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Ellipticity statistic as measure of MIMO multipath richness

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'Multipath richness' characterises the capability of a propagation environment to support parallel communication modes. The so-called ellipticity statistic is proposed as a measure of multipath richness of a MIMO radio channel. A measurement example clarifies the proposed figure of merit.

Introduction: It is well known that multipath propagation in a radio channel has a beneficial effect on the capacity of a multiple-input multiple-output (MIMO) communication system, especially at high SNR [1]. For example, a MIMO transceiver operating in a rich scattering radio propagation environment can employ some form of spatial multiplexing in order to approach its theoretical spectral efficiency. In another setting, in the analysis of radio channel measurements it is often of interest to investigate how multipath richness depends on the propagation environment, antenna properties, carrier frequency, or some other parameter. However, the very question of how to quantify multipath richness is still in many ways an open problem. In this Letter, we show that a parameter from multivariate statistical analysis, the ellipticity statistic, is a natural measure for multipath richness and illustrate its use with a real-world measurement example.

Mutual information (MI): The concept of multipath richness provides a propagation-based explanation for parallel information channels in a MIMO system. Therefore, to quantify multipath richness, we focus on the mutual information of the MIMO radio channel. Consider a set of $n_r \times n_t$ random channel matrices, $\{\mathbf{H}^{(i)}\}_{i=1}^{\infty}$, where each channel realisation is independently drawn from a probability distribution that satisfies $\mathrm{E}[\|\mathbf{H}^{(i)}\|_F^2] = n_r n_t$. For the ith realisation, MI between the $n_t \times 1$ channel input and the $n_r \times 1$ noisy channel output is $I_{\mathbf{H}^{(i)}} = \log_2|\mathbf{I}n_r + \rho/n_t \mathbf{H}^{(i)}\bar{\mathbf{H}}^{(i)}|$ [1], where ρ is the average SNR at the output of each of the n_r receiver antennas, and $\bar{\mathbf{H}}$ is the conjugate transpose of $\mathbf{H}$. The channel input and the additive noise are assumed to be isotropic complex Gaussian random variables of appropriate dimensions [1]. For this channel input distribution $I_{\mathbf{H}^{(i)}}$ is the upper bound on the highest achievable rate of error-free communication over the channel realisation $\mathbf{H}^{(i)}$, which is assumed known by the receiver. For the set $\{\mathbf{H}^{(i)}\}_{i=1}^{\infty}$, the probability that a randomly drawn channel matrix does not support rate I_p, called outage mutual information [1], is $\mathrm{Prob}(I_{\mathbf{H}^{(i)}} < I_p) = p$.

Ellipticity statistic: It is well understood that spatial multiplexing – facilitated by multipath richness – is a useful technique mainly at high SNR, where the channel eigenvalues are above the receiver noise power level. Thus, to find a measure for multipath richness it seems logical to examine mutual information in the high-SNR regime. We denote $K = \min(n_r, n_t)$ and assume, for now, that $\mathrm{rank}(\mathbf{H}^{(i)}) = K$. Further, we denote the nonzero eigenvalues of $\mathbf{H}^{(i)}\bar{\mathbf{H}}^{(i)}$ with $\{\lambda_k^{(i)}\}_{k=1}^{K}$, their geometric mean with $m_g^{(i)} = (\prod_{k=1}^{K} \lambda_k^{(i)})^{1/K}$, their arithmetic mean with $m_a^{(i)} = 1/K \sum_{k=1}^{K} \lambda_k^{(i)}$, and their ratio with $\gamma^{(i)} = m_g^{(i)}/m_a^{(i)}$. It has been shown that MI can be decomposed at high SNR as [2]

$$I_{\mathbf{H}^{(i)}} \simeq I_{\mathrm{sup}} + I_{\mathrm{fad}}^{(i)} + I_{\mathrm{mux}}^{(i)} \tag{1}$$

where

$$I_{\mathrm{sup}} = K \log_2\left(1 + \frac{\rho n_r}{K}\right) \simeq K \log_2\left(\frac{\rho n_r}{K}\right) \tag{2}$$

$$I_{\mathrm{fad}}^{(i)} = K \log_2\left(\frac{\|\mathbf{H}^{(i)}\|_F^2}{n_r n_t}\right) \tag{3}$$

