined in Chapter 5, see equations (5.11), (5.12) and (5.13). As regards to the rejection of step disturbances and modification of the robust MPC CD algorithm, see equation (5.28). Weighting matrices of the objective function are $R = 0.000001I$ and $Q = I$. These parameters are the same in all simulations.

6.2.2 Comparison with the primal robust MPC CD algorithm

In order to test the effectiveness of the fast robust MPC algorithm it is compared to *the primal robust MPC CD algorithm*, which was studied earlier in Chapter 5. The fast robust MPC CD algorithm is comprised of the *off-line robust processing algorithm* (*Algorithm 6.1*) and the *continuous feedback matrix algorithm* (*Algorithm 6.2*). We choose the x_1 -axis as an one-dimensional subspace, and discretize it into thirteen points x_1^{set} = [1, 0.9, 0.75, 0.65, 0.52, 0.4, 0.28, 0.18, 0.1, 0.05, 0.02, 0.01, 0.001]. In the *Algorithm 6.1* the minimizers γ_i, Q_i , X_i , Y_i , Z_i at x_i are calculated off-line based on the discretization of the subspace x_1 (i = 1,…,13) and saved into the data base. In the on-line part of the *Algorithm 6.1*, the control law is defined after the acceptance of initial state $x(0)$ and implementation of the bisection search over Q_i^{-1} in the data base. The additional constraint (6.1) in the *Algorithm 6.2* ensures that the defined feedback matrix is continuous. Algorithm 6.1 comprises also the modification of primal robust MPC CD algorithm, which is used for rejection of step disturbances as defined in Chapter 5.4.2. Structure of the fast robust MCP CD algorithm is presented in Fig. 6.1.

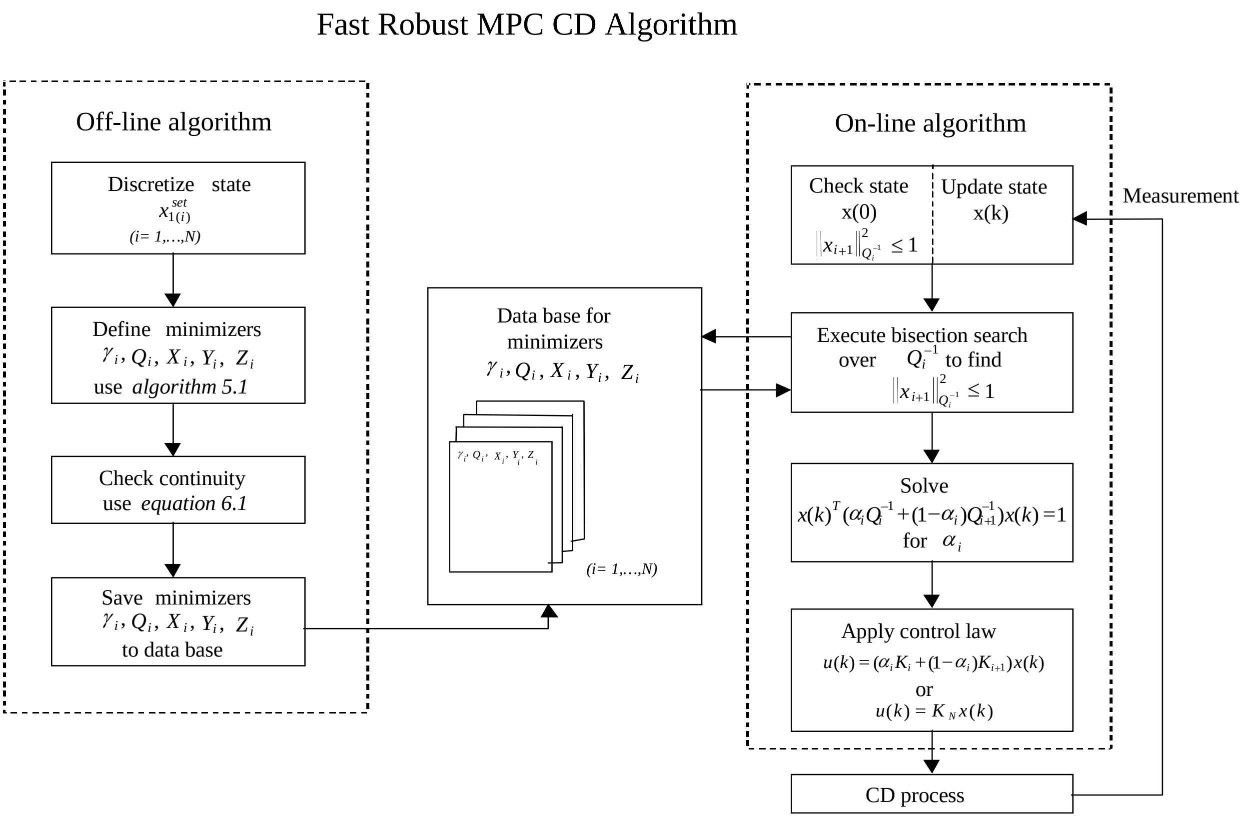

Figure 6.1: Structure of the fast robust MPC CD algorithm

In the first simulation the number of actuators is 20, thus the response matrix is $G^{20,20}$. Actuator spacing is 150 mm, and as previously mentioned no delay is present ($\tau = 0$). The width of the sheet is 3000 mm. The traversing speed of the measuring head is assumed to be 500 mm per sec. Thus a scanning time of 6 sec for the profile measurement is achieved. This is the sampling time for the simulation. Tustin's method is used to accomplish the transformation from *s*-domain to *z*-domain.

In Fig. 6.2 the comparison of the primal MPC CD algorithm with the fast MPC CD algorithm is presented. The response and control actions of the closed-loop systems to the step disturbance d_4 are shown. Similarly the development of standard deviation is presented. Simulated model is the CD response model G_{12}.

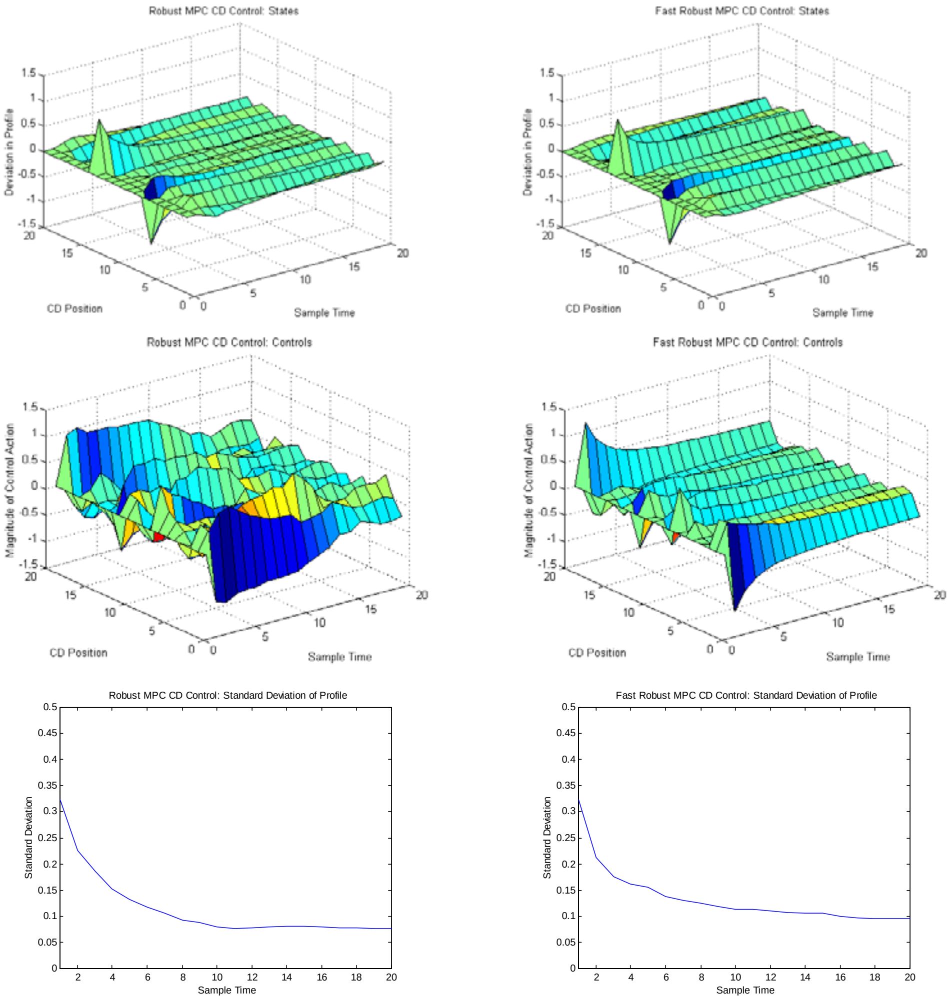

Figure 6.2: Response, control actions and development of standard deviation of the closed-loop system to step disturbance d_4. CD response model G_{12}. Left the primal robust MPC CD algorithm. Right the fast robust MPC CD algorithm.

