Department of the Built Environment

Geoid and postglacial rebound related gravity change in Finland

Mirjam Bilker-Koivula

Aalto University

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Mirjam Bilker-Koivula

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Abstract

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Abstract
Positioning using Global Navigation Satellite Systems (GNSS) is widely used nowadays and it is getting more and more accurate. This requires also better geoid models for the transformation between heights measured with GNSS and heights in the national height system. In Finland heights are continuously changing due to the Fennoscandian postglacial rebound. Land uplift models are developed for the Fennoscandian land uplift area, not only for the vertical velocities, but also for the gravity change related to postglacial rebound.

In this dissertation geoid studies were carried out in search of the geoid model that is most suitable for the conversion of GNSS heights in the EUREF-FIN coordinate system to heights in the Finnish height system N2000 on land as well as on sea. In order to determine the relationship between gravity change rates and vertical velocities, time series of absolute gravity measurements were analysed.

Methods were tested for fitting a geoid model to GNSS-levelling data. The best method for Finland was found to be least-squares collocation in combination with cross-validation. The result was the height conversion surface FIN2005N00, the official model for Finland. Then, high-resolution global gravity field models were tested in geoid modelling for Finland. The resulting geoid models were better than the earlier geoid models for Finland. After correcting for an offset and tilt, the differences with other models disappeared. Also, a method was developed to validate geoid models at sea using GNSS measurements collected on a vessel. The method was successful and key elements were identified for the process of reducing the GNSS observations from the height of observation down to the geoid surface.

Possible offsets between different types of absolute gravimeters were investigated by looking at the results of international comparisons, bi-lateral comparisons and of trend calculations. The trend calculations revealed significant offsets of 31.4 ± 10.9 μGal, 32.6 ± 7.4 μGal and 6.8 ± 0.8 μGal for the IMGC, GABL and JILAg-5 instruments. The time series of absolute gravity measurements at 12 stations in Finland were analysed. At seven stations reliable trends could be determined. Ratios between -0.206 ± 0.017 and -0.227 ± 0.024 μGal/mm and axis intercept values between 0.248 ± 0.089 and 0.335 ± 0.136 μGal/yr were found for the relationship between gravity change rates and vertical velocities. These values are larger than expected based on results of others.

The knowledge obtained in the geoid studies will be of benefit in the determination of the next generation geoid models and height conversion surfaces for Finland. Before clear conclusions can be drawn from the absolute gravity results, more studies related to glacial isostatic adjustment, and longer high-quality time series from more stations in Finland, as well as the whole of the uplift area

Keywords Geoid modelling, validation, FIN2005N00, absolute gravimetry, gravity trend, postglacial rebound, glacial isostatic adjustment, Finland

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Tiivistelmä

Satelliittipaikannus (GNSS) on nykyään laajasti käytössä ja sen tarkkuus paranee koko ajan. Tämä vaatii myös parempia geoidimalleja, joita tarvitaan kun satelliittipaikannuksella mitattuja korkeuksia muunnetaan kansallisessa korkeusjärjestelmässä oleviksi korkeuksiksi. Suomessa korkeudet muuttuvat jatkuvasti jääkauden jälkeisen maannousun johdosta. Maannousumalleja kehitetään Fennoskandian maannousualueelle, ei pelkästään koordinaattien pystykomponentin nopeuksille, vaan myös jääkauden jälkeisen maannousuun liittyville painovoimamuutoksille.

Tässä väitöskirjassa tehtiin geoiditutkimuksia etsittäessä geoidimallia, joka parhaiten sopii muuntamaan EUREF-FIN koordinaattijärjestelmässä olevat GNSS-korkeudet Suomen N2000-korkeusjärjestelmässä oleviksi korkeuksiksi sekä maalla että merellä. Absoluuttipainovoimamittauksien aikasarjoja analysoitiin painovoiman muutosnopeuksien ja pystysuuntaisten nopeuksien väliseen suhteen määrittämiseksi.

Geoiditutkimuksessa testattiin menetelmiä, joilla geoidimalli sovitaan GNSS-vaaitus aineistoon. Parhaaksi menetelmäksi Suomessa osoittautui pienimmän neliösumman kollokaatio yhdessä ristivalidoinnin kanssa. Tulos oli FIN2005N00-korkeuden muunnospinta, josta tuli Suomen virallinen geoidimalli. Seuraavaksi testattiin korkean resoluution globaaleja painovoimakenttämalleja Suomen geoidilaskennassa. Tuloksena olevat geoidimallit olivat parempia kuin aiemmat geoidimallit Suomen alueelle. Vakioeron ja kallistuksen poistamisen jälkeen erot muihin malleihin hävisivät. Lisäksi kehitettiin menetelmä, jolla geoidimalleja voidaan tarkistaa merellä laivan GNSS-mittauksien avulla. Menetelmä onnistui ja tunnistettiin keskeiset elementit prosessille, jolla muunnetaan GNSS havainnot mittauskorkeudelta geoidipintaan.

Absoluuttigravimetrityyppien välisiä vakioeroja etsittiin tutkimalla kansainvälisten vertailujen, kahdenvälisten vertailujen ja trendilaskennan tuloksia. Trendilaskennan tulokset paljastivat merkittäviä vakioeroja IMGC, GABL ja JILAg-5 laitteille: 31.4 ± 10.9 μGal, 32.6 ± 7.4 μGal ja 6.8 ± 0.8 μGal. 12 aseman absoluuttipainovoimamittauksien aikasarjat analysoitiin ja seitsemälle asemalle saatiin luotettavat trendit. Painovoiman muutosnopeuksien ja pystysuuntaisten nopeuksien väliselle suhteelle estimoidut suhdeluvut vaihtelivat 0.206 ± 0.017 ja -0.227 ± 0.024 μGal/mm välillä ja akselin leikkausarvot 0.248 ± 0.089 ja 0.335 ± 0.136 μGal/v välillä. Nämä arvot ovat suurempia kuin odotettiin aikaisempien tulosten perusteella.

Geoiditutkimuksista saatu tieto hyödynnetään Suomen seuraavien geoidimallien ja korkeuden muunnospintojen määrittämisessä. Ennen kuin absoluuttipainovoiman tuloksista voidaan tehdä selkeitä johtopäätöksiä, tarvitaan lisää maannousututkimusta ja pidemmät korkealaatuiset aikasarjat useammalta asemalta Suomesta, sekä koko maannousun alueelta ja sen reunalta.

Avainsanat Geoidimallinnus, FIN2005N00, absoluuttipainovoima, painovoimatrendi, jääkauden jälkeinen maannousu, Suomi

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I would like to recognize the financers that made part of the work possible: The Academy of Finland (grant 117132), the Finnish Transport Agency and the European Union that co-financed the FAMOS-FREJA project within the framework of the Connecting Europe Facility.

Special thanks go to my research group that I have neglected badly in this last stage of the work. You have been very supportive and understanding, giving me time to concentrate on the work.

For my family and friends: thank you for believing in me. The compilation part of this thesis was written under exceptional circumstances, working long days from home during a worldwide pandemic. I sincerely thank my husband Hannu and son Tino who had to put up with me during this time. Thank you for your love, patience and support during the process.

Espoo, 3 December 2020
Mirjam Bilker-Koivula

Contents

List of Abbreviations and Symbols

ADT	Absolute Dynamic Topography
BSCD2000	Baltic Sea Chart Datum 2000
BSHC	Baltic Sea Hydrographic Commission
C	Geopotential number
CCM	CIPM Consultative Committee for Mass and Related Quantities
CIPM	International Committee for Weights and Measures (Comité international des poids et mesures)
CIPM MRA	Mutual Recognition agreement of the CIPM
CMC	Calibration and Measurement Capability
CMEMS	Copernicus Marine Environment Monitoring Service
DI	Designated Institute
DTU	Technical University of Denmark
ECAG	European Comparison of Absolute Gravimeters
ETRF	European Terrestrial Reference Frame
ETRS	European Terrestrial Reference System
EURAMET	European Association of National Metrological Institutes
EUREF-FIN	Coordinate reference system for Finland
EVRS	European Vertical Reference System
FAMOS	Finalising Surveys for the Baltic Motorways of the Sea
FGI	Finnish geodetic Institute/Finnish Geospatial Research Institute
FMI	Finnish Meteorological Institute
FOGN	First Order Gravity Net
g	Gravity
$\dot{g}$	Gravity change rates

GIA	Glacial Isostatic Adjustment
GNSS	Global Navigation Satellite Systems
H	Orthometric height
H^*	Normal height
h	Ellipsoidal height
$\dot{h}$	Vertical velocities
ICAG	International Comparison of Absolute Gravimeters
ICGEM	International Centre for Global Earth Models
IfE	Institute für Erdmessung
IGRS	International Gravity Reference System
ISG	International Service for the Geoid
ITRF	International Terrestrial Reference Frame
KC	Key Comparison
LSC	Least-squares collocation
MRO	Regional Metrological Organization
N	Geoid height
N2000	National height system of Finland
N60	Former national height system of Finland
NAP	Normaal Amsterdams Peil
NKG	Nordic Geodetic Commission
NMBU	Norwegian University of Life Sciences
NMI	National Metrological Institute
PGR	Postglacial Rebound
PS	Pilot Study
SMHI	Swedish Meteorological and Hydrological Institute
SSH	Sea Surface Height
T	Disturbing potential
TC	MRO Technical Committee
U	Normal gravity potential
W	Gravity potential

γ	Normal gravity
Δg	Gravity anomaly
ΔN	Geoid difference
$\Delta\zeta$	Height anomaly difference
ζ	Height anomaly

List of Publications

This doctoral dissertation consists of a summary and of the following publications which are referred to in the text as publication 1-5.

Publication 1: Bilker-Koivula M (2010) Development of the Finnish height conversion surface FIN2005N00*, Nordic Journal of Surveying and Real Estate Research,* vol. 7, no. 1, pp. 76–88, 2010.
http://www.njsr.fi/issues/2010/7_1_bilker_koivula.pdf

Publication 2: Bilker-Koivula M (2014) Assessment of high resolution global gravity field models for geoid modelling in Finland. In Marti U. (Eds.): Gravity, Geoid and Height Systems - Proceedings of the IAG Symposium GGHS2012, October 9-12, 2012, Venice, Italy, *International Association of Geodesy Symposia*, vol. 141, pp. 51-58. Springer, Cham, DOI: 10.1007/978-3-319-10837-7_7.

Publication 3: Nordman M, Kuokkanen J, Bilker-Koivula M, Koivula H, Häkli P, Lahtinen S (2018) Geoid validation on the Baltic Sea using ship-borne GNSS data, *Marine Geodesy*, vol. 41, no. 5, pp. 457-476, 2018.
DOI: 10.1080/01490419.2018.1481160

Publication 4: Pettersen BR, Bilker-Koivula M, Breili K, Engfeldt A, Falk R, Gitlein O, Gjevestad JGO, Hoppe W, Lysaker DI, Mäkinen J, Omang OCD, Reinhold A, Timmen L (2010) An accuracy assessment of absolute gravimetric observations in Fennoscandia, *Nordic Journal of Surveying and Real Estate Research,* vol. 7, no. 1, pp. 7–14, 2010.
http://www.njsr.fi/issues/2010/7_1_pettersen_et_al.pdf

Publication 5: Bilker-Koivula M, Mäkinen J, Ruotsalainen H, Näränen J, Saari T (2021) Forty-three years of absolute gravity observations of the Fennoscandian postglacial rebound in Finland, *Journal of Geodesy*, vol. 95, nr. 24, 18 pp., 2021. DOI: 10.1007/s00190-020-01470-9

The publications were published in channels that have JUFO Publication Forum level 1. Publications 1 and 3-5 are peer-reviewed journal articles. Publication 2 is a peer-reviewed conference article.

Author’s Contribution

Publication 1: Development of the Finnish height conversion surface FIN2005N00.

The author was solely responsible for the publication.

Publication 2: Assessment of high resolution global gravity field models for geoid modelling in Finland.

The author was solely responsible for the publication.

Publication 3: Geoid validation on the Baltic Sea using ship-borne GNSS data.

The author initiated, planned and supervised the project, participated in analysing and interpreting the results and writing the publication. The first author was in charge of publication. The first, second, fifth and sixth authors did the calculations. The fourth author participated in planning and interpretation. All authors participated in writing the publication.

Publication 4: An accuracy assessment of absolute gravimetric observations in Fennoscandia.

The author provided the results for the comparisons in Finland and was involved in interpretation of the results and writing of the publication. The first author initiated the article and did the main work. All authors contributed with measurement results.

Publication 5: Forty-three years of absolute gravity observations of the Fennoscandian postglacial rebound in Finland.

The author did the final processing of all Finnish data from 2003 onwards, performed the time series and gravity change versus height change studies and wrote the publication. The second author initiated the work and participated in analysis and interpretation. All authors carried out observations and data processing.

1. Introduction

1.1 Background and research environment

During the last two decades, determining coordinates and heights using the Global Navigation Satellite Systems (GNSS) has become the general practice in surveying, positioning and navigation. GNSS positioning is not only used on a large scale, but is also becoming more and more accurate. With that, also better accuracy is demanded for the geoid models that provide the link between heights measured by GNSS and heights in the national height system, which have been measured by levelling. At the same time, more and more GNSS measurements are being processed with respect to a global reference system, resulting in coordinates in a global reference frame at the epoch of observation. To transform theses coordinates to the national coordinate and height systems of a country, a set of transformations is needed which also account for continental plate motions. Things are even more complicated in the Fennoscandian area, where the land is continuously rising as a result of the disappearance of the ice load that covered the area during the last Ice Age. At the maximum of the uplift area the land rises by about 1 cm/yr. When working with heights defined at different epochs the land uplift must be taken into account.

Under the umbrella of the Nordic Geodetic Commission (NKG), the Nordic and Baltic countries work together to provide accurate geoid models and models of the land uplift for the area. During the last two decades the NKG has provided two geoid models, NKG2004 (Forsberg et al. 2004) and NKG2015 (Ågren et al. 2016b), and two land uplift models, NKG2005LU (Vestøl 2006, Ågren and Svensson 2007) and NKG2016LU (Vestøl et al. 2019, Olsson et al. 2019). The latest land uplift model does not only provide vertical velocities, but also gravity change rates. For this purpose the ratio between gravity change rate and vertical velocities was determined by repeated absolute gravity measurements (Olsson et al. 2019). The NKG2015 model is the most accurate NKG geoid model in the series of geoid models developed within the NKG throughout the years. It is also the first model to have an epoch attached to it. Before the geoid modelling, all terrestrial gravity data used in the modelling were converted from the epochs of the national gravity networks to the common epoch of 2000.0 using the NKG2005LU land uplift model and the relation between gravity change rates and vertical velocities.

On a global level, gravity field research underwent a big change with the launch of the dedicated gravity satellites CHAMP, GRACE and GOCE, and in recent years the GRACE-FO. Accuracies of the global gravity and geoid models improved considerably for resolutions from 500 km down to 80 km. This also reduced long-wavelength errors in regional geoid models as in their calculation, global gravity field models are combined with local terrestrial gravity data. The GRACE and GRACE-FO missions also made it possible to measure gravity changes in time from space. In the NKG2015 geoid modelling this was utilized as the global gravity field model used could be defined in the same epoch as the terrestrial gravity data. Also high-resolution global gravity field models with resolutions as small as 10 km became available and their accuracy and suitability for geoid modelling in Finland had to be tested.

On a national level, Finland established its GNSS-based 3D coordinate system EUREF-FIN in 1998 (Ollikainen et al. 2000). It is the Finnish realisation of the European Terrestrial Reference System ETRS89 at epoch 1997.0 The new height system, N2000, based on the third precise levelling of Finland, was introduced in 2007 (JHS163 2007; Lehmuskoski et al. 2008). The N2000 height system is connected to the datum European Vertical Reference System EVRS at epoch 2000.0 and has the zero level at NAP (Normaal Amsterdams Peil). The introduction of the new height system called for a new geoid model or conversion surface for the transformation of heights between EUREF-FIN and N2000.

In recent years the countries around the Baltic Sea have decided to introduce a common vertical reference for the Baltic Sea, the Baltic Sea Chart Datum 2000 (BSCD2000, Ågren et al. 2016a, BSHC Chart Datum Working Group 2018). Like the N2000 system on land, the BSCD2000 is based on the definition of the EVRS at epoch 2000.0. As such, the height system at sea will be the same as on land and as on land, the zero level will be defined by the geoid. Between 2014 and 2019 the project FAMOS, Finalising Surveys for the Baltic Motorways of the Sea (FAMOS Consortium 2014–2017) took place co-financed by the European Union within the framework of the Connecting Europe Facility. The project aimed to support the introduction of the BSCD2000 by improving important geodetic tools and infrastructure. Marine gravimetry measurements were carried out with the goal of calculating a Baltic Sea geoid model with an accuracy of better than 5 cm. Quality control of the geoid models at sea was one of the challenges in the project.

The current official first-order gravity network of Finland is FOGN. It was measured in the early 1960s with a relative gravimeter and is referred to the epoch 1963.0. In 2009 the renewal of the first-order gravity net started with the re-measurement of the FOGN points with an absolute gravimeter. On a global level, preparations are made for the establishment of the International Gravity Reference System (IGRS). The IGRS will be realized by absolute gravity observations, taking into account the comparisons between the absolute gravimeters that carry out the observations (Wilmes et al. 2016, Wziontek et al. 2020).