$$I_{\mathrm{mux}}^{(i)} = K \log_2(\gamma^{(i)}) \tag{4}$$

Here I_{sup} is the mutual information of the ideal channel with equal eigenvalues, and $I_{\mathrm{fad}}^{(i)}$ is the MI owing to the SNR variation of the ith channel realisation, given with respect to the average channel power gain $n_r n_t$. The parameter $\gamma^{(i)} \in [0, 1]$, i.e. the ratio of arithmetic and geometric means of the channel eigenvalues, is called the ellipticity statistic [3]. It measures the ellipticity of the hyperellipsoid the axis lengths of which are the eigenvalues of $\mathbf{H}^{(i)}\bar{\mathbf{H}}^{(i)}$, or, equivalently, the power gains of the K parallel information pipes of the MIMO channel. Note that $I_{\mathrm{mux}}^{(i)}$ is nonpositive, which at first seems counter-intuitive for an information measure. However, $I_{\mathrm{mux}}^{(i)}$ can be interpreted as MI degradation from the ideal case (I_{sup}) owing to eigenvalue dispersion. If all eigenvalues are equal, then $\gamma^{(i)} = 1$, and consequently $I_{\mathrm{mux}}^{(i)} = 0$ with no MI loss. This information theoretic relation of $I_{\mathrm{mux}}^{(i)}$ to spatial multiplexing capability of a MIMO channel makes $\gamma^{(i)}$ a natural measure for multipath richness.

Properties: The ellipticity statistic $\gamma^{(i)}$ satisfies the following desirable properties:

– Its logarithm has the appealing interpretation as the mutual information loss from the ideal unitary channel owing to eigenvalue dispersion.
– It is scale-invariant with respect to average SNR (ρ) and fading of SNR ($\|\mathbf{H}^{(i)}\|_F^2$). One would expect multipath richness to be a property of propagation environment, and therefore independent of average or instantaneous SNR.
– It does not require the selection of subjective parameters, such as SNR or a dynamic range window for the eigenvalue spectrum.

Multipath richness can also be measured using the condition number [4]. The condition number, however, depends only on the largest and the smallest eigenvalue, and also lacks an operational meaning. In contrast, $\gamma^{(i)}$ is a function of all eigenvalues, and its logarithm has the interpretation as the MI loss relative to the ideal environment. Another measure of multipath richness is the effective degrees of freedom (EDOF) [5]. EDOF, however, depends on SNR, which has to be selected subjectively. Moreover, at high SNR, all full rank channels have the same EDOF, i.e. $\lim_{\rho\to\infty} \mathrm{EDOF} = K$. Therefore, all rank-$K$ channels are equal in this sense, which is a drawback of EDOF.

Like any multipath richness measure based on mutual information, the ellipticity statistic is also a function of channel eigenvalues and hence it depends on n_r and n_t. As an extreme example, when one eigenvalue is exactly zero, the channel is rank deficient and consequently $\gamma^{(i)} = 0$ and $I_{\mathrm{mux}}^{(i)} = -\infty$. The interpretation in this case is that the channel cannot support K parallel modes of communication. However, this does not mean that the channel mutual information itself is zero, but simply indicates that one cannot approximate it using (1), even at the high SNR limit. In practice, this has little significance since real-world MIMO channels are rarely, if ever, strictly rank deficient. Further, practical channel models produce channel matrices that have full rank with probability one. For the Rayleigh channel statistics, the distribution of $I_{\mathrm{mux}}^{(i)}$ has been derived in [2].

