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Public Transit Access and Income Segregation

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1 | Introduction

Residential segregation by income is an increasingly studied feature of cities around the world. [1] A growing literature on the topic shows that low-income families tend to be segregated in lower-opportunity neighborhoods with worse access to good schools and desirable urban amenities (Bergman et al. 2019; Fernandez and Rogerson 1996), and the spatial concentration of poverty leads to more persistent income and racial inequality and intergenerational poverty (Chetty et al. 2014; Fogli et al. 2024; Sethi and Somanathan 2004). Public transportation is often expected to bridge inequalities in access to urban goods and services by disproportionately benefiting low-income households. However, it is also often held responsible for concentrating poverty in cities (Kalachek 1968; Glaeser et al. 2008). Because public transit networks tend to be sparse relative to the road network, residential neighborhoods that are close to desirable destinations by driving may be relatively poorly connected by public transit (and vice versa). So, when high- and low-income households differ in their travel mode choices, they also tend to differ in preferences over transit- and driving-accessible neighborhoods. While carefully planned public transit expansions can improve low-income households' access to otherwise high-income residential neighborhoods, they also have the potential to segregate the poor further in low-income (and potentially low-quality) neighborhoods.

In this paper, I study the implications of mass transit expansions for residential income segregation within cities. I start by documenting systematic differences in high- and low-income commuters' usage of and residential proximity to road transit (such as buses) versus rail transit (such as trains). Consistent with these empirical observations, I propose a stylized model of high- and low-income households choosing where to reside and how to travel. I use this theoretical framework to characterize the relationship between income segregation and the spatial distribution of transit and travel mode choices. Then, I endogenize the location of public transit improvements as the choice of a transit planner trying to maximize transit ridership in the city, and I show that such a planner will expand "fast" transit (such as subways and trains) where it exacerbates income segregation but expand "slow" transit (such as buses) where it minimizes segregation.

My results serve to highlight the tension between efficient and equitable policy outcomes. To relieve increasing levels of urban

congestion and environmental pollution, local governments and financial institutions like the World Bank are slated to spend large sums on expanding mass transit infrastructure around the world.[2] As fast growing cities implement new high-speed transit infrastructure to efficiently move drivers off the streets, will they lead to more income-segregated cities? Will improved mobility leave poorer households with worse access to high-income neighbors and associated endogenous amenities? Given the persistence of such transport infrastructure (Brooks and Lutz 2019) and their role in shaping the long-run structure of cities (Heblich et al. 2020), we need to better understand how the spatial distribution of mass transit infrastructure affects income segregation in the presence of alternative policy objectives such as increasing mass transit ridership and battling negative externalities of driving.

While real cities have many correlates of public transit and income segregation that are difficult to disentangle, using a parsimonious model to study a highly stylized city allows me to isolate and better understand a single channel through which transit access is commonly believed to shape income segregation within cities: differences in the opportunity cost of travel time lead high-income households to ride the faster travel mode more and low-income households to ride the cheaper mode more. As a result, conditional on other neighborhood differences, high-income households are willing to pay more to reside in the neighborhood that is more accessible by the faster mode of travel, while low-income households are willing to pay more to reside in the neighborhood that is more accessible by the cheaper mode. Housing prices capitalize the demand for neighborhoods and lead to residential sorting by income. I show that the effect of new transit improvements on residential income segregation is minimized either when high- and low-income households use both modes of travel equally often or when both neighborhoods are equally accessible by transit relative to driving. When public transit is slow relative to driving and attracts more low-income riders, income segregation is minimized by improving transit access from the richer neighborhood. Whereas, when public transit is relatively fast (such as subways) and attracts more high-income riders, income segregation is minimized by improving transit access from the poorer neighborhood. I show that a transit planner trying to maximize ridership will improve transit access from the neighborhood with higher aggregate incomes, where there are more drivers likely to benefit from switching to riding transit. However, these transit improvements inadvertently exacerbate income segregation when transit has more high-income ridership and is relatively fast but reduce income segregation when transit has more low-income ridership and is relatively slow.

1.1 | Related Literature

The literature has long recognized that the availability, speed, and price of public transportation facilities may disproportionately affect employment opportunities for black and low-income households (Kalachek 1968). More recently, Glaeser et al. (2008) compare metropolitan areas in the United States and find that the urbanization of poverty comes mainly from better access to public transportation in central cities. The intuition behind this claim is that the opportunity for poor and rich households to sort into different travel mode choices allows them to avoid competing for the same housing stock and instead segregate across more and less transit-accessible neighborhoods. As such, urban public transportation acts as a poverty magnet. In contrast, I show that access to public transportation need not lead to greater income segregation for a host of possible travel configurations. In fact, the income sorting on public transit access can reduce income segregation arising from other spatial features of the city (such as the distribution of amenities and driving times), making urban public transportation a potential tool for income desegregation. However, I also show that transit authorities trying to increase the city's overall transit ridership are likely to expand the transit network in ways that further segregate the rich and poor, consistent with the empirical observations of Glaeser et al. (2008).

This paper also helps reconcile a growing body of empirical evidence from evaluations of new urban transportation infrastructure. The vast majority of these studies focus on high-speed mass transit expansions and often find evidence of more income segregation (Heilmann 2018; Glaeser et al. 2008; Warnes 2020), but not always (Kahn 2007; Craig 2019; Pathak et al. 2017).[3] And even when not explicitly studying segregation, researchers have found large disparities in welfare implications for the rich and the poor that are correlated with gains in overall transit ridership in ways consistent with this paper's findings (Brooks and Denoeux 2021). For instance, bus rapid transit (BRT) expansions in Bogotá (Tsivanidis 2019) led to large accessibility gains for the high-skilled and the rich as well as large gains in overall ridership. Whereas with a similarly large-scale BRT expansion in Jakarta (Gaduh et al. 2020), the gains in ridership were modest and mostly concentrated among middle and low-income households. My proposed model can rationalize these differences in transit ridership and transit-induced segregation from the perspective of income sorting on "fast" versus "slow" transit relative to alternative travel modes. More broadly, my framework allows me to offer generalizable insights about counterfactual transit expansions that would be difficult to glean from empirical case studies trying to match data moments of a specific realized transit expansion (such as Tsivanidis 2019 or Gaduh et al. 2020). While this paper focuses on one specific channel through which mass transit expansions may affect welfare, it does not study how income segregation affects welfare. Overall welfare also depends on mass transit's effects on other urban attributes (such as population decentralization, road congestion, and air pollution) that are beyond the scope of this paper but highly relevant to the discussion on optimal transit policy.

Methodologically, this paper contributes to the literature modeling the distribution of transportation infrastructure and populations within cities. A large class of spatial equilibrium theory models study income sorting within cities (Wheaton 1977 and variants) and its interaction with travel mode choice (LeRoy and Sonstelie 1983 and variants). These models tend to offer households a continuum of residential location choices that mechanically causes a discrete number of household types (such as rich and poor) to segregate perfectly across space. As a result, this framework does not allow studying segregation at the intensive margin. Empirical studies have traditionally built on this framework to document changes in population and property prices primarily as a function of proximity to the city center (Gonzalez-Navarro and Turner 2018; Baum-Snow 2007). Instead, I let the cityscape be defined flexibly using only a set of travel times with no underlying topographic constraints. Among

other things, doing so allows me to consider scenarios where mass transit networks are sparse and travel times by transit are uncorrelated (or even negatively correlated) with driving times. I also deviate from these spatial equilibrium models by having a continuum of household types sorting into a discrete number of choices, similar to Epple and Platt (1998), Sethi and Somanathan (2004), and related equilibrium sorting models where segregation can be an outcome of interest at the intensive margin.[4] While I focus on segregation by income, the framework developed in this paper could be adapted to model transit-induced segregation along other demographic dimensions of interest.[5]

The rest of this paper is organized as follows. Section 2 motivates with key relevant facts about mass transit ridership in US cities. Section 3 proposes a model of travel mode and residential location choices by heterogeneous households in a stylized city. Section 4 studies the effect of improving transit access on income segregation in a static equilibrium of the model, and characterizes how the effect varies with the type and location of transit improvements. Section 5 shows how and when income segregation arises as a by-product of transit planners try to maximize transit ridership or improve travel times for incumbent riders.

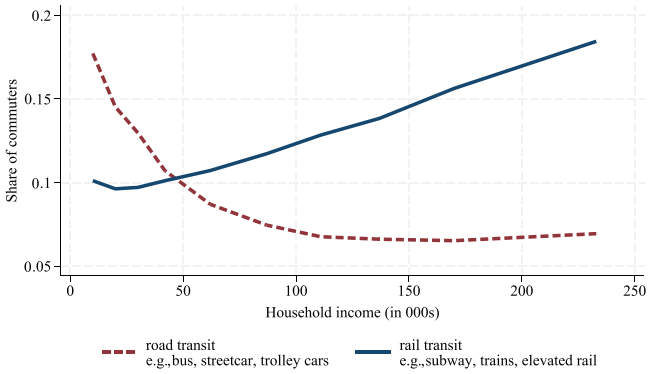
2 | Empirical Observations

Below, I highlight six key empirical facts about mass transit ridership and accessibility within US cities that motivate the rest of the paper. I focus on individual choices and census tract-level outcomes as observed in the American Community Survey (ACS) 2013–2017, ACS 2008–2012, and the 2000 Census for large US cities (Ruggles et al. 2019). I obtain census tract-level housing prices from Davis et al. (2021) and locations of mass transit stops from Transitland and Mobility Database, open-source distributors of General Transit Feed Specification (GTFS) data. See Appendix A.1 for additional data details.

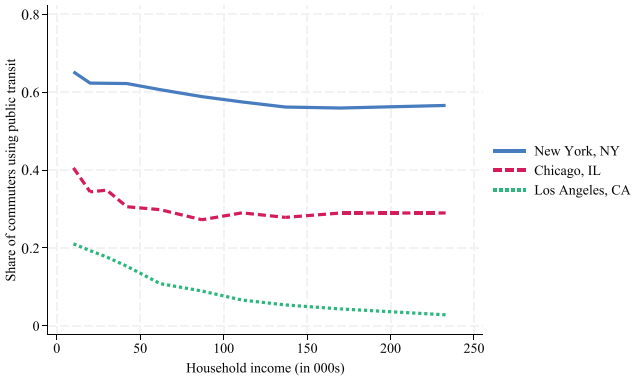
Fact 1. *Buses are used more by low-income commuters, whereas subways and rail transit are used more by high-income commuters.*

Figure 1a uses microdata for a 5% sample of individuals to plot as a function of annual household income the fraction of workers in US cities for whom the primary means of commute is either road-based (dashed red line) or rail-based (solid blue line) public transportation. I classify a commute as using road transit if the primary means are buses, streetcars, or trolley cars. I classify the commute as using rail transit if the primary means are subways, trains, or other rail transit. I find that ridership on road transit like buses is decreasing with household income, whereas ridership on rail transit is increasing with income.[6]

That bus ridership declines with income is consistent with our theoretical priors (LeRoy and Sonstelie 1983; Glaeser et al. 2008). Higher-income commuters have a higher opportunity cost of travel time and are less likely to choose slower modes of travel like buses. Whereas, lower-income commuters are more likely to choose the cheaper mode of travel, which is typically public transit. However, when mass transit is fast relative to driving (e.g., rail transit during commuting hours), we expect higher-income commuters to ride public transit more. Next, I show how transit ridership varies across cities.



(a) By mode type (road vs. rail transit)



(b) By city (the three largest US cities)

FIGURE 1 | Share of commuters by household income whose primary means of commute is either road- or rail-based public transit (a), or any public transit (b), based on a 5% sample of individuals from the 2013–2017 American Community Survey. Household incomes are censored at $300,000 per year and discretized into 10 bins, starting at $1, $15,000, $25,000, $35,000, $50,000, $75,000, $100,000, $125,000, $150,000, and $200,000. Lines connect at the observed median of each bin. I restrict the sample to all survey respondents who lived in a Census-designated city, were above 16 years of age and were working at the time of the survey. See Appendix A.1 for data preparation details.

Fact 2. *Cities with more expansive transit networks (and more rail transit) have higher transit ridership as well as a higher share of high-income commuters using mass transit.*

Figure 1b shows overall transit ridership as a function of household income separately for the three largest US cities. The city of New York, often ranked the most transit friendly large city in the United States, has a dense enough subway network that it is the most popular means of transportation for both high- and low-income commuters.[7] Public transportation in Los Angeles, on the other hand, is mostly made up of a relatively sparse network of buses. Transit ridership on commutes is much lower and is also mostly made up of low-income commuters. Chicago is somewhere in between with a dense bus network and a relatively sparse rail transit network. In all three cities, lower-income commuters are more likely to use road transit and higher-income

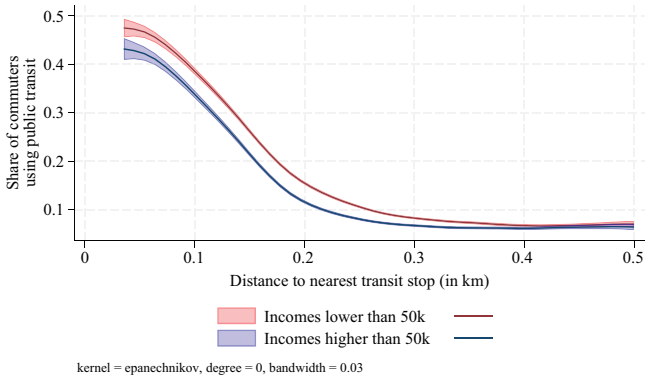


FIGURE 2 | **Share of high- and low-income commuters using mass transit, by proximity to nearest transit stops.** Figure plots kernel-smoothed functions of distance to nearest mass transit stop from the residential tract of households earning either above (in blue) or below (in red) $50,000 per year. Shaded areas depict 95% confidence intervals.

commuters are equally or more likely to use rail transit (see Figure A.2). So, unsurprisingly, overall transit ridership decreases with income more steeply in Los Angeles than in Chicago and New York.[8]

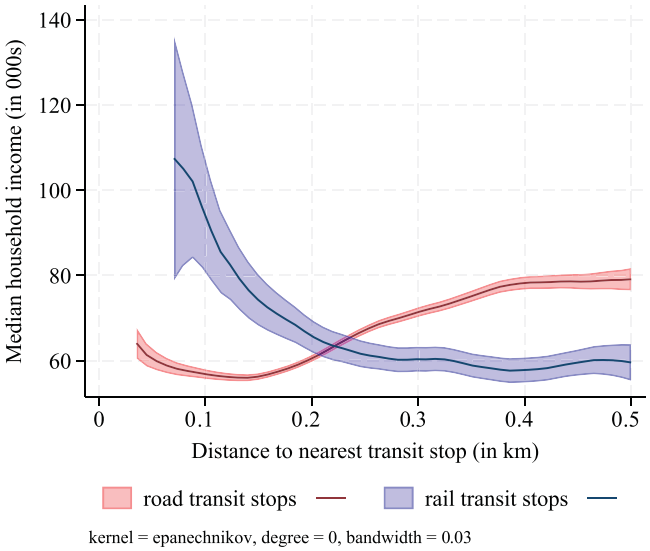
The rest of this section focuses on variation across neighborhoods within cities, where neighborhoods refer to census tracts and the extent of "cities" are defined by their corresponding metropolitan areas (CBSAs). My full sample of tracts encompass the 54 CBSAs with population over 300,000 where at least 2% of commuters use public transportation. Facts 3 and 6 are based on these 54 CBSAs. Facts 4 and 5 are based on the 30 CBSAs where at least 5% of the transit commutes use rail transit (but are robust to considering all 54 CBSAs).

Fact 3. *Commuters who reside closer to transit stops tend to ride transit more.*

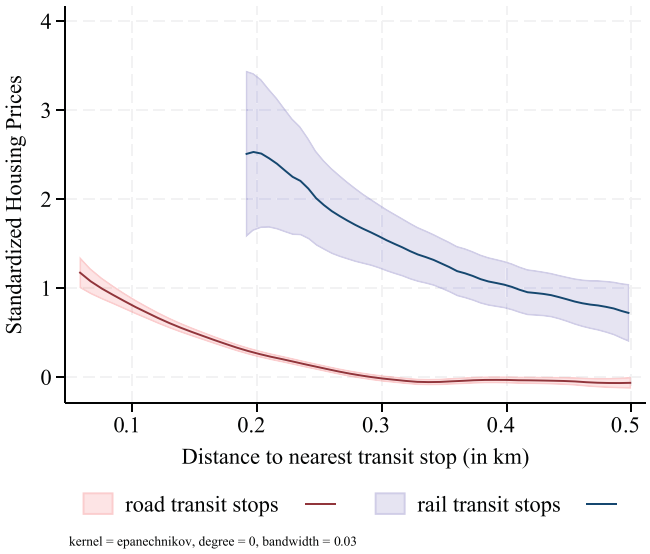
Proximity to transit stop determines how much a neighborhood's residents commute via mass transit. Figure 2 plots the share of commuters who use public transit as a function of the neighborhood's average (population-weighted) distance to the nearest transit stop. Transit ridership drops sharply with distance.

Commuters who reside in neighborhoods that are 100 m away from the nearest transit stops are four times more likely to ride transit than those in neighborhoods that are 250 m away. This relationship between transit ridership and proximity is true for both high- and low-income commuters. In fact, conditional on residing in a neighborhood that is close to both rail and bus transit stops, the rich and poor are equally likely to ride public transit (see Figure A.3), which is in stark contrast to Figure 1a where I do not condition on residential proximity to mass transit.[9] This last observation suggests that high- and low-income residential neighborhoods have systematically different access to rail versus bus transit.

Fact 4. *Higher-income households reside closer to rail transit stops. Lower-income households reside closer to road transit stops.*



(a) Median household income in census tract



(b) Housing prices

FIGURE 3 | **Neighborhood median incomes and housing price** indices (at the level of census tracts) as kernel-smoothed functions of distance to nearest mass transit (either road or rail transit) stop for the average household in the census tract (horizontal axis). Shaded areas depict 95% confidence intervals. Sample is restricted to the 30 CBSAs where at least 5% of transit commutes use rail transit. House price indices are standardized (to mean 0 and standard deviation 1) within each city, and are only available for 62% of the tracts for which incomes are reported. Therefore, sample sizes differ across the two figures.

Figure 3a plots median household incomes in census tracts by their proximity to the nearest mass transit stop. Median household incomes are lower near road transit stops and increase further away. However, median household incomes are higher near rail transit stops and decrease further away.[10] Most transit

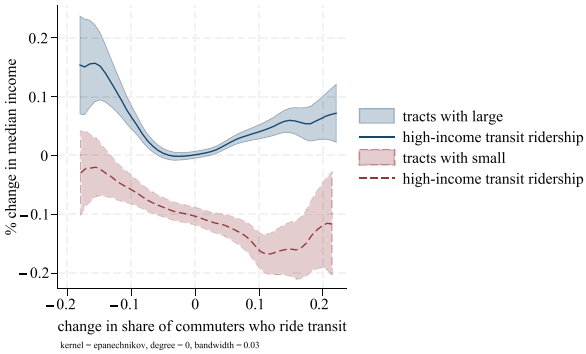
stops in cities comprise of bus stops: they make up over 97% of stops in my sample (even in a restricted sample of cities where more than half the transit commutes use rail transit). So, understandably, the literature has typically found lower-income households to reside closer to mass transit (Barton and Gibbons 2017; Pathak et al. 2017), and has deemed urban public transportation a poverty magnet in US cities (Glaeser et al. 2008). Yet, as I show here, residential sorting by income depends on the type of transit: higher-income households are sorted into neighborhoods close to rail transit stops but further away from bus stops.[11]

Fact 5. *Housing prices are higher near (both rail and road) transit stops.*

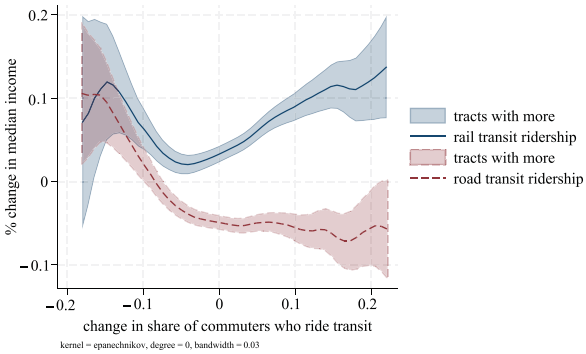
Figure 3b plots standardized house price indices for each census tract (obtained from Davis et al. 2021) against their distance to the nearest rail transit stop and the nearest road transit stop. Regardless of the type of mass transit service using the stop, census tracts closer to transit stops are likely to have higher housing prices than tracts further away.[12] This relationship is neither surprising nor unique to US cities. Mass transit expansions have been associated with increases in local housing prices in cities around the world.[13] The higher housing prices typically imply a higher willingness to pay for better transit access (and correlated urban amenities). And Fact 4 suggests that low-income households are more likely to pay this premium for better access to road transit while higher-income households are more likely to pay the premium for better access to rail transit. Such preference heterogeneity by household income may have important implications for how neighborhood income compositions respond to changes in the mass transit network. In the following empirical fact, I document how changes in transit ridership correlate with changes in neighborhood incomes within US cities between the 2000 and 2010 censuses.

Fact 6. *In neighborhoods where transit ridership is sufficiently low-income or mostly road-based, changes in transit ridership are negatively correlated with changes in median incomes. In neighborhoods where transit ridership is sufficiently high-income or mostly rail-based, increases in transit ridership are positively correlated with changes in median incomes.*

Figure 4 plots the relationship between changes in a neighborhood's transit ridership and changes in the neighborhood's median income (conditional on CBSA fixed effects). I restrict my sample to the neighborhoods where at least 5% of the commutes used public transit and median household incomes were above $5000 in the year 2000.[14] In Figure 4a, I plot this relationship separately for census tracts with small versus large high-income transit ridership, where the latter are ones where at least 15% of the public transit commuters have annual household incomes above $35,000.[15] In Figure 4b, I plot separate relationships for census tracts based on whether or not they have more rail or more road transit ridership. In neighborhoods where transit ridership is more road-based or predominantly low income, changes in public transit ridership are negatively correlated with changes in neighborhood income (relative to the average neighborhood in the CBSA), potentially due to poorer households relocating to more transit-accessible neighborhoods. But a large enough increase in transit ridership (such as by greater than 10 percentage points)



(a) By high-income transit ridership. Solid blue line: tracts where at least 15% of the public transit commuters have annual household incomes above $35,000. Dashed red line: all other tracts.



(b) By road vs. rail transit ridership. Solid blue line: tracts with higher rail transit ridership. Dashed red line: tracts with higher road transit ridership.

