Turbo Equalization with Low Complexity Decoder

Markku Pukkila[1] and Jan C. Olivier[2]

[1]Nokia Research Center, P.O. Box 407, FIN-00045 Nokia Group, Finland
[2]Nokia Research Center, 6000 Connection Drive, Irving, TX 75039, USA
E-mail: {markku.pukkila; jan.olivier}@nokia.com

***Abstract* - Turbo equalization (TE) is an iterative receiver algorithm repeating equalization and channel decoding to improve receiver performance. As equalization and decoding are performed several times, the complexity of the TE receiver increases respectively. Furthermore, the decoder needs to provide probability information as output, which increases the computational burden even more. In this paper we introduce a new method to reduce the decoding complexity of the TE receiver. This soft trellis decoding (STD) technique utilises information obtained from the previous decoding step to select those trellis metrics that need to be recalculated. Hence, the most unlikely trellis branches are eliminated. We evaluate STD performance in the Enhanced General Packet Radio System (EGPRS) platform by simulations. We show that STD reaches performance close to full trellis decoding, but requires only a fraction of the computational power.**

I. INTRODUCTION

The TE method performing iteratively equalization and channel decoding was introduced in [2] soon after the famous turbo coding principle was published [1]. The EGPRS system is an interesting application for TE due to significant performance gain as shown in [3-6], suitable frame structure and rectangular interleaving. However, TE increases complexity due to the repeated equalization and decoding processes.

In this paper we propose a novel STD method to decrease the decoder complexity in the TE receiver. The number of states is reduced adaptively, which leads to smaller number of branch metrics to be calculated and thereby to lower receiver complexity. Our method resembles adaptive T-algorithm presented in [7], but we exploit soft values from the previous TE iteration instead of short term channel impulse response (CIR) that is used in T-algorithm. Based on the soft values STD selects, which branches are computed in trellis search and which can be neglected as highly improbable.

There are several low complexity equalizers available, e.g., decision feedback equalizer (DFE) [8], decision feedback sequence estimation (DFSE) [9] or reduced-state sequence estimation (RSSE) [10]. All of these algorithms can be used as a part of the TE receiver. Our objective is to present such TE receiver that maintains good iterative performance gain, but meets low computational requirements.

We apply STD for the EGPRS system and evaluate the performance of the reduced complexity TE scheme. Moreover, we analyse the STD complexity compared to full trellis decoding during the TE iterations.

The paper is organised as follows. After the introduction the transmission system model is presented in Chapter II. The TE scheme is discussed in Chapter III where the turbo principle in general and related receiver algorithms are presented. In Section IV the STD technique is discussed in detail. After that simulation results are given and finally some conclusions are drawn.

II. SYSTEM MODEL

The transmission system in this paper follows the EGPRS platform, which is modelled in Fig. 1. In the transmitter side a block of data bits $\mathbf{u}$ is protected by a convolutional encoder and punctured to desired data rate. These bits $\mathbf{c}$ are interleaved over four successive transmission bursts and grouped into 8-PSK symbols. Every three bits form a symbol $\mathbf{a}_k = (a_{k,1}, a_{k,2}, a_{k,3})$ and symbols are organised as bursts $\mathbf{a} = (\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_K)^T$. Moreover, the 8-PSK modulator associates a complex-valued symbol z_k for each input symbol $\mathbf{a}_k$.

The modulated signal is transmitted over a frequency selective fading channel and thermal noise at the receiver is modelled as additive white Gaussian noise (AWGN).

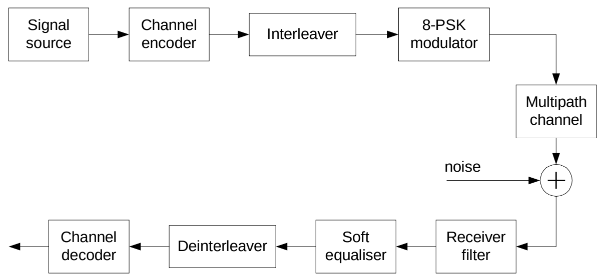

Fig. 1. Transmission system model.

