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Control of a two-electron quantum ring with an external magnetic field

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ABSTRACT

We investigate the use of external time-dependent magnetic field for the control of the quantum states in a two-electron quantum ring. The hyperfine interaction of the confined electrons with surrounding nuclei couples the singlet state with the three triplet states. When the external magnetic field is changed, the singlet ground state becomes degenerate with the triplet states allowing singlet–triplet transitions. By choosing different speeds for the magnetic field switching the final quantum state of the system can be manipulated. We evaluate suitable magnetic field values and time scales for efficient quantum ring control.

1. Introduction

The construction of a working quantum computer is one of the most fascinating aims of modern science. The building block of a quantum computer is qubit, a quantum bit. During recent years, several proposals for a qubit have been studied experimentally and theoretically. One of the most popular candidates has been electron spin in semiconductor nanostructures.

The control of a two-electron double quantum dot has been realized experimentally by using electric fields [1] and has also been studied theoretically [2,3]. When the electron density is concentrated over a certain radial distance from the dot center, the quantum dot is transformed to a quantum ring. The tuning of the quantum states of quantum rings by electric fields has been demonstrated experimentally [4], the energy structure of two-electron quantum rings has been focus of theoretical investigations [5–7], and feasible qubit operations for the quantum ring have also been discussed [8]. The decoherence caused by the interaction between the confined electron spin and the hyperfine spins of surrounding nuclei has been widely studied [9–11]. Recently, the control of two-electron double quantum dot by time-dependent external magnetic field has been addressed [12]. Here we apply this control method in a quantum ring. In our analysis, the electrons are confined by a harmonic radial potential. In addition, there is a Gaussian potential at the dot center. This causes the electron density to concentrate in a ring-shaped area around the origin, making a quantum ring.

In this paper, we study the control of a two-electron quantum ring using magnetic field. The lowest-lying energy states of this system are the singlet state $|S\rangle$ and three triplet states $|T_-\rangle$, $|T_0\rangle$, and $|T_+\rangle$. The hyperfine interaction couples the singlet and triplet states. When the singlet and triplet states are degenerate, singlet–triplet transitions are possible. We show that the time-dependent external magnetic field can be used to control the singlet and triplet energies in order to induce transitions from singlet to triplet state.

The rest of the manuscript is organized as follows. The physical model is discussed in Section 2. In Section 3 the numerical results are presented and conclusions are made in Section 4.

2. Model

We model the two-electron system with the Hamiltonian

$$H=\sum_{i=1}^{2}\left(\frac{(-i\hbar\nabla_i-\frac{e}{c}\mathbf{A}_i)^2}{2m^*}+V(\mathbf{r}_i,\mathbf{s}_i)\right)+\frac{e^2}{\varepsilon r_{12}}, \tag{1}$$

where the effective mass $m^*=0.067m_e$ and permeability $\varepsilon=12.7$ are material parameters for GaAs. The external potential V consists of two parts,

$$V=V_Z+V_C. \tag{2}$$

The first part, V_Z, is the potential caused by the Zeeman interaction

$$V_Z(\mathbf{r},\mathbf{s})=g^*\mu_B\mathbf{B}(\mathbf{r})\mathbf{s}, \tag{3}$$

where the Landé factor of GaAs is $g^*=-0.44$. The magnetic field can be divided into a homogeneous external magnetic field $\mathbf{B}_{ext}=\nabla\times\mathbf{A}$ and an inhomogeneous random hyperfine field $\mathbf{h}(\mathbf{r})$. The hyperfine field strength is determined from a fit to experimental data [3]. The second part in the external potential is the confinement potential V_C, that depends only on the radial coordinate. In our case it has two parts, harmonic potential

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and Gaussian peak at the center of the ring, resulting to the expression [13]

$$V_C(r) = \tfrac{1}{2}m^*\omega_0^2 r^2 + V_0 \exp(-r^2/r_0^2), \quad (4)$$

where the Gaussian peak has height $V_0 = 34$ meV, width $r_0 = 20$ nm, and the strength of the harmonic potential is $\hbar\omega_0 = 3.0$ meV. The radial potential is illustrated in Fig. 1.

