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ASPECTS OF SECOND-ORDER FEEDBACK AM SYNTHESIS

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ABSTRACT

The technique of Feedback Amplitude Modulation (FBAM) is introduced in its second-order form. The basic aspects of the novel algorithm are discussed and its interpretation as a special case of a recursive periodic linear time-varying filter is explored. Extensions to the basic method are introduced, first with reference to existing variants of the first-order case, which have been previously studied. Second-order specific variations, such as pole-angle modulated resonator and frequency modulation, are discussed in some detail. The spectra produced with the proposed second-order methods are generally richer than those obtained using first-order methods, which is very desirable. Applications of the second-order FBAM methods complement the paper.

1. INTRODUCTION

Digital Feedback Amplitude Modulation (FBAM) synthesis is a class of synthesis techniques based on a self-modulating oscillator [1], [2]. It consists of a straightforward arrangement in which a delayed output is added to the oscillator amplitude. The simplest case, using a sinusoidal oscillator and a unit-sample delay, is defined by the following equation:

$$y(n) = \cos(\omega_0 n)\left[1 + y(n-1)\right], \qquad (1)$$

where $\omega_0 = 2\pi f_0/f_s$, f_0 is the fundamental frequency, and f_s is the sampling rate.

This basic equation can then be used as the germ for a number of variations, which include, for instance, the addition of extra terms, waveshaping, heterodyning and longer feedback periods. FBAM can also be described a special case of a first-order recursive periodically linear time-variant (PLTV) filter [3], if Eq. (1) is recast as

$$y(n) = x(n) + a(n)y(n-1), \qquad (2)$$

with $x(n) = a(n) = \cos(\omega_0 n)$. Regarding FBAM as a PLTV filter proves to be very useful for the understanding of the system, as well as for developing variants to the basic technique, as extensively discussed in [2].

In this paper, we will extend the FBAM method from its first-order formulation (FBAM-1) to the second-order form (FBAM-2) and its variants. We will first examine the basic attributes of a straight extension of Eq. (1) into second-order and the definition of a basic FBAM-2 algorithm. This will be followed by a study of some of its derivative forms and applications.

2. SECOND-ORDER FBAM

A simple FBAM-2 expression can be defined as

$$y(n) = \cos(\omega_0 n)\left[1 + y(n-1) + y(n-2)\right], \qquad (3)$$

where the amplitude of an oscillator is modulated by both its one-sample delay and its two-sample delay.

As with the first-order FBAM, this feedback expression can be expanded into an infinite sum of products given by

$$\begin{aligned}
y(n) &= \cos(\omega_0 n) + \cos(\omega_0 n)\cos(\omega_0[n-1]) \\
&+ \cos(\omega_0 n)\cos(\omega_0[n-1])\cos(\omega_0[n-2]) + \ldots + \\
&\cos(\omega_0 n)\cos(\omega_0[n-2]) \\
&+ \cos(\omega_0 n)\cos(\omega_0[n-2])\cos(\omega_0[n-4]) + \ldots + \\
&\cos(\omega_0 n)\cos(\omega_0[n-2])\cos(\omega_0[n-3]) + \\
&\cos(\omega_0 n)\cos(\omega_0[n-2])\cos(\omega_0[n-3])\cos(\omega_0[n-4]) + \ldots \\
&= \sum_{j=0}^{\infty}\left\{\begin{array}{l}
\prod_{k=0}^{j}\cos\left[\omega_0(n-k)\right] + \\
\prod_{l=0}^{j}\cos\left[\omega_0(n-2l)\right] + \\
\cos(\omega_0 n)\prod_{m=0}^{j}\cos\left[\omega_0(n-m-2)\right]
\end{array}\right\},
\end{aligned} \qquad (4)$$

which defines a pulse-like waveform made up of harmonics of the fundamental f_0. When compared to the first-order feedback oscillator,

$$\begin{aligned}
y(n) &= \cos(\omega_0 n)\left[1 + y(n-1)\right] \\
&= \sum_{k=0}^{\infty}\prod_{m=0}^{k}\cos\left[\omega_0(n-m)\right],
\end{aligned} \qquad (5)$$

we can see that a number of extra terms exist in the expansion. These will give rise to a narrower pulse and a richer spectrum with a wider bandwidth (see Fig. 1).