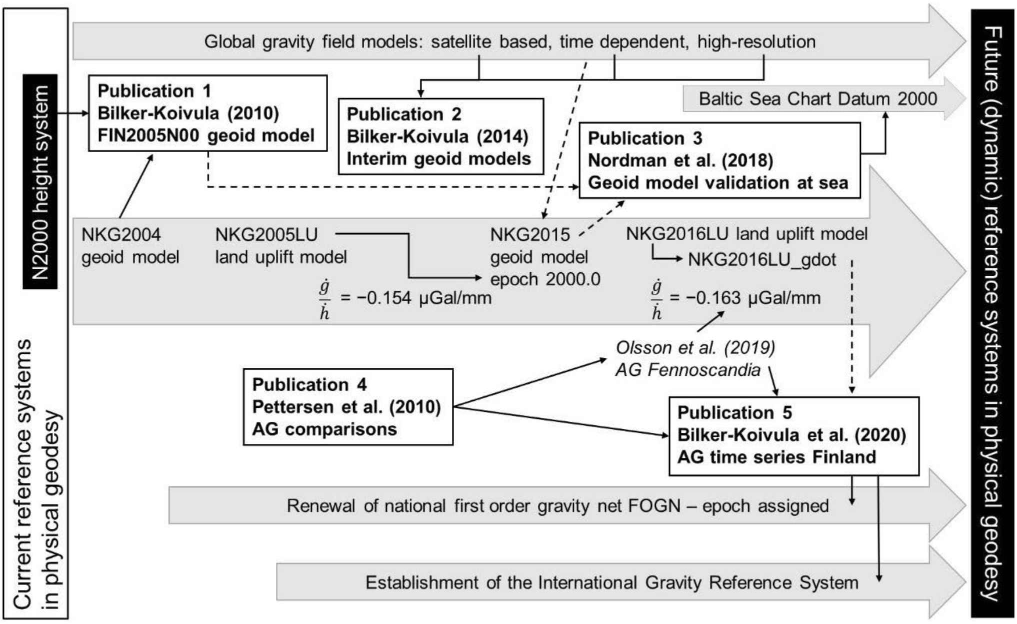

Figure 1. Schematic overview of the Publications of the thesis and their relations to each other and to the processes, which are draw on the background with grey arrows. $\dot{g}/\dot{h}$ is the ratio between the gravity change rate and vertical velocity and absolute gravimetry is denoted by AG.

Figure 1 shows the above mentioned processes that are the background for the research presented in this thesis. All these processes, starting from the current reference systems in physical geodesy, those for height and gravity, lay the foundation for future reference systems that may be time dependent, so-called dynamic reference systems.

1.2 Objectives and research questions

The objectives of the studies performed in this thesis can be found from the processes described in the previous section, which are also shown in Figure 1. The figure shows also where the different studies of the thesis fit into the processes and how they are connected to each other.

The first objective came up when the NKG2004 geoid model was released and the new height system for Finland, N2000, was to be introduced. There was a direct need to develop a new height conversion surface for the transformations between GNSS-derived heights and levelled heights. It initiated the first study of this thesis, the objective of which was to find the best way of fitting the NKG2004 geoid model to GNSS-levelling data in order to obtain a height conversion surface for Finland (Publication 1).

Shortly after the first study, the high-resolution global gravity field models, EGM2008 (Pavlis et al. 2008, 2012) and EIGEN-6C (Förste et al. 2011), were released. This led to the second study of this thesis, where the objective was to validate these models with Finnish data and investigate their use for geoid modelling in Finland (Publication 2).

The foreseen introduction of the BSCD2000 and in particular the related FAMOS project opened the possibility for the third geoid-related study of the thesis. The objective of this study was to find out if it is possible to use GNSS measurements from a vessel for geoid validation at sea (Publication 3).

The last two objectives are related to gravity and arose from the absolute gravity measurements that have been carried out in Finland and the whole Fennoscandian area. These allow the study of the relation between the observed gravity changes and the land uplift. When investigating time series it is important that there are no jumps occurring in time due to instrument offsets and that there are no offsets between different instruments, if more than one instrument is involved. The objective of the fourth study was therefore to detect possible offsets between the different absolute gravimeters involved (Publications 4 and 5).

The fifth study then analysed the Finnish absolute gravity time series with the objective to find the relationship between the gravity change rates and the vertical velocities caused by the land uplift (Publication 5).

The objectives led to the follow five research questions:

1. What is the best way of fitting the NKG2004 geoid model to GNSS-levelling data to establish a conversion surface for the transformation between EUREF-FIN ellipsoidal heights and N2000 normal heights?
2. Can the geoid model for Finland be improved using the high-resolution, partly satellite-based, global gravity field models EGM2008 and EIGEN6C?
3. Can a geoid model be validated by GNSS measurements at sea?
4. What is the magnitude of possible offsets between the absolute gravimeters that have been used for measurements in Finland?
5. What do the time series of absolute gravity in Finland tell us about the relationship between postglacial rebound induced gravity change rates and vertical velocities?

The research questions are answered by five publications (1-5). The answers to research questions 1, 2 and 3 are given in publications 1, 2 and 3, respectively. Publication 4 and a part of publication 5 give the answer to question 4. Question 5 is answered in publication 5.

1.3 Scope of the thesis

The focus of this thesis is on geoid models and absolute gravity data analysis. Although GNSS data is used in the thesis, GNSS and the processing of GNSS data are outside the scope of the thesis. Also, the Fennoscandian land uplift will be discussed, but GIA modelling is not part of the thesis. The main study area is Finland and for the marine geoid study the Bothnian Sea between Finland and Sweden.

1.4 Dissertation structure

This thesis consists of a summary and five peer-reviewed publications. In this introductory section the background, objectives, research questions and scope of the thesis were presented. Section 2 lays the theoretical basis for the studies performed. The materials and methods used are presented in Section 3. Next, Section 4 gives the main results from the publications related to each research question. Section 5 discusses the results and their implications as well as recommendations for further studies. Finally, a summary is given in Section 6.

The publications can be found at the end of the thesis.

2. Theoretical foundation

2.1 Gravity, geoid and heights – a short introduction

Gravity is the force experienced by a mass on the surface of the Earth, which is the vector sum of the gravitational force and the centrifugal force. The gravitational force is a result of the direct attraction of the Earth and the centrifugal force is caused by the Earth's rotation.

The gravity field in three dimensions is a geopotential field, where the gravity potential is denoted by W. The derivative of W in three directions is the gravity vector $\vec{g}$. The magnitude of $\vec{g}$ is the gravity g. In SI units the unit of gravity is that of acceleration: m/s^2. If the shape of the Earth is approximated by an ellipsoid of revolution, at the equator g = 9.780 m/s^2 and at the poles g = 9.832 m/s^2. In physical geodesy it is common to use the unit Gal, where:

1 Gal = 10^{-2} m/s^2, 1 mGal = 10^{-5} m/s^2, and 1 μGal = 10^{-8} m/s^2.

The surfaces in a geopotential field on which W is constant are called equipotential surfaces. The geoid is such an equipotential surface and its potential value is denoted by W_0. Heights are directly related to the gravity potential. The difference between the geopotential value in a point P, W_P, and W_0 is the geopotential number C_P, which can be expressed as:

$$C_\mathrm{P} = W_0 - W_\mathrm{P} = \int_0^H g\, dH = \bar{g} H_\mathrm{P}, \tag{1}$$

where H_P is the height of point P above the geoid and $\bar{g}$ the average gravity along the plumb line between the geoid and point P. H is the orthometric height (see Figure 2). Orthometric heights are commonly used in national height systems and are obtained by levelling. For example the previous height system of Finland, N60, was based on orthometric heights.

Heights measured with a Global Navigation Satellite System (GNSS) refer to an ellipsoid instead of the geoid. These are called ellipsoidal heights h. The relationship between orthometric heights and ellipsoidal heights is given by the height between the geoid surface and the ellipsoid called the geoid height N (see Figure 2):

$$N = h - H \tag{2}$$

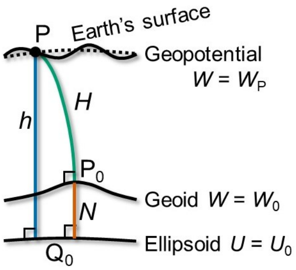

Figure 2. Orthometric height (*H*), ellipsoidal height (*h*) and geoid height (*N*)

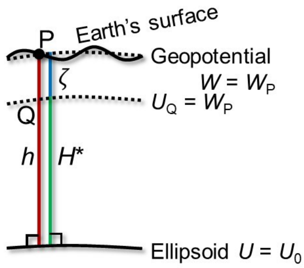

Figure 3. Normal height (*H**), ellipsoidal height (*h*) and height anomaly (ζ)

The Earth can be approximated by a rotationally symmetric ellipsoid of which the surface is the equipotential surface of a normal gravity potential field. The normal gravity potential is denoted by U, with $U = U_0$ at the surface of the ellipsoid. The derivative of the normal potential in three directions gives the normal gravity vector, of which the magnitude is the normal gravity γ. The difference between the real gravity potential and the approximate normal potential is the disturbing potential $T = W - U$.
The geoid height, N, is directly related to the disturbing potential and can be given by Bruns' formula:

$$N = \frac{T}{\gamma} = \frac{W - U}{\gamma} \tag{3}$$

When determining orthometric heights, knowledge of the local gravity is required. If we replace the gravity in equation (1) by the normal gravity we get the equation for normal heights H^* (see Figure 3):

$$C_{\mathrm{P}} = \int_0^{H^*} \gamma \, dH^* = \bar{\gamma} H^*, \tag{4}$$

with $\bar{\gamma}$ the average of the normal gravity along the plumb line. Because the ellipsoid is a simple geometric figure, also the expression for the normal potential in space, and that for the normal gravity, can be obtained in simple mathematical from and calculated. Also normal heights are commonly used. For example the current height system of Finland, N2000, is based on normal heights.

Figure 3 shows how the normal height is measured from the ellipsoid. The surface where the normal potential, U, is equal to the potential, W, at the Earth's surface is called the telluroid. The difference between the ellipsoidal height and the normal height is the height anomaly ζ:

$$\zeta = h - H^* \tag{5}$$

Like the geoid heights, the height anomalies, ζ, can also be drawn above the ellipsoid, see Figure 4. Then, they form a surface called the quasi-geoid and the normal heights are heights above the quasi-geoid.

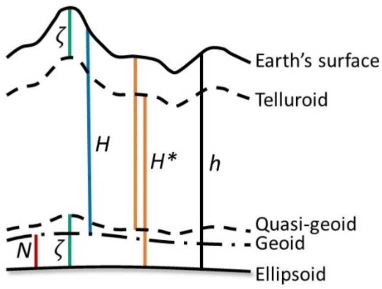

Figure 4. Overview of the different heights and their relation the geoid, quasi-geoid and telluroid. $\boldsymbol{H}$ is the orthometric height, $\boldsymbol{H}^*$ the normal height, $\boldsymbol{h}$ the ellipsoidal height, $\boldsymbol{N}$ the geoid height and ζ the height anomaly.

The quasi-geoid is not as smooth as the geoid as it correlates with the heights of the terrain. The quasi-geoid is close to the geoid and at sea they are equal.

As shown above, gravity, heights and the geoid are linked together. Geoid heights are directly linked to gravity by Stokes' integral (e.g. Heiskanen and Moritz 1967, equation 2-163b):

$$N = \frac{R}{4\pi G} \iint_{\sigma} \Delta g \, S(\psi) \, d\sigma, \tag{6}$$

where $\Delta g = g_P - \gamma_Q$ is the gravity anomaly, the difference between the gravity at point P and the normal gravity at point Q, where $U_Q = W_P$ (see Figure 2 and Figure 3). R is the radius of the Earth, G is the universal, or Newton's, gravitational constant, ψ is the angular distance on the sphere. $S(\psi)$ is Stokes's kernel, which can be expressed in terms of Legendre polynomials $P_n(\cos\psi)$ (e.g. Heiskanen and Moritz 1967, equation 1-57′):

$$S(\psi) = \sum_{n=2}^{\infty} \frac{2n+1}{n-1} P_n\,(\cos\psi). \tag{7}$$

Detailed descriptions of the Legendre polynomials can be found in textbooks on physical geodesy and geoid modelling (e.g. Heiskanen and Moritz 1967, Sansò and Sideris 2013).

2.2 Geoid models

Geoid and quasi-geoid models are given at different scales. There are global models that cover the whole Earth, and regional models that cover only a certain region or country.

Global gravity field models are usually presented as coefficients of a global spherical harmonic expansion of the global gravity potential:

$$V(\phi,\lambda,r) = \frac{GM}{r}\left(1 + \sum_{n=2}^{n} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \bar{P}_{nm}(\sin\phi)\,[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda]\right), \tag{8}$$

where (ϕ,λ,r) are spherical coordinates, a is usually the equatorial radius of the reference ellipsoid used, $\bar{P}_{nm}(\sin\phi)$ are the normalized associated Legendre functions (see textbooks on physical geodesy and geoid modelling for a detailed explanation), and $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the normalized spherical harmonic coefficients for degree n and order m, that define the global model. By changing the first part of equation (8) the coefficients $\bar{C}_{nm}$ and $\bar{S}_{nm}$ can also be used to calculate geoid heights N or gravity anomalies Δg.

In the era before the gravity satellites, global models would be developed to a maximum degree and order of 360, like for example the widely used EGM96 (Lemoine et al. 1998). These models were prone to long-wavelength errors as they were calculated mainly from terrestrial gravity observations that are not able to capture the long-wavelength information of the gravity field. The gravity satellites CHAMP, GRACE and GOCE changed this. For example Saari and Bilker-Koivula (2015) showed for Finland that the GOCE-satellite based models performed better than the EGM96 model when looking at the long wavelength signal, i.e., the coefficients up to degree and order 200. EGM2008 (Pavlis et al. 2008; 2012) was the first global gravity model to go to higher degrees and orders, going up to degree 2190 and with a complete set of coefficients up to degree and order 2159. In the making of the model, some data from the GRACE satellite were used. The next high-resolution model published was the EIGEN-6C model (Förste et al. 2011), going up to degree and order 1420. It included data from GOCE, LAGEOS and GRACE. Both EGM2008 and EIGEN-6C were evaluated and used in local geoid modelling in Publication 2. Since, many more satellite-only gravity field models were produced as well as numerous high-resolution models combining gravity satellite data with high-resolution terrestrial gravity data. All models are collected at the International Centre for Global Earth Models (ICGEM) (Barthelmes and Köhler 2016, Ince et al. 2019), where a good overview can be found.

Regional geoid models are usually calculated combining a global gravity field model with local terrestrial gravity data. The models are provided as geoid heights, or in the case of a quasi-geoid as height anomalies, in a grid format. Examples of such models are the European quasi-geoid model EGG2015 (Denker 2013, 2015) and the geoid models covering the Nordic and Baltic countries, calculated under the umbrella of the Nordic Geodetic Commission (NKG), the latest being the NKG2015 quasi-geoid (Ågren et al. 2016b).

How a regional model is calculated is described in section 2.2.3 and applied in Publication 2. But first, section 2.2.1 describes the validation of geoid models and section 2.2.2 the fitting of a geoid model to the national height systems. Section 2.2.4 examines the accuracy of geoid models.

2.2.1 Geoid model validation

To validate (quasi-)geoid models on land is rather straight forward. On sea it is more complicated. On land, GNSS and levelling data can be used to calculate geoid heights that can then be compared to geoid heights from a geoid model:

$$\Delta N = h - H - N, \qquad (9)$$

and likewise height anomaly differences can be calculated in the case of normal heights and a quasi-geoid:

$$\Delta\zeta = h - H^{*} - \zeta. \tag{10}$$

The statistics of ΔN or $\Delta\zeta$ give information on the accuracy of the (quasi-)geoid model. It must be noted that it is the combined accuracy, including the uncertainties of the ellipsoidal heights, the orthometric or normal heights as well as the (quasi-)geoid heights.

At sea the validation of models is less trivial as no GNSS-levelling data is available. In theory the sea level coincides with the geoid when the sea is at rest. In practice this is not the case, due to, for example, variations in water temperature, salinity, tides, currents and winds. As a consequence, for geoid model validation at sea, the sea surface height, SSH, must be measured and the difference between the sea surface and the geoid model, the absolute dynamic topography, ADT, modelled, see Figure 5.

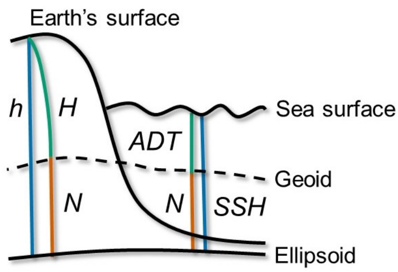

Figure 5. Relationship between the different heights on land and at sea, where $\boldsymbol{h}$ is the ellipsoidal height, $\boldsymbol{H}$ the orthometric height, $\boldsymbol{N}$ the geoid height, $\boldsymbol{ADT}$ the absolute dynamic topography, and $\boldsymbol{SSH}$ the sea surface height

Analogously to equations (9) and (10) the geoid height differences are now given by:

$$\Delta N = SSH - ADT - N \tag{11}$$

At sea the geoid coincides with to the quasi-geoid, $N = \zeta$, and likewise the geoid height differences are equal to the height anomaly differences $\Delta N = \Delta\zeta$.

Satellite altimetry, which has successfully been used for geoid modelling over the oceans, is suitable for geoid model validation. For example, it was used to validate geoid models in the North Sea by Schall et al. (2016). An alternative to satellite altimetry is to measure the sea surface from vessels using GNSS observations. This was successfully done on the river Waser in Germany by Lavrov et al. (2015), and on the Baltic Sea by Jürgenson et al. (2008) and Varbla et al. (2017 and 2020). Lavrov et al. (2016) even used marine GNSS measurements to improve the geoid model in the coastal areas of Israel.

The challenge for geoid validation with GNSS observations on vessels is not so much the GNSS observations themselves, but the reduction of the GNSS observations to SSH. The processing of GNSS observations from moving platforms has been studied a lot, but is out of the scope of this thesis.

Reinking et al. (2012) and Roggenbuck et al. (2014) addressed the problem of reducing the GNSS observations to sea level with the purpose of validating altimeter data on the oceans. Their main concerns were changes in the static draft of the vessels, due to changes in the load (fuel and water) during the voyage, the dynamic draft or squat, which is the draft of the vessel related to its velocity, and the heave, which are height variations of the vessel caused by waves. For the comparison with SSH from altimeter data, they also looked at ocean tide, tidal loading and the inverse barometric effect. In our approach these last three were not considered as we looked at the instantaneous SSH and used dynamic models that gave us the ADT at the observation epoch and location.

