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Degree of polarization in tightly focused optical fields

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We analyze the degree of polarization of random, statistically stationary electromagnetic fields in the focal region of a high-numerical-aperture imaging system. The Richards–Wolf theory for focusing is employed to compute the full 3×3 electric coherence matrix, from which the degree of polarization is obtained by using a recent definition for general three-dimensional electromagnetic waves. Significant changes in the state of partial polarization, compared with that of the incident illumination, are observed. For example, a wave consisting of two orthogonal and uncorrelated incident-electric-field components produces rings of full polarization in the focal plane. These effects are explained by considering the distribution of the spectral densities of the three electric field components as well as the correlations between them. © 2005 Optical Society of America

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1. INTRODUCTION

The state of polarization of tightly focused light exhibits a multitude of features that pertain to the three-dimensional (3D) character of the electromagnetic field. A striking peculiarity of such a focused wave field is the longitudinal electric field component, which may even exceed in strength the two transversal components.[1] Recently, tightly focused fields have found particular importance in, e.g., optical microscopy,[1,2] data storage,[3] and the use of optical tweezers for particle manipulation.[4]

The 3D profiles of both the intensity and the polarization of monochromatic (deterministic) light in the focal region have been the subject of intense study. The field distribution for focused, linearly polarized light was calculated already quite some time ago by Richards and Wolf, who found that the rotational symmetry of the intensity distribution is broken in the focal region.[5] More recently, the focusing of nonuniformly polarized optical beams has been examined both theoretically and experimentally.[6,7] It was shown that the shape of the focal spot can be controlled by using azimuthally or radially polarized illumination[8] and that the area of the focus may be extremely small when radial polarization is employed.[9] Other topics of recent research interest have been highly focused electromagnetic fields,[10] variation of the polarization state near the focus,[11] and focusing of electromagnetic beams into uniaxial and biaxial crystalline media.[12–14]

Although the focusing of monochromatic fields has been examined in detail, the analysis of randomly fluctuating focused fields has attracted less attention. Previous studies on the subject have commonly been performed within the framework of the scalar theory, thereby omitting the vectorial nature of light.[15–17] In this work, we investigate the focusing of uniformly partially polarized radiation and, in particular, the variation of the polarization properties of the light field in the focal region. As the degree of polarization, we employ the definition put forward in Ref. 18; it is suitable for dealing with 3D nonparaxial fields such as those encountered in the image space of a high-numerical-aperture (high-NA) imaging system.

The paper is organized as follows. In Section 2, we present the theory for the calculation of the electric spectral coherence matrix in the image space. The behavior of the 3D degree of polarization in the focal region is then analyzed and discussed in Section 3. The main conclusions are summarized in Section 4.

2. THEORY

The degree of polarization is a quantity that reflects the correlations between the electric field components of a fluctuating electromagnetic wave at a single point. The traditional formulation of this quantity is valid only for beamlike fields, for which the information on the state of partial polarization may be expressed by the 2×2 coherence (or polarization) matrix constructed from the two transversal electric field components.[19,20] Recently, the concept of the degree of polarization was extended to also deal with general 3D electromagnetic wave fields that have a longitudinal electric field component and so must be represented by the full 3×3 coherence matrix.[18,20–22] We consider the field in the space-frequency domain, where the elements of the 3×3 (spectral) coherence matrix $\Phi_3(\mathbf{r}, \omega)$, at a point $\mathbf{r}$ and at frequency ω, are given by

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$$\phi_{ij}(\mathbf{r}, \omega) = \langle E_i^*(\mathbf{r}, \omega) E_j(\mathbf{r}, \omega)\rangle, \qquad (i, j) = (x, y, z). \tag{1}$$

In Eq. (1), $E_i(\mathbf{r}, \omega)$ and $E_j(\mathbf{r}, \omega)$ represent Cartesian components of a realization of the stationary electromagnetic field, and the angle brackets and the asterisk denote ensemble averaging and complex conjugation, respectively. According to Eq. (1), the coherence matrix is Hermitian, i.e., $\phi_{ij}(\mathbf{r}, \omega) = \phi_{ji}^*(\mathbf{r}, \omega)$.

The 3D degree of polarization, $P_3(\mathbf{r}, \omega)$, is given by the formula[18]

$$P_3^2(\mathbf{r}, \omega) = \frac{3}{2}\left\{\frac{\mathrm{tr}[\Phi_3^2(\mathbf{r}, \omega)]}{\mathrm{tr}^2[\Phi_3(\mathbf{r}, \omega)]} - \frac{1}{3}\right\}, \tag{2}$$

where tr denotes the trace of a matrix. The 3D degree of polarization is independent of the orientation of the coordinate system and bounded between 0 and 1, which correspond to unpolarized and fully polarized 3D fields, respectively. Furthermore, as its two-dimensional (2D) counterpart, the 3D degree of polarization is a measure of the correlations that exist between the orthogonal electric field components.[18] We remark here that recently an alternative definition for the degree of polarization of 3D electromagnetic fields has been proposed.[23]

It is insightful to express the 3D degree of polarization explicitly in terms of the complex correlation coefficients $\mu_{ij}(\mathbf{r}, \omega)$, defined as

$$\mu_{ij}(\mathbf{r}, \omega) \equiv \frac{\phi_{ij}(\mathbf{r}, \omega)}{[\phi_{ii}(\mathbf{r}, \omega)\phi_{jj}(\mathbf{r}, \omega)]^{1/2}}. \tag{3}$$