As with the original FBAM-1, it is useful to regard FBAM-2 as a second-order PLTV filter. In this case, Eq. (3) becomes

$$y(n) = x(n) + a_1(n)y(n-1) + a_2(n)y(n-2), \qquad (6)$$

with $x(n) = a_1(n) = a_2(n) = \cos(\omega_0 n)$. Of course, when developing the algorithm fully as a PLTV there will be no need to force the two coefficients $a_1(n)$ and $a_2(n)$ to be the same periodic signal or the filter input to be a sinusoid.

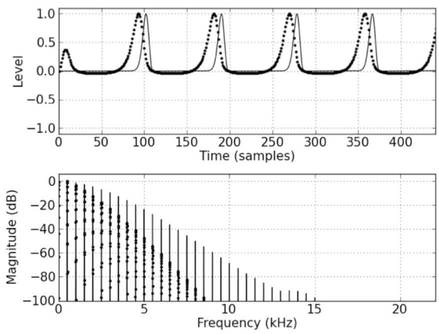

Figure 1. Comparison of FBAM-1 (dots) and FBAM-2 (continuous line) waveforms and spectra. $f_0 = 500$ Hz.

Finally, to complete the basic FBAM-2 algorithm, it is useful to include scaling parameters for the two feedback terms,

$$y(n) = \cos(\omega_0 n)\left[1 + \beta_1 y(n-1) + \beta_2 y(n-2)\right], \quad (7)$$

following the form seen in [1] for the first-order case, where it is called the 'theme' on which subsequent 'variations' are based. The flowchart of this basic FBAM-2 algorithm is shown in Fig. 2.

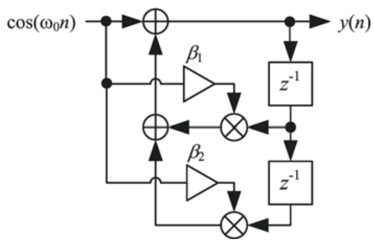

Figure 2. Flowchart of the basic FBAM-2 algorithm.

3. VARIANTS

3.1. Second-order versions of FBAM-1 variations

The FBAM-1 coefficient-modulated allpass filter variation can be transformed into a second-order configuration by connecting two allpass stages into a cascade, and furnishing the latter stage with two feedback terms, as in

$$\begin{aligned} w(n) &= x(n-1) - a(n)\left[x(n) - w(n-1)\right] \\ y(n) &= w(n-1) - a(n)\left[w(n) - \beta_1 y(n-1) - \beta_2 y(n-2)\right], \end{aligned} \quad (8)$$

with $x(n) = a(n) = \cos(\omega_0 n)$. The effect of the added allpass stage ($\beta_2 = 0$) is depicted in Fig. 3, which shows a modest increase in bandwidth when compared to the original FBAM-1 form. However, increasing β_2 towards unity will gradually widen the bandwidth, until the spectrum reaches the shape shown in Fig. 1 ($\beta_2 = 1$).

Other second-order FBAM-1 variants present similar characteristics. For example, the waveshaping variation, defined as

$$y(n) = \cos(\omega_0 n)\left\{1 + f\left[\beta_1 y(n-1) + \beta_2 y(n-2)\right]\right\} \quad (9)$$

and shown in Fig. 4 using a cosine waveshaper, behaves accordingly.

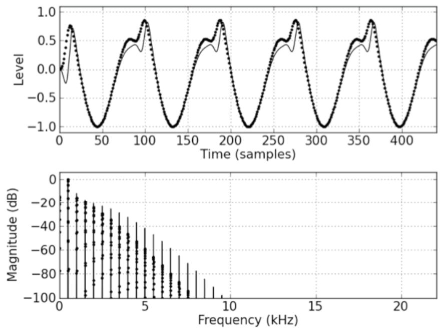

Figure 3. Allpass variation, comparison of FBAM-1 (dots) and FBAM-2 (continuous line). $f_0 = 500$ Hz, $\beta_1 = 1$, $\beta_2 = 0$.

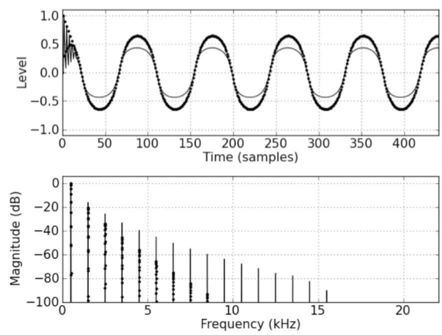

Figure 4. Waveshaping variation – using $f = \cos[.]$ waveshaper, comparison of FBAM-1 (dots) and FBAM-2 (continuous line). $f_0 = 500$ Hz, $\beta_1 = \beta_2 = 1$.

