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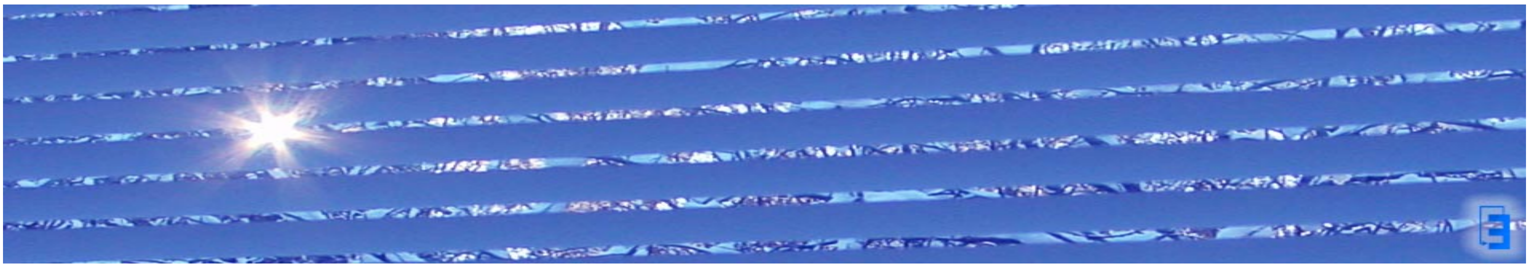
SIMULTANEOUS SYNTHESIS OF FLEXIBLE HEAT EXCHANGER NETWORKS

Doctoral Thesis

Juha Aaltola



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Juha Aaltola

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ABSTRACT

In industry there is still lot of potential to make an energy system more efficient and thereby reduce the waste heat available. On the other hand there is an option to export the waste heat to another industry or to society. When the use of a heat exchanger network is considered for these tasks the optimization framework developed in this work can be implemented to calculate the cost of optimal investments.

This thesis presents a framework for generating flexible heat exchanger networks (HEN) over a specified range of variations in the flow rates and temperatures of the streams, so that the total annual costs (TAC) as a result of utility charges, exchanger areas and selection of matches are minimized. The proposed framework includes (i) an initialization stage to reduce the problem size, (ii) a multiperiod simultaneous MINLP model to synthesize a flexible HEN configuration, (iii) a multiperiod LP feasibility test model to check the operability and identify critical conditions which are to be included in the possible resolve stage of the MINLP model, and (iv) an NLP improvement model for further optimization by partly removing simplifications related to the MINLP model. This framework results in a HEN which can work in varying conditions without losing stream temperature targets and can keep an economically optimal energy integration.

This thesis also shows how the simplified superstructure presentation proposed by Yee and Grossmann (1990a) can be applied for generating flexible heat exchanger networks. Furthermore, this thesis presents a scheme which eliminates the modeling of bypasses, so that the nonlinear heat balances, binary variables, temperature variables and flow variables related to each bypass in the superstructure are no longer needed in the model. The elimination of bypass modeling, a stage-wise superstructure presentation and isothermal mixing assumption, make the MINLP model more robust and efficient to solve. Since this MINLP model is not solved until a problem is well prepared by the other parts of the developed optimization framework, the methodology presented in this thesis is applicable to solve industrial size grassroot design cases of flexible heat exchanger network problems

Lastly, the proposed HEN synthesis strategy has been successively applied to two industrial

problems where the industrial waste heat streams have been cooled down, forming a local and site level energy integration to gain savings in steam consumption and to avoid cooling tower investment. Both these problems represent the special case of correlated uncertain parameters, which here means that there is a relationship between uncertain parameters given in the stream data.

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Nomenclature

Abbreviations:

TAC	total annual costs
MINLP	mixed integer nonlinear programming
MILP	mixed integer linear programming
NLP	nonlinear programming
LP	linear programming
HEN	heat exchanger network
HRAT	heat recovery approach temperature
EMAT	exchanger minimum approach temperature
LMTD	logarithmic mean temperature difference
AMTD	arithmetic mean temperature difference

Subscripts:

i	hot process or utility stream
j	cold process or utility stream
k	index for stage 1,…, NOK (number of stages) and temperature location 1,…, NOK + 1
p	operation period
HU	hot utility
CU	cold utility

Superscripts:

max	maximum
min	minimum
u	utility
c	cold stream or utility
h	hot stream or utility

IN	inlet
OUT	outlet
UP	upper bound
F	fixed
b	bypass
a	after exchanger before mixing bypass stream

Sets:	
CP	set of a cold process stream *j*
CU	set of a cold utility
HP	set of a hot process stream *i*
HU	set of a hot utility
PR	set of a operation period, $p = 1, \ldots,$ NOP
ST	set of a stage in the superstructure, $k = 1, \ldots,$ NOK

Parameters:	Units	
AF	-	annualization factor
$A_{i,j,k,p}$	[m^2]	heat transfer area on exchanger connecting streams *i* and *j* in stage *k* in period *p*
B	-	exponent for area cost
C	€/unit	area cost coefficient for heat exchanger
C^{CU}	€/unit	per unit cost for cold utility
C^{F}	€/unit	fixed charge for heat exchanger unit
C^{HU}	€/unit	per unit cost for hot utility
Cw	-	weight for decrease of areas
DOP	-	duration of period
$dt^{ca}_{j,k,p}$	[ºC]	temperature difference on exchanger before mixing bypass stream when bypass is on cold stream *j*
$dt^{ha}_{i,k,p}$	[ºC]	temperature difference on exchanger before mixing bypass stream when bypass is on hot stream *i*

DT^{UP}	[ºC]	an upper bound on temperature difference
F	[kW/K]	heat capacity flow rate
$F^{cb}_{i,j,k,p}$	[kW/K]	cold side bypass fraction over exchanger connecting streams *i* and *j* in stage *k* in period *p*
$F^{cin}_{i,j,k,p}$	[kW/K]	cold side fraction in exchanger connecting streams *i* and *j* in stage *k* in period *p*
$F^{hb}_{i,j,k,p}$	[kW/K]	hot side bypass fraction over exchanger connecting streams *i* and *j* in stage *k* in period *p*
$F^{hin}_{i,j,k,p}$	[kW/K]	hot side fraction in exchanger connecting streams *i* and *j* in stage *k* in period *p*
CN^{UP}	[W/K]	an upper bound on conductance
HU_p^{UP}	[kW]	an upper bound on total hot utility available
$MAXAREA_{i,j,k}$	[m²]	an upper bound on heat transfer area for exchanger connecting streams *i* and *j* in stage *k*
NOK	-	number of stages
NOP	-	number of periods
NU	-	total number of units
Q^{UP}	[kW]	an upper bound on heat exchange capacity
$SPLIT^{C}$	-	existence of split on cold stream *j* at stage *k*
$SPLIT^{H}$	-	existence of split on hot stream *i* at stage *k*
$t^{ca}_{j,k,p}$	[ºC]	temperature after exchanger before mixing bypass stream when bypass is on cold stream *j*
$t^{ha}_{i,k,p}$	[ºC]	temperature after exchanger before mixing bypass stream when bypass is on hot stream *i*
T^{IN}	[ºC]	inlet temperature of stream
T^{OUT}	[ºC]	outlet temperature of stream
U	[kW/m²K]	overall heat transfer coefficient
$Z_{i,j,k}$	-	existence of match (*i*,*j*) in stage *k*
Z^{HU}_{j}	-	existence of match between cold stream *j* and hot utility
Z^{CU}_{i}	-	existence of match between hot stream *i* and cold utility
ε	[ºC]	exchanger minimum approach temperature

Variables:	Units	
$q_{i,j,k,p}$	[kW]	Heat flow exchanged between hot stream i and cold stream j in period p
$dt^{cu}_{j,k,p}$	[ºC]	temperature difference for match of hot stream i and cold utility in period p
$dt^{hu}_{i,k,p}$	[ºC]	temperature difference for match of cold stream *j* and hot utility in period *p*
$dt_{i,j,k,p}$	[ºC]	temperature difference for match (*i*,*j*) at temperature location *k* in period *p*
$f^{c}_{i,j,k,p}$	[kW/K]	heat capacity flow rate of cold stream fraction related to exchanger *i*,*j*,*k*
$f^{h}_{i,j,k,p}$	[kW/K]	heat capacity flow rate of hot stream fraction related to exchanger *i*,*j*,*k*
Obj	-	sum of linearly weighted objectives
$q^{cu}_{i,p}$	[kW]	Heat flow exchanged between hot stream *i* and cold utility in period *p*
$q^{hu}_{j,p}$	[kW]	heat flow exchanged between cold stream *j* and hot utility in period *p*
$sdt_{i,j,k,p}$	-	slack variable for temperature approach violations related to match (*i*,*j*) at temperature location *k* in period *p*
$s_{i,j,k,p}$	-	slack variable
$s^{min}_{i,j,k}$	-	minimum of $s_{i,j,k,p}$ for match *i*,*j*, *k*
$t^{c}_{j,k,p}$	[ºC]	temperature of cold stream *j* at hot end of stage *k* in period *p*, exchanger outlet
$t^{c}_{j,k+1,p}$	[ºC]	temperature of cold stream j at hot end of stage *k* in period *p*, exchanger inlet
$t^{cs}_{i,j,k,p}$	[ºC]	temperature of cold stream fraction after exchanger *i*,*j*,*k* in period *p*
$t^{h}_{i,k,p}$	[ºC]	temperature of hot stream *i* at hot end of stage *k* in period *p*, exchanger inlet

$t^{h}_{i,k+1,p}$	[ºC]	temperature of hot stream *i* at hot end of stage *k* in period *p*, exchanger outlet
$t^{hs}_{i,j,k,p}$	[ºC]	temperature of hot stream fraction after exchanger *i*,*j*,*k* in period *p*

Binary variables

z^{cu}_{i}	-	existence of match between cold stream *j* and hot utility
z^{hu}_{j}	-	existence of match between hot stream *i* and cold utility
$z_{i,j,k}$	-	existence of match connecting streams *i* and *j* in stage *k*

1 INTRODUCTION

1.1 Background

Heat exchanger network (HEN) synthesis has been one of the most well studied issues within process synthesis during the last three decades. Process synthesis, a part of process design, has the objective of developing systematically a flowsheet which describes the overall process system and which meets certain specified performance criteria, and is ultimately able to transform the raw materials into the desired products (Floudas (1995)). As shown in Figure 1 the overall process system consists of three main interactive components, which can be integrated into an operable plant. These three components are:

(i) Industrial processes
(ii) Heat recovery system
(iii) Utility system.

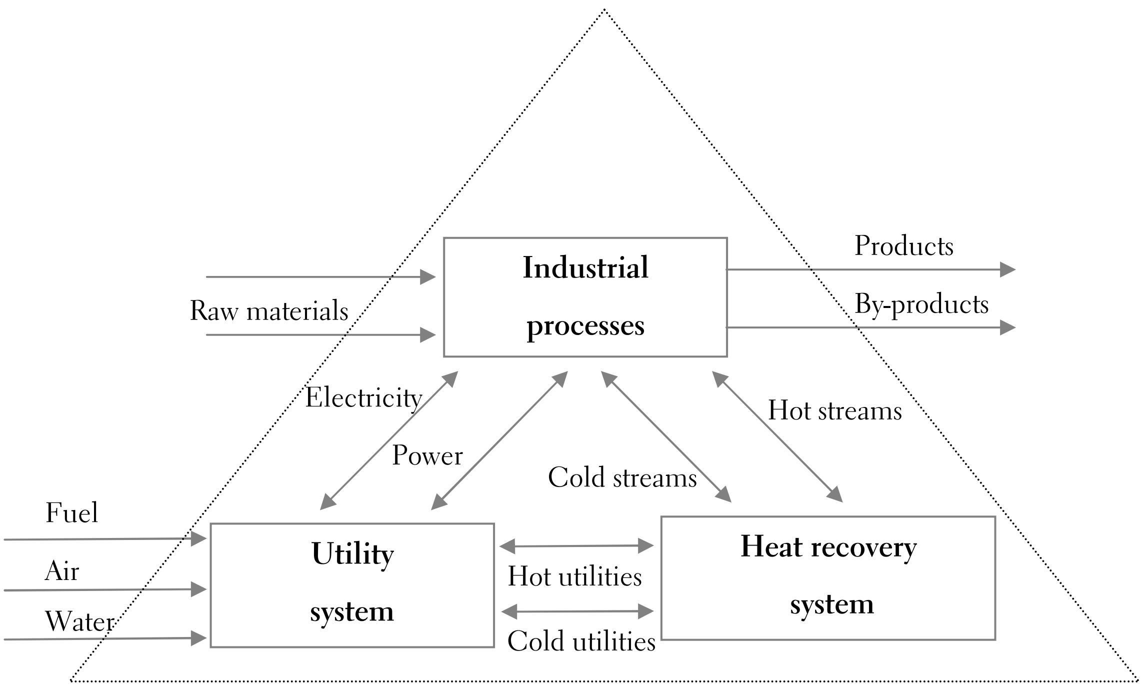

Figure 1. An overall process system.

The industrial processes may consist of reactors, separators and recycle systems that transform the raw materials into the desired products. The utility system consists of hot and cold utility units. Typical hot utility units are turbines, generators, motors and boilers providing the required electricity, steam and hot water. Cold water from external sources is used as the cold utility, providing the necessary cooling in the processes. In the heat recovery system, the process streams exchange heat so as to reduce the hot and cold utility requirements. The only units in a heat recovery system are the heat exchangers.

The major challenge within the heat exchanger network synthesis problem is to identify the best pair of process streams to be connected with the heat exchangers, so as to maximize economical energy recovery. This pairing problem is a potentially explosive combinatorial problem, which includes nonlinear models describing each unit and its sizing, resulting in MINLP (mixed integer nonlinear programming) models and in general to local solutions to the problem.

When the environment in the plant introduces significant changes in the operating conditions, in contrast to most designs which have been developed with the assumption of fixed design parameters, a synthesized HEN should not only be optimal in nominal conditions, but also operable in the specified changing environment. In other words, a HEN should remain operable under variations without losing stream temperature targets while, at the same time, keeping an economically optimal energy integration. The previously mentioned HEN synthesis problem field is considered in this dissertation. Specifically, a thorough study has been conducted into the special class of the general problem: How to formulate an efficient optimization model for simultaneous optimization of the flexible HEN synthesis problem.

1.2 Research problem

The flexible HEN synthesis problem to be addressed in this thesis can be stated as follows: A set of hot streams to be cooled and cold streams to be heated are given which include multiperiod stream data with inlet and outlet stream temperatures, heat capacity flow rates and heat transfer coefficients. In addition, a set of hot and cold utilities are specified. The objective then is, within the range of the operating conditions, to determine the heat exchanger network for energy recovery between the given set of hot and cold streams, so that the annualized cost of the equipment plus the annual cost of utilities will be minimized. The special case of correlated uncertain parameters, which here means that there is a relationship between uncertain parameters given in the stream data, is considered as a part of a feasibility test.

There have been a large number of methodologies proposed for HEN synthesis. One of the latest developments was during the early 1990s, when research efforts moved away from decomposition based approaches and focused instead on simultaneous optimization approaches. There have also been several systematic approaches to HEN synthesis where the flow rate and temperature variations have been considered. One of the first approaches considered variations after a design stage, leading to uneconomical HEN costs. In subsequent studies, the flexibility issue was included in the iterative scheme in the last part of the design stage where the HEN structure had already been designed for the nominal condition. Therefore, in order to achieve a HEN with the required level of flexibility, the original economical structure would be lost and the result would no longer be preferable. The most recently published works in this field are:

(i) Papalexandri and Pistikopoulos (1994a, b) introduced a systematic framework for the synthesis of a flexible HEN based on a multiperiod hyperstructure network presentation. The proposed framework results in a flexible and structurally controllable HEN featuring minimum annualized costs. In this formulation, detailed modeling was used for all the components and interconnections of the HEN, which naturally improves the possible attainable solution, but on the other hand leads to very tight limits on the size of a problem.

(ii) Tantimuratha *et al.* (2001) proposed a decomposition based iterative optimization scheme where the flexibility issue is addressed at a targeting stage. With the use of targets the network is optimized with a systematic iterative approach to minimize the annual costs and to ensure that the HEN is structurally capable of handling variations. However, the problem has been decomposed into subproblems; these being economical and flexible screening and screening and cost optimization. Early decisions in the selection of ΔT_{min} and primal network configurations affect the resulting HEN configuration, thus the procedure may lead to suboptimal solutions.

Since the benefits of simultaneous HEN synthesis approaches versus sequential approaches have been demonstrated, and while the sizes of the simultaneously solved flexible HEN synthesis problems have been very small, there is a demand for a method which could solve larger size problems and still retain the characteristics of simultaneous optimization.

1.3 Objective

The main objective of this thesis is to develop a method for solving flexible HEN synthesis problems, so that the trade-offs between energy costs, fixed charges for units and costs for the exchanger area can be simultaneously accounted. Unlike the previous simultaneous methods the developed method should be capable of solving industrial problems with available commercial mathematical programming solvers.

1.4 Scope of the research

The part of the framework, where the network feasibility is tested, has been developed especially for flexible HEN synthesis problems in local process integration cases where fully correlated variations and a few short term disturbances take place e.g., seasonal and short term changes in district heating systems and seasonal changes in pulp mills.

A discrete time representation is used in all models in the presented framework, therefore for all problems time sub-intervals (called periods) must be defined. There is a connection between the model size and the number of periods so that model size increases linearly as more periods are specified because most of the variables and constraints are defined for each period.

This work mainly considers grassroot design cases, although the presented framework allows for the predefining of the existence of heat exchanger units. Stream repiping and exchanger reassignments are not considered.

The following general assumptions are related to all the models:

(i) utility duties, split fractions and bypasses can be adjusted
(ii) perfect control, i.e., control can be adjusted to compensate for uncertain parameters and no delays in the measurements, or adjustments in the control are considered
(iii) constant heat capacities.

1.5 Outline of this work

In Chapter 2, the literature addressing heat exchanger network grassroot design and the flexibility problems is reviewed.

In Chapter 3 a framework is presented for generating flexible heat exchanger networks (HEN) over a specified range of variations in the flow rates and temperatures of the streams, so that the total annual costs (TAC) as a result of utility charges, exchanger areas and selection of matches are minimized. The proposed framework includes:

(i) An initialization stage to find a good initial point for the start of the optimization and to identify adequate bounds for the problem to reduce the model size for the later optimization stages.

(ii) A multiperiod simultaneous MINLP model to provide the optimal HEN structure, which minimizes costs when the conductance of every match is allowed to change separately under each specified period. The areas of heat exchangers depend on the conductance ($\frac{Q}{LMTD}$) where the LMTD is logarithmic mean temperature difference and the Q is heat load, thus the optimal solution may represent different sizes of heat exchangers for each match. However, there is only one investment decision to make and the maximum area is chosen for periods with the largest exchanger area requirements. For those operating conditions with a smaller area than the maximum, a bypass fraction is calculated to keep the stage temperatures at an optimal level.

(iii) A feasibility test model to analyze the results of the MINLP model for the temperature approach violations. The idea behind this LP formulation is to minimize temperature approach violations. The LP model can be solved with multiperiod data required in order to expose infeasibilities i.e. temperature approach violations. This data is formed by making denser discretization of the problem data for correlated parameters and also with the addition of periodical data for short term disturbances. After solving the LP feasibility test, the multiperiod MINLP model is resolved with data that includes additional periods, representing the worst temperature approach violation as identified by the LP feasibility test. The loop, including the MINLP and LP stage, continues until the MINLP model has sufficient data to provide a network without temperature approach violations under a whole specified range of parameter variations.

(iv) An NLP improvement model for the further optimization of the HEN. After achieving the solution from the MINLP stage, the NLP improvement model takes account of maximum areas so as to obtain the real area investment costs. This is done to achieve an optimal trade-off between capital and operating cost, leading to the real minimum total annual costs. The NLP model also takes account of the non-isothermal mixing of the streams after the parallel exchangers. In the NLP improvement model, the structure of the HEN is fixed and the areas of the exchangers are limited by setting the upper limit for each of them to correspond to the result of the MINLP.

Chapter 4 presents the applications of the optimization framework. Industrial size problems are illustrated and discussed.

Finally, the conclusion of the work in Chapter 5 and the suggested future work in Chapter 6 are presented.

2 State of the art of HEN synthesis

2.1 Introduction

Heat exchanger network (HEN) synthesis is one of the most extensively studied problems in industrial process synthesis. This is attributed to the importance of determining the energy costs for a process and improving the energy recovery in industrial sites. The first systematic method to consider energy recovery was the thermodynamic approach of the concept of pinch, introduced during the 1970s. This was followed by mathematical programming, stochastic optimization approaches and hybrid methods developed from these. Furman and Sahinidis (2002) reported that over 400 papers have been published on the subject over the last 40 years. Gundersen and Naess (1988) and Ježowski (1994a,b) have also contributed thorough reviews on HEN synthesis.

2.2 Targets and decomposition-based approaches

The first approaches in the 1960s and early 1970s treated the HEN synthesis problem without applying decomposition into sub-tasks. The limitations of optimization techniques were the bottleneck of the mathematical approaches at that time. For the synthesis problem of the HEN, the thermodynamic approach of pinch analysis was introduced by the work of Hohmann (1971) and Linnhoff and Flower (1978a, b). As a result of the pinch concept, the single task approaches were shifted to procedures introducing techniques for decomposing the problem into three subtasks (i.e., targets); minimum utility cost, minimum number of units and minimum investment cost network configurations. The main advantage of decomposing the HEN synthesis problem is that sub-problems can be treated in a much easier fashion than the original single-task problem. The sub-problems are the following:

(i) *Minimum utility* cost target corresponds to the maximum energy recovery that can be achieved in a feasible HEN for a fixed heat recovery approach temperature (HRAT), allowing for the elimination of several non-energy efficient HEN structures. Minimum utility cost was first introduced by Hohmann (1971) and Linnhoff and Flower (1978a) and later as an LP transportation model by Cerda et al. (1983), being an improvement of the LP transshipment model of Papoulias and Grossmann (1983).