The received signal **r**, which is sampled at the symbol rate, is given as

$$\mathbf{r} = \mathbf{Z}\mathbf{h} + \mathbf{w} \,, \tag{1}$$

where symbol matrix **Z** is defined as

$$\mathbf{Z} = \begin{bmatrix} z_{L+1} & z_L & \cdots & z_1 \\ z_{L+2} & z_{L+1} & & z_2 \\ \vdots & & \ddots & \vdots \\ z_{K+L} & z_{K+L-1} & \cdots & z_K \end{bmatrix} \tag{2}$$

and CIR is represented by a tapped delay line $\mathbf{h} = (h_0, h_1, \ldots, h_L)^T$. The white Gaussian noise samples having variance $\sigma^2 = N_0/2$ are denoted by **w**.

III. TURBO EQUALIZATION

A. Principle of Turbo Equalization

Fig. 2 illustrates the iterative receiver structure, which is employed in this paper. The equalizer calculates log-likelihood values $\lambda_{eq}(\mathbf{a})$ using a priori values $\lambda^a(\mathbf{a})$ originated from the previous iteration. During the initial iteration we set $\lambda^a(\mathbf{a}) \equiv 0$, since there is no a priori information available yet.

During metrics calculation the actual a priori probabilities $\Pr(a_{k,j})$ for each transition are extracted from the log-likelihood ratio

$$\lambda^a(a_{k,j}) = \ln \frac{\Pr(a_{k,j} = 0)}{\Pr(a_{k,j} = 1)} \,. \tag{3}$$

The equalizer output $\lambda_{eq}(\mathbf{a})$ consists of intrinsic and extrinsic information. The latter is the incremental information obtained in the equalization and it is extracted from the output as follows [2]

$$\lambda_{eq}^{ext}(a_{k,j}) = \lambda_{eq}(a_{k,j}) - \lambda^a(a_{k,j}) \,. \tag{4}$$

The extrinsic information is deinterleaved to achieve a priori information $\lambda^a(\mathbf{c})$ on the coded data. These values are provided for the Soft-in-Soft-out (SISO) channel decoder, which calculates soft outputs $\lambda_d(\mathbf{c})$ for the coded bits. The feedback into the equalizer contains only incremental information that is obtained from the surrounding bits in the channel decoding. This extrinsic information is obtained as [2]

$$\lambda_d^{ext}(c_{k,j}) = \lambda_d(c_{k,j}) - \lambda^a(c_{k,j}) \,. \tag{5}$$

The turbo equalization technique is based on utilising the extrinsic information at the subsequent iteration round [2]. Thus it is interleaved and provided for the equalizer as a priori information $\lambda^a(\mathbf{a})$ on the bit reliabilities. As there is now new information available in the detection, more reliable decisions are achieved. At the final stage, the SISO decoder is not any more needed, since only hard decisions $\hat{\mathbf{u}}$ on the information bits are of interest.

B. Soft-in-Soft-out Decoder

Since the feedback information consists of probabilities, SISO decoder is needed. A suitable algorithm is BCJR-max-log-MAP, which provides a posteriori probability (APP) information for each bit [11,12]. The state probability for trellis state *s* at time *k* in the forward direction is denoted by

$$\alpha_k(s) = p\left(s, \mathbf{r}_{j \le k}\right) \tag{6}$$

and in the backward direction by

$$\beta_k(s) = p\left(\mathbf{r}_{j>k} \middle| s\right) \,. \tag{7}$$

The log-probability for the transition between states *s'* and *s* is given as

$$\ln \gamma_k(s', s) = \sum_{j=1}^{M} \lambda^a\left(c_{k,j}\right) c_{k,j} \,, \tag{8}$$

assuming coding rate of 1/*M*. Using the given definitions the output of BCJR-max-log-MAP is the following [12]