The condition number of this model is very high 464.37 and the system is not decentralized integral controllable, see Table 5.1. We can see that the fast robust MPC CD controller gives nearly the same performance as the primal robust MPC CD controller. As can be seen the control actions of primal robust MPC CD algorithm are more aggressive and extensive concluding to slightly better control performance. On the other hand, the fast robust MPC CD controller acts consistently but less aggressively. However, it works rather successfully decreasing the standard deviation of the profile acceptably, which mean that the control actions have been effective and actuator profile throughout the simulation has been physically realistic by the use of actuator constraints. However, a slight overshoot of control actions can be seen in the beginning of procedure. An apparent reason for difference in the control performance of algorithms results from the harsh discretization of state subspace of the fast robust MPC CD algorithm only into thirteen points.

Of course the results of the simulations can not be exactly the same, because the applied process disturbances during the simulation are generated randomly. The development of standard deviation indicates that in terms of efficiency the difference between algorithms is minor. However, as regards to the performance both algorithms could be slightly faster. In terms of computation time the fast robust MPC CD algorithm is superior to the primal algorithm. On a 3 GHz Pentimum 4 PC with 2 GB of RAM the average time for the fast robust MPC CD algorithm to compute a feedback gain for our reference model was 1.0-1.4 seconds. This is about 100 times faster than the 120-160 seconds it takes for the primal robust MPC CD algorithm.

In Fig. 6.3 the response of the closed-loop system to the step disturbances d_1 and d_3 is shown. Simulated response model is now G_{10}. The condition number of this CD model is also rather high 23.43 and element values of RGA are large and negative. In addition, the response model G_{10} is not decentralized integral controllable. Simulation parameters were the same as in the previous simulation and control actions and outputs were limited.

We can see that the fast robust MPC CD controller works analogously with the primal robust MPC CD controller and the performance of controllers is quite similar. However, as in the case of primal robust MPC CD algorithm, the behavior of the closed-loop response is dependent on the selected CD model type and the type of disturbances. It is obvious that complex and demanding CD responses like model G_{10} need aggressive controls and more time steps to stabilize step disturbances. However, the complexity of the response model does not affect significantly to the computation time.

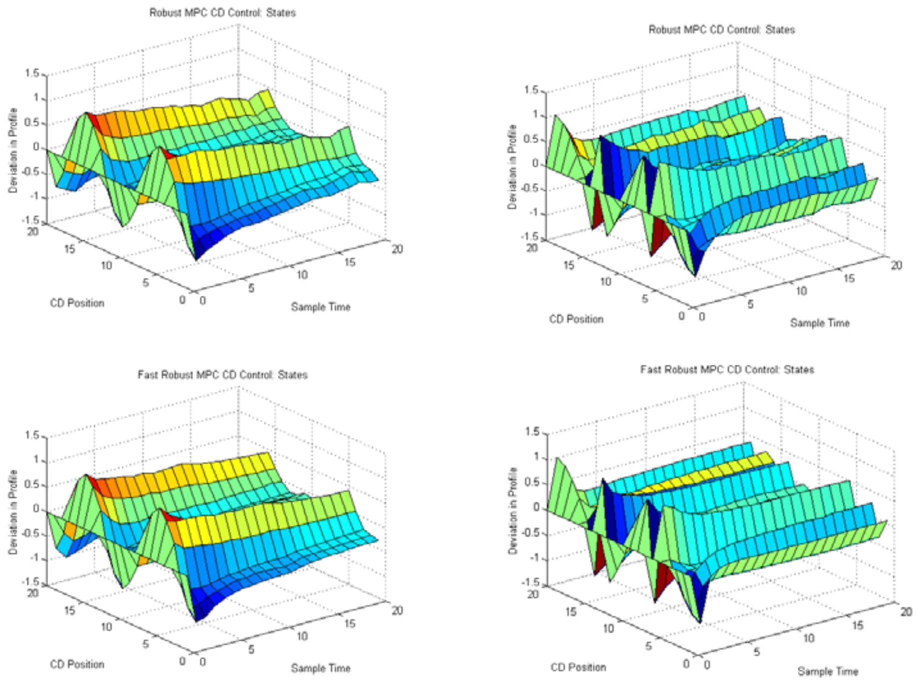

Figure 6.3: Response of the closed-loop system to step disturbances d_1 and d_3 respectively. CD response model G_{10}. Above the primal robust MPC CD algorithm. Below the fast robust MPC CD algorithm.

In Fig. 6.4 the norm of K as function of iterations for the fast robust MPC CD controller is shown. Simulated response model was G_{10} and the disturbance profile was d_2. We can see that in the beginning K must be small because x(k) is large for small k. But as x(k) approaches zero, K can be made larger while still meeting the input constraint $|u(k)| = |Kx(k)|$. Although the changes of the norm of K are moderate they clearly indicate functioning of the CD algorithm.

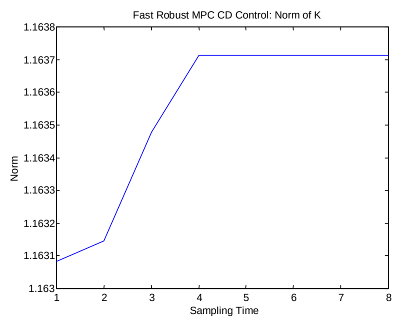

Figure 6.4: Norm of the feed back matrix K as a function of time steps.

6.2.3 Simulation of the large scale response models

The size of the CD response model is an important factor affecting to the computation time. This is examined in Fig.6.5 where the closed-loop simulation of model size $G^{86,86}$ is presented. The comparison of the primal robust MPC CD algorithm with the fast robust MPC CD algorithm was accomplished for the response model G_{12} with the impulse disturbance d_1. In this simulation the number of actuators was 86, actuator spacing 100 mm and web width 8600 mm. The sampling time for the simulation was likewise the same 6 seconds as before, because we wanted to use equal actuator dynamics as earlier. In practice sampling time of 6 seconds for web width of 8.6 meter is unrealistic but for comparative simulation purposes it is justified. Also now the Tustin's method was used to accomplish the transformation from *s*-domain to *z*-domain. Simulation parameters and constraints are as defined earlier.

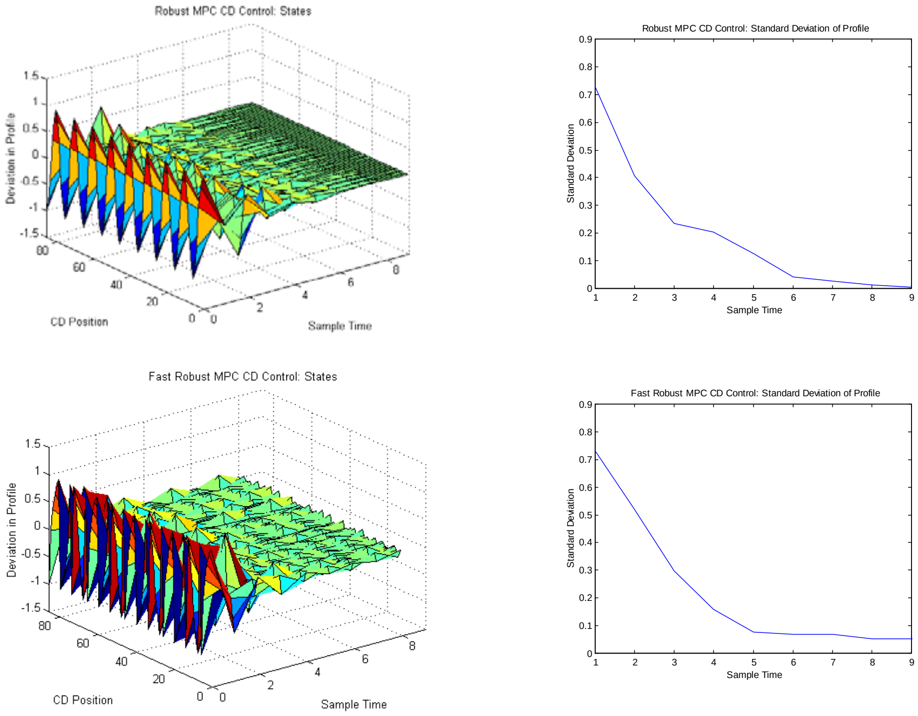

Figure 6.5: Response of the closed-loop system of size $G^{86,86}$ to impulse disturbance d_3. Response model G_{12}. Above the primal robust MPC CD algorithm. Below the fast robust MPC CD algorithm.