(ii) *Minimum number of units* target determines the match combination with the minimum number of units and their load distribution for a fixed utility cost. The MILP transportation model of Cerda and Westerberg (1983) and the MILP transshipment model of Papoulias and Grossmann (1983) are the most common, while the vertical heat transfer formulation of Gundersen and Grossmann (1990) and Gundersen, Duvold and Hashemi-Ahmady (1996) are also used.

(iii) *Minimum investment cost network configurations* is based on the heat load and match information of previous targets. Using the superstructure-based formulation, developed by Floudas *et al.* (1986), the NLP problem is formulated and optimized for the minimum total cost of the network. The objective function in this model is the investment cost of the heat exchangers (i.e., heat transfer area since utility loads and matches are fixed) that are postulated in a superstructure. The objective function can be defined as a function of temperatures, using driving temperature forces expressed in the logarithmic mean temperature difference (LMTD) form, which is nonlinear and convex. Also, the energy balance constraints for the mixers and heat exchangers are nonlinear since they have bilinear products of unknown flow rates times corresponding to unknown temperatures. Because of the bilinear energy balance equalities the NLP problem formulation is nonconvex, which means that use of local NLP solvers (e.g., CONOPT and MINOS) yield local solutions (Floudas and Ciric (1989)).

The HEN synthesis strategy developed by Floudas et al. (1986), is a decomposition-based method i.e., a sequential approach. This synthesis strategy involves partitioning the problem into temperature intervals and, if possible, into subnetworks according to the pinch method. Next, the problem is decomposed into three sub-problems (i), (ii) and (iii), which are then solved according to the heuristic of finding the minimum cost network subject to the minimum number of units, which is subject to the minimum utilities costs. The HRAT is the only fixed parameter in the three sequential stages and can subsequently be updated by performing some search algorithm. This HEN synthesis strategy is recommended to be applied for all global solutions of a minimum number of units.

A more recent sequential approach proposed by Zhu (1995a, b, c, d) decomposes the problem into a set of enthalpy intervals reducing the dimensionality on the problem. Based on this decomposition an automated synthesis method is proposed by Zhu (1997) where targeting principles and heuristic rules are used with an MILP model for a selection of matches, and an MINLP model for determination of the final HEN configuration.

The decomposition-based approaches have proved in many case studies to be powerful HEN synthesis tools. The main shortcoming of the sequential approaches is the fact that the three-way trade-off between energy, units and area are not considered rigorously. Furthermore, the decision to decompose the original problem into sub-problems relying on pinch analysis may result in sub-optimal networks. Therefore, the HEN synthesis problem should be treated as a single-task problem.

2.3 Simultaneous approaches

The primary limitation of decomposition-based methods is that costs due to energy, units and area cannot be optimized simultaneously, and as a result the trade-offs are not taken into account appropriately. The selection of HRAT and partition into subnetworks affects the number of units and the heat exchange area in the final configuration. Therefore, the decomposition of the HEN synthesis problem may lead to suboptimal networks.

Simultaneous heat exchanger network synthesis methods aim to find the optimal network without or with some decomposition of the problem. Simultaneous optimization normally results in MINLP formulations, which include assumptions to simplify these complex models.

Floudas and Ciric (1989) proposed a match-network hyperstructure model to simultaneously optimize all of the capital costs related to the heat exchanger network. This MINLP formulation is based on the combination of the transshipment model of Papoulias and Grossmann (1983) for match selection, and the minimum investment cost network configuration model of Floudas and Grossmann (1986) for determining the heat exchanger areas, temperatures, and the flowrates in the network. The proposed simultaneous synthesis may still lead to suboptimal networks, since the value for HRAT must be specified before the design stage. The match-network hyperstructure model was then further modified by Ciric and Floudas (1991) to treat HRAT as an explicit optimization variable (i.e., the optimization of the minimum utilities cost is included as well). This MINLP formulation included any decomposition into design targets and simultaneously optimizes trade-offs between energy, units and area. Ciric and Floudas (1991) also demonstrated the benefit of a simultaneous approach versus sequential methods.

Another simultaneous synthesis formulation was proposed by Yee and Grossmann (1990b) where the model is based on the stage-wise superstructure representation of Yee *et al.* (1990a). The simplified superstructure consists of a number of stages, and in each stage many different possibilities for stream matching are allowed to take place. The assumptions of

(i) isothermal mixing,
(ii) no split stream flowing through more than one exchanger,
(iii) utilities are located in the ends of the superstructure, and
(iv) no stream by-pass

make the constraint set linear, while the objective function remains a nonlinear and nonconvex one. Daichendt and Grossmann (1994a, b, c and d) have developed a preliminary screening procedure for reducing the number of binary variables needed and finding bounds on the objective, in order to decrease the size and increase the robustness of the MINLP model of Yee and Grossmann (1990b).

Most recently there has been work by Soršak and Kravanja (2002) where the stage-wise superstructure by Yee *et al.* (1990a) is extended to alternative exchanger types. In this MINLP formulation additional constraints are specified to provide a feasible temperature distribution in the HEN, since different types of heat exchangers influence the inlet and outlet temperatures. As the consideration of different exchanger types drastically increases the combinatory, the integer-infeasible path MINLP approach has been applied to perform an initialization scheme and to decrease the computation effort required to solve the MILP master problem of the modified OA/ER (outer-approximation/equality-relaxation) algorithm proposed by Kocis and Grossmann (1987). Also, a multilevel MINLP procedure in reduced integer space has been proposed to solve industrial size HEN problems.

2.4 Multiperiod approaches

Previous sections dealt with heat exchanger network synthesis under the assumptions of fixed operating parameters at a nominal condition for given specifications of a process design. When the environment introduces significant changes in the operating conditions, a synthesized HEN must also be thermodynamically feasible for different operating modes i.e., it must be flexible.

In this work flexibility refers to the ability to handle a range of steady state operating conditions which a particular process design can achieve (Biegler *et al.* (1997)). Other related terms commonly used by researchers are *switchability*, *resilience*, *controllability*, *sensitivity*, and *operability*. Switchability is defined as the ability of a plant to move from one steady state condition to another. Controllability is the ability of a particular design, normally including the control system, to maintain safe and stable operating conditions during disturbances. All these other aspects are equally important, but flexibility is the first step that must be considered for any safe and operable design. Reviews of research into flexibility and operability can be found in Furman and Sahinidis (2002).

Marselle *et al.* (1982) defined resilience for heat exchanger networks and stated other properties of resilience. They proposed a heuristic design method for structurally resilient networks with respect to inlet parameter variations. They also identified a number of worst possible operating conditions: maximum heating, maximum cooling, maximum total heat exchange and minimum temperature difference. All these conditions were designed individually and later combined into a flexible design to handle all situations. One shortcoming of the presented approach is that due to the non-linearities of the problems, the claimed critical conditions, which are the operation conditions limiting the flexibility of the exchanger network, might not in fact reflect the real critical ones.

Swaney and Grossmann (1985a) introduced a flexibility index, which defines the maximum parameter range that can be achieved for a feasible operation. A flexibility index allows the designer to compare the degrees of flexibility for various design configurations and gives information on the critical points of uncertain parameters that limit the flexibility of a design. For the calculation of the flexibility index Swaney and Grossmann (1985b) presented a direct search procedure and implicit enumeration scheme. Later, Grossmann and Floudas (1987) introduced an active set strategy for the automated solution of the flexibility test and the flexibility index of Swaney and Grossmann (1985a). A mixed integer formulation was presented to identify the potential active constraints that limit the flexibility of a design. As the constraints for the feasible operation of the HEN result in a nonlinear region, the corresponding problem for feasibility analysis becomes an MINLP formulation.

Kotjabasakis and Linnhoff (1986) introduced sensitivity tables to find which heat exchanger areas should be increased and which heat exchanger should be bypassed in order to make a nominal design sufficiently flexible, and for making decisions for the trade-offs between cost effectiveness and flexibility of the design.

Floudas and Grossmann (1986) introduced a multiperiod MILP model for the minimum utilities cost and minimum number of matches target problems, based on Papoulias and Grossmann's (1983) transshipment model. In this model the changes in the pinch point and utility required at each time period are taken into account. Extensions were presented first by Floudas and Grossmann (1987a), an NLP formulation based on a superstructure presentation of possible network topologies to derive automatically network configurations that feature minimum investment cost, fewest number of units, and minimum utility cost for each time period. This was followed by Floudas and Grossmann (1987b) introducing a systematic two-stage procedure with

(i) prediction of matches coupled with a feasibility test at the level of matches and
(ii) derivation of the network configuration where the flexibility analysis has been made at the level of the structure.

The sequential targeting, fixing and optimization approach presented has the advantage of decomposing the synthesis problem. However, it does have the disadvantage that trade-offs between energy, number of units, area and flexibility aspect are not rigorously taken into account.

Papalexandri and Pistikopoulos (1994a, b) introduced a systematic framework for the multiperiod operation of the synthesis and retrofit heat exchanger network design. An iterative design procedure involves a multiperiod hyperstructure representation and an MINLP formulation, allowing both structural and control alternatives to be explored simultaneously. In this formulation, detailed modeling was used for all components and interconnections of the HEN, which naturally improves the possibly attainable solution, but on the other hand leads to very tight limits on the size of a problem.

Aguilera and Nasini (1995) proposed an MILP formulation for testing the flexibility of the HEN for flowrate variation, and later Aguilera and Nasini (1996) introduced a flexibility test for the HEN with non-overlapping inlet temperature variations. This was an MILP formulation which takes into account all the range of operation conditions at the same time.

Tantimuratha *et al.* (2001) proposed a screening and targeting process for the HEN design with flexibility consideration in both grassroots and retrofit cases. The screening stage is based on the screening models of Briones and Kokossis (1999a, b, c) and it considers both economic and flexibility aspects prior to network development. The cost and flexibility targets can be combined to compare the problem trade-offs. The model of Floudas and Grossmann (1986) is expanded, with additional constraints, to exploit the flexibility potential of selected match combinations for flexible network development via an iterative procedure. For a selected set of matches from the targeting stage the network is optimized with a systematic iterative approach to minimize the annual costs and to ensure that the HEN is structurally capable of handling variations. Decomposition of

(i) the screening stage into economic and flexibility aspects and
(ii) the overall problem into selecting superstructures and minimizing total costs

may lead to suboptimal solutions. This work does not consider process stream temperature variations.

One of the most recently published works is Konukman *et al.* (2002), which introduces simultaneous flexibility targeting and the synthesis of the minimum utility heat exchanger networks. In the article, the superstructure-based simultaneous MILP formulation is solved successively for increasing values of the targeted flexibility, to reveal the necessary structural modifications and their corresponding minimum utility consumption levels. This work is applied to the HEN superstructure formulation proposed by Yee *et al*.(1990a).

In this work the simplified superstructure presentation proposed by Yee *et al*.(1990a) is applied to generating flexible heat exchanger networks over a specified range of variations in the flow rates and temperatures of the streams, so that the total annual costs as a result of utility charges, exchanger areas and selection of matches are minimized. The presented optimization scheme eliminates the modeling of bypasses, so that the nonlinear heat balances, binary variables, temperature variables and flow variables related to each bypass in the superstructure are no longer needed in the model. The elimination of bypass modeling a stage-wise superstructure presentation and an isothermal mixing assumption make the MINLP model more robust and efficient to solve. In particular, the number of binary variables is reduced due to the elimination of the modeling of bypasses, thus making it possible to solve the MILP master problem faster. Therefore, unlike the previous methods the developed method is capable of solving industrial heat exchanger network problems simultaneously as shown in Table 1.

Table 1. The earlier key developments compared to this work.

Considers:	Floudas and Grossmann (1987b)	Papalexandri and Pistikopoulos (1994a, b)	Tantimuratha *et al.* (2001)	This work
Energy	x	x	x	x
Number of units	x	x	x	x
Area	x	x	x	x
Flexibility aspect	x	x	x	x
Controllability		x		
Temperature variations	x	x		x
Flowrate variations	x	x	x	x
Industrial size problems			x	x
Retrofit designs		x	x	
Simultaneously		x		x

This optimization framework results in a flexible HEN working under variations without losing stream temperature targets while keeping an economically optimal energy integration. The part of the framework where the network feasibility is tested is developed specifically for the special case of correlated uncertain parameters.

3 Framework for Flexible HEN synthesis

In this chapter a framework is presented for generating flexible heat exchanger networks (HEN) over a specified range of variations in the flow rates and temperatures of the streams, so that the total annual costs (TAC) as a result of utility charges, exchanger areas and selection of matches are minimized. First, a short step by step description of the flexible HEN synthesis framework is presented to give an overview of the proposed method.

3.1 Overview of the presented synthesis framework

To make the synthesis framework easier to follow the basic information about the different optimization stages included are presented here. The most important information about the presented optimization scheme is summarized by the following steps. These steps are also described by the flowchart in Figure 2:

(i) The basic multiperiod stream data is defined. This data related to the most common conditions includes inlet and outlet stream temperatures, heat capacity flow rates and heat transfer coefficients for each hot and cold stream.

(ii) The initial estimates for hot utility upper bounds for each period (HU^{UP}), required by the multiperiod stage-wise MILP model and the multiperiod simultaneous MINLP model, are defined by the LP transshipment model (Papoulias and Grossmann (1983)).

(iii) The initial bounds on the allowed number of units (MinNU) and the minimum number of stages (MinNST) are obtained by the multiperiod stage-wise MILP model.

(iv) The multiperiod simultaneous MINLP model is solved with initial periodical data and bounds based on HU^{UP} and MinNU.

(v) The LP feasibility test defines the critical conditions that limit the flexibility on a design.

(vi) The multiperiod MINLP model is resolved with data including additional periods representing the worst temperature approach violation. This loop, including the MINLP stage and LP stage, as is shown in Figure 2, continues until the resulting network is feasible for the whole specified range of parameter variations.

(vii) The NLP improvement model takes account of maximum areas and of the non-isothermal mixing of the streams after the parallel exchangers.

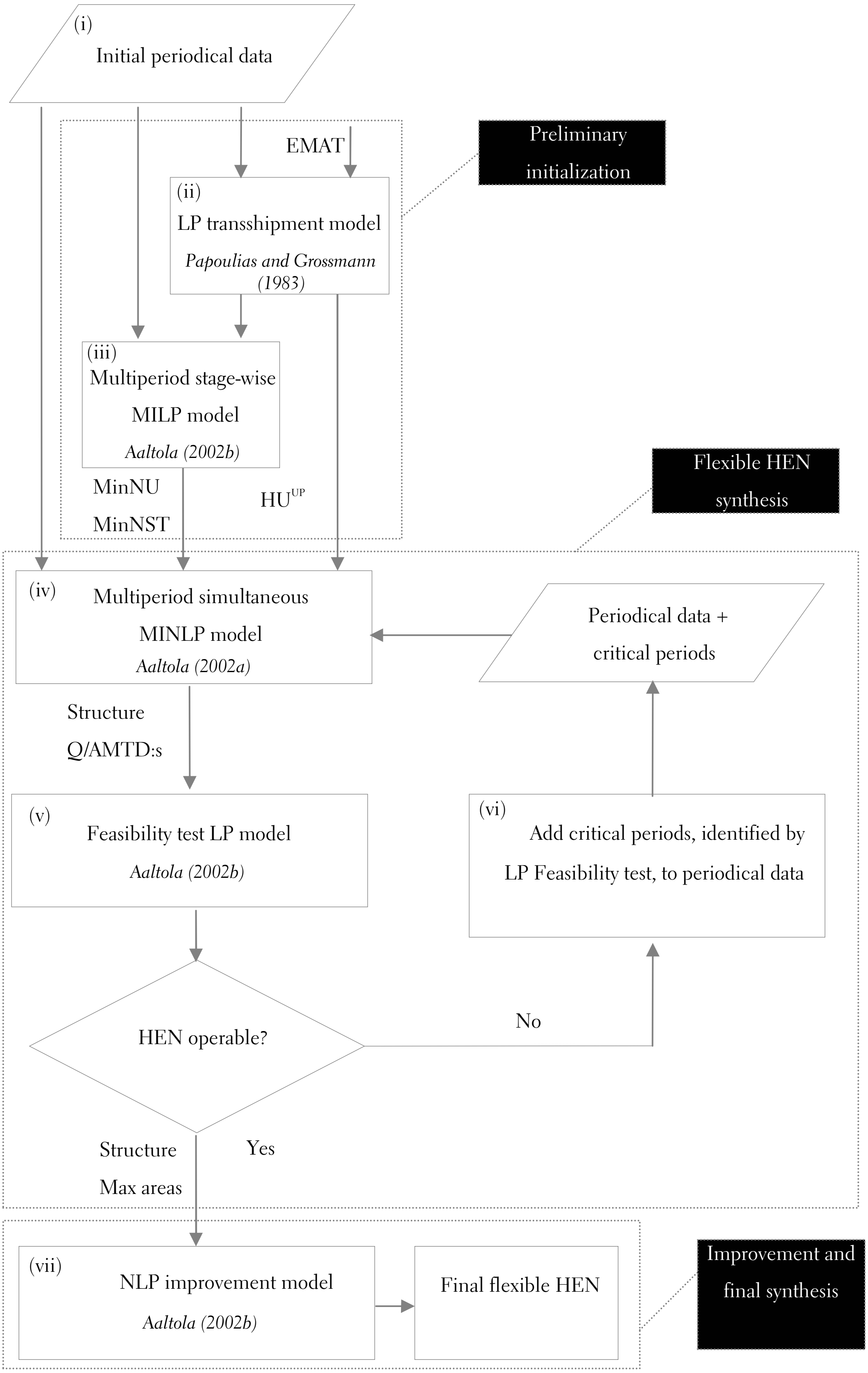

Figure 2. Flowchart for the flexible HEN synthesis procedure (Aaltola (2002b)).

3.2 HEN synthesis problem statement

3.2.1 The basic HEN synthesis problem statement

The basic heat exchanger problem can be stated as follows: Given are a set of hot process streams to be cooled and a set of cold process streams to be heated. All these process streams have a specified heat capacity flowrate and inlet and outlet temperatures. Also given is a set of hot and cold utilities with their corresponding temperatures.

The objective, is to determine the heat exchanger network for energy recovery that will minimize the annualized cost of the equipment plus the annual cost of utilities.

3.2.2 The flexible HEN synthesis problem statement

The flexible HEN synthesis problem to be addressed in this thesis can be stated as follows: Given are a set of hot streams to be cooled and cold streams to be heated, including a multiperiod stream data with inlet and outlet stream temperatures, heat capacity flow rates and heat transfer coefficients. Also, a set of hot and cold utilities are specified.

The objective is, under the range of operating conditions, to determine the heat exchanger network for energy recovery between the given set of hot and cold streams, so that the annualized cost of the equipment plus the annual cost of utilities will be minimized.

3.2.3 Special case of correlated parameters

This dissertation work started from the case where the properties of all the streams except one are dependent on outside temperature. It was soon realized that these kinds of HEN problems with correlated changes can be found widely in the chemical, pulp and paper and metal industries, in fact wherever the stream characteristics are dependent on weather, production volume etc. Furthermore, there is already a good concept of an active set strategy, published by Grossmann and Floudas (1987), which takes account of all variations, but ends up in an MINLP formulation. Thus, the main aim of this work has been to develop a method specifically for flexible HEN synthesis problems in process integration cases where correlated variations and a few disturbances take place. The special case of correlated parameters here means that there is a relationship between uncertain parameters given in stream data. In this way, an increase in the temperature of one cold stream will imply a linear increase or decrease in a hot stream.

The correlation between parameters only affects the feasibility test part of the framework, which includes an LP feasibility test and an NLP improvement model. The multiperiod simultaneous MINLP model can be used regardless of which method is used to find out the critical conditions, if the feasibility test method takes account of the stage-wise structure. Also, the NLP improvement model can be successfully used, but if the periods forming the data do not represent the full operating range, the feasibility must be ensured afterwards with another test.

In the case of uncorrelated disturbances, the critical conditions can be revealed by an active set strategy (Grossmann and Floudas (1987)), since the LP and NLP model would use y^n periods, forming the data where y is the number of the uncertain parameters and n is the number of values the parameters have been discretized into. The active set strategy should be used instead of an LP feasibility test model after a multiperiod simultaneous MINLP model and after an NLP improvement model. In the case of correlated parameters, the data is formed by the n periods, thus, because the active set strategy for feasibility analysis of the HEN becomes the MINLP formulation, the LP feasibility model is preferable, allowing a larger problem size.

In the case of "use of process heat of a pulp mill in a district heating system", the variations of the stream properties in the pulp mill are relatively large and long term, without significant short term disturbances. In a district heating system there are both long and short term variations, so the short term disturbances are related to only one stream. Thus, this is a good example as it represents a special class of problems where the properties of different streams are correlated.

The flexible HEN synthesis examples satisfying the conditions of correlated stream characteristics were not found from previously published journal papers. The second case study is based on a conference paper Manninen *et al.* (2000).