$$\begin{aligned} \lambda_d\left(c_{k,j}\right) = & \max_{\substack{(s',s) \\ c_{k,j}=+1}} \left\{ \ln \alpha_{k-1}(s') + \ln \gamma_k(s', s) + \ln \beta_k(s) \right\} \\ & - \max_{\substack{(s',s) \\ c_{k,j}=-1}} \left\{ \ln \alpha_{k-1}(s') + \ln \gamma_k(s', s) + \ln \beta_k(s) \right\} \end{aligned} \tag{9}$$

where the forward recursion gives

$$\ln \alpha_k(s) = \max_{s'} \left\{ \ln \gamma_k(s', s) + \ln \alpha_{k-1}(s') \right\} \tag{10}$$

and the backward

$$\ln \beta_k(s') = \max_{s} \left\{ \ln \gamma_{k+1}(s', s) + \ln \beta_{k+1}(s) \right\}. \tag{11}$$

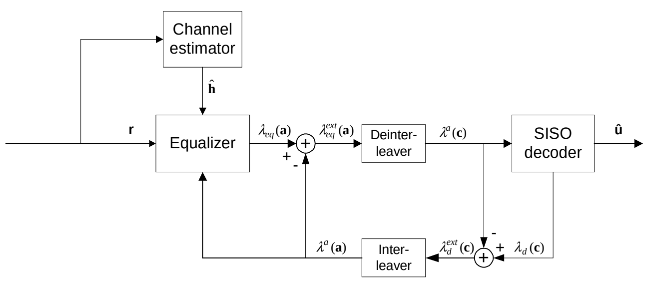

Fig. 2. Turbo equaliser structure.

IV. Soft Trellis Decoding

The complexity reduction of STD is based on a priori knowledge on the trellis transition probabilities in decoding. As transitions are related to the information bits **u**, the STD module needs a priori input $\lambda^a(\mathbf{u})$ and it provides a posteriori output $\lambda_d(\mathbf{u})$, which will be used in the next iteration. Moreover, STD uses information related to coded bits, i.e., $\lambda^a(\mathbf{c})$ and $\lambda_d(\mathbf{c})$, like the SISO decoder presented in Ch. III. The inputs and outputs of STD are illustrated in Fig. 3.

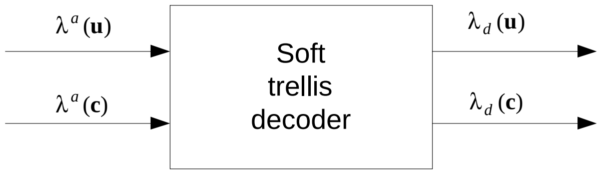

Fig. 3. Input/output model of soft trellis decoder.

STD uses the a priori knowledge on the information bits

$$\lambda^a(u_k) = \ln\frac{\Pr(u_k = 1)}{\Pr(u_k = 0)} \qquad (12)$$

to adjust the trellis size accordingly. Since the TE receiver performs multiple decoding steps, this a priori information can be obtained from the previous decoding step. Mathematically, a priori information at iteration round $n+1$ is given by

$$\lambda^{a,(n+1)}(\mathbf{u}) = \lambda_d^{(n)}(\mathbf{u}) \quad , \qquad (13)$$

where $\lambda_d^{(n)}(\mathbf{u})$ denotes the decoder soft output for the information bits at n^{th} iteration. This soft information is defined as follows

$$\lambda_d(u_k) = \ln\frac{\Pr(u_k = 1 \,|\, \hat{\mathbf{c}})}{\Pr(u_k = 0 \,|\, \hat{\mathbf{c}})} \quad . \qquad (14)$$