FIGURE 4 | Change in neighborhood incomes versus change in transit ridership between 2000 and 2010. Neighborhood units are census tracts where at least 5% of commuters ride transit and median household incomes are above $5000 in the year 2000. Vertical axis depicts the percentage change in median household incomes net of average change in median incomes in the CBSA. Horizontal axis depicts percentage point change in the share of commutes by public transit. Shaded areas depict 95% confidence intervals.

breaks that correlation, potentially because transit now attracts higher-income riders. However, in neighborhoods where transit ridership is more high-income or more rail-based to begin with, the relationship between changes in ridership and incomes is U-shaped: large decreases in transit ridership coincide with larger increases in neighborhood incomes, but so do large increases in transit ridership. Decreases in transit ridership probably coincide with neighborhoods becoming less accessible to lower-income commuters, whereas the large increases in ridership are likely generated by new high-income residents (given Facts 2–4). These patterns are consistent with case studies of new mass transit infrastructure where the rich typically gentrify new high-speed transit neighborhoods (e.g., Kahn 2007) while incomes tend to fall in neighborhoods with new bus transit services (e.g., Pathak et al. 2017).

The following section proposes a model of travel mode and residential location choices that formalizes these empirical facts to generate insights about the implications of counterfactual

transit expansions for income segregation and citywide transit ridership. I then use my theoretical framework to characterize how and when income segregation arises as a by-product of transit planners trying to maximize transit ridership.

3 | Model

3.1 | The City and Its Residents

Consider a city with two generic residential neighborhoods, $n \in \{1, 2\}$. The neighborhoods may be East and West, downtown and suburb, or some other spatially separated areas of the city. The city has a continuum of measure 1 of heterogeneous households. All households are exogenously assigned to one of two income groups $g \in \{L, H\}$. There are Λ households with low income w_L and $1-\Lambda$ households with high income w_H. Each household experiences an idiosyncratic benefit ξ from choosing to living in neighborhood 1 over neighborhood 2, where ξ is independent of income and distributed uniformly between $-\Xi$ and Ξ. Households in the city are characterized completely by their income w_g and preference ξ for neighborhood 1. The city is "closed" in that there is no possibility of migration in or out of the city and the distribution of household characteristics (w_g, ξ) are fixed. However, households may relocate costlessly across residential neighborhoods within the city.

Conditional on residing in neighborhood n, households enjoy local amenities whose consumption benefits A_{gn} may vary across income groups. The provision of these amenities is exogenously fixed. All households consume a single unit of identical housing space at (endogenously determined) market price p_n.[16] The neighborhoods have an identical and fixed supply of housing that accommodates exactly half of the city's households, and the housing prices ensure that aggregate demand for housing in each neighborhood matches the fixed supply of 1/2.[17] So, the difference in housing prices $\Delta p \equiv p_1 - p_2$ essentially captures the attractiveness of residential neighborhood 1 relative to 2.

3.2 | Travel in the City

Households in the city are faced with a continuum of measure 1 trips of different types (such as to work, to grocery stores, etc.) indexed by q. The trip destinations could be located anywhere in the continuous space surrounding the neighborhoods. For each trip, a household may use one of two modes of travel $m \in \{\textit{transit}, \textbf{car}\}$. Travel times τ_n^m depend on the household's residential neighborhood n and mode of travel m.[18] Assume, without loss of generality, that neighborhood 1 is at least as accessible by *car* as neighborhood 2: $\tau_1^c \leq \tau_2^c$. Households also incur a monetary cost of κ^m on each trip that we can assume (without loss of generality) to be larger for trips by **car** than for trips by *transit* ($\kappa^c > \kappa^t$).[19] The total travel cost of a trip is the sum of this monetary cost and the household's opportunity cost of the time spent traveling:

$$\tilde{C}_{gn}^m = \beta \cdot w_g \cdot \tau_n^m + \kappa^m, \tag{1}$$

where $\beta > 0$ is the value of a unit of time on the road as a fraction of income.[20] There are also trip-specific benefits/costs γ_q to using ***car*** over ***transit*** (such as from having a car to carry back large groceries, or from taking the transit where parking spots are scarce) that are drawn from independent and identical uniform distributions over a broad support $[-\Gamma, \Gamma]$. Let Ψ and ψ denote the c.d.f. and p.d.f. of the distribution, respectively. So, for each trip q from neighborhood n, a household from income group g chooses travel mode $m_{gn}^\star(\gamma_q)$ to minimize $\tilde{C}_{gn}^m - \gamma_q \cdot \mathbf{I}_{m=c}$. The household chooses to ride ***transit*** on trips where it is cheaper to do so:

$$\tilde{C}_{gn}^t < \tilde{C}_{gn}^c - \gamma_q$$

$$\Leftrightarrow \gamma_q < \tilde{C}_{gn}^c - \tilde{C}_{gn}^t \equiv \tilde{\gamma}(w_g; \Delta\tau_n, \kappa) \equiv \tilde{\gamma}_{gn}, \tag{2}$$

where, henceforth, $\Delta\tau_n \equiv \tau_n^t - \tau_n^c$ denotes travel time by ***transit*** relative to ***car***, $\kappa \equiv \kappa^c - \kappa^t$ denotes the difference in monetary cost of travel, and $\tilde{\gamma}(w_g; \Delta\tau_n, \kappa)$, or $\tilde{\gamma}_{gn}$ for short, is the γ needed to make a group g household indifferent between choosing ***car*** over ***transit***. The fraction of trips that is taken by ***transit*** from neighborhood n is $\Psi(\tilde{\gamma}_{gn})$. Note from (2) that ***transit*** ridership is decreasing with income w_g when ***transit*** is slower than **car** ($\Delta\tau_n > 0$), and vice versa. This relationship between transit ridership and incomes is consistent with what we observed for US cities under empirical Fact 2.

3.3 | Household Preferences

Besides travel and housing costs, households spend the rest of their incomes on consuming x of some composite good at a normalized price of 1. Conditional on living in neighborhood n, each household chooses their consumption x and distribution of travel modes $m^\star$ to maximize the following linear utility function:

$$U_{gn}(x, m^\star; \xi) \equiv x + A_{gn} + \xi \cdot \mathbf{I}_{n=1} + \int_{\gamma : m_{gn}^\star(\gamma) = c} \gamma \cdot d\Psi(\gamma) \tag{3}$$

subject to the budget constraint

$$x = w_g - p_n - \int_\gamma \tilde{C}_{gn}^{m_{gn}^\star(\gamma)} \cdot d\Psi(\gamma). \tag{4}$$

In words, households derive utility from local amenities, private consumption, idiosyncratic taste ξ for neighborhood 1 over 2, and net benefits from choosing to ride **car** over ***transit*** (aggregated across all their trips). The household's consumption is constrained by their income w net of expenditure on housing and travel (resulting from optimal travel mode choices). We can substitute the budget constrained consumption x from (4) into (3) to yield the following indirect utility from choosing to live in neighborhood n:

$$V_{gn} \equiv w_g - p_n - C_{gn} + A_{gn} + \xi \mathbf{I}_{n=1}, \tag{5}$$

where C_{gn} is the minimized total travel cost (both monetary and psychological) resulting from optimal travel mode choices on each trip.[21] Households reside in neighborhood 1 in equilibrium when they have (w_g, ξ) such that $V_{g1} \geq V_{g2}$ or, equivalently:

$$\xi > C_{g1} - C_{g2} - A_{g1} + A_{g2} + p_1 - p_2 \equiv \bar{\xi}(w_g; \Delta p) \equiv \bar{\xi}_g, \tag{6}$$

that is, if the household has a high enough relative taste for living in neighborhood 1 given its travel and housing costs and gains

from local amenities. The expression for $\bar{\xi}(w_g; \Delta p)$, or $\bar{\xi}_g$ for short, denotes the taste difference ξ needed to make a household with income w_g indifferent between the two neighborhoods. $\bar{\xi}_g$ also characterizes the mapping of household characteristics to their choice of residential neighborhood for any given pair of housing prices. Let F denote the c.d.f. of the uniformly distributed ξ. Then, the measure (and fraction) of households of type g living in neighborhoods 1 and 2 are $1 - F\left(\bar{\xi}_g\right)$ and $F\left(\bar{\xi}_g\right)$, respectively. For narrative convenience, I assume from here on that tastes ξ have broad enough distributional support so that neither income group is perfectly segregated in one of the residential neighborhoods (i.e., $\bar{\xi}_g$ is always strictly within the support of the distribution of ξ).[22]

3.4 | Equilibrium

A competitive equilibrium of the model is characterized by: (i) exogenously fixed parameters $\left\{\beta, \Lambda, \kappa, \{w_g, A_{g1} - A_{g2}\}_{g\in\{L,H\}}, \{\tau_n^m\}_{m\in\{c,t\},n\in\{1,2\}}\right\}$, (ii) households making optimal travel mode choices according to (2) given trip-specific travel costs, (iii) households making optimal neighborhood choices according to (6) given aggregate travel costs, local amenities, and housing prices, and (iv) housing price difference Δp across neighborhoods such that the aggregate demand for housing in each neighborhood equals the fixed housing supply:

$$\Lambda \cdot F\left(\bar{\xi}_L\right) + (1 - \Lambda) \cdot F\left(\bar{\xi}_H\right) = \frac{1}{2}. \tag{7}$$

It is easy to show that for any set of exogenous parameters, a unique value of Δp clears this housing market and there exists a unique analytically solvable equilibrium.[23] In later sections, I allow relative ***transit*** travel times $\Delta\tau_n$ to be updated by a hypothetical urban planner who is trying to either minimize segregation or maximize transit ridership.

3.5 | Travel Mode Choices

I now characterize households' optimal choices of travel mode m and neighborhood n. We know from (2) that $\tilde{\gamma}_{gn}$ and, hence, the fraction $\Psi\left(\tilde{\gamma}_{gn}\right)$ of a household's trips by ***transit*** is decreasing in income when ***transit*** is slower than **car** ($\Delta\tau_n > 0$) and increasing in income when ***transit*** is faster ($\Delta\tau_n < 0$). In other words, the higher the household's income the more they ride the faster mode of travel. So, as in empirical Fact 1, high-income households ride transit more when it is faster than driving (as is typically the case with rail-based transit) and low-income households ride transit more when it is slower than driving (as is typically the case with road-based transit).

As we decrease travel times by ***transit*** (relative to **car**), all households increase their ***transit*** ridership. But because higher-income households have a higher opportunity cost of time, $\tilde{\gamma}_{Hn}$ increases more than $\tilde{\gamma}_{Ln}$ and high-income households switch to riding ***transit*** more than low-income households. As a result, like in empirical Fact 2, when ***transit*** is faster, all households ride ***transit*** more but a higher share of the ***transit*** rides are by high-income households.[24]

Finally, note that the ***transit*** ridership for both income groups are higher in neighborhood 1, $\Psi\left(\tilde{\gamma}_{g1}\right) > \Psi\left(\tilde{\gamma}_{g2}\right)$, when ***transit*** travel times (relative to **car**) are lower in neighborhood 1 than in neighborhood 2 ($\tilde{\gamma}_{g1} > \tilde{\gamma}_{g2} \Leftrightarrow \Delta\tau_1 < \Delta\tau_2$). In other words, consistent with empirical Fact 3, regardless of income, households ride transit more when they reside in a neighborhood that is easier to access by transit.[25]

3.6 | Residential Location Choices

Residential location choices are dictated by the parameter $\bar{\xi}_g$. At any given income w_g, households with high $\xi > \bar{\xi}_g$ reside in neighborhood 1, while those with low $\xi < \bar{\xi}_g$ reside in neighborhood 2. First, consider the more straightforward case where local amenities are the same everywhere ($A_{g1} = A_{g2}$ for $g \in \{L, H\}$), **car** is infinitely slow and everyone always rides ***transit***. Then, $\bar{\xi}_g$ is a linear function of income and the difference in ***transit*** travel times: $\bar{\xi}_g = \beta \cdot w_g \cdot (\tau_1^t - \tau_2^t) + A_2 - A_1 + \Delta p$.[26] Suppose neighborhood 1 offers shorter travel times: $\tau_1^t < \tau_2^t$. Because higher-income households have a higher opportunity cost of travel time, they require a smaller ξ to make them choose the closer neighborhood 1 over the distant one 2. Thus, $\bar{\xi}(w_g)$ is decreasing in income, and the closer neighborhood 1 has more high-income households, while the distant neighborhood 2 has more low-income households.[27] The larger the difference in travel times between the two neighborhoods, the steeper is the slope of $\bar{\xi}(w_g)$ with respect to income, and the worse is the sorting by income (with a slope of 0 indicating perfect income mixing). With two modes of travel, $\bar{\xi}(w_g)$ is not a linear function of income but its gradient is still proportional to the difference in *average* travel times (weighted by travel mode share) between the neighborhoods.

Note that the differences in housing prices Δp and neighborhood amenities $A_{g2} - A_{g1}$ do not affect the degree of income sorting. They determine the intercept of $\bar{\xi}(w_g)$, but not its gradient relative to income. Even so, endogenous housing prices make sure to set the intercept of $\bar{\xi}(w_g)$ where it distributes the households evenly across neighborhoods and clears the housing market according to (7). So, if amenities and **car** travel times are the same across neighborhoods, then the neighborhood with the shorter ***transit*** time would compensate for their attractiveness by having higher housing prices.[28] This is true regardless of how ***transit*** travel times compare to **car** travel times and is consistent with empirical Fact 5.

Finally, how are high- and low-income households sorted between the two neighborhoods in an equilibrium where both modes of travel are in use? For narrative convenience, I deem a neighborhood n "richer" or "higher-income" if the share of the city's high-income households residing in n is larger than the share of its low-income households in n. Conversely, the "poorer" or "lower-income" neighborhood has a higher share of the city's low-income households than the city's high-income households.[29] For example, $\bar{\xi}_L > \bar{\xi}_H$ is equivalent to a richer neighborhood 1 and a poorer neighborhood 2, and vice versa.

Proposition 1. *Suppose $\tilde{\gamma}_{gn} > 0$ for all g, n. There exist constants $\bar{\tau}$ and $\bar{K}$ such that neighborhood 1 is higher-income ($\bar{\xi}_L > \bar{\xi}_H$) if and*

only if

$$(\Delta\tau_2 - \Delta\tau_1) \cdot (\Delta\tau_1 + \Delta\tau_2 - 2\bar{\tau}) < \bar{K},$$

where $\bar{\tau} \equiv (\kappa + \Gamma)/(\beta \cdot (w_H + w_L))$, $\bar{K} \equiv Q^{-1} \cdot \left[A_{H1} - A_{H2} - A_{L1} + A_{L2} + \beta \cdot (w_H - w_L) \cdot (\tau_2^c - \tau_1^c)\right]$ *is a function of how much more attractive amenities* A_{gn} *and car travel times* τ_n^c *are in neighborhood 1 (relative to neighborhood 2) for high-income households (relative to low-income households), and* $Q \equiv (1/2)\beta \cdot \psi \cdot (w_H^2 - w_L^2)$.

The proof follows in Appendix A.2.3. Suppose neighborhoods 1 and 2 have the same difference in *car* travel times and local amenities ($K = 0$). Then, Proposition 1 implies that if average *transit* travel times are small ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$), neighborhood 1 is higher income when *transit* is more accessible from neighborhood 1 ($\Delta\tau_2 > \Delta\tau_1$) and neighborhood 2 is higher income when *transit* is more accessible from neighborhood 2 ($\Delta\tau_2 < \Delta\tau_1$). On the other hand, if average *transit* travel times are large ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$), neighborhood 1 is higher income when *transit* is less accessible from neighborhood 1 ($\Delta\tau_2 < \Delta\tau_1$) and neighborhood 2 is higher income when *transit* is less accessible from neighborhood 2 ($\Delta\tau_2 > \Delta\tau_1$).

Remark 1. *Suppose neighborhoods are equally attractive in their car travel times and local amenities* ($\bar{K} = 0$). *When transit is slow on average* ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$), *the more transit-accessible neighborhoods are lower-income. When transit is fast on average* ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$), *the more transit-accessible neighborhoods are higher-income.*

Intuitively, when *transit* is fast relative to *car*, high-income households ride *transit* more and are more willingness to outbid low-income households for housing in the neighborhood where *transit* is faster. But, when *transit* is slow, high-income households ride *car* more and are less willing to outbid low-income households for housing where *transit* is faster. The implications of this result are consistent with empirical Fact 4 from earlier that median neighborhood incomes are higher closer to rail transit ("fast" on average) but lower closer to road transit ("slow" on average).

Of course, what entails "slow" versus "fast" transit in this context depends on the parameter $\bar{\tau}$, which is a function of the difference in time-invariant costs (κ) of *car* versus *transit* and the opportunity costs of travel time (βw_g). Increasing the time-invariant cost of *car* or decreasing the opportunity costs of time (without altering *transit* times) can make a previously "slow" *transit* fast enough to attract high-income households to more *transit*-accessible neighborhoods. More generally, Proposition 1 implies that if local amenities and/or *car* travel times make neighborhood 1 either more ($\bar{K} > 0$) or less ($\bar{K} < 0$) attractive than neighborhood 2 to high-income households, then the *transit* travel time difference between neighborhoods need to compensate for the additional (un)attractiveness of neighborhood 1 in order to induce the income sorting described above.

Having characterized the city in equilibrium, the following section explores how changes in *transit* access shift household choices and income segregation. From here on, changes in *transit* access or transit improvements are presented in terms of a change in $\Delta\tau_n$.[30]

4 | Income Segregation

As a quantifiable measure of income segregation, I rely on the commonly used dissimilarity index, which is the minimum fraction of the city's population that needs to relocate to achieve perfect income mixing (i.e., the same distribution of incomes in each neighborhood). In the context of my stylized city, income dissimilarity, denoted *DISS*, is computed as follows:

$$DISS \equiv \Lambda \cdot \left|F(\bar{\xi}_L) - \frac{1}{2}\right| + (1 - \Lambda) \cdot \left|F(\bar{\xi}_H) - \frac{1}{2}\right| \tag{8}$$

$$= 2\Lambda \cdot (1 - \Lambda) \cdot f \cdot \left|\bar{\xi}_L - \bar{\xi}_H\right|, \tag{9}$$

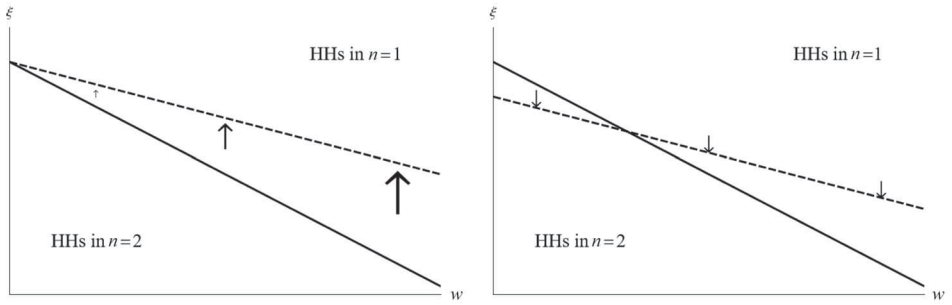
where the second equality follows from substituting in the housing market clearing constraint (7) and $f = 1/2\Xi > 0$ as the probability density over tastes ξ. Note that *DISS*imilarity is minimized when $\bar{\xi}_L = \bar{\xi}_H$, which is when low- and high-income households face similar differences in travel costs and amenity benefits across neighborhoods.

4.1 | Marginal Effect of Changes in Transit Travel Times

How do changes in travel times by *transit* affect income sorting? First, suppose we lower (relative) *transit* travel times $\Delta\tau_2$ from neighborhood 2. Lower travel costs C_{g2} shifts $\bar{\xi}_g$ up for all households with positive *transit* ridership. The magnitude of the shift will differ by income. Higher-income households gain more from shorter travel times on each trip (as they have a larger opportunity cost), but lower-income households may take more trips by *transit*. Then, as long as there is nonzero *transit* ridership in neighborhood 2, aggregate housing demand goes up, forcing housing prices to also go up (and prices to go down in neighborhood 1). Higher housing prices in neighborhood 2 result in some marginal households across all income groups moving out to neighborhood 1, shifting $\bar{\xi}_g$ down by the same amount for all households. Figure 5 demonstrates how $\bar{\xi}(w_g)$ would shift for households with different incomes following a reduction in *transit* travel times that benefits the high-income households more.[31]

In general, the income group with the larger reduction in travel costs sees a net increase in neighborhood population whereas the income group with the smaller reduction in travel costs sees a net decline in neighborhood population. To simplify the narrative, assume henceforth that the support $[-\Gamma, \Gamma]$ of the distribution of γ is broad enough to include $\tilde{\gamma}_{gn}$ for all g and n. In other words, there is positive *car* and *transit* ridership among both income groups in both neighborhoods.

Lemma 1. *Changes in the high-income share of a neighborhood's population is positively correlated with changes in transit ridership in the neighborhood when transit is faster than car, but negatively correlated with changes in transit ridership when transit is slower than car.*



(a) Travel cost effect: lower C_{g2} increases demand for neighborhood 2

(b) Housing price effect: lower Δp reduces demand for neighborhood 2

FIGURE 5 | $\bar{\xi}(w;\Delta p)$ when we reduce travel times τ_2^t from the poorer neighborhood 2.

The proof follows in the Appendix (as with all formal results from here on) but the intuition is straightforward. When *transit* is faster than *car*, marginal transit riders are more likely to be high-income. Any change that makes *transit* more popular in the neighborhood also makes the neighborhood more attractive to the high-income marginal transit riders from the other neighborhood. The opposite holds true when *transit* is slower than *car* and marginal transit riders are more likely to be low-income. Since *transit* ridership itself is a function of travel time differences across modes, the following equivalent statement emerges from the same proof.

Corollary 1. *Changes in the high-income share of a neighborhood's population is positively correlated with changes in transit ridership in the neighborhood when transit ridership is larger among high-income households (than among low-income households) in the neighborhood, but is negatively correlated with changes in transit ridership when transit ridership is larger among low-income households.*

The relationships described above are also what I document earlier in empirical Fact 6. Note that Lemma 1 and Corollary 1 only characterize correlations between changes at the margin. A large enough change can flip the direction of the correlations, such as by changing *transit* from faster to slower than *car* (or the other way around). Such cases might explain the U-shaped relationships we had observed in Figure 4, as I show later in Section 4.2. So, at the margin, what do these relationships mean for how *transit* improvements affect income segregation?

Lemma 2. *Assume* $\Psi\left(\tilde{\gamma}_{gn}\right) > 0$ *for some g (nonzero transit ridership) in neighborhood n. A marginal reduction in transit travel times* ***decreases*** *income segregation* ($dDISS/d\Delta\tau_n \geq 0$) *if and only if either:*

- *n is the* ***richer*** *neighborhood and has large enough* ***low-income*** *transit* ***ridership*** ($w_H \cdot \Psi(\tilde{\gamma}_{Hn}) < w_L \cdot \Psi(\tilde{\gamma}_{Ln})$), *OR*
- *n is the* ***poorer*** *neighborhood and has large enough* ***high-income*** *transit* ***ridership*** ($w_H \cdot \Psi(\tilde{\gamma}_{Hn}) > w_L \cdot \Psi(\tilde{\gamma}_{Ln})$).