In Publication 3 we developed a procedure for the validation of geoid models at sea using marine GNSS observations. The method is described in section 3.2.3.

2.2.2 Fitting a geoid model to national height systems

Pure gravimetric geoid models never perfectly fit to the GNSS-levelling data. There is usually an offset due to the fact that the W_0 of the national height system does not coincide with the W_0 of the geoid model, that inherited the zero level from the global gravity field model (Sánchez and Sideris 2017). There may be other systematic differences, like different treatment of the permanent tide (Mäkinen and Ihde 2009) for the geoid model and the height systems involved (e.g. Slobbe et al 2019), and effects due to different epochs of the different datasets. The latter is for example the case in most Nordic countries where the GNSS-based coordinate systems have a different epoch than the height systems. Due to the land uplift in the Nordic area, with vertical change rates of up to 1 cm/yr, geoid height differences obtained from GNSS and levelling in these countries include a height difference due to the land uplift. In Finland the EUREF-FIN GNSS data have epoch 1997.0 (Ollikainen et al. 2000) and the N2000 levelling data have epoch 2000.0. Depending on location the effect of the 3 years land uplift on the geoid heights differences is between 8.5 mm and 28.1 mm (Publication 2).

For national use in combination with national height systems, (quasi-)geoid models are usually fitted to GNSS-levelling data. Strictly speaking the models are then not anymore gravimetric (quasi-)geoid models, but conversion surfaces for the transformation between ellipsoidal heights, measured by GNSS in the national 3D reference system, and orthometric or normal heights, as obtained by levelling in the national height system. The Finnish model FIN2005N00 is such a height conversion surface and its development is described in Publication 1. The International Service for the Geoid (ISG) hosts a geoid repository, where regional geoid models and their meta-data are collected (Reguzzoni et al. 2016). The ISG calls a geoid model that was fitted to geoid-levelling data a hybrid model.

When fitting a geoid model to GNSS-levelling, the geoid height differences, ΔN, or height anomaly differences, $\Delta\zeta$, are the starting point. A surface is fitted through the differences. This correction surface is then added to the geoid model to form the conversion surface.

A method for surface fitting is for example fitting of a polynomial:

$$\Delta N = \sum_{i=0}^{n} \sum_{j=0}^{n-i} a_{ij} \Delta\varphi^{i} \Delta\lambda^{j}\,, \qquad (12)$$

where n is the order or degree of the polynomial, and $\Delta\varphi$ and $\Delta\lambda$ are coordinates scaled according to:

$$\Delta\varphi = \frac{2\varphi - (\varphi_{\max} + \varphi_{\min})}{\varphi_{\max} - \varphi_{\min}} \text{ and } \Delta\lambda = \frac{2\lambda - (\lambda_{\max} + \lambda_{\min})}{\lambda_{\max} - \lambda_{\min}} \qquad (13)$$

This method was applied in the calculation of the previous geoid model for Finland, the FIN2000 geoid model (Ollikainen 2002) and in the tests of publication 1.

Another widely used method for surface fitting is least-squares collocation (LSC) (Moritz 1989). The values of the estimated surface are estimated at grid points from the geoid height differences, ΔN, by solving the following equation:

$$\hat{s} = C_{sx} C_{xx}^{-1} x, \qquad (14)$$

where x is the input vector of known geoid height differences, ΔN, and $\hat{s}$ is the vector of the unknown correction surface (grid) values. $C_{xx} = C(x_i, x_j)$ is the auto-covariance matrix of the values in x, and $C_{sx} = C(s_i, x_j)$ is the cross-covariance matrix of the values in x and $\hat{s}$. The values in the covariance matrices are given by the covariance function, $C(r)$, that describes the relationship between two points separated by a distance r. This method is widely used (e.g. Ellmann et al. 2020). A combination of the two methods is also possible.

2.2.3 Regional geoid modelling

A common way to calculate regional geoid models is by the remove-compute-restore technique. The process of the technique is described below and a schematic overview is given see Figure 6.

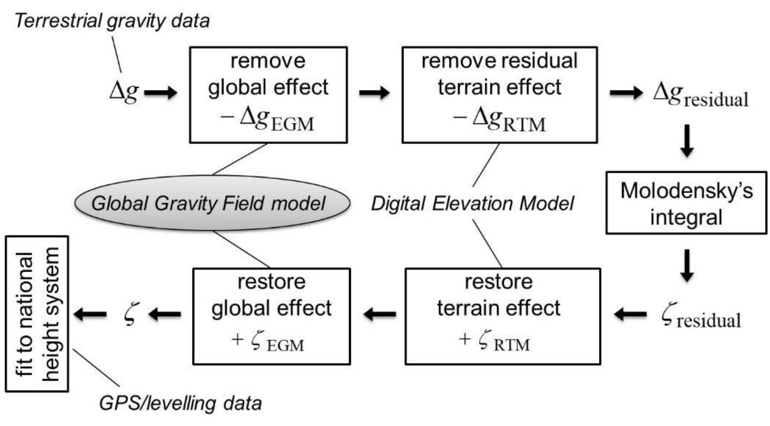

Figure 6. Schematic of the remove-calculate-restore technique for geoid modelling.

The process in Figure 6 starts with terrestrial data from which free-air gravity anomalies have been calculated:

$$\Delta g_{\mathrm{FA}} = g - \gamma_0 + \frac{\partial \gamma}{\partial h} H = g - \gamma_0 + 0.3086H \text{ mGal}, \tag{15}$$

where g is the gravity observation, γ_0 the normal gravity on the ellipsoid, H the height of the point above the geoid in m, and $\frac{\partial \gamma}{\partial h}$ the normal gradient of gravity, that is the change of gravity as a function of height along the plumb line from the point where g is observed down to the point on the ellipsoid, where γ_0 is calculated. The normal gradient can for most applications be approximated by its value at 45° latitude: 0.3086 mGal/m.

Starting from the free-air gravity anomalies, Δg_{FA}, first the known long-wavelength effect of the global gravity field model, Δg_{EGM}, is removed. Then the known shorter wavelength signal is calculated from the terrain models in the form of residual terrain corrections, Δg_{RTM}, and removed. This results in residual gravity anomalies, $\Delta g_{\mathrm{residual}}$:

$$\Delta g_{\mathrm{residual}} = \Delta g_{\mathrm{FA}} - \Delta g_{\mathrm{EGM}} - \Delta g_{\mathrm{RTM}} \tag{16}$$

Long-wavelength terrain corrections calculated from a height reference model with the same spatial resolution as the EGM are subtracted from the terrain corrections calculated from a high resolution digital elevation model. The result are residual terrain corrections, Δg_{RTM}, that contain only the short-wavelength signal of the terrain. The residual gravity anomalies are then gridded to a regular grid of gravity anomalies, $\Delta g_{\mathrm{residual}}$.

Then, residual height anomalies, $\zeta_{\mathrm{residual}}$, can be obtained by evaluation of Moldensky's integral (Heiskanen and Moritz 1967, equations 8-52 and 8-70):

$$\zeta = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g + G_1)\, S(\psi)\, d\sigma, \tag{17}$$

where G_1 may be given by:

$$G_1 = \frac{R^2}{2\pi} \iint_{\sigma} \frac{H - H_P}{l_0^3} \Delta g\, d\sigma. \tag{18}$$

Here, R is the radius of the Earth and γ the normal gravity above the ellipsoid. $S(\psi)$ is Stokes' kernel, as given in (7). G_1 can be approximated by the linear approximation of the terrain correction, which works for terrain with inclinations smaller than 45° (Tziavos and Sideris 2013, equation 8.11):

$$c(x_P, y_P) = \frac{1}{2} G \iint_{E} \frac{\rho(x,y)[H(x_P, y_P) - H(x,y)]^2}{\left[(x_P - x)^2 + (y_p - y)^2\right]^{3/2}} dxdy \tag{19}$$

The evaluation of Molodensky's integral results in residual height anomalies, $\zeta_{\mathrm{residual}}$. Now the height-anomaly effect of the residual terrain corrections, ζ_{RTM}, and of the global gravity field model, ζ_{EGM}, are added back.

The result is a grid with final height anomalies or quasi-geoid heights, ζ, that is the quasi-geoid model:

$$\zeta = \zeta_{\mathrm{residual}} + \zeta_{\mathrm{RTM}} + \zeta_{\mathrm{EGM}} \tag{20}$$

The quasi-geoid model can then be compared to GNSS-levelling data to assess its accuracy with respect to the national heights systems. This is discussed in the next section.

The remove-restore technique in combination with Molodensky's integral is widely used in regional and national geoid calculations. It is also the technique that is used in Publication 2 and in the calculation of the NKG2004 geoid model that was used in Publication 1. Other geoid modelling methods and variations to the method described above exist and can be found in the textbooks on geoid modelling (e.g. Sansò and Sideris 2013). For calculation of the NKG2015 geoid model, used in Publication 3, least squares modification of Stokes' formula was applied, see for example Sjöberg (2003) for a compact overview and Ellmann (2004), Ågren (2004) for more detailed applications. The gridding of the residual anomalies is also not arbitrary as was shown in Märdla et al. (2017).

2.2.4 Geoid model accuracy

With the increasing use of GNSS for height determination, the demands are rising for geoid models to become more accurate. In a comparison of the NKG2015 model with GNSS-levelling data over the whole area, standard deviations of 2.85 cm were found, but, for individual countries with a smooth gravity field, lower values between 1.5 cm and 2.0 cm were found (Ågren et al. 2016b).

The NKG has put the goal on a 1 cm accurate geoid model, while some even talk about 0.5 cm geoid models. Also the goal for Finland is to achieve a 1 cm geoid model (Poutanen et al. 2017). The question is if this is feasible. The current NKG2015 model is already a step in the right direction: When taking the uncertainty of the GNSS-levelling datasets into account, the relative accuracy of the NKG2015 model was estimated to be ~1.5-2.0 cm on land (Ågren et al. 2016b).

In areas with high-resolution good-quality gravity and GNSS-levelling data it has proven to be possible to reach 1 cm or even 0.5 cm: Ågren and Sjöberg (2014) conclude that for Sweden calculation of a 5 mm model is possible if the resolution of the gravity data would be at least 5 km with no gaps and the uncorrelated gravity anomalies should have standard errors below 0.5 mGal and the correlated anomalies below 0.1 mGal. In some flat parts they were able to achieve this, but they foresee it being difficult in the mountainous areas even if the data fulfils the requirements. Ellmann et al. (2020) have produced a 5 mm geoid model for Estonia, a flat country with high-resolution gravity data and a high-quality GNSS-levelling dataset. Slobbe et al. (2019) calculated the quasi-geoid model for the Netherlands and Belgium and estimated the relative accuracy of the model to be 0.7 cm for the Netherlands and 1.0 cm for Belgium, when the uncertainties of the GNSS-levelling data were taken into account.

2.3 Absolute gravimetry

The absolute value of gravity, i.e., the acceleration of free fall, at a certain moment in time at a certain location can be measured by an absolute gravimeter. Early absolute measurements of gravity were made with pendulums. For example in Finland already in the 1960s absolute gravity measurements were made with a long wire pendulum (Hytönen 1972). Since then, different types of gravimeters were developed, see Marson (2012) for an overview. During the second half of the twentieth century different free-fall type gravimeters were developed, of which the early IMGC and GABL instruments visited Finland in 1976 (Cannizzo et al. 1978) and 1980 (Arnautov et al. 1982), respectively. In the 1980s a series of six JILAg-type absolute gravimeters were produced (Faller et al. 1983, Niebauer et al. 1986), making absolute gravimeters and their measurements available to a wider range of scientists. The FGI acquired the JILAG-5 in 1988. In the beginning of the 1990s a commercial absolute gravimeter was developed, the FG5 (Niebauer et al. 1995), which spread the use of absolute gravimeters around the world. Also an instrument for use outdoors, having a larger measurement uncertainty, was developed, the A10 (Micro-g LaCoste 2008).

The future of absolute gravimetry is in quantum physics. At several places around the world atom gravimeters are being developed (see e.g. Zou et al. 2011, Wu et al. 2014, Gillot et al. 2016, Ménoret et al. 2018).

The widely used FG5 instrument is described in short in the following subsection 2.3.1. Thereafter, subsection 2.3.2 will give an introduction to the comparisons of absolute gravimeters and subsection 2.3.3 discusses the time series of absolute gravimeter observations.

2.3.1 The absolute gravimeter FG5

A schematic overview of the FG5 absolute gravimeter is shown in Figure 7. The FG5 instrument is a free-fall type gravimeter (Niebauer et al. 1995). The upper part of the instrument is the dropping chamber where a test mass containing a corner-cube reflector is dropped in vacuum. In the middle is a Mach-Zender type interferometer. A laser beam from an iodine-stabilized laser comes into the interferometer and is split into two beams. One beam, the reference beam, travels straight through to the photo-detector. The other beam, the so-called test beam, first travels up to the falling corner cube. There it is reflected back down through the interferometer to the corner cube hanging from the super spring in the bottom part of the instrument. Again it is reflected back up to the interferometer, where it is reflected to reunite with the reference beam before reaching the photo-detector. The superspring in the bottom of the instrument is a system of springs that isolates the measurements from high-frequent ground motions.

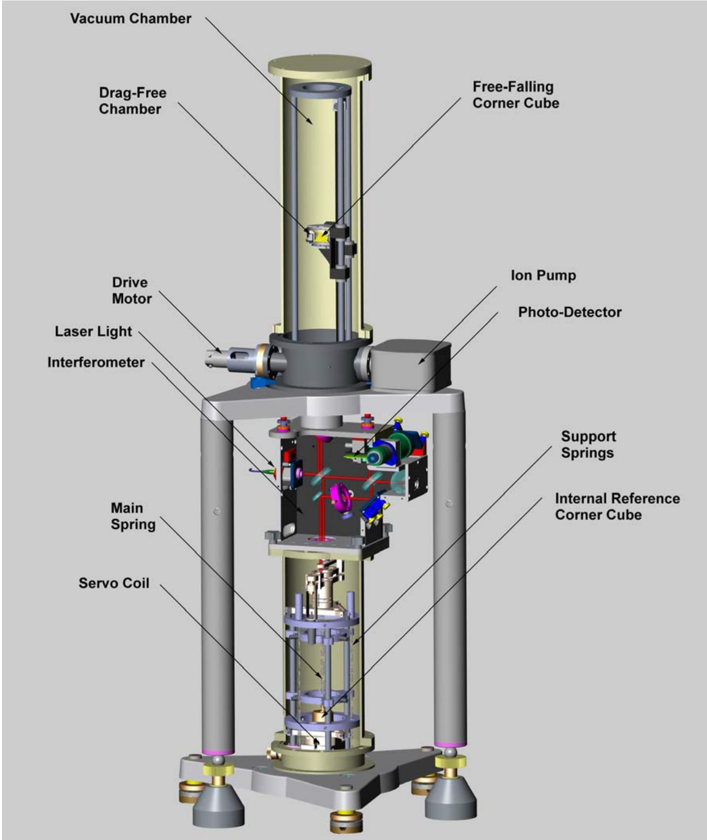

Figure 7. Schematic of the FG5 absolute gravimeter (Micro-g LaCoste 2006), courtesy of Micro-g LaCoste, Inc.

During the fall of the test mass, the length of the test beam will change, which results in interferometric fringes once the laser beams are combined. From the fringes in combination with accurate time by means of a Rubidium clock, an accurate set of time-stamped positions of the test mass is obtained. In principle the gravity value can then be obtained by a least-squares solution of the following equation (Micro-g LaCoste 2006):

$$x_i = x_0 + v_0 + \frac{1}{2} g_0 t_i^2, \tag{21}$$

where (t_i, x_i) are the time-position pairs and x_0, v_0 and g_0 are parameters that are estimated. Of these, g_0 is the gravity value we are interested in. In practice, the equation is more complicated as gravity changes a little bit along the distance of the fall due to the vertical gradient of gravity, and as the length of the reference laser beam gets shorter during the fall. For a more detailed explanation and equations the reader is referred to Micro-g LaCoste (2006, 2007).

After the g-value is obtained, it is corrected for polar motion, Earth tide and ocean loading, and air pressure, and then transferred from the top of the drop trajectory to a pre-defined height.

From each drop a gravity value is obtained. Typically the test mass is dropped every 10 seconds in a set that consists of 50 consecutive drops. This is then repeated every 30 minutes. Ideally, 24 to 48 sets are measured for each setup. In the Nordic countries we adopted a measurement scheme where we do two setups per station with the instrument turned 180 degrees between setups.

In the early 2010s an upgrade for the FG5 was developed, the FG5X (Niebauer et al. 2011). The new version has a new dropping chamber that incorporates a recoil compensating driving mechanism of the wagon that carries the test mass before and after the drop. It also has a longer dropping distance. The new instrument has a better performance and smaller uncertainties.

For the quality of the final gravity value, three parameters are available from the g-software (Micro-g LaCoste 2007): the set scatter, the measurement precision and the total uncertainty. The measurement precision is the standard deviation of the measurements and combined with instrumental-related uncertainties they form the total uncertainty. The instrumental part of the uncertainty is estimated at 1.1 μGal by Niebauer et al. (1995). However, not all instrumental components nor site-dependent uncertainties are covered by this uncertainty budget.

A more complete uncertainty estimate is provided by the instrument owners at international comparisons of absolute gravimeters with their gravity measurements. These uncertainties combine the standard deviations of the measurements with a full set of instrument-related uncertainties as well as site-dependent uncertainties. These uncertainty estimates have been compulsory since the international comparison of 2015, ICAG2015 (Jiang et al. 2011). For the instrument operated by the FGI, the FG5-221 and its upgrade, the FG5X-221, these total uncertainty values are mostly 2.6 μgal and 2.3 μGal, respectively.