The magnitude of the correlation coefficients, i.e., the degree of correlation, is bounded to the interval $0 \leq |\mu_{ij}(\mathbf{r}, \omega)| \leq 1$, with the lower limit indicating uncorrelated field components and the upper limit corresponding to complete correlation between the components. Unlike the degree of polarization, the (complex) correlation coefficients depend on the orientation of the Cartesian coordinate system. In terms of the degrees of correlation, we can write the 3D degree of polarization as[18]

$$P_3^2(\mathbf{r}, \omega) = 1 - \frac{\sum_{ij} 3[1 - |\mu_{ij}(\mathbf{r}, \omega)|^2]\phi_{ii}(\mathbf{r}, \omega)\phi_{jj}(\mathbf{r}, \omega)}{[\phi_{xx}(\mathbf{r}, \omega) + \phi_{yy}(\mathbf{r}, \omega) + \phi_{zz}(\mathbf{r}, \omega)]^2}, \tag{4}$$

where the summation is carried out over index pairs $(ij) = (xy, xz, yz)$. In this work, Eq. (4) is particularly useful, since in analyzing the 3D degree of polarization of light in the focus, we will emphasize the role of the changes in the correlation coefficients and in the spectral densities associated with the field components.

To study the 3D degree of polarization in the image space of a rotationally symmetric aplanatic optical system, we make use of the classical Richards–Wolf theory.[5,24] The Richards–Wolf model for focusing is based on the Debye approximation, and it is valid for systems of large NA. In short, it consists of expressing the electric field converging toward the focus as an electromagnetic angular-spectrum decomposition with the plane-wave components corresponding to the rays of geometrical optics. These wave components are superposed with the correct phases and directions of polarization to produce the electromagnetic field in the focal region.

Consider a paraxial optical field incident along the z axis on an imaging system, illustrated in Fig. 1. The refractive index of the medium surrounding the system is equal to unity. The electric field components of the incident wave are written as

$$\mathbf{E}_i^0(\mathbf{r}, \omega) = E_i^0(\omega)A^0(x, y)\exp(ikz)\mathbf{e}_i, \qquad i = (x, y), \tag{5}$$

where the (spatially independent) incident-field amplitude $E_i^0(\omega)$ is a random variable and $A^0(x, y)$ is the (deterministic) transverse profile of the wave, taken to be cylindrically symmetric. Here, and henceforth, the superscript 0 refers to quantities of the incident wave. Furthermore, $k = 2\pi/\lambda$ is the vacuum wave number, λ is the wavelength of light, and $\mathbf{e}_i$, with $i = (x, y)$, are the unit vectors along the Cartesian coordinate axes. In this work, we use the value $\lambda = 633\,\mathrm{nm}$ for the wavelength. Throughout our analysis, we assume that the x component of the incident beam is a completely coherent scalar wave field and that, likewise, the y component is fully coherent. Instead, the degree of correlation between the x and y components is a spatially independent parameter that will be varied. The field traversing the optical system converges toward the focus O from the focal sphere $\mathcal{F}$ (see Fig. 1). The coordinate system is defined by the orthogonal axes OX, OY, and OZ, with the focus O in the origin. The axis of revolution of the optical system is the z axis. Spherical polar coordinates (r, θ, ϕ) are introduced, with the polar axis along the axis of revolution of the system and the azimuth measured from the OX direction.

If the polarization of the incident wave is taken to be along the x axis, the realization at frequency ω of the electric field at point $\mathbf{r}$ in the focal region is expressed as

$$\mathbf{E}(\mathbf{r}, \omega) = E_x(\mathbf{r}, \omega)\mathbf{e}_x + E_y(\mathbf{r}, \omega)\mathbf{e}_y + E_z(\mathbf{r}, \omega)\mathbf{e}_z, \tag{6}$$

where the complex amplitudes $E_i(\mathbf{r}, \omega)$, with $i = (x, y, z)$, are of the form[5,24]

$$E_x(\mathbf{r}, \omega) = -E_x^0(\omega)\frac{ik}{2}[I_0(\mathbf{r}, \omega) + I_2(\mathbf{r}, \omega)\cos 2\phi], \tag{7}$$

$$E_y(\mathbf{r}, \omega) = -E_x^0(\omega)\frac{ik}{2}I_2(\mathbf{r}, \omega)\sin 2\phi, \tag{8}$$

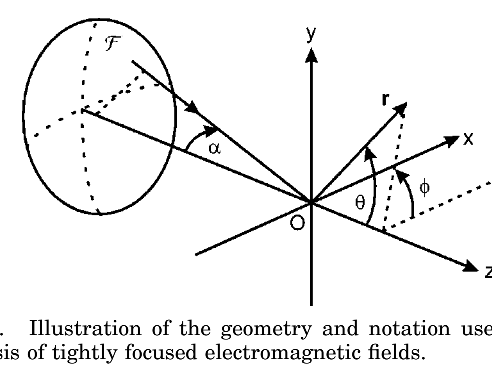

Fig. 1. Illustration of the geometry and notation used in the analysis of tightly focused electromagnetic fields.

$$E_z(\mathbf{r}, \omega) = -E_x^0(\omega) k I_1(\mathbf{r}, \omega)\cos\phi. \quad (9)$$