The proof follows in Appendix A.3.3. When we reduce *transit* travel times from some neighborhood *n* (keeping *car* travel times the same), we make *n* more attractive to potential *transit* riders. Greater demand for residing in *n* (along with fixed housing supply) would drive up housing prices and drive out the households with relatively low *transit* usage (and, hence, low marginal willingness to pay for faster *transit* travel). A marginal reduction in *transit* travel times benefits more the income group that spends more on *transit* travel. So, improving *transit* travel times from a neighborhood where *transit* ridership is sufficiently high-income would attract disproportionately more high-income households to the neighborhood. There is increased income segregation if the neighborhood is already the richer of the two neighborhoods but reduced income segregation if the neighborhood is the poorer one to begin with. Similarly, improving *transit* travel times from a neighborhood where *transit* ridership is sufficiently low-income would attract disproportionately more low-income households, increasing income segregation if the neighborhood was relatively poor to begin with but reducing income segregation if the neighborhood was relatively rich to begin with. We can rewrite *transit* ridership in terms of travel times to further refine the result.

Proposition 2. *A marginal reduction in transit travel times from neighborhood n* ***decreases*** *income segregation* ($dDISS/d\Delta\tau_n > 0$) *if and only if either:*

- *n is the* ***richer*** *neighborhood and has sufficiently* ***slow*** *transit* ($\Delta\tau_n > \bar{\tau}$), *OR*
- *n is the* ***poorer*** *neighborhood and has sufficiently* ***fast*** *transit* ($\Delta\tau_n < \bar{\tau}$).

Extending Lemma 2 to Proposition 2 is straightforward. We know that *transit* riders are more likely to be high-income the faster *transit* is. Given nonzero *transit* ridership in the neighborhood, small improvements in *transit* travel times from a neighborhood with fast (slow) *transit* must attract disproportionately more high (low) income households to the neighborhood. Thus, improving fast (slow) *transit* in the richer neighborhood or slow (fast) *transit* in the poorer neighborhood would increase (decrease) income segregation.[32] This result implies that, if a transit planner wants to minimize any income segregation resulting from *transit* travel times improvements, then they would choose to improve travel times in the low-income neighborhood when transit is "fast" (like subways) and to travel times in the high-income

neighborhood when transit is "slow" (like buses on shared lanes).[33] Corollary 2 states this more formally.

Corollary 2. *A marginal reduction in transit travel time from neighborhood 2 minimizes DISS*imilarity *more than a marginal reduction in transit travel time from neighborhood 1 ($dDISS/d\Delta\tau_1 > dDISS/d\Delta\tau_2$) if and only if:*

- *neighborhood 2 is poorer ($\bar{\xi}_L > \bar{\xi}_H$) and the transit network is fast enough to attract more high-income ridership ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$), OR*
- *neighborhood 2 is richer ($\bar{\xi}_L < \bar{\xi}_H$) and the transit network is slow enough to attract more low-income ridership ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$).*

Note that the choice of neighborhood in Corollary 2 depends on average *transit* travel times in the entire city. In contrast, the result of Proposition 2 on whether or not travel time changes in a neighborhood increase segregation in the city depends only on *transit* travel times in that neighborhood.

4.2 | Income Segregation as a Function of Travel Times

The previous section focused only on *marginal* changes in travel times. But a large enough reduction in travel times (such as the opening of the city's first subway line) might turn a slow *transit* ($\Delta\tau_n > \bar{\tau}$) into a fast *transit* ($\Delta\tau_n < \bar{\tau}$) or tip the composition of *transit* riders from predominantly poor to rich. A large enough change could also flip a neighborhood from being the richer (poorer) one to the poorer (richer) one. Arguably, such events are sparse and most transit development happens incrementally. In Appendix A.2.3, I solve for a general closed-form relationship between *DISS*imilarity and travel times.

To illustrate this relationship, suppose local amenities and car travel times are the same across neighborhoods ($\tau_1^c = \tau_2^c$, $A_{H1} = A_{H2}$, $A_{L1} = A_{L2}$). Then, Figure 6a shows a contour plot of *DISS* as a function of $\Delta\tau_1$ and $\Delta\tau_2$ with darker shades indicating greater income segregation. Figure 6b graphs the travel times that minimize income segregation ($\bar{\xi}_L = \bar{\xi}_H$), depicted by two straight lines. As we move further away from these lines, income segregation increases. The upward sloping line is $\Delta\tau_1 = \Delta\tau_2$, when *transit* is equally fast/slow (relative to *car*) in both residential neighborhoods. There is perfect income mixing along this line. The downward sloping line is $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$. Recall that $\bar{\tau}$ is the cutoff for whether *transit* is fast enough (relative to *car*) to attract more high-income than low-income ridership. When $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$, the marginal benefit from shorter *transit* times (as well as *transit* ridership in the city) is the same for high- and low-income households in the city.[34] More generally, along $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$, high- and low-income households experience the same difference in travel costs between the two neighborhoods so that differences between *transit* and *car* travel times do not induce any income sorting. Consequently, income segregation is minimized either when equalizing travel times by *transit* (relative to *car*) between the residential neighborhoods, or when equalizing the difference in the relative travel costs across rich and poor households.

When *transit* is slow on average ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$, as in the top or right quadrants of Figure 6b) but relatively faster from neighborhood n ($\Delta\tau_n < \Delta\tau_j$ for $j \neq n$), low-income households are willing to pay more for the faster *transit* access from neighborhood n. As a result, n is the poorer neighborhood. Similarly, when *transit* is fast on average ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$, as in the bottom or left quadrants) and relatively faster from neighborhood n ($\Delta\tau_n < \Delta\tau_j$ for $j \neq n$), high-income households are willing to pay more for the faster *transit* access from neighborhood n, making n the richer neighborhood. Given this income sorting on *transit* access, an immediate implication of Corollary 2 is as follows.

Remark 2. *Segregation is minimized when (marginal) improvements in transit access take place in the neighborhood with relatively slower transit.*

As depicted by the arrows in Figure 6b, income segregation is minimized when:

- improving *transit* access from neighborhood 1 when $\Delta\tau_1 > \Delta\tau_2$ OR
- improving *transit* access from neighborhood 2 when $\Delta\tau_1 < \Delta\tau_2$.

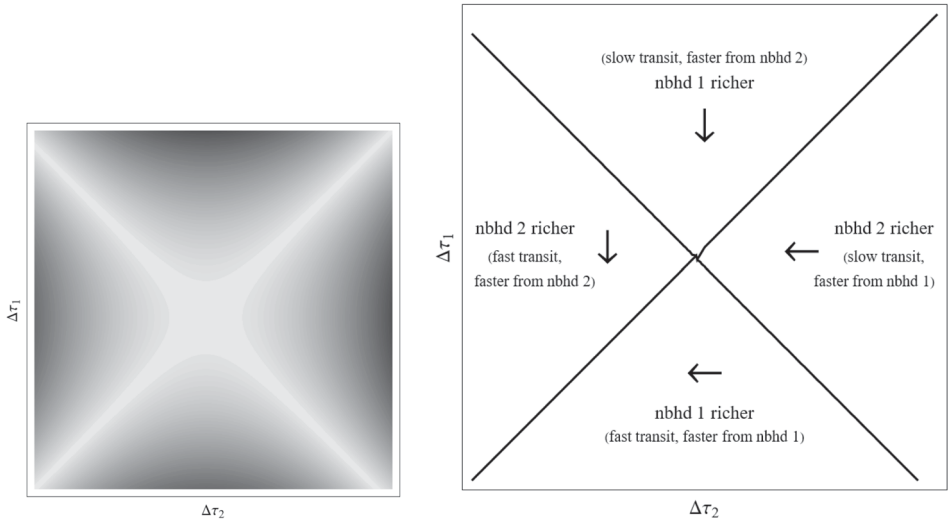
Doing so will move the city's network of travel times toward $\Delta\tau_2 = \Delta\tau_1$, where *transit* is equally fast across residential neighborhoods.

5 | Transit Ridership Trade-Off

While so far we have only focused on mass transit's effect on income segregation, the first-order outcomes of interest for transit planners are likely to be improving travel times for incumbent riders and increasing transit ridership. Besides ridership affecting private revenues, many publicly desirable outcomes, like reducing road congestion or air pollution, depend crucially on transit's ability to get drivers to switch travel modes. Even if their incentives are not directly aligned with social welfare or revenue maximization, transit planners may face political pressure to favor transit improvements that lead to immediate gains in transit ridership (though not necessarily long-term gains).[35] In this section, I endogenize the location of marginal transit improvements as the choice of a transit planner with one of three stylized outcomes of interest: (1) maximizing "short-term" *transit* ridership (ignoring induced changes in residential locations), (2) maximizing "long-term" *transit* ridership, and (3) shorter *transit* travel times for incumbent transit riders (ignoring induced changes in ridership). Then I analyze how the planner's choices affect income segregation in the city. To constrain the set of changeable parameters, assume (i) the transit planner is restricted to only improving travel times by *transit* (and not by *car*), and (ii) the marginal cost of improving travel times is constant.[36]

5.1 | Maximizing Short-Term Transit Ridership

Let R denote the fraction of trips by *transit* in the city across all households and neighborhoods. First, consider a "short-term" transit planner who only cares about immediate gains in transit



(a) Contour plot of *DISS* as a function of $\Delta\tau_1$ and $\Delta\tau_2$. Darker shades correspond to larger *DISS*imilarity.

(b) $DISS = 0$ (or $\bar{\xi}_L = \bar{\xi}_H$), corresponding to either $\Delta\tau_1 = \Delta\tau_2$ or $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$.

FIGURE 6 | Income segregation at different travel time combinations (assuming $\tau_1^c = \tau_2^c$, $A_{H1} = A_{H2}$, and $A_{L1} = A_{L2}$)

ridership, and maximizes ridership taking the income distribution in each residential neighborhood as fixed (and exogenous to travel time changes).[37] Formally, the short-term planner assumes

$$\frac{dF(\bar{\xi}_g)}{d\Delta\tau_n} = 0 \; \forall g \in \{L, H\} \text{ and } n \in \{1, 2\}$$

and chooses to improve *transit* travel times from the neighborhood n where the marginal effect (of *transit* time) on R is larger.

Proposition 3. *The short-term transit planner maximizes transit ridership R by reducing transit travel times from the residential neighborhood with* ***higher*** *aggregate income.*

The intuition behind this result is straightforward. Households with higher opportunity cost of travel time benefit more from shorter travel times. Given constant marginal *transit* ridership (due to uniformly distributed γ), the households who switch more from *car* to *transit* are the high-income households. In contrast, low-income households generally have large *transit* ridership and are less responsive to changes in *transit* travel times (as in Figure A.7). So it is optimal for the transit planner to target ridership gains among high-income households. Since the planner ignores the fact that these households might instead relocate to a different neighborhood in response, they would reduce *transit* times from the neighborhood that already has higher residential incomes.

Recall from Proposition 2 that when *transit* is relatively slow ($\Delta\tau_n > \bar{\tau}$) from the high-income neighborhood, such a policy will attract more low-income riders to the richer neighborhood and reduce income segregation. Whereas when *transit* is relatively fast ($\Delta\tau_n < \bar{\tau}$) from the high-income neighborhood, the policy will exacerbate income segregation. Continued *transit* improvements (or a large enough improvement) in the richer neighborhood can make a slow *transit* fast enough to attract more high-income riders to the already rich neighborhood. Once *transit* is fast enough everywhere, income segregation will only continue to increase with subsequent travel time reductions.[38]

There is one important caveat to this policy implication for income segregation. Because the two residential neighborhoods equally split the population of the city (in the baseline model), the neighborhood with the higher share of high-income households is also the one with higher aggregate incomes and higher marginal *transit* ridership. When neighborhood sizes are allowed to vary (as in Appendix A.2.5), a neighborhood with a significantly larger share of the city's overall population (and trips) could be relatively poor (with a lower mean income) and yet have higher aggregate residential income (and marginal *transit* ridership), so that the short-term planner would choose to improve high-speed *transit* from this neighborhood and do so without increasing income segregation.[39] In real cities, neighborhoods that have seen more *transit* development (such as downtowns) do tend to be more densely populated, consistent with Proposition 3. My results on income segregation suggests that when these neighborhoods are relatively poor, improving low-speed transit would exacerbate income segregation while improving high-speed transit would reduce income segregation.

5.2 | Maximizing Long-Term Transit Ridership

Where do *transit* travel time improvements maximize long-term *transit* ridership? Households that stay in the same neighborhood in response to marginal changes in *transit* face the same changes in long-term *transit* ridership as in the short term. What about the change in ridership among households that move?

Lemma 3. *A marginal reduction in transit travel times* $\Delta\tau_n$ *maximizes transit ridership among households that move (neigh-*

borhoods) if n is the neighborhood with smaller relative transit travel time $\Delta\tau_n$.

This is because movers increase their *transit* usage when they move in to a neighborhood with relatively shorter *transit* times, but decrease their *transit* usage when they move to a neighborhood with longer *transit* times. Now we can compare the sum of the changes in ridership among stayers and movers to characterize when lower *transit* times from neighborhood 1 increases the city's "long-term" *transit* ridership more than lower *transit* times from neighborhood 2.

Proposition 4. *A marginal reduction in transit travel times from neighborhood 1 increases transit ridership more than a marginal reduction in transit travel times from neighborhood 2* ($dR/d\Delta\tau_1 < dR/d\Delta\tau_2$) *if and only if*

$$\bar{\xi}_L - \bar{\xi}_H > \frac{K}{2},$$

where $K \equiv A_{H1} - A_{H2} - A_{L1} + A_{L2} + \beta \cdot (w_H - w_L) \cdot (\tau_2^c - \tau_1^c)$.

The constant K denotes the extent to which income sorting across neighborhoods is driven by factors other than *transit*: differences in neighborhood amenities and *car* travel times. Proposition 4 implies

Remark 3. *When* $K = 0$ *and income sorting is determined only by differences in transit access, long-term transit ridership is maximized by targeting richer neighborhoods (just as with short-term transit ridership).*

Intuitively, because high-income households have higher marginal transit ridership than low-income households, the planner's optimal policy is to nudge more high-income households to the neighborhood with the smaller $\Delta\tau$, where they will have higher transit usage. The planner does so by either: (a) improving "fast" transit in the neighborhood with the smaller $\Delta\tau$, or (b) improving "slow" transit in the neighborhood with the larger $\Delta\tau$. In the case of $K = 0$, both options correspond to improving *transit* in the richer neighborhood. As a result, like with short-term ridership, long-term *transit* ridership is also maximized by improving fast transit where it exacerbates income segregation and improving slow transit where it reduces segregation. However, when income sorting depends on non*transit* neighborhood attributes ($|K| > 0$), there are additional scenarios where improving transit in the richer neighborhood is not always the optimal policy. Appendix A.2.4 characterizes the planner's long-term ridership-maximizing policy more exhaustively.

To visualize the outcome trade-off for $K \approx 0$, Figure 7 extends Figure 6b to distinguish the optimal choice of a transit planner deciding where to improve *transit* times while maximizing *transit* ridership R as in Proposition 3 (dashed red arrows) from the optimal choice of a planner minimizing *DISS* as in Corollary 2 (solid black arrows).

When *transit* is fast enough on average to attract more high-income riders ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$), the *DISS*-minimizer would improve (fast) *transit* from the poorer neighborhood while the *R*-maximizer would do the opposite and improve (fast) *transit* from the neighborhood with higher aggregate incomes. This is true for nonmarginal/large reductions in travel times too. Whereas, when *transit* is slow on average and attracts primarily low-income riders ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$), it is optimal for the planner to keep improving *transit* from the richer neighborhood, regardless of whether they are minimizing *DISS* or maximizing *R*.[40] But once *transit* has become fast enough relative to *car* (on average) to be attractive to both high- and low-income households ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$), maximizing *transit* ridership will inevitably induce more income segregation.

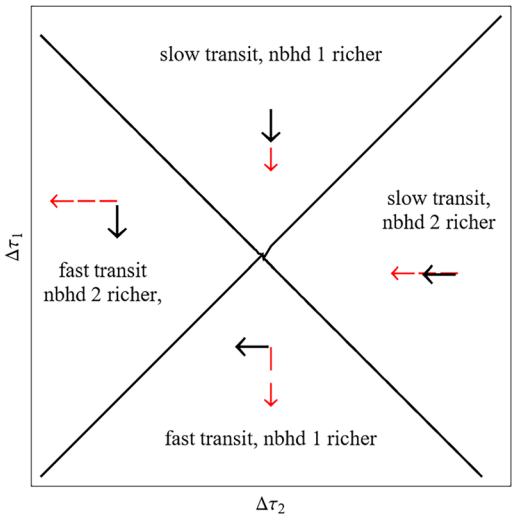


FIGURE 7 | Optimal directions of marginal *transit* improvements to minimize *DISS* (solid, black arrows) or to maximize *R*idership (red, dashed arrows). Arrows depict where to reduce travel times based on Corollary 2 and Proposition 3. Solid lines correspond to $DISS = 0$ and $\bar{\xi}_L = \bar{\xi}_H$.

5.3 | Improving Travel Times for Incumbent Transit Riders

In the United States, there is a long history of routine opposition to new transit infrastructure (both "slow" and "fast") by local residents who fear increased traffic, crime, and/or changes in the demographic composition of their neighbors (Altshuler and Luberoff 2003). On the other hand, the most ardent proponents of new transit infrastructure tend to be communities that already rely heavily on public transit access. Understandably, transit planners may have an incentive to expand transit in neighborhoods with already high transit ridership instead of trying to maximize new ridership. Consider a "naive" transit planner who only cares about reducing travel times for incumbent riders taking both the income distribution and the transit ridership in each residential neighborhood as fixed.

Remark 4. *When* $K = 0$ *and income sorting is determined only by differences in transit access, the "naive" transit planner will improve "slow" transit from the low-income neighborhood and "fast" transit from the high-income neighborhood.*

This follows directly from transit riders in "slow" transit neighborhoods being more low-income and transit riders in "fast"

transit neighborhoods being more high-income. As a result, a transit location policy that prioritizes existing ridership always exacerbates segregation when the income sorting is driven primarily by differences in transit access. Improving "slow" transit from the low-income neighborhood attracts even more low-income households to the neighborhood and improving "fast" transit from the high-income neighborhood attracts even more high-income households.

6 | Conclusion

In this paper, I document income differences in "fast" and "slow" transit ridership and in willingness to pay to reside in neighborhoods with varying access to public transit. Consistent with these observations, I propose a novel theoretical framework for characterizing travel mode choices and income segregation within cities. My model predicts that, while transit is slow relative to driving (e.g., buses on shared lanes), improving transit access to a residential neighborhood attracts more low-income residents to the neighborhood. Whereas, improving access via relatively fast transit attracts more high-income residents. Consequently, from any static equilibrium of the model, improving access by either fast transit to the high-income neighborhood or slow transit to the low-income neighborhood ends up exacerbating income segregation in the city. Whereas, improving access by either slow transit to the high-income neighborhood or fast transit to the low-income neighborhood reduces income segregation.

An important implication of this relationship between transit access and income segregation is that policies that try to benefit existing transit riders or to maximize new transit ridership often (but not always) lead to higher-income segregation. For example, improving fast transit in high-income neighborhoods helps generate new transit ridership but at the cost of displacing low-income transit riders in the neighborhood with high-income transit riders (thereby increasing segregation). On the other hand, improving low-speed transit in high-income neighborhoods is likely to increase overall transit ridership as well as reduce segregation. So, while both high- and low-speed transit access can be improved in a way that minimizes income segregation as a by-product, high-speed transit planners are likely to trade off lower income segregation for higher transit ridership.

What these trade-offs ultimately mean for social welfare (as opposed to segregation) is a bigger question beyond the scope of this paper. However, they tell us that transit's role in income segregation relies importantly on the past and ongoing choices of transit planners (and not just on incomes differences in individual demand for transit). The distinction between "fast" and "slow" transit developed in this paper is particularly relevant given powerful political incentives to overspend on salient mass transportation infrastructure such as rail transit or BRT when the negative externalities for voters are less salient (Glaeser and Ponzetto 2018), such as the long-term costs of income segregation. There are similarly strong incentives for these politicized investments to appear efficient at generating new transit ridership. Consequently, could faster travel by mass transit leave poorer households with better mobility but worse access to high-income neighbors and high-quality amenities (often funded through taxation on local income)?

While this paper has focused only on the relationship between public transit access and income segregation, future research can study the effect of segregation on endogenous amenities and other welfare-relevant outcomes. It can also distinguish the gentrification effects of trying to reduce income segregation by displacing high-income versus low-income incumbent households in a neighborhood. For example, if transit planners are averse to displacing low-income households with high-income transit riders, they may be biased toward improving only slow transit (in high-income neighborhoods) as opposed to fast transit (in low-income neighborhoods).

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Data Availability Statement

The data that support the findings of this study are openly available in OpenICPSR at https://doi.org/10.3886/E224942V1 (Akbar 2025).

Endnotes

[1] A search for "Residential segregation by income" on Google Scholar (as of July 2024) returns 20,800 articles for 2023 up from 7300 articles for 2003. Even as a share of the number of search results for "Residential segregation" (29,800 in 2023 and 13,800 in 2003), search results for "Residential segregation by income" went up from 53% in 2003 to 70% in 2023.

[2] For instance, China spent over USD 100 billion on rail transit infrastructure in 2017 alone (OECD 2019) and opened over 45 subway lines across 25 cities just between August 2016 and December 2017.

[3] There are also cross-sectional studies that have linked transit availability to greater income mixing (Blumenberg and Ong 2001; Sanchez 2002).

[4] In doing so, this work connects to a number of theoretical and empirical papers outside of the transportation literature that have modeled residential location choices similarly over a discrete set of neighborhoods to study, for instance, the effect of residential segregation on labor market efficiency (Benabou 1993) and inequality (Fogli et al. 2024) or the effect on segregation of local amenities such as public school financing (Fernandez and Rogerson 1996) and air pollution (Banzhaf and Walsh 2013).

[5] For instance, Baum-Snow and Kahn (2000) document disproportionately higher mass transit usage among black, female, college-educated, and unmarried urban residents.

[6] To be sure that this income difference in transit ridership is not due to higher incomes in cities with more rail transit, Figure A.1 restricts the sample to only the cities with both high road and rail transit ridership.