We solve the lowest-energy singlet and triplet states using finite difference method with Lanczos diagonalization. The density of the singlet state of quantum ring is depicted in Fig. 2. The maximum of the singlet density is attained at the radius $r \approx 40$ nm, where the confining potential has its minimum value. The Gaussian peak added to the confinement potential at the center of the dot creates a hole in the singlet density of the quantum ring. The singlet density vanishes for radii larger than 60 nm.

The energies of the two lowest energy states as a function of the external magnetic field are shown in Fig. 3. In the calculation, the hyperfine field and the Zeeman term in the Hamiltonian are not taken into account, because these terms bring at most 0.1 meV contribution to the total energy for magnetic fields smaller than 5 T. Hence, the two lowest states are singlet state and triplet state. For zero magnetic field singlet is the ground state. As the magnetic field increases, singlet and triplet states alternate in ground state and excited state.

The singlet–triplet transitions are possible only for such magnetic field values when the singlet and triplet become degenerate. Thus, the relevant energy for the following analysis is the difference $\Delta E = E_T - E_S$ between singlet and triplet energies E_S and E_T. The energy differences between the three triplet states and the singlet state as a function of magnetic field are shown in Fig. 4. Here the Zeeman term is included in the Hamiltonian, but hyperfine term is omitted, as it does not contribute significantly to the energy difference. The mechanism of the quantum ring control is based on the fact that the energy differences change sign when the magnetic field increases. When the magnetic field

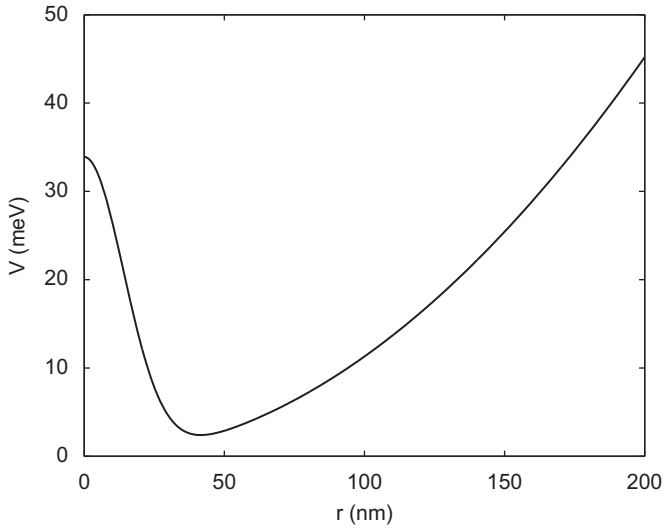

Fig. 1. The potential of the quantum ring, given by Eq. (4).

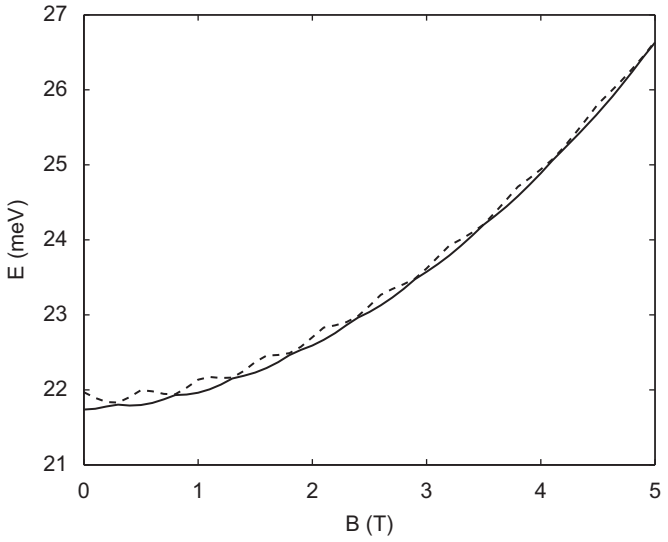

Fig. 3. Energies of the ground state (solid) and first excited state (dashed) of the quantum ring as a function of the external magnetic field. In the calculation of these energies, the hyperfine field is set to zero and the Zeeman term is neglected.