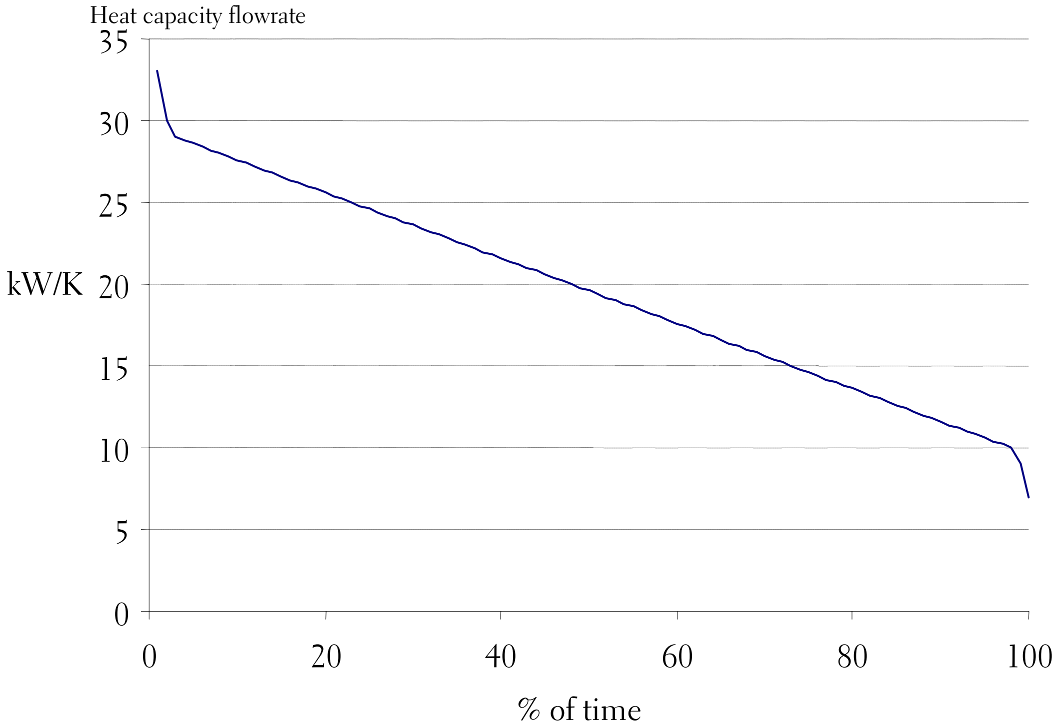

Figure 3. Duration curve of heat capacity flow rate with seasonal changes and some operational faults.

In both case studies calculated in Section 4.1, the operational fault situations are ignored since the normal operation represents about 99% of the total operation. In other words, if the duration of a certain heat capacity flow rate is like the one shown in,Figure 3. the network should be designed for normal operation, or values between 10 – 29 kW/K because the penalty from the possible increase of utility consumption represents a very small amount of energy. Furthermore, it is not economical to build additional utilities or exchangers to be used only for a few days or hours during the year. It is possible to include the extreme periods in the optimization but since model size increases as more periods are specified, it is essential to keep the number of periods as few as possible.

3.3 Stage-wise superstructure presentation

This section presents the stage-wise heat exchanger network superstructure presentation of Yee *et al.*(1990a), which is used in all optimization model formulations proposed in this thesis. The simplified superstructure consists of a number of stages, and in each stage many different possibilities for stream matching are allowed. For each stage, the corresponding stream is split and directed to a heat exchanger for each potential match between each hot and cold stream. The outlets of the exchangers are then mixed and the mixed stream is directed to the next stage. The outlet temperatures of each stage are treated as variables. Heaters and coolers are located at the outlets of the streams. This superstructure presentation does not require the identification of the pinch point or partitioning into subnetworks. A superstructure example for two hot and cold streams is shown in Figure 4. Two stages are used, so the superstructure consists of

(i) four utilities placed at the ends of the superstructure,
(ii) eight exchangers, with four possible matches in each stage and
(iii) four temperature variable between each stage.

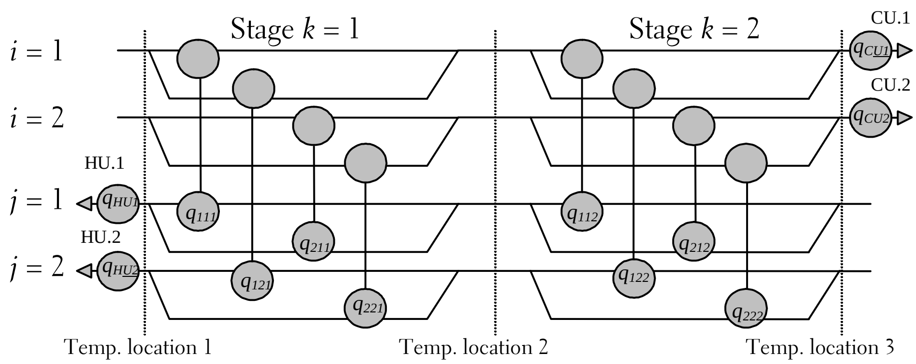

Figure 4. HEN superstructure is simplified by dividing it to stages.

The assumption of mixing the outlets of the exchangers simplifies the model formulation significantly. This isothermal mixing specifies that the outlet temperatures of each exchanger at a particular stream in one stage are the same. As is shown in Figure 5, the outlet temperatures of both exchangers (H1-C1 and H1-C2) at one stage are considered as a single variable i.e., $t_{H1,C1} = t_{H1,C2} = t_{H1,2}$. By setting these temperatures to be the same, the nonlinear heat balance around each heat exchanger can be eliminated and only an overall heat balance must be defined for each stream within each stage. Furthermore, heat capacity flowrates of the streams can be fixed and flow variables are no longer needed in the model. These simplifications are possible, since by determining the optimal temperatures of each stage, it is possible to backtrack and calculate the flowrates for each split fraction by using energy balances for each stream at each stage of the superstructure.

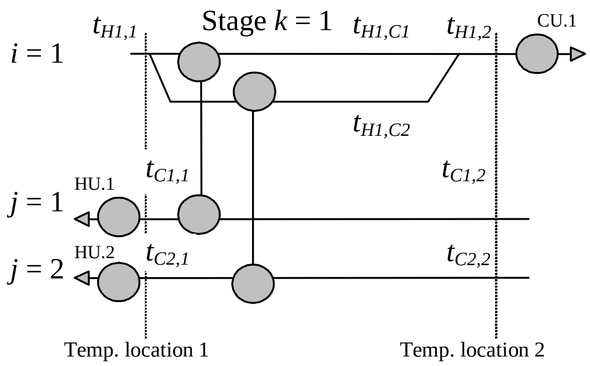

Figure 5. Isothermal mixing at the outlets of the exchangers.

As a result, the feasible space of the problem can be defined by a set of linear constraints, and the dimensionality of the problem is reduced. The nonlinearities of the problem, involving the heat exchanger area calculations using stage temperatures, are included in the objective function. This simplification, due to the stage-wise superstructure presentation, makes the model more robust and easier to solve (Yee *et al.*(1990a)).

It is important to note, however, that the simplified HEN superstructure presentation has a few limitations:

(i) For HEN configurations including split streams an over-estimation of area may occur because the trade-off of area, among the exchangers that are associated with split streams, may be restricted (Yee *et al.*(1990a)).

(ii) Resulting HEN structure may feature more exchanger units than required if a split stream take place (Floudas (1995)).

(iii) HEN structures that are only feasible with nonisothermal mixing may be excluded (Floudas (1995)).

(iv) HEN structures in which a split stream goes through several exchangers in series are neglected, as is shown in Figure 6 (Floudas (1995)).

The significance of these limitations depends on the characteristics of the problem and it is hard to predict in advance. The limitation (ii) can be analyzed beforehand at the initialization stage, but there is still the possibility that the area is over-estimated or the optimal solution may represent a suboptimal solution since some of the structures are neglected. However, the model size will be smaller and larger problem sizes can be tackled.

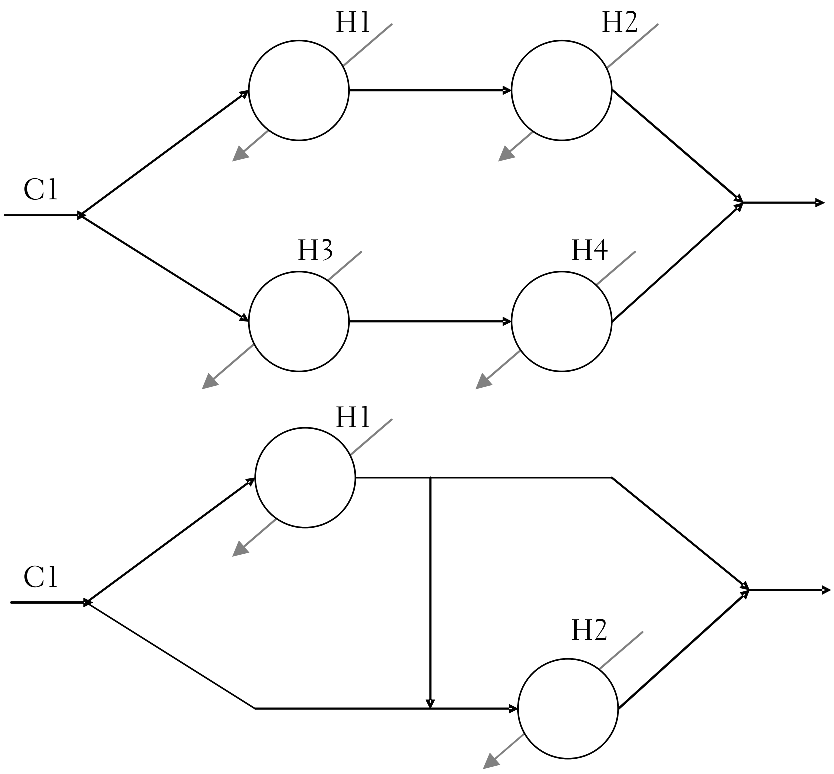

Figure 6. Structures not considered by the simplified superstructure presentation.

3.4 Preliminary initialization

The aim of initialization is to prepare a problem to be successively solved in the later stages. The initialization becomes more important when the problem size increases. Providing the bounds for the problem reduces the combinatory and search space of the problem and gives a point from where to start the optimization. Since the main goal of the optimization is to obtain a minimum total annual cost for the HEN, the most common operation conditions are crucial. This is why the preliminary initialization, as well as the later MINLP stage, takes into account only the most common conditions, and excludes all short term disturbances.

3.4.1 LP transshipment model to solve minimum utility problem

The concept of pinch point gives a limit for the maximum energy integration and hence allows the determination of the minimum utility cost prior to knowing the structure of the HEN. This minimum utility cost target can be formulated as an LP transshipment model that corresponds to a well-known network model in operations research. Papoulias and Grossmann (1983) applied this transshipment model to a minimum utility problem where heat is considered as a commodity, which is transferred from the hot process and hot utilities to the cold process streams and cold utilities via the temperature intervals, as is shown in Figure 7. Basically, the model is formulated by

(i) introducing variables for all potential heat flows,
(ii) writing the overall energy balances around each temperature interval,
(iii) writing the optimization model that minimizes the utility costs subject to the energy balance constraints.

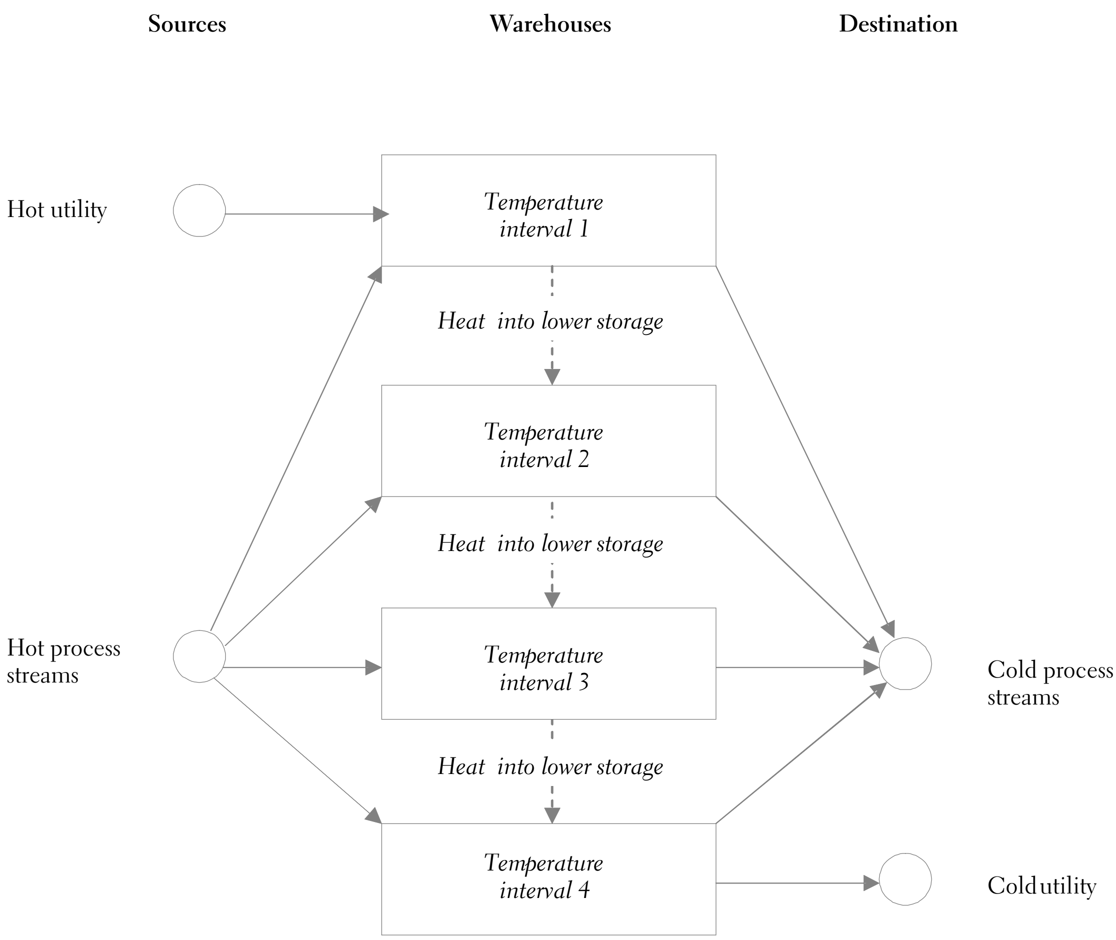

Figure 7. HEN as a transshipment model.

The objective function of the MINLP model is non-linear and non-convex and that is why the solution represents a local optimum. Because of the resulting local optimum the initial point, from where the optimization will be started, may affect the final solution. However, it is possible to generate good HEN structures by performing several runs with different bounds on the hot utility availability. In many cases it can be seen that solving the HEN synthesis problem leads to a large pool of local optima with values of the same magnitude. Therefore, a combination of HU^{UP}:s (an upper bound on total hot utility available) leading to good HEN configurations can easily be found by starting with proper initials. The initial estimates for minimum hot utility upper bounds for each period can be achieved from the transshipment model by Papoulias and Grossmann (1983). In order to achieve HU^{UP}:s the EMAT value should be chosen as an initial to the LP model. Relatively small

EMAT values give a near minimum HU^{UP}:s providing the tight limits for the search space of the first MINLP problem, which is advantageous to the solving time. Also, the concept of supertargeting (Ahmad and Linnhoff (1984)) can be used to find the approximate optimal EMAT, since the minimum total capital cost of the network is a function of this parameter.

3.4.2 MILP transportation model for minimum number of units

The solution of the LP transshipment model provides the loads of hot and cold utilities and the location of pinch points (i.e., subnetworks). Using this information from the LP transshipment model, the MILP transshipment model of Papoulias and Grossmann (1983) determines the minimum number of units and heat load on each unit for each subnetwork. The objective function is the sum of the binary variables representing all possible matches i.e., match of hot and cold process streams, match of hot utilities and cold process stream, and match of cold utilities and hot process stream. Since the model type is the MILP, the global solution can be guaranteed. However, the model can have several global solutions that have the same minimum number of units. It is possible to generate all such solutions, resolving the model with additional integer cuts corresponding to each optimal solution.

3.4.3 Multiperiod stage-wise MILP model for minimum number of units of flexible configuration

In this subsection the new multiperiod model for the minimum number of units using a stage-wise superstructure is introduced. The model provides the HEN structure with a minimum number of units capable of keeping the outlet temperatures of the network at their target values during specified periods when the HRAT and EMAT are given. In this model the HRAT is given for each period in the form of the upper bound of hot utility load (HU^{UP}). The information on the minimum number of units is essential in solving the MINLP model successfully, because many problems are characterized by a wide range of same level local optimums. For example, the case introduced in Section 4.1 results in about the same annual costs, using 22 or 17 units. When considering operability and the practical issues of implementing the HEN, the preferred choice is the HEN that uses fewer exchanger units. That is why the MINLP optimization is started with additional constraints, limiting the number of matches close to the minimum found in the MILP model. This "close to minimum value" can also be defined for each unit type i.e., process to process, hot utility and cold utility matches. More accurately, the close to minimum value in this case represents the maximum number of certain types of units required in order to achieve the minimum total number of units. When solving the MINLP, the maximum allowed number of units is increased to check if the objective shows marked improvements.

In the MINLP optimization it is possible to either add constraints for the maximum number of units or raise the unit price, in order to find solutions with a near minimum number of units. In the case described in Section 4.1, the unit price was multiplied by 10 to find a solution featuring the minimum number of units. Additional constraints are preferable for the solving time and the robustness of the model.

One necessary feature of the MILP model is the stage-wise superstructure (the isothermal mixing assumption), which enables the use of the resulting information in the MINLP model. With the MILP model, the lower bound of the minimum number of stages can easily be analyzed by simply increasing the number of stages and choosing the smallest one, resulting in a minimum value for the objective. This lower bound value is a good initial value to be used in the MINLP model, since the model size is minimal. After this the number of stages should be increased and the MINLP model be resolved until the value of the objective function improves. This leads to the point where, despite increasing the number of stages, the network configuration stays the same and stages without matches occur. There is the possibility that this kind of result can be further improved by adding even more stages, therefore it is always beneficial to try add as many stages as the problem size and the solving capability allows. However, the maximum number of stages can be considered to be those recommended by Yee and Grossmann (1990b), that is the maximum of the number of hot or cold streams or, alternatively, the one discussed in Daichendt and Grossmann (1994a) i.e., the number of temperature intervals. Notice that choosing a larger number of stages will lead to more combinations of stream matches and will significantly reduce the solvable problem size.

The last useful application of the multiperiod MILP model is the opportunity it offers to examine the suitability of a stage-wise superstructure for a problem. The isothermal mixing assumption may result in a penalty for the hot utility consumption or, on the other hand, for the minimum number of units, as is mentioned in limitation (ii) in Section 3.3. By comparing the results from the stage-wise MILP and the transshipment MILP model, the amount of penalty can be found and the suitability of a stage-wise superstructure can be tested. The comparison must be done for one period at a time.

The proposed MILP model for minimizing the number of units for a flexible heat exchanger network has been formulated by using the same set of constraints, from (3.6.1) to (3.6.22), and variables as will be later determined for the MINLP formulation. The only difference is the objective function, which in the MILP model is defined as a summation of binary variables representing the unit existences. The objective function is

$$\min NU = \sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST} z_{i,j,k} + \sum_{i\in HP}\sum_{CU} z_i^{cu} + \sum_{j\in CP}\sum_{HU} z_j^{hu} \qquad (3.4.1)$$

where NU is the total number of units. Upper bounds for EMAT and HRAT are given by constraints (3.6.21) and (3.6.22).

3.5 Heat transfer area calculations

The heat transfer area $A_{i,j,k}$ for match i,j in stage k for the corresponding

(i) heat exchanger load $Q_{i,j,k}$,
(ii) hot stream inlet $t_{i,k}$ and outlet temperatures $t_{i,k+1}$,
(iii) cold stream inlet $t_{j,k+1}$ and outlet temperatures $t_{j,k}$ and
(iv) the heat transfer coefficient $U_{i,j}$ for match i,j

can be calculated by the use of LMTD method as follows:

$$A_{i,j,k} = \frac{Q_{i,j,k}}{U_{i,j} LMTD_{i,j,k}} \qquad (3.5.1)$$

where the $LMTD_{i,j,k}$ is

$$LMTD_{i,j,k} = \frac{(t_{i,k} - t_{j,k}) - (t_{i,k+1} - t_{j,k+1})}{\ln \frac{t_{i,k} - t_{j,k}}{t_{i,k+1} - t_{j,k+1}}} . \tag{3.5.2}$$

When the temperature approaches are equal in both ends of the exchanger, the logarithmic mean temperature difference calculation causes numerical difficulties as a result of division by zero. However, good approximations have been developed for the LMTD calculation to avoid such difficulties. In this work the following approximation published by Paterson (1984) is used. Paterson approximation tends to slightly underestimate the heat transfer area.

$$LMTD_{i,j,k,p} = \frac{2}{3}\left[\left(t_{i,k,p} - t_{j,k,p}\right)\left(t_{i,k+1,p} - t_{j,k+1,p}\right)\right]^{\frac{1}{2}} + \frac{\left(t_{i,k,p} - t_{j,k,p}\right) + \left(t_{i,k+1,p} - t_{j,k+1,p}\right)}{6} \tag{3.5.3}$$

3.6 Multiperiod simultaneous MINLP model

3.6.1 Introduction

The main objective of this thesis is to develop a method to solve flexible HEN synthesis problems so that the trade-offs between energy costs, fixed charges for units and costs for the exchanger area can be simultaneously optimized. The main part of this method is a simultaneous optimization model with necessary simplifications to make the formulation robust and efficient enough to solve industrial size problems. These simplifications are:

(i) The stage-wise superstructure representation of Yee *et al.*(1990a) is used

(ii) The modeling of the possible bypass streams are excluded by considering conductance changes of heat exchangers as changes in bypass fractions.

3.6.2 Model formulation

The existence of each potential heat exchanger in the superstructure is represented by binary variables. This means that if a heat exchanger exists in any period, it must exist in every period, and the fixed cost is charged accordingly. The formulation includes continuous variables assigned to temperatures and to the heat loads of each period. The model is solved to minimize the annualized cost of the equipment plus the annual cost of utilities. This model provides matches that take place ($z_{i,j,k}$, z_i^{cu} and z_j^{hu}) and for every period, areas of each exchanger ($A_{i,j,k,p}$, $A_{i,p}^{cu}$ and $A_{j,p}^{hu}$), corresponding exchanger loads ($q_{i,j,k,p}$, $q_{i,p}^{cu}$ and $q_{j,p}^{hu}$) and possible bypass flow for each exchanger. Later, examples will be presented to clarify the basic ideas behind this design method.