[7] New York City is rated by walkscore.com as having a Transit Score of 89 out of 100 (as of July 2024). In comparison, Los Angeles has a Transit Score of 53 and Chicago has a Transit Score of 65. The methodology behind Transit Scores is described at this link: https://www.walkscore.com/transit-score-methodology.shtml.

[8] Figure A.2 shows that the shares of commuters riding buses are comparable across the three cities, and much of the level differences in overall transit ridership are due to differences in rail transit ridership.

[9] The ACS does not make publicly available the breakdown of rail transit ridership by incomes at the granularity of census tracts or comparable subgeographies of cities: only overall public transit ridership.

[10] To make sure we are not just comparing downtowns to suburbs, I restrict the graph to only the tracts where the average census block is within 0.5 km of a transit stop. But even beyond half a kilometer, median household incomes continue to differ for road versus rail transit stops, as seen in Figure A.4a. To make sure the patterns are not driven by income differences across cities, Figure A.4c standardizes the neighborhood incomes within each city (to have mean 0 and standard deviation 1). Figure A.4b shows that the neighborhood incomes differ near rail and road transit stops (but slightly less) even when considering a wider set of CBSAs where I do not require rail transit usage to be at least 5% of transit commutes. The pattern of income sorting by proximity to mass transit stays the same.

[11] Since proximity affects transit ridership (Fact 3) and higher-income commuters reside closer to rail transit and away from road transit (Fact 4), it makes sense that Figure 1a documents smaller income differences in overall public transit ridership in a city like New York where both rail and bus transit are widespread compared to a city like Los Angeles where the rail transit network is sparse.

[12] Figure A.5 shows this downward sloping relationship with housing prices also extends to census tracts that are up to a kilometer away from the nearest transit stops.

[13] For example, see (among others): Gupta et al. (2022) in New York, Zhou et al. (2021) in Shanghai, Fesselmeyer and Liu (2018) in Singapore, Harjunen (2018) in Helsinki, Hess and Almeida (2007) in Buffalo, Baum-Snow and Kahn (2005) in 16 US cities, McMillen and McDonald (2004) in Chicago, and Dewees (1976) in Toronto.

[14] I also exclude neighborhoods with the top and bottom 1 of transit ridership changes.

[15] In Appendix A.6, I show that the relationships noted in Figure 4a are robust to alternative definitions of "high-income transit ridership."

[16] Prices p_n could be interpreted as housing rent to absentee landlords or recurring costs of homeownership (e.g., mortgage payments, maintenance, etc.). But, more generally, p_n is the price of residing in neighborhood n. In a more realistic world, this could reflect more than just the monetary cost of housing: we can interpret p_n as endogenous local (dis)amenities while A_{gn} are the exogenous local amenities.

[17] In Section A.2.5, I generalize the model to allow housing consumption and populations to vary across neighborhoods by letting housing demand be price elastic (instead of fixed). Doing so does not change the paper's main results. I also discuss the implications of allowing a price-elastic housing supply function.

[18] Appendix A.2.6 extends the model to let travel destinations (and travel times) vary with household income.

[19] Think of ***t**ransit* as a generic label for a publicly provided mass transit, such as a subway or a city bus, that runs on a fixed network of routes. Similarly, ***c**ar* could be any alternative mode of travel with a wider network of routes such as a taxi or a private vehicle.

[20] The analysis is robust to alternative functional forms of this travel cost as long as the opportunity cost and the monetary cost are additively separable. For instance, we can include a monetary cost that varies with travel time (e.g., for gas). Doing so would not change the model's qualitative predictions in any meaningful way.

[21] More specifically,

$$C_{gn} \equiv \int_{\gamma} \left(\tilde{C}_{gn}^{m_{gn}^{\star}(\gamma)} - \gamma \cdot \mathbf{I}_{[m_{gn}^{\star}(\gamma)=c]} \right) \cdot \partial \Psi(\gamma).$$

[22] This assumption is simply for narrative convenience. As I show in Appendix A.3.10, when either $\bar{\xi}_L$ or $\bar{\xi}_H$ is outside the distribution's support $[-\Xi, \Xi]$, travel time changes have no effect on income segregation. Intuitively, when $\bar{\xi}_g$ is outside of the support, all households of income w_g choose to reside in the same neighborhood ($F\left(\bar{\xi}_g\right)$ is either 0 or 1) and marginal changes in neighborhood characteristics (and $\bar{\xi}_g$) do not alter their choice.

[23] Uniqueness of Δp follows from the fact that (i) F is the c.d.f. of a uniformly distributed variable and (ii) Δp is additively separable from all other parameters in $\bar{\xi}_g$. Similarly, linearity of (2) and (6) ensure that household choices are unique given housing prices and exogenous parameters.

[24] Figure A.7 illustrates how the share $\Psi\left(\bar{\gamma}_{gn}\right)$ of a household's trips by transit would vary for households with different incomes. I show that this model relationship closely resembles the empirical relationship depicted in Figures A.3 and 1a for US cities.

[25] While Fact 3 is about a neighborhood's proximity to transit stops (because counterfactual transit and driving travel times are not observable), I assume that proximity is strongly positively correlated with transit travel times from the neighborhood. This is a fairly common assumption in the empirical literature (e.g., Baum-Snow and Kahn 2005; Glaeser et al. 2008; Heilmann 2018)).

[26] Figure A.8 provides an illustration of the function $\bar{\xi}_g = \bar{\xi}(w_g)$ at different incomes.

[27] If neighborhood 2 were more accessible ($\tau_1^t > \tau_2^t$), the reverse would be true and $\bar{\xi}(w_g)$ would be increasing in income. Let me draw some comparisons with the income sorting in standard monocentric city models (Alonso 1964; Mills 1967; Muth 1969). In these class of models, housing demand is not fixed but both price- and income-elastic (unlike in my model). So higher-income households consume more housing and have a stronger preference for locations with cheaper housing. But they also have a higher opportunity cost of travel time. Because more accessible locations are also more expensive, where the high-income residents live depends on how the income elasticity of housing demand compares to the income elasticity of travel costs. The standard monocentric city models typically frame this comparison as an empirical problem, allowing the models to be consistent with either direction of observed income sorting: the rich living in more remote locations (when income elasticity of housing demand dominates) or the rich living in more accessible locations (when income elasticity of travel costs dominates). In contrast, the model in this paper unambiguously predicts that the rich prefer the more accessible location (conditional on local amenity differences). That said, empirical estimates of the income elasticity of housing demand and land tend to be relatively small (Glaeser et al. 2008) and the empirical Facts in Section 2 indicate that higher-income households do locate in more rail-transit-accessible neighborhoods despite housing in these neighborhoods being much more expensive.

[28] Here is a quick proof. If we eliminate differences in ***c**ar* travel times ($\tau_1^c = \tau_2^c$), then $C_{g1} \leq C_{g2}$ if and only if $\tau_1^t \leq \tau_2^t$ for all income groups g. If we eliminate differences in amenities ($A_{g1} = A_{g2}$), then $\bar{\xi}_g = C_{g1} - C_{g2} + p_1 - p_2$ determines the household sorting across neighborhoods for each group g. For housing markets to clear (the same number of households residing in either neighborhood), we need housing prices such that $p_1 \geq p_2$ if and only if $C_{g1} \leq C_{g2} \Leftrightarrow \tau_1^t \leq \tau_2^t$.

[29] Note that, given the population in each neighborhood is exactly half, the richer neighborhood also has higher aggregate residential income than the poorer household.

[30] Throughout the paper, *car* travel times are fixed across neighborhoods. So a level change in relative *transit* times $\Delta\tau_n$ is equivalent to a level change in *transit* times τ_n^t.

[31] Figure 5 allows a continuous range of incomes on the horizontal axis in order to better illustrate the functional form of $\bar{\xi}(w;\Delta p)$. But note that incomes in the model are discrete and only take one of two values: w_L and w_H.

[32] The $\bar{\tau}$ in these results is the same as the one in Proposition 1, and hence so is the definition of "fast" and "slow" transit.

[33] Note that such a policy does not necessarily reduce overall income segregation because it may be that *transit* is already sufficiently fast in the richer neighborhood and slow in the poorer neighborhood, in which cases any marginal improvement in *transit* times will only induce more income segregation.

[34] Along this line, it may be that *transit* is much faster in 1 than in 2 so that high-income households choose to ride *transit* more (than the low-income households) from neighborhood 1 and ride the *car* more from 2, whereas low-income households ride the *car* more (than high-income households) from 1 and the *transit* more from 2. But the travel cost savings to the high-income households from riding the *transit* from 1 (as opposed to riding the *car* from 2) are the same as the savings to the low-income households from riding the *car* from 1 (as opposed to the *transit* from 2).

[35] The motives of transit planners are usually closely aligned with those of local public authorities. The US Federal Transit Administration (2018) reported that 54% of the transit authorities in its database are departments of local (city or county) governments. Another 28% are independent public agencies and 2% are operated by state departments of transportation.

[36] In reality, marginal costs vary a lot across travel times and space. For instance, extending a subway line may be a lot more expensive than extending a bus route. This paper abstracts away from these context-specific marginal cost differences.

[37] Alternatively, we may assume the transit planner is myopic or believes household relocation across neighborhoods to be a much slower process (e.g., due to moving costs, lease contracts, etc.) relative to households switching modes of travel across trips. In the latter case, when convergence to the new residential sorting equilibrium is slow, transit planners may be able to maximize ridership better even in the very long run by implementing the "short-term" policy.

[38] This result holds only as long as $\tilde{\gamma}_{gn}$ are in the support of γ, as we assumed earlier in the paper. For short enough travel times by *transit*, the high-income households in the richer neighborhood n may already be riding *transit* on every trip (since $\tilde{\gamma}_{Hn} > \Gamma$) and there would be no more new ridership to attract in n ($\Psi(\tilde{\gamma}_{Hn}) = 1$). At this point. the short-term planner switches to improving *transit* in the poorer neighborhood. This boundary case is not yet relevant for most real cities where *transit* is far from the strictly dominant mode of choice for high-income households.

[39] The assumption of equally sized neighborhoods serves purely to simplify exposition. All of the paper's results continue to hold for, say, arbitrary housing supplies H_n such that $H_1 + H_2 = 1$.

[40] However, a large enough improvement in even the poorer neighborhood might make *transit* fast and attractive enough for high-income households to gentrify the neighborhood (i.e., flip the income sorting across neighborhoods) and achieve the intended effect.

[41] To compare mass transit across neighborhoods within a city, I use the following crude measure of transit access at the level of census tracts. From each residential census block, I compute the straight-line distance to the nearest mass transit stop, and take its mean within each census tract weighted by block population. This is the proximity to mass transit for the average household living in the tract.

[42] In the figure, I allow incomes to vary continuously (instead of discretely between two values) on the horizontal axis in order to illustrate the functional form of $\Psi(\tilde{\gamma}_{gn})$. But note that incomes in the model actually vary discretely between values w_L and w_H.

[43] In a monocentric city, neighborhood 1 might correspond to the central business district.

[44] Traditional urban spatial equilibrium models often suppose the demand for residential land is income-elastic so that the rich have a relatively lower willingness to trade off housing space than the poor. However, empirical estimates of the income elasticity of housing demand are typically too low to explain the observed sorting of rich and poor households within cities (see Glaeser et al. 2008). Regardless, an income-elastic housing demand would change the cutoffs for when improvements in high-speed transit access minimizes income segregation more than improvements in low-speed transit access. But, qualitatively, the relationship between *transit* changes and income segregation would remain the same.

[45] The price elasticity of housing demand is $-(1+\sigma)$.

[46] For instance, Tsivanidis (2019) finds that Bogotá's new BRT service, the Transmilenio, manages to better connect residences to high-skilled job sectors as opposed to low-skilled job sectors, which are more dispersed throughout the city. Because low-skilled commuters could access low-skilled jobs relatively easily from neighborhoods further away from the Transmilenio, higher-income commuters benefited more from the transit expansion and gentrified neighborhoods closer to Transmilenio stops.

[47] θ_k could also be interpreted as the outcome of some underlying trip destination choice model. For instance, conditional on residence and travel mode, each income group can choose their destination to maximize some destination-specific utility shifter net of any additional destination-specific travel cost, where θ_k captures how costly it is (in terms of travel) to switch trip destinations.

[48] Note that in (A.10) θ_k does not depend on the destination types g that are getting shorter transit times. As such, high- and low-income destinations are symmetrically distributed with respect to each other. This assumption is simply for notational convenience. One could easily accommodate a different parameter θ_{kg} for each destination type g.

[49] Note that, unlike the results in Section 4, the outcome for income segregation does not depend on the level of *transit* ridership (as long as it is positive) or the *transit* travel time. In the baseline model, the speed of *transit* travel relative to *car* determined who benefited more from reductions in *transit* travel times and who were more likely to move in response. Now that travel time improvements can be targeted directly at a particular income group, the effect on income segregation depends only on who are being targeted.

[50] Quantitatively, the outcome depends on the destinations being connected and the magnitude of the network spillover effects.

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Appendix A

A.1 | Data and Additional Empirical Evidence

Data for the empirical facts in Section 2 and this Appendix include household survey data from American Community Survey (ACS) 2013–2017, ACS 2008–2012, and the 2000 Census. I obtain census-tract level house prices from Davis et al. (2021) and locations of mass transit stops from Transitland, an open-source distributor of General Transit Feed Specification (GTFS) data. In addition, I use a crosswalk between 2010 census blocks, census tracts, and corresponding metropolitan areas (CBSAs) (downloaded from the Missouri Census Data Center's Geocorr 2014 application) to define stable CBSA boundaries. The 2013–2017 ACS data include: (a) data downloaded from IPUMS that corresponds to microdata on individuals for a 5% sample of the population, and (b) data downloaded from NHGIS that includes aggregate measures at the level of census tracts.

A.1.1 | 2013–2017 ACS Microdata

The ACS microdata is used to construct graphs like the ones in Figures 1a,b, A.1, and A.2. It identifies the type of public transportation used by each commuters. However, the microdata do not identify the census tract or any comparable subgeography of cities and cannot be used for the subsequent graphs that investigate variation across neighborhoods within cities. I discard individuals who do not reside in a census-designated city or whose annual household income is missing/censored or above 300,000 USD. While household incomes are reported precisely, for ease of visualization, I discretize them into 10 bins starting at: $1, $15,000, $25,000, $35,000, $50,000, $75,000, $100,000, $125,000, $150,000, and $200,000. Each commuter is assigned at most a single primary means of transportation to work. Commuters include everyone for whom a means of transportation to work is nonmissing and not labeled "N/A" or "Worked at home." I identify the means of transportation to work as public transit if the commuter uses a road transit ("Bus or streetcar," "Bus or trolleybus," or "Streetcar or trolleycar"), a rail transit ("Subway or elevated" or "Railroad") or a "Ferryboat." All aggregations use the person weights included with the microdata. There are 1224 cities in the full sample and 144 cities in the sample where at least 5% of commuters use rail transit and at least 5% of commuters use road transit.

A.1.2 | 2013–2017 ACS Census Tract Data

The ACS census tract-level measures are used to construct graphs like the ones in Figures 2, 3a, A.3, and A.4a,b,c. These measures do not include a breakdown of the type of public transportation by income bracket, and thus cannot be used to construct Figure 2.1. I observe counts of commuters by means of transportation (including type of public transportation) and by income group interacted with a coarser categorization of means of transportation (that does not distinguish the type of public transportation). I also observe median household incomes in the census tract. For graphs where incomes appear on the horizontal axis, I use annual household incomes that are already binned into eight categories, starting at $0, $10,000 $15,000 $25,000, $35,000, $50,000, $65,000, and $75,000. For graphs where incomes appear on the vertical axis, I use the median neighborhood income, which is not binned but top-coded at $250,000.

A.1.3 | Time-Series Data From 2000 Census and 2008–2012 ACS

The 2000 Census and the 2008–2012 ACS data were downloaded from NHGIS' Time Series Tables collection already integrated into a panel data at the level of census tracts where tracts have been merged nominally and assigned to an integrated geography. See https://www.nhgis.org/time-series-tables for details. The time-series variables include counts of public transit commuters and all commuters (over 16 years old) who did not work from home. I use these data to plot the figures related to Fact 6. I ignore census tracts that do not coincide (by FIPS code) with tracts in my 2010 CBSA definition. The time-series variables do not include counts of transit riders interacted with income, so to identify tracts with large versus small high-income transit ridership (as in Figure 4a), I merge them in from the 2013–2017 ACS census tract data. The time-series variables also do not include counts of commuters by type of public transportation (as used in Figure 4b), so I merge them in from the (non-time-series version of the) 2000 Census. I also use the 2008–2012 public transit ridership data to determine the 54 CBSAs where at least 2% of the commuters use public transportation.

A.1.4 | Housing Prices and Distances to Transit Stops

Housing price data at the census tract level come from Davis et al. (2021) and correspond to the property value of a standardized single-family house. Prices are not available for all census tracts. Within each CBSA, I further normalize the housing prices to have mean 0 and standard deviation 1.

Finally, open-source GTFS data on mass transit stops are available by transit agency on Transitland (https://www.transit.land/) and Mobility Database (https://mobilitydatabase.org/). For most cities, this datum is only available starting in the late-2010s. First, I downloaded the transit stop data for all agencies based in the United States from both GTFS sources (Transitland and Mobility Database), and identified the ones with transit stops within my CBSA boundaries. For each CBSA, I compared the coverage of transit agencies and stops between the two GTFS sources and picked the one that is more comprehensive—in most cases, they are nearly identical. I removed agencies that operated across too many CBSAs. These included companies like Greyhound and FlixBus. I also removed agencies with too few stops or sparse networks to avoid including private shuttle services. I consolidated duplicate stops that are repeated when multiple agencies are sharing them.

For each stop, I used the GTFS route data to identify the type of transit that stopped there (list of GTFS route types available here: https://gtfs.org/schedule/reference/#routestxt). I classify route types "subway," "rail," and "monorail" as using rail transit. I classify route types "bus," "tram," "streetcar," "light rail," "cable tram," and "trolleybus" as using road transit. I leave the remaining route types ("ferry," "aerial lift," "funicular") as unclassified. These unclassified route types encompass less than 0.06% of active stops in my CBSAs. I drop 9.5% of the stops that have no routes going through. They are presumably no longer in use, but may also include cases where the GTFS information provided by the transit agencies is incomplete. I use the remaining sample of stops to compute straight-line distances from each census block to the nearest stop, the nearest rail transit stop, and the nearest road transit stop. Then, for each census tract, I compute the (block-)population-weighted average of distances to the nearest stops from each census block in the tract.

A.1.5 | Transit Ridership by Household Income

In Figure A.1, I further decompose the relationship shown earlier in Figure 1b by separately plotting the share of commuters using road transit (panel a) and rail transit (panel b) by household income and doing so separately for each of the three largest US cities. The share of commuters using road transit is comparable across the three cities. It is slightly lower in Los Angeles than in Chicago, but the differences are not as stark as for the share of commuters using rail transit. So, much of the differences in overall transit ridership between New York, Chicago, and Los Angeles depicted in Figure 1b appear to be due to rail transit ridership.

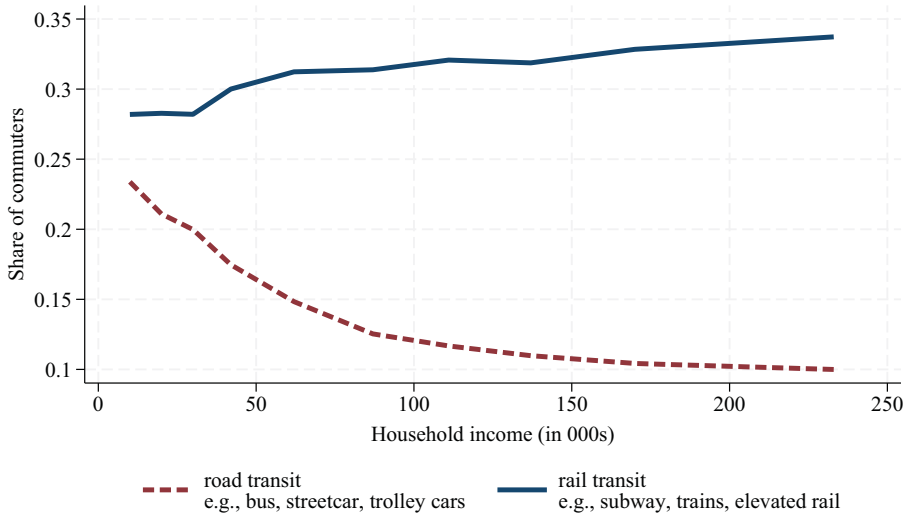
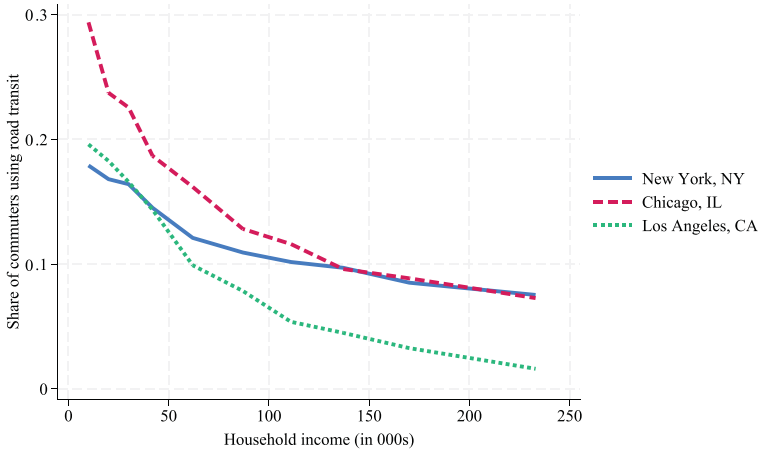
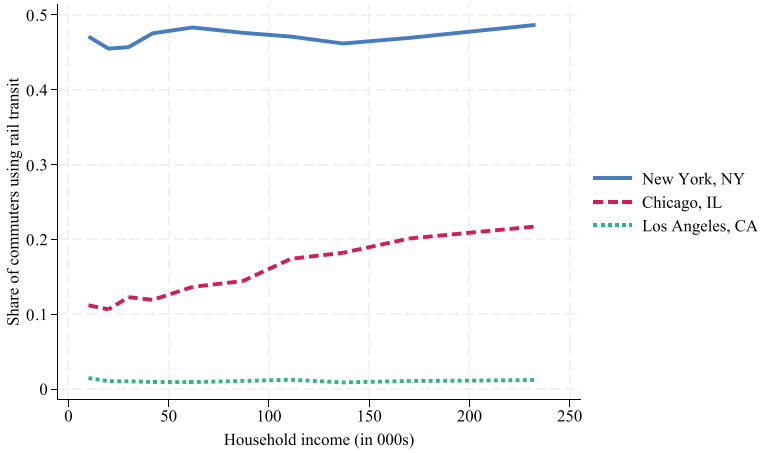


FIGURE A.1 | Version of Figure 1a but restricted to cities with both high bus and rail transit ridership (>5% of commutes using each), spanning 30 metropolitan areas. Source: 2013–2017 American Community Survey. Household income is censored at $300,000 per year and discretized into 10 bins. Lines connect points at the median of each income bin.