We note that a longitudinal electric field component (z component) is generated in the focal region. In Eqs. (7)–(9), ϕ is the azimuth at the position $\mathbf{r}$, and

$$I_0(\mathbf{r}, \omega) = \int_0^{\alpha_0} A(\alpha)\sin\alpha(1 + \cos\alpha) J_0\left(v\frac{\sin\alpha}{\sin\alpha_0}\right) \times \exp\left(iu\frac{\cos\alpha}{\sin^2\alpha_0}\right)\mathrm{d}\alpha, \quad (10)$$

$$I_1(\mathbf{r}, \omega) = \int_0^{\alpha_0} A(\alpha)\sin^2\alpha J_1\left(v\frac{\sin\alpha}{\sin\alpha_0}\right) \times \exp\left(iu\frac{\cos\alpha}{\sin^2\alpha_0}\right)\mathrm{d}\alpha, \quad (11)$$

$$M(\mathbf{r}, \omega) = -\frac{ik}{2}\begin{bmatrix} I_0(\mathbf{r}, \omega) + I_2(\mathbf{r}, \omega)\cos 2\phi & I_2(\mathbf{r}, \omega)\sin 2\phi \\ I_2(\mathbf{r}, \omega)\sin 2\phi & I_0(\mathbf{r}, \omega) - I_2(\mathbf{r}, \omega)\cos 2\phi \\ -2iI_1(\mathbf{r}, \omega)\cos\phi & -2iI_1(\mathbf{r}, \omega)\sin\phi \end{bmatrix} \quad (17)$$

and T denotes the transpose. The elements of the coherence matrix related to the incident field are explicitly written as

$$\phi_{ij}^0(\omega) = \langle E_i^{0*}(\omega) E_j^0(\omega)\rangle, \qquad (i, j) = (x, y). \quad (18)$$

If we make use of Eq. (3) for the off-diagonal elements and the Hermiticity property $\phi_{xy}^0(\omega) = \phi_{yx}^{0*}(\omega)$, the coherence matrix of the incident field takes on the form

$$\Phi^0(\omega) = \begin{bmatrix} \phi_{xx}^0(\omega) & |\mu_{xy}^0(\omega)|[\phi_{xx}^0(\omega)\phi_{yy}^0(\omega)]^{1/2}\exp[i\beta^0(\omega)] \\ |\mu_{xy}^0(\omega)|[\phi_{xx}^0(\omega)\phi_{yy}^0(\omega)]^{1/2}\exp[-i\beta^0(\omega)] & \phi_{yy}^0(\omega) \end{bmatrix}, \quad (19)$$

$$I_2(\mathbf{r}, \omega) = \int_0^{\alpha_0} A(\alpha)\sin\alpha(1 - \cos\alpha) J_2\left(v\frac{\sin\alpha}{\sin\alpha_0}\right) \times \exp\left(iu\frac{\cos\alpha}{\sin^2\alpha_0}\right)\mathrm{d}\alpha, \quad (12)$$

where J_0, J_1, and J_2 are Bessel functions of the first kind and we have introduced the usual optical coordinates u and v through the definition

$$u = kz\sin^2\alpha_0, \quad (13)$$

$$v = k\rho\sin\alpha_0, \quad (14)$$

where $\rho = (x^2 + y^2)^{1/2}$. The parameter α_0 is the angular semiaperture of the optical system, and $\sin\alpha_0$ is its NA. The explicit form of the function $A(\alpha)$ in Eqs. (10)–(12) depends on the optical system and the transverse profile $A^0(x, y) = A^0(\rho)$ of the incident electromagnetic field. Since the imaging system is assumed to fulfill the sine condition, the expression for $A(\alpha)$ is[24]

$$A(\alpha) = A^0(f\sin\alpha)\cos^{1/2}\alpha, \quad (15)$$

where f is the focal length. In this paper, we assume that the incident optical beam is well collimated, so that it can be approximated by a plane wave, i.e., $A^0(\rho) = 1$.

By repeating the above analysis also for a y-polarized incident field and combining the results for the two orthogonal polarizations, we obtain an expression for the electric field in the focal region for a realization of the incident wave of any state of polarization. Hence we find the following compact formula for the 3×3 coherence matrix $\Phi_3(\mathbf{r}, \omega)$ of the focal field in terms of the 2×2 coherence matrix $\Phi^0(\omega)$ of the incident wave:

$$\Phi_3(\mathbf{r}, \omega) = M^*(\mathbf{r}, \omega)\Phi^0(\omega)M^{\mathrm{T}}(\mathbf{r}, \omega), \quad (16)$$

where $M(\mathbf{r}, \omega)$ is the 3×2 matrix

where $\phi_{xx}^0(\omega)$ and $\phi_{yy}^0(\omega)$ are the intensities of the x and y components, respectively, and $|\mu_{xy}^0(\omega)|$ and $\beta^0(\omega)$ are the magnitude and the phase of their complex correlation coefficient. In the following, we restrict our study to incident fields for which $\phi_{xx}^0(\omega) = \phi_{yy}^0(\omega)$. Note that for such a field the degree of correlation $|\mu_{xy}^0(\omega)|$ is equal to the conventional two-dimensional (2D) degree of polarization.[20] By performing the integrations in Eqs. (10)–(12) numerically, we obtain the 3×3 coherence matrix in the image space from Eqs. (16), (17), and (19). The intensity distribution (spectral density) then is $I(\mathbf{r}, \omega) = \mathrm{tr}[\Phi_3(\mathbf{r}, \omega)]$. The 3D degree of polarization and the correlation coefficients in the image space are calculated from Eqs. (2) and (3). The results are obtained as a function of the incident wave correlation coefficient and the NA of the imaging system.