In order to formulate the MINLP model for total annual cost comprising of utility cost, fixed charges for exchanger units and areas, the following definitions are necessary:

(i) Indices:

i = hot process or utility stream,

j = cold process or utility stream,

k = index for stage 1,…, NOK

and temperature location 1,…, NOK + 1,

p = operation period;

(ii) Sets:

HP = set of a hot process stream i,,

CP = set of a cold process stream j,

HU = hot utility,

CU = cold utility,

ST = set of a stage in the superstructure, k = 1,…, NOK,

PR = set of a operation period, p= 1,…, NOP;

(iii) Parameters:

T^{IN} = inlet temperature of stream,

T^{OUT} = outlet temperature of stream,

F = heat capacity flow rate,

U = overall heat transfer coefficient,

C^{CU} = per unit cost for cold utility,

C^{HU} = per unit cost for hot utility,

C^F = fixed charge for heat exchanger unit,

C = area cost coefficient for heat exchanger,

B = exponent for area cost,

AF = annualization factor,

NOK = number of stages,

NOP = number of periods,

DOP = duration of period,

Q^{UP} = an upper bound on heat exchange,

DT^{UP} = an upper bound on temperature difference,

HU^{UP} = an upper bound on total hot utility available,

ε = exchanger minimum approach temperature;

(iv) Positive variables:

$t_{i,k,p}$ = temperature of hot stream *i* at hot end of stage *k* in period *p*,

$t_{j,k,p}$ = temperature of cold stream *j* at hot end of stage *k* in period *p*,

$dt_{i,j,k,p}$ = temperature difference for match (*i*,*j*) at temperature location *k* in period *p*,

$dt^{hu}_{i,k,p}$ = temperature difference for match of cold stream *j* and hot utility in period *p*,

$dt^{cu}_{j,k,p}$ = temperature difference for match of hot stream *i* and cold utility in period *p*,

$q_{i,j,k,p}$ = heat exchanged between hot stream *i* and cold stream *j* in period *p*,

$q^{hu}_{j,p}$ = heat exchanged between cold stream *j* and hot utility in period *p*,

$q^{cu}_{i,p}$ = heat exchanged between hot stream *i* and cold utility in period *p*;

(v) Binary variables:

$z_{i,j,k}$ = existence of match (*i*,*j*) in stage *k*,

z^{hu}_{j} = existence of match between cold stream *j* and hot utility,

z^{cu}_{i} = existence of match between hot stream *i* and cold utility;

(vi) Variables:

TAC = total annual costs for the network.

The set of constraints consists of:

Overall heat balances are to ensure sufficient heating and cooling of each process stream in each period. These equality constraints specify that the heat content of each stream equals the sum of heat exchanged with other streams at each stage plus the exchange with the utility.

$$(T_{i,p}^{IN} - T_{i,p}^{OUT})F_{i,p} = \sum_{k\in ST}\sum_{j\in CP} q_{i,j,k,p} + q_{i,p}^{cu}, \quad i \in HP, \quad p \in PR, \tag{3.6.1}$$

$$(T_{j,p}^{OUT} - T_{j,p}^{IN})F_{j,p} = \sum_{k\in ST}\sum_{i\in HP} q_{i,j,k,p} + q_{j,p}^{hu}, \quad j \in CP, \quad p \in PR. \tag{3.6.2}$$

Heat balance is required for each stream at each stage of the superstructure in each period to determine temperatures. To properly define the temperature variables and stages, the index k is used. The set k = 1…NOK is used to represent the NOK stages while the set k = 1…NOK+1 is for temperature locations in the superstructure. The heat balances are as follows:

$$(t_{i,k,p} - t_{i,k+1,p})F_{i,p} = \sum_{j\in CP} q_{i,j,k,p}, \quad k \in ST, \quad i \in HP, \quad p \in PR, \tag{3.6.3}$$

$$(t_{j,k,p} - t_{j,k+1,p})F_{j,p} = \sum_{i\in HP} q_{i,j,k,p}, \quad k \in ST, \quad j \in CP, \quad p \in PR. \tag{3.6.4}$$

Assignment of inlet temperatures in each period. The superstructure inlet corresponds to temperature location k = 1 for hot streams, while for cold streams to location k = NOK + 1:

$$T_{i,p}^{IN} = t_{i,1,p}, \quad i \in HP, \quad p \in PR, \tag{3.6.5}$$

$$T_{j,p}^{IN} = t_{j,NOK+1,p}, \quad j \in CP, \quad p \in PR. \tag{3.6.6}$$

Constraints for *feasibility of temperatures* in each period are needed to specify a monotonic decrease of temperature at each stage. Also, an upper bound for the outlet temperature of each stage is set at the respective stream's outlet temperature. Note that the outlet temperature does not necessarily correspond to the stream's target temperature, since heating and cooling using utilities may take place at the superstructure outlet.

$$t_{i,k,p} \geq t_{i,k+1,p}, \quad k \in ST, \quad i \in HP, \quad p \in PR, \tag{3.6.7}$$

$$t_{j,k,p} \geq t_{j,k+1,p}, \quad k \in ST, \quad j \in CP, \quad p \in PR, \tag{3.6.8}$$

$$T_{i,p}^{OUT} \leq t_{i,NOK+1,p}, \quad i \in HP, \quad p \in PR, \tag{3.6.9}$$

$$T_{j,p}^{OUT} \geq t_{j,1,p}, \quad j \in CP, \quad p \in PR. \tag{3.6.10}$$

Energy balances for utility matches are determined for each process stream and period in terms of the corresponding outlet temperature in the last stage and the corresponding target temperature.

$$(t_{i,NOK+1,p} - T_{i,p}^{OUT})F_{i,p} = q_{i,p}^{cu}, \quad i \in HP, \quad p \in PR, \tag{3.6.11}$$

$$(T_{j,p}^{OUT} - t_{j,1,p})F_{j,p} = q_{j,p}^{hu}, \quad j \in CP, \quad p \in PR. \tag{3.6.12}$$

Logical constraints are used for existence of matches (*i,j*) in stage *k* and for utilities. In addition, the upper bound on heat exchange capacity Q^{UP} for each period can be set to the smallest heat content of corresponding period of the two streams involved in the match.

$$q_{i,j,k,p} - Q_p^{UP} z_{i,j,k} \leq 0, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.6.13}$$

$$q_{i,p}^{cu} - Q_p^{UP} z_i^{cu} \leq 0, \quad i \in HP, \quad p \in PR, \tag{3.6.14}$$

$$q_{j,p}^{hu} - Q_p^{UP} z_j^{hu} \leq 0, \quad j \in CP, \quad p \in PR, \tag{3.6.15}$$

$$z_{i,j,k}, z_i^{cu}, z_j^{hu} \in \{0,1\}. \tag{3.6.16}$$

Calculation of temperature differences for each temperature location in each period are used to ensure feasible driving forces for exchangers. The binary variables are used to activate these constraints. When a match (*i,j*) in stage *k* occurs, $z_{i,j,k}$ equals one and the constraint becomes active, so that the temperature difference is properly calculated. However, when the match does not take place ($z_{i,j,k}$ equals zero), DT_p^{UP} sets the upper bound for temperature approach. Here DT_p^{UP} is defined as a maximum positive temperature difference between hot and cold stream for each period. Similar constraints are also used for utilities:

$$dt_{i,j,k,p} \le t_{i,k,p} - t_{j,k,p} + DT_p^{UP}(1 - z_{i,j,k}), \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.6.17}$$

$$dt_{i,j,k+1,p} \le t_{i,k+1,p} - t_{j,k+1,p} + DT_p^{UP}(1 - z_{i,j,k}), \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.6.18}$$

$$dt_{i,p}^{cu} \le t_{i,NOK+1,p} - T_{CU}^{OUT} + DT_p^{UP}(1 - z_i^{cu}), \quad i \in HP, \quad p \in PR, \tag{3.6.19}$$

$$dt_{j,p}^{hu} \le T_{HU}^{OUT} - t_{j,1,p} + DT_p^{UP}(1 - z_j^{hu}), \quad j \in CP, \quad p \in PR. \tag{3.6.20}$$

These constraints can be expressed as inequalities, since the minimization of the objective function, where the exchanger areas are calculated using the temperature approaches, drives the temperature approaches upwards.

The objective function is non-linear and non-convex and hence, despite the linear set of constraints, the solution of the resulting optimization model represents a local optimum. However, it is possible to generate a pool of the local optima by performing several runs with different upper bounds on the hot utility availability. Also, a value for the exchanger minimum approach temperature (EMAT) can be defined, setting the upper bound for a crisscross heat transfer i.e. nonvertical heat transfer on the composite curves.

The lowest allowable exchanger minimum approach temperature is defined as:

$$dt_{i,j,k,p} \ge \varepsilon, \tag{3.6.21}$$

Total hot utility availability is limited by following constraints:

$$\sum_{j \in CP} q_{j,p}^{hu} \le HU_p^{UP}, \quad j \in CP, \quad p \in PR, \tag{3.6.22}$$

In order to avoid the modeling of bypasses in the MINLP model, the objective function considers the area of one match to be the mean value of areas in different periods, hence the model under-estimates total area costs and over-estimates exchanger areas. The exact method of calculating the real area costs is to add up the costs related to the maximum match areas. The methods for searching for maximum areas and obtaining real area investment costs in an objective function, introduce non-linearities in constraints or non-linearities with discontinuous derivatives in the objective function. A model with these methods would be more difficult to solve and less robust. Despite the area cost under-estimation, the multiperiod model finds good partial solutions for separate periods, which can be seen when the results of the multiperiod model are compared to the corresponding results of the single period model. The real total annual costs are calculated after the optimization, within a separate part of the model.

TAC (total annual costs for the network) can be defined as the summation of:

(i) Unit costs for all matches,
(ii) mean area costs for matches (i,j,k),
(iii) mean area costs for cold utility matches,
(iv) mean area costs for hot utility matches,
(v) weighted cold utility costs and
(vi) weighted hot utility costs.

Thus, the *objective function* is

$$\min TAC =$$

$$AF\left[\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST} C_{i,j}^{F} z_{i,j,k} + \sum_{i\in HP}\sum_{CU} C_{i,CU}^{F} z_{i}^{cu} + \sum_{j\in CP}\sum_{HU} C_{i,HU}^{F} z_{j}^{hu}\right] \quad \text{(i)}$$

$$+ AF\sum_{p\in PR}\frac{1}{NOP}\sum_{i\in HP}\sum_{j\in CP}\sum_{k\in ST} C_{i,j}\left[\frac{q_{i,j,k,p}}{LMTD_{i,j,k,p}U_{i,j}}\right]^{B_{i,j}} \quad \text{(ii)}$$

$$+ AF\sum_{p\in PR}\frac{1}{NOP}\sum_{i\in HP} C_{i,CU}\left[\frac{q_{i,p}^{cu}}{LMTD_{i,CU,p}U_{i,CU,p}}\right]^{B_{i,CU}} \quad \text{(iii)}$$

$$+ AF\sum_{p\in PR}\frac{1}{NOP}\sum_{j\in CP} C_{j,HU}\left[\frac{q_{j,p}^{hu}}{LMTD_{j,HU,p}U_{j,HU,p}}\right]^{B_{j,HU}} \quad \text{(iv)}$$

$$+ \sum_{p\in PR}\frac{DOP_{p}}{NOP}\sum_{i\in HP} C^{CU} q_{i,p}^{cu} \quad \text{(v)}$$

$$+ \sum_{p\in PR}\frac{DOP_{p}}{NOP}\sum_{j\in CP} C^{HU} q_{j,p}^{hu}. \quad \text{(vi)} \qquad (3.6.23)$$

This optimization formulation is able to take into account weighted periods, so that the most common operating condition can dominate, while the uncommon one is still considered. This can be done by defining the duration of periods DOP_p for utility costs.

3.6.3 Illustrative example for MINLP stage

Next, the basic example, consisting of two hot and two cold streams, (Saboo and Morari (1984)) is considered where the heat capacity flowrate F_{H2} is an uncertain parameter. The problem data for the multiperiod problem is selected as is shown in Table 2. In addition to data from the original example:

- the annual costs of unit duty for hot utility is 115.2 €/(kW·a) and for cold utility 1.3 €/(kW·a),
- the costs equation for exchangers is 8333.3 · unit+641.7 · Area (€),
- lifetime used is 3 a and rate of interest 18%,
- overall heat transfer coefficients for all matches are 4 kW·m^{-2}·K^{-1}.

Table 2. Operating conditions for example.

	Period 1			Period 2		
Stream	T^{IN} (K)	T^{OUT} (K)	F (kW/K)	T^{IN} (K)	T^{OUT} (K)	F (kW/K)
H1	723	553	2	723	553	2
H2	583	323	1	583	323	1.8
C1	388	563	2	388	563	2
C2	313	393	3	313	393	3

The number of stages and hot utility limits were defined by the initialization procedure. In solving the multiperiod MINLP model by using 3 stages, setting the hot utility limits (HU_1^{UP} and HU_2^{UP}) to less than zero and forbidding stream splitting, the results shown in Table 3 and Figure 8 are obtained.

Table 3. Results of MINLP model.

Match	Period 1				Period 2			
i.j.k	1.1.3	2.1.1	2.2.2	CU.1	1.1.3	2.1.1	2.2.2	CU.1
$A_{i,j,k,p}$ [m^2]	0.97	0.69	2.09		0.22	9.03	4.14	
$q_{i,j,k,p}$ [kW]	330	20	240	10	122	228	240	218
$F^{h}_{i,j,k,p}$ [kW/K]	2	1	1		2	1.8	1.8	
$F^{c}_{i,j,k,p}$ [kW/K]	2	2	3		2	2	3	
$T^{INh}_{i,k,p}$ [ºC]	723	583	563		723	583	456	
$T^{OUTh}_{i,k,p}$ [ºC]	558	563	323		662	456	323	
$T^{INc}_{j,k,p}$ [ºC]	388	553	313		388	449	313	
$T^{OUTc}_{j,k,p}$ [ºC]	553	563	393		449	563	393	

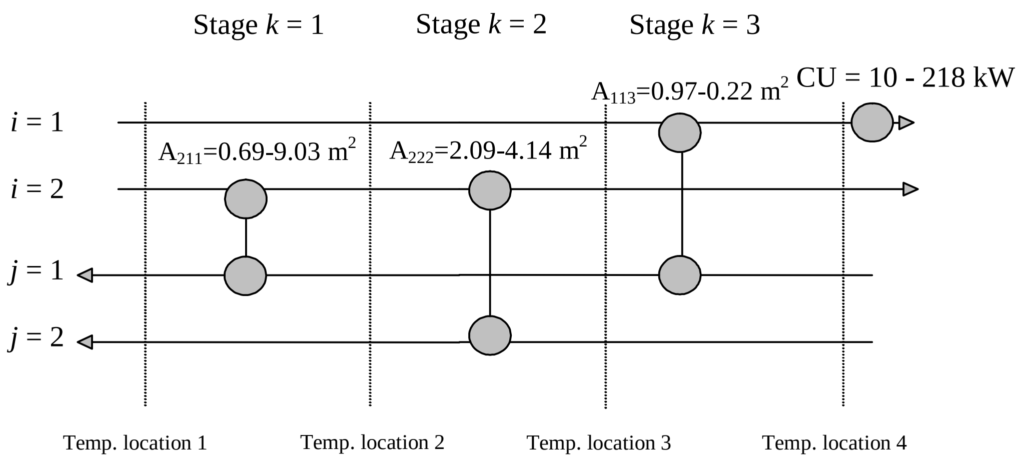

Figure 8. The resulting HEN after the first MINLP.

The HEN resulting from the MINLP optimization stage consists of three process to process heat exchangers and one cold utility. For each exchanger there are two different areas and for the cold utility there is a heat load for both periods. If it was possible to increase and decrease the areas of the exchangers and regulate the cold utility load, this configuration would be able to operate under these two periods with the set up values shown in Figure 8.

3.6.4 The elimination of the bypass modeling

When designing a HEN, the required area for the exchanger is calculated from the heat duties, which is the only degree of freedom for a single heat exchanger. During the operation the heat duty must be varied in order to meet certain specifications. This may be done by manipulating the exchanger area directly or, as in most cases, the bypass stream must be installed and manipulated.

The multiperiod simultaneous MINLP model provides the optimal HEN structure, which minimizes costs when the conductance of every match is allowed to change separately under each specified period. The heat exchanger areas are dependent on the conductance, thus, the optimal solution may represent more than one value of area for each match. However, there is only one investment decision to make and the maximum area is chosen to meet the requirements for the periods which need the largest exchanger area. For those operating conditions with a smaller area $A_{i,j,k,p}$ than maximum $A_{i,j,k}^{\max}$, a bypass is needed to keep the stage temperatures at an optimal level. In order to calculate these bypass fractions, we need first to establish the performance equations. The exchanger load for periods when, according to the optimal solution, the required area is smaller than $A_{i,j,k}^{\max}$ is given by:

$$q_{i,j,k,p} = A_{i,j,k,p} \cdot LMTD_{i,j,k,p} \cdot U_{i,j} \tag{3.6.24}$$

The optimal heat load $q_{i,j,k,p}$ and stage temperatures must remain the same as defined by the optimization, although the existing area is $A_{i,j,k}^{\max}$ instead of $A_{i,j,k,p}$. Therefore the area increase will decrease the values of temperature approaches and this way it will also decrease the value of LMTD. Thus, the new log mean temperature difference $LMTD_{i,j,k,p}^{b}$ (when the bypass is open) is introduced. Now, the optimal exchanger load will be given by:

$$q_{i,j,k,p} = A_{i,j,k}^{\max} \cdot LMTD_{i,j,k,p}^{b} \cdot U_{i,j} \tag{3.6.25}$$

Substituting Equation (3.6.24) in Equation (3.6.25) then yields the log mean temperature difference when the bypass is open:

$$LMTD_{i,j,k,p}^{b} = \frac{A_{i,j,k,p}}{A_{i,j,k}^{\max}} \cdot LMTD_{i,j,k,p} \tag{3.6.26}$$

In order to derive the bypass fraction equation, the temperature after the exchanger (before mixing the bypass stream) $t_{i,k,p}^{ha}$ needs to be computed. In order to calculate the hot side bypass, $t_{i,k+1,p}$ in Equation (3.5.3) is replaced by $t_{i,k,p}^{ha}$ and $LMTD_{i,j,k,p}$ is replaced by $LMTD_{i,j,k,p}^{b}$ to make the expression solvable for $t_{i,k,p}^{ha}$.

The equation that represents heat balance for the combination of the heat exchanger and the bypass (Figure 9) is:

$$\left(F_{i,j,k,p}^{h} - F_{i,j,k,p}^{hb}\right)\left(t_{i,k,p} - t_{i,k,p}^{ha}\right) = \left(t_{i,k,p} - t_{i,k+1,p}\right)F_{i,j,k,p}^{h} \tag{3.6.27}$$

Where $F^{h}_{i,j,k,p}$ is the hot side heat capacity flow rate addressed to match (*ijk*) in a period *p* and $F^{hb}_{i,j,k,p}$ is the hot side bypass fraction over exchanger (*ijk*) in a period *p*. Finally, the equation for calculating the hot side bypass fractions over exchanger (*ijk*) can be written as follows:

$$F^{hb}_{i,j,k,p} = F^{h}_{i,j,k,p} - \frac{\left(t_{i,k,p} - t_{i,k+1,p}\right)F^{h}_{i,j,k,p}}{\left(t_{i,k,p} - t^{ha}_{i,k,p}\right)} \tag{3.6.28}$$

The bypasses considering the cold side are calculated correspondingly. The proposed optimization framework does not consider whether the bypass should be placed on the hot or cold stream.

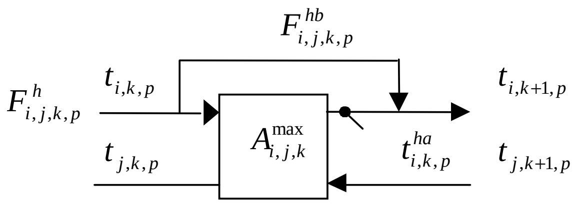

Figure 9. Bypass placed on hot stream.

3.6.5 Illustrative example for applying bypass calculations

From the results from Subsection 3.6.3 in Table 3, it follows that if we properly adjust the load in the cooler and in the exchangers, the network shown in Figure 8 can operate feasibly for specified periods. Finally, it is of interest to define for each period the mass flow rates in the split fractions, and in order to clarify this the same example is revisited to apply bypass calculations. The final network with bypasses placed on cold streams is shown in Figure 10.

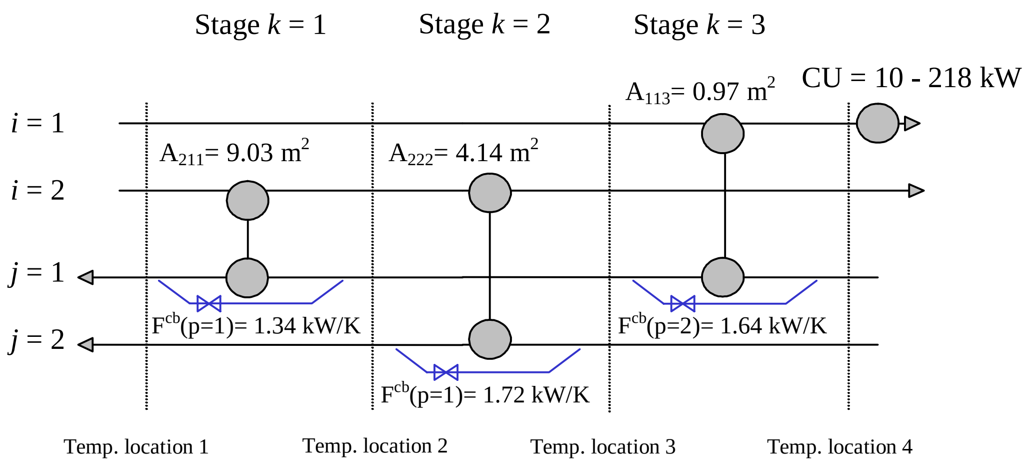

Figure 10. The resulting HEN from first MINLP with bypass set up information.