(a) by road transit, e.g., buses



(b) by rail transit, e.g., subways and trains

FIGURE A.2 | **Share of commuters using mass transit by household income**, based on a 5% sample of individuals from the 2013–2017 American Community Survey. Household incomes are censored at $300,000 per year and discretized into 10 bins, starting at $1, $15,000, $25,000, $35,000, $50,000, $75,000, $100,000, $125,000, $150,000, and $200,000. Lines connect at the observed median of each bin. I restrict the sample to all survey respondents who lived in a Census-designated city, were above 16 years of age and were working at the time of the survey.

A.1.6 | Income Differences in Transit Ridership Across Neighborhoods

Figure A.3 plots transit ridership in census tracts at varying proximity to transit stops.[41] Note that the level of transit ridership is higher in census tracts close to bus stops (and even higher closer to rail transit stops) than those farther away. Accordingly, cities with denser transit networks and more rail transit tend to have higher overall transit ridership (as in Figure 1b).

Note that Figure A.3 is unable to to show how ridership varies with income near rail stops far away from bus stops because there are very few census tracts that are close to rail transit but far from bus (only 53 census tracts across 54 CBSAs—and mostly very concentrated in specific areas of New York, Chicago, and DC). It makes sense for transit planners to place bus connections near rail transit stops.

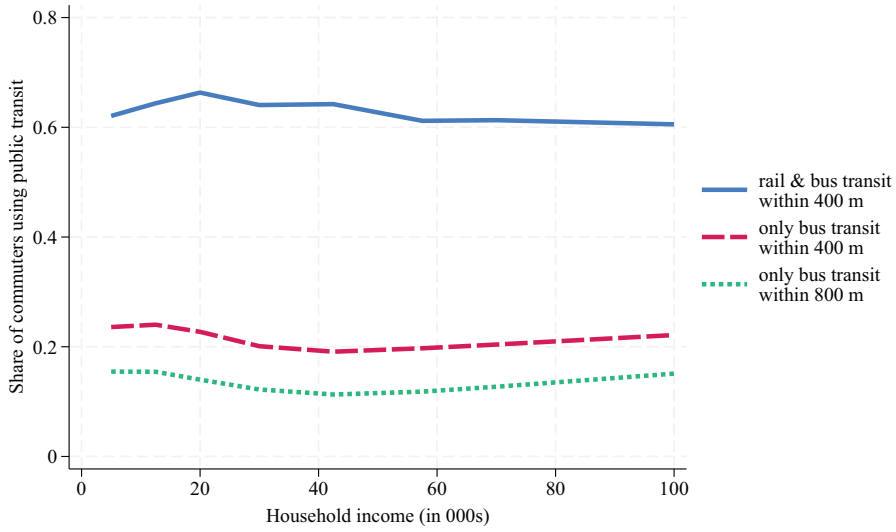
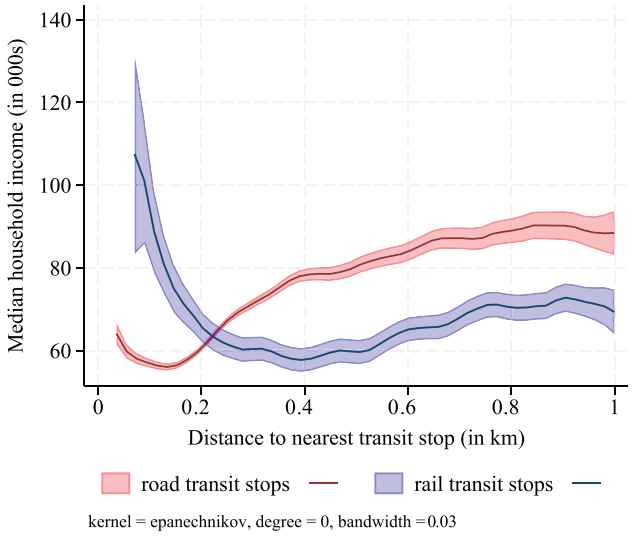
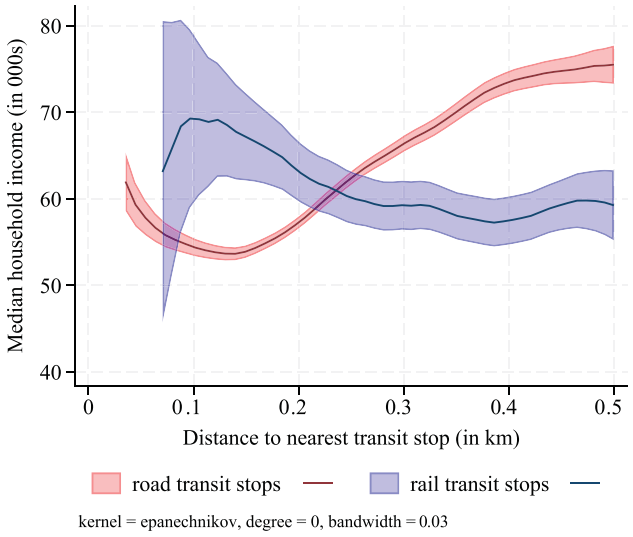


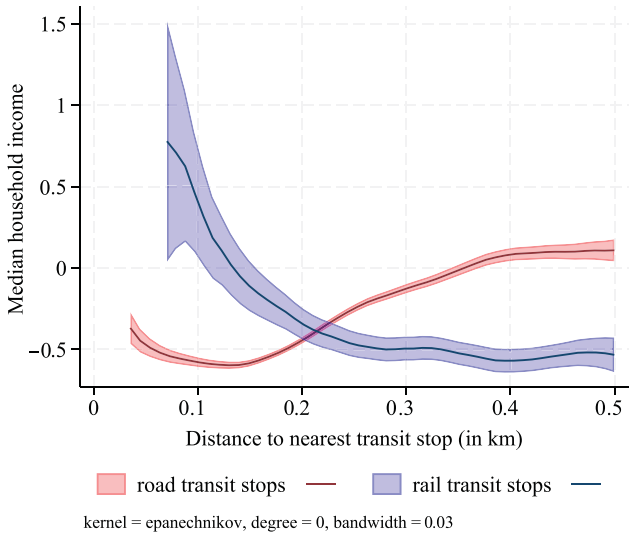
FIGURE A.3 | Share of commuters using mass transit by income and proximity to transit stops, based on census tract-level counts of commuters who use public transportation within each of eight household income brackets, starting at $0, $10,000, $15,000, $25,000, $35,000, $50,000, $65,000 and $75,000 in annual earnings. Lines on the graph connect at the midpoint of each income bracket, with $100,000 denoting the top income bracket. The blue solid line represents tracts where the average household lives within 400 m of a subway/railway stop. The dashed red line represents tracts where the average household has a bus stop but no subway/railway stop within 400 m. The dotted green line represents tracts where the average household has a bus stop but no subway/railway stop within 800 m. Sample is restricted to CBSAs where at least 20% of the transit commutes use rail transit.



(a) Beyond 0.5 km from nearest transit stops



(b) All 54 CBSAs



(c) Incomes standardized (mean 0, std. dev. 1) within cities

FIGURE A.4 | **Median household incomes in neighborhoods** as kernel-smoothed functions of distance to nearest mass transit (either road or rail transit) stop for the average household in the census tract (horizontal axis). Shaded areas depict 95% confidence interval. Figures are different variations of Figure 3a. In panels (a) and (c), the sample is restricted to the 30 CBSAs where at least 5% of transit commutes use rail transit. Panel (b) includes all 54 CBSAs where the location of transit stops are known.

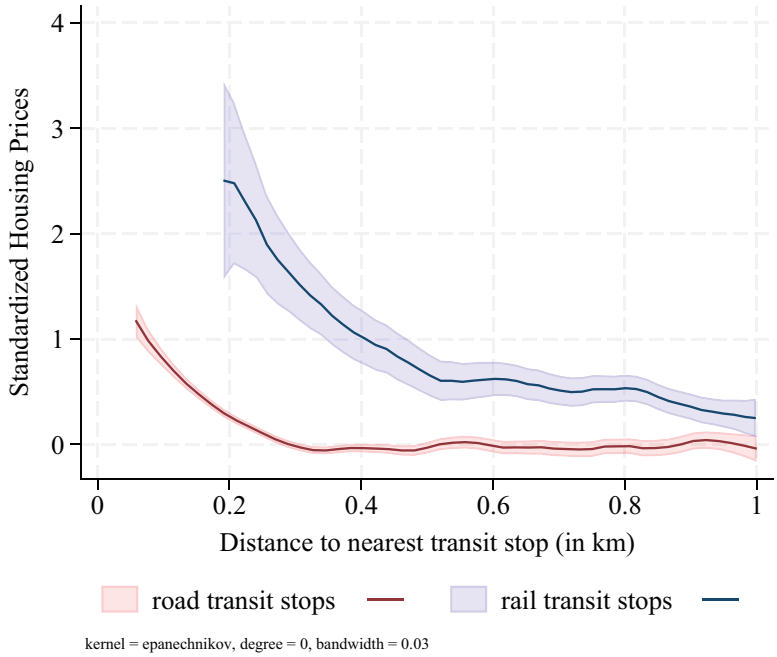
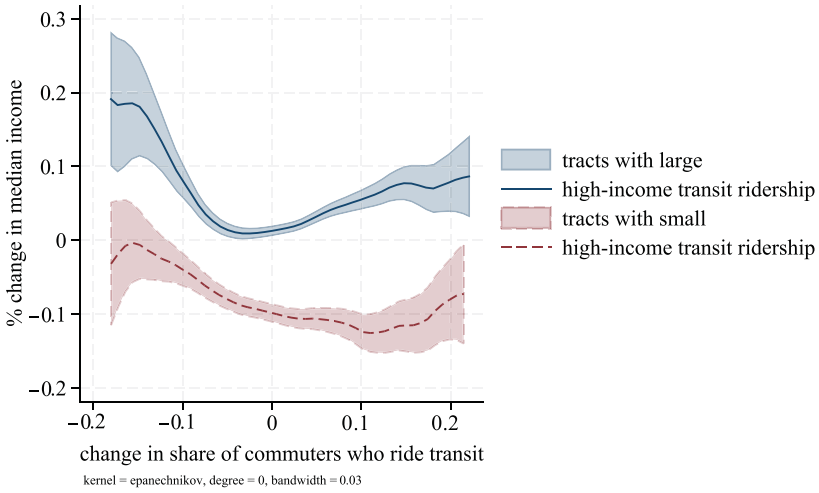
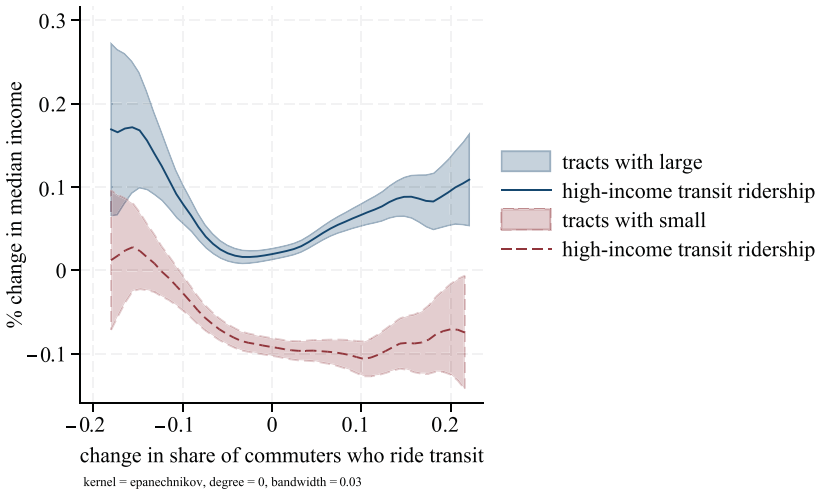


FIGURE A.5 | **Housing price indices** (for census tracts up to 1 km away from the nearest transit stops) as kernel-smoothed functions of distance to nearest road and rail transit stops for the average household in the census tract (horizontal axis). Shaded areas depict 95% confidence interval. Sample is restricted to the 30 CBSAs where at least 5% of transit commutes use rail transit. House price indices are standardized (to mean 0 and standard deviation 1) within each city, and are only available for 75% of the tracts for which incomes are reported.



(a) High-income transit ridership: at least 25% of public transit commuters have household incomes greater than $35,000



(b) High-income transit ridership: at least 15% of public transit commuters have household incomes greater than the 20th percentile of median neighborhood incomes in the CBSA

FIGURE A.6 | Change in neighborhood incomes versus change in transit ridership between 2000 and 2010. Neighborhood units are census tracts where at least 5% of commuters ride transit and median household incomes are above $5000 in the year 2000. Vertical axis depicts the percentage change in median household incomes net of average change in median incomes in the CBSA. Horizontal axis depicts percentage point change in the share of commutes by public transit. Shaded areas depict 95% confidence intervals. Solid blue line corresponds to tracts with a high share of high-income transit riders. Dashed red line corresponds to all other tracts.

A.2 | Additional Theoretical Results

A.2.1 | Transit Ridership in the Baseline Model

Figure A.7 illustrates how the share $\Psi\left(\tilde{\gamma}_{gn}\right)$ of a household's trips by transit would vary with household income.[42] Note that this model relationship between transit ridership and incomes closely resembles the empirical relationship depicted in Figures A.3 (and 1a) for US cities. Just as in empirical Fact 2, high- and low-income households differ more in their transit ridership when transit is slow and differ less when transit is fast. As *transit* becomes much faster than *car*, income differences in transit ridership widen once more (this time with the higher-income households riding *transit* more). Accordingly, when ***transit*** is faster than ***car***, transit ridership resembles the observed rail transit ridership in Figure 1a. But this is not the reality in most US neighborhoods and, hence, the upward sloping relationship is not observed empirically once we condition on the neighborhood's proximity to transit stops.

A.2.2 | Income Sorting in the Baseline Model

Figure A.8 provides an illustration of the function $\bar{\xi}_g = \bar{\xi}(w_g)$ at different incomes.

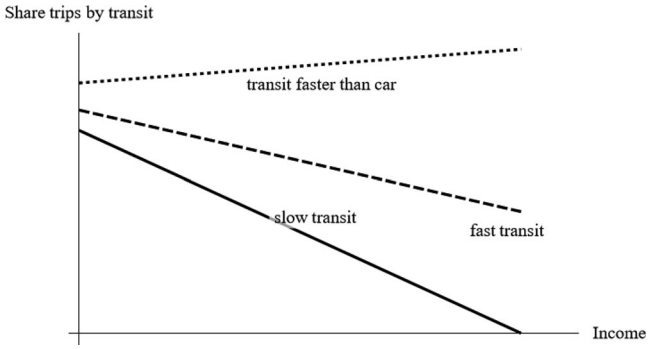


FIGURE A.7 | Fraction $\Psi\left(\tilde{\gamma}_{gn}; w_g\right)$ of trips taken by ***transit*** as a function of income when *transit* is: slow ($\Delta\tau_n \gg 0$), fast but slower than ***car*** ($\Delta\tau_n > 0$), and faster than ***car*** ($\Delta\tau_n < 0$).

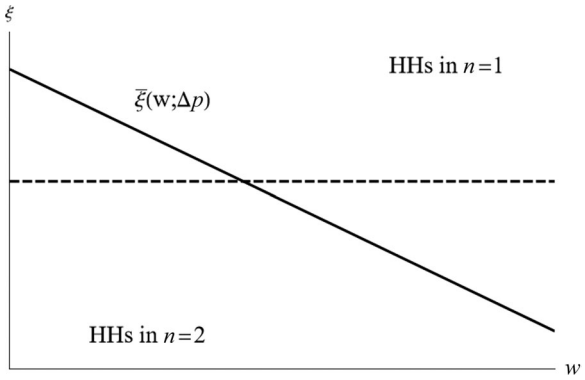


FIGURE A.8 | Graph of $\bar{\xi}(w;\Delta p)$ with $\tau_n^c \ggg \tau_n^t$ in both n, $\tau_1^t < \tau_2^t$ and $A_{g1} - p_1 < A_{g2} - p_2$. Households with $\xi_g > \bar{\xi}(w_g)$ live in neighborhood 1, while those with $\xi_g < \bar{\xi}(w_g)$ live in neighborhood 2. Dashed line depicts zero income segregation.

A.2.3 | Income Dissimilarity

This section characterizes a closed-form solution for income *DISS*imilarity as a function of *transit* ridership, travel times, and exogenous parameters of the model. I continue to assume that there is nonzero high and low *transit* ridership in both neighborhoods ($\tilde{\gamma}_{gn} \in [-\Gamma, \Gamma] \forall g \in \{L, H\}$ and $\forall n \in \{1, 2\}$).

Lemma A.1. *DISS* is proportional to $|\bar{\xi}_L - \bar{\xi}_H|$ where

$$\bar{\xi}_L - \bar{\xi}_H = \frac{1}{2\psi} \cdot \left(\Psi(\tilde{\gamma}_{L2})^2 - \Psi(\tilde{\gamma}_{H2})^2 - \Psi(\tilde{\gamma}_{L1})^2 + \Psi(\tilde{\gamma}_{H1})^2\right) + K \tag{A.1}$$

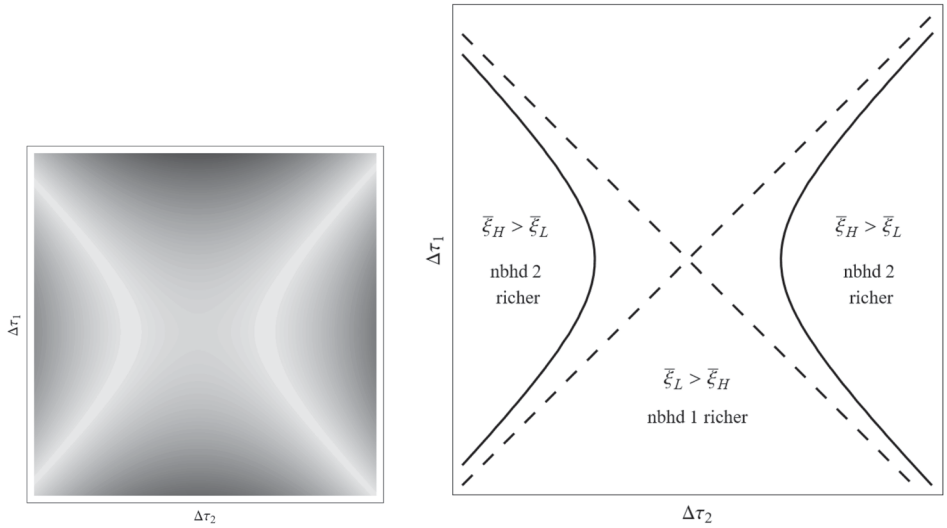
$$= Q \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot (\Delta\tau_1 + \Delta\tau_2 - 2\bar{\tau}) + K, \tag{A.2}$$

where $K \equiv A_{H1} - A_{H2} - A_{L1} + A_{L2} + \beta \cdot (w_H - w_L) \cdot (\tau_2^c - \tau_1^c)$ and $Q \equiv (1/2)\beta \cdot \psi \cdot (w_H^2 - w_L^2)$ are constants.

The proof follows in Appendix A.3.4.

That *DISS* is proportional to $|\bar{\xi}_L - \bar{\xi}_H|$ follows trivially from its definition in (9). The first equality, (A.1), characterizes income sorting across neighborhoods as a function of differences in high- and low-income *transit* ridership (squared) between the two neighborhoods. Greater differences across neighborhoods in the income sorting into travel mode choices translates to greater income sorting across the residential neighborhoods themselves. In other words, *DISS* is higher when the difference in *transit* ridership between high- and low-income households is much larger in

one neighborhood than the other. The second equality, (A.2), of Proposition A.1 substitutes in households' optimal mode choices to rewrite *DISS* purely as a function of travel time differences (between *transit* and *car*) and other exogenous parameters of the model.



(a) Contour plot of *DISS* as a function of $\Delta\tau_1$ and $\Delta\tau_2$. Darker shades correspond to greater *DISS*imilarity.

(b) Solid lines correspond to $\bar{\xi}_L = \bar{\xi}_H$ (or $DISS = 0$). Dashed lines are asymptotes, which correspond to either $\Delta\tau_1 = \Delta\tau_2$ or $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$.

FIGURE A.9 | Income segregation at different travel time combinations, when $\mathbf{K} > \mathbf{0}$.

In Section 4, I illustrated a special case of $K = 0$ where the income sorting is driven solely by differences in travel times by *transit*. When $K \neq 0$, there is a baseline level of income segregation that arises even when *transit* is the same in both neighborhoods ($\tau_1^t = \tau_2^t$): high-income households find neighborhood 1 more attractive when $K > 0$ and neighborhood 2 more attractive when $K < 0$. Assume, without loss of generality, that $K > 0$. In other words, neighborhood 1 offers better high-income amenities and shorter travel times by *car*.[43] As shown in Figure A.9, equalizing travel costs across neighborhoods (as in along the lines $\Delta\tau_1 = \Delta\tau_2$ and $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$ and nearby) is no longer the optimal solution for minimizing income segregation. Minimizing *DISS* now involves attracting some more high-income households to 2 or some more low-income households to 1 in order to offset the baseline level of income segregation. The former requires *transit* to be faster in 2 (i.e., a lower $\Delta\tau_2$) than before when *transit* is generally fast ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$ and *transit* riders are relatively high-income) while the latter requires *transit* to be slower in 2 (i.e., a higher $\Delta\tau_2$) than before when *transit* is generally slow ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$ and *transit* riders are relatively low-income). The opposite would be true for $K < 0$.

When $K < 0$, neighborhood 2 offers better high-income amenities and shorter travel times by *car*. As shown in Figure A.10, even when *car* and *transit* cost differences are the same across neighborhoods (as in along the dashed lines $\Delta\tau_1 = \Delta\tau_2$ and $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$ and nearby), minimizing *DISS* involves attracting some more high-income households to 1 or some more low-income households to 2 (relative to when $K = 0$) in order to offset the baseline level of income segregation. The former requires *transit* to be faster in 1 (i.e., a lower $\Delta\tau_1$) than before when *transit* is generally fast ($\Delta\tau_1 + \Delta\tau_2 < 2\bar{\tau}$) and *transit* riders are relatively high-income while the latter requires *transit* to be slower in 1 (i.e., a higher $\Delta\tau_1$) than before when *transit* is generally slow ($\Delta\tau_1 + \Delta\tau_2 > 2\bar{\tau}$) and *transit* riders are relatively low-income.