3. RESULTS

We apply the theory in Section 2 to study the changes in the 3D degree of polarization and in the intensity distribution of an electromagnetic field when it is focused to a small spot. Figure 2 displays the total intensity $I(\mathbf{r}, \omega)$ in the focal plane of an imaging system of NA = 0.9 for

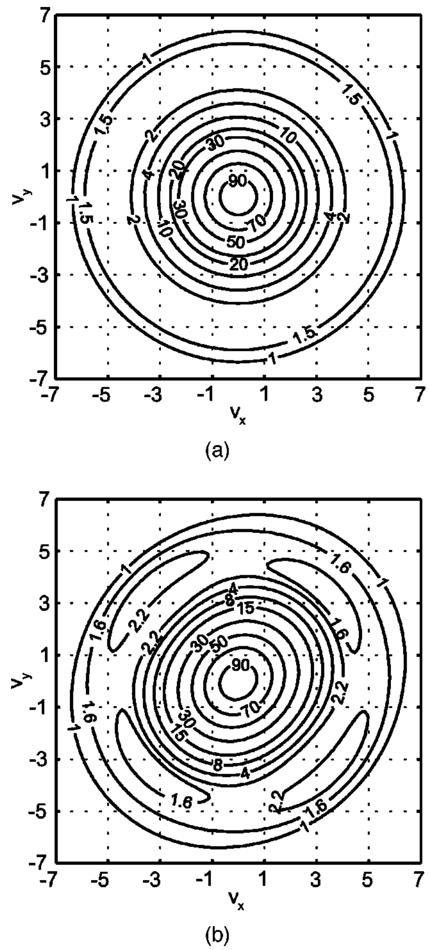

Fig. 2. Contour plots of the intensity distribution $I(\mathbf{r},\ \omega)$ in the focal plane ($u = 0$) for (a) 2D-unpolarized light [$|\mu_{xy}^{0}(\omega)| = 0$] and (b) partially polarized light with $|\mu_{xy}^{0}(\omega)| = 0.5$ and $\beta^{0}(\omega) = 0$. The NA of the optical system is 0.9. The value of $I(\mathbf{r},\ \omega)$ has been normalized to 100 at the origin. The axes are $v_x = v \cos \phi$ and $v_y = v \sin \phi$.

two different incident fields: a field with (a) $|\mu_{xy}^{0}(\omega)| = 0$ and (b) $|\mu_{xy}^{0}(\omega)| = 0.5$ and $\beta^{0}(\omega) = 0$. Note that the uncorrelated case $|\mu_{xy}^{0}(\omega)| = 0$ corresponds to a field that is unpolarized in the 2D sense, i.e., its conventional degree of polarization is zero. From now on, we will refer to such an incident field as 2D unpolarized, although one should recall that its 3D degree of polarization assumes the value $P_3^{0}(\omega) = 1/2$. This illustrates the fact that the value of the degree of polarization, in general, depends on the dimensionality of the analysis.[18,25] Figure 2 shows that for the 2D-unpolarized incident field the focal-plane intensity distribution is symmetric, whereas for the partially polarized illumination the distribution is elongated along one direction and compressed in the orthogonal direction. The most significant difference between the focus of 2D-unpolarized and partially polarized light is the appearance of interference lobes in the latter case. As the degree of correlation between the field components of the incident light increases, the intensity distribution in the focal plane eventually approaches that produced by linearly polarized illumination as a result of the choice $\beta^{0}(\omega) = 0$.

When the paraxial incident wave is focused, a z component of the electric field is produced that may be comparable in strength with the x and y components. All three components of a field realization in the image space are formed as linear combinations of the two components of an incident-field realization, $E_x^{0}(\omega)$ and $E_y^{0}(\omega)$. Therefore it may happen that at some points in the focal region, the x, y, and z components of a realization are all proportional to the same random variable. At these points in the image space, the field components are completely correlated, and, consequently in view of Eq. (4), the 3D degree of polarization is equal to unity. We prove in Appendix A that these are the only points at which the degree of correlation between the field components is unity. This behavior is evidenced in Fig. 3(a), where we have plotted the 3D degree of polarization in the focal plane ($u = 0$) when the incident wave is unpolarized in the 2D sense [$|\mu_{xy}^{0}(\omega)| = 0$]. We note that the focus is surrounded by a set of concentric rings of full polarization. The cross sections along the azimuth $\phi = \pi/4$ shown in Fig. 3(b) display the 3D degree of polarization and the degrees of correlation $|\mu_{xy}(v,\ \omega)|$ and $|\mu_{xz}(v,\ \omega)|$. In this particular direction, the degree of correlation between the y and z components is the same as that between the x and z components, i.e., $|\mu_{yz}(v,\ \omega)| = |\mu_{xz}(v,\ \omega)|$. We observe that on the ring where the field is fully polarized there is complete correlation between the different field components, as discussed above. We also note that the value $P_3(v,\ \omega) = 0.5$ in the focus ($v = 0$) is a consequence of the field having only two components, which are uncorrelated and of equal intensity. However, at the two other points where $P_3(v,\ \omega) = 0.5$ the situation is more complicated, since the field, for this orientation of the coordinate system, consists of three nonzero components for which the degrees of correlation are in the interval $0 < |\mu_{ij}(v,\ \omega)| < 1\ (ij = xy, xz, yz)$. Figure 3(c) displays the intensities of the x and z components of the electric field as a function of v when $\phi = \pi/4$. We remark that the intensities of the x and y components are identical along this line. It is seen that a system with NA = 0.9 produces a z component with a maximum intensity equal to 7% of the total intensity at the focus. Furthermore, on the rings where the field is fully polarized all three field components are nonzero. The total intensity on the first ring of full polarization is approximately 5% of the intensity at the focus. For incident light of more complex spatial profile than the plane wave used here, e.g., for an annular beam, the intensity of the field on the rings of fully polarized light can be considerably higher.