The condition number of this response model is 631.44 and DIC analysis in Chapter 4 revealed that model is not decentralized integral controllable. The time for the primal robust MPC CD algorithm to compute the closed-loop response, with default values for all LMI functions, was 85.16 hours per sample and the full simulation took time over four weeks on a 3 GHz 4 Pentium PC with 2 GB of RAM. This was also the main reason to use impulse disturbance in this comparative simulation because algorithm modifications required by step disturbance rejection approach would have doubled the computation time. For the fast robust MPC CD algorithm the average time to compute the same response was only 12.95 seconds per sample. However, as can be seen a slight offset remained into the profile, even with the impulse disturbance. Anyhow, the fast robust MPC CD algorithm was over 24000 times quicker than the original robust MPC CD algorithm.

The behavior of the model size $G^{86,86}$ was also studied with response models G_{10} and G_7 which represent less complex cases than model G_{12}. In Fig.6.6 the response of closed-loop system to step disturbances d_4 and d_1 is shown.

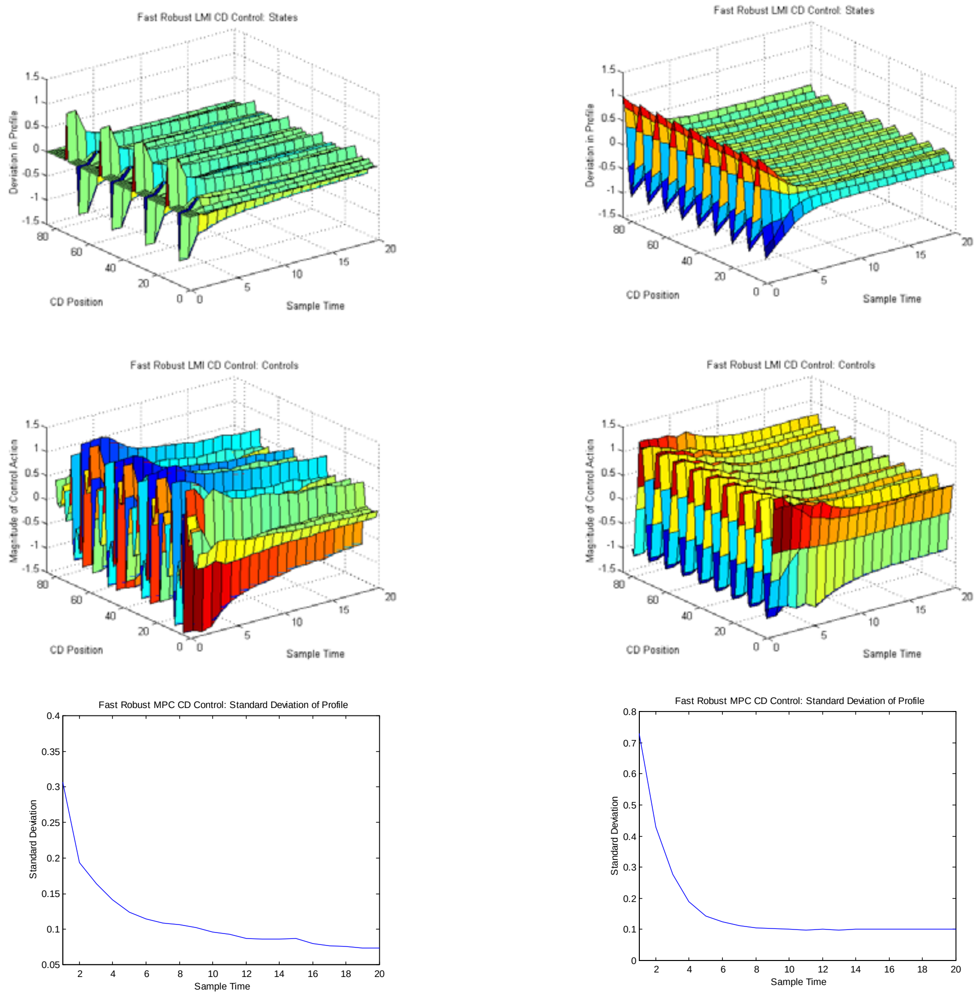

Figure 6.6: Response, control actions and development of standard deviation of the closed-loop system. Left response model G_{10} with step disturbance d_4. Right response model G_7 with step disturbance d_1.

In this simulation the general simulation parameters and constraints were the same as before and the modification of system and objective function was done as defined in Chapter 5.4.2. The condition number of matrixes G_{10} and G_7 is 315.50 and 93.26 respectively. Therefore it is not a surprise that the fast robust MPC CD algorithm was able to handle also these less complex cases consistently. This can also be seen from the improvement of standard deviation during simulation. However, a characteristic feature seemed to be that the fast robust MPC CD algorithm could not completely eliminate step disturbances. Control actions remained too faint and they did not even reach their limits at the end of simulation run. Anyhow, for the CD response model G_{10} the average time to compute the response was 11.32 seconds per sample and for the model G_7 only 9.85 seconds. We may observe that the complexity of the CD response model affects to some extent the computation time. Also the selection of profile disturbance has an influence to the computation time, but it is less significant that the complexity of the model.

It is obvious that actual time spent for closed-loop simulations will be completely dependent on the computer system that is being used. Fig.6.7 shows how the computation time per sample for the robust MPC CD algorithm grows as function of the number of actuators. Simulated response was model G_{12} and the time values were defined at three different points. Number of discretization points was in every simulation the same (N = 13). As we can see the slope follows the rule of quadratic growth based on the formulation of the total number of flops required to calculate one control action $O(n_s^2 \log_2 S)$. Therefore we may say that the size of the model defines the computation time of the algorithm and the number of discretization points is more like a constant tuning parameter.

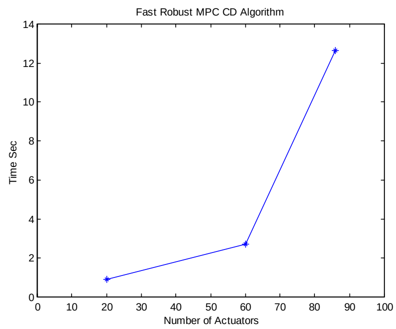

Figure 6.7: Computation time per sample for the fast robust MPC CD algorithm as function of the number of actuators.

6.2.4 Some observations of the standard MPC CD algorithm

Next we will study how a standard, off-the-self MPC algorithm can handle our uncertain CD control problem. Our intention is just to clarify applicability of these algorithms, because as previously mentioned these algorithms are based on nominal models, which do not necessarily guarantee system robustness in the presence of model mismatch. Also these simulations will take place in Matlab 5.2 environment and we will use the previous large scale models in the simulations.

Model Predictive Control Toolbox (Morari and Ricker, 1994) provides several efficient functions and methods for analysis and design of standard MPC systems. However, basis of this approach is slightly different than ours and MPC Toolbox assumes that the quadratic objective function, which consists of two additive terms, does not penalize particular values of the input vector $u(k)$, but only changes of the input vector, $\Delta u(k)$. This approach is very typical in the majority of the predictive control literature. In addition, the second quadratic term of the objective function is a difference between predicted outputs and future reference values. However, MPC Toolbox enables a convenient way to simulate closed-loop systems with hard bounds on manipulated variables and outputs. MPC Control Toolbox's *cmpc* -function can be used to simulate performance of the closed-loop CD system. This function solves a quadratic programming problem to accomplish the simulation and it enables a possibility to incorporate model mismatch to the simulated CD system by using a different model for controller design and for plant design.