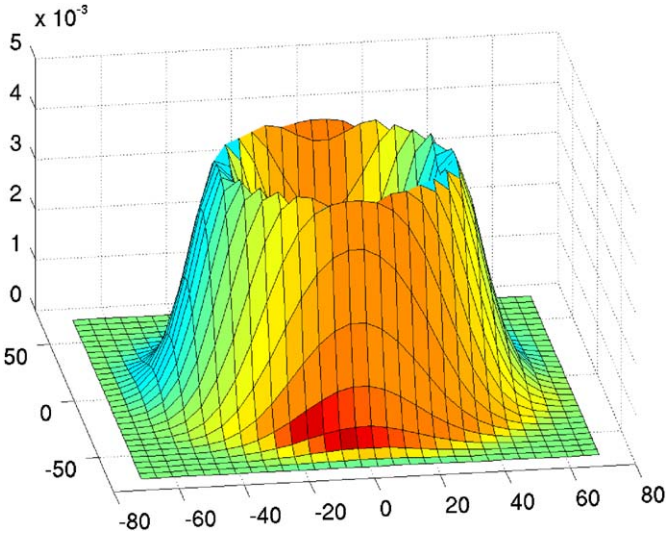

Fig. 2. Electron density of the singlet state in a quantum ring. Unit of the x and y coordinates is nm. Magnetic field strength is 1.3 T.

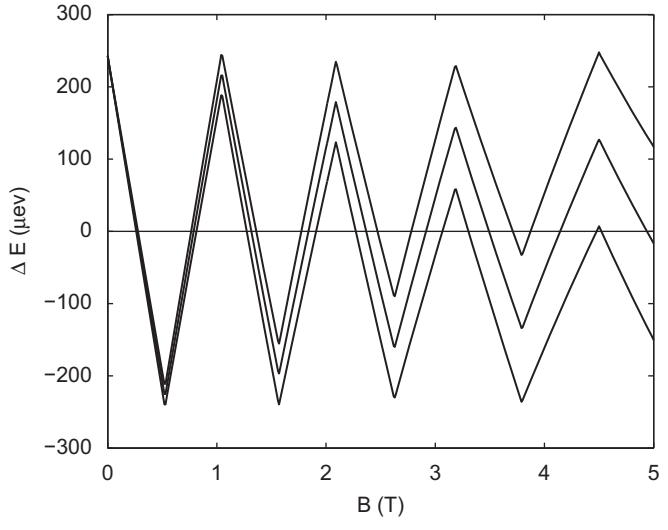

Fig. 4. The energy difference ΔE between the singlet state S and triplet states T_-, T_0 and T_+ of a double quantum dot as a function of the external magnetic field. In the calculation of these energies, the hyperfine field is set to zero.

is increased, the singlet and triplet states are degenerate for certain magnetic field values. The degeneracies of the three triplet states occur at differing magnetic fields, and the differences between the degeneracy points increase for larger magnetic fields due to the Zeeman energy.

There are several singlet–triplet degeneracies for different magnetic field values, enabling the control of the speed of the singlet–triplet transition not only by changing the sweeping time of the magnetic field, but also by choosing different magnetic field intervals, as the energy changes more slowly as a function of magnetic field for larger magnetic field values and the three degeneracy points are further apart from each other. If the confining potential is stronger than the one used in our analysis, the maximum of the energy difference increases. In such a case, the singlet–triplet degeneracies are closer to each other. Hence, the interval of the magnetic field values used in the switching would need to be much smaller.

The energy difference ΔE depends linearly on the magnetic field under many intervals. If the magnetic field is changed linearly as a function of time inside such an interval, ΔE depends linearly on time. If ΔE changes sign in the interval, the occurring transition is a Landau-Zener transition. For a two-level quantum system, the probability of a diabatic transition depends only on the off-diagonal element of the two-level Hamiltonian α, and on the time derivative of the energy difference K, according to the Landau–Zener formula [14,15]

$$P = \exp\left(-\frac{2\pi\alpha^2}{|K|}\right). \tag{5}$$

If the hyperfine field coupling the two states is random, one has to integrate the probability over Gaussian distributed α, resulting to a more complicated expression [12].