3.2. Pole-angle modulated resonator

As discussed above, extensions of the basic FBAM-2 algorithm can be created by considering it as a second-order PLTV system. An interesting synthesis case is to consider an LTI resonator filter structure [4], defined by

$$y(n) = x(n) + 2R\cos(\theta)y(n-1) - R^2 y(n-2), \quad (10)$$

where the two complex-conjugate filter poles have radius R and angle $\pm\theta$. In the PLTV case, our synthesis equation can be written as

$$y(n) = x(n) + 2R\cos(\pi[\beta a(n) + \alpha])y(n-1) - R^2 y(n-2), \tag{11}$$

where $x(n) = a(n) = \cos(\omega_0 n)$ and we are able to implement the filter pole-angle modulation.

Now we have three parameters to play with, the filter radius *R,* the modulation amount β, and the angle offset α. Various waveform shapes and spectra can be obtained with different values for these parameters, within their stability range. Fig. 5 shows the synthesis of a quasi-bandlimited square wave, generated by setting $R = 0.5$, $\beta = 1$ and $\alpha = 0$. Higher values of *R* will produce more harmonics, but with aliasing becoming more prominent.

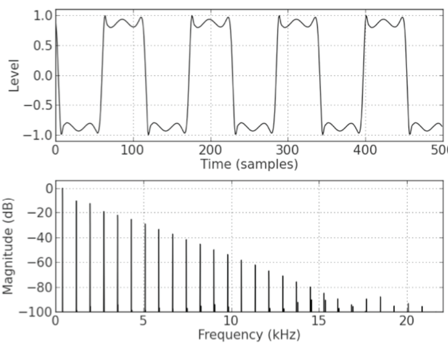

Figure 5. Pole-angle modulation synthesis, with $R = 0.5$, $\beta = 1$ and $\alpha = 0$.

The resonator structure also allows for classic frequency (or phase) modulation [5] to be implemented, within a narrow range of parameters. By setting *R* to close or equal to 1 and limiting β to small values (around 0.01), we can then use *α* to denote a carrier frequency *c* in Hz:

$$\alpha = \frac{c}{f_s/2}. \tag{12}$$

This combination of parameters is very unstable and some *c:m* ratios are impossible (*m* defined as in [5] to be the modulator frequency). In particular, the cases $c \leq m$ are problematic. Some ratios of small numbers are also unstable: 3:2, 2:1. The FM spectrum will be present for the duration of the envelope of the resonator impulse response, which is a decaying exponential defined by R^n. This allow us to generate an inharmonic attack based on a certain *c:m* ratio, which leads into an harmonic tone defined by the pole angle modulation synthesis after a certain amount of time.

Of course, since the pole-angle modulated resonator is PLTV, we can use distinct signals for its input $x(n)$ and modulator $a(n)$. An interesting case arises when we have a sinusoidal modulator and an arbitrary monophonic pitched input. In this case, we will be able to add components to the signal, creating a distorted output which is reminiscent of adaptive FM (AdFM) [6] and Adaptive Phase Distortion synthesis [7]. An example is shown on Fig. 6, where a C4 flute tone is used as an input to a pole-angle modulated resonator.

By setting the modulator frequency in relation to the input fundamental, it is possible to create harmonic or inharmonic spectra, depending on the modulator to input f_0 ratio. This follows the similar principles of *c*:*m* ratios in FM (and AdFM) synthesis.

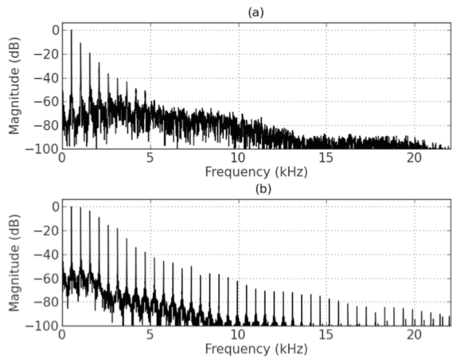

Figure 6. Pole-angle modulated resonator with a C4 flute tone as input: (a) original steady-state spectrum and (b) spectrum of the pole-angle modulated filter output.