We will utilize these features to construct our standard MPC CD simulation application for the discrete-time state-space model description. Standard MPC CD controller is constructed based on the nominal model of Table 4.1 and fixed constant parameter values (k_a = 1.0 and τ_a = 8.0) of actuator dynamics in the equation (5.30). Sampling time is as before 6 seconds. This nominal model is used for the state estimation in standard MPC CD controller, see also Fig. 5.2. However, structure of the MPC Toolbox's function requires that the model mismatch is constant during simulation while in the previous fast robust MPC CD simulation it was changed randomly by the process generator at each sampling time. Therefore we have to use fixed values for all varying parameters: $g_i \in [g_{imin}, g_{imax}]$, $k_a \in [k_{amin}, k_{amax}]$ and $\tau_a \in [\tau_{amin}, \tau_{amax}]$. We will choose maximum values for these parameters. Thus the uncertainty is equal (25%) for all elements g_i, excluding the diagonal elements that represent the center of the response and stay unchanged. For actuator dynamics $k_a \in [0.9, 1.1]$ and $\tau_a \in [6, 10]$, we will choose the maximum values of the range, see equation (5.30). This model is used to represent the plant, and it incorporates the model mismatch to the standard MPC CD simulation. Therefore the simulation will take place at the corner point of the multi-model polytope set Ω_{sim}.

We will use default values for all parameters of the *cmpc* –function. Thus the prediction horizon is infinite and only one control move is calculated and implemented. In the objective function, equal unity weighting for all outputs over the entire prediction horizon is used. Similarly zero weighting to the changes of the controls is used. Future reference value is zero and all control actions $|u(k)| \leq 1_{max}$ and outputs $|y(k)| \leq 1_{max}$ are limited to the same value. In addition the rate of change of the input vector $\Delta u(k)$ is unbounded. The step disturbance is connected to the matrix of disturbances to the plant and no noise filtering is used. See in details (Morari and Ricker, 1994).

In the simulation we will apply the standard MPC CD control algorithm for the same CD response models as presented in Chapter 4. As previously, we will use CD response models from the Table 4.1 and four step disturbances for the simulation, see equation (5.32). In Fig. 6.8 behavior of states of the standard MPC CD control system is presented in the case when the complexity of the response model $G^{86,86}$ is low or none model mismatch is included to the system.

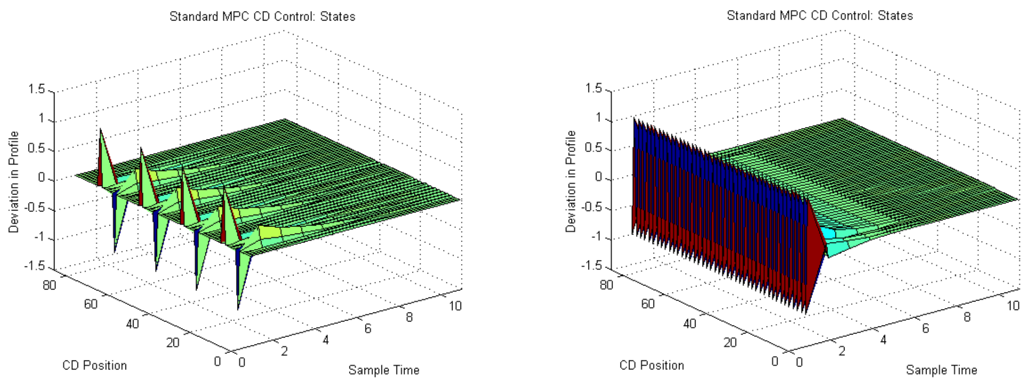

Figure 6.8: Standard MPC CD control algorithm. Left response of the closed-loop system with model mismatch. CD model G_2 with step disturbance d_4. Right response of the closed-loop system without model mismatch. CD model G_{12} with step disturbance d_2.

All CD response models of size $G^{86,86}$ were analyzed and likewise all step disturbance options were studied. Simulations indicated that irrespective of the step disturbance, CD response models G_1, G_2, G_3, G_6, G_8 and G_9 behaved in a similar way. In these cases the standard MPC CD algorithm was able to overcome the control task effectively and quickly in spite of the defined model mismatch. Similarly, if no model mismatch was present, the standard MPC CD algorithm was able to solve all CD response model control tasks. Time required to compute the closed-loop responses was less than 9 seconds per sample. The complexity of the response model affected the computation time, but anyhow the standard MPC CD algorithm turned out to be fast.

On the other hand, in Fig. 6.9 behavior of states and control actions of the standard MPC CD algorithm are presented in the case when the complexity of the response model $G^{86,86}$ is high.

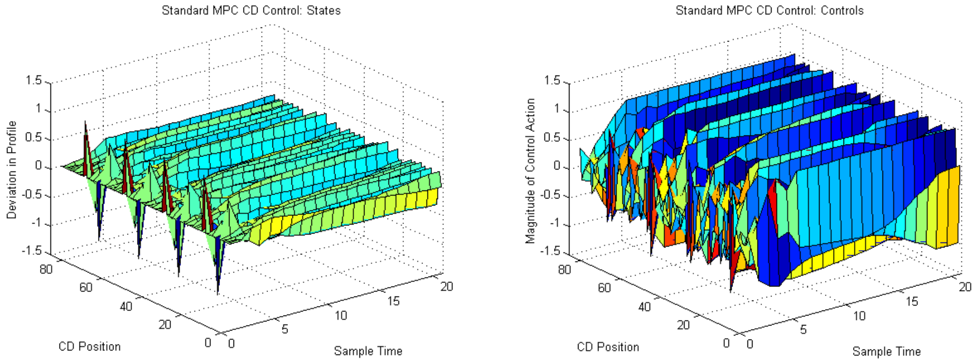

Figure 6.9: Standard MPC CD control algorithm. Response of the closed-loop system with model mismatch. CD model G_{10} with step disturbance d_4.

All complex CD response models $G_{4,}$ G_5, G_7, G_{10}, G_{11} and G_{12} behaved similarly and the type of step disturbance had a minimal effect on performance. In the beginning the standard MPC CD control algorithm worked effectively and was almost able to eliminate disturbances. However, soon controls reached their minimum or maximum limits and were saturated to these values. Behavior of the fast robust MPC CD control with response model G_{10} and step disturbance d_4 was shown earlier in Fig. 6.6. As a remark may be highlighted that the fast robust MPC CD control behaved similarly also with these fixed model mismatch values which were now used for standard MPC CD simulation.

If we look back to the values of Table 4.2 we observe that all badly behaved CD response models have high condition numbers and large element values of RGA. In addition both DIC analysis and $\mu(E(0))$ rule indicate that the decentralized integral controllability requirement is not satisfied with these CD response models. Only exception is CD response model G_9 which is DIC but not $\mu(E(0))$ rule compatible. It is clear that the strong interaction between adjacent zones in the complex response models is a reason for higher RGA values. Similarly only small uncertainties in the elements of $G^{86,86}$ are allowed without worsening performance of the control. Therefore this simulation confirms our previous claim that the inherent property of the plant $G^{86,86}$ has a crucial influence on the controllability of the uncertain CD system. If we increase the value of fixed uncertainty of varying elements of $G^{86,86}$ from 25% to 50% and maintain the error of actuator dynamics at the same values as before, the set of not badly behaving standard MPC CD response models will reduce to G_1, G_3, G_6 and G_8. From Table 4.2 and Appendix B we see that they represent less complex, diagonal dominant models with reasonable RGA values and rather insignificant off-diagonal elements. On the other hand, if we reduce the value of fixed uncertainty of varying elements of $G^{86,86}$, the set of not badly behaving models will be wider that the original six models. Naturally, if we continue this procedure we will end up to the CD response models without mismatch.

Based on straightforward simulations, it seems that the standard MPC CD algorithm is able to handle less complex CD response models with realistic uncertainty effectively and fast, at least when the size of the model $G^{86,86}$ is moderate. On the other hand, when the complexity of the model is high and interactions between adjacent zones are strong the algorithm went of the course and it could not guarantee system robustness in the presence of model mismatch and similarly it could not fulfill satisfactorily the control task.