2.3.2 Metrology and the comparison of absolute gravimeters

Absolute gravimeters can be the national standard for free-fall acceleration when they are owned by a National Metrological Institute (NMI) or Designated Institute (DI) and their measurements can be traced to the SI units. Traceability is obtained through the calibration of the frequencies of the laser and the reference clock (Marti et al. 2014). Currently, 7 NMIs and DIs have a guaranteed traceability of the gravity acceleration measurements with a declared CMC (Calibration and Measurement Capability) approved within the CIPM MRA (Mutual Recognition Agreement of the International Committee for Weights and Measures) (CIPM 1999). The FGI is one of these institutes and the FG5X-221 is the national standard for free-fall acceleration in Finland. In the CMC an uncertainty for the absolute gravity measurement is declared. To maintain the CMC it is important that the absolute gravimeter associated to it participates in comparisons to validate its measurement uncertainty. Absolute gravimeters other than those with declared CMCs can obtain traceability from comparison against a gravimeter of a NMI or DI with declared CMCs or from comparison against a gravity value at a reference station (Marti et al. 2014).

Also for observing time series it is important to monitor the stability of the instrument in time, especially when more than one instrument is involved. For the above mentioned purposes international comparisons of gravimeters (ICAG) are organised. The first international comparisons were organized in the 1980s: ICAG81-82, ICAG1985 and ICAG1991 (Boulanger et al. 1983, 1986 and 1991). This was the era before the FG5s, when only few absolute gravimeters existed. FG5 instruments participated for the first time in the ICAG1994 (Marson et al. 1995) and from the ICAG2001 (Vitushkin et al. 2002) onwards the comparisons have been dominated by FG5 instruments. In 2003 the first European comparison of absolute gravimeters, ECAG2003 (Francis et al. 2005), took place. From then onwards International and European comparisons took place in turn two years from each other.

Starting with the international comparison of 2009 the international and regional comparisons have been divided into a CIPM Key Comparison (KC) and Pilot Study (PS) as defined by the CIPM MRA (CIPM 1999). The international CCM.G comparisons are approved by the CIPM Consultative Committee for Mass and Related Quantities. Regional comparisons, e.g. EURAMET.M.G, are agreed with the Regional Metrological Organization (MRO), for Europe the European Association of National Metrological Institutes (EURAMET), and approved by the CCM. Only NMIs and DIs can participate in KCs. PSs are open for any institute (Jiang et al. 2012, CIPM 1999). The results of the regional KCs and PSs are linked to the last CCM.G comparison through the instruments that participated in both KCs.

International comparisons can be complemented by bilateral comparisons and double occupations. This is especially important for instruments that do not participate in international comparisons. In bilateral comparisons two absolute gravimeters measure simultaneously at neighbouring pillars and then swap places. In the case of double occupations, the measurements are not performed simultaneously, but in different days on the same station. The time span between the measurements is preferably no more than two weeks to avoid differences in measured values due to changes in environmental parameters.

2.3.3 Time series of absolute gravity

Absolute gravity measurements can be used to monitor and study geophysical processes that involve mass changes. These processes are, for example, tectonics (e.g. Van Camp et al. 2016), non-tidal sea level variations (Olsson et al. 2009), hydrological mass variations (Pálinkáš et al. 2013) and glacial isostatic adjustments (GIA) (e.g. Lambert et al. 2001, Teferle et al. 2009, Olsson et al. 2019, also section 2.4). An extensive overview of geophysical processes that can be observed with gravity observations, is presented in Van Camp et al. (2017).

When analysing time series of absolute gravity observations it is necessary to make sure the absolute instruments involved are stable in time and no offsets occur. It is therefore important that absolute gravimeters are taken to comparisons as described in section 2.3.2. Offsets for instruments can also be calculated as part of the trend calculation. The latter is applied in Publication 5.

Van Camp et al. (2005) studied the noise that affects absolute gravity measurements and the trends derived from them. They estimated that, depending on the noise model, a time span of 15-25 years is needed to determine a gravity trend with an uncertainty of 0.1 μGal/yr. The uncertainty of the trend did not differ much for annual, semi-annual or quarterly measurements. Gitlein (2009) and Timmen et al. (2012) did yearly measurements over a time span of 4-5 years and concluded they could detect gravity trends with an average uncertainty of 0.6 μGal/yr. In Publication 5 trends are determined from measurements over time spans between 4 and 43 years.

In the analysis of absolute time series it is also important to realize that a gravimeter senses all mass changes that take place in its surroundings. It may not be possible to separate the signal of the phenomenon under study from other mass-related processed that are going on. For example correcting gravity time series for hydrological signals can successfully reduce the variation of these time series as is shown, for example, in Ophaug et al. (2016), Lambert et al. (2006) and Mikolaj et al. (2015). Also studies with the superconducting gravimeter in Metsähovi show that a large part of the variations in gravity can be explained by variations in the level of the Baltic Sea and groundwater storage (Mäkinen et al. 2014, Virtanen et al. 2014).

2.4 Fennoscandian land uplift and gravity change

The Fennoscandian crust is continuously rising due to ongoing glacial isostatic adjustment (GIA) since the last ice age. Vertical velocities due to the postglacial rebound (PGR) reach vertical velocities up to 1 cm/yr (Milne et al. 2001). The PGR has been studied for decades with different types of observations: e.g., tide gauges (Ekman 1996), repeated precise levelling (Mäkinen and Saaranen 1998), and continuous observations by permanent GNSS stations (Johansson et al. 2002, Kierulf et al. 2014, Lahtinen et al. 2019). The land uplift is not only observed by height observations, but also by observing gravity changes due to the PGR. Observations of the gravity changes can give additional information on the GIA processes, as gravity changes include the effect of the vertical motion of the Earth's surface as well as the effect of mass changes below the surface. Gravity observations on the surface can help us understand what happens under the surface when it is going up.

2.4.1 Gravity observations in the Fennoscandian land uplift area

Studies of the PGR in Fennoscandia using gravity observations started already in the 1960s when a relative gravity line was established at 63° latitude crossing the Fennoscandian uplift area (Kiviniemi 1974). More lines at 56°, 61° and 65° latitude (Figure 8) were established in the 1970s (Mäkinen et al. 1986). Repeated relative gravity measurements were performed on these lines up till 2003 (Ekman and Mäkinen 1996, Mäkinen et al. 2005).

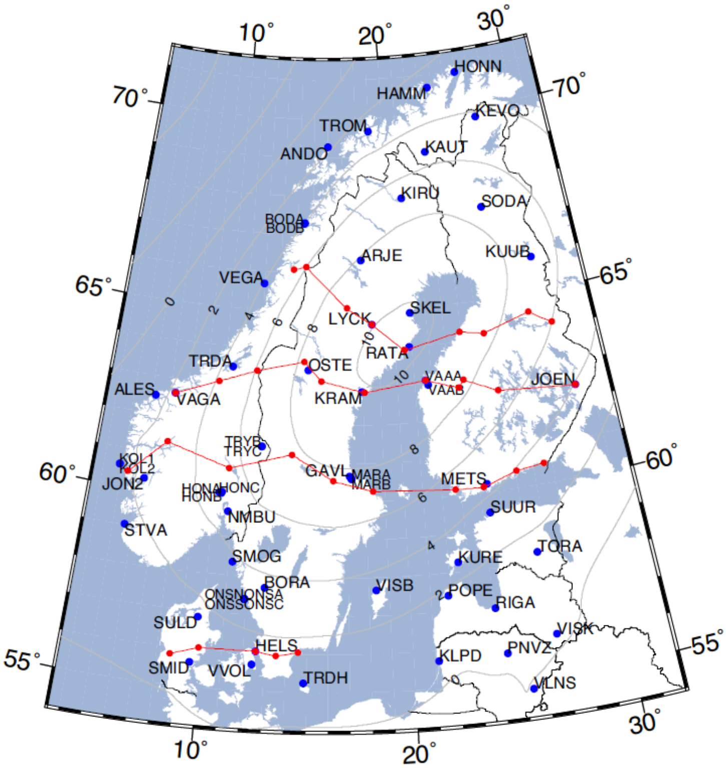

Figure 8. The Fennoscandian land uplift relative gravity lines (red) and absolute gravity stations with repeated measurements (blue). Contour lines are vertical velocities in mm/yr derived from the NKG2016LU_abs land uplift model. Courtesy of Olsson et al. (2019).

Repeated absolute gravity measurements started in the 1980s with the JILAg-5 instrument of the FGI, which measured repeatedly at several locations in the Fennoscandian uplift area between 1988 and 2002. During the 1990s FG5 measurements were also performed by the National Oceanic and Atmospheric Administration (NOAA) and the Bundesamt für Kartographie und Geodäsie (BKG). In 2003 the Nordic Absolute Gravity Project of the working group for Geodynamics of the Nordic Geodetic Commission was started. In that framework the FGI did measurements with the FG5-221 starting 2003 and with the FG5X-221 from 2013 onwards. Teams of the Institute für Erdmessung (IfE) of the Leibnitz Universität Hannover measured an extensive network of absolute gravity stations all over the Fennoscandian uplift area repeatedly with an FG5 between 2003 and 2008. During that period also the Norwegian University of Life Sciences (NMBU), the Swedish mapping, cadastral and land registration authority, Lantmäteriet, and Technical University of Denmark (DTU) started measurements. The NMBU and Lantmäteriet measure with an FG5 and DTU with an A10. Figure 8 shows the Fennoscandian stations with repeated gravity measurements up till 2015. Results of the measurements by NOAA, BKG and IfE are presented in Gitlein (2009), measurements by NMBU in Ophaug et al. (2016) and by Lantmäteriet in Olsson et al. (2016). Results of all repeated absolute gravity measurements in the Fennoscandian PGR area between 1988 and 2015 are presented in Olsson et al. (2019).

Gitlein (2009) and Timmen et al. (2012) concluded that after a period of 4 to 5 years of yearly absolute gravity measurements PGR-related gravity change rates could be determined and that they could be considered reliable, because of their agreement with land uplift models when taking their uncertainties, 0.6 μGal/yr on average, into account. Ophaug et al. (2016) applied refined corrections for ocean tide loading, atmosphere and global hydrology to the absolute gravity change rates and found that refined gravity rates agree better with the land uplift model than standard rates at 9 out of 20 sites. However, they also found that at sites where the GIA-signal is dominant, the refinement does not improve the agreement. At the sites, where the refinement improves the agreement, the GIA-signal is less dominant and part of the observed gravity change signal may be of other origin, e.g., tectonics or groundwater (Ophaug et al. 2016). Olsson et al. (2015) concluded that their solution of the absolute gravity trends in Sweden derived from the measurements with the FG5-233 improved when applying biases of the instrument, taken from the results of the ICAGs and ECAGs. These offsets were also applied for to the FG5-233 observations for the final solution in Olsson et al. (2019). In this solution, only FG5 measurements were taken into account and only the trends of 21 out of 43 stations were included.

Publication 5 deals with the absolute gravity data in Finland. Compared to Olsson et al. (2019) it contains an extra 6 years of data and improves the spatial coverage by adding 5 more stations.

2.4.2 Relationship between gravity change and land uplift

There is a relationship between the gravity change rates and the vertical velocities. The relationship is not only interesting for the understanding of the present PGR processes, but it is also used to separate the effect of past and present ice-mass changes (e.g. Van Dam et al. 2017; Omang and Kierulf 2011), and to evaluate the motion of the origin of the global terrestrial reference frame with respect to the centre of mass (e.g. Mazotti et al. 2011; Lambert et al. 2013). The ratio for gravity change rates versus vertical velocities, $\dot{g}/\dot{h}$, has also practical applications in geodesy: The ratio was for example used in combination with a land uplift model to convert the gravity data of the Nordic and Baltic countries to the same epoch for the calculation of the NKG2015 geoid model (Ågren et al. 2016b). And likewise, in the establishment of the new Swedish reference frame and system for gravity, RG2000, a land uplift model in combination with a $\dot{g}/\dot{h}$ -ratio was used to bring all observations to the same epoch (Engfeldt 2000).

The relationship between the gravity change and vertical velocities can be given as:

$$\dot{g} = a + b\,\dot{h}, \tag{22}$$

with the gravity change rates $\dot{g}$, the vertical velocities $\dot{h}$, the intercept $a = \dot{g}_{\dot{h}=0}$, and slope b, which is the ratio $\dot{g}/\dot{h}$ for $a = 0$. In early determinations of the ratio, no intercept values were estimated. With the models and measurements becoming more accurate, also intercept values need to be considered. Intercept values can tell something about the misalignment between the origins of the reference frames of the vertical velocities and the centre of mass. Also in case there are mass movements that are not related to the vertical movements or even GIA, this will result in non-zero intercept values.

At the time the Fennoscandian relative gravity land uplift lines were established the $\dot{g}/\dot{h}$ ratio was suggested to be -0.171 µGal/mm in Kiviniemi (1974). Wahr et al. (1995) estimated the ratio to be -0.154 µGal/mm by modelling over Antarctica and Greenland. Since then ratios for different PGR regions have been determined from gravity and GNSS observations. For Fennoscandia estimates can be found in Ekman and Mäkinen (1996), Mäkinen et al. (2005), Timmen et al. (2012), Ophaugh et al. (2016) and Olsson et al. (2019).

Based on GIA modelling, Olsson et al (2015) estimated the relation to be:

$$\dot{g} = 0.03 - 0.163\dot{h}.$$

When no axis intercept is estimated the values of Mäkinen et al. (2005), Timmen et al. (2012) and Olsson et al. (2019) for the ratio agree with the value of -0.16 µGal/mm. However, when also intercepts are estimated the ratio values vary between -0.13 µGal/mm and -0.18 µgal/mm in Ophaug et al. (2016) and Olsson et al. (2019), and intercept values vary between -0.21 µGal/yr and +0.14 µGal/yr.

Publication 5 looks at the relationship of equation (22) using the absolute gravity trends determined for the Finnish stations.

2.4.3 Fennoscandian land uplift models

GIA models are geophysical models of the GIA-induced land uplift. The main components in GIA modelling are Earth models, describing the geometry and rheology of the Earth, ice models, giving the ice load history, and the sea-level equation, describing the distribution of melt-water into the oceans (Steffen 2017). An extensive overview of data and GIA modelling in Fennoscandia is given by Steffen and Wu (2011). GIA models can be constrained with the uplift rates determined with the different techniques (e.g. Milne et al 2004).

Although geophysical models of PGR are constrained by geodetic measurements, they are not very suitable as models to be used in the transformation of coordinates from one epoch to another. For that purpose models are needed that fit better to the data. Therefore empirical and semi-empirical models, combining geophysical models with empirical models, are created for Fennoscandia.

Vestøl (2006) created an empirical model from tide-gauge, levelling and GPS data using least-squares collocation. Ågren and Svensson (2007) then combined the empirical model of Vestøl (2006) with a geophysical model based on Lambeck et al. (1998). The resulting model was adopted by the NKG as the NKG2005LU land uplift model.

Currenty the best representation of the GIA-induced land uplift is given by the NKG2016LU land uplift model. The model is again a semi-empirical model (Vestøl et al. 2019), but this time a new GIA model as well as a new empirical model were computed as an NKG cooperation. This time more GNSS and levelling data were used for the calculations of the empirical model, but no tide-gauge data. Based on the different input components an uncertainty grid was determined for the model. Three versions of the model are provided: NKG2016LU_abs containing the absolute vertical velocities relative to the ellipsoid, NKG2016LU_lev containing vertical velocities relative to the geoid, which experiences uplift as well, and NKG2016LU_gdot giving the gravity change rates. The latter was calculated by multiplying NKG2016LU_abs with the factor -0.163 µGal/mm determined by Olsson et al. (2015) and confirmed by Olsson et al. (2019). An uncertainty value of ±~0.016 µgal/mm was determined for the ratio by Ophaug et al. (2016).

3. Materials and Methods

3.1 Materials

In order to answer the research questions, a lot of different datasets and models were used in publications 1-5. The datasets and models are given in Table 1 together with some basic information and the number of the publication they were used in.

Table 1. Datasets and models used in the thesis, their source or reference, additional information, and number of Publication in which they were used (#).