A.2.4 | Transit Ridership

The fraction of trips taken by *transit* in the city (across all households and neighborhoods) is given by

$$R \equiv \Lambda \cdot r_L + (1 - \Lambda) \cdot r_H, \tag{A.3}$$

$$\text{where } r_g \equiv \Psi\left(\tilde{\gamma}_{g2}\right) \cdot F\left(\bar{\xi}_g\right) + \Psi\left(\tilde{\gamma}_{g1}\right) \cdot \left[1 - F\left(\bar{\xi}_g\right)\right], \tag{A.4}$$

where r_g denotes the total *transit* ridership among households of type g. The marginal effect on total *transit* ridership R of a *transit* time change in neighborhood n is given by

$$\begin{aligned}\frac{dR}{d\Delta\tau_n} &= \mathbf{E}\left[\frac{dr_g}{d\Delta\tau_n}\right] \\ &= \underbrace{\mathbf{E}\left[\frac{d\Psi\left(\tilde{\gamma}_{gn}\right)}{d\Delta\tau_n} \cdot \left(\mathbf{I}_{n=2} \cdot F\left(\bar{\xi}_g\right) + \mathbf{I}_{n=1} \cdot \left[1 - F\left(\bar{\xi}_g\right)\right]\right)\right]}_{\text{change in } \textit{transit} \text{ ridership among stayers, } S_n}\end{aligned}$$

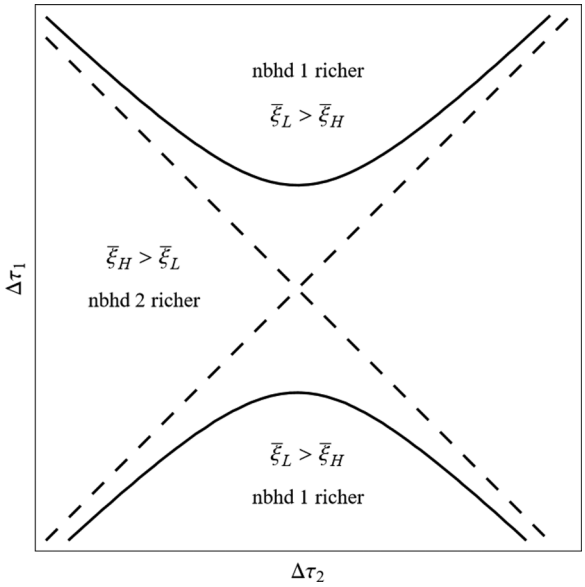


FIGURE A.10 | Areas with $DISS = 0$ for $K < 0$. Solid lines correspond to $\bar{\xi}_L = \bar{\xi}_H$. Dashed lines are asymptotes of $\bar{\xi}_L = \bar{\xi}_H$, which corresponds to either $\Delta\tau_1 = \Delta\tau_2$ or $\Delta\tau_1 + \Delta\tau_2 = 2\bar{\tau}$.

$$+ \underbrace{\mathbf{E}\left[\frac{dF(\bar{\xi}_g)}{d\Delta\tau_n} \cdot \left(\Psi(\tilde{\gamma}_{g2}) - \Psi(\tilde{\gamma}_{g1})\right)\right]}_{\text{change in } transit \text{ ridership among movers, } M_n}, \tag{A.5}$$

where $\mathbf{E}[\cdot]$ denotes the expectation over the distribution of w. The first additive component of $dR/d\Delta\tau_n$ in (A.5) is the marginal change in *transit* ridership in neighborhood n among households that continue to stay in the same neighborhood or the change in overall *transit* ridership assuming no households move ($\partial F\left(\bar{\xi}_g\right)/\partial\Delta\tau_n = 0$). Let S_n denote this "short-run" change in *transit* ridership. The second additive component, denoted M_n, is the net change in *transit* ridership among households moving neighborhoods in response to changing travel costs and housing prices. Together, they sum up to the total "long-run" change in *transit* ridership in the city. In this section, I identify when the marginal effect on *transit* ridership is larger from *transit* time reductions in neighborhood 1 rather than in neighborhood 2. In other words, when is $(dR/d\Delta\tau_2) > (dR/d\Delta\tau_1)$? Alternatively, when is the following expression positive?

$$\frac{dR}{d\Delta\tau_2} - \frac{dR}{d\Delta\tau_1} = \underbrace{\mathbf{E}\left[\frac{d\Psi(\tilde{\gamma}_{g2})}{d\Delta\tau_2} \cdot F(\bar{\xi}_g) - \frac{d\Psi(\tilde{\gamma}_{g1})}{d\Delta\tau_1} \cdot \left[1 - F(\bar{\xi}_g)\right]\right]}_{S_2 - S_1}$$

$$+ \underbrace{\mathbf{E}\left[\left(\frac{dF(\bar{\xi}_g)}{d\Delta\tau_2} - \frac{dF(\bar{\xi}_g)}{d\Delta\tau_1}\right) \cdot \left(\Psi(\tilde{\gamma}_{g2}) - \Psi(\tilde{\gamma}_{g1})\right)\right]}_{M_2 - M_1}. \tag{A.6}$$

Proposition 3 claims $S_2 > S_1$ (i.e., *transit* ridership among nonmovers is maximized from a *transit* time reduction in neighborhood 1) if and only if neighborhood 1 has higher residential incomes than neighborhood 2.

Marginal Effect of Transit Travel Times on Long-Term Transit Ridership

While a formal proof of Proposition 4 (where to improve *transit* to maximize long-term ridership) is in Appendix A.3.8, this section goes over the intuitions underlying the effect of marginal *transit* time reductions on overall "long-term" *transit* ridership R. In the long term, since households can relocate costlessly and compete for the housing stock in either neighborhood, the planner would have to improve *transit* in the neighborhood where: (a) the households that would switch more to *transit* (the high-income group) would have higher willingness to pay for housing and (b) the reduction in transit ridership among households that get priced out would be low. When $K = 0$, there are two possible scenarios:

a. *transit* is faster from the richer neighborhood and more attractive to high-income households. In this case, the transit planner maximizes ridership by improving *transit* access from the richer neighborhood as it attracts more high-income *transit* ridership to the neighborhood and prices out (primarily) low-income households who would retain high *transit* ridership even in the poorer neighborhood.

b. *transit* is faster from the poorer neighborhood and more attractive to low-income households. The transit planner still maximizes ridership by improving *transit* access from the richer neighborhood as both the (primarily) high-income households that move out in response as well as the households that stay would increase their *transit* ridership.

Given the income sorting across neighborhoods is determined only by differences in *transit* access ($K = 0$), these are the only two possible scenarios. When *transit* is more attractive to the poor, the neighborhood with faster *transit* must be poorer. Whereas, when *transit* is more attractive to the rich, the neighborhood with faster *transit* must be richer. As a result, *transit* ridership is maximized by targeting neighborhoods with higher residential incomes.

However, if income sorting across neighborhoods also depends importantly on factors other than *transit* access ($|K| > 0$), there are two additional possible income-sorting scenarios:

c. *transit* is faster from the richer neighborhood and more attractive to low-income households, so improving *transit* access from the richer neighborhood would decrease *transit* ridership among the (primarily) high-income households that move out.

d. *transit* is faster from the poorer neighborhood and more attractive to high-income households, so improving *transit* access from the richer neighborhood would decrease *transit* ridership for the high-income households that move in.

In both cases, the policy that maximizes long-term *transit* ridership depends on how the change in ridership among households that move compare to that of households that stay in their original neighborhood. Whether or not a household moves depends not just on differences in *transit* access (as in the baseline model) but also on differences in *car* access and preferences over residential amenities.

A.2.5 | Model Generalization: Elastic Housing Demand and Neighborhood Populations

I assumed housing consumption per household and neighborhood populations are fixed. But, in reality, households may choose smaller apartments/dwellings to continue to reside in more easily accessible neighborhoods. City centers and more transit-accessible neighborhoods do indeed tend to be more densely populated. So, when travel times by *transit* are reduced and capitalized into higher housing prices, we may wonder whether the opportunity to trade off housing space (instead of moving out of the neighborhood) dampens the effect on income segregation. Below, I present a generalization of the model where a price-elastic housing demand implies population (and density) varies across neighborhoods. Changes in housing prices do lead to smaller population shifts (crowded out by changes in housing consumption) but continue to have no effect on income segregation, as in the baseline model. This is because the marginal utility gain from additional housing space is still the same for both low- and high-income households. So price changes shift $\bar{\xi}_g$ (where household type g is indifferent between neighborhoods) by the same amount for all households and does not affect its gradient with respect to income, which is what determines income segregation.[44]

Let us relax the assumption of fixed housing consumption and fixed neighborhood populations in the baseline model. Suppose households now gain utility from consuming more housing and choose their housing consumption h to maximize the following utility function:

$$U_{gn}(x, h, m^\star; \xi) \equiv x + h^{\sigma/(1+\sigma)} + A_{gn} + \xi \cdot \mathbf{I}_{n=1} + \int_\gamma \gamma \mathbf{I}_{[m^\star_{gn}(\gamma)=c]} \cdot \partial\Psi(\gamma) \tag{A.7}$$

subject to budget constraint

$$x = w_g - hp_n - \int_\gamma \tilde{C}_{gn}^{m^\star_{gn}(\gamma)} \cdot \partial\Psi(\gamma). \tag{A.8}$$

In this specification, $\sigma > 0$ determines the constant price elasticity of housing demand.[45] First-order condition with respect to h lets us solve for the optimal housing consumption as a function of housing price:

$$0 = \frac{\sigma}{1+\sigma} h^{-1/(1+\sigma)} - p_n$$

$$\Rightarrow h^\star = \left(\frac{(1+\sigma)p_n}{\sigma} \right)^{-(1+\sigma)}.$$

After substituting in optimal numeraire consumption and mode choices, the indirect utility from choosing to reside in neighborhood n is

$$V_{gn} \equiv w_g + \sigma^\sigma (1+\sigma)^{-(1+\sigma)} p_n^{-\sigma} - C_{gn} + A_{gn} + \xi \mathbf{I}_{n=1}. \tag{A.9}$$

Households are indifferent between the two neighborhoods when their taste ξ for neighborhood 1 is equal to

$$\bar{\xi}_g \equiv V_{g2} - V_{g1} = \Delta_\sigma p + C_{g1} - C_{g2} + A_{g2} - A_{g1},$$

where $\Delta_\sigma p \equiv \sigma^\sigma (1+\sigma)^{-(1+\sigma)} (p_2^{-\sigma} - p_1^{-\sigma})$ is the difference in the utility from housing prices in each neighborhood. The baseline scenario corresponds to $\Delta_\sigma p = \Delta p$.

We will also drop the assumption of equally sized neighborhoods, and assume the housing stock in neighborhood n is fixed at H_n. For the housing market to clear in each neighborhood, aggregate housing demand needs to equal the fixed housing supply:

$$h^\star(p_1) \cdot \left(\Lambda \cdot \left(1 - F(\bar{\xi}_L)\right) + (1-\Lambda) \cdot \left(1 - F(\bar{\xi}_H)\right) \right) = H_1$$

$$h^\star(p_2) \cdot \left(\Lambda \cdot F(\bar{\xi}_L) + (1-\Lambda) \cdot F(\bar{\xi}_H) \right) = H_2.$$

The clearing conditions for the two housing markets are equivalent since the total population of the city is still fixed at 1, so that

$$\frac{H_2}{h^\star(p_2)} + \frac{H_1}{h^\star(p_1)} = 1.$$

The population in each neighborhood is now a function of the housing price. Our dissimilarity measure (the fraction of population that needs to move to achieve perfect income mixing) is

$$DISS \equiv \Lambda \cdot \left| F(\bar{\xi}_L) - \frac{H_2}{h^\star(p_2)} \right| + (1-\Lambda) \cdot \left| F(\bar{\xi}_H) - \frac{H_2}{h^\star(p_2)} \right|.$$

We can substitute in $H_2/h^\star(p_2)$ from the housing market clearing condition and solve for the same dissimilarity measure as before

$$DISS = \Lambda \cdot \left| F\left(\bar{\xi}_L\right) - \Lambda \cdot F(\bar{\xi}_L) - (1-\Lambda) \cdot F(\bar{\xi}_H) \right| + (1-\Lambda) \cdot \left| F\left(\bar{\xi}_H\right) - \Lambda \cdot F(\bar{\xi}_L) - (1-\Lambda) \cdot F(\bar{\xi}_H) \right|$$

$$= 2\Lambda \cdot (1-\Lambda) \cdot \left| F\left(\bar{\xi}_L\right) - F\left(\bar{\xi}_H\right) \right|.$$

Given $\bar{\xi}_g$ are always in the support of ξ, income dissimilarity

$$DISS = 2\Lambda \cdot (1-\Lambda) \cdot f \cdot \left| \bar{\xi}_L - \bar{\xi}_H \right|$$

is, as before, no longer a function of housing prices. We can substitute Δp with $\Delta_\sigma p$ in the proofs and results on transit's effect on income segregation and they still hold. Income segregation does not depend on neighborhood population (density) or housing prices in this model.

Price-Elastic Housing Supply

Note that I do not explicitly model a flexible/price-elastic housing supply function. A price-elastic housing supply function might alter how many households move neighborhoods in response to travel time changes but does not affect the income composition of who moves. As long as the gradient of $\bar{\xi}_g$ still depends only on the difference in travel costs and not on housing supply, the qualitative relationship between *transit* access and income segregation remains unaffected.

A.2.6 | Model Generalization: Heterogeneous Trips and Network Externalities

The baseline model simplified the city's geography by assuming all households in a neighborhood have the same travel destinations, allowing me to characterize each mode's travel network with a pair of travel times. In reality, rich and poor households may commute to very different work or consumption locations as parts of a city specialize in different economic sectors. As a result, mass transit services may differ significantly in how well they connect a residence to high- versus low-income destinations.[46]

Suppose high- and low-income households in my model city have different trip destinations, and let τ^m_{ng} denote the travel time to destinations of type g (corresponding to income group g) from neighborhood n using mode m. Planners can now improve travel times to particular destination types from each neighborhood, but with some positive spillovers for everyone else in the neighborhood (through improved access to the broader transit network). More precisely, suppose *transit* travel times from any residential neighborhood n are related such that

$$\frac{d\tau^t_{nk}}{d\tau^t_{ng}} = \theta_k \geq 0 \text{ for } k \neq g \; \forall g \; \forall n, \tag{A.10}$$

where θ_k measures how additionally accessible group k destinations are following a marginal reduction in *transit* travel times to other destinations from the same neighborhood. Intuitively, θ_k captures how well the existing transit network connects the routes between neighborhood n and group g destinations also to group k destinations.[47] For example, if low-income jobs are more dispersed than high-income jobs and have relatively smaller travel time gains from improved *transit* connectivity to a particular neighborhood, θ_L must be smaller than θ_H.[48]

What is the marginal effect of lower *transit* travel times from neighborhood n to type g destinations on income segregation?

$$\frac{dDISS}{d\tau^t_{ng}} = \frac{\partial DISS}{\partial \tau^t_{ng}} + \theta_k \cdot \frac{\partial DISS}{\partial \tau^t_{nk}} \text{ where } k \neq g. \tag{A.11}$$

The first additive component is the direct effect of lower τ^t_{ng} for the targeted destination type while the second additive component is the spillover effect on the other destination type. Note that when $\theta_k = 1$, the spillover effect (of lower τ^t_{ng}) is the same as the direct effect and all travel destinations benefit equally from the improved *transit* access. The baseline model from Section 3 is an extreme case of perfect network spillovers ($\theta_H = \theta_L = 1$).

For small enough travel time complementarities θ_k, the direct effect dominates and the effect on *DISS*imilarity is straightforward. Improving travel times to low-income destinations attracts more low-income households to the neighborhood, whereas improving travel times to high-income destinations attracts more high-income households.

Lemma A.2. When $\theta_k \approx 0$, a marginal reduction in travel time τ^m_{ng} to destinations of type $g \neq k$ decreases income segregation ($dDISS/d\tau^t_{ng} > 0$) if and only if income group k is more likely to reside in neighborhood n.

In other words, improving travel times from either the poorer neighborhood to low-income destinations or the richer neighborhood to high-income destinations increases income segregation by attracting more low-income households to the poorer neighborhood or more high-income households to the richer neighborhood. Similarly, improving travel times from either the poorer neighborhood to high-income destinations or the richer neighborhood to low-income destinations decreases income segregation.[49]

On the other hand, for high enough θ_k, the spillover effect from lower τ^t_{nk} can sometimes dominate the direct effect of lower τ^t_{ng} to make the *transit* improvement more attractive to the untargeted income group.

Proposition A.1. A marginal reduction in travel time τ^t_{ng} to destinations of type $g \neq k$ decreases income segregation ($dDISS/d\tau^t_{ng} > 0$) if and only if either:

- income group k is more likely to reside in neighborhood n but income group g benefits more from the shorter *transit* times: $w_g \Psi\left(\bar{\gamma}_{gn}\right) > \theta_k w_k \Psi(\bar{\gamma}_{kn})$ OR
- income group g is more likely to reside in neighborhood n but income group k benefits more from the shorter *transit* times: $w_g \Psi\left(\bar{\gamma}_{gn}\right) < \theta_k w_k \Psi(\bar{\gamma}_{kn})$

In other words, suppose high (low) income households have higher *transit* ridership, *transit* connectivity, and/or value of travel time so that they benefit more from shorter *transit* times. Then, improving *transit* access from the high (low) income neighborhood increases income segregation by disproportionately attracting more high (low) income households to the neighborhood. In contrast, improving *transit* from the low (high) income

neighborhood decreases income segregation by disproportionately attracting more high (low) income households.1. Note that, qualitatively, this effect of lower *transit* travel times on income segregation is the same as in the baseline model, as depicted in Lemma 2.[50]

Corollary A.1. Given nonnegative travel time complementarities θ_H and θ_L, a marginal reduction in travel time τ^I_{ng} to destinations of type $g \neq k$ minimizes *DISS*imilarity more from the residential neighborhood with a larger share of

- income group k when $w_g\left[\Psi(\tilde{\gamma}_{1g}) + \Psi(\tilde{\gamma}_{2g})\right] > \theta_k w_k\left[\Psi(\tilde{\gamma}_{1k}) + \Psi(\tilde{\gamma}_{2k})\right]$
- income group g when $w_g\left[\Psi(\tilde{\gamma}_{1g}) + \Psi(\tilde{\gamma}_{2g})\right] < \theta_k w_k\left[\Psi(\tilde{\gamma}_{1k}) + \Psi(\tilde{\gamma}_{2k})\right]$

In other words, to minimize income segregation, a transit planner would reduce travel times from the high (low) income neighborhood when overall *transit* ridership in the city is sufficiently large among low (high) income households. Once again, this result reinforces that of Corollary 2.

More generally, whether the rich or the poor benefit more from *transit* improvements depends both on the income composition of *transit* ridership (and, hence, on *transit* travel times relative to driving) as in the baseline model as well as on the destinations being connected. All else equal, better access to low-income destinations attracts more low-income households to the neighborhoods, whereas better access to high-income destinations attracts more high-income households. I show that, qualitatively, the effect of lower *transit* travel times on income segregation is the same as for the baseline model and as depicted in Section 4. Quantitatively, the conditions for when *transit* improvements increase or decrease segregation additionally depend on the magnitude of the network spillover effects.

A.3 | Mathematical Derivations and Proofs

A.3.1 | Changes in Transit Ridership Versus Changes in Neighborhood Incomes (Proof of Lemma 1 and Corollary 1)

Proof. Overall transit ridership in neighborhood n is given by

$$\Psi(\tilde{\gamma}_{Hn}) \cdot (\text{High-income share of neighborhood's population})$$

$$+\Psi(\tilde{\gamma}_{Ln}) \cdot (1 - \text{High-income share of neighborhood's population})$$

Total population in neighborhood n is one-half and the total population of high-income households is $(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)$ in neighborhood 2 and $(1-\Lambda)\cdot\left[1-F\left(\bar{\xi}_H\right)\right]$ in neighborhood 1. So, the high-income share of the neighborhood population is $2(1-\Lambda)\cdot\left[1-F\left(\bar{\xi}_H\right)\right]$ in neighborhood 1 and $2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)$ in neighborhood 2.

Let $\bar{r}_n$ denote the overall transit ridership in neighborhood n. In neighborhood 2:

$$\bar{r}_2 \equiv 2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)\cdot\Psi(\tilde{\gamma}_{H2}) + \left[1-2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)\right]\cdot\Psi(\tilde{\gamma}_{L2}),$$

which can be rewritten as

$$0 = 2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)\cdot\left[\Psi(\tilde{\gamma}_{H2}) - \Psi(\tilde{\gamma}_{L2})\right] + \Psi(\tilde{\gamma}_{L2}) - \bar{r}_2.$$

We can then apply the implicit function theorem to solve for the gradient of the high-income share of neighborhood 2's population with respect to its transit ridership:

$$\frac{\partial\left[2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)\right]}{\partial\bar{r}_2} = -\left(\Psi(\tilde{\gamma}_{H2}) - \Psi(\tilde{\gamma}_{L2})\right)^{-1}\cdot(-1) = \left(\Psi(\tilde{\gamma}_{H2}) - \Psi(\tilde{\gamma}_{L2})\right)^{-1}. \tag{A.12}$$

This gradient is positive if and only if $\Psi(\tilde{\gamma}_{H2}) > \Psi(\tilde{\gamma}_{L2})$. In other words, when a larger fraction of high-income households ride transit in the neighborhood than the fraction of low-income households, changes in overall transit ridership $\bar{r}_2$ are positively correlated with changes in the neighborhood's high-income share $2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)$. And the correlation is negative when a larger fraction of low-income households ride transit in the neighborhood: $\Psi(\tilde{\gamma}_{H2}) < \Psi(\tilde{\gamma}_{L2})$. The same analysis yields the equivalent result for neighborhood 1:

$$\frac{\partial\left(2(1-\Lambda)\cdot\left[1-F\left(\bar{\xi}_H\right)\right]\right)}{\partial\bar{r}_1} \text{ if and only if } \Psi(\tilde{\gamma}_{H1}) > \Psi(\tilde{\gamma}_{L1}).$$

This proves Corollary 1.