3.6.6 Computational issues

To solve the MINLP model the DICOPT++ algorithm published by Viswanathan and Grossmann (1990) is used in the general algebraic modeling system GAMS (Brooke *et al.* (1988)).

The MINLP algorithm OA/ER/AP (outer approximation with equality relaxation and augmented penalty) proposed by Viswanathan and Grossmann (1990) is described in Figure 11 (minimization problem). This algorithm can not guarantee global optimality unless the problem is convex. The first step of the proposed algorithm is to solve the NLP relaxation, which treats the binary variables as continuous. If the solution found is not an integer, a relaxed master MILP problem is formulated by linearization of nonlinearities and is thus solved. The master problem predicts the values for binary variables, which are used in the primal NLP problem leading to next linearization and solving of the master problem. This sequence of solving the primal NLP and formulating and solving the master MILP problem continues until there is an increase in the optimal value of the feasible primal NLP problem. The proposed MILP master problem is based on the OA/ER (outer-approximation/equality-relaxation) algorithm proposed by Kocis and Grossmann (1987), allowing violations of linearizations of nonconvex constraints by penalizing these violations. This method has proved to be effective in solving nonconvex MINLP problems and has a high degree of reliability for finding the global optimum. In general, the quality of the solution of the relaxed NLP has been noticed to be an important factor in solving a problem to a globally optimal solution.

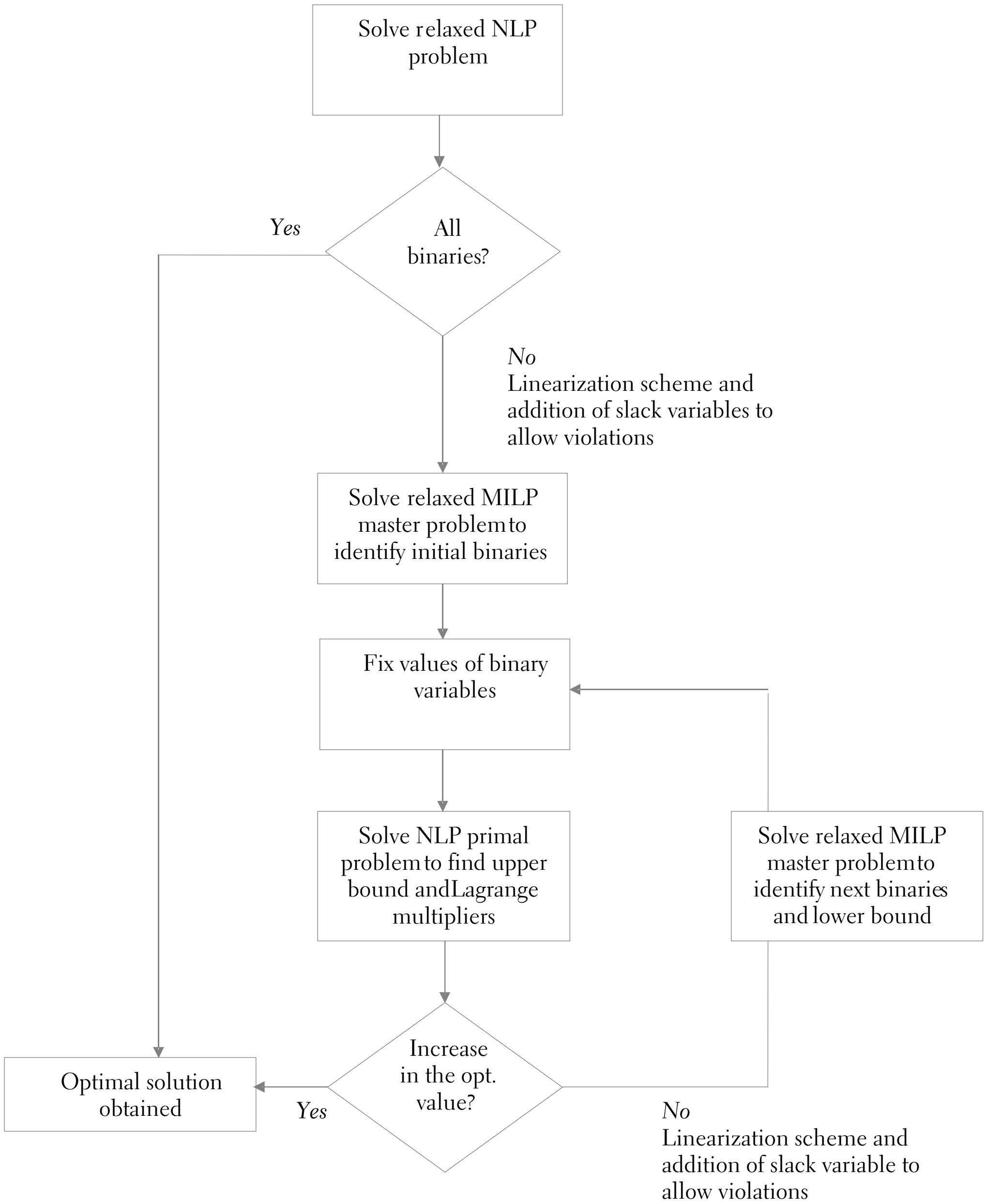

Figure 11. The solution scheme of the combined penalty function and outer-approximation algorithm (Viswanathan and Grossmann (1990)).

Due to the elimination of the bypass modeling, the stage-wise superstructure representation and isothermal mixing the proposed multiperiod MINLP model defines the feasible space with linear constraints equations (3.6.1)-(3.6.22). Therefore this formulation excludes the need for any linearization scheme leading to a reducing computational time required when solving MINLP problems.

It should be noticed in this part of the work that the objective function in the MINLP model is non-linear and non-convex and hence, despite the linear set of constraints, the solution of the resulting optimization model represents a local optimum. However, it is possible to generate a pool of local optima by performing several runs with different upper bounds on the availability of hot utility. With further analysis focused on the best solutions in the pool, good solutions can be achieved.

3.7 Feasibility test

After the multiperiod simultaneous MINLP model has provided the optimal HEN structure for certain periods, one may ask if the configuration between the defined conditions is feasible. In this section the task, how to ensure that the network is feasible in operating not only over these specified periods, but also over the whole range of the specified parameters, is discussed. This is referred to the task of keeping the outlet temperatures in the network, defined by the MINLP model, at their target values during a short and long time horizon. Note that there is an assumption of perfect control, i.e., control can be adjusted depending on the realization of uncertain parameters and no delays in the measurements, or adjustments in the control are considered.

3.7.1 Illustrative example for demonstrating nonconvexities

The previous example in Section 3.6 is revisited to illustrate the nonconvexities of feasibility problems. The example is analyzed to see whether the resulted network is feasible for the whole range $1 < F_{H2} < 1.8$ kW/K.

In order to address the above mentioned question, first the energy balance equations are established for each exchanger:

Exchanger 113: $2(723\text{-}th_{14})=2(tc_{13}\text{-}388)$ (3.7.1)

Exchanger 211: $F_{H2}(583\text{-}th_{22})=2(563\text{-}tc_{13})$ (3.7.2)

Exchanger 222: $F_{H2}(th_{22}\text{-}th_{24})=3(393\text{-}313)$ (3.7.3)

Exchanger Q^{CU}: $Q^{CU}=2(th_{14}\text{-}553)$ (3.7.4)

For feasible operation of this network the inequalities for limiting temperature locations are:

Temperature location 2: $th_{22}\text{-}tc_{13} \geq 0$ (3.7.5)

Temperature location 2: $th_{22}\text{-}393 \geq 0$ (3.7.6)

Temperature location 4: $th_{24} \leq 323$ (3.7.7)

In the above, inequalities (3.7.5) and (3.7.6) guarantee network feasibility with a zero temperature approach, while (3.7.7) states that the outlet temperature of hot stream 2 must be equal to, or lower than, 323 K. All temperatures can be regarded as state variables, F_{H2} being an uncertain parameter and Q^{CU} a control variable. By adjusting the cold utility load Q^{CU} the network, independent of area choices, should be capable of handling variations.

By eliminating the state variables th_{14}, th_{22}, th_{24} and tc_{13} in Equations (3.7.1) - (3.7.4) and substituting them into Equations (3.7.5) - (3.7.7) the following inequalities can be solved in terms of F_{H2} and Q^{CU}:

$$Q^{CU} \leq -10\frac{5F_{H2}-2}{F_{H2}-2} \quad (3.7.8)$$

$$Q^{CU} \leq 190F_{H2}-10 \quad (3.7.9)$$

$$Q^{CU} \geq 260F_{H2}-250 \quad (3.7.10)$$

These constraints (3.7.8) – (3.7.10) can then be plotted, as is shown in Figure 12, where the nonconvex feasible region can be seen. This network has an infeasible operation for $1.12 \leq F_{H2} \leq 1.65$. This example shows that the results of the multiperiod model should always be tested for infeasibilities.

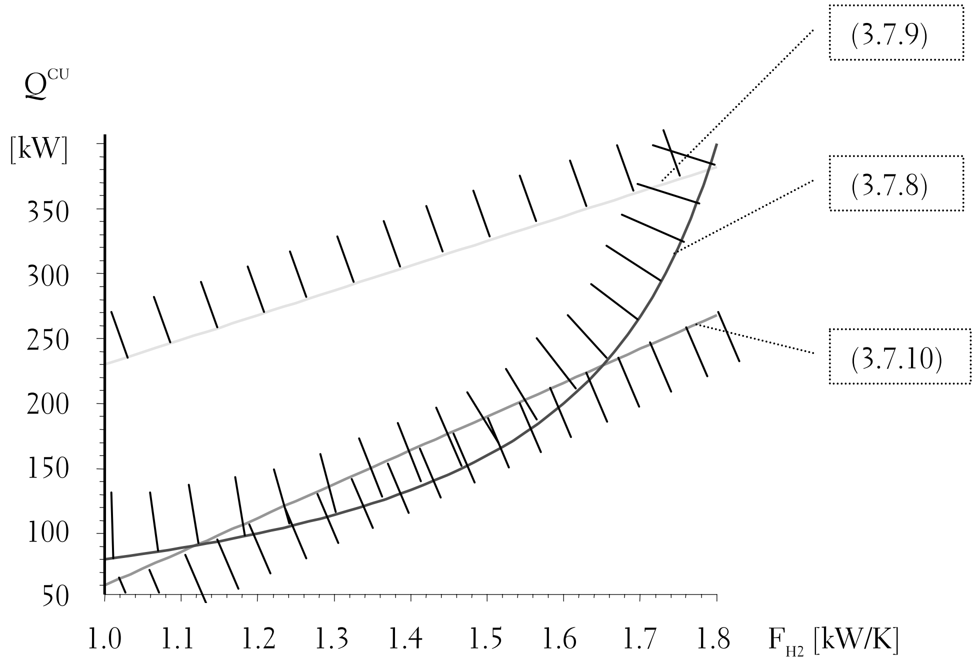

Figure 12. Inequality constraints for temperature approaches.

3.7.2 Multiperiod LP feasibility test model

The LP formulation has been developed to analyze the structural and final flexibility of the HEN. The isothermal mixing assumption is used in the formulation to maintain the linear constraints and make the model compatible with MINLP model, so that critical conditions found with the LP model are suitable for further MINLP optimization. The proposed model is especially suitable for determining network feasibility in the cases where, in addition to a large number of correlated uncertain parameters, only a few independent variations take place. Therefore, it is a practical tool for cases featuring large long term variations with only a few short term disturbances occurring at the same time e.g., the integration of waste heat streams of pulp mills and district heating systems. Note that this LP model takes account of not only structural feasibility but also feasibility depending on individual exchanger conductance.

The LP formulation for minimum temperature approach violations of a given network configuration is formulated by using the same indices and sets as used by the MINLP model. In addition to definitions of the MINLP model the following ones are necessary:

(i) Parameters:

DT^{UP} = an upper bound on temperature difference,

CN^{UP} = an upper bound on conductance,

Z = existence of match (i,j) in stage k (MINLP results),

Z^{CU} = existence of match between cold stream j and hot utility (MINLP results),

Z^{HU} = existence of match between hot stream i and cold utility (MINLP results);

(ii) Positive variables:

$sdt_{i,j,k,p}$ = slack variable for temperature approach violations related to match (i,j) at temperature location k in period p;

(iii) Variables:

z = the summation of temperature approach violations.

With these additional definitions, the model can now be formulated. Equations (3.6.1)-(3.6.12), which appeared in Subsection 3.6.2, are restated without the discussions. The *set of constraints* consists of:

Overall heat balances:

$$(T_{i,p}^{IN} - T_{i,p}^{OUT})F_{i,p} = \sum_{k\in ST}\sum_{j\in CP} q_{i,j,k,p} + q_{i,p}^{cu}, \quad i \in HP, \quad p \in PR, \qquad (3.7.11)$$

$$(T_{j,p}^{OUT} - T_{j,p}^{IN})F_{j,p} = \sum_{k\in ST}\sum_{i\in HP} q_{i,j,k,p} + q_{j,p}^{hu}, \quad j \in CP, \quad p \in PR. \qquad (3.7.12)$$

Heat balances:

$$(t_{i,k,p} - t_{i,k+1,p})F_{i,p} = \sum_{j\in CP} q_{i,j,k,p}, \quad k \in ST, \quad i \in HP, \quad p \in PR, \qquad (3.7.13)$$

$$(t_{j,k,p} - t_{j,k+1,p})F_{j,p} = \sum_{i\in HP} q_{i,j,k,p}, \quad k \in ST, \quad j \in CP, \quad p \in PR. \qquad (3.7.14)$$

Assignment of inlet temperatures:

$$T_{i,p}^{IN} = t_{i,1,p}, \quad i \in HP, \quad p \in PR, \tag{3.7.15}$$

$$T_{j,p}^{IN} = t_{j,NOK+1,p}, \quad j \in CP, \quad p \in PR. \tag{3.7.16}$$

Feasibility of temperatures:

$$t_{i,k,p} \geq t_{i,k+1,p}, \quad k \in ST, \quad i \in HP, \quad p \in PR, \tag{3.7.17}$$

$$t_{j,k,p} \geq t_{j,k+1,p}, \quad k \in ST, \quad j \in CP, \quad p \in PR, \tag{3.7.18}$$

$$T_{i,p}^{OUT} \leq t_{i,NOK+1,p}, \quad i \in HP, \quad p \in PR, \tag{3.7.19}$$

$$T_{j,p}^{OUT} \geq t_{j,1,p}, \quad j \in CP, \quad p \in PR. \tag{3.7.20}$$

Energy balances for utility matches:

$$(t_{i,NOK+1,p} - T_{i,p}^{OUT})F_{i,p} = q_{i,p}^{cu}, \quad i \in HP, \quad p \in PR, \tag{3.7.21}$$

$$(T_{j,p}^{OUT} - t_{j,1,p})F_{j,p} = q_{j,p}^{hu}, \quad j \in CP, \quad p \in PR. \tag{3.7.22}$$

Logical constraints for existence of matches (*i*,*j*) in stage *k* and utilities:

$$q_{i,j,k,p} - Q_p^{UP} Z_{i,j,k} \leq 0, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.7.23}$$

$$q_{i,p}^{cu} - Q_p^{UP} Z_i^{CU} \leq 0, \quad i \in HP, \quad p \in PR, \tag{3.7.24}$$

$$q_{j,p}^{hu} - Q_p^{UP} Z_j^{HU} \leq 0, \quad j \in CP, \quad p \in PR, \tag{3.7.25}$$

Calculation of temperature differences for each temperature location in each period are used to ensure feasible driving forces for existing exchangers. The fixed binary variables Z, Z^{CU} and Z^{HU} are used to define whether the constraint is involved or not (exchangers exist if the parameter $Z_{i,j,k}$ = 1). The temperature differences are expressed as equalities to avoid the miscalculation of the exchanger areas, since the objective function does not have strong terms in relation to the temperature differences. Additional slack variables are added to temperature approach Equations (3.7.26) and (3.7.27). The slack variables are added to allow violations for feasible driving forces for existing exchangers.

$$dt_{i,j,k,p} = t_{i,k,p} - t_{j,k,p} + sdt_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR$$

only when ($Z_{i,j,k}$ = 1). (3.7.26)

$$dt_{i,j,k+1,p} = t_{i,k+1,p} - t_{j,k+1,p} + sdt_{i,j,k+1,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR$$

only when ($Z_{i,j,k}$ = 1). (3.7.27)

$$dt^{cu}_{i,p} = t_{i,NOK+1,p} - T^{OUT}_{CU}, \quad i \in HP, \quad p \in PR,$$

Only when (Z^{CU}_{i} = 1). (3.7.28)

$$dt^{hu}_{j,p} \le T^{OUT}_{HU} - t_{j,1,p}, \quad j \in CP, \quad p \in PR.$$

Only when (Z^{HU}_{j} = 1). (3.7.29)

The maximum allowed conductance for each exchanger $CN^{UP}_{i,j,k}$ is limited by the inequality constraints (3.7.30) where the arithmetic mean of temperature differences are used instead of LMTD to maintain the linear nature. In Equation (3.7.31), $q^{MINLP}_{i,j,k,p}$ and $dt^{MINLP}_{i,j,k,p}$ are parameters, obtained from the MINLP model. These parameters are connected with matches($Z^{MINLP}_{i,j,k,p}$ =1) and periods with *maximum exchanger area*, thus the conductance of each match in the LP model is limited so as to be smaller than or equal to conductance related to the maximum areas of each match in the MINLP model.

$$q_{i,j,k,p} \le \left(dt_{i,j,k,p} + dt_{i,j,k+1,p}\right) CN^{UP}_{i,j,k}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR$$

only when ($Z_{i,j,k}$ = 1), where, (3.7.30)

$$CN^{UP}_{i,j,k} = \frac{q^{MINLP}_{i,j,k,p}}{dt^{MINLP}_{i,j,k,p} + dt^{MINLP}_{i,j,k+1,p}}, \quad i \in HP, \quad j \in CP, \quad k \in ST \qquad (3.7.31)$$

After this, the objective function is written in the following manner to minimize the summation of additional slack variables i.e., temperature approach violations:

$$\text{Min}\, z = \sum_{p \in PR} \sum_{i \in HP} \sum_{j \in CP} \sum_{k \in ST} (sdt_{i,j,k,p} + sdt_{i,j,k+1,p}) \qquad (3.7.32)$$

The idea of this LP formulation is to minimize temperature approach violations, which can be solved with multiperiod data representing information, something that is required to expose infeasibilities. This data is formed by making denser discretization of the problem data for correlated parameters and the addition of periodical data for short term disturbances.

After solving the LP feasibility test, the multiperiod MINLP model is resolved with data that includes additional periods, representing the worst temperature approach violation i.e., critical conditions that limit the flexibility of a design. This loop, including the MINLP stage and LP stage, as is shown in Figure 13, continues until the resulting network is feasible for the whole specified range of parameter variations. If the problem involves many streams with uncorrelated disturbances, an active set strategy for the automated solution of the flexibility test (Grossmann and Floudas (1987)) should be used to identify the potential active constraints that limit the flexibility of a design. In this work cases with large seasonal fully correlated variations and a few short term disturbances are considered.

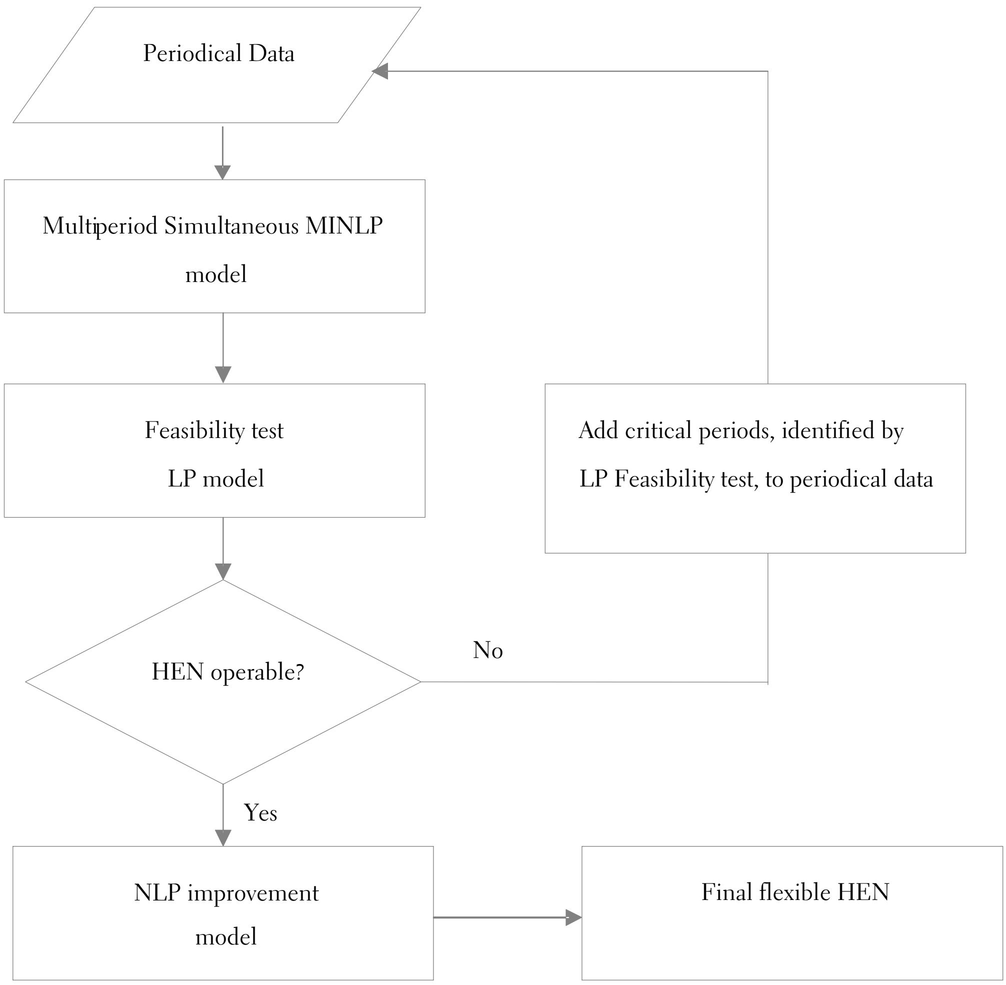

Figure 13. Description of overall method identifying the required periods for multiperiod MINLP stage and removing simplifications related to MINLP model.