Dataset/model	Source/Reference	Additional information	#
GPS-levelling data			
EUVN-DA	FGI, Ollikainen (2006)	50 points, 1st order EUREF-FIN & N2000	1,2
NLS	NLS, benchmark register 2011 (Puupponen 2011, personal communication)	526 points classes 1-3 EUREF-FIN & N2000	2
Marine data			
Airisto GNSS/IMU data	2015 Airisto gravity survey	Applanix POS MV 320 System, 1 Hz GNSS, 200 Hz IMU	3
Airisto auxiliary data	2015 Airisto gravity survey, Meritaito Ltd.	Vessel's instrumentation coordinates in internal coordinate frame, squat function, wind speed and direction, static draft readings	3
GNSS permanent stations data FinnRef	NLS FinnRef network	1 Hz, 3 stations	3
GNSS permanent stations data TrimNet	GeoTrim Oy Trimnet network	1 Hz, 5 stations	3
GNSS permanent stations data Swepos	Lantmäteriet Swepos network	1 Hz, 3 high class stations, 10 lower class stations	3
Tide gauge data Finland	Finnish Meteorological Institute (FMI)	10 Tide gauge stations on Bothnian Bay coast, hourly data, referenced to N2000	3
Tide gauge data Sweden	Swedish Meteorological and Hydrological Institute (SMHI)	6 Tide gauge stations on Bothnian Bay coast, hourly data, referenced to RG2000	3
Baltic Sea physics analysis and forecast-model	Copernicus Marine Environment Monitoring Service (CMEMS)	CMEMS (2016), hourly grids, resolution 2 km x 2 km	3

Dataset/model	Source/Reference	Additional information	#
Geoid models			
NKG2004	Forsberg et al. (2004)	Developed in working group for geoid determination of the Nordic Geodetic Commission (NKG), grid, Nordic and Baltic	1
NKG2015	Ågren et al. (2016b)	Developed in working group for geoid and height systems of the Nordic Geodetic Commission (NKG). ETRF2000, epoch 2000.0, grid, Nordic and Baltic	3
FIN2005N00	Publication 1	Conversion surface between EUREF-FIN and N2000, grid, Filand	3
Relative gravity data			
FGI dataset	FGI gravity database	Point data	2
NKG2004 dataset	NKG gravity database, situation 2004	Point data, maintained by the NKG working group on geoid and height systems	2
Russia dataset		Gridded data, 5' x 7.5′	2
Arctic GP	Kenyon et al. (2008)	Gridded data, 5' x 5′	2
Global gravity field models			
EGM2008	Pavlis et al. (2008, 2012)	$(n_{\max}, m_{\max})$ = (2190,2159)	2
EIGEN-6C	Förste et al. (2011)	$(n_{\max}, m_{\max})$ = (1420,1420), time-dependency coefficients up to degree and order 50. Evaluated at epoch 2006.25 using linear time-dependencies.	2
Digital elevation models			
SCANDEM	NKG	Compiled in working group for geoid determination of the Nordic Geodetic Commission (NKG) from national models for NKG2004 geoid model, Nordic and Baltic	2
Korkeusmalli 25	National Land Survey of Finland (NLS)	25 meter resolution, Finland	2
EGM2008 height model	Pavlis et al. (2008, 2012)	Spherical harmonic coefficients of elevation $(n_{\max}, m_{\max})$ = (2190,2190), provided with EGM2008 global gravity field model	2
Absolute gravity data			
IMGC data	Istituto di Metrologia “G. Colonnetti”(IMGC), Cannizzo et al. (1978)	IMGC instrument 1976, 1 station, corrected afterwards for polar motion and atmospheric pressure changes	5
GABL data	Soviet Academy of Sciences (ANSSR), Arnautov et al. (1982)	GABL instrument 1980, 2 stations, corrected afterwards for polar motion and atmospheric pressure changes	5
NOAA data	National Oceanic and Atmospheric Administration (NOAA), Gitlein (2009)	FG5-102 instrument 1993, 1 station, FG5-111 instrument 1995, 3 stations	4,5

Dataset/model	Source/Reference	Additional information	#
FG5-101 data	Bundesamt für Kartographie und Geodäsie (BKG), Falk (Personal communication 2003)	FG5-201 instrument 2000, 1 station	
JILAg-5 data	FGI	JILAg-5 instrument 1988-2002, original processing results, except for unification of vertical gravity gradients	5
FG5-220 data	Institut für Erdmessung (IfE), Gitlein (2009)	FG5-220 instrument 2003-2008, 3 stations	4,5
FG5(X)-221 data	NLS-FGI	Re-processed for Publication 5, FG5-221 instrument 2003-2012, FG5X-221 instrument 2013-2019, 12 stations	4,5
Land uplift velocities			
NKG2005LU land uplift model	Vestøl (2006), Ågren and Svensson (2007)	Uplift rates: absolute, relative	2,3
NKG_RF03vel velocity model	Nørbech et al. (2008)	Developed by the NKG working group for Reference Frame and Positioning, 3D velocity grids	3
NKG2016LU land uplift model	Vestøl et al. (2019), Olsson et al. (2019)	Uplift rates: absolute (_abs), relative (_lev), gravity (_gdot) ITRF2008	5
GNSS vertical rates 2	Kierulf et al. (2014)	ITRF2008	5
GNSS vertical rates 3	Vestøl et al. (2019)	ITRF2008, uncertainties multiplied by 1.41 as suggested by authors	5
GNSS vertical rates 4	Lahtinen et al. (2019)	ITRF2014	5

3.2 Methods

3.2.1 Height conversion surface calculation

The height conversion surface was calculated for Finland for the conversion between EUREF-FIN ellipsoidal heights and N2000 heights as obtained by levelling. The NKG2004 geoid model was used in combination with the EUVN-DA GPS-levelling dataset (Figure 9).

Height-anomaly differences were calculated according to equation (10) and then a surface was fitted to the differences. Polynomial fitting and least-squares collocation (LSC) were tested (see section 2.2.2). Software for polynomial fitting was written by the author according to equations (12) and (13). Polynomials of degrees 0 to 5 were tested. For LSC the GEOGRID routine of the GRAVOSOFT software package (Forsberg 2003; Tscherning et al. 1992) was used.

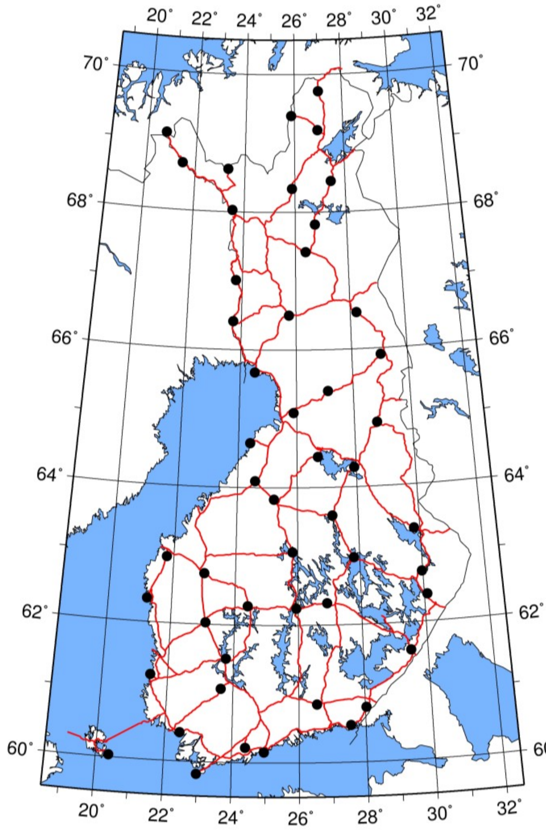

Figure 9. Study area for Publications 1 and 2 and EUVN-DA GPS-levelling data used for geoid model validation.

The covariance function, $C(r)$, in the LSC equation (14), was approximated by a second order Gauss-Markov covariance function, which is implemented in the GEOGRID routine:

$$C(r) = C_0\left(1+\frac{r}{\alpha}\right)e^{(-r/\alpha)}. \qquad (23)$$

The correlation length α is an input parameter. The scale C_0 is the signal variance of the data and is calculated by the program from the input $\Delta\zeta$. Another input parameter is the noise level of the input data, which determines how tight the surface is fitted to the input point values. Different noise levels were tested.

For both methods the fit statistics give an indication of how a calculated surface fits to the given data, but do not give any information of how good the fit will be for other locations not included in the given data. At the time of the study the EUVN-DA data was the only GPS-levelling dataset available. Due to the lack of data for testing, cross-validation was applied to test the actual performance of the calculated surfaces. The so-called leave-one-out cross-validation was applied. In that method one point at a time is left out of the calculations. Then the value for the left out point was interpolated from the fitted surface and compared with the known value. All points were left out in turn, which resulted in cross-validation residuals for all points.

3.2.2 Method used for geoid modelling

The geoid modelling in Publication 2 was done as described in section 2.2.3 using the GRAVSOFT software package (Forsberg 2003; Tscherning et al. 1992). The geoid model was calculated for Finland. Gravity datasets that were used and the area they covered are shown in Figure 10.

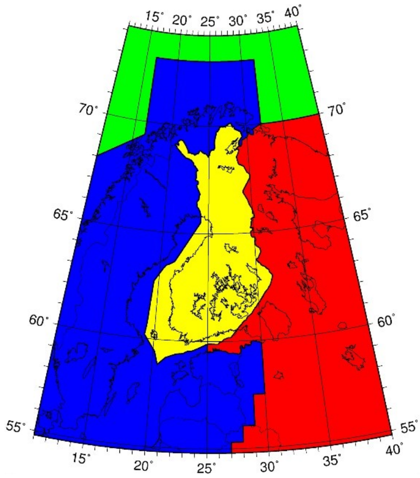

Figure 10. Areas of the gravity datasets used for geoid modelling in Publication 2: Yellow – FGI database, Blue – NKG database, Green – Arctic GP, Red – Russian data.

For the remove-compute-restore procedure, the effects of the global model, Δg_{EGM} and ζ_{EGM}, were calculated from the EGM2008 model to degree 2190 and order 2159 and from the EIGEN-6C model up to degree and order 1420. The effect of the residual terrain corrections, Δg_{RTM} and ζ_{RTM}, were determined using the SCANDEM digital elevation model with the area for Finland replaced by the korkeusmalli 25 elevation model. As a reference model the EGM2008 height model was used, developed to the same degree and order as the global gravity field model used. The long-wavelength terrain corrections calculated from the reference model were subtracted from the terrain corrections calculated from the digital elevation model to result in Δg_{RTM}. The residual gravity anomalies were gridded using LSC to create an evenly distributed set of gravity anomalies as input to Moledensky's integral, equation (17).

The integral was evaluated in Publication 2 using multi-banded spherical FFT (Forsberg and Sideris 1993), which is implemented in the SPFOUR routine of the GRAVSOFT software package. In the SPFOUR routine it is possible to apply the Wong-Gore modification of the Stokes kernel (Wong and Gore 1969), where the lowest harmonics of the kernel function are set to zero. This is justified by the fact that the lower harmonic signals, i.e., the long-wavelength signals, have already been removed from the data. By applying the modification, the effect of local data on the longer wavelengths is eliminated. In Finland this helped to prevent the effect of remaining terrain effects in the Norwegian mountains to creep into the geoid model solution in Finland. We applied a tapered version of the Wong-Gore modification. Here, the lowest harmonics are set to zero up to degree N_1, and then gradually and linearly go to full power at degree N_2 (Forsberg 2003):

$$S_{\text{mod}}(\psi) = S(\psi) - \sum_{n=2}^{N_2} \alpha(n) \frac{2n+1}{n-1} P_n(\cos\psi) \tag{24}$$

$$\alpha(n) = \begin{cases} 1 & \text{for} \quad 2 \le n \le N_1 \\ \dfrac{N_2 - n}{N_1 - N_2} & \text{for} \quad N_1 \le n \le N_2 \\ 0 & \text{for} \quad N_2 \le n \le N \end{cases} \qquad n = 2, \dots, N$$

Different values for N_1 and N_2 were tested with N_2 being always 10 higher than N_1. The quasi-geoid models thus obtained were then compared with the GPS-levelling datasets to find the optimal combination of N_1 and N_2. For these comparisons, the GNSS datasets were corrected for land uplift to bring the epoch of the GNSS data from 1997.0 to the epoch of the levelling data: 2000.0. This was done with the NKG2005LU land uplift model.

3.2.3 Geoid model validation at sea

In publication 3 we developed a method to validate geoid models at sea. The method was tested with GNSS/IMU data that was collected on board the survey vessel MSV Airisto, owned by Meritaito Ltd., during a dedicated gravity campaign in the Bothean Sea (Figure 11). A squat function was available for the vessel and readings of the depth scales of the vessel at the stern and bow were taken always right before leaving a harbour and immediately after arriving to a harbour.

The procedure we followed for geoid validation is explained hereafter. The heights involved are shown in Figure 12 and a schematic overview of the procedure is given in Figure 13.

At first, a homogeneous set of coordinates was calculated for the FinnRef, Swepos and TrimNet permanent GNSS reference stations on land. This was done with the Bernese 5.2 software (Dach et al. 2015). The final solution was constrained to the IGb08 coordinates in epoch 2015.75 of 6 datum points that were obtained from the GNSS Analysis Centre of the Finnish Geospatial Institute. Then, the shipborne GNSS/IMU data were processed for each of the 46 lines of the trajectory separately with the Applanix POSPac MMS Version 7.1 software.

Now the trajectory coordinates were translated from the origin of the vessel to the bottom of the vessel using her internal coordinate frame and the pitch and roll information obtained from the IMU. The translation of the coordinates from the bottom of the vessel to the sea level were then calculated using the static and dynamic draft information. The static draft information was derived from the stern and bow readings and the dynamic draft from the squat function in combination with the velocity readings of the vessel.

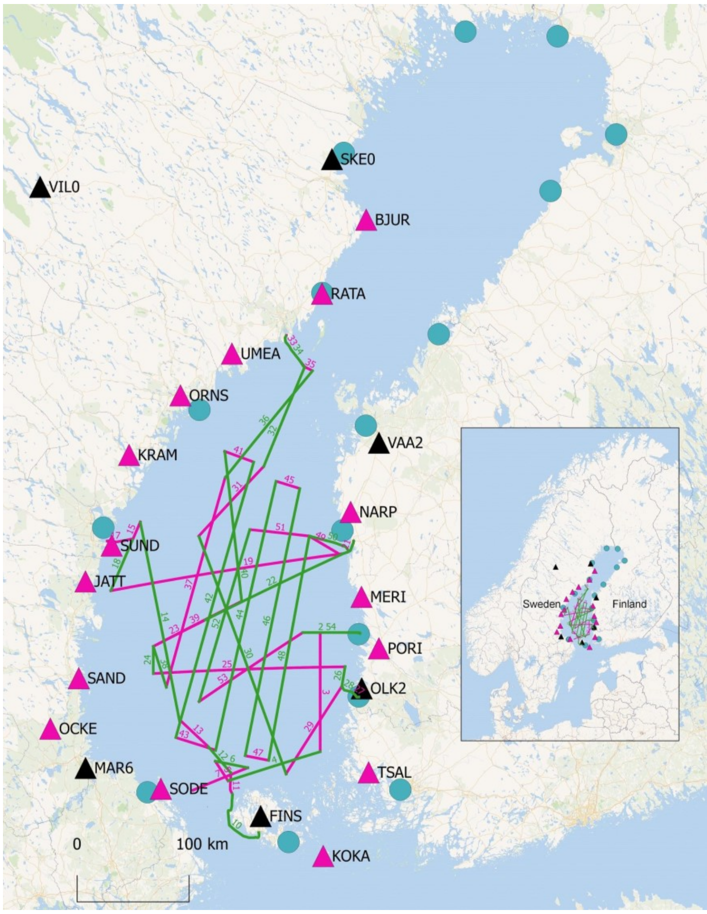

Figure 11. Study area for publication 3: The Bothnian Sea with the lines of the Airisto campaign trajectory that were processed separately. Triangles are permanent GNSS base stations that served as reference stations in the trajectory calculations, with black triangles for the datum points in the reference coordinates calculations. Blue circles are tide gauges used in the sea-surface modelling.

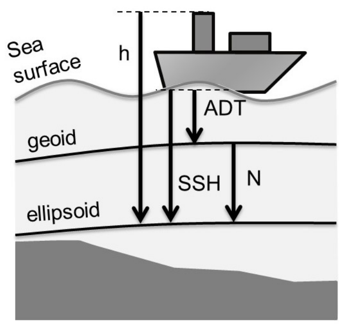

Figure 12. Heights involved in geoid validation at sea using GNSS observations: h is the ellipsoidal height measureed with GNSS, SSH the sea surface height, ADT the absolute dynamic topography and N the geoid height.

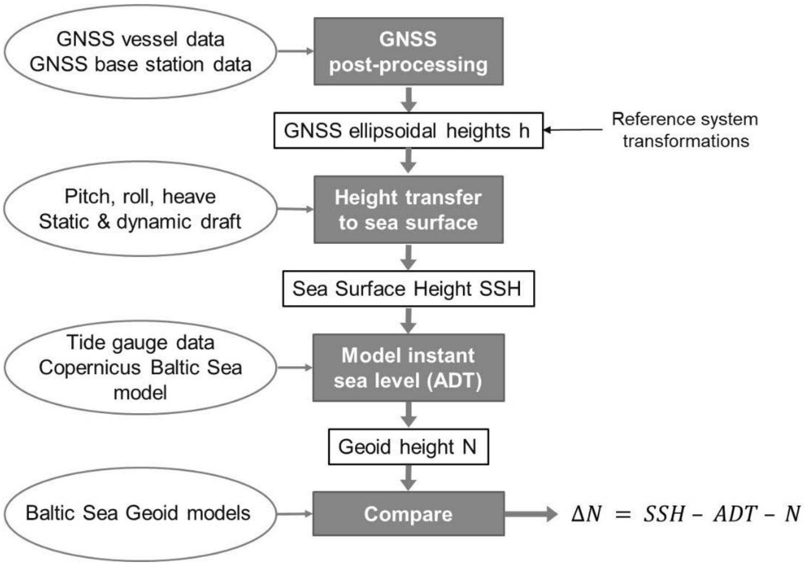

Figure 13. Schematic overview of the steps involved in geoid validation at sea using GNSS.

We then transformed the trajectory coordinates at sea level to the reference frames and epochs related to the geoid models. This was necessary, because our test area is close to the maximum of the Fennoscandian land uplift area, where heights change up to 0.01 m/year. The IGb08 (2015.75) coordinates were transformed into ETRF96 (1997.0) for the FIN2005N00 geoid model and ETRF2000 (2000.0) for the NKG2015 geoid model. It was done with a method that uses the NKG_RF03vel deformation model for intra-plate corrections (Häkli et al. 2016).

Now we have a SSH trajectory. Next we reduced the SSH to the geoid surface applying two different methods for the determination of the ADT: the tide-gauge method and the physical-model method. In the tide-gauge method the data of the tide gauges around the Bothnian Sea were interpolated to hourly sea surfaces with 2 km x 2 km resolution using thin-plate spline regression.

For the physical-model method sea-level data of the Baltic Sea physics analysis and forecast model (CMEMS 2016) were taken. Because this model is not fixed to a geodetic height reference, the model had to be aligned to the height reference of this study. In this procedure we took sea-level data from points close to the tide gauges and fitted surfaces through these points the same way as was done for the tide-gauge method. Then we subtracted these surfaces from the tide-gauge surfaces to create correction surfaces. The correction surfaces were in turn added to the original model surfaces, resulting in the physical-model surfaces.

The ADT at each epoch and GNSS location was derived from the surfaces of both methods using the nearest-neighbour interpolation method in space and time. The resulting heights were the GNSS-derived geoid heights that could be compared to the geoid models in the area: The FIN2005N00 model and the NKG2015 model.

3.2.4 Analysis of absolute gravimeter comparisons

To study the offsets of absolute gravimeters that operated in Finland we looked at the performance of these instruments in the international comparisons (Publication 5) and in bi-lateral comparisons (Publications 4 and 5).