We can substitute the expression for $\tilde{\gamma}_{gn}$ into (A.12) to rewrite the gradient as a function of travel times:

$$\frac{\partial\left[2(1-\Lambda)\cdot F\left(\bar{\xi}_H\right)\right]}{\partial\bar{r}_2} = -\left(\psi\cdot\beta\cdot(w_H - w_L)\cdot\Delta\tau_2\right)^{-1}.$$

The resulting expression on the right is negative when transit is slower than car ($\Delta\tau_2 > 0$) and positive when transit is faster than car ($\Delta\tau_2 < 0$). In other words, **the high-income share of a neighborhood's population is positively correlated with transit ridership in the neighborhood when transit is fast, but negatively correlated when transit is slow**. This concludes the proof of Lemma 1. □

A.3.2 | Aggregate Travel Cost Function

We can rewrite the aggregate travel cost function C_{gn} as follows:

$$C_{gn} \equiv \int_{\gamma}\tilde{C}_{gn}^{m^{\star}_{gn}(\gamma)}\cdot\partial\Psi(\gamma) - \int_{\gamma:m^{\star}_{gn}(\gamma)=c}\gamma\cdot d\Psi(\gamma)$$

$$= \int_{-\infty}^{\tilde{\gamma}_{gn}} \tilde{C}^t_{gn} \cdot \partial\Psi(\gamma) + \int_{\tilde{\gamma}_{gn}}^{+\infty} \left(\tilde{C}^c_{gn} - \gamma\right) \cdot d\Psi(\gamma)$$

$$= \left[1 - \Psi\left(\tilde{\gamma}_{gn}\right)\right] \cdot \tilde{C}^c_{gn} + \Psi\left(\tilde{\gamma}_{gn}\right) \cdot \tilde{C}^t_{gn} - \int_{\tilde{\gamma}_{gn}}^{+\infty} \gamma \cdot d\Psi(\gamma). \tag{A.13}$$

A.3.3 | Marginal Effect of Transit Times on Income Segregation

Proof of Lemma 2

Proof. First, let us determine the marginal effect of *transit* times on overall travel costs C_{gn} from (A.13):

$$\frac{dC_{gn}}{d\Delta\tau_n} = \beta \cdot w_g \cdot \left[\psi\left(\tilde{\gamma}_{gn}\right) \cdot \tilde{C}^c_{gn} - \psi\left(\tilde{\gamma}_{gn}\right) \cdot \tilde{C}^t_{gn} + \Psi\left(\tilde{\gamma}_{gn}\right) - \tilde{\gamma}_{gn} \cdot \psi\left(\tilde{\gamma}_{gn}\right)\right],$$

where the last component follows from the Fundamental Theorem of Calculus.

$$\Rightarrow \frac{dC_{gn}}{d\Delta\tau_n} = \beta \cdot w_g \cdot \left[\psi\left(\tilde{\gamma}_{gn}\right) \cdot \left(\tilde{C}^c_{gn} - \tilde{C}^t_{gn} - \tilde{\gamma}_{gn}\right) + \Psi\left(\tilde{\gamma}_{gn}\right)\right].$$

Then, plugging in $\tilde{\gamma}_{gn}$ from (2) yields

$$\frac{dC_{gn}}{d\Delta\tau_n} = \beta \cdot w_g \cdot \Psi\left(\tilde{\gamma}_{gn}\right). \tag{A.14}$$

Now, we can take the derivative of *DISS* from (9) with respect to $\Delta\tau_n$:

$$\frac{dDISS}{d\Delta\tau_n} = 2\Lambda \cdot (1-\Lambda) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H])$$

$$\cdot \left[(1 - 2\mathbf{I}_{n=2}) \cdot \left(f \cdot \frac{dC_{Ln}}{d\Delta\tau_n} - f \cdot \frac{dC_{Hn}}{\Delta d\tau_n}\right) + (f - f) \cdot \frac{d\Delta p}{d\tau^t_n}\right] \tag{A.15}$$

$$= 2\Lambda \cdot (1-\Lambda) \cdot f \cdot (1 - 2\mathbf{I}_{n=2}) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot \left(\frac{dC_{Ln}}{d\Delta\tau_n} - \frac{dC_{Hn}}{d\Delta\tau_n}\right), \tag{A.16}$$

where

$$\frac{dC_{Ln}}{d\Delta\tau_n} - \frac{dC_{Hn}}{d\Delta\tau_n} = \beta \cdot (\Psi(\tilde{\gamma}_{Ln}) \cdot w_L - \Psi(\tilde{\gamma}_{Hn}) \cdot w_H). \tag{A.17}$$

In (A.16), $(dDISS/d\tau^t_n) < 0$ if and only if one of the following holds true:

a. $n = 2$, $\bar{\xi}_L < \bar{\xi}_H$ and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L < \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$
b. $n = 1$, $\bar{\xi}_L > \bar{\xi}_H$ and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L < \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$
c. $n = 2$, $\bar{\xi}_L > \bar{\xi}_H$ and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L > \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$
d. $n = 1$, $\bar{\xi}_L < \bar{\xi}_H$ and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L > \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$

Recall that $F\left(\bar{\xi}_g\right)$ is the fraction of type g households that live in neighborhood 2. Since there are only two residential neighborhoods, a larger fraction of high-income households in neighborhood 2 (i.e., $F\left(\bar{\xi}_H\right) > F\left(\bar{\xi}_L\right)$ or $\bar{\xi}_H > \bar{\xi}_L$) automatically implies a larger fraction of low-income households in neighborhood 1 (i.e., $1 - F\left(\bar{\xi}_L\right) > 1 - F\left(\bar{\xi}_H\right)$). So, the first two conditions correspond to n being the richer neighborhood and the last two conditions correspond to n being the poorer neighborhood. Lumping them together, $(dDISS/d\Delta\tau_n) < 0$ if and only if either:

- n is the richer neighborhood and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L < \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$ OR
- n is the poorer neighborhood and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L > \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$

Similarly, $(dDISS/d\tau^t_n) > 0$ if and only if either:

- n is the richer neighborhood and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L > \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$ OR
- n is the poorer neighborhood and $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L < \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$

□

Proof of Proposition 2

Proof. Note that $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L < \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$ is equivalent to

$$\psi \cdot (\kappa - \beta \cdot w_L \cdot \Delta\tau_n + \Gamma) \cdot w_L < \psi \cdot (\kappa - \beta \cdot w_H \cdot \Delta\tau_n + \Gamma) \cdot w_H$$

$$\Leftrightarrow \beta \cdot \Delta\tau_n \cdot (w_H - w_L) \cdot (w_H + w_L) < (w_H - w_L) \cdot (\kappa + \Gamma)$$

$$\Leftrightarrow \Delta\tau_n < \frac{\kappa + \Gamma}{\beta \cdot (w_H + w_L)}.$$

Conversely, $\Psi(\tilde{\gamma}_{Ln}) \cdot w_L > \Psi(\tilde{\gamma}_{Hn}) \cdot w_H$ is equivalent to

$$\Delta\tau_n > \frac{\kappa + \Gamma}{\beta \cdot (w_H + w_L)}.$$

Let

$$\bar{\tau} \equiv \frac{\kappa + \Gamma}{\beta \cdot (w_H + w_L)}.$$

Then Lemma 2 can be restated as follows.

Improved *transit* access increases income segregation ($dDISS/d\Delta\tau_n < 0$) if and only if either:

- n is the richer neighborhood and $\Delta\tau_n < \bar{\tau}$ OR
- n is the poorer neighborhood and $\Delta\tau_n > \bar{\tau}$

Whereas, improved *transit* access decreases income segregation ($dDISS/d\Delta\tau_n > 0$) if and only if either:

- n is the richer neighborhood and $\Delta\tau_n > \bar{\tau}$ OR
- n is the poorer neighborhood and $\Delta\tau_n < \bar{\tau}$

□

Proof of Corollary 2

Proof. Take the derivative of *DISS* with respect to $\Delta\tau_n$ and compare it for the two neighborhoods:

$$\frac{dDISS}{d\Delta\tau_1} - \frac{dDISS}{d\Delta\tau_2} = 2\Lambda \cdot (1-\Lambda) \cdot f \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot G \tag{A.18}$$

$$\text{where } G \equiv \frac{dC_{L1}}{d\Delta\tau_1} - \frac{dC_{H1}}{d\Delta\tau_1} + \frac{dC_{L2}}{d\Delta\tau_2} - \frac{dC_{H2}}{d\Delta\tau_2}.$$

Improving *transit* times in neighborhood 1 would induce more income mixing (or less income segregation) than doing so in neighborhood 2 if and only if $(dDISS/d\Delta\tau_1) - (dDISS/d\Delta\tau_2)$ is positive. The sign depends on G. If we plug in $(dC_{gn})/(d\Delta\tau_n)$ from (A.14),

$$G = \beta \cdot w_L \cdot (\Psi(\tilde{\gamma}_{L1}) + \Psi(\tilde{\gamma}_{L2})) - \beta \cdot w_H \cdot (\Psi(\tilde{\gamma}_{H1}) + \Psi(\tilde{\gamma}_{H2})).$$

Since $\tilde{\gamma}_{gn} \in [-\Gamma, \Gamma]\ \forall n, g$, we can plug in $\Psi(\tilde{\gamma}_{gn}) = \psi \cdot (\tilde{\gamma}_{gn} + \Gamma)$ into G:

$$G = \beta \cdot \psi \cdot [w_L \cdot (\tilde{\gamma}_{L1} + \tilde{\gamma}_{L2} + 2\Gamma) - w_H \cdot (\tilde{\gamma}_{H1} + \tilde{\gamma}_{H2} + 2\Gamma)].$$

Plug in $\tilde{\gamma}_{gn} \equiv \kappa - \beta \cdot w_g \cdot \Delta\tau_n$:

$$\Rightarrow G = \beta \cdot \psi \cdot \left[\beta \cdot (\Delta\tau_1 + \Delta\tau_2) \cdot (w_H^2 - w_L^2) - 2(\kappa + \Gamma) \cdot (w_H - w_L)\right]$$

$$= \beta \cdot \psi \cdot (w_H - w_L) \cdot [\beta \cdot (\Delta\tau_1 + \Delta\tau_2) \cdot (w_H + w_L) - 2(\kappa + \Gamma)].$$

Finally, on plugging this expression for G back into (A.18), we get that $(dDISS/d\Delta\tau_1) - (dDISS/d\Delta\tau_2)$ is positive if and only if either:

- neighborhood 2 is poorer ($\bar{\xi}_L > \bar{\xi}_H$) and *transit* is sufficiently slow

$$\Delta\tau_1 + \Delta\tau_2 > \frac{2(\kappa + \Gamma)}{\beta \cdot (w_H + w_L)} = 2\bar{\tau}$$

- OR neighborhood 2 is richer ($\bar{\xi}_L < \bar{\xi}_H$) and *transit* is sufficiently fast

$$\Delta\tau_1 + \Delta\tau_2 < \frac{2(\kappa + \Gamma)}{\beta \cdot (w_H + w_L)} = 2\bar{\tau}.$$

□

A.3.4 | Income Sorting as a Function of Travel Times

In this section, I provide a proof of the income sorting described in Proposition 1 and derive a closed-form solution of income *DISS*imilarity in the city purely in terms of travel times and exogenous parameters of the model. Income segregation is driven by the difference in travel costs between the neighborhoods, and how it differs between high- and low-income households. First, I characterize how this difference in travel costs varies with *transit* ridership and travel times. Then, I show how neighborhood differences in travel costs and independent amenities affect endogenously determined housing prices and ultimately the income sorting across neighborhoods (and the resulting level of income *DISS*imilarity) in a competitive equilibrium. Throughout this section, assume that $\tilde{\gamma}_{gn}$ lies within the support of the distribution of γ for all n, g.

The Aggregate Travel Cost

Based on the travel cost function specified in (A.13), we have

$$C_{g1} - C_{g2} = \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[\tilde{C}^c_{gn} + \Psi(\tilde{\gamma}_{gn}) \cdot (\tilde{C}^t_{gn} - \tilde{C}^c_{gn})\right] + \int_{\tilde{\gamma}_{g2}}^{\tilde{\gamma}_{g1}} \gamma \cdot d\Psi(\gamma).$$

Substitute $\tilde{C}^c_{gn} \equiv \kappa + \beta \cdot w_g \cdot \tau^c_n$ from (1) and $\tilde{\gamma}_{gn} \equiv \tilde{C}^c_{gn} - \tilde{C}^t_{gn}$ from (2) into $C_{g1} - C_{g2}$

$$= \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[\beta \cdot w_g \cdot \tau^c_n - \Psi(\tilde{\gamma}_{gn}) \cdot \tilde{\gamma}_{gn}\right] + \int_{\tilde{\gamma}_{g2}}^{\tilde{\gamma}_{g1}} \gamma \cdot d\Psi(\gamma). \tag{A.19}$$

Because γ is uniformly distributed with constant density ψ and $\tilde{\gamma}_{gn}$ lie within the support $[-\Gamma, \Gamma]$, we know $\Psi(\tilde{\gamma}_{gn}) = \psi \cdot [\tilde{\gamma}_{gn} + \Gamma]$ and we can substitute $d\Psi(\gamma) = \psi \cdot d\gamma$ into the integral to get

$$\int_{\tilde{\gamma}_{g2}}^{\tilde{\gamma}_{g1}} \gamma \cdot d\Psi(\gamma) = \frac{\psi}{2} \cdot \left[(\tilde{\gamma}_{g1})^2 - (\tilde{\gamma}_{g2})^2\right].$$

Now plug them into (A.19) to get

$$C_{g1} - C_{g2} = \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[\beta \cdot w_g \cdot \tau^c_n - \psi \cdot (\tilde{\gamma}_{gn} + \Gamma) \cdot \tilde{\gamma}_{gn} + \frac{\psi}{2} \cdot (\tilde{\gamma}_{gn})^2\right]. \tag{A.20}$$

Add $0 = \frac{\psi}{2} \cdot \left(3\Gamma^2 - 3\Gamma^2 + \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot 2\tilde{\gamma}_{gn} \cdot (\Gamma - \Gamma)\right) = \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[-\psi \cdot \Gamma \cdot (\tilde{\gamma}_{gn} + \Gamma) + \frac{\psi}{2} \cdot (\Gamma^2 + 2\tilde{\gamma}_{gn} \cdot \Gamma)\right]$ to get:

$$\begin{aligned} C_{g1} - C_{g2} &= \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[\beta \cdot w_g \cdot \tau^c_n - \psi \cdot (\tilde{\gamma}_{gn} + \Gamma) \cdot (\tilde{\gamma}_{gn} + \Gamma) + \frac{\psi}{2} \cdot (\tilde{\gamma}_{g1} + \Gamma)^2\right] \\ &= \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[\beta \cdot w_g \cdot \tau^c_n - \frac{1}{2\psi} \cdot \Psi(\tilde{\gamma}_{gn})^2\right]. \end{aligned} \tag{A.21}$$

Income Sorting (Proofs of Lemma A.1 and Proposition 1)

Lemma A.1 proposes a closed-form solution of $\bar{\xi}_L - \bar{\xi}_H$, first as a function of each income group's transit ridership in each neighborhood, and then as a function of travel times and other exogenous parameters of the model. I derive these closed-form solutions below.

Proof. Recall that $\bar{A} \equiv A_{H1} - A_{H2} - A_{L1} + A_{L2}$. Then,

$$\bar{\xi}_L - \bar{\xi}_H = C_{L1} - C_{L2} - C_{H1} + C_{H2} + \bar{A}.$$

Plug in $C_{g1} - C_{g2}$ from (A.21) to get

$$\bar{\xi}_L - \bar{\xi}_H = \sum_g \sum_n (1 - 2\mathbf{I}_{g=H}) \cdot (1 - 2\mathbf{I}_{n=2}) \cdot \left[\beta \cdot w_g \cdot \tau^c_n - \frac{1}{2\psi} \cdot \Psi(\tilde{\gamma}_{gn})^2\right] + \bar{A}$$

and substitute in $K \equiv \beta \cdot (w_H - w_L) \cdot (\tau^c_2 - \tau^c_1) + \bar{A}$:

$$= K - \frac{1}{2\psi} \cdot \sum_g \sum_n (1 - 2\mathbf{I}_{g=H}) \cdot (1 - 2\mathbf{I}_{n=2}) \cdot \Psi(\tilde{\gamma}_{gn})^2 \tag{A.22}$$

which can be rewritten as

$$\bar{\xi}_L - \bar{\xi}_H = \frac{1}{2\psi} \cdot \left(\Psi(\tilde{\gamma}_{L2})^2 - \Psi(\tilde{\gamma}_{H2})^2 - \Psi(\tilde{\gamma}_{L1})^2 + \Psi(\tilde{\gamma}_{H1})^2\right) + K.$$

Thus, we have (A.1), the first equality of Lemma A.1. Now plug in $\tilde{\gamma}_{gn}$ from (2) into (A.22) to get $\bar{\xi}_L - \bar{\xi}_H$

$$= K - \sum_g (1 - 2\mathbf{I}_{g=H}) \cdot \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \left[\frac{\psi}{2} \cdot (\kappa - \beta \cdot w_g \cdot \Delta\tau_n + \Gamma)^2\right]$$

and expand the square

$$= K - \sum_g (1 - 2\mathbf{I}_{g=H}) \cdot \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot \frac{\psi}{2} \cdot \left[2(\kappa + \Gamma) \cdot (\beta \cdot w_g \cdot \Delta\tau_n) - (\beta \cdot w_g \cdot \Delta\tau_n)^2\right],$$

where $\sum_g (1 - 2\mathbf{I}_{g=H}) \cdot \sum_n (1 - 2\mathbf{I}_{n=2}) \cdot (\kappa + \Gamma)^2 = 0$.

$$= \beta \cdot \frac{\psi}{2} \cdot \left[\beta \cdot (w_H^2 - w_L^2) \cdot \left[(\Delta\tau_1)^2 - (\Delta\tau_2)^2\right] - 2(\kappa + \Gamma) \cdot (w_H - w_L) \cdot (\Delta\tau_1 - \Delta\tau_2)\right] + K$$

$$= \beta \cdot \frac{\psi}{2} \cdot (w_H - w_L) \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot (\beta \cdot (w_H + w_L) \cdot (\Delta\tau_1 + \Delta\tau_2) - 2(\kappa + \Gamma)) + K.$$

Substitute in $\bar{\tau} \equiv (\kappa + \Gamma)/(\beta \cdot (w_H + w_L))$ to get

$$\bar{\xi}_L - \bar{\xi}_H = \beta \cdot \frac{\psi}{2} \cdot (w_H - w_L) \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot \beta \cdot (w_H + w_L) \cdot (\Delta\tau_1 + \Delta\tau_2 - 2\bar{\tau}) + K.$$

Substitute in $Q \equiv (1/2)\beta \cdot \psi \cdot (w_H^2 - w_L^2) = (1/2)\beta \cdot \psi \cdot (w_H - w_L) \cdot (w_H + w_L)$ to derive our second equality, (A.2), from Lemma A.1:

$$\bar{\xi}_L - \bar{\xi}_H = Q \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot (\Delta\tau_1 + \Delta\tau_2 - 2\bar{\tau}) + K.$$

□

Then, based on our definition of income *DISS*imilarity from (9),

$$DISS = 2\Lambda \cdot (1 - \Lambda) \cdot f \cdot \left| Q \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot (\Delta\tau_1 + \Delta\tau_2 - 2\bar{\tau}) + K \right|.$$

Proposition 1 dictates when neighborhood 1 is high income ($\bar{\xi}_L > \bar{\xi}_H$) as a function of travel times and exogenous parameters of the model.

Proof. The proof follows directly from the closed-form solution derived in Lemma A.1 with $\bar{K} \equiv K/Q$. □

A.3.5 | Maximizing Short-Term Transit Ridership (Proof of Proposition 3)

Proposition 3 claims $S_2 > S_1$ (i.e., *transit* ridership among nonmovers is maximized from a *transit* time reduction in neighborhood 1) if and only if neighborhood 1 has higher residential incomes than neighborhood 2.

Proof. Since $\tilde{\gamma}_{gn}$ lie in the support $[-\Gamma, \Gamma]$ of the distribution of γ for both g and both n, the marginal effect of *transit* times on *transit* ridership of households of a given type is the same across neighborhoods:

$$\frac{d\Psi(\tilde{\gamma}_{g1})}{d\Delta\tau_1} = \frac{d\Psi(\tilde{\gamma}_{g2})}{d\Delta\tau_2} = -\psi \cdot \beta \cdot w_g. \tag{A.23}$$

Plug this into $S_2 - S_1$ from (A.6) to get

$$S_2 - S_1 = \psi \cdot \beta \cdot \mathbf{E}\left[w_g \cdot \left(1 - 2F\left(\bar{\xi}_g\right)\right)\right]. \tag{A.24}$$

We can expand the expectation over the two possible household types to get $S_2 > S_1$ if and only if

$$\Lambda \cdot w_L \cdot \left(1 - 2F\left(\bar{\xi}_L\right)\right) + (1 - \Lambda) \cdot w_H \cdot \left(1 - 2F\left(\bar{\xi}_H\right)\right) > 0$$

$$\Leftrightarrow F\left(\bar{\xi}_L\right) \cdot w_L \cdot \Lambda + F\left(\bar{\xi}_H\right) \cdot w_H \cdot (1 - \Lambda) < \frac{1}{2} \cdot (w_H \cdot (1 - \Lambda) + w_L \cdot \Lambda).$$

In the inequality above, on the left is the total income of households in neighborhood 2, and on the right is half the total income of households in the city. In other words, when the aggregate income of residents of neighborhood 2 is smaller than that of the residents of neighborhood 1, it is optimal for the transit provider to reduce *transit* times in neighborhood 1 (and vice versa). □

A.3.6 | Marginal Effect of Transit Times on Transit Ridership Among Stayers

Since the distribution of incomes across neighborhoods are an endogenous function of the distribution of travel times, we can continue to develop the expression in (A.24) to rewrite it as a function of exogenous parameters of the model. This will be useful when characterizing changes in *transit* ridership in the long run. So, plug in $\bar{\xi}_g$ from (6):

$$S_2 - S_1 = \psi \cdot \beta \cdot \mathbf{E}\left[w \cdot \left(1 - 2f \cdot \left(C_{g1} - C_{g2} - A_{g1} + A_{g2} + \Delta p + \Xi\right)\right)\right]$$

$$= \psi \cdot \beta \cdot \mathbf{E}\left[w \cdot \left(1 - 2f \cdot \left(C_{g1} - C_{g2} - A_{g1} + A_{g2} + \Delta p + \Xi\right)\right)\right]. \tag{A.25}$$

From the housing market clearing condition in (7), we have

$$\Delta p = \frac{1}{2f} - \Xi - \mathbf{E}\left[C_{g1} - C_{g2} - A_{g1} + A_{g2}\right]. \tag{A.26}$$

Plug in this Δp into (A.25) to get

$$S_2 - S_1 = \psi \cdot \beta \cdot \mathbf{E}\left[w_g \cdot \left(1 - 2f \cdot \left(C_{g1} - C_{g2} - A_{g1} + A_{g2} + \frac{1}{2f} - \mathbf{E}\left[C_{g1} - C_{g2} - A_{g1} + A_{g2}\right]\right)\right)\right]$$

$$= 2f \cdot \psi \cdot \beta \cdot \left[\mathbf{E}\left[w_g \cdot \left(C_{g1} - C_{g2} - A_{g1} + A_{g2}\right)\right] - \mathbf{E}[w_g] \cdot \mathbf{E}\left[C_{g1} - C_{g2} - A_{g1} + A_{g2}\right]\right].$$

We can simplify the expectation over incomes to get

$$S_2 - S_1 = 2f \cdot \psi \cdot \beta \cdot \Lambda \cdot (1 - \Lambda) \cdot (w_H - w_L) \cdot \left[C_{H1} - C_{H2} - C_{L1} + C_{L2} + \bar{A}\right] \tag{A.27}$$

where $\bar{A} \equiv A_{H1} - A_{H2} - A_{L1} + A_{L2}$.