For the case of 2D-unpolarized illumination, $\Phi^{0}(\omega)$ is proportional to the 2×2 unit matrix. It then follows from Eqs. (2) and (16) that the 3D degree of polarization in the focal region can be expressed by the formula

$$P_3(\mathbf{r},\ \omega) = \left(\frac{1}{4} + 3\left\{\frac{|I_1(\mathbf{r},\ \omega)|^2 + \mathrm{Re}[I_0^{*}(\mathbf{r},\ \omega)I_2(\mathbf{r},\ \omega)]}{|I_0(\mathbf{r},\ \omega)|^2 + 2|I_1(\mathbf{r},\ \omega)|^2 + |I_2(\mathbf{r},\ \omega)|^2}\right\}^2\right)^{1/2}, \quad (20)$$

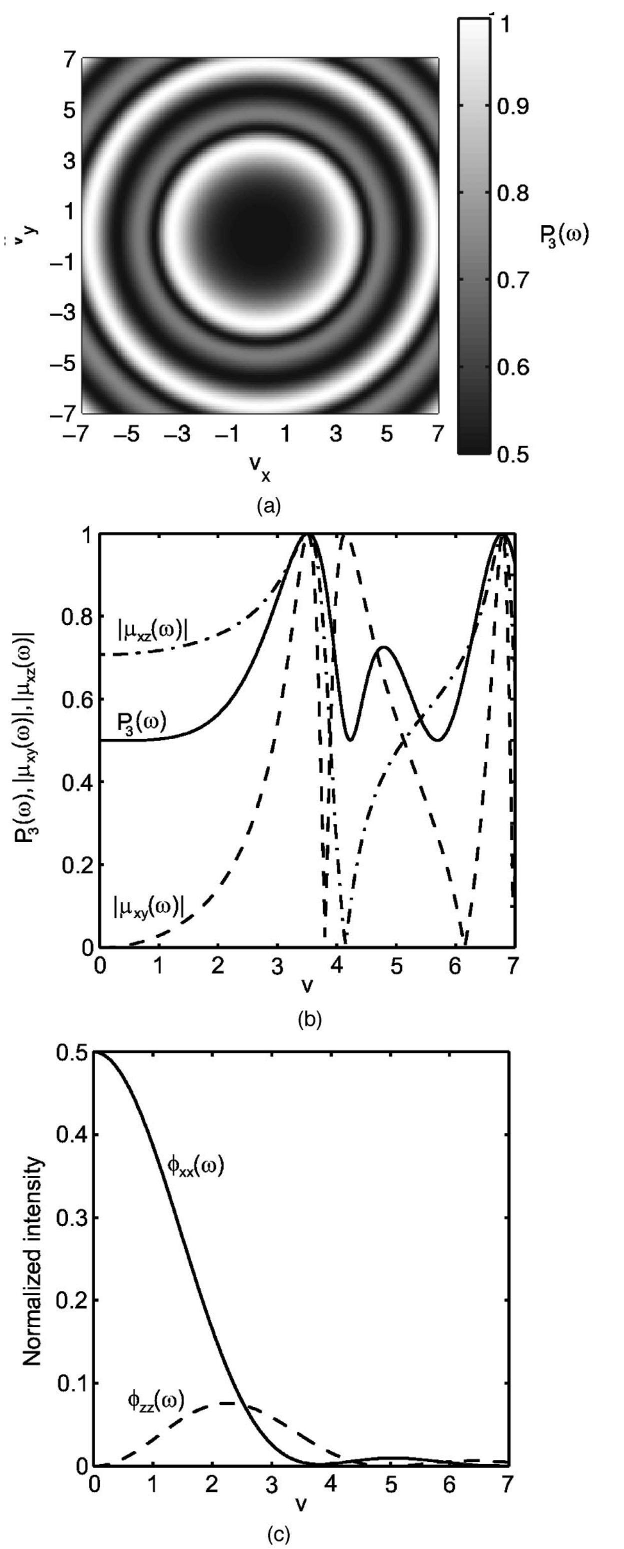

Fig. 3. Polarization of the incident 2D-unpolarized light when focused through an optical system of NA = 0.9: (a) degree of polarization $P_3(v, \phi, \omega)$ in the focal plane ($u = 0$), (b) $P_3(v, \omega)$ (solid curve) and the degrees of correlation $|\mu_{xy}(v, \omega)|$ (dashed curve) and $|\mu_{xz}(v, \omega)|$ (dotted–dashed curve) along the azimuth $\phi = \pi/4$ in the focal plane, (c) intensities $\phi_{xx}(v, \omega)$ (solid curve) and $\phi_{zz}(v, \omega)$ (dashed curve) on the same azimuth as that in (b). The intensities have been normalized by the total intensity at the origin. The axes in (a) are defined as $v_x = v \cos \phi$ and $v_y = v \sin \phi$.