In our setup, the energies of the states above the four lowest-lying states are considerably larger than their coupling with the lowest-lying states induced by the hyperfine field. Hence, we approximate the quantum dynamics of the system using a 4×4 Hamiltonian, constructed in the basis of the singlet and three triplet states [16]. The system is initialized in the singlet state. The time dependence of the quantum states is obtained from the relation $\psi(t) = \exp(-iHt/\hbar)\psi(0)$. The randomness of the hyperfine field is

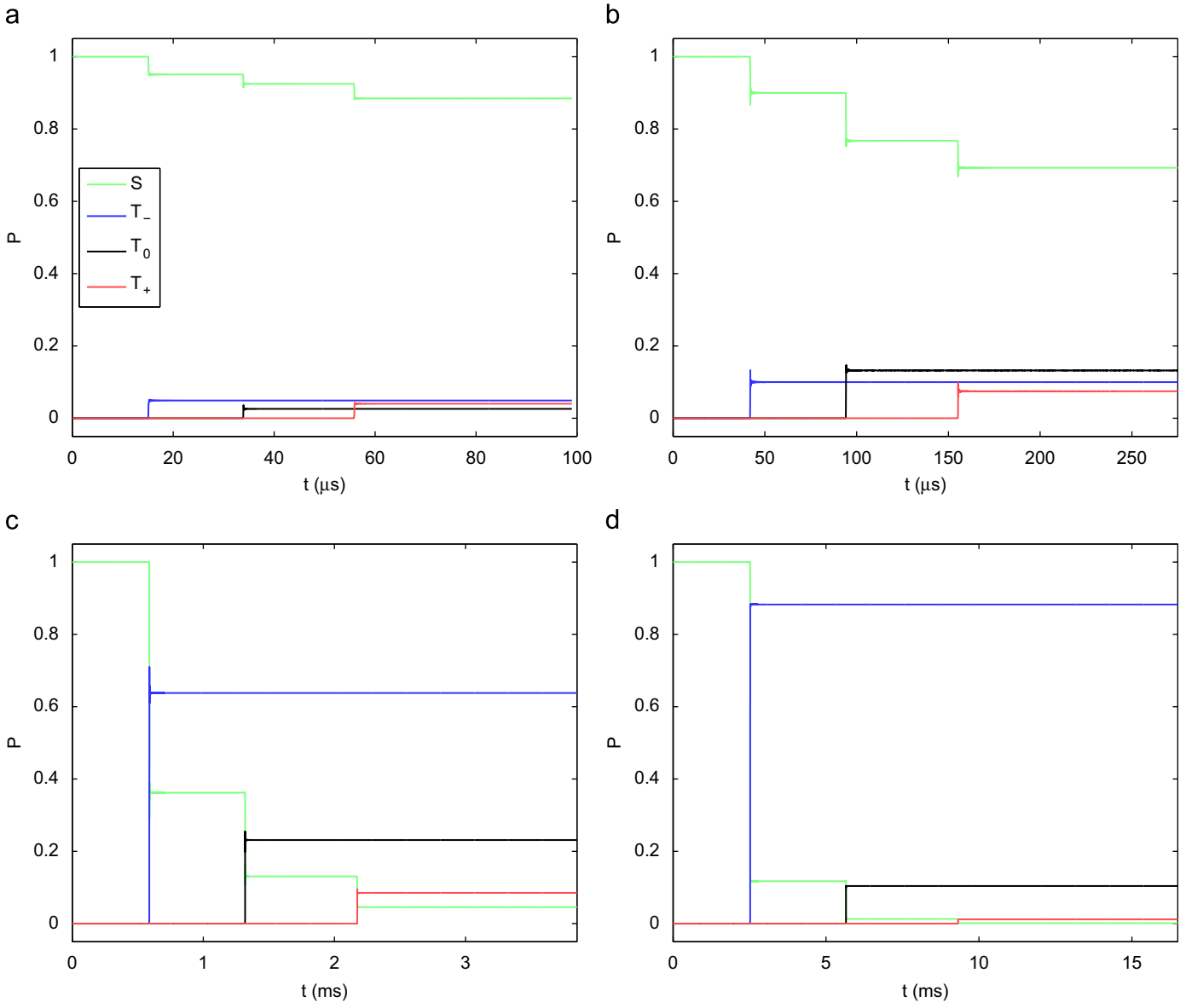

Fig. 5. Probabilities P of the singlet state S and triplet states T_-, T_0 and T_+ of a quantum ring as a function of time averaged over 1000 realizations. The time used to switch the magnetic field from 1.2 to 1.5 T is 100 μs in (a), 275 μs in (b), 3.85 ms in (c), and 17 ms in (d).