3.7.3 Illustrative example for feasibility test

As an illustrative example this feasibility test is applied into the example presented first in Subsection 3.6.3. The new multiperiod data for the LP model is produced by dividing the parameter changes into ten steps. In this small example it means that data is similar for each period, except when F_{H2} increases from 1 to 1.8 with the steps of 0.089 kW/K. Solving this LP model yields the following values for slack variables $sdt_{i,j,k,p}$ representing temperature approach violations related to match *i*,*j*, temperature interval *k* and period *p* [K],

$sdt_{2.1.1.8}$ = 1.792

$sdt_{2.1.1.9}$ = 1.859

$sdt_{2.1.2.3}$ = 3.115

$sdt_{2.1.2.4}$ = 5.860

$sdt_{2.1.2.5}$ = 6.729

$sdt_{2.1.2.6}$ = 6.068

$sdt_{2.1.2.7}$ = 4.145

$sdt_{2.1.2.8}$ = 1.166

which corresponds to the infeasible region 1.178 · F_{H2} · 1.622, as is shown in Figure 12. The worst temperature approach violation is in period p = 5 being 6.7 K corresponding to flow rate F_{H2} = 1.35 kW. This period is added to the data of the MINLP model, which after addition involves three periods as is shown in Table 4.

Table 4. Operating conditions after added critical period.

	Period 1			Period 2			**Period 3**		
	T^{IN}	T^{OUT}	F	T^{IN}	T^{OUT}	F	$\mathbf{T^{IN}}$	$\mathbf{T^{OUT}}$	**F**
Stream	(K)	(K)	(kW/K)	(K)	(K)	(kW/K)	**(K)**	**(K)**	**(kW/K)**
H1	723	553	2	723	553	2	**723**	**553**	**2**
H2	583	323	1	583	323	1.8	**583**	**323**	**1.35**
C1	350	563	2	350	563	2	**350**	**563**	**2**
C2	313	393	3	313	393	3	**313**	**393**	**3**

Resolving the multiperiod MINLP model gives the results shown in Table 5 and

Figure 14. After this, analysis of the resulting network feasibility with the proposed LP model yields zero for all slack variables ($sdt_{i,j,k,p}$), which means that proper adjustment of bypasses (over matches 212 and 223) and cold utility load (in stream H2) leads to a feasible operation of the network over the specified range of uncertain parameters.

Table 5. Results of MINLP model with additional critical period.

Match	Period 1				Period 2				Period 3			
i.j.k	1.1.1	2.1.2	2.2.3	CU.2	1.1.1	2.1.2	2.2.3	CU.2	1.1.1	2.1.2	2.2.3	CU.2
$A_{i,j,k,p}$ [m^2]	1.06	0.03	1.64		1.06	0.03	0.77		1.06	0.03	0.94	
$q_{i,j,k,p}$ [kW]	340	10	240	10	340	10	240	218	340	10	240	102.6
$F^{h}_{i,j,k,p}$ [kW/K]	0.48	0.24	0.24		0.48	0.43	0.43		0.48	0.32	0.32	
$F^{c}_{i,j,k,p}$ [kW/K]	0.48	0.48	0.71		0.48	0.48	0.71		0.48	0.48	0.71	
$T^{INh}_{i,k,p}$ [°C]	723	583	573		723	583	577.4		723	583	575.6	
$T^{OUTh}_{i,k,p}$ [°C]	553	573	333		553	577.4	444.1		553	575.6	398.6	
$T^{INc}_{j,k,p}$ [°C]	393	388	313		393	388	313		393	388	313	
$T^{OUTc}_{j,k,p}$ [°C]	563	393	393		563	393	393		563	393	393	

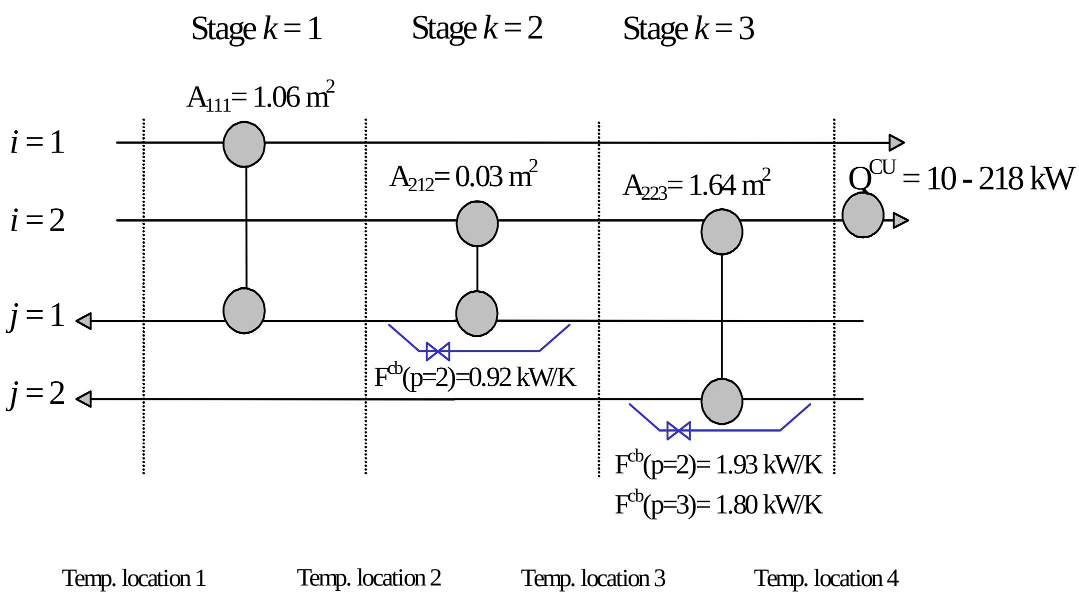

Figure 14. Network configuration capable of handling variations.

3.8 Multiperiod NLP improvement model

In this section the NLP improvement model is introduced for further optimization. Further optimization here means removing simplifications from the MINLP model, and at the same time checking whether the final HEN configuration is feasible to operate. After achieving the solution from the MINLP model, the NLP improvement model takes account of maximum areas to obtain real area investment costs and, furthermore, to achieve an optimal trade-off between capital and operating cost, leading to the real minimum total annual costs. The NLP model also takes account of the non-isothermal mixing of the streams after the parallel exchangers.

It would also be possible to take account of the maximum heat transfer areas already in the MINLP model, but this introduced non-linearities in constraints or non-linearities with discontinuous derivatives in the objective function. This would make an MINLP model more difficult to solve and would decrease the manageable problem size. In any case, when using a stage-wise superstructure, the additional NLP stage is required to remove the assumption of isothermal mixing.

Further, if the maximum area was to be introduced into the objective function of the NLP model, it would introduce discontinuous derivatives. Instead, the slack variables representing the decrease of exchanger area are introduced in the set of maximum area constraints. This set of maximum area constraints is written so that the heat load participates linearly to make the manageable problem size as large as possible. The maximum heat transfer areas are minimized, introducing the minimum slack variables of each match in the objective function. The sum of the objectives to be minimized in the NLP improvement model involves costs resulting from utility consumption and the decrease of exchanger areas i.e., the minimum slack variables of each match.

In the NLP improvement model the structure of the HEN is fixed and the areas of the exchangers are limited by setting the upper limit to correspond to the result obtained from the MINLP stage. The feasibility issue in the NLP stage is naturally less critical than in the LP feasibility test, since structure at the level of matches is fixed and structural feasibility already guaranteed. However, the data used in the NLP improvement model should be discretized, so that the whole range of the most important operating conditions is represented, since there might be changes in areas, flow rate fractions and utility loads.

The following definitions diverge from the ones determined for the MINLP model and are necessary in order to formulate the NLP improvement model:

(i) Parameters:

$MAXAREA_{i,j,k}$ = an upper bound on area (MINLP results),

Cw = weight for decrease of areas,

Z = existence of match (i,j) in stage k (MINLP results),

Z^{CU} = existence of match between cold stream j and hot utility (MINLP results),

Z^{HU} = existence of match between hot stream i and cold utility (MINLP results);

$SPLIT^{H}$ = existence of split on hot stream i at stage k,

$SPLIT^{C}$ = existence of split on cold stream j at stage k,;

(ii) Positive variables:

$t^{hs}_{i,j,k,p}$ = temperature of hot stream fraction after exchanger i,j,k in period p,

$t^{cs}_{i,j,k,p}$ = temperature of cold stream fraction after exchanger i,j,k in period p,

$f^{h}_{i,j,k,p}$ = heat capacity flow rate of hot stream fraction related to exchanger i,j,k,

$f^{h}_{i,j,k,p}$ = heat capacity flow rate of cold stream fraction related to exchanger i,j,k,

$s_{i,j,k,p}$ = slack variable,

$s^{\min}_{i,j,k}$ = minimum of $s_{i,j,k,p}$ for match (i,j) at temperature location k;

(iii) Variables:

Obj = sum of linearly weighted objectives.

The set of constraints consists of:

Overall heat balances exist to ensure sufficient heating and cooling of each process stream in each period. These constraints specify that the heat content of each stream equals the sum of the heat exchanged with other streams at each stage plus the exchange with the utility.

$$(T_{i,p}^{IN} - T_{i,p}^{OUT})F_{i,p} = \sum_{k\in ST}\sum_{j\in CP} q_{i,j,k,p} + q_{i,p}^{cu}, \qquad i \in HP, \quad p \in PR, \tag{3.8.1}$$

$$(T_{j,p}^{OUT} - T_{j,p}^{IN})F_{j,p} = \sum_{k\in ST}\sum_{i\in HP} q_{i,j,k,p} + q_{j,p}^{hu}, \qquad j \in CP, \quad p \in PR. \tag{3.8.2}$$

Heat balance is required to determine the temperatures for each stream at each stage in each period. To properly define the temperature variables and stages, the index k is used. The set k=1…NOK is used to represent the NOK stages while the set k = 1…NOK+1 is for temperature locations in the superstructure. The heat balances are as follows:

$$(t_{i,k,p} - t_{i,k+1,p})F_{i,p} = \sum_{j\in CP} q_{i,j,k,p}, \qquad k \in ST, \quad i \in HP, \quad p \in PR, \tag{3.8.3}$$

$$(t_{j,k,p} - t_{j,k+1,p})F_{j,p} = \sum_{i\in HP} q_{i,j,k,p}, \qquad k \in ST, \quad j \in CP, \quad p \in PR. \tag{3.8.4}$$

Energy balances for utility matches are determined for each process stream and period in terms of the corresponding outlet temperature in the last stage and the corresponding target temperature.

$$(t_{i,NOK+1,p} - T_{i,p}^{OUT})F_{i,p} = q_{i,p}^{cu}, \qquad i \in HP, \quad p \in PR, \tag{3.8.5}$$

$$(T_{j,p}^{OUT} - t_{j,1,p})F_{j,p} = q_{j,p}^{hu}, \qquad j \in CP, \quad p \in PR. \tag{3.8.6}$$

The mass balance for stage k is needed when parallel exchangers exists:

$$\sum_{j\in CP} f_{i,j,k,p}^{h} = F_{i,p}, \qquad i \in HP, \qquad k \in ST, \quad p \in PR, \tag{3.8.7}$$

$$\sum_{i\in HP} f_{i,j,k,p}^{c} = F_{j,p}, \qquad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.8}$$

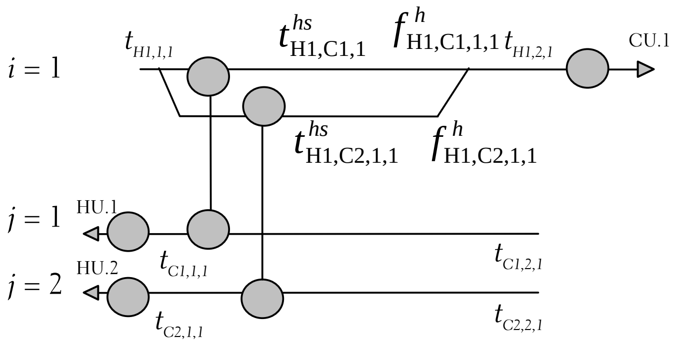

Figure 15. Variables related to non-isothermal mixing.

To calculate the fraction temperatures in inputs of mixers after the exchangers, energy balances for each stream in each period are needed at stages where the stream is split, as is shown in Figure 15:

$$f^{h}_{i,j,k,p}\left(t_{i,k,p} - t^{hs}_{i,j,k,p}\right) = q_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.9}$$

$$f^{c}_{i,j,k,p}\left(t^{cs}_{i,j,k,p} - t_{j,k+1,p}\right) = q_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.10}$$

The interval temperatures, after the stage with a split in the output of mixers, are calculated as follows:

$$t_{i,k+1,p} = \frac{\sum_{j \in CP} f^{h}_{i,j,k,p} t^{hs}_{i,j,k,p}}{F_{i,p}}, \quad i \in HP, \quad k \in ST, \quad p \in PR, \tag{3.8.11}$$

$$t_{j,k,p} = \frac{\sum_{i \in CP} f^{c}_{i,j,k,p} t^{cs}_{i,j,k,p}}{F_{j,p}}, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.12}$$

Stage temperatures are assigned straight to fraction temperature if a split stream does not occur.

$$t_{i,k+1,p} = t^{hs}_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.13}$$

$$t_{j,k,p} = t^{cs}_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.14}$$

Stream mass flow rates are also assigned to fraction mass flow rates if a stream split does not occur.

$$f^{h}_{i,j,k,p} = F_{i,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.15}$$

$$f^{c}_{i,j,k,p} = F_{j,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.16}$$

Assignment of inlet and outlet temperatures and constraints for *feasibility of temperatures* are similar to the corresponding constraints in the MINLP model.

$$T^{IN}_{i,p} = t_{i,1,p}, \quad i \in HP, \quad p \in PR, \tag{3.8.17}$$

$$T^{IN}_{j,p} = t_{j,NOK+1,p}, \quad j \in CP, \quad p \in PR. \tag{3.8.18}$$

$$T^{OUT}_{i,p} \le t_{i,NOK+1,p}, \quad i \in HP, \quad p \in PR, \tag{3.8.19}$$

$$T^{OUT}_{j,p} \ge t_{j,1,p}, \quad j \in CP, \quad p \in PR. \tag{3.8.20}$$

$$t_{i,k,p} \ge t_{i,k+1,p}, \quad k \in ST, \quad i \in HP, \quad p \in PR, \tag{3.8.21}$$

$$t_{j,k,p} \ge t_{j,k+1,p}, \quad k \in ST, \quad j \in CP, \quad p \in PR, \tag{3.8.22}$$

Constraints for *feasibility of temperatures* in each period are also needed for fraction temperatures.

$$t_{i,k,p} \ge t^{hs}_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.23}$$

$$t^{cs}_{i,j,k,p} \ge t_{j,k+1,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.24}$$

Logical constraints for the existence of matches (*i,j*) in stage *k* and utilities:

$$q_{i,j,k,p} - Q^{UP}_{p} Z_{i,j,k} \le 0, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.25}$$

$$q_{i,p}^{cu} - Q_p^{UP} Z_i^{CU} \le 0, \quad i \in HP, \quad p \in PR, \tag{3.8.26}$$

$$q_{j,p}^{hu} - Q_p^{UP} Z_j^{HU} \le 0, \quad j \in CP, \quad p \in PR, \tag{3.8.27}$$

Calculation of temperature differences for each temperature location in each period are used to ensure feasible driving forces for existing exchangers. The use of a specified constraint depends on the existence of a split stream related to the specified exchanger. Thus, the parameters for the split stream $SPLIT^H$ and $SPLIT^C$ are used to define whether the constraint is involved in the model or not. We first define those constraints, activated when the match does not connect the split fraction to either a hot or cold stream i.e., $SPLIT^H = 0$ and $SPLIT^C = 0$:

$$dt_{i,j,k,p} = t_{i,k,p} - t_{j,k,p} + DT_p^{UP}(1 - Z_{i,j,k}), \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.28}$$

$$dt_{i,j,k+1,p} = t_{i,k,p} - t_{j,k+1,p} + DT_p^{UP}(1 - Z_{i,j,k}), \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.29}$$

When the hot or cold stream is divided into two or more fractions, temperature approaches are calculated by using both the fraction temperatures $t_{i,j,k,p}^{hs}$ and $t_{i,j,k,p}^{cs}$, and stage temperatures $t_{i,k,p}$ and $t_{j,k,p}$. The following constraints are activated when $SPLIT^H = 1$ or $SPLIT^C = 1$:

$$dt_{i,j,k,p} = t_{i,k,p} - t_{i,j,k,p}^{cs} + DT_p^{UP}(1 - Z_{i,j,k}), \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.30}$$

$$dt_{i,j,k+1,p} = t_{i,j,k,p}^{hs} - t_{j,k+1,p} + DT_p^{UP}(1 - Z_{i,j,k}), \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.31}$$

The temperature approaches are calculated in a similar way to the MINLP model, but with the difference being that parameters indicate match existences, and the temperature approaches are expressed as equalities to avoid the miscalculation of the exchanger areas. The *temperature approaches for utilities* are:

$$dt_{i,p}^{cu} = t_{i,NOK+1,p} - T_{CU}^{OUT} + DT_p^{UP}(1 - Z_i^{CU}), \quad i \in HP, \quad p \in PR, \tag{3.8.32}$$

$$dt_{i,p}^{hu} = T_{HU}^{OUT} - t_{j,1,p} + DT_{p}^{UP}(1 - Z_{j}^{HU}), \quad j \in CP, \quad p \in PR. \tag{3.8.33}$$

The lowest allowable exchanger minimum approach temperature is defined as:

$$dt_{i,j,k,p} \geq \varepsilon, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.34}$$

The maximum allowed exchanger areas are limited by the inequality constraint (3.8.35), where $MAXAREA_{i,j,k}$ are parameters obtained from the MINLP model. These parameters are connected with matches ($z_{i,j,k,p}^{MINLP}=1$) and periods with *maximum exchanger area*. Sometimes a slight increase (1-5%) of these area parameters is favorable as it also allows for the increase of the exchanger area, if this is seen as being beneficial from the view-point of the overall network area. Thus, on the RHS a multiplier (1.01-1.05) can be used.

$$q_{i,j,k,p} = MAXAREA_{i,j,k} LMTD_{i,j,k,p} U_{i,j,k} - s_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.35}$$

For each match and for each period, the positive slack variable $s_{i,j,k,p}$ is added to represent the decrease of the exchanger area *i,j,k* in a given period *p*. Minimum values of $s_{i,j,k,p}$ for each match are found by introducing the new variable $s_{i,j,k}^{\min}$ and further setting $s_{i,j,k}^{\min}$ to be lower or equal to any of the slack variables of one match over each of the periods. Thus, if all slack variables related to a certain match take the positive value, the smallest of these values represents the decrease of the exchanger area. Because the model drives the smallest value upwards, the following relaxation can be used:

$$s_{i,j,k}^{\min} \leq s_{i,j,k,p}, \quad i \in HP, \quad j \in CP, \quad k \in ST, \quad p \in PR, \tag{3.8.36}$$

The *objective* is to minimize the weighted sum of the quantities of the utility costs and achieve a decrease in the exchanger area. The Objective function can be written as follows,

$$\min Obj = \sum_{p\in PR} \frac{DOP_p}{NOP} \sum_{i\in HP} C^{CU} q_{i,p}^{cu} + \sum_{p\in PR} \frac{DOP_p}{NOP} \sum_{j\in CP} C^{HU} q_{j,p}^{hu} - \sum_{i\in HP} \sum_{j\in CP} \sum_{k\in ST} Cw \cdot s_{i,j,k}^{\min}, \tag{3.8.37}$$

where *Cw* is the weight, which expresses the importance of the decrease of the exchanger area. The solving procedure involves the interactions with the calculation of the total annual costs. The improvement of the TAC is achieved by altering the weight *Cw* and rerunning the model until the TAC first improves and then deteriorates. The bypass fractions are calculated as described in section 3.6.4.

3.9 Summary of the synthesis framework

This section presents a short step by step summary of the flexible HEN synthesis framework, so that the most important information from Chapter 3 is summarized. The framework can be described by the following steps. These are also described by the flowchart in Figure 2:

(i) The multiperiod stream data is defined, including inlet and outlet stream temperatures, heat capacity flow rates and heat transfer coefficients for each hot and cold stream. Data also includes the required information to specify hot and cold utilities. This basic data includes the most common conditions and is used by the following models: the LP transshipment model, the multiperiod stage-wise MILP model and the first run of the multiperiod simultaneous MINLP model.

(ii) The initial estimates for hot utility upper bounds for each period (HU^{UP}) are defined by the LP transshipment model (Papoulias and Grossmann (1983)). HU^{UP}:s are required by the multiperiod stage-wise MILP model and the multiperiod simultaneous MINLP model. The EMAT value should be defined as an initial to the LP model. It should be noticed that the decision of EMAT value does not have a straight effect on the final results, but it affects the number of MINLP solutions required for finding the optimal utility level.

(iii) The initial bounds on allowed number of units (MinNU) for the MINLP model are obtained by the multiperiod stage-wise MILP model. Also, the minimum number of stages (MinNST) required by the problem is defined by this model. The MILP model requires HU^{UP} :s be defined as initial values for each period.