The available international comparisons are given in Table 2 together with references to the publications where the results can be found. The results of the comparisons could not in all cases be used as given in the publications. We made an attempt to show all results in the same way as the results of the comparisons of 2013, 2015 and 2017, where the offset is the average of the differences between the measurements and the reference station value. The uncertainties are the RMS values of the expanded uncertainties of the differences, where the uncertainties of both the measurement and the reference value are taken into account. For the early measurements 1981-1997 only offsets are given. Of the instruments that have measured in Finland, the FG5(X)-221 has, as the national standard for free-fall acceleration, participated in all KCs, whereas the other FG5(X) instruments have participated in the PSs.

Table 2. International (ICAG, CCM.G) and European (ECAG, EURAMET.M.G) absolute gravimeter comparisons and their publications.

Comparison	Publication
ICAG81-82	Boulanger et al. (1983)
ICAG1985	Boulanger et al. (1986)
ICAG1989	Boulanger et al. (1991)
ICAG1994	Marson et al. (1995)
ICAG1997	Robertsson et al. (2001)
ICAG2001	Vitushkin et al. (2002)
ECAG2003	Francis et al. (2005)
ICAG2005	Jiang et al. (2011)
ECAG2007	Francis et al. (2010)
CCM.G-K1 (2009)	Jiang et al. (2012)
EURAMET.M.G-K1 (2011)	Francis et al. (2013)
CCM.G-K2 (2013)	Francis et al. (2015)
EURAMET.M.G-K2 (2015)	Pálinkáš et al. (2017)
CCM.G-K2.2017	Shuqing et al. (2020)
EURAMET.M.G-K3 (2018)	Falk et al. (2020)

In the study of the bi-lateral comparisons we analysed comparisons of absolute gravity instruments operating in the Fennoscandian land-uplift area between 2003 and 2006 in Publication 4. We looked at simultaneous observations, non-simultaneous observations weeks or months apart and non-simultaneous observations from adjacent years at sites where the land uplift can be neglected. In Publication 5 we concentrated on bi-lateral comparison results of absolute gravity instruments that operated in Finland. There, we mainly looked at simultaneous observations taken 1-3 days apart and non-simultaneous observations one or two weeks apart.

3.2.5 Trend estimation from absolute gravimeter time series

For the analysis of the gravity time series in Finland we used all available absolute gravity data for 12 absolute gravity stations in Finland (see Figure 14 and Table 1). The data of the FG5(X)-221 instrument were re-processed by the author for this investigation using the g7 or g9 Absolute Gravity Data Acquisition and Processing Software by Micro-g LaCoste (2007).

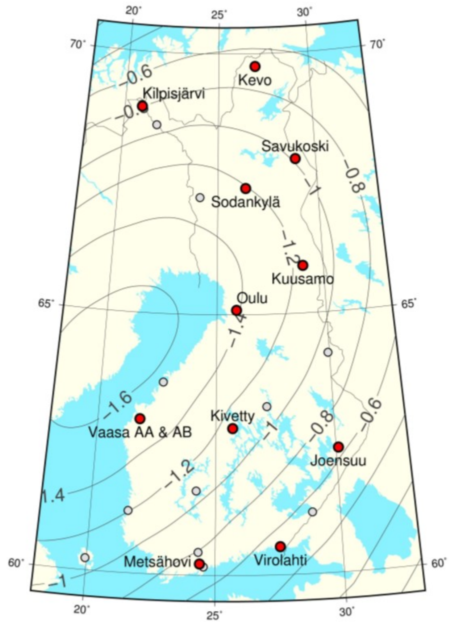

Figure 14. Absolute gravity stations in Finland. Red dots are stations used in this study with more than three observations over a time span of at least 3 years. Grey dots are stations with less observations, which are not used in the study. The contour lines show the GIA induced gravity change rate according to the NKG2016LU_gdot model (Olsson et al. 2019).

The trends were calculated from the time series in a weighted least-squares adjustment. In the adjustment we used the following weights for the absolute gravity observations of the different instruments:

IMGC	10.0 μGal
GABL	10.0 μGal
JILAg-5	7.0 μGal
FG5	2.6 μGal
FG5X	2.3 μGal

The observation equation is given by:

$$g_{ijk} = a_j + b_j t_i + c_k + \varepsilon_{ijk}, \quad (25)$$

where g_{ijk} is the gravity value at epoch i, station j and measured with instrument k. a_j and b_j are the constant and trend for station j, and c_k the offset for instrument k.

First, trends were fitted for each station separately. No offsets were estimated in this stage, $c_k = 0$. But, at stations with data from more than one type of instrument, also separate trends were calculated from only JILAg-5 measurements (Metsähovi), only FG5(X) measurements (Joensuu, Metsähovi, Sodankylä, Vaasa AA and AB, and Virolahti), or only FG5 and JILAg-5 measurements (Metsähovi and Sodankylä).

In a second approach, we estimated the trends of all stations with observations of other instruments in addition to FG5(X) observations in one common adjustment, where offsets, c_k, were introduced for instruments other than the FG5 and FG5X. Once the trends were obtained, they could be compared to gravity change rates derived from the NKG2005LU-gdot land-uplift model.
For the trends that were found reliable, we investigated how long it took for the trends to stabilize. This was done so that for each station, time series of trends were calculated starting with two measurements and adding one new measurement for each new trend calculation. The time series of trends were calculated without and with applying the offsets determined earlier for the IMGC, GABL and JILAg-5 instruments.

3.2.6 Calculation of ratio between gravity change rates and vertical velocities

Once a reliable set of gravity trends is obtained, we can look at the relationship between the gravity change rates and vertical velocity. The relationship is given by equation (22). Standard regression methods cannot be used to solve equation (22), because both $\dot{g}$ and $\dot{h}$ contain errors. Instead we use the FITEXY routine of Press et al.(2012), where the straight line is fitted to the pairs $(\dot{h}_i, \dot{g}_i)$ by minimizing the orthogonal weighted distances between the line and the pairs ("orthogonal regression"). Both the observed gravity trend at station i and the vertical velocities at the same station are subject to error. The estimates a^* and b^* are calculated by minimizing the following χ^2 merit function with respect to a and b:

$$\chi^2(a,b) = \sum_{i=1}^{n} \frac{(\dot{g}_i - a - b\dot{h}_i)^2}{\sigma_{\dot{g}_i}^2 + b^2\sigma_{\dot{h}_i}^2}, \tag{26}$$

where the $\sigma_{\dot{g}_i}$ are the uncertainties of the $\dot{g}_i$ and the $\sigma_{\dot{h}_i}$ the uncertainties of the $\dot{h}_i$. Here, we do not assume that the estimated line goes through the origin, where $a = \dot{g}_{\dot{h}=0} = 0$.

As a second option, we force the line to go through the origin. This is, however, not possible with the FITEXY routine. Instead, we use the FREML algorithm by AMC (2002). This algorithm is based on the same equation (26), but it has the option to fit a line going through the origin, where $\dot{g}_{\dot{h}=0} = 0$. In the results we denote the estimates a^* by $\dot{g}_{\dot{h}=0}$ and b^* by $\dot{g}/\dot{h}$. Instead of the intercept $\dot{g}_{\dot{h}=0}$ the intercept $\dot{h}_{\dot{g}=0}$ can also be given.

We used four different sets of vertical velocities for the estimation of the relationship of the gravity change rates and the vertical velocities: the vertical rates of the NKG2016LU-abs model (Vestøl et al. 2019) and GNSS-datasets by Kierulf et al. (2014), Vestöl et al. (2019), and Lahtinen et al. (2019). We used two sets of uncertainties with the vertical rates. The first set of uncertainties, σ_1, were those that are provided with the vertical rates. The second set of uncertainties, σ_2, was formed by including the uncertainties of the reference frame origin motion in the original uncertainties by error propagation. In the case of ITRF2008 we use uncertainty values for the origin motion by Wu et al. (2011): 0.5 mm/yr for drift and 0.2 mm/yr for scale. For ITRF2014 we us a value of 0.33 mm/yr, which was determined by Riddell et al. (2017).

4. Results

In section 1.2 the following research questions were defined:

1. What is the best way of fitting the NKG2004 geoid model to GNSS-levelling data to establish a conversion surface for the transformation between EUREF-FIN ellipsoidal heights and N2000 normal heights?
2. Can the geoid model for Finland be improved using the high-resolution, partly satellite-based, global gravity field models EGM2008 and EIGEN6C?
3. Can a geoid model be validated by GNSS measurements at sea?
4. What is the magnitude of possible offsets between the absolute gravimeters that have been used for measurements in Finland?
5. What do the time series of absolute gravity in Finland tell us about the relationship between postglacial rebound induced gravity change rates and vertical velocities?

In the subsections below the results are given for each research question.

4.1 Research question 1: Geoid model fitting

What is the best way of fitting the NKG2004 geoid model to GNSS-levelling data to establish a conversion surface for the transformation between EUREF-FIN ellipsoidal heights and N2000 normal heights?

The answer to question 1 above can be found in publication 1. This publication studied different methods and parameters to calculate the correction surface to the NKG2004 geoid model to best fit the Finnish GNSS and levelling data. Geoid height differences were calculated from the 50 EUVEN-DA GNSS-levelling data points and the NKG2004 geoid model. The geoid-height differences had an offset of -0.032 m and standard deviation of 0.041 m. Correction surfaces were fitted to these geoid-height differences. Polynomial surfaces were tested by fitting polynomials of degrees from zero to five. Alternatively, least-squares collocation was tested varying the input parameters for correlation length, α, between 50 and 250 km and noise level, σ_0, between 0.008 and 0.027 m, respectively (see sections 2.2.2 and 3.2.1).

At the time of the calculations the EUVN-DA dataset was the only GNSS dataset available. The statistics of the fit residuals only give information on how well the surfaces fit internally to the given data, but not how well they fit to locations elsewhere. No other independent dataset was available for testing. Therefore the surfaces were also tested using cross-validation, leaving one point at a time out of the surface calculation.

Table 3 summarizes the best results for each calculation method and for the internal fit as well as the cross-validation.

Table 3. Standard deviations of the fit residuals and the cross-validation residuals for best performing correction surfaces calculated using polynomial fitting and least-squares collocation (LSC) with correlation length α and noise level $\boldsymbol{\sigma_0}$.

Fitting method	Fitting parameters	σ of fit (m)	σ of cross-validation (m)
No correction surface		0.041	
Polynomial			
	5 degrees	0.013	0.033
	3 degrees	0.017	0.021
LSC			
	offset, α = 50 km, σ_0 = 0.01 m	0.002	0.025
	offset, α = 200 km, σ_0 = 0.02 m	0.011	0.019

Best internal fits to the data were obtained using the highest possible polynomial degrees (5) or short correlation lengths (50 km) and low noise levels (0.01 m) in the least-squares collocation calculations, with collocation giving the best results with a standard deviation of the fit of 2 mm. These correction surfaces fit well to the data, but the surfaces are not very smooth and change rapidly outside the area covered by the dataset. These are good examples of overfitting, where the surface is too tightly fit to the data leading to unrealistic surfaces.

The cross-validation results favour lower degree polynomials and collocation using longer correlation length and higher noise levels (see Table 3). These correction surfaces are much smoother, but in case of the 3rd degree polynomial, the surface still changes rapidly outside the area of the dataset which is unrealistic. It may give problems in the vicinity of the Finnish borders. The best correction surface was found with least-squares collocation using a correlation length of 200 km and a noise level of 0.02 m. In that case the standard deviation is below 2 cm for the whole country. This conversion surface was added to the NKG2004 geoid model to form the official Finnish height conversion model FIN2005N00.

The best way of fitting the NKG geoid model to GNSS-levelling data is by surface fitting using least-squares collocation and applying cross-validation for testing.

4.2 Research question 2: Using global models

Can the geoid model for Finland be improved using the high-resolution, partly satellite-based, global gravity field models EGM2008 and EIGEN6C?

The high-resolution global gravity models EGM2008 and EIGN-6C were analysed and used in quasi-geoid model calculation in publication 2. The global models were combined with the terrestrial gravity datasets of the FGI within the Finnish borders and of the NKG, the Arctic GP and Russia outside the borders. The high-resolution global models were used as background models in the remove-calculate-restore restore calculations as described in section 3.2.2. To eliminate the influence of local effects, such as the terrain effects of the Norwegian mountains, on the longer wavelengths, a tapered version of the Wong-Gore modification of the Stokes integral kernel (Wong and Gore 1969) was applied. The models calculated were then compared with the EUVN-DA and NLS GNSS-levelling data.

Different tapering degrees of the Wong-Gore modification were tested. Best results were obtained when the Wong-Gore modification was tapered between degrees $N_1 = 50$ and $N_2 = 60$. These results are shown in Table 4.

Table 4. Statistics of the differences between the GPS-levelling data and the final height anomalies calculated using the EGM2008 global model (developed to degree 2190 and order 2159) and the EIGEN-6C global model (developed to degree and order 1420) as background models and with Wong-Gore modification of the Stokes kernel at degree numbers 50 to 60.

Global model	GNSS-lev. dataset	Differences (m)				Offset and tilt removed (m)			
		Mean	σ	Min.	Max.	Mean	σ	Min.	Max.
EGM2008									
	EUVN-DA	-0.561	0.028	-0.635	-0.499	0.000	0.023	-0.084	0.048
	NLS	-0.556	0.035	-0.695	-0.443	0.000	0.028	-0.164	0.075
EIGEN-6C									
	EUVN-DA	-0.608	0.024	-0.678	-0.552	0.000	0.024	-0.076	0.053
	NLS	-0.601	0.031	-0.572	-0.516	0.000	0.029	-0.164	0.069

Standard deviations between 2.3 and 3.5 cm were obtained. When no trend was removed the quasi-geoid model with the EIGEN-6C background model performed a little better than that with the EGM2008 background model. After correcting for an offset and a tilt in two directions, the differences in performance between the models disappear. For the EGM2008 case the trend removal reduced the standard deviation. For the case with the EIGEN-6C model, the standard deviation does stay the same, only the offset is gone. One can conclude that EGM2008 may still have had long-wavelength errors, like those common in global gravity field models of the pre- gravity satellite era. In the EIGEN-6C model the long-wavelength errors seem to have disappeared.

In section 4.1 we saw that the differences between the EUVN-DA GNSS-levelling data and the NKG2004 quasi-geoid model had a standard deviation of 0.041 m. Without removal of a trend, the new models presented here give better results when compared to the same EUVN-DA dataset, with standard deviations of 0.028 and 0.024 m. Once an offset and trend is removed the standard deviations are around 0.024 for all models, also the NKG2004 model (see publication 1).

The new quasi-geoid models calculated using the high-resolution global gravity field models EGM2008 and EIGEN6C are an improvement over earlier quasi-geoid models available for Finland. However, once corrected for an offset and tilt, no difference can be seen anymore when compared to GNSS-levelling data.

4.3 Research question 3: Geoid-model validation at sea

Can a geoid model be validated by GNSS measurements at sea?

The answer to this question is given in publication 3. We followed the procedure given in section 3.2.3 using the marine datasets described in section 3.1. Here I describe the main findings related to each step of the process, as all influence the final solution.

4.3.1 GNSS data handling and reduction to sea level

A homogeneous set of coordinates were calculated for the reference stations on land. The final daily solutions had an RMS of about 1 mm for the horizontal and 3 mm for the vertical coordinates.

After processing of the GNSS/IMU data, tracks were available for 46 lines of the trajectory. For most of the lines, the solution of the height component was good, with the vertical accuracy being 1.5-2 cm. However, some lines showed jumps in the height series correlating with high standard deviation values for the heights. These are related to bad ambiguity fixes that are probably caused by disturbing signals in the GPS frequency range. 8 lines that showed these jumps in the height series were not used in the final solution.

Next, the trajectory coordinates were translated from the origin of the vessel via the bottom of the vessel to the sea surface using the pitch and roll information from the IMU and the static and dynamic draft information. The effects of the pitch and roll were 4.2 and 11.0 cm at most, but within 0.1 and 0.3 cm for 95% of the cases. Changes of the static draft were 1 cm per day accounting for 8 cm change in height in total. The dynamic draft was 15 cm when the vessel was moving at her average speed.

We found that it is essential in our area to transform the coordinates to the reference frame and epoch of the geoid models. The influence of the reference frame and epoch transformations on the heights of the trajectory were between 10 and 15 cm.

4.3.2 Translation from sea level to geoid surface and comparison with geoid models

In the last step of the process the trajectory heights were reduced to the geoid surface using the sea level surfaces created from the tide gauges and those created from the physical model (see section 3.2.3 for details). The geoid heights, N, thus created were then compared to the geoid models FIN2005N00 and NKG2015. The geoid-height differences created, ΔN, showed high-frequency variations, with amplitudes up to tens of centimetres that are not geoid related. Therefore we did a 10 min moving average smoothing of the data to get a smoother signal. The smoothing reduced the grand mean standard deviations from 11 to 4 cm. Statistics of the ΔN for each line are given in Table 2 of publication 3. Some lines show mean ΔN below 1 cm and standard deviations of many lines are below 3 cm. Here, we present the combined results for the smoothed dataset in Table 5. We see that overall the NKG2015 model performs slightly better than the FIN2005N00 model and the physical sea-level model gives better results than the tide-gauge model in our test area. Best results were obtained with the physical sea-level model and the NKG2015 geoid model, with a grand mean of 2.0 cm and a standard deviation of 4.0 cm.

Table 5. Grand means and standard deviations of the geoid height differences calculated from the GNSS trajectory and the geoid models FIN2005N00 and NKG2015 using either the tide-gauge sea surface or the physical-model sea surface for the reduction from sea surface to the geoid. The data was smoothed with a 10 minutes moving average.

	Tide gauge		Physical model	
	Mean (cm)	**Std.dev. (cm)**	**Mean (cm)**	**Std.dev. (cm)**
FIN2005N00	4.5	4.2	3.6	4.1
NKG2015	2.9	4.1	2.0	4.0

All values in Table 5 are below 5 cm, which was the goal of the FAMOS project (FAMOS Consortium. 2014–2017, Bilker-Koivula et al. 2017) for the accuracy of the final geoid model for the Baltic Sea.