where Re denotes the real part. We observe that $P_3(\mathbf{r}, \omega)$ in this particular case is independent of the azimuth angle ϕ and always greater than 1/2, as also evidenced by Figs. 3(a) and 3(b). Hence the 3D degree of polarization (of a 2D-unpolarized beam) cannot be decreased through the process of focusing. Furthermore, it can be deduced from Eq. (20) that the condition for the field to be fully polarized at a point is that $I_0(\mathbf{r}, \omega) = I_2(\mathbf{r}, \omega)$. Solving this equation at $u = 0$ gives $v \approx 3.5$ and $v \approx 6.8$ for the radii of the first two rings of full polarization [see Fig. 3(b)].

The dependence of the 3D degree of polarization on the NA of the optical system is illustrated in Fig. 4, where the distribution of $P_3(v, \omega)$ is shown as a function of NA and coordinate v. The incident field is taken to be unpolarized in the 2D sense, and so the 3D degree of polarization in the focal region does not depend on the azimuth ϕ, as

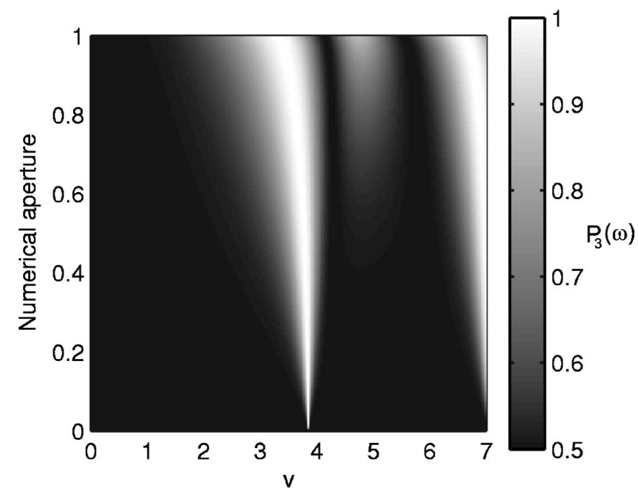

Fig. 4. Distribution of the 3D degree of polarization in the focal plane ($u = 0$) as a function of the NA of the optical system and the coordinate v for incident 2D-unpolarized light.

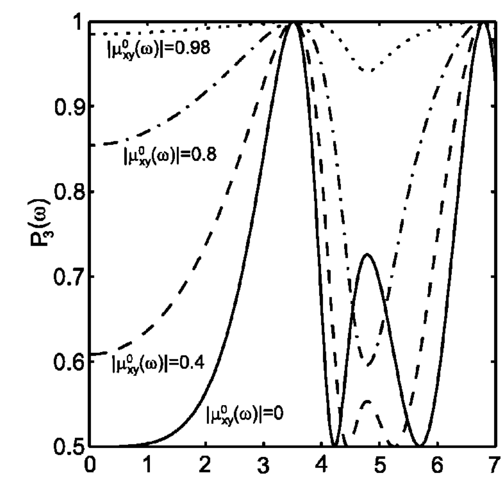

Fig. 5. Plots of the 3D degree of polarization for varying degrees of correlation of the incident field. The curves are along the azimuth $\phi = \pi/4$ in the focal plane ($u = 0$) of an imaging system with NA = 0.9. The solid curve is for 2D-unpolarized incident light [$|\mu^0_{xy}(\omega)| = 0$], whereas the others are for $|\mu^0_{xy}(\omega)| = 0.4$ (dashed curve), $|\mu^0_{xy}(\omega)| = 0.8$ (dotted–dashed curve), and $|\mu^0_{xy}(\omega)| = 0.98$ (dotted curve). For the cases $|\mu^0_{xy}(\omega)| > 0$, we have chosen $\beta^0(\omega) = 0$.