3.3. Frequency-modulated filters

Following these principles, it is possible to modulate the filter frequency and bandwidth directly, by converting the modulating signal into the filter pole parameters *R* and θ. In this case, in order to obtain a more stable behaviour from the filter, we keep a fixed *Q* ratio, which ultimately means that both *R* and θ are modulated. To do that, we use Eq. (10) and the following identities:

$$\theta = \frac{2\pi f_c(n)}{f_s} \text{ and } R = \exp\left[-\frac{\pi}{f_s}\frac{f_c(n)}{Q}\right], \tag{13}$$

with $Q = f_c{:}B$, where *B* is the –3 dB bandwidth in Hz and $f_c(n)$ is the time-varying centre frequency in Hz. The time-varying centre frequency can then be generated by sinusoidal modulation as

$$f_c(n) = f_c + A\cos\left[\frac{2\pi f_m n}{f_s}\right], \tag{14}$$

where f_m is the modulation frequency in Hz. Care needs to be taken with *Q* and the frequency deviation *A* to keep the filter stable and reduce aliasing. The latter can be set to the product If_m, where *I* is a modulation index, as in classic FM synthesis. This set-up is much more stable than the basic pole-angle modulation FM and allows for yet another range of synthesis and processing effects.

4. APPLICATIONS

The Chamberlin state variable filter is a widely used second-order topology that enables decoupled control over the center frequency F_c and Q factor, $Q_c = 1/Q$, of the filter [8], [9]. Applying the frequency-modulation variant of 3.3, we keep Q_c fixed within the range 0…2, and modulate the center frequency of the filter using

$$f_c(n) = 2\sin\left(\frac{\pi F_c}{f_s}\right) + A\cos\left(\frac{2\pi f_m}{f_s}n\right). \qquad (15)$$

The lowpass output of the Chamberlin filter can then be written in PLTV form as

$$\begin{aligned} y(n) = {} & b_1(n)x(n-1) + \beta_1 a_1(n)y(n-1) \\ & - \beta_2 a_2(n)y(n-2), \end{aligned} \qquad (16)$$

with $b_1(n) = f_c(n)^2$, $a_1(n) = 2 - f_c(n)Q_c - f_c(n)^2$, and $a_2(n) = 1 - f_c(n)Q_c$. Setting β_2 close or equal to 1, Eq. (16) produces a formant whose bandwidth and magnitude can be controlled using Q_c and A. Parameter β_1 controls the center frequency of the resonance peak, as depicted in Fig. 7. The waveform plot shows also an initial transient, which damps rapidly with low Q_c values, but stays more pronounced when Q_c is increased. This is useful in inharmonic attack segment generation.

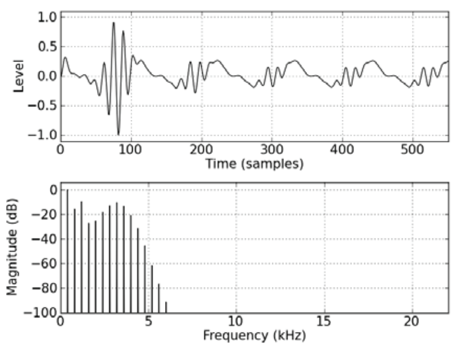

Figure 7. Coefficient-modulated Chamberlin lowpass filter output, with $f_0 = f_m = 400$ Hz, $F_c = 100$ Hz, $Q_c = 1.3$, $A = 0.2$, $\beta_1 = 0.9$, and $\beta_2 = 1$.

5. CONCLUSIONS

In this paper, we have studied some basic aspects of the second-order FBAM. We have presented it as a novel and natural extension of the first-order version of the synthesis method. It was demonstrated that the spectra of second-order FBAM variants are in general wider and richer than their first-order counterparts. This is a definite improvement on the original method, as it allows a more complex output without any further modifications to the process. Following the methodology of previous studies for first-order cases, we have looked at the technique as a form of PLTV filtering with a sinusoidal input and modulator.

As a further extension of this principle, we proposed some new variants based on standard second-order filters, in particular looking at ways of modulating resonator parameters. This leads to novel possibilities, based on pole-angle and center-frequency modulation of second-order filters. The principles of second-order PLTV can be useful in the construction of interesting adaptive effects.

A remaining issue, currently under investigation, regards the filter stability, which is more complex here than in the first-order cases. Although beyond the scope of this initial study, this forms an important research question that will be tackled in subsequent work.

6. ACKNOWLEDGMENTS

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