We proved that geoid models can be validated at sea. How well this can be done, depends on the different steps of the process: the reliability of coordinates of the permanent GNSS base stations, the quality of the GNSS/IMU data and trajectory solutions, the reference frame and epochs related to the geoid models, the quality of the internal coordinate system of the vessel, the accuracy of the height transfers to the sea level, which include information on the static draft, the squat and pitch and roll, and the quality of the sea-surface modelling.

4.4 Research question 4: Offsets between absolute gravimeters

What is the magnitude of possible offsets between the absolute gravimeters that have been used for measurements in Finland?

This question is studied in Publication 4 as well as in Publication 5, where the latter concentrated on the instruments that operated in Finland only. The question can be addressed by looking at international comparisons, bi-lateral comparisons and by simultaneous estimation in trend calculations. These different angles are described in the following sub-sections.

4.4.1 Results of international comparisons

Results of the international comparisons are shown in Table 6 for the instruments that have operated in Finland (see Olsson et al. (2019) for a similar overview considering all instruments operating in the whole Fennoscandian uplift area and international comparisons until 2015). The way the offsets with respect to the comparison reference values are presented differs between comparisons and the results were homogenized for Table 6. For the early comparisons, where only offsets are given, IMGC and GABL offsets are estimated to have uncertainties in the range of 20 µGal and the JILAg-5 offsets have uncertainties around 10 µGal. FG5 offset uncertainties would be around 5 µGal.

Table 6. International (ICAG, CCM.G) and European (ECAG, EURAMET.M.G) absolute gravimeter comparisons and the deviations from the reference value in μGal of participating instruments that operated in Finland between 1976 and 2019. Values in bold are relevant for the measurements in Finland. Uncertainties are 2σ.

	IMGC	GABL	JILAg-5	FG5-101	FG5-102	FG5-220	FG5-221
ICAG81-82[1]	**-6**	**+7**					
ICAG1985[2]	-2.3	+4.6					
ICAG1989[3]	-12.6	+9.1	**-8**				
ICAG1994[4]	-1.3 ± 6		**9[§]**	-0.6 ± 6.4	**1.4 ± 6**		
ICAG1997[5]	9.7		**0.5**	**-2.7**			
ICAG2001[6]			**(5.7 ± 12)[a]**	**(2.9 ± 4.3)[a]** **2.3 ± 4.3[b]**			
ECAG2003[7]						**-1.6 ± 4.6**	**1.0 ± 5.6**
ICAG2005[8]				-2.5 ± 5.2[c]			**-0.5 ± 5.4[c]**
ECAG2007[9]				1.8 ± 6.0		**2.3 ± 6.4**	**-0.2 ± 7.8**
CCM.G-K1 (2009)[10]				0.5 ± 3.8	-6.0 ± 4.8	**1.7 ± 4.8**	**1.6 ± 5.4**
EURAMET.M.G-K1 (2011)[11]					-6.3 ± 5.0	1.1 ± 5.3	**0.5 ± 6.2**
						FG5X-220	
CCM.G-K2 (2013)[12]					-5.6 ± 5.1	2.3 ± 5.3	**1.5 ± 5.7[d]**
					FG5X-102		**FG5X-221**
EURAMET.M.G-K2 (2015)[13]					0.2 ± 5.1	5.2 ± 5.9	**-2.1 ± 5.7[d]**
CCM.G-K2.2017[14]					1.3 ± 3.7		**0.9 ± 4.7[d]**
EURAMET.M.G-K3 (2018)[15]				-2.7 ± 6.6		-1.5 ± 5.6	**1.1 ± 5.3[d]**

[a]Results with all data included. [b]Results of final solution where the data of JILAg-5 were excluded. [c]Results multiplied by -1 to correct for the different sign-definition in 2005. [d]Result of Key Comparison. [§]New value calculated by authors after publication.

[1]Boulanger et al. (1983), [2]Boulanger et al. (1986), [3]Boulanger et al. (1991), [4]Marson et al. (1995), [5]Robertsson et al. (2001), [6]Vitushkin et al. (2002), [7]Francis et al. (2005), [8]Jiang et al. (2011), [9]Francis et al. (2010), [10]Jiang et al. (2012), [11]Francis et al. (2013), [12]Francis et al. (2015), [13]Pálinkáš et al. (2017), [14]Shuqing et al. (2020), [15]Falk et al. (2020)

The IMGC and GABL instruments differ by 13 μGal in the first comparison (Boulanger et al. 1983) and by 6.9 μGal in the ICAG1985 (Boulanger et al. 1986). Considering that the individual offsets were estimated to have uncertainties in the range of 20 μGal, the instruments were in agreement. These instruments measured in Finland in 1976 and 1980, respectively, and due to the continuous developments of these instruments, it is unclear if the results of the comparisons are valid to assess their measurements in Finland. In addition, the relation of these early-generation instruments to later instruments (JILAg, FG5 and FG5X) cannot be established through the comparisons.

Based on the results shown in Table 6 no conclusions can be made on the possible offsets between the JILAg-5 instrument and the FG5 instruments during the time that these instruments participated together in comparisons (1994-2001).

We can, based on the comparison results shown in Table 6, conclude that the instruments that operated in Finland were in agreement with the comparison reference values and therefore it can be assumed that there were no offsets.

4.4.2 Bi-lateral comparisons

Detailed results of the bi-lateral comparisons are given in Publication 4 and the supplementary material of Publication 5. Tables 1 to 3 in Publication 4 give the gravity differences for the instrument pairs involved. Likewise, Table S4 of the supplement of Publication 5 contains the gravity differences for the instruments involved in that publication. The results party overlap for the differences between the FG5-221 and FG5-220, but values may differ due to the recalculations done for Publication 5. From year to year the gravity differences between the instruments vary. When considering the differences between the FG5-221 and the FG5-220, instruments that both operated in Finland, the differences are between -4.1 and +3.5 μGal for the simultaneous observations in publication 4 and between -4.1 and +2.3 μGal in the supplement of publication 5. It must be noted that neither in publication 4 (Table 7) nor in the supplement of publication 5 (Table 8) uncertainties are given for the individual comparison results. Also, subsequent measurements at different pillars at the same location were not combined, but instead treated as individual comparisons.

The results of the bi-lateral comparisons in publication 4 are summarized in Table 7 and of those in publication 5 in Table 8.

Table 7. Average gravity differences and their standard deviations in μGal between FG5 absolute gravimeters at bi-lateral comparisons in the Fennoscandian area (and Bad Homburg) between 2003 and 2006. The number of observations is given in brackets. Last column shows statistics for the whole dataset. (Publication 4)

Type of comparison	FG5-221 – FG5-220	FG5-220 – FG5-226	FG5-220 – FG5-301	FG5-226 – FG5-301/101	all
Simultaneous	-0.4 ± 2.7 (15)	1.9 ± 2.5 (6)	0.9 ± 2.6 (8)	-1.0 ± 0.6 (2)	0.7 ± 2.5 (31)
Non-simultaneous	-1.6 ± 2.8 (6)	1.8 ± 3.0 (4)	-0.1 ± 4.1 (7)		0.9 ± 3.4 (17)
(Non-)Simultaneous	-0.7 ± 2.7 (21)	1.9 ± 2.6 (10)	0.4 ± 3.3 (15)	-1.0 ± 0.6 (2)	0.8 ± 2.8 (48)
≥1 year apart		1.2 ± 5.6 (6)	6.5 (1)	2.2 ± 4.3 (3)	2.1 ± 4.9 (10)

Table 8. Average gravity differences and their standard deviations in μGal at bi-lateral comparisons between the absolute gravimeters that have operated in Finland. The number of observations is given in brackets as well as the years the comparisons took place. (Summary of data Table S4 in the supplement of Publication 5)

Type of comparison	FG5-221 – FG5-220 (2004-2008)	FG5-111 – FG5-101 (1995)	FG5-101 – JILAg-5 (2000)	FG5-111 – JILAg-5 (1995)	FG5-102 – FG5-101 (1993)
Simultaneous	-2.3 ± 2.5 (12)	2.5 (1)	-8.5 ± 0.7 (2)	-1.0 ± 4.6 (4)	-4.3 ± 0.1 (2)
Non-simultaneous	-2.1 ± 0.8 (3)	6.1 (1)			-0.4 ± 0.8 (2)
(Non-)Simultaneous	-2.2 ± 2.3 (15)	4.3 ± 2.5 (2)	-8.5 ± 0.7 (2)	-1.0 ± 4.6 (4)	-2.4 ± 2.3 (4)

Based on Table 7 there seem to be no offsets between the instruments. The difference between the FG5-221 and FG5-220 is of particular interest for the measurements in Finland. In Table 7 their difference is negative, but small and insignificant. However, in Table 8 their differences are much larger, also negative, and almost significant when considering the average values with respect to their uncertainties and the uncertainty of 2.6 μGal used for FG5 observations in publication 5. The negative value suggests the FG5-220 values are higher than the FG5-221 values in the timespan 2004-2008 covered by the comparisons. Of the international comparisons falling within this timespan only the ECAG2007 was visited by the FG5-220 and then its value was 2.3 μGal higher than the comparison reference value and even a little higher than the value of the FG5-221. However the value difference was well within the 2σ range (Table 6). In later comparisons the FG5-220 consistently showed positive values, except for the EURAMET.M.G-K3 (2018) comparison (Table 6). But, these results are not relevant for Finland, as the FG5-220 did not visit Finland after 2008.

The JILAg-5 instrument showed a difference of 8.5 μGal with the FG5-101, but only -1.0 μGal difference with the FG5-111. This difference in outcome cannot be explained by the difference between the FG5-101 and Fg5-111, which, based on the one simultaneous comparison shown in Table 4, was +2.5 μGal. The FG5-111 was also found to be in agreement with other FG5s in comparisons and monitoring in North America (Klopping et al. 1997; Sasagawa et al. 1995; Lambert et al. 2001).

Overall, the bi-lateral comparisons show no significant consistent differences between the instruments involved.

4.4.3 Offset estimation combined with trend calculations

In Publication 5 the possible offset of pre-FG5 instruments is further investigated by estimating their offsets in a calculation where the trend parameters of all stations were determined in combined adjustment (see section 3.2.5). In the combined solution the offsets were calculated with respect to the FG5 and FGX-221 instruments, for which no offsets were calculated. One test was made where also an offset was calculated for the FG5X-221 instrument. The results are summarized in Table 9.

Table 9. Offsets with respect to FG5(X) instruments in μGal estimated for the IMGC, GABL, JILAg-5 and FG5X-221 in combined trend calculations for Finnish stations. For comparison the JILAg-5 offset determined by Olsson et al. (2019) is also given.

Trends calculated for/in	IMGC	GABL	JILAg-5	FG5X-221
All stations	31.39 ± 10.90	32.59 ± 7.36	6.76 ± 0.81	
All stations, not Metsähovi	33.56 ± 11.05	28.66 ± 10.83	8.99 ± 2.24	
All stations	34.55 ± 10.98	35.42 ± 7.45	7.66 ± 0.89	-1.41 ± 0.58
Olsson et al. 2019			7.74 ± 0.78	

Large significant offsets of more than 30 μGal were found for the IMGC and GABL instruments. For the JILAg-5 offsets between 6.76 ± 0.81 μGal and 8.99 ± 2.24 μGal were found. The JILAg-5 offset of 7.66 ± 0.89 μGal determined in Olsson et al. (2019) falls in between these values. In the solution containing all stations including Metsähovi, the large amount of data in Metsähovi dominates the solution. Therefore, also a calculation was made where the Metsähovi data was left out. It resulted in a larger offset value of 8.99 ± 2.24 μGal for the JILAG-5. Nevertheless, the solution including also Metsähovi was preferred, because of the lower uncertainty value for the JILAg-5 offset. When also an offset was estimated for the FG5X-221 instrument, an offset of -1.41 ± 0.58 μGal was obtained. Although this offset is significant with respect to its own uncertainty, it is smaller than the uncertainty of single FG5X observations.

Also no group effects can be seen in the international comparisons where both FG5s and FG5Xs were present and the Fg5X-221 shows no consistent offset between the comparisons. Therefore, at this stage, the FG5X offset was not taken into account.

To conclude: the combined trend solutions result in offsets of 31.4 μGal, 32.6 μgal and 6.8 μgal for the IMGC, GABL and JILAG-5 instruments, respectively.

4.5 Research question 5. Land-uplift mechanism

What do the time series of absolute gravity in Finland tell us about the relationship between postglacial rebound induced gravity change rates and vertical velocities?

In Publication 5 trends are determined from the time series of absolute gravity data in Finland and the ratio between the gravity trends and land-uplift rates is investigated. In the following sections first the gravity trends that were found are discussed and then the ratios found will be presented.

4.5.1 Absolute gravity trends

Trends were estimated for 12 stations (Figure 8) in Finland in two ways. The first estimations were performed in a station-wise adjustment using observations of all instruments or only the observations of the JILAg and/or FG5(X) instruments of one station at a time. The second estimation was a combined adjustment, where trends for all stations and offsets of the instruments other than FG5(X) were estimated in one solution.

The trends of five stations were found not suitable for further analysis. Out of these, the stations Kilpisjärvi, Kivetty, Oulu and Savukoski do not have enough observations yet to result in reliable trends. The Kevo station has a long time series, but shows a trend that is much lower than expected based on the land-uplift figures. The reason can be found in the complex hydrological setting of the station. Also at other stations, such as Joensuu, effects of changes in local hydrology between years are visible, but time series are long enough for extreme hydrology events to show up as outliers.

Publication 5 showed that the trends of the seven stations in the final solution have stabilized in time. Trends were found to stabilize between 15 to 20 years for the trend of JILAg and FG5(X) combined with an offset applied for the JILAg-5 data, and generally within 10 years when the trends were derived from FG5(X) data alone.

Table 10 shows the results for the combined solution where all data is included and offsets were used for the instruments other than FG5(X). The results in the shaded rows are left out of the final solution.

Table 10. Gravity trends $\dot{g}$ in µGal/yr. $\dot{g}_{NKG}$ are derived from the NKG2016LU_gdot land-uplift model (Olsson et al. 2019). $\dot{g}$ were estimated in a combined adjustment, where all station trends and offsets of instruments other than FG5(X) were estimated in one solution. $\Delta\dot{g}$ and δ are differences and relative errors with respect to the $\dot{g}_{NKG}$ values. ΔT and # are the time span and the number of the observations used in the trend estimation. Shaded rows are not included in the final solution.

Station	Instrument	$\dot{g}_{NKG}$	ΔT	#	$\dot{g}$	$\Delta\dot{g}$	δ
Joensuu	JILAg & FG5(X)	-0.64 ± 0.12	16.7	10	-0.58 ± 0.20	0.06 ± 0.23	-9%
Kevo	FG5(X)	-0.68 ± 0.12	12.1	8	-0.22 ± 0.24	0.46 ± 0.27	-68%
Kilpisjärvi	FG5X	-0.76 ± 0.13	4.0	3	-1.38 ± 0.77	-0.62 ± 0.78	82%
Kivetty	FG5X	-1.13 ± 0.15	3.7	3	-0.69 ± 0.88	0.44 ± 0.89	-39%
Kuusamo AB	FG5(X)	-1.16 ± 0.15	11.0	5	-1.06 ± 0.29	0.10 ± 0.33	-9%
Metsähovi AB	all	-0.73 ± 0.12	39.4	314	-0.63 ± 0.03	0.10 ± 0.13	-14%
Oulu	FG5X	-1.42 ± 0.17	4.0	3	-2.28 ± 0.81	-0.86 ± 0.83	61%
Savukoski	FG5X	-0.99 ± 0.15	5.1	3	-1.77 ± 0.64	-0.78 ± 0.66	79%
Sodankylä	all	-1.21 ± 0.16	43.0	16	-1.26 ± 0.13	-0.05 ± 0.21	4%
Vaasa AA	JILAg & FG5(X)	-1.53 ± 0.18	31.2	18	-1.70 ± 0.10	-0.17 ± 0.21	11%
Vaasa AB	JILAg & FG5(X)	-1.51 ± 0.18	21.0	17	-1.61 ± 0.14	-0.10 ± 0.23	7%
Virolahti	JILAg & FG5(X)	-0.52 ± 0.11	18.0	4	-0.58 ± 0.38	-0.06 ± 0.40	12%

All trends of the final solution shown in Table 10 are in agreement with the uplift model when the uncertainties of the trends of the uplift-model as well as of the estimated trends are taken into account. Although the differences of the estimated trends at the Vaasa stations agree with the uplift model considering their uncertainties, the estimated trends at these stations are larger than the trends of the uplift model by 0.17 and 0.10 µGal/yr. Several possible reasons for the difference are discussed in Publication 5, but all of them predict differences to the standard uplift models smaller than the differences found.

4.5.2 Ratio between gravity change rates and land uplift rates

The ratio, $\dot{g}/\dot{h}$, was calculated using the gravity change rates of the final solution in section 4.5.1 and the uplift rates from the NKG2016LU_abs land uplift model (Vestøl et al. 2019) and the GNSS datasets of Kierulf et al. (2014), Vestøl et al. (2019), and Lahtinen et al. (2019). The calculation method is described in section 3.2.6. A summary of the resulting ratios is given in Table 11 together with the ratios that were found by others.

The ratio solutions found in this study are best when the intercept is not fixed, with goodness-of-fit values, q, between 0.7 and 1. The best fit, with q = 0.980, was obtained with the vertical velocities of the NKG2016LU model. It resulted in a $\dot{g}/\dot{h}$ ratio of -0.211 ± 0.019 µGal/mm and $\dot{g}_{\dot{h}=0}$ intercept value of 0.309 ± 0.116 µGal/yr (see Figure 15). The second best fit, with q = 0.959, a $\dot{g}/\dot{h}$ ratio of -0.206 ± 0.017 µGal/mm and $\dot{g}_{\dot{h}=0}$ intercept value of 0.248 ± 0.089 µGal/yr was obtained with the GNSS rates of Lahtinen et al. (2019).