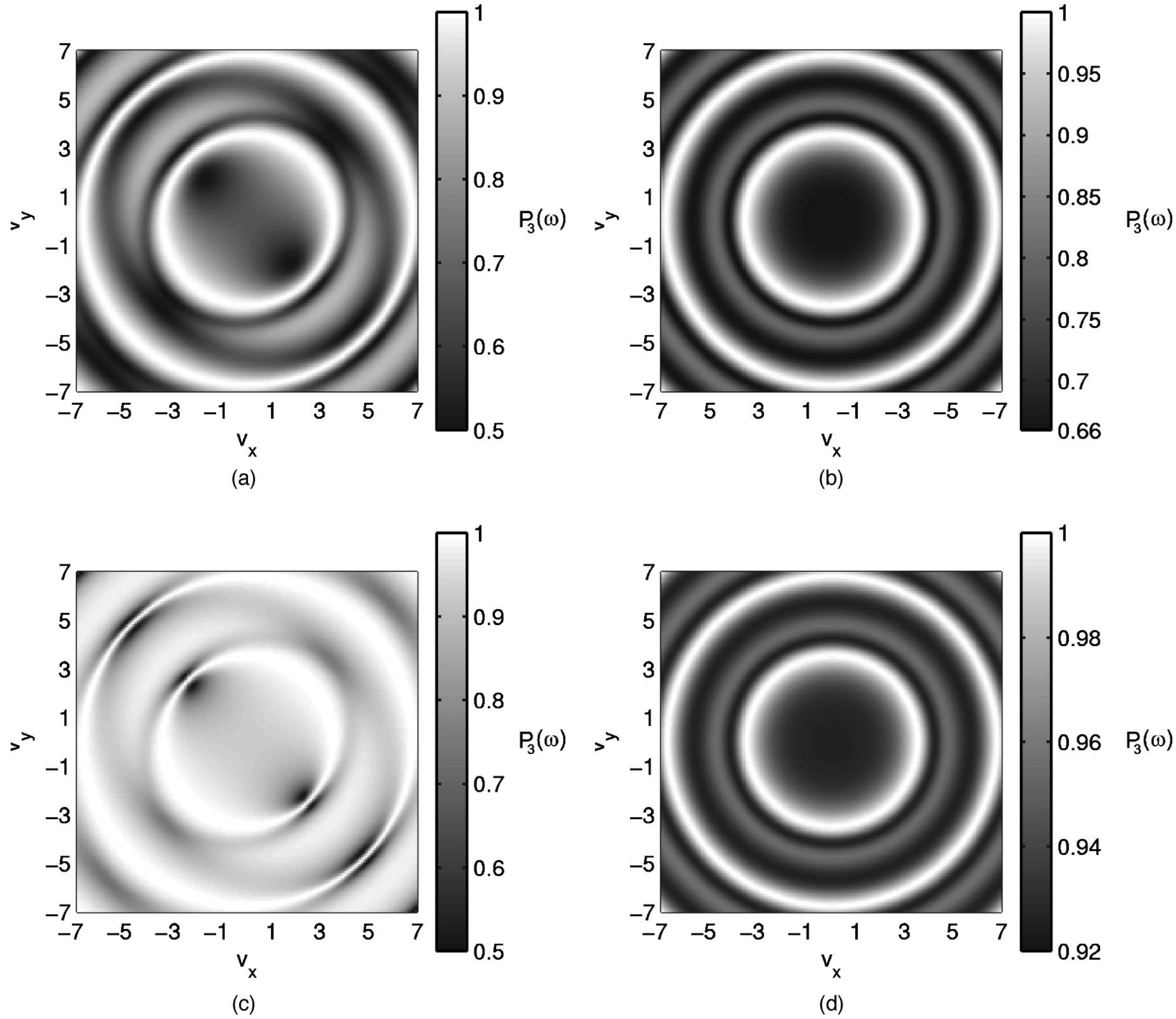

Fig. 6. Distribution of the 3D degree of polarization in the focal plane for various partially polarized incident beams. In (a) and (c), the phase of the correlation coefficient is $\beta^0(\omega) = 0$, while in (b) and (d) $\beta^0(\omega) = \pi/2$. In (a) and (b), the degree of correlation of the incident field is $|\mu_{xy}^0(\omega)| = 0.5$, whereas in (c) and (d) $|\mu_{xy}^0(\omega)| = 0.9$. The axes are defined as $v_x = v \cos \phi$ and $v_y = v \sin \phi$.

discussed in connection with Eq. (20). We observe that the field is fully polarized on certain rings around the focus already for small NA.[26] As the numerical aperture of the optical system is increased, the rings become thicker and their radii decrease.

The degree of correlation of the incident-field components strongly affects the distribution of the 3D degree of polarization in the image space. Figure 5 displays $P_3(v, \omega)$ along the azimuth $\phi = \pi/4$ in the focal plane of an imaging system of NA = 0.9 for different values of the degree of correlation $|\mu_{xy}^0(\omega)|$ of the incident field. As above, we have set $\beta^0(\omega) = 0$. We see that in all cases there exist rings of full polarization, between which the behavior of the 3D degree of polarization is different for the incident fields of different degrees of correlation. For the two lowest values of $|\mu_{xy}^0(\omega)|$, a local maximum of the degree of polarization is formed between the rings of full polarization. As the degree of correlation of the incident-field components (i.e., the usual 2D degree of polarization of the incident wave) increases, this maximum decreases and eventually completely vanishes. The observed behavior in this case mainly follows from the change in the distribution of the degree of correlation between the x and y components of the focal field. As a general remark on Fig. 5, we point out that when the degree of correlation of the incident-field components approaches unity, the field in the focal plane becomes fully polarized.

As pointed out above, the degree of polarization of the incident field is equal to $|\mu_{xy}^0(\omega)|$ if $\phi_{xx}^0 = \phi_{yy}^0$; hence it is independent of the phase of the correlation coefficient. However, $\beta^0(\omega)$ has a significant influence on the 3D degree of polarization of the focused field. In Figs. 6(a) and 6(b), $P_3(v, \phi, \omega)$ is shown in the focal plane for incident light with $\beta^0(\omega) = 0$ and $\beta^0(\omega) = \pi/2$, respectively, when $|\mu_{xy}^0(\omega)| = 0.5$. For both incident fields, $P_3^0(\omega) \approx 0.66$. In the case of fully polarized illumination $[|\mu_{xy}^0(\omega)| = 1]$, the values chosen for the phase of the correlation coefficient would correspond to linear $[\beta^0(\omega) = 0]$ and circular $[\beta^0(\omega) = \pi/2]$ polarization. From Fig. 6(a), we observe that there exist regions in the focal plane in which the 3D degree of polarization is lower than that of the incident field, whereas in Fig. 6(b) $P_3(v, \phi, \omega)$ is nowhere less than 0.66. By a calculation analogous to that leading to Eq. (20), it can be shown that for $\beta^0(\omega) = \pi/2$ the minimum value of the 3D degree of polarization in the focal region is equal to $P_3^0(\omega)$. For the other choice of the phase, however, there are points at which $P_3(v, \phi, \omega) = 0.5$. When $|\mu^0_{xy}(\omega)|$ increases, the light field in the focal plane eventually becomes strongly polarized, as illustrated in Figs. 6(c) and 6(d), which, respectively, are for incident fields with $\beta^0(\omega) = 0$ and $\beta^0(\omega) = \pi/2$. For both cases, $|\mu^0_{xy}(\omega)| = 0.9$, corresponding to $P^0_3(\omega) \approx 0.93$. We observe that when $\beta^0(\omega) = 0$, the 3D degree of polarization in the focal plane is considerably lower than $P^0_3(\omega)$ at points in the direction of the interference lobes.