6.2.5 Conclusions of the CD simulations

Numerous simulations were performed to demonstrate the efficiency of the new fast robust MPC CD control strategy. Algorithm was applied to the large scale CD processes with general, realistic uncertainty structures. The fast robust MPC CD algorithm is successful at controlling all the above examples with time-varying uncertainties and input and output constraints. The new robust MPC CD algorithm increases the tolerance of the controller to the model uncertainty conveniently and concurrently provides that the decrease in the nominal performance is relatively small. Simulation results of the closed-loop system show that the fast robust MPC CD algorithm achieves acceptable steady-state performance when extensive model uncertainty is present.

In spite of the achieved good results some additional observations may be enounced. Although the calculation of minimizers γ_i, Q_i , X_i , Y_i , Z_i at discretization points was done off-line, it required considerably CPU time. This was especially true for the large scale $G^{86,86}$ models. Because the calculation of minimizers is based on the LMI Matlab algorithm (Gahinet *et al.,* 1996), the limitations of LMI optimization software are present. Therefore the average computation time for one large scale minimizer set, with a priori values for the LMI functions of the used primal robust MPC algorithm, was approximately 24 hours on a 3 GHz Pentium 4 PC with 2 GB of RAM in Matlab 5.2 environment. Definition of a priori values for the LMI functions is possible after calculation of the first feasible set of minimizer, see in details Gahinet *et al.* (1996). In spite of all, the computation time of the whole subspace x_1 (i = 1,…,13), for one CD response model, was almost 2 weeks. As regards to the minimizers calculated by using the modification of primal robust MPC algorithm, the average computation time of one set was actually over 150 hours. Although the use of 2.2 GHz AMD Athlon 64 Dual Core 4400 PC with 2 GB of RAM in Matlab 7.1 environment bisected this computation time, it was an extensive task to accomplish. In addition, attempts to increase the dimension of the large scale CD response model $G^{86,86}$ led immediately to the memory problems, as could be expected with the used hardware. Anyhow, for practical applications the computation of minimizers can surely be arranged by using more powerful computers or by using several CPUs for the task.

Comparison of the fast robust MPC CD algorithm with the primal robust MPC CD algorithm revealed that the loss of performance was minor and the advantage in on-line computation was over four orders of magnitude. However, a characteristic feature of the present fast robust MPC CD algorithm was that it could not completely eliminate step disturbances even during long-lasting simulation. An apparent reason for this is the harsh logarithmic scale discretization of state subspace x_1 into fixed thirteen points and applied search method. Increased number of points would definitely improve accuracy and performance of the algorithm. On the other hand, it would also increase search time of the used bisection method, which turned out to be straightforward and reliable but not necessarily extremely fast. Thus a consideration of more powerful search methods would evidently offer better results (Press *et al.* 1992). Worthwhile would also be the method proposed by Kothare *et al.* (1996), which influences to the minimum decay rate of the states x: $\|x(k)\| \leq c\rho^k \|x(0)\|$ and to the speed of closed-loop response by defining an additional tuning parameter ρ and a set of new LMIs. Anyhow, we have shown for quadratic CD response systems with up to six models in the uncertainty set and models up to 86 states and inputs, that the fast robust MPC CD algorithm can be applied on-line in a reasonable amount of time.

On the other hand, study of the standard MPC CD algorithm with fixed uncertainty values for all varying parameters indicated that the current MPC algorithms (Morari and Ricker, 1994) are fast and efficient when the complexity of the CD response model of size $G^{86,86}$ is low or none model mismatch is present. Thus the development work which has been done during the last decades in this field has not been fruitless. However, when the complexity of the uncertain models was high the standard MPC algorithm could not guarantee system robustness and execute the control task well. First indication of this was already seen in Chapter 4 from the results of RGA and DIC analysis, which implied to the fundamental control problems with these complex CD response models and suggested carefulness in the selection of controller structure. On the other hand, if we would accept integral controllability (IC) to our control criterion, we could use the IRA method proposed by Yu and Luyben (1987) to analyse how much variation in the CD response model parameters can be tolerated before the system becomes uncontrollable in terms of IC. As referred in Chapter 4, the IRA approach is applicable to any controller with integral action, including also Dynamic Matrix Controller. Anyhow, it seems that the standard MPC CD algorithm is applicable to less complex CD response models. Likewise it is clear that more complex and demanding CD models require other advanced control methods such as the robust MPC CD controller or some kind of adaptation mechanism to be connected to the standard MPC CD algorithm.

We may conclude, that in simulations the new fast robust MPC CD algorithm provided robustness to model uncertainties and computation efficiency in under 20 CPU sec, which is in order of magnitude identical to sampling time for the real-time control algorithms. The developed new CD algorithm looks very promising and efficient enough to be implemented on real paper machines, even those of high dimensionality and complex CD response model structures.

Chapter 7

Conclusions and Further Research

This work has concentrated on the analysis and design of industrial robust model predictive control (MPC) for paper machine cross-directional (CD) processes, which are known as large scale, ill-conditioned, and inherently uncertain. Addressing model uncertainty for these processes is essential because model mismatch can cause the closed loop system to perform poorly. An approach was developed which exploits the structure of generic sheet process models to design a robust MPC controller for uncertain CD processes. First, we studied the paper machine CD process, its characteristics, and existing CD control applications. Then we derived a structure of CD response model uncertainty, and proposed a new method to evaluate complexity of the response models. After that we presented the robust model predictive CD control algorithm using linear matrix inequalities (LMIs). This method was utilized to formulate the fast robust MPC CD algorithm.The following is a summary of our main results.

Traditionally, the complexity of the steady state CD response model is evaluted by using its condition number. Response models are generally considered ill-conditioned and difficult to control if their condition numbers are large. However, taking into account inherent characteristics of the CD process, such as diagonal type input uncertainty of actuators, a better indicator to describe model complexity is the relative gain array (RGA) method. Together with the concept of decentralized integral controllable (DIC) system RGA provides a functional analysis and screening method to classify CD responses and controller structures. By applying these methods to the practical industrial CD response models we have shown that clear classification can be done, and approach may be used further for controller design analysis.

In practical CD applications several reasons may cause errors to the CD response models. For instance, such reasons include process disturbances, errors in the response estimation procedure, varying operating conditions in the paper machine, actuator failures, and MD-CD cross coupling errors in the measurements. All these features of the uncertain process are connected to the CD response model. We proposed a structured CD response model uncertainty, and use of linear time-varying (LTV) system with polytopic uncertainty description (Kothare *et al.*,1996) to the solution of this problem.

We adopted the MPC approach to develop present inadequate CD control strategies, and combined it with a general polytopic uncertainty to formulate a new robust CD control strategy. We formulated an infinite horizon MPC problem with input and output constraints and system uncertainty as a convex optimization problem involving linear matrix inequalities (LMIs). The on-line optimization involved the solution of an LMI-based linear objective minimization. The resulting time-varying state-feedback control law minimized, at each time-step, an upper bound on the robust performance objective, subject to input and output constraints. The feasible receding horizon control algorithm robustly asymptotically stabilized the set of uncertain CD models under consideration, which was verified performing several simulations based on the industrial paper machine models. Because the achieved computation times of the developed primal robust MPC CD algorithm were too high for the real-time applications, a computationally efficient approach was adopted (Wan and Kothare, 2003).

We developed an efficient robust constrained state feedback MPC CD algorithm for linear time-varying (LTV) systems, which produces a series of explicit control laws corresponding to a series of controlled invariant ellipsoids calculated off-line one within another in the state space. We suggested an off-line robust constrained MPC CD algorithm with guaranteed robust stability of closed-loop system for the polytopic uncertainty description. The concept of an asymptotically stable invariant ellipsoid enabled us to provide robust stability without the demand of finding an optimum of the system at each sampling time. In addition, the formulation of a series of asymptotically stable invariant ellipsoids one within another in state space provided more degree of freedom to improve control performance. The clear advantage of this algorithm is that it gives off-line a set of stabilizing state feedback laws, and because no optimization is required except a simple bisection search, the on-line computation time of the robust MPC CD algorithm is significantly reduced. We applied the fast robust MPC CD algorithm to selected industrial large scale CD processes and verified the efficiency of the algorithm by numerous simulations. We compared the fast algorithm with the primal robust MPC CD algorithm and addressed that the loss of performance was minor, and the benefit in on-line computation was over four orders of magnitude. In the simulations algorithm provided robustness to model uncertainties and computational efficiency in under 20 CPU sec. The fast robust MPC CD algorithm is highly promising and efficient enough to be implemented on real paper machines with complex CD response models and similar decentralized processes.