To be concise, I use $\bar{A}$ to denote the effect of exogenously distributed amenities in the city. Note that $S_2 - S_1$ is proportional to income *DISS*imilarity as defined in (9):

$$S_2 - S_1 = 2f \cdot \psi \cdot \beta \cdot \Lambda \cdot (1 - \Lambda) \cdot (w_H + w_L) \cdot \left(\bar{\xi}_L - \bar{\xi}_H\right). \tag{A.28}$$

In words, the greater the difference in incomes across residential neighborhoods, the larger are the gains in ("short-term") *transit* ridership from improving *transit* in the richer neighborhood.

A.3.7 | Marginal Effect of Transit Times on Transit Ridership Among Movers (Proof of Lemma 3)

What is the change in *transit* ridership M_n among households that move?

Proof. Plug in $\bar{\xi}_g$ from (6) into $M_2 - M_1$ from (A.6) and take its derivative with respect to τ_1^t and τ_2^t to get

$$M_2 - M_1 = f \cdot \mathbf{E}\left[\left(\frac{d\Delta p}{d\Delta\tau_2} - \frac{dC_{g2}}{d\Delta\tau_2} - \frac{d\Delta p}{d\Delta\tau_1} - \frac{dC_{g1}}{d\Delta\tau_1}\right) \cdot \left(\Psi\left(\tilde{\gamma}_{g2}\right) - \Psi\left(\tilde{\gamma}_{g1}\right)\right)\right].$$

The derivative of Δp from (A.26) with respect to $\Delta\tau_n$ is

$$\frac{d\Delta p}{d\Delta\tau_n} = -\mathbf{E}\left[\frac{dC_{gn}}{d\Delta\tau_n} \cdot (1 - 2\mathbf{I}_{n=2})\right],$$

which we can plug into $M_2 - M_1$

$$= f \cdot \mathbf{E}\left[\left(\mathbf{E}\left[\frac{dC_{g2}}{d\Delta\tau_2} + \frac{dC_{g1}}{d\Delta\tau_1}\right] - \frac{dC_{g2}}{d\Delta\tau_2} - \frac{dC_{g1}}{d\Delta\tau_1}\right) \cdot \left(\Psi\left(\tilde{\gamma}_{g2}\right) - \Psi\left(\tilde{\gamma}_{g1}\right)\right)\right].$$

Substitute in $(dC_{gn}/d\Delta\tau_n) = \beta \cdot w_g \cdot \Psi\left(\tilde{\gamma}_{gn}\right)$ from (A.14) and plug in $\Psi\left(\tilde{\gamma}_{gn}\right) = \psi \cdot \left(\kappa - \beta \cdot w_g \cdot \Delta\tau_n + \Gamma\right)$ to get

$$\begin{aligned} M_2 - M_1 = {} & f \cdot \beta \cdot \mathbf{E}\left[\psi \cdot \beta \cdot w_g^2 \cdot (\Delta\tau_1 + \Delta\tau_2)\right] \cdot \mathbf{E}\left[\psi \cdot \beta \cdot w_g \cdot (\Delta\tau_1 - \Delta\tau_2)\right] \\ & - f \cdot \beta \cdot \mathbf{E}\left[w_g \cdot \left[\psi \cdot \beta \cdot w_g \cdot (\Delta\tau_1 + \Delta\tau_2)\right] \cdot \left[\psi \cdot \beta \cdot w_g \cdot (\Delta\tau_1 - \Delta\tau_2)\right]\right] \\ = {} & f \cdot \psi^2 \cdot \beta^3 \cdot \left(\mathbf{E}[w_g^3] - \mathbf{E}[w_g^2] \cdot \mathbf{E}[w_g]\right) \cdot \left[(\Delta\tau_2)^2 - (\Delta\tau_1)^2\right]. \end{aligned}$$

Finally, simplify the expectation over the two possible incomes:

$$M_2 - M_1 = f \cdot \beta^3 \cdot \psi^2 \cdot \Lambda \cdot (1 - \Lambda) \cdot (w_H^2 - w_L^2) \cdot (w_H - w_L) \cdot \left[(\Delta\tau_2)^2 - (\Delta\tau_1)^2\right] \tag{A.29}$$

$$M_2 > M_1 \Leftrightarrow \Delta\tau_1 < \Delta\tau_2.$$

In other words, ridership among movers increases more from *transit* improvements in neighborhood 1 (as opposed to 2) if and only if *transit* is relatively faster from neighborhood 1 than from 2. This is because movers increase their *transit* usage when they move into a neighborhood with relatively shorter *transit* times, but decrease their *transit* usage when they move to a neighborhood with longer *transit* times. Similarly, households that move out of the neighborhood with faster *transit* had relatively lower *transit* usage (and lower willingness to pay the higher housing prices for it) to begin with. So, overall *transit* ridership among the city's movers is maximized by marginal improvements in *transit* from the neighborhood that has the relatively shorter *transit* travel time. □

A.3.8 | Maximizing Long-Term Transit Ridership (Proof of Proposition 4)

Let me finally compare the change in *transit* ridership among all households in the city.

Proof. *Recall from (A.6) that $(dR/d\Delta\tau_2) - (dR/d\Delta\tau_1)$ equals the sum of $S_2 - S_1$ and $M_2 - M_1$. So, plug in $\bar{\xi}_L - \bar{\xi}_H$ from (A.2) into $S_2 - S_1$ from (A.28) and add them to $M_2 - M_1$ from (A.29) to get $(dR/d\Delta\tau_2) - (dR/d\Delta\tau_1)$*

$$\begin{aligned} = {} & 2f \cdot \psi \cdot \beta \cdot \Lambda \cdot (1 - \Lambda) \cdot (w_H - w_L) \cdot [\beta \cdot (w_H - w_L) \cdot [\psi \cdot (\kappa + \Gamma) \cdot (\Delta\tau_1 - \Delta\tau_2)] \\ & - \psi \cdot \beta^2 \cdot (w_H^2 - w_L^2) \cdot \left[(\Delta\tau_1)^2 - (\Delta\tau_2)^2\right] + K], \end{aligned}$$

which is positive if and only if

$$\beta \cdot (w_H - w_L) \cdot \psi \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot (\kappa + \Gamma - \beta \cdot (w_H + w_L) \cdot (\Delta\tau_1 + \Delta\tau_2)) + K > 0$$

$$\Leftrightarrow 2Q \cdot (\Delta\tau_1 - \Delta\tau_2) \cdot (\bar{\tau} - (\Delta\tau_1 + \Delta\tau_2)) + K > 0$$

$$2(\bar{\xi}_L - \bar{\xi}_H - K) + K > 0,$$

where I substituted in Q, K, and $\bar{\xi}_L - \bar{\xi}_H$ from Proposition A.1. So, improving transit travel times from neighborhood 1 increases transit ridership more than improving transit times from neighborhood 2 if and only if

$$\bar{\xi}_L - \bar{\xi}_H > \frac{K}{2}.$$

□

A.3.9 | Generalization: Travel Times to Income-Specific Destinations

Suppose the travel network is characterized by travel times τ^m_{ng} to destinations of type g (for income group g). What is the marginal effect of transit travel times on income segregation? When is $(dDISS/d\tau^t_{ng}) < 0$? There are four marginal *transit* travel time changes to evaluate. Let $\Delta\tau_{ng} \equiv \tau^c_{ng} - \tau^t_{ng}$ denote the travel time difference between *car* and *transit* for each commute ng. Households are indifferent between *car* and *transit* on trips with γ equal to

$$\tilde{\gamma}_{ng} = \kappa - \beta \cdot w_g \cdot \Delta\tau_{ng}$$

Zero Network Spillovers (Proof of Lemma A.2)

Proof. To compute $dDISS/d\tau^t_{ng}$, first, derive the marginal effect of travel times on travel costs:

$$\frac{dC_{ng}}{d\tau^t_{ng}} = \beta \cdot w_g \cdot \left[\psi(\tilde{\gamma}_{ng}) \cdot \tilde{C}^c_{ng} - \psi(\tilde{\gamma}_{ng}) \cdot \tilde{C}^t_{ng} + \Psi(\tilde{\gamma}_{ng}) - \tilde{\gamma}_{ng} \cdot \psi(\tilde{\gamma}_{ng})\right],$$

where the last component follows from the Fundamental Theorem of Calculus.

$$\Rightarrow \frac{dC_{ng}}{d\tau^t_{ng}} = \beta \cdot w_g \cdot \left[\psi(\tilde{\gamma}_{ng}) \cdot (\tilde{C}^c_{ng} - \tilde{C}^t_{ng} - \tilde{\gamma}_{ng}) + \Psi(\tilde{\gamma}_{ng})\right].$$

Then, plugging in $\tilde{\gamma}_{ng}$ from (2) yields

$$\frac{dC_{ng}}{d\tau^t_{ng}} = \beta \cdot w_g \cdot \Psi(\tilde{\gamma}_{ng}). \tag{A.30}$$

Now, we can take the derivative of *DISS* from (9) with respect to τ^t_{ng}:

$$\frac{dDISS}{d\tau^t_{ng}} = 2\Lambda \cdot (1-\Lambda) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot$$

$$\left[(1 - 2\mathbf{I}_{n=2}) \cdot (1 - 2\mathbf{I}_{g=H}) \cdot f \cdot \frac{dC_{ng}}{d\tau^t_{ng}} + (f - f) \cdot \frac{d\Delta p}{d\tau^t_{ng}}\right] \tag{A.31}$$

$$\Rightarrow \frac{dDISS}{d\tau^t_{ng}} = 2\Lambda \cdot (1-\Lambda) \cdot f \cdot (1 - 2\mathbf{I}_{n=2}) \cdot (1 - 2\mathbf{I}_{g=H}) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot \frac{dC_{ng}}{d\tau^t_{ng}}. \tag{A.32}$$

In (A.32), $(dDISS/d\tau^t_{ng}) < 0$ if and only if there is nonzero *transit* ridership among the target group ($\Psi(\tilde{\gamma}_{ng}) > 0$) and one of the following holds true:

- $n = 2$, $\bar{\xi}_L < \bar{\xi}_H$, and $g = H$
- $n = 1$, $\bar{\xi}_L > \bar{\xi}_H$, and $g = H$
- $n = 2$, $\bar{\xi}_L > \bar{\xi}_H$, and $g = L$
- $n = 1$, $\bar{\xi}_L < \bar{\xi}_H$, and $g = L$

Recall that $F(\bar{\xi}_g)$ is the fraction of group g households that live in neighborhood 2. Since there are only two residential neighborhoods, a larger fraction of high-income households in neighborhood 2 (i.e., $F(\bar{\xi}_H) > F(\bar{\xi}_L)$ or $\bar{\xi}_H > \bar{\xi}_L$) automatically implies a larger fraction of low-income households in neighborhood 1 (i.e., $1 - F(\bar{\xi}_L) > 1 - F(\bar{\xi}_H)$). So, the first two conditions correspond to n being the richer neighborhood and the last two conditions correspond to n being the poorer neighborhood. Lumping them together, $(dDISS/d\tau^t_{ng}) < 0$ if and only if either:

- n is the richer neighborhood and $g = H$ OR
- n is the poorer neighborhood and $g = L$

Similarly, $(dDISS/d\tau^t_{ng}) > 0$ if and only if either:

- n is the richer neighborhood and $g = L$ OR
- n is the poorer neighborhood and $g = H$

□

Positive Network Spillovers (Proof of Proposition A.1)

Proof. When is $(dDISS/d\tau^t_{ng}) < 0$?

$$\frac{dDISS}{d\tau^t_{ng}} = \frac{\partial DISS}{\partial \tau^t_{ng}} + \theta_k \cdot \frac{\partial DISS}{\partial \tau^t_{nk}} \text{ where } k \neq g,$$

where we can plug in the derivative from (A.32) to get

$$\frac{dDISS}{d\tau^t_{ng}} = 2\Lambda \cdot (1-\Lambda) \cdot f \cdot (1 - 2\mathbf{I}_{n=2}) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot (1 - 2\mathbf{I}_{g=H}) \cdot \left(\frac{dC_{ng}}{d\tau^t_{ng}} - \theta_k \frac{dC_{nk}}{d\tau^t_{nk}} \right)$$

$$= 2\Lambda \cdot (1-\Lambda) \cdot f \cdot (1 - 2\mathbf{I}_{n=2}) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot (1 - 2\mathbf{I}_{g=H}) \beta \cdot \left(w_g \Psi(\tilde{\gamma}_{ng}) - \theta_k w_k \Psi(\tilde{\gamma}_{nk}) \right).$$

Then $(dDISS/d\tau^t_{ng}) < 0$ if and only if one of the following holds true:

a. n is the richer neighborhood and $g = H$ and $w_g \Psi(\tilde{\gamma}_{gn}) > \theta_k w_k \Psi(\tilde{\gamma}_{kn})$
b. n is the poorer neighborhood and $g = L$ and $w_g \Psi(\tilde{\gamma}_{gn}) > \theta_k w_k \Psi(\tilde{\gamma}_{kn})$
c. n is the richer neighborhood and $g = L$ and $w_g \Psi(\tilde{\gamma}_{gn}) < \theta_k w_k \Psi(\tilde{\gamma}_{kn})$
d. n is the poorer neighborhood and $g = H$ and $w_g \Psi(\tilde{\gamma}_{gn}) < \theta_k w_k \Psi(\tilde{\gamma}_{kn})$

The first two conditions are consistent with the results of Lemma 2. They state that improving travel times from the high (low) income neighborhood to high (low) income destinations increases income segregation when there is large enough high (low) income transit ridership in the neighborhood and sufficiently small θ_k. The last two conditions state that improving travel times from the high (low) income neighborhood even to low (high) income destinations increase income segregation when there is large enough high (low) income transit ridership and sufficiently large θ_k.

Similarly, $(dDISS/d\tau^t_{ng}) > 0$ if and only if one of the following holds true:

a. n is the richer neighborhood and $g = L$ and $w_L \Psi(\tilde{\gamma}_{Ln}) > \theta_H w_H \Psi(\tilde{\gamma}_{Hn})$
b. n is the poorer neighborhood and $g = H$ and $w_H \Psi(\tilde{\gamma}_{Hn}) > \theta_L w_L \Psi(\tilde{\gamma}_{Ln})$
c. n is the richer neighborhood and $g = H$ and $w_H \Psi(\tilde{\gamma}_{Hn}) < \theta_L w_L \Psi(\tilde{\gamma}_{Ln})$
d. n is the poorer neighborhood and $g = L$ and $w_L \Psi(\tilde{\gamma}_{Ln}) < \theta_H w_H \Psi(\tilde{\gamma}_{Hn})$.

□

Positive Network Spillovers (Proof of Corollary 3)

From where does shorter travel times by *transit* minimize *DISS*imilarity more?

Proof. Take the derivative of *DISS* with respect to τ^t_{gn} and compare $n = 1$ to $n = 2$:

$$\frac{dDISS}{d\tau^t_{g1}} - \frac{dDISS}{d\tau^t_{g2}} = 2\Lambda \cdot (1-\Lambda) \cdot f \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H]) \cdot (1 - 2\mathbf{I}_{g=H}) \cdot G_1, \tag{A.33}$$

$$\text{where } G_1 \equiv \frac{dC_{g1}}{d\tau^t_{g1}} - \theta_k \frac{dC_{k1}}{d\tau^t_{k1}} + \frac{dC_{g2}}{d\tau^t_{g2}} - \theta_k \frac{dC_{k2}}{d\tau^t_{k2}} \text{ where } k \neq g.$$

Improving *transit* times in neighborhood 1 would induce more income mixing (or less income segregation) than doing so in neighborhood 2 if and only if $(dDISS/d\tau^t_{g1}) - (dDISS/d\tau^t_{g2})$ is positive. The sign depends on G_1. Based on (A.14),

$$G_1 = w_g \Psi(\tilde{\gamma}_{1g}) - \theta_k w_k \Psi(\tilde{\gamma}_{1k}) + w_g \Psi(\tilde{\gamma}_{2g}) - \theta_k w_k \Psi(\tilde{\gamma}_{2k})$$

$$= w_g \left(\Psi(\tilde{\gamma}_{1g}) + \Psi(\tilde{\gamma}_{2g}) \right) - \theta_k w_k (\Psi(\tilde{\gamma}_{1k}) + \Psi(\tilde{\gamma}_{2k}))$$

$G_1 < 0$ implies that the nontargeted benefits from expanding the transit network outweighs the direct benefits from shorter travel times to targeted destinations. So, $(dDISS/d\tau^t_{1g})$ is larger than $(dDISS/d\tau^t_{2g})$ if and only if either

a. neighborhood 1 is richer, $g = L$, and sufficiently large low-income *transit* ridership such that $G_1 > 0$, OR
b. neighborhood 1 is poorer, $g = H$, and sufficiently large high-income *transit* ridership such that $G_1 > 0$, OR
c. neighborhood 1 is richer, $g = H$, and sufficiently large low-income *transit* ridership such that $G_1 < 0$, OR
d. neighborhood 1 is poorer, $g = L$, and sufficiently large high-income *transit* ridership such that $G_1 < 0$

The cases for improving *transit* times in neighborhood 2 follow the same reasoning. □

Qualitatively, we get lower income segregation on improving *transit* access between the high (low) income neighborhood and

- low (high) income destinations when *transit* ridership is sufficiently large among low (high) income households
- high (low) income destinations when *transit* ridership is sufficiently large among low (high) income households

More concisely, we get lower income segregation on improving *transit* access from the high (low) income neighborhood when *transit* ridership is sufficiently large among low (high) income households, just like in the baseline model.

A.3.10 | Perfect Income Segregation ($\bar{\xi}_g \notin [-\Xi, \Xi]$ for Some g)

The paper's main results focus on cases where both $\bar{\xi}_L$ and $\bar{\xi}_H$ lie in the support $[-\Xi, \Xi]$ of the distribution of ξ. In this section, I consider the alternative where $\bar{\xi}_g \notin [-\Xi, \Xi]$ for some $g \in \{L, H\}$. In other words, all households in one of the income groups sort perfectly into one neighborhood. Let $f(\xi)$ denote the probability density of ξ such that $f(\xi) = 0$ for $\xi \notin [-\Xi, \Xi]$ and $f(\xi) = 1/(2\Xi)$ for $\xi \in [-\Xi, \Xi]$.

After substituting the housing market clearing constraint (7) into the dissimilarity index (8), we can rewrite our measure of segregation as

$$DISS = 2\Lambda \cdot (1 - \Lambda) \cdot \left| F\left(\bar{\xi}_L\right) - F\left(\bar{\xi}_H\right) \right|. \tag{A.34}$$

Lemma A.3. *Suppose $\bar{\xi}_g \notin [-\Xi, \Xi]$ for some $g \in \{L, H\}$. Then marginal changes in travel times do not affect income segregation in the city*

$$\frac{dDISS}{d\tau_n^m} = 0 \; \forall m \in \{t, c\}, \; \forall n \in \{1, 2\}.$$

Proof. Take the derivative of $DISS$ in (A.34) with respect to travel time τ_n^m to get

$$\frac{dDISS}{d\tau_n^m} = 2\Lambda \cdot (1 - \Lambda) \cdot (1 - 2\mathbf{I}[\bar{\xi}_L < \bar{\xi}_H])$$

$$\cdot \left[(1 - 2\mathbf{I}_{n=2}) \cdot \left(f\left(\bar{\xi}_L\right) \cdot \frac{dC_{Ln}}{d\tau_n^m} - f\left(\bar{\xi}_H\right) \cdot \frac{dC_{Hn}}{d\tau_n^m} \right) + \left(f\left(\bar{\xi}_L\right) - f\left(\bar{\xi}_H\right) \right) \cdot \frac{d\Delta p}{d\tau_n^m} \right].$$

When $\bar{\xi}_g \notin [-\Xi, \Xi]$ for both $g \in \{L, H\}$, $f\left(\bar{\xi}_g\right) = 0 \; \forall g$. So none of the households move in response to marginal changes (in $\bar{\xi}$) and $(dDISS/d\tau_n^m) = 0$.

Let us then consider the case where at least one of $\bar{\xi}_L$ or $\bar{\xi}_H$ lie in the support. Suppose $\bar{\xi}_g \in [-\Xi, \Xi]$ and $\bar{\xi}_k \notin [-\Xi, \Xi]$ for $g \neq k$. The housing market clearing condition in (7) boils down to

$$\frac{1}{2} = \Lambda \cdot \mathbf{I}[\bar{\xi}_k > \Xi] + (1 - \Lambda) \cdot \frac{\bar{\xi}_g + \Xi}{2\Xi}$$

and we can decompose $\bar{\xi}_g$ to solve for the equilibrium housing price difference across neighborhoods:

$$\Leftrightarrow \Delta p = -\Xi + \left(1 - 2\Lambda \cdot \mathbf{I}[\bar{\xi}_k > \Xi]\right) \cdot \frac{2\Xi}{2(1 - \Lambda)} - \left[C_{g1} - C_{g2} - A_{g1} + A_{g2}\right].$$

Take the derivative with respect to τ_n^m to get

$$\frac{d\Delta p}{d\tau_n^m} = -(1 - 2\mathbf{I}_{n=2}) \cdot \frac{dC_{gn}}{d\tau_n^m}$$

$$\Rightarrow \frac{dDISS}{d\tau_n^m} = -\frac{2\Lambda \cdot (1 - \Lambda)}{2\Xi} \cdot \left[(1 - 2\mathbf{I}_{n=2}) \cdot \frac{dC_{gn}}{d\tau_n^m} + \frac{d\Delta p}{d\tau_n^m} \right] = 0.$$

Since only households in income group g relocate in response to travel time changes and since the number of group g households that move out is the same as the number of group g households that move in (given fixed housing supply), the income compositions of the neighborhoods remain the same in equilibrium.

Thus, marginal changes in travel times do not affect $DISS$ when either $\bar{\xi}_L$ or $\bar{\xi}_H$ (or both) does not lie in the support $[-\Xi, \Xi]$ of the distribution of ξ. □