4. CONCLUSIONS

We have studied the state of partial polarization of a fluctuating electromagnetic field in the focal region of an optical imaging system of high NA. We find marked changes in the 3D degree of polarization as the light field is focused. For a 2D-unpolarized incident beam, the field may be locally fully polarized in the focal plane. More precisely, for such an incident field, the focus is surrounded by rings of full polarization. When the components of the incident beam are partially correlated, the phase of the complex correlation coefficient has a strong effect on the shape of the distribution of the 3D degree of polarization in the focal plane. In all cases, as the degree of correlation between the incident-field components increases, the image-space electric field, on average, gradually becomes fully polarized. The enhanced 3D degree of polarization is a consequence of the increased degree of correlation between the electric field components. On the basis of our results, the focal region of a high-NA imaging system is a good candidate for experimental studies of the polarization properties of 3D electromagnetic fields, e.g., using scanning-probe techniques.[27]

APPENDIX A: CONDITION FOR FULL THREE-DIMENSIONAL POLARIZATION

If the incident field is completely polarized, the 3D electromagnetic field in the focal region is, of course, everywhere fully polarized. Therefore we consider an incident field that is not completely polarized, i.e., we assume that $\phi^0_{xx}(\omega),\ \phi^0_{yy}(\omega) > 0$ and $|\mu^0_{xy}(\omega)| < 1$. Under these assumptions, we next derive a sufficient and necessary condition for full polarization at a point in the focal region. First, we note that Eq. (4) implies that

$$P_3(\mathbf{r}, \omega) = 1 \Leftrightarrow |\mu_{xy}(\mathbf{r}, \omega)|^2 = |\mu_{xz}(\mathbf{r}, \omega)|^2 = |\mu_{yz}(\mathbf{r}, \omega)|^2 = 1. \tag{A1}$$

Second, the three electric field components in the focal region are generated as a linear combination of the amplitudes $E^0_x(\omega)$ and $E^0_y(\omega)$ of the incident field, i.e.,

$$E_i(\mathbf{r}, \omega) = a_i(\mathbf{r}, \omega)E^0_x(\omega) + b_i(\mathbf{r}, \omega)E^0_y(\omega), \quad i = (x, y, z), \tag{A2}$$

where $a_i(\mathbf{r}, \omega)$ and $b_i(\mathbf{r}, \omega)$ are complex-valued functions. From Eqs. (1), (3), and (A2), we straightforwardly obtain the following equivalence relation:

$$|\mu_{ij}(\mathbf{r}, \omega)|^2 = 1 \Leftrightarrow |a_i(\mathbf{r}, \omega)b_j(\mathbf{r}, \omega) - b_i(\mathbf{r}, \omega)a_j(\mathbf{r}, \omega)|^2 \times [|\mu^0_{xy}(\omega)|^2 - 1]\phi^0_{xx}(\omega)\phi^0_{yy}(\omega) = 0 \tag{A3}$$

for $(ij) = (xy, xz, yz)$. We see that when the conditions assumed for the incident field hold, the electric field in the focal region is fully polarized if and only if

$$a_i(\mathbf{r}, \omega)b_j(\mathbf{r}, \omega) = b_i(\mathbf{r}, \omega)a_j(\mathbf{r}, \omega) \quad \text{for all } (ij) = (xy, xz, yz). \tag{A4}$$

This result is equivalently expressed as

$$\frac{a_x(\mathbf{r}, \omega)}{b_x(\mathbf{r}, \omega)} = \frac{a_y(\mathbf{r}, \omega)}{b_y(\mathbf{r}, \omega)} = \frac{a_z(\mathbf{r}, \omega)}{b_z(\mathbf{r}, \omega)}, \tag{A5}$$

which implies that the field at a point in the focal region is fully polarized if and only if all three electric field components are proportional to the same random function, which is a linear combination of the random amplitudes $E^0_x(\omega)$ and $E^0_y(\omega)$. This result holds also when $a_i(\mathbf{r}, \omega) = 0$ [or $b_i(\mathbf{r}, \omega) = 0$] for all $i = (x, y, z)$, in which case the three components are proportional to $E^0_y(\omega)$ [or $E^0_x(\omega)$]. If the field at some point in the image space has only two nonzero components, the above analysis implies that they must be proportional to the same random function in order for the field to be completely polarized. Thus, in all cases, the field is fully polarized at some point in the focal region if and only if the field components at that point are proportional to the same random function. The random function may, of course, be different from point to point.

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