Table 11. Relationship between gravity rates and vertical rates found for Fennoscandia in the literature and in Publication 5 of this thesis. $\dot{g}_{\dot{h}=0}$ is the intercept where $\dot{h} = 0$.

Publication	$\dot{g}_{\dot{h}=0}$ (µGal/yr)	$\dot{g}/\dot{h}$ (µGal/mm)	Input data	Time span	Area
Mäkinen et al. (2005)	forced through origin	-0.16 ± 0.04 … -0.18 ± 0.06	relative gravity lines, precise levelling, continuous GPS	1966-2003	Finland 63° line
Timmen et al. (2012)	forced through origin	-0.163 ± 0.020	Absolute gravity, continuous GPS	2003-2008	Fennoscandia 10 stations
Olsson et al. (2019)	forced through origin	-0.163 ± 0.005 -0.164 ± 0.006	Absolute gravity, NKG2016LU_abs	1995-2015	Fennoscandia 21 stations
Publ. 5	forced through origin	-0.170 ± 0.009 … -0.180 ± 0.010	Absolute gravity, NKG2016LU_abs, continuous GNSS	1976-2019	Finland 7 stations
Ophaug et al. (2016)	−0.097 ± 0.196 … −0.210 ± 0.183	-0.133 ± 0.030 … -0.167 ± 0.045	Absolute gravity, empirical land uplift model	1993-2014	Norway 10 stations
Olsson et al. (2015)	0.030	-0.163 ± 0.016	Geophysical GIA model		Fennoscandia
Olsson et al. (2019)	0.04 ± 0.12 … 0.14 ± 0.14	-0.167 ± 0.020 … -0.177 ± 0.013	Absolute gravity, NKG2016LU_abs, continuous GNSS	1995-2015	Fennoscandia 21 stations
Publ. 5	0.248 ± 0.089 … 0.335 ± 0.136	-0.206 ± 0.017 … -0.227 ± 0.024	Absolute gravity, NKG2016LU_abs, continuous GNSS	1976-2019	Finland 7 stations

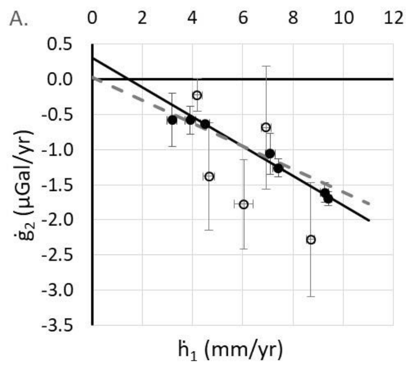

Figure 15. Observed gravity trends plotted versus uplift rates from the NKG2016LU_abs land uplift model (Vestøl et al. 2019). Closed dots represent the stations of the final $\dot{g}_2$ solution and open dots the other stations that were not used in the ratio estimation. The linear relation between the $\dot{g}_2$ gravity rates and the $\dot{h}_1$ vertical rates, the solution for $\dot{g} = a + b\,\dot{h}$, is shown calculated with the σ_1 uncertainties and the intercept $\dot{g}_{\dot{h}=0}$ estimated (solid line). For comparison, the modelled relationship, $\dot{g} = 0.030 - 0.163\,\dot{h}$, of Olsson et al. (2015) is also plotted (dashed line).

When the intercept $\dot{g}_{\dot{h}=0}$ is estimated together with the ratio, the ratios obtained in Publication 5 are all higher than those found by others for Fennoscandia (see Table 11, lower half). Also the intercept values are significantly non-zero. When no intercept value is estimated, forcing the slope to go through the origin, $\dot{g}_{\dot{h}=0} = 0$, smaller ratios are found and they are in agreement with the ratios found in the literature (Table 11, upper half). However, these solutions with a forced zero-intercept are less good, with goodness of fit values, q, between 0.520 and 0.747. The gravity data in Finland favours a non-zero intercept and larger ratios between the gravity change rates and the vertical velocities.

For the linear relationship a non-zero intercept is predicted based on GIA modelling (Olsson et al. 2015), however the intercept values found in Publication 5 are ten time larger than the intercept value of Olsson et al. (2015).

A misalignment of the origins of the reference frames related to the vertical velocities and the centre of mass will translate in to a non-zero intercept. However, for the reference frames related to the vertical velocities used in Publication 5 (ITRF2008 and ITRF2014, see Table 1) no misalignment with the centre of mass exists according to Mazzotti et al. (2011), Altamimi et al. (2011) and Altamini (2016). Also, the uncertainties of the origin motion found by Wu et al. (2011) and Riddell et al. (2017) are smaller than the intercept values found in Publication 5 (see Table 11). In North Amerika no evidence was found for misalignment of the ITRF2005/2008 origin based on an analysis of absolute gravity and GNSS time series by Mazzotti et al. (2011) and Lambert et al. (2013).

The uncertainties of the GNSS velocities should be considered as well, as they may be underestimated. Increasing these uncertainties did not change the values of the ratios and the intercepts, but their uncertainties got larger. When the uncertainties of the GNSS velocities were doubled, the uncertainties of the intercepts grew so that the intercept values were no longer significant.

5. Discussion

5.1 Scientific implications

In Publication 1, the accuracy of the FIN2005N00 model was determined to be 2 cm based on cross validation. At sea the model agreed with GNSS-derived geoid heights within 5 cm (Publication 3), which is considered to be the desirable accuracy level at sea by the FAMOS Project (FAMOS Consortium. 2014–2017).

The quasi-geoid models calculated in Publication 2 showed, after removal of an offset and tilt, accuracy levels a little poorer than those of the FIN2005N00 model, but were at the same level as the NKG2004 model after removal of an offset and tilt in the test calculations of Publication 1. Therefore it is expected that, would these models be fitted to the same GNSS-levelling data as in Publication 1, the resulting model accuracies would be at the same level as the FIN2005N00. No noticeable improvement would probably be obtained.

Publication 2 was the start for more investigations studying the impact of the new global gravity models that included more and more gravity satellite data. For example Saari and Bilker-Koivula (2015) evaluated GOCE-based Global Geoid Models using Finnish GNSS-levelling and gravity data. The Finnish gravity data, as well as the data of the other Nordic and Baltic countries, was for the calculations of the NKG2015 geoid model transformed to epoch 2000.0 and the zero tide system. When this data was used together with the high resolution global model EIGEN6C4, a standard deviation of 1.8 cm was obtained when comparing the resulting quasi-geoid model, after removing an offset with the EUVN-DA GPS-levelling data (Saari and Bilker-Koivula 2017). This is already at the level of the NKG2005N00 model or even a little better. It is also slightly better than the NKG2015 model that has after removing a tilt a standard deviation of 2 cm when compared with the same GPS-levelling dataset.

The least-squares collocation in combination with cross-validation proved to be a suitable way for fitting a gravimetric geoid model to the national height systems in Finland (Publication 1). It should also be applied in the future when new geoid models are calculated for Finland.

The method for validation of geoid models with GNSS observations at sea proved to be successful in Publication 3 and was already implemented in combination with a next marine gravity campaign in the Eastern part of the Gulf of Finland (Saari et al. 2020).

No conclusions could be made regarding offsets between the absolute gravimeters that have measured in Finland based on the ICAGs, ECAGs, and bilateral comparisons. In the trend calculations an offset of 6.76 ± 0.81 µgal was found for the JILAg-5 instrument. This offset can now also be used for the JILAg-5 measurements at other stations in the Fennoscandian area. It is then possible to include these JILAg-5 measurements in the time series. Especially the stations in the Baltic countries, that have relatively few observations, will benefit from offset-corrected measurements that will improve their time series. As the Baltic countries are close or on the border of the land-uplift area, good time series for their stations are essential for the determination of the intercept values in the relation between gravity change rates and vertical velocities.

Based on seven stations in Finland, Publication 5 found $\dot{g}/\dot{h}$ ratios and $\dot{g}_{\dot{h}=0}$ intercept values that are larger than values found by Ophaug et al. (2016), based on Norwegian data, and Olsson et al. (2019), based on 21 stations over the whole Fennoscandian area. In the final solution of Olsson et al. (2019) 22 out of 43 absolute gravity stations were not included and in Publication 5, 5 out of 12 stations were left out. Once more stations become available with long and high-quality time series, more insight can be gained on the relation between the gravity change and land-uplift rates.

If, in the future, the larger ratios and intercept values are confirmed by new data, it may indicate that there may be an additional gravity change due to mass movements that are not related to the vertical movements associated with glacial isostatic adjustment. We may also have to look at more refined models of GIA.

5.2 Practical implications

The FIN2005N00 height conversion surface, calculated in Publication 1, is the official national geoid model for Finland. It was widely taken into use in all GNSS-based surveying, where heights are needed in the National height system N2000. The model is implemented in many commercial GNSS measurement and processing software. A technical report was written giving information of the model and giving advice and examples how to use it (Bilker-Koivula and Ollikainen, 2009).

Publication 3 showed not only that it is possible to validate geoid models by GNSS measurements at sea, but also that at the Bothnian Sea the geoid models FIN2005N00 and NKG2015 have an accuracy of better than 5 cm. For mariners this also implies that GNSS height determination at sea would be possible with that accuracy.

The absolute gravity stations and measurements described in Publication 5 will contribute to the International Gravity Reference Frame (IGRF) of the International Association of Geodesy (IAG) (Wziontek et al. 2020). All the absolute gravity stations in Finland will together form a high-quality national network, while the Metsähovi station is intended to be a reference station as well as a comparison site of the IGRF.

5.3 Reliability and validity

At the time the FIN2005N00 height conversion surface was calculated in Publication 1, a limited set of high-accurate GPS-levelling data was available. In Publication 2 an additional dataset (NLS) with varying quality was available for validation of the geoid models calculated in that publication. When the FIN2005N00 model was afterwards tested with this NLS dataset, it was found that, based on that dataset, the accuracy of the model was 3 cm on average for the whole country, but close to 2 cm in the Southern half of the country and close to 4 cm in the Northern part. One must, however, take into account that these accuracy values not only describe the accuracy of the geoid model, but also contain the uncertainties of the GNSS and levelling data. The accuracy of GNSS and levelling data can easily reach values of 1 cm or more when measurement classes higher than one are considered. In GNSS, especially the height is the weakest component of the GNSS coordinates.

Our method for geoid validation at sea was performed in the Bothnian Sea, but the method can be applied anywhere in the Baltic Sea. The Baltic Sea is a well monitored closed sea area, with tide gauges all around and the Baltic Sea physics analysis and forecast-model (CMEMS 2016) is available for the whole area. In fact, the method can be applied on any well monitored sea area. In the open oceans, it would be more difficult, as the modelling of the absolute dynamic topography would be a problem. Also, tides should be considered. The Baltic Sea has a very small tidal signal and sea level variations are dominated by other phenomena such as winds and salinity. The tide gauge observations and sea surface models include also the small tidal effect. In other areas tides may have to be modelled. The method also requires knowledge of the internal coordinates and static and dynamic draft of the vessel. The test of Publication 3 was done with a surveying vessel, which has all these parameters standardly available. But for most vessels, this is not the case.

At 7 out of the 12 stations in Publication 5 we believe to have reliable absolute gravity trends. The trends were found to stabilize between 15 to 20 years for the trends of JILAg and FG5(X) combined with an offset applied for the JILAg-5 data, and generally within 10 years when the trends were derived from FG5(X) data alone. We did not correct the time series for any seasonal effects from hydrology, although at some stations we can see from groundwater readings that the there is a correlation between groundwater level and the variations in the gravity signal. As is shown, for example, by Ophaugh et al. (2016), Lambert et al. (2006) and Mikolaj et al. (2015) the variation in time series can be reduced by correcting the time series for hydrological signals. It is to be expected that the convergence of a time series to a stable trend goes faster when the hydrological signal is removed.

The absolute gravity results presented in Publication 5 cover only part of the uplift area. There were stations close to the centre of the uplift area, but none close to the border. The data must be combined with data from other countries, like it was done in Olsson et al. (2019), to get the whole picture.

5.4 Recommendations and further research

The height conversion surface and quasi-geoid models calculated in Publications 1 and 2, and also later models calculated for Finland in Saari and Bilker-Koivula (2017) do not yet reach the 1 cm accuracy level that is set for Finland in Poutanen et al. (2017). To reach the goal of a 1 cm geoid model, the following studies could be conducted:

First, the currently available models for Finland should be tested using the new GNSS-levelling dataset, containing 100 points, that was measured for this purpose by the National Land Survey of Finland in 2016-2017. A new conversion surface for the country can then be calculated as was done in Publication 1 using least-squares collocation and cross-validation.

Second, it would be good to study if it is, in theory, even possible to calculate a 1 cm geoid model for Finland based on the current datasets. Requirements for the datasets in Finland could be determined, e.g. like it was done in Farahani et al. (2017) for the Netherlands and in Ågren and Sjöberg (2014) for Sweden.

It could be worth looking again at the simultaneous bi-lateral comparisons of Publications 4 and 5, but taking the uncertainties of the measurements into account when calculating the difference between the instrument results. Also, the difference between the instruments could be given as a combined outcome of the measurements on both pillars, instead of providing the outcome for each pillar separately as was done in Publications 4 and 5. In an additional test offsets could be estimated for different FG5(X) instruments in the combined trend adjustment.

At several absolute gravity stations variations in gravity may be explained by changes in local hydrology. An attempt should be made to model the hydrological signal and correct the gravity time series. A special case is Metsähovi, where a lot could be gained by the combined analysis of absolute gravimeter and superconducting gravimeter observations. Also gravity changes caused by geophysical processes other than the post glacial rebound should be studied.

And last, but not least, repeated absolute measurements should be continued in Finland as well as in the other Nordic and Baltic countries to increase the number of stations with long and good time series and strengthen the solution of the relationship between gravity change rates and land uplift vertical velocities.

6. Summary

This thesis gave answers to five research questions. The first three questions dealt with geoid models in Finland and were answered in Publications 1 to 3:

1. What is the best way of fitting the NKG2004 geoid model to GNSS-levelling data to establish a conversion surface for the transformation between EUREF-FIN ellipsoidal heights and N2000 normal heights?
2. Can the geoid model for Finland be improved using the high-resolution, partly satellite-based, global gravity field models EGM2008 and EIGEN6C?
3. Can a geoid model be validated by GNSS measurements at sea?

Publication 1 found that the best way to fit the NKG2004 geoid model to GNSS-levelling data in Finland was by least-squares collocation in combination with cross-validation. The outcome was the height conversion surface FIN2005N00. It is currently the official geoid model for Finland and used widely in applications where transformations are needed between EUREF-FIN heights, as measured by GNSS, and N2000 heights, as measured by levelling. The model was found to have an accuracy of 2 to 3 cm on land and better than 5 cm at sea.

In Publication 2, the high-resolution global gravity field models were successfully used in quasi-geoid calculations for Finland. The resulting models performed better than earlier gravimetric geoid models available for Finland. In the comparisons with GPS-levelling data the differences with other models disappeared after correcting for an offset and tilt. Therefore, no new height conversion surface was calculated from these new gravimetric geoid models.

Question 3 was answered in Publication 3, where a method was developed for geoid validation at sea using GNSS measurements collected on a vessel. The method was successful and proved that it is possible to validate geoid models at sea. The following things should be taken into account: the reliability of coordinates of the permanent GNSS base stations, the quality of the GNSS/IMU data and trajectory solutions, the reference frame and epochs related to the geoid models, the quality of the internal coordinate system of the vessel, the accuracy of the height transfers to sea level, which include information on the static draft, the squat and pitch and roll, and the quality of the sea-surface modelling.

The last two questions related to gravity change and land uplift in Finland and were dealt with in Publications 4 and 5:

4. What is the magnitude of possible offsets between the absolute gravimeters that have been used for measurements in Finland?
5. What do the time series of absolute gravity in Finland tell us about the relationship between postglacial rebound induced gravity change rates and vertical velocities?

The results of international comparisons and bi-lateral comparisons of absolute gravimeters were analysed in Publications 4 and 5. Based on these comparisons no conclusions could be made on offsets between instruments that have measured in Finland. Offsets for instruments were also estimated as part of the trend calculations in Publication 5. Significant offsets of 31.4 ± 10.9 μGal and 32.6 ± 7.4 μGal were found for the IMGC and GABL instruments. For the JILAg-5 instrument, that did the majority of the measurements between 1988 and 2002, an offset of 6.8 ± 0.8 μGal was determined. These values were used in the further time series analysis that were done to answer research question 5.

For seven out of 12 stations stable trends were derived from the absolute gravity time series. Statistically these trends were in agreement with the NKG2016LU_gdot uplift model. However, the model seems to underestimate the trend in the Vaasa area. The trend from FG5(X) data was found to stabilize generally within 10 years. Ratios between -0.206 ± 0.017 and -0.227 ± 0.024 μGal/mm and axis intercept values between 0.248 ± 0.089 and 0.335 ± 0.136 μGal/yr were estimated for the relationship between gravity change rates and vertical velocities. These values are larger than those found by others. The axes intercept values are extrapolated, because no point in Finland is close to the border of the uplift area. More robust solutions the Finnish part of the land uplift will be obtained in the future when more data becomes available at more stations in Finland. It is also foreseen that, when more stations with high-quality long time series become available for the whole of the Fennoscandian land uplift area and its borders, it will be possible, together with a new refined NKG semi-empirical land uplift model, to better explain the findings of Publication 5.

In the geoid-related part of the thesis important knowledge was gained on geoid modelling using high resolution global gravity field models, geoid model validation and fitting of a gravimetric geoid model to the national reference systems for heights in Finland. The knowledge obtained will be of benefit in the determination of the next generation geoid models and height conversion surfaces for Finland.

In the second part, dealing with gravity and land uplift, larger values than expected were found for the relationship between gravity change rates and vertical velocities. More GIA-related studies in addition to high-quality longer time series from more stations in Finland, as well as the whole of the uplift area and its boundaries, are needed before clear conclusions can be drawn from these results.

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