From (12) we extract the actual a priori probabilities

$$\Pr(u_k = 1) = \frac{\exp(\lambda^a(u_k))}{1 + \exp(\lambda^a(u_k))} \qquad (15)$$

and

$$\Pr(u_k = 0) = \frac{1}{1 + \exp(\lambda^a(u_k))} \quad . \qquad (16)$$

The STD algorithm requires a predetermined threshold probability δ, by which we can control the trade-off between receiver performance and complexity reduction. Once a priori probability exceeds the threshold δ, i.e., $\Pr(u_k = i) > \delta$, only transition metrics $\gamma_k(u_k = i)$, which correspond to bit $u_k = i$, need to be calculated and the other metrics $\gamma_k(u_k \neq i)$ are neglected as highly improbable. Furthermore, if during the previous time instants some transitions are neglected, the starting state in forward direction $\alpha_k(s)$ or in backward direction $\beta_k(s')$ could have probability zero and hence there is no need to compute the outgoing transition metrics $\gamma_k(s, s')$. Equivalently, we may give the metric value $-\infty$ for a neglected transition in the max-log-MAP decoder and summarise STD rules as follows:

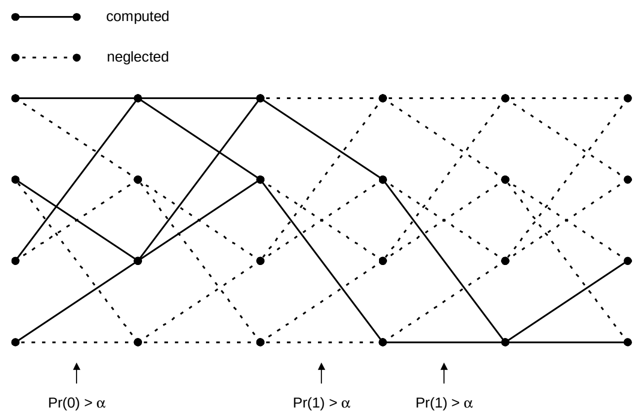

Fig. 4. Example of STD trellis structure, only part of transition metrics are calculated.

$$\gamma_k(s, s') = \gamma_k(u_k = i) = -\infty \ \text{ if } \begin{cases} \Pr(u_k = i) < 1 - \delta \\ \quad OR \\ \alpha_k(s) = -\infty \\ \quad OR \\ \beta_k(s') = -\infty \end{cases} \quad . \qquad (17)$$

Fig. 4 shows an example of trellis structure, where highly improbable bits are encountered at three different stages. STD calculates only metrics related to solid lines and dashed lines are eliminated without any computation. In this example only fraction of the original metrics is recalculated. The reduction depends both on the signal quality and the threshold δ. The decoder complexity reduces even further in the following iterations, since SNR improves during the TE process. The threshold can be set beforehand and it can be fixed for various situations. By adjusting threshold properly we can find a reasonable compromise between receiver performance and complexity reduction.

V. Performance Results

TE performance exploiting STD is evaluated in the EGPRS system using 8-PSK modulation. We use the strongest MCS-5 coding scheme, which has the coding rate of 0.37. DFSE equalizer is used and Typical Urban channel profile with speed of 3-km/h (TU3) and ideal frequency hopping are assumed. At the first iteration a conventional max-log-MAP decoder is applied and during further iterations low-complexity decoder is used. The first iteration denotes conventional equalization and decoding, i.e., there is not yet any feedback information available. STD utilises a fixed threshold during all simulations.

Fig. 5 shows block error rates (BLER) for TE receiver after iterations 1-4 using STD (asterisk) or full trellis decoder (line). The STD technique achieves performance, which is very close to full trellis. Only small degradation is observed with low BLER levels, but at the interesting range around BLER 10^{-1} there is no practical degradation due to complexity reduction. Equivalent performance after first iteration is achieved, since STD performs full trellis calculation as well.

Fig. 6 describes computational efforts of STD during the different iterations. The curves show average amount of metrics that STD computes compared to full trellis, e.g., value 0.2 corresponds to 20 % of the full trellis decoder complexity. The complexity depends heavily on the signal quality and iteration number. In the presence of low E_b/N_o STD has to recalculate more trellis paths, since only a few decisions are certain enough to exceed the STD threshold. Iterative processing improves signal quality; thus STD recalculates fewer paths when the next iteration is considered.