(iv) The multiperiod simultaneous MINLP model is solved with initial periodical data and bounds based on HU^{UP} and MinNU. The MINLP model is solved a number of times with different HU^{UP} mixes to define the behavior of TAC as a function of a HU^{UP} mixes, as is seen in Figure 16.

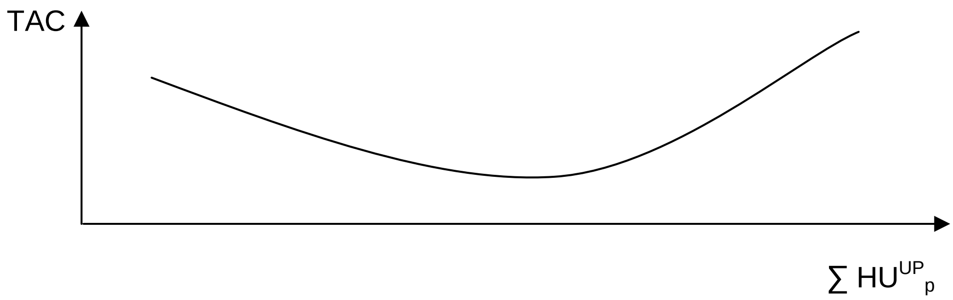

Figure 16. Total annual costs as a function of hot utility consumption level.

(v) The feasibility test LP model analyzes the structural and final flexibility of the HEN after the multiperiod simultaneous MINLP model has provided the optimal HEN structure with exchanger conductance approximations ($\frac{Q}{AMTD}$), where LMTD is replaced with arithmetic mean of temperature differences in the heat exchanger (AMTD). The LP feasibility test defines the critical conditions that limit the flexibility on a design. This feasibility test analysis is focused on the best solution in the pool (minimum TAC:s).

(vi) After solving the LP feasibility test, the multiperiod MINLP model is resolved with data including additional periods representing the worst temperature approach violation i.e., critical conditions. This loop, including the MINLP stage and LP stage, as is shown in Figure 2, continues until the resulting network is feasible for the whole specified range of parameter variations.

(vii) Simplifications of the MINLP model are partly removed by the NLP improvement model. The NLP improvement model takes account of maximum areas and of the non-isothermal mixing of the streams after the parallel exchangers. In the NLP improvement model the structure of the HEN is fixed and the areas of the exchangers are limited by setting the upper limit for each of them to correspond to the result of the final MINLP.

4 APPLICATIONS OF THE FRAMEWORK

4.1 Use of process heat of a pulp mill in a district heating system

4.1.1 Introduction and the problem data

The integrated Kymi industry plant at Kuusankoski in Finland consists of a fine paper mill, a coating plant, and a pulp mill. The integrated system, shown in Figure 17, is part of the UPM-Kymmene fine paper division that produces both uncoated and coated fine paper. Its customers are paper merchants and office supplies wholesalers, printers and converters.

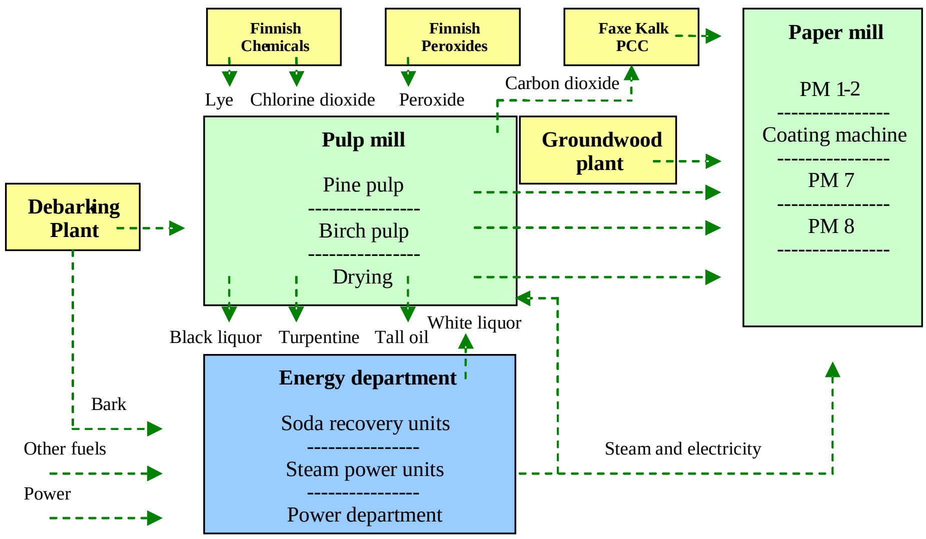

Figure 17. Production flow sheet of integrated pulp and paper mill.

This case study is about how to utilize the waste heat of the pulp mill in the district heating network of two cities. The energy integration is formed via a flexible heat exchanger network that exports heat from a pulp mill to a district heating system all year round. This network is designed by applying the optimization framework developed, so that annual costs will be minimized.

To find out the potential of the pulp mills waste heat streams several hot process streams have been analyzed. The common criteria for a stream to be waste heat was that the stream is cooled by an external utility. Data extraction has been done using a plant wide control system and making additional temperature measurements. Calculations based mass and energy balances were used to gain some values which could not be measured and to ensure the reliability of the data.

Under normal production the properties of waste heat, such as temperature and mass flow, mainly depend on outside temperature. Therefore, the variations of waste heat streams are seasonal. Data has been extracted on a one-year time scale, in winter and summer ,as is listed in Table 6. The target temperatures of all waste heat streams are set to 37°C, since this is the highest temperature allowed by the biological waste water treatment plant.

Table 6. Problem data for the case.

		winter			summer		
Stream		T^{IN} (°C)	T^{OUT} (°C)	F (kW/K)	T^{IN} (°C)	T^{OUT} (°C)	F (kW/K)
Flue gas scrubbers	H1	65	37	323	65	37	913
Hot water	H2	65	37	599	65	37	356
Acidic effluent, line 3	H3	65	37	645	65	37	582
Acidic effluent, line 4	H4	65	37	503	65	37	478
Alkaline effluent, line 4	H5	65	37	42	65	37	75
Secondary condensate	H6	70	37	126	70	37	134
Alkaline effluent, line 3	H7	75	37	172	75	37	247
Hexen effluent, line 4	H8	84	37	218	84	37	218
Wash liquor, line 3	H9	84	75	318	84	75	293
O-stage effluent, line 3	H10	91	75	155	95	75	113
District heating	C1	48	91	1475	48	79	550
Chemical water	C2	5	55	670	15	55	691
ClO_2	C3	5	37	159	15	40	159

The first cold stream in Table 6 considers two different district heating systems belonging to the cities of Kuusankoski and Kouvola. These cities are near enough to utilize the pulp mill's secondary heat in their district heating network. The annual heat energy consumptions for the district heating systems are 100 GWh for Kuusankoski and 264 GWh for Kouvola. The combined annual heat energy for both cities is 364 GWh. In both systems the maximum heat loads normally appear between November and February and the minimum between June and August, being approximately 90 MW and 15 MW respectively. In winter the entry temperatures of the district heating water ranges around 91°C, and in summer around 79°C. The average exit temperature of both district heating systems stays mainly between 48°C and 55°C. The heat capacity flow rate in winter is 1475 kW/K with ± 30% short term variation, while in summer the minimum guaranteed heat capacity flow rate in the system is 550 kW/K, as is shown in Figure 18. Even though the heat capacity flow rate exceeds 1475 kW/K, the disturbance for the district heating stream has been limited to $550 \leq F_{c1} \leq 1475$ kW/K, since 1475 kW/K is large enough to utilize all the waste heat available. Further, the rest of the heat content must come from external sources. The rest of the cold streams, ClO_2 and chemical water, represent the streams of the pulp mill that requires heating and therefore should be heated by secondary heat.

The demand for district heating depends on the season, the day of the week and the time of the day. Consequently, the properties on the heat exchanger network changes due to short term changes in district heating network properties and also due to the seasonal changes in both systems.

Cost information used in the case is:

- the annual costs of unit duties are 115.2 €/(kW·a) and 1.3 €/(kW·a) for hot and cold utility,
- the costs equation for the exchangers is 8333.3 €/unit + 641.7 €/m^2,
- lifetime used is 3 a and rate of interest 18%,
- overall heat transfer coefficients for all matches are 4 $kW \cdot m^{-2} \cdot K^{-1}$.

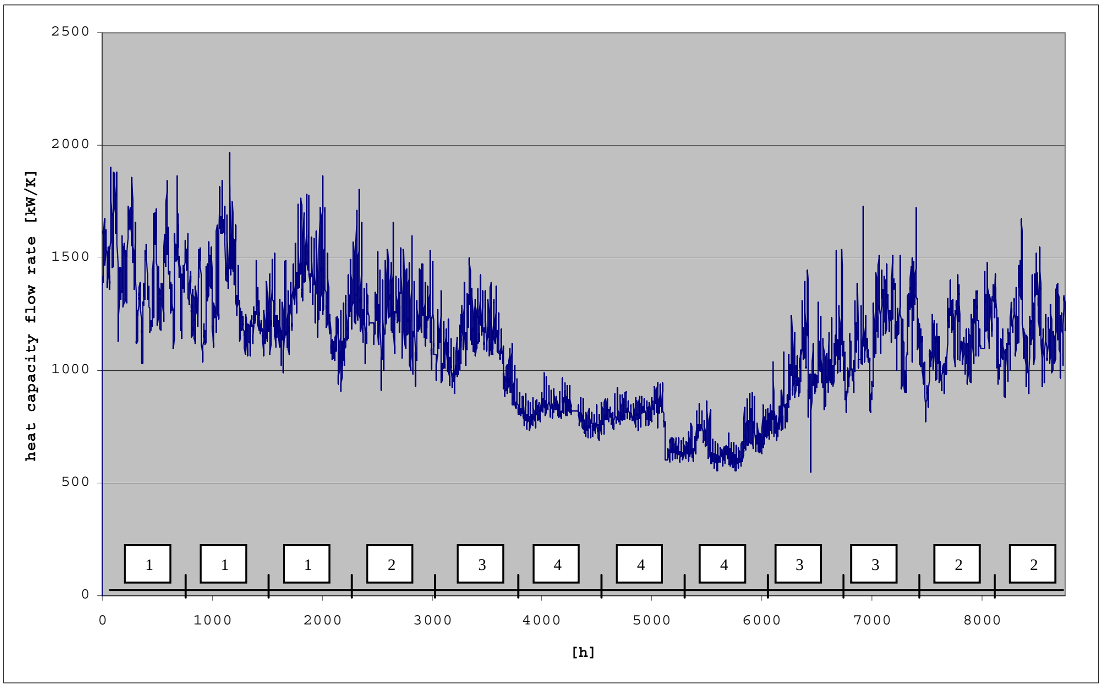

Figure 18. Annual heat capacity flowrate curve and the numbers of time period (boxes) included in MINLP.

4.1.2 Multiperiod simultaneous MINLP model and LP feasibility test

The data from winter to summer has been discretized into four periods, linearising the seasonal changes between these situations. These four periods represent certain times of the year, as the numbers on the boxes show in Figure 18.

The MILP target model was used to find the lower bound for the superstructure stages and the maximum number of units. The number of stages for this example was set at 3 and the constraints for the maximum number of units at (≤ 17). As the maximum hot utility loads the upper bounds corresponding to HRAT 6°C were used. To describe the heat recovery levels here the composite curves with HRAT 6°C for summer and winter situations are shown in Figure 19 and Figure 20. The maximum hot utility limits for different periods are HU^{UP}_1= 35500 kW, HU^{UP}_2= 19200 kW, HU^{UP}_3= 8500 kW, HU^{UP}_4= 0 kW, while the minimum approach temperature was set at ε = 0.5 °C.

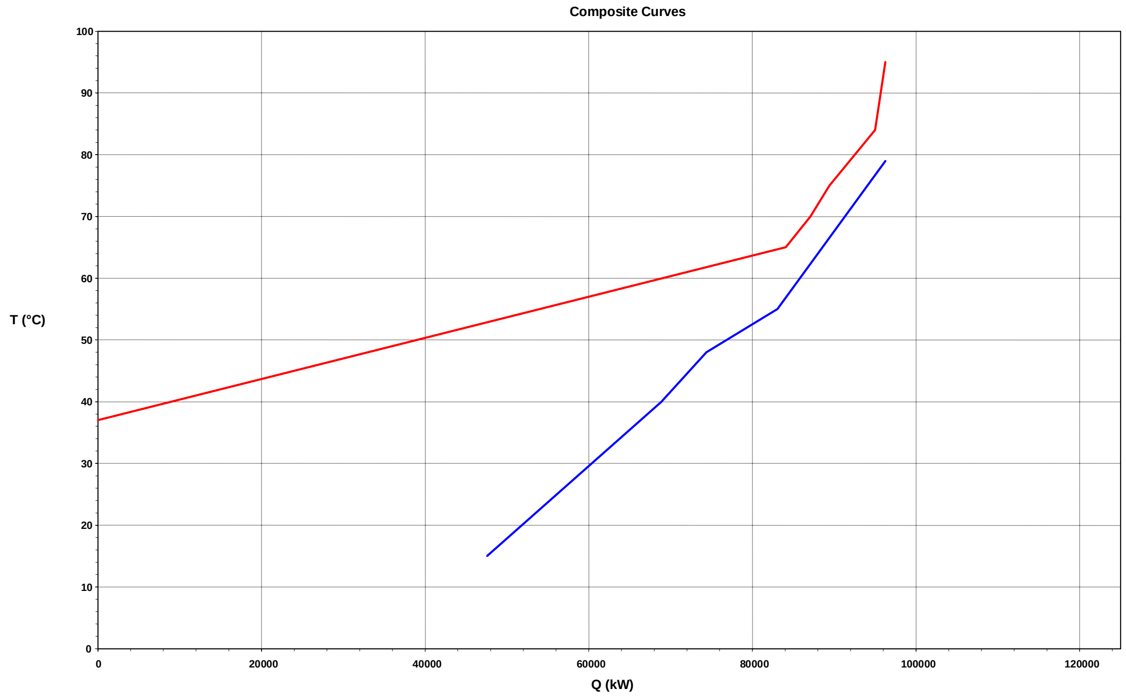

Figure 19. Composite curves for summer situation.

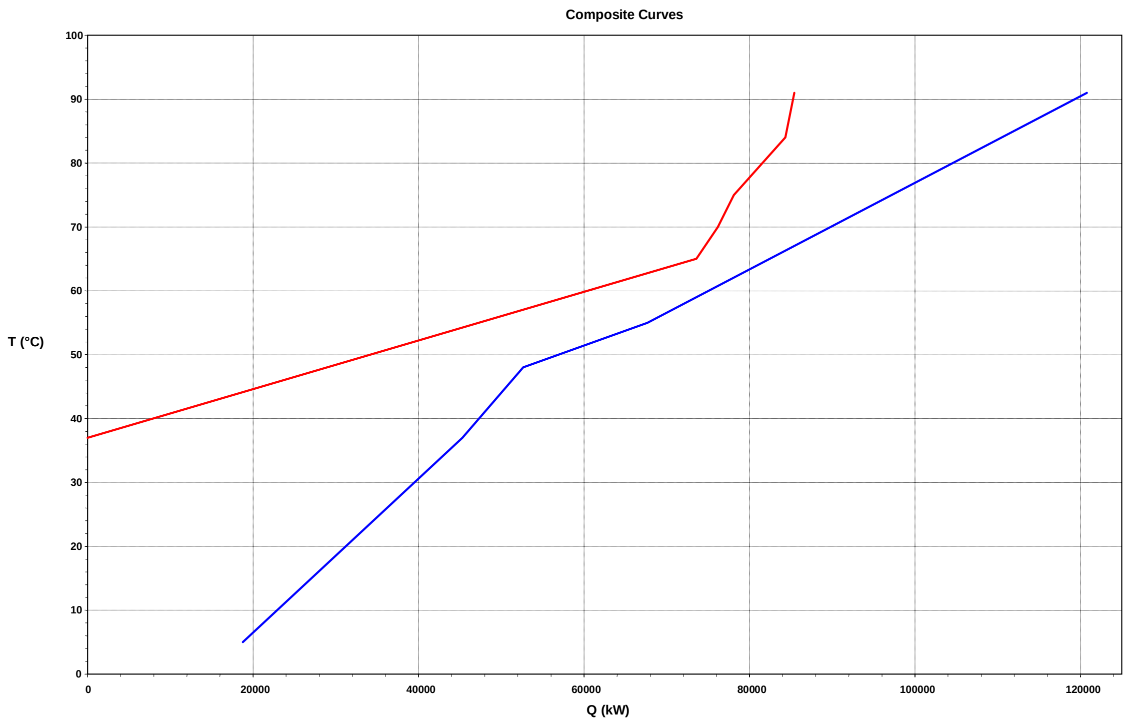

Figure 20. Composite curves for winter situation.

The problem contained 2684 single equations, 1376 single variables and 103 binary variables. Solving the multiperiod MINLP model with the limits defined above gives the results shown in Table 7 and Table 8. The model was solved in 10 minutes on a mobile Intel® P III 1GHz using DICOPT++ (Viswanathan and Grossmann, 1990) with MINOS (Murtaugh and Saunders, 1985) and CPLEX® via GAMS (Brooke et al. 1988).

Process to process matches are defined by indexes of hot stream, cold stream, stage and period respectively (e.g. 3.1.3.1 stands for match H3-C1 in stage 3 in the first period). The total annual costs, after the MINLP optimization, is 3221 k€ when the maximum area is 4425.9 m^2 distributed among the 16 units. Notice that the hot utility is supplied by the existing boiler and is not considered to be a unit.

The resulting network was checked with the LP feasibility test model to ensure that the network could be operated feasibly under the specified conditions. In addition to the seasonal correlated uncertain parameters, the disturbances of the district heating stream in the LP model were modeled, allowing the heat flow capacity rate $Fc_{1,p}$ and inlet temperature $T^{IN}c_{1,p}$ to change ±30% in each seasonal period, so that $48 \leq T^{IN}c_{1,,p} \leq 55$ °C and $550 \leq Fc_{1,p} \leq 1475$ kW/K.

The temperature was discretized into two situations; 0% and max, the heat flow capacity rate was discretized into the three situations (min, 0 %, max) and seasonal parameters were discretized into 20 periods. The resulting LP model included all the combinations of short term variations (except for situations used in the first MINLP model) for all 20 seasonal periods, making the total sum of periods equal to 100.

To test the whole range of disturbances (from 0 to ±30%) of the F and T^{IN}, the LP model was executed 10 times by increasing the disturbance % each time. The solutions of the LP model yields zero for all slack variables (sdt_{ijkp}), which means that the HEN is structurally flexible, and also when considering exchanger conductance approximations, can be feasibly operated over the specified range of short and long term variations. It took 2 sec to solve the feasibility test LP model (GAMS, CPLEX®, mobile Intel® P III 1GHz).