taken into account by averaging $\psi(t)$ over 1000 different realizations of the hyperfine field. The probabilities of the singlet and triplet states are obtained from the squares of the expansion coefficients of the wave function in the singlet–triplet basis.

3. Results

In the control scheme we use, the magnetic field is changed linearly from 1.2 to 1.5 T. All three triplet states have degeneracies with the singlet state inside this interval, see Fig. 4. Hence, the singlet probability decreases three times, resulting in a graph where the singlet probability has a step function form with three steps. In Fig. 5 are shown the probabilities of the singlet and triplet states as a function of time, averaged over 1000 random hyperfine field realizations, for four different magnetic field switching times: 100 μs (a), 275 μs (b), 3.85 ms (c), and 17 ms (d). All four figures indicate that the singlet–triplet transitions occur at the close neighborhood of the degeneracy points, as the probabilities stay constant until the degeneracy point is reached. The oscillations of singlet and triplet states are so rapidly damped that they are not visible in Fig. 5. Only peaks at the transition points remain as traces of the oscillations. Hence, the behavior of the probabilities resembles step function. In Fig. 5(a), the switching is done in 100 μs. Now the switching is so fast that the singlet probability does not change considerably at the degeneracy points, hence the triplet probabilities are quite small in the end. When the switching takes 275 μs in Fig. 5(b), we observe that the steps of the singlet probability are now larger, but still the singlet probability is in the end larger than the triplet probabilities. If the switching time is increased to 3.85 ms, see Fig. 5(c), we denote that the singlet probability changes considerably already at the first singlet–triplet transition. The following two transitions are thus smaller, as the singlet probability has a diminished value before the transitions take place. Finally in the Fig. 5(d), the switching time is increased to 17 ms. Now the slow switching process enables a large singlet–triplet transition at the first degeneracy point and T_+ state gains a probability close to 1. Thus, the other triplet states can only have small probabilities in the end. These differences in these four figures are caused by the exponential dependence of the singlet–triplet transition probability on the switching speed. For a single hyperfine field realization, the behavior of the singlet probability gives a qualitatively similar step function graph, where the transition probabilities are given exactly by the Landau-Zener formula. For slower switching speeds in Figs. 5(c) and (d) the T_- state has a large probability after the transition. This limits the probabilities of the T_0 and T_+ states due to the conservation of the total probability.

4. Conclusions

In conclusion, we analyzed the use of an external time-dependent magnetic field for the control of a two-electron quantum ring. The energy structure of the singlet and triplet states allows singlet–triplet transitions for certain magnetic field values. By initializing the system in the singlet state and changing the external magnetic field value continuously, so that singlet–triplet degeneracy points are crossed, all three triplet states have nonzero probabilities. By choosing different switching times for the magnetic field, one obtains a variety of final states, as they are different superpositions of the singlet and three triplet states. In the present analysis, the speed of the magnetic field change was constant during the switching process. This speed could be changed between different singlet–triplet transitions. Thus, the probabilities of the different triplet states can be easily adjusted, making this setup feasible for the construction of a qubit. Our results give the relevant time scales and magnetic field values for the experimental realization of quantum ring control. The singlet–triplet energy behaves piecewise linearly as a function of the external magnetic field. The Landau–Zener formula, valid for linear energy crossings, can be used for accurate description of singlet–triplet transition processes. The time scale of the magnetic field change from 1.2 to 1.5 T in this control scheme is of the order of 0.1–1 ms. For the time being, such a magnetic field control is quite demanding to do experimentally, but the development of experimental techniques may solve this problem in the near future.

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