Further research

There are numerous ways in which this research may be continued. All of the presented ideas are examined by simulations; therefore an obvious step in the near future would be an implementation on real production machines. However, before that some practical and also theoretical issues may be necessary to investigate. In the following, some examples of important further work are presented, hopefully encouraging continued efforts in the area of robust MPC CD control and implementations in the near future.

Mapping errors are not taken into account in the robust MPC CD simulation algorithm, and similarly they are not included into the defined polytopic uncertainty description. However, this shortcoming is easy to solve by extending the set of polytope with required minimum and maximum limits and updating the number of vertexes in the equations (5.19) and (5.22).

One interesting theoretical subject, which should be studied, is the variable delay. In addition, constant delay should be incorporated into the fast robust MPC CD algorithm, because for the meantime it is excluded. A distinct reason for this is the formulation of augmented state and expanded dimension of the LMI optimization problem. With the present off-the-shelf LMI optimization software (Gahinet *et al.*, 1996) a convenient solution to this limitation is the utilization of more powerful hardware for the off-line calculation of minimizers used in *Algorithm 6.1*.

On the other hand, as regards the computation, advanced computer technology or more specified LMI optimization solvers would fairly provide means to overcome problems related to the computation time of proposed large scale robust CD algorithm. This is an entirely unexplored field of research, and it is anticipated that rather fast results could be achieved with a reasonable contribution.

Apart from the application to CD control, the method proposed here has more general applicability, since the same kind of problems arise in a variety of process industry. Cross-directional models represent general plants with dynamics multiplied by a constant response matrix. Similar model formulations can easily be found from elsewhere process industry. For instance, a simplified distillation column studied by Skogestad *et al*. (1988) has exactly the same model structure.

Therefore, it might be worthwhile to evaluate the applicability of developed efficient robust MPC algorithm for these processes.

This study is still in an early stage and much more work is required before proceeding to practical implementation. However, the fast robust MPC CD control strategy provides a very potential method to improve the control of CD processes, and to increase product quality of paper manufacturing. Therefore this subject should prove to be very fruitful for research in the coming years.

Appendix A

Mathematics

This appendix summarizes some results from mathematics.

A.1 Relative gain array (RGA)

According to Bristol (1966) relative gain array (RGA) is a tool for pairing controlled and manipulated variables in decentralized (multi-loop SISO) control systems. We will consider a linear *nxn* system described by the model

$$y(\mathrm{s}) = G(\mathrm{s})u(\mathrm{s}) \qquad \text{(A.1-1)}$$

Where u(s) and y(s) are n-dimensional vectors of inputs and outputs, respectively, and G(s) is a matrix of transfer functions. It is assumed that G(s) is stable and strictly proper matrix, and the steady-state gain matrix $G = G(0)$ is nonsingular.

RGA defines how an apparent transfer function between a given output variable (y_i) and a given input variable (u_j) is affected by control of other output variables. This measure is expressed as the ratio (λ_{ij}) of the transfer function between the two variables with all other outputs uncontrolled, and the transfer function between the same variables when all other outputs are perfectly controlled (Skogestad and Postlethwaite, 1996). Mathematically, the *relative gain* is expressed as

$$\lambda_{ij} = \left(\frac{\partial y_i}{\partial u_j}_{u_{k,k \neq j}} \right) \left(\frac{\partial y_i}{\partial u_j}_{y_{k,k \neq i}} \right)^{-1} \qquad \text{(A.1-2)}$$

The relative gains λ_{ij} for all possible variable pairings define a matrix, the Relative Gain Array (RGA), Λ. The partial derivatives in Eq. (A.1-2) can be related to the open-loop transfer functions of a system. From the definition of the relative gain, Eq. (A.1-2), it follows that the RGA for the system can be expressed as

$$\Lambda(s) = G(s)\otimes(G(s)^{-1})^{T} \qquad \text{(A.1-3)}$$

where $\otimes$ is the Hadamard of Schur product, which denotes element-by-element multiplication. A steady state RGA is obtained when the transfer functions are evaluated at $s = 0$. A frequency-dependent dynamic version of the RGA is obtained when $s = j\omega$.

A.1.1 Interpretations and properties of the RGA

The RGA matrix (A.1-2) has some interesting interpretations and properties. Interpretations (Häggblom, 1995) may be listed as follows:

If $0 < \lambda_{ij} < 1$, the control loop will interact with other loops.

If $\lambda_{ij} = 1$ the relative gain indicates a desirable variable pairing since interaction does not affect the open-loop gain between y_i and u_j. This is often interpreted so that the loop y_i-u_j does not interact with the rest of the system.

If $\lambda_{ij} > 1$, the open-loop gain between y_i and u_j will decrease when the other loops are closed. If the RGA contains values $\lambda_{ij} \gg 1$, the closed-loop system is very sensitive to parameter changes. Such a system may be uncontrollable.

If $\lambda_{ij} < 0$, the sign of the open-loop gain between y_i and u_j is changed when the other control loops are closed. Normally this results in an inverse response of y_i to changes in u_j.

If $\lambda_{ij} = 0$, the relative gain does not indicate whether the corresponding variable pairing is feasible or not. Control depends entirely on the other control loops.

Properties of the RGA may be analyzed as follows. RGA-elements (A.1-2) can be written in the following form

$$\lambda_{ij} = g_{ij} \cdot \hat{g}_{ji} = g_{ij} \frac{c_{ij}}{\det G} = (-1)^{i+j} \frac{g_{ij} \det G^{ij}}{\det G} \tag{A.1-4}$$

where $\hat{g}_{ji}$ denotes the *ji*'th element of the matrix $\hat{G} \triangleq G^{-1}$, G^{ij} denotes a matrix G with row *i* and column *j* deleted, and $c_{ij} = (-1)^{i+j}\det G^{ij}$ is the *ij*'th cofactor of the matrix G.

For any non-singular *nxn* matrix G, the following algebraic properties hold (Skogestad and Postlethwaite, 1996).

1. $\Lambda(G^{-1}) = \Lambda(G^T) = \Lambda(G)^T$

2. The sum of the elements of each row and each column of the relative gain array is always unity. That is $\sum_{i=i}^{n} \lambda_{ij} = 1$ and $\sum_{j=1}^{n} \lambda_{ij} = 1$.

3. Any permutation of rows and columns in transfer matrix G results in the same permutation in the RGA. Mathematically, if Λ = RGA(G), and P_1 and P_2 are permutation matrices, and if Λ' = RGA(P_1GP_2), then $\Lambda' = P_1 \Lambda P_2$.

4. The RGA is invariant under input and output scaling. Thus $\Lambda(D_1GD_2) = \Lambda(G)$ where D_1 and D_2 are diagonal matrices.

5. $\Lambda(G) = I$ if and only if G is a lower or upper triangular matrix; and in particular the RGA of a diagonal matrix is the identity matrix.

6. RGA is a measure of sensitivity to relative element-by-element uncertainty in the matrix. Matrix G becomes singular if a single element in G is perturbed from g_{ij} to $g'_{ij} = g_{ij}\,(1- \frac{1}{\lambda_{ij}})$.

A.2 Singular value decomposition (SVD)

Singular Value Decomposition (SVD) is an important tool in analyzing multivariable systems. It is a key to the formulation of robust multivariable control problems, and it provides information on system gain and measures of input and output interactions. In mathematics, SVD provides a sensible method of dealing with the concept of matrix rank. An important result of the SVD is the condition number, which describes how near the system is to singularity (Skogestad and Postlethwaite, 1996).