Measurement example: To illustrate the use of ellipticity statistic, we provide a measurement example in an urban microcell scenario. The measurements, which were conducted in the centre of Helsinki at 2.1 GHz, are documented in [6], where the measurement route used here is designated by 'Rout'. The custom-build spherical receiver array enables estimation of the 3D polarimetric channel response and embedding of arbitrary 3D antenna patterns at the receiver during post-processing, as detailed in [7]. Dipole antennas are used at the receiver. We denote a cross-dipole with vertically (V) and horizontally (H) polarised feeds with '+', while an array of two vertically polarised dipoles is denoted '| |'. The transmit antennas are dual-polarised patch antennas, also with V and H feeds [6]. For all antenna configurations the element spacing is half a wavelength both at the receiver and the transmitter. The total number of channel snapshots was 2500, corresponding to about 86 m measurement distance. The receiver moved along a street perpendicular to the line-of-sight (LOS) street, crossing it in the middle of the route. Results for three $n_r \times n_t$ MIMO systems are shown in Fig. 1: two 2×2 systems with orthogonally polarised (+, +) and co-polarised (| |,| |) transmit and receive antennas, and a 4×4 system with two orthogonally polarised antennas at both ends (++, ++). Fig. 1 shows the sliding mean (6 m) of $\{\log_2(\gamma^{(i)})\}_{i=1}^{2500}$, which is the MI loss normalised by K, i.e. the dimension of the MIMO system. In LOS, eigenvalue dispersion decreases for the 2×2 system with (+, +) polarisations. With ideal cross-polarisation discrimination I_{mux} would be zero, but in practice the signal is depolarised in reflections, which induces crosstalk between the channels. In contrast, in the other two cases the multipath richness is considerably reduced owing to high correlation. After the receiver again enters NLOS there is a decreasing trend in the multipath richness, which illustrates the well-known fact that the number of significant propagation paths decreases in the deeply shadowed region of the street canyon [6]. Fig. 2 shows empirical distributions of $\log_2(\gamma^{(i)})$ for the NLOS parts of the route. In all cases the transmitter and the receiver arrays consist of cross-polarised elements; e.g. the 2×8 system has the (+, ++++) configuration. The 2×8 system has the smallest MI penalty per dimension; this exemplifies how adding more antennas in one end of the link mitigates the effect of channel fading on the spatial multiplexing properties of a MIMO system. The theoretical result shown for the 2×2 channel indicates that the multipath richness of the examined measurement scenario is clearly worse than that of the Rayleigh IID fading case analysed in [2].

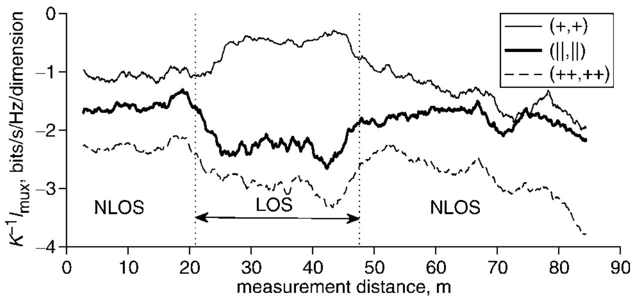

Fig. 1 *$K^{-1}I^{(i)}_{mux}$ against measurement distance*

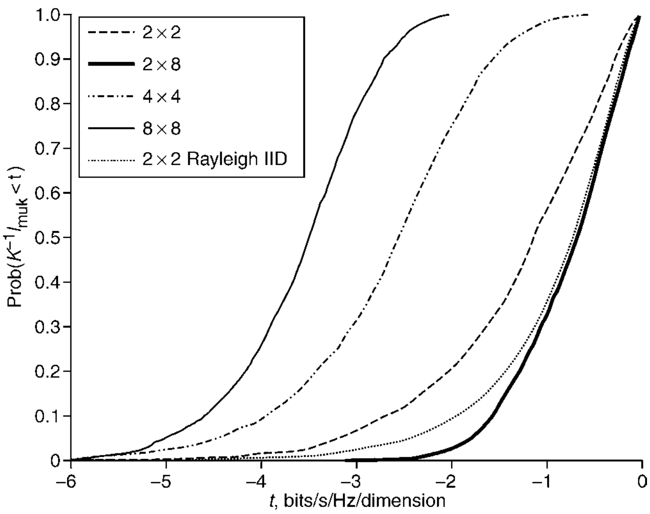

Fig. 2 *Empirical CDF of $K^{-1}I^{(i)}_{mux}$ for various $n_r \times n_t$ systems*

Conclusion: Ellipticity statistic, which quantifies the dispersion of the channel eigenvalues, is proposed as a yardstick for measuring multipath richness. A measurement example illustrates its use in comparison with the spatial multiplexing capability of MIMO antenna systems.

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