As a practical example, let us consider operation point of E_b/N_0 = 6 dB, which provides BLER under 10^{-1} for the TE receiver. The first iteration requires full metrics calculation in the decoder, but already the second iteration is reduced to 20 % and third and fourth iteration to 10 % of the original decoder complexity. Hence, TE with four iterations needs totally 1.4 times full max-log-MAP decoder complexity at E_b/N_0 6 dB. However, the total TE complexity increases also due to equalization at each iteration round.

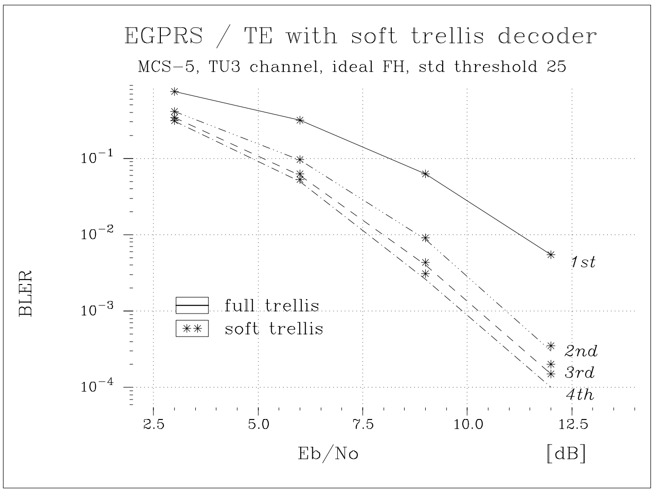

Fig. 5. Sensitivity in MCS-5.

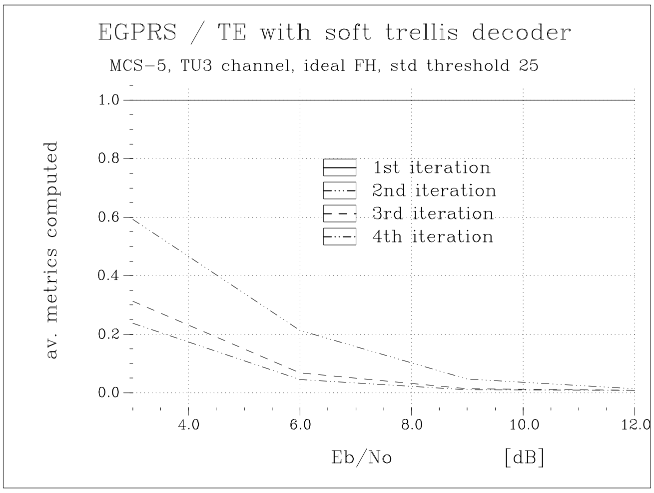

Fig. 6. Complexity of STD.

VI. Discussion

TE provides a significant performance gain in packet data applications, but at the cost of more complex receiver. The equalization and decoding are repeated several times and the decoder has to provide soft outputs. In this paper we propose a novel STD method to decrease the decoder complexity of TE receiver. Basically, STD utilises reliability information from the previous TE iteration to neglect some of the most unlikely trellis transitions and thereby reduces the number of the metrics calculations needed.

Simulations in EGPRS/MCS-5 system show a dramatic complexity reduction for TE receiver with a negligible performance loss compared to conventional channel decoder. The reduction is larger, when signal quality improves or further iteration round is considered. For example at the operation point E_b/N_0 = 6 dB only 1.4 times full trellis decoding complexity is needed to perform four STD decodings totally.

Since TE is based on the iterative use of both equalization and decoding, low-complexity algorithms for equalization are also worth considering for TE. Furthermore, the first iteration of STD spends a lot of processing power. Hence, the total complexity of TE may still be reduced.

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