Singular Value Decomposition. For any complex *nxm* matrix G, there exist unitary matrices

$$U = [u_1\ u_2\ \dots\ u_n] \in C^{nxn} \tag{A.2-1}$$

and

$$V = [v_1\ v_2\ \dots\ v_m] \in C^{mxm} \tag{A.2-2}$$

such that

$$G = U\Sigma V^T = \sum_{i=1}^{r} \sigma_i(G) u_i v_i^T \ ; \quad r = \mathrm{rank}(G) \leq \min\{m,n\} \tag{A.2-3}$$

where $$\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \in R^{nxm} \tag{A.2-4}$$

and Σ_1 is a diagonal matrix

$$\Sigma_1 = \mathrm{diag}\{\sigma_1, \sigma_2, \dots, \sigma_r\} \tag{A.2-5}$$

containing the ordered non-negative *singular values* of G, as

$$\bar{\sigma} \equiv \sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r \equiv \underline{\sigma} \tag{A.2-6}$$

U is a unitary matrix referred to as the *output rotation matrix*, V is a unitary matrix referred to as the *input rotation matrix*, and u_i and v_i are referred to as the *i*th *output* and *input singular vector*, respectively. Because U and V are unitary, G transforms an input vector v_i into a vector with gain σ_i in the direction of u_i. Following (A.2-3), this can be written for i = 1,2, …, r as

$$Gv_i = \sigma_i u_i \quad \text{or equivalently} \quad G^T u_i = \sigma_i v_i. \tag{A.2-7}$$

The *condition number* of a *nxn* matrix G is defined as

$$\gamma(G) \equiv \frac{\sigma_1}{\sigma_n} \tag{A.2-8}$$

Geometric interpretation of SVD (Featherstone, 1997) is that the singular values of the matrix G are precisely the lengths of the semi-axes of the hyperellipsoid *E* defined by

$$E = \{y \mid y = Gx, x \in C^{m}, \|x\| = 1\}. \tag{A.2-9}$$

Therefore an input in the direction v_1 results in the largest $\|y\|$ for all $\|x\| = 1$; while an input in the direction v_m results in the smallest $\|y\|$ for all $\|x\| = 1$. In terms of the input and output, v_1 (v_m) is the *maximum* (*minimum*) gain input direction, while u_1 (u_m) is the *maximum* (*minimum*) gain output direction.

Appendix B

RGA Tables of the CD Response Models

This appendix describes relative gain array (RGA) tables of the studied CD response models. Original size of the RGA matrix is (86*x*86). Presented dimension of the tables depicts (11*x*16) first elements of the RGA matrix.

RGA table of CD model 1

1,044	-0,044	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,044	1,089	-0,045	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	-0,045	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,046	1,091	-0,046	0,000	0,000	0,000	0,000

RGA table of CD mode2

1,250	-0,250	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,250	1,563	-0,313	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	-0,313	1,641	-0,328	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	-0,328	1,660	-0,332	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,332	1,665	-0,333	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,333	1,666	-0,333	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,333	1,667	-0,333	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	-0,333	1,667	-0,333	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,333	1,667	-0,333	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,333	1,667	-0,333	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,333	1,667	-0,333	0,000	0,000	0,000	0,000

RGA table of CD model 3

1,150	-0,020	-0,130	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,020	1,185	-0,028	-0,137	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,130	-0,028	1,348	-0,033	-0,157	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	-0,137	-0,033	1,366	-0,035	-0,160	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	-0,157	-0,035	1,392	-0,037	-0,163	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,160	-0,037	1,398	-0,037	-0,164	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,163	-0,037	1,403	-0,037	-0,165	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,164	-0,037	1,404	-0,038	-0,165	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	-0,165	-0,038	1,405	-0,038	-0,165	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,165	-0,038	1,406	-0,038	-0,165	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,165	-0,038	1,406	-0,038	-0,165	0,000	0,000	0,000

RGA table of CD model 4

0,197	0,344	0,459	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,344	-0,285	0,520	0,421	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,459	0,520	-0,571	0,428	0,164	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,421	0,428	0,097	0,142	-0,089	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,164	0,142	0,487	0,113	0,093	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,089	0,113	0,157	0,385	0,434	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,093	0,385	-0,443	0,548	0,417	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,434	0,548	-0,429	0,368	0,079	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,417	0,368	0,194	0,107	-0,086	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,079	0,107	0,492	0,153	0,168	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,086	0,153	0,029	0,438	0,466	0,000	0,000	0,000

RGA table of CD model 5

1,741	-0,460	-0,280	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,460	1,457	-0,192	0,195	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,280	-0,192	0,737	0,265	0,471	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,195	0,265	0,208	0,208	0,124	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,471	0,208	0,832	-0,247	-0,264	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,124	-0,247	1,654	-0,411	-0,119	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,264	-0,411	1,346	-0,018	0,346	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,119	-0,018	0,418	0,316	0,402	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,346	0,316	0,308	0,069	-0,039	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,402	0,069	1,189	-0,373	-0,288	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,039	-0,373	1,687	-0,320	0,044	0,000	0,000	0,000

RGA table of CD model 6

1,023	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,023	1,046	-0,023	0,000	0,000	0,000	0,000

RGA table of CD model 7

1,450	-0,237	-0,227	0,013	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,237	1,203	-0,096	0,089	0,041	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,227	-0,096	0,756	0,166	0,347	0,054	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,013	0,089	0,166	0,247	0,229	0,229	0,026	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,041	0,347	0,229	0,442	0,027	-0,086	-0,001	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,054	0,229	0,027	1,101	-0,195	-0,220	0,005	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,026	-0,086	-0,195	1,356	-0,147	0,010	0,037	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,001	-0,220	-0,147	0,900	0,108	0,306	0,054	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,005	0,010	0,108	0,307	0,244	0,292	0,034	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,037	0,306	0,244	0,335	0,089	-0,014	0,003	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,054	0,292	0,089	0,948	-0,160	-0,223	0,000	0,000	0,000

RGA table of CD model 8

1,068	-0,048	-0,014	-0,006	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,048	1,121	-0,054	-0,013	-0,007	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,014	-0,054	1,140	-0,052	-0,012	-0,007	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,006	-0,013	-0,052	1,144	-0,053	-0,013	-0,007	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	-0,007	-0,012	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	-0,007	-0,013	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,007	-0,013	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,007	-0,013	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,007	-0,013	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	-0,007	-0,013	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	0,000	-0,007	-0,013	-0,053	1,145	-0,053	-0,013	-0,007	0,000	0,000

RGA table of CD model 9

1,572	-0,301	-0,111	-0,157	-0,003	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,301	1,932	-0,407	-0,074	-0,154	0,004	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,111	-0,407	2,129	-0,352	-0,075	-0,164	-0,020	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,157	-0,074	-0,352	2,227	-0,350	-0,078	-0,197	-0,019	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,003	-0,154	-0,075	-0,350	2,227	-0,350	-0,079	-0,197	-0,019	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,004	-0,164	-0,078	-0,350	2,230	-0,344	-0,079	-0,198	-0,020	0,000	0,000	0,000	0,000	0,000	0,000
0,000	0,000	-0,020	-0,197	-0,079	-0,344	2,300	-0,346	-0,083	-0,210	-0,022	0,000	0,000	0,000	0,000	0,000
0,000	0,000	0,000	-0,019	-0,197	-0,079	-0,346	2,300	-0,346	-0,082	-0,210	-0,022	0,000	0,000	0,000	0,000
0,000	0,000	0,000	0,000	-0,019	-0,198	-0,083	-0,346	2,305	-0,343	-0,083	-0,210	-0,023	0,000	0,000	0,000
0,000	0,000	0,000	0,000	0,000	-0,020	-0,210	-0,082	-0,343	2,317	-0,341	-0,083	-0,213	-0,023	0,000	0,000
0,000	0,000	0,000	0,000	0,000	0,000	-0,022	-0,210	-0,083	-0,341	2,318	-0,341	-0,084	-0,214	-0,024	0,000

RGA table of CD model 10

1,940	-2,384	0,894	0,816	-0,421	0,000	0,155	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-2,384	2,349	-1,351	1,106	1,228	-0,060	0,000	0,111	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,894	-1,351	0,407	-0,517	1,036	0,370	0,175	0,000	-0,015	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,816	1,106	-0,517	0,575	-1,155	0,301	-0,245	0,065	0,000	0,053	0,000	0,000	0,000	0,000	0,000	0,000
-0,421	1,228	1,036	-1,155	1,282	-0,277	-0,566	-0,288	0,093	0,000	0,068	0,000	0,000	0,000	0,000	0,000
0,000	-0,060	0,370	0,301	-0,277	0,755	-0,699	0,359	0,344	-0,128	0,000	0,035	0,000	0,000	0,000	0,000
0,155	0,000	0,175	-0,245	-0,566	-0,699	1,984	-1,697	0,941	1,131	-0,309	0,000	0,131	0,000	0,000	0,000
0,000	0,111	0,000	0,065	-0,288	0,359	-1,697	0,983	-0,547	1,112	0,785	0,103	0,000	0,015	0,000	0,000
0,000	0,000	-0,015	0,000	0,093	0,344	0,941	-0,547	0,203	-0,845	0,709	0,026	0,086	0,000	0,007	0,000
0,000	0,000	0,000	0,053	0,000	-0,128	1,131	1,112	-0,845	1,354	-1,179	-0,280	-0,421	0,086	0,000	0,117
0,000	0,000	0,000	0,000	0,068	0,000	-0,309	0,785	0,709	-1,179	1,038	0,071	-0,302	-0,005	0,093	0,000

RGA table of CD model 11

0,413	0,853	-0,012	0,410	-0,541	-0,053	-0,070	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,853	-2,432	2,853	-0,110	0,236	-0,445	0,276	-0,231	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
-0,012	2,853	-3,141	2,109	0,024	0,146	-1,275	0,331	-0,035	0,000	0,000	0,000	0,000	0,000	0,000	0,000
0,410	-0,110	2,109	-1,231	0,604	-0,240	0,138	-0,842	-0,033	0,195	0,000	0,000	0,000	0,000	0,000	0,000
-0,541	0,236	0,024	0,604	-0,445	0,971	-0,135	0,047	0,303	-0,172	0,108	0,000	0,000	0,000	0,000	0,000
-0,053	-0,445	0,146	-0,240	0,971	-0,351	0,866	-0,045	0,201	0,039	-0,174	0,086	0,000	0,000	0,000	0,000
-0,070	0,276	-1,275	0,138	-0,135	0,866	-1,297	1,390	0,203	0,082	0,861	-0,227	0,188	0,000	0,000	0,000
0,000	-0,231	0,331	-0,842	0,047	-0,045	1,390	-1,381	0,619	0,291	-0,127	0,887	-0,232	0,295	0,000	0,000
0,000	0,000	-0,035	-0,033	0,303	0,201	0,203	0,619	0,184	-0,856	0,296	0,062	-0,045	-0,173	0,273	0,000
0,000	0,000	0,000	0,195	-0,172	0,039	0,082	0,291	-0,856	0,780	-0,215	0,381	0,201	0,355	-0,153	0,072
0,000	0,000	0,000	0,000	0,108	-0,174	0,861	-0,127	0,296	-0,215	-0,013	0,185	-0,023	-0,193	0,334	0,017

RGA table of CD model 12

2,700	-2,865	0,976	-0,872	1,895	-0,722	0,075	-0,261	0,342	-0,269	0,000	0,000	0,000	0,000	0,000	0,000
-2,865	4,578	-1,533	0,959	-2,042	1,909	-0,124	0,697	-1,010	0,411	0,020	0,000	0,000	0,000	0,000	0,000
0,976	-1,533	0,094	-0,077	0,744	-0,666	0,464	0,554	1,019	-0,458	0,003	-0,120	0,000	0,000	0,000	0,000
-0,872	0,959	-0,077	-0,380	-0,280	0,398	-0,159	-0,132	0,908	0,855	-0,097	0,120	-0,243	0,000	0,000	0,000
1,895	-2,042	0,744	-0,280	0,632	-0,520	0,087	0,364	-1,273	1,167	0,530	-0,522	0,480	-0,262	0,000	0,000
-0,722	1,909	-0,666	0,398	-0,520	0,115	0,094	-0,366	0,887	-0,939	0,584	0,853	-0,801	0,227	-0,052	0,000
0,075	-0,124	0,464	-0,159	0,087	0,094	-0,236	0,249	-0,255	0,391	-0,453	0,517	0,566	-0,287	0,062	0,009
-0,261	0,697	0,554	-0,132	0,364	-0,366	0,249	-0,200	-0,156	0,194	0,209	-0,064	-0,072	0,013	-0,039	0,026
0,342	-1,010	1,019	0,908	-1,273	0,887	-0,255	-0,156	0,874	-0,837	0,018	-0,573	1,832	-0,604	-0,131	-0,057
-0,269	0,411	-0,458	0,855	1,167	-0,939	0,391	0,194	-0,837	0,717	-0,038	0,531	-1,324	0,890	-0,087	0,095
0,000	0,020	0,003	-0,097	0,530	0,584	-0,453	0,209	0,018	-0,038	0,044	0,127	-0,067	0,132	-0,274	0,167

Appendix C

An Example of the Discrete-time CD Response Model

This appendix presents, as an example, a simulated discrete-time response model of the Toeplitz symmetric matrix $G_2^{20,20}$ = toeplitz$_{20}$\{1.0, 0.4, 0, …, 0\}.

First-order actuator dynamics with uncertain model parameters and a constant time-delay are used:

$$g_a(s) = \frac{k_a e^{-\tau s}}{\tau_a s + 1}$$

$$k_a \in [0.9, 1.1]; \quad \tau_a \in [6.0, 10.0]; \quad \tau \in Z \qquad \text{(C.1-1)}$$

The time-delay τ is an integer number of sample times $\{\tau \in Z\}$. However, in this example it is not used ($\tau = 0$). The gain k_a and the time constant τ_a are allowed to vary randomly and independently between the specified upper and lower bounds.

A steady-state assumption is presumed and Tustin's method is used to accomplish the transformation from *s*-domain to *z*-domain. Sampling time is 6 sec. We have three pairs of varying parameters: $g_i \in [g_{imin}, g_{imax}]$, $k_a \in [k_{amin}, k_{amax}]$ and $\tau_a \in [\tau_{amin}, \tau_{amax}]$, therefore $L = 6$ and the multi-model polytope is $\Omega_{sim} = \mathrm{Co}\{[A_1\ B_1],[A_2\ B_2],\ldots,[A_6\ B_6]\}$. Discrete-time response model will be of the form

$$y(z) = G(z)u(z)\ , \qquad G(z) = g_a(z)\, G_2^{20,20} \qquad \text{(C.1-2)}$$

where the elements of the transfer matrix *G(z)* are given by

$$G_i(z) = C[zI - A_i\,]^{-1}B_i\, z^{-\tau}\ ,\ i = 1,2,\ldots,6 \qquad \text{(C.1-3)}$$

and (A_i, B_i, C) are defined in equations (5.7) and (5.8). Discrete-time state space matrices are

$$A_i = \begin{pmatrix} a_{1_i} & 0 & 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & a_{1_i} & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & a_{1_i} & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & a_{1_i} & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & a_{1_i} & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 & a_{1_i} & 0 \\ 0 & \dots & \dots & 0 & 0 & 0 & 0 & a_{1_i} \end{pmatrix}_{20x20} \quad B_i = \begin{pmatrix} b_{1_i} & b_{2i} & 0 & 0 & 0 & \dots & \dots & 0 \\ b_{2_i} & b_{1_i} & b_{2_i} & 0 & \dots & \dots & \dots & 0 \\ 0 & b_{2_i} & b_{1_i} & b_{2_i} & 0 & \dots & \dots & 0 \\ 0 & 0 & b_{2_i} & b_{1_i} & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & b_{2_i} & 0 & 0 \\ 0 & \dots & \dots & \dots & b_{2_i} & b_{1_i} & b_{2_i} & 0 \\ 0 & \dots & \dots & \dots & 0 & b_{2_i} & b_{1_i} & b_{2_i} \\ 0 & \dots & \dots & 0 & 0 & 0 & b_{2_i} & b_{1_i} \end{pmatrix}_{20x20}$$

$a_{1i} \in [0.3333 \ \ 0.5384], \quad i = 1,2,\dots,6$

$b_{1_i} \in [0.5455 \ \ 0.6667]$

$b_{2_i} \in [0.1636 \ \ 0.2727], \quad i = 1,2,\dots,6$

$$C_i = \begin{pmatrix} c_{1_i} & 0 & 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & c_{1_i} & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & c_{1_i} & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & c_{1_i} & \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & c_{1_i} & 0 & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 & c_{1_i} & 0 \\ 0 & \dots & \dots & 0 & 0 & 0 & 0 & c_{1_i} \end{pmatrix}_{20x20}$$

$c_{1_i} = 1, \quad i = 1,2,\dots,6$

where the elements of the matrices A_i and B_i vary between the defined upper and lower limits depending on the multi-model polytope $\Omega_{sim} = \mathrm{Co}\{[A_1\ B_1],[A_2\ B_2],\dots,[A_6\ B_6]\}$.

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