Table 7. Results after the MINLP stage, periods 1, 2, 3 and 4.

p = 1

	Match i.j.k	123	223	313	413	533	623	712	733	812	823	911	1011
$A_{i,j,k,p}$	[m^2]	219.3	443.1	795.7	630.0	44.7	121.0	305.7	71.1	331.7	206.7	266.3	120.4
$q_{i,j,k,p}$	[kW]	7528	16772	8392	6544	1176	4158	2558	3912	5204	5042	2862	2480
$F^h_{i,j,k,p}$	[kW/K]	323.0	599.0	645.0	503.0	42.0	126.0	172.0	172.0	218.0	218.0	318.0	155.0
$F^c_{i,j,k,p}$	[kW/K]	150.6	335.5	828.7	646.2	36.8	83.2	486.0	122.3	988.7	100.8	790.0	684.6
$t^h_{i,k,p}$	[ºC]	65.0	65.0	65.0	65.0	65.0	70.0	75.0	60.1	84.0	60.1	84.0	91.0
$t^h_{i,k+1,p}$	[ºC]	41.7	37.0	52.0	52.0	37.0	37.0	60.1	37.4	60.1	37.0	75.0	75.0
$t^c_{j,k+1,p}$	[ºC]	5.0	5.0	48.0	48.0	5.0	5.0	58.1	5.0	58.1	5.0	63.4	63.4
$t^c_{j,k,p}$	[ºC]	55.0	55.0	58.1	58.1	37.0	55.0	63.4	37.0	63.4	55.0	67.0	67.0
$F^{hb}_{i,j,k,p}$	[kW/K]	94.0	-	-	6.7	20.7	34.5	12.3	-	1.5	56.9	173.7	53.7
$F^{hin}_{i,j,k,p}$	[kW/K]	228.9	-	-	496.3	21.3	91.5	159.7	-	216.5	161.1	144.2	101.3
$t^{ha}_{i,k,p}$	[ºC]	32.1	-	-	51.8	9.8	24.6	59.0	-	60.0	28.8	64.2	66.5
$dt^{ha}_{i,k,p}$	[ºC]	27.1	-	-	3.8	4.8	19.6	0.9	-	1.8	23.8	0.8	3.1
$F^{cb}_{i,j,k,p}$	[kW/K]	10.8	-	-	15.8	15.9	10.4	211.5	-	123.0	4.4	637.2	550.7
$F^{cin}_{i,j,k,p}$	[kW/K]	139.8	-	-	630.5	20.9	72.7	274.5	-	865.7	96.5	152.8	133.9
$t^{ca}_{j,k,p}$	[ºC]	58.8	-	-	58.4	61.4	62.2	67.5	-	64.1	57.3	82.1	81.9
$dt^{ca}_{j,k,p}$	[ºC]	6.2	-	-	6.6	3.7	7.8	7.6	-	19.9	2.9	1.9	9.1

p = 2

$q_{i,j,k,p}$	[kW]	9699	14504	7735	6123	1484	4224	2586	3127	4823	5423	2790	2256
$F^h_{i,j,k,p}$	[kW/K]	520.0	518.0	624.0	494.0	53.0	128.0	197.0	197.0	218.0	218.0	310.0	141.0
$F^c_{i,j,k,p}$	[kW/K]	194.0	290.1	651.3	515.6	51.2	84.5	407.2	107.8	759.6	108.4	645.1	521.6
$t^h_{i,k,p}$	[ºC]	65.0	65.0	65.0	65.0	65.0	70.0	75.0	61.9	84.0	61.9	84.0	91.0
$t^h_{i,k+1,p}$	[ºC]	46.4	37.0	52.6	52.6	37.0	37.0	61.9	46.0	61.9	37.0	75.0	75.0
$t^c_{j,k+1,p}$	[ºC]	5.0	5.0	48.0	48.0	8.0	5.0	59.9	8.0	59.9	5.0	66.2	66.2
$t^c_{j,k,p}$	[ºC]	55.0	55.0	59.9	59.9	37.0	55.0	66.2	37.0	66.2	55.0	70.6	70.6
$F^{hb}_{i,j,k,p}$	[kW/K]	-	103.8	-	-	22.2	33.8	5.8	106.5	-	61.2	140.7	36.2
$F^{hin}_{i,j,k,p}$	[kW/K]	-	414.1	-	-	30.8	94.2	191.2	90.5	-	156.8	169.3	104.7
$t^{ha}_{i,k,p}$	[ºC]	-	30.0	-	-	16.8	25.2	61.5	27.3	-	27.3	67.5	69.5
$dt^{ha}_{i,k,p}$	[ºC]	-	25.0	-	-	8.8	20.2	1.6	19.3	-	22.3	1.3	3.2
$F^{cb}_{i,j,k,p}$	[kW/K]	-	17.1	-	-	20.7	10.2	59.6	34.5	-	6.6	458.8	369.1
$F^{cin}_{i,j,k,p}$	[kW/K]	-	273.0	-	-	30.5	74.3	347.6	73.3	-	101.9	186.3	152.6
$t^{ca}_{j,k,p}$	[ºC]	-	58.1	-	-	56.7	61.9	67.3	50.6	-	58.3	81.2	81.0
$dt^{ca}_{j,k,p}$	[ºC]	-	6.9	-	-	8.3	8.2	7.7	11.2	-	3.6	2.8	10.0

p = 3

$q_{i,j,k,p}$	[kW]	5275	12236	5633	4540	1792	4323	2719	2660	4720	5526	2718	2540
$F^{h}_{i,j,k,p}$	[kW/K]	715.9	437.0	603.0	485.9	64.0	131.0	222.0	222.0	218.0	218.0	302.0	127.0
$F^{c}_{i,j,k,p}$	[kW/K]	131.9	305.9	475.1	382.9	64.0	108.1	313.6	95.0	544.3	138.1	443.4	414.4
$t^{h}_{i,k,p}$	[ºC]	65.0	65.0	65.0	65.0	65.0	70.0	75.0	62.8	84.0	62.4	84.0	95.0
$t^{h}_{i,k+1,p}$	[ºC]	57.6	37.0	55.7	55.7	37.0	37.0	62.8	50.8	62.4	37.0	75.0	75.0
$t^{c}_{j,k+1,p}$	[ºC]	15.0	15.0	48.0	48.0	12.0	15.0	59.9	12.0	59.9	15.0	68.5	68.5
$t^{c}_{j,k,p}$	[ºC]	55.0	55.0	59.9	59.9	40.0	55.0	68.5	40.0	68.5	55.0	74.7	74.7
$F^{hb}_{i,j,k,p}$	[kW/K]	568.1	48.8	219.6	174.9	15.9	3.6	-	147.3	-	-	94.3	11.2
$F^{hin}_{i,j,k,p}$	[kW/K]	147.8	388.1	383.4	311.0	48.1	127.4	-	74.7	-	-	207.7	115.8
$t^{ha}_{i,k,p}$	[ºC]	29.3	33.5	50.3	50.4	27.8	36.1	-	27.2	-	-	70.9	73.1
$dt^{ha}_{i,k,p}$	[ºC]	14.3	18.5	2.3	2.4	15.8	21.1	-	15.2	-	-	2.4	4.5
$F^{cb}_{i,j,k,p}$	[kW/K]	23.8	14.9	117.6	93.6	15.9	1.9	-	34.1	-	-	207.8	170.6
$F^{cin}_{i,j,k,p}$	[kW/K]	108.1	291.0	357.4	289.3	48.1	106.2	-	60.9	-	-	235.7	243.9
$t^{ca}_{j,k,p}$	[ºC]	63.8	57.1	63.8	63.7	49.2	55.7	-	55.7	-	-	80.1	78.9
$dt^{ca}_{j,k,p}$	[ºC]	1.2	8.0	1.2	1.3	15.8	14.3	-	7.1	-	-	3.9	16.1

p = 4

$q_{i,j,k,p}$	[kW]	7325	9968	2906	2386	2100	4422	2540	1875	4321	5925	2637	2260
$F^{h}_{i,j,k,p}$	[kW/K]	912.9	356.0	581.9	477.9	75.0	134.0	247.0	247.0	218.0	218.0	293.0	113.0
$F^{c}_{i,j,k,p}$	[kW/K]	183.1	249.2	301.9	248.0	84.0	110.5	203.6	75.0	346.3	148.1	296.1	253.8
$t^{h}_{i,k,p}$	[ºC]	65.0	65.0	65.0	65.0	65.0	70.0	75.0	64.7	84.0	64.2	84.0	95.0
$t^{h}_{i,k+1,p}$	[ºC]	57.0	37.0	60.0	60.0	37.0	37.0	64.7	57.1	64.2	37.0	75.0	75.0
$t^{c}_{j,k+1,p}$	[ºC]	15.0	15.0	48.0	48.0	15.0	15.0	57.6	15.0	57.6	15.0	70.1	70.1
$t^{c}_{j,k,p}$	[ºC]	55.0	55.0	57.6	57.6	40.0	55.0	70.1	40.0	70.1	55.0	79.0	79.0
$F^{hb}_{i,j,k,p}$	[kW/K]	609.4	89.5	410.0	336.5	-	-	64.1	204.2	38.2	6.8	-	-
$F^{hin}_{i,j,k,p}$	[kW/K]	303.5	266.5	171.9	141.4	-	-	182.8	42.8	179.8	211.2	-	-
$t^{ha}_{i,k,p}$	[ºC]	40.9	27.6	48.1	48.1	-	-	61.1	20.9	60.0	36.1	-	-
$dt^{ha}_{i,k,p}$	[ºC]	25.9	12.6	0.1	0.1	-	-	3.5	5.9	2.3	21.1	-	-
$F^{cb}_{i,j,k,p}$	[kW/K]	22.8	29.1	131.0	107.6	-	-	36.7	35.8	129.3	1.8	-	-
$F^{cin}_{i,j,k,p}$	[kW/K]	160.4	220.1	170.9	140.4	-	-	167.0	39.1	217.0	146.3	-	-
$t^{ca}_{j,k,p}$	[ºC]	60.7	60.3	65.0	65.0	-	-	72.8	62.9	77.5	55.5	-	-
$dt^{ca}_{j,k,p}$	[ºC]	4.3	4.7	0.0	0.0	-	-	2.2	1.8	6.5	8.7	-	-

Table 8. Utility related results from MINLP stage.

		H1	H3	H4	H7	C1
A^u	[m^2]	341.8	239.1	196.4	92.9	-
q^u_1	[kW]	1516	9668	7540	66	35384
q^u_2	[kW]	4861	9737	7709	1773	19200
q^u_3	[kW]	14773	11251	9068	3057	7160
q^u_4	[kW]	18239	13391	10998	4971	0

4.1.3 NLP improvement model

In the NLP improvement model the structure of the HEN is fixed (Figure 21) and the areas of the exchangers are limited by setting upper limits for each of them ($Amax_{i,j,k}$) to correspond to the result of the MINLP stage.

The temperature was discretized into the two situations, 0% and max, the heat flow capacity rate into the three situations (min, 0%, max), and seasonal parameters into 10 periods. The resulting NLP model included all the combinations of short term variations for all the 10 seasonal periods, making the total sum of periods 60.

Total annual costs after solving the NLP stage for disturbance ±30% is 3084 k€, when the maximum total area is 4411.0 m^2. The NLP improvement model was executed in 65 minutes on a mobile Intel® P III 1GHz using MINOS via GAMS. The results are shown in Table 9 and Table 10.

Table 9. Results for winter and summer after NLP improvement, periods 1 and 4.

Winter, p=1	Match i.j.k	123	223	313	413	533	623	712	733	812	823	911	1011
$A_{i,j,k,p}$	[m^2]	178.3	443.1	795.7	630.0	42.9	113.5	305.7	71.1	331.7	170.1	266.3	120.4
$q_{i,j,k,p}$	[kW]	7579	16772	8403	6587	1176	4158	2558	3912	5255	4991	2862	2480
$F^h_{i,j,k,p}$	[kW/K]	323.0	599.0	645.0	503.0	42.0	126.0	172.0	172.0	218.0	218.0	318.0	155.0
$F^c_{i,j,k,p}$	[kW/K]	154.8	335.5	827.8	647.2	36.8	74.7	278.3	122.3	1196.7	105.1	491.7	983.3
$t^h_{i,k,p}$	[ºC]	65.0	65.0	65.0	65.0	65.0	70.0	75.0	60.1	84.0	59.9	84.0	91.0
$t^h_{i,k+1,p}$	[ºC]	41.5	37.0	52.0	51.9	37.0	37.0	60.1	37.4	59.9	37.0	75.0	75.0
$t^c_{j,k+1,p}$	[ºC]	5.0	5.0	48.0	48.0	5.0	5.0	58.2	5.0	58.2	5.0	63.5	63.5
$t^c_{j,k,p}$	[ºC]	54.0	55.0	58.2	58.2	37.0	60.7	67.4	37.0	62.6	52.5	82.1	81.8
$F^{cb}_{i,j,k,p}$	[kW/K]	-	-	-	-	15.7	-	-	-	-	5.0	338.0	848.4
$F^{cin}_{i,j,k,p}$	[kW/K]	-	-	-	-	21.0	-	-	-	-	100.2	153.6	134.9
$t^{ca}_{j,k,p}$	[ºC]	-	-	-	-	60.8	-	-	-	-	54.8	82.1	81.9
$dt^{ca}_{j,k,p}$	[ºC]	-	-	-	-	4.2	-	-	-	-	5.1	1.9	9.1
Summer, p=4													
$q_{i,j,k,p}$	[kW]	6540	9968	4725	349	2100	4422	3543	1875	3536	6710	2637	2260
$F^h_{i,j,k,p}$	[kW/K]	913.0	356.0	582.0	478.0	75.0	134.0	247.0	247.0	218.0	218.0	293.0	113.0
$F^c_{i,j,k,p}$	[kW/K]	149.0	220.2	286.3	263.7	92.0	116.9	406.1	66.9	143.9	205.0	296.2	253.8
$t^h_{i,k,p}$	[ºC]	65.0	65.0	65.0	65.0	65.0	70.0	75.0	60.7	84.0	67.8	84.0	95.0
$t^h_{i,k+1,p}$	[ºC]	57.8	37.0	56.9	64.3	37.0	37.0	60.7	53.1	67.8	37.0	75.0	75.0
$t^c_{j,k+1,p}$	[ºC]	15.0	15.0	48.0	48.0	15.0	15.0	57.2	15.0	57.2	15.0	70.1	70.1
$t^c_{j,k,p}$	[ºC]	58.9	60.3	64.5	64.5	37.8	52.8	66.0	43.0	81.8	47.7	79.0	79.0
$F^{cb}_{i,j,k,p}$	[kW/K]	1.8	-	-	242.0	-	-	-	23.6	-	14.2	-	-
$F^{cin}_{i,j,k,p}$	[kW/K]	147.1	-	-	21.7	-	-	-	43.4	-	190.8	-	-
$t^{ca}_{j,k,p}$	[ºC]	59.5	-	-	64.1	-	-	-	58.2	-	50.2	-	-
$dt^{ca}_{j,k,p}$	[ºC]	5.6	-	-	0.9	-	-	-	2.4	-	17.6	-	-

Table 10. Utility related results from NLP improvement.

		H1	H3	H4	H7	C1
A^u	[m^2]	351.54	254.96	222.24	113.53	-
q^u_{winter}	[kW]	1465	9657	7497	66	35279
q^u_{summer}	[kW]	19024	11571	13035	3968	0

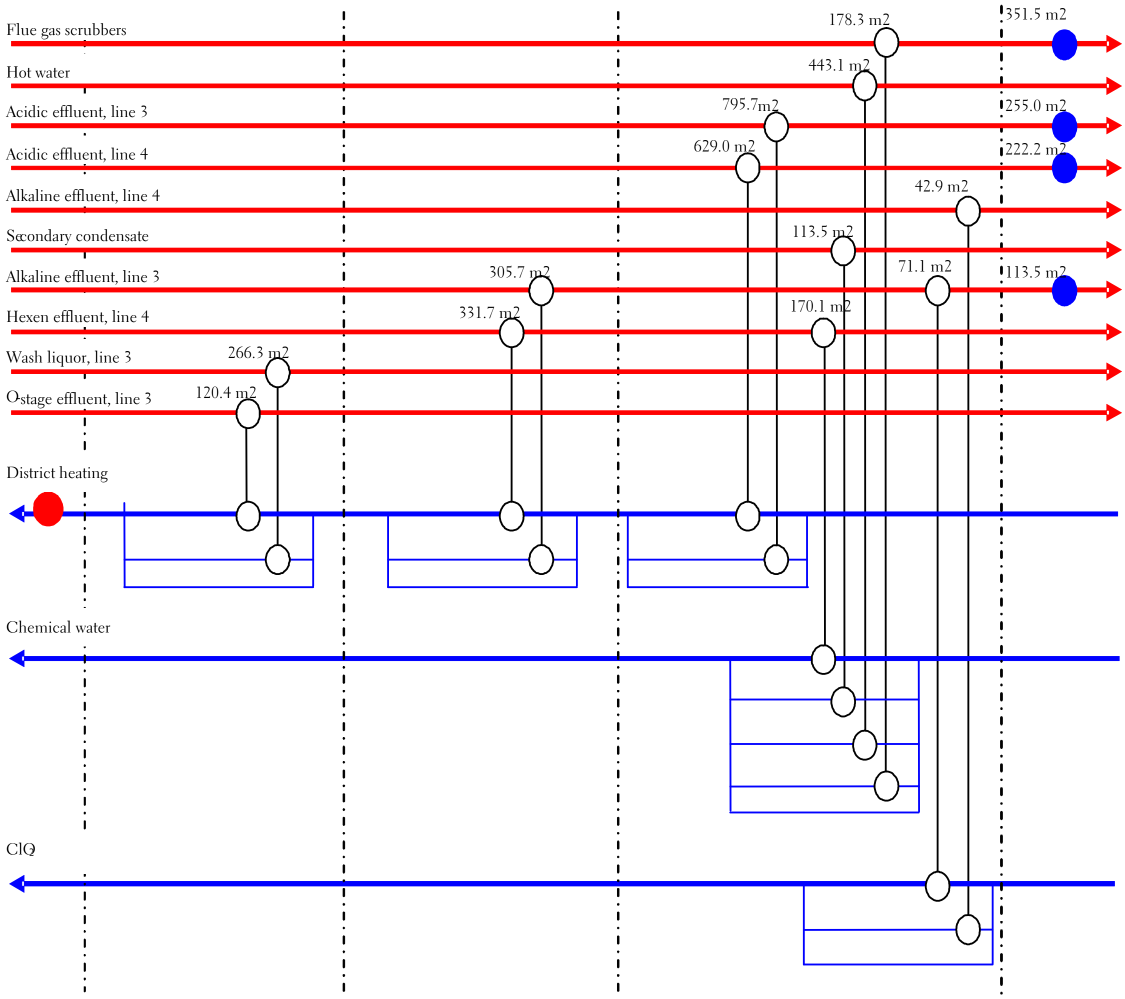

Figure 21. Final network configuration between a pulp mill and the district heating system.

The final optimized heat exchanger network between two district heating systems and a pulp mill is shown in Figure 21. Five bypasses are installed on cold streams. The maximum cold utility load required is now 51.5 MW instead of 96.3 MW, that is the maximum energy content of all the hot streams if they were cooled down to 37°C.

In Figure 22 the combined district heating load curve (including both cities) and approximation for the corresponding load curve, after all the available waste heat is utilized, is shown. The optimized annual external utility energy is now 94 GWh instead of 364 GWh, that is the amount of steam required without the heat exchanger network. These utility related savings, 270 GWh annual steam and 45 MW cooling capacity, require a network investment of 2988 k€.

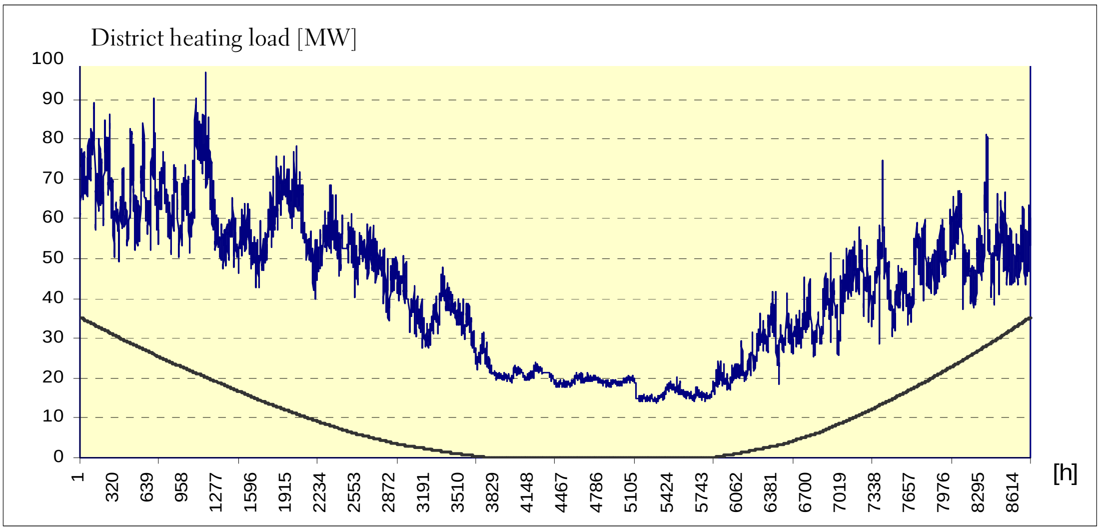

Figure 22. Load curves for district heating systems of two cities before and after the HEN investment (approximation).

4.2 Rationalisation of external cooling demand on a paper mill site

4.2.1 Introduction and the problem data

The following case study was published by Manninen *et al.* (2000) where Pinch analysis based HEN design methods were applied to the analysis and design of the heat recovery system and the utility system of a paper mill to provide a cost-effective solution to the minimization of external cooling demand, thus potentially eliminating the need for a cooling tower. The case study comprises a paper mill consisting of:

- Two thermo mechanical pulping lines (TMP 1 and 2),
- Four paper machines (PM A to D),
- Debarking station,
- Process water preparation station,
- Effluent treatment station,
- Power plant.

The proposed HEN design framework is applied to construct a heat exchanger network for this same case, which could operate in both summer and winter, and in all conditions between these two seasonal norms. The data for the winter and summer situations are shown in Table 11. The inlet temperature and heat capacity flow rate changes in the data are considered to be seasonal and thus fully correlated. The design scheme in this work has not been applied in order to compare the results between the two methods, but to test and show the ability of the presented method in tackling larger multiperiod problems. The case study was simplified by not examining lay-out issues and making an assumption that all the process water needed could be heated up before distribution into process specific fractions.

Table 11. Problem data for the case Manninen *et al.* (2000).

		winter			summer		
		T^{IN}	T^{OUT}	F	T^{IN}	T^{OUT}	F
Stream		(°C)	(°C)	(kW/K)	(°C)	(°C)	(kW/K)
TMP 1 clear filtrate	H1	70.0	25.0	137.3	70.0	25.0	137.3
TMP 2 clear filtrate	H2	70.0	25.0	249.7	70.0	30.0	249.7
PM A cooling 1	H3	50.0	49.0	800.0	50.0	49.0	800.0
PM A cooling 2	H4	60.0	50.0	90.0	60.0	50.0	90.0
PM B cooling 1	H5	50.0	49.0	1000.0	50.0	49.0	1000.0
PM B cooling 2	H6	60.0	50.0	86.0	60.0	50.0	86.0
PM C cooling 1	H7	50.0	49.0	4200.0	50.0	49.0	4200.0
PM C cooling 2	H8	60.0	50.0	155.0	60.0	50.0	155.0
PM D cooling 1	H9	50.0	49.0	4525.0	50.0	49.0	4525.0
PM D cooling 2	H10	60.0	50.0	545.9	60.0	50.0	545.9
PM A waste water	H11	-	-	0.0	38.0	37.0	120.0
PM B waste water	H12	39.2	37.0	246.4	42.3	37.0	247.7
PM C waste water	H13	40.5	37.0	265.1	45.4	37.0	272.1
PM D waste water	H14	-	-	0.0	40.1	37.0	173.2
PM D centricleaner reject	H15	55.0	37.0	66.7	55.0	37.0	66.7
Condenser	H16	24.0	23.9	115120.0	-	-	0.0
Process water	C1	2.0	53.7	1274.5	22.0	54.2	1249.3
Debarking station circulation water	C2	30.0	42.0	594.2	30.0	42.0	116.7

The winter and summer situations were both threshold problems, since only a hot utility is needed when $HRAT_{winter}$ stays below 13°C corresponding to HU^{UP}_{winter} = 22111kW, and $HRAT_{summer}$ stays below 3°C, corresponding to HU^{UP}_{summer} = 697kW, as is shown in the composite curves for winter (Figure 23) and for summer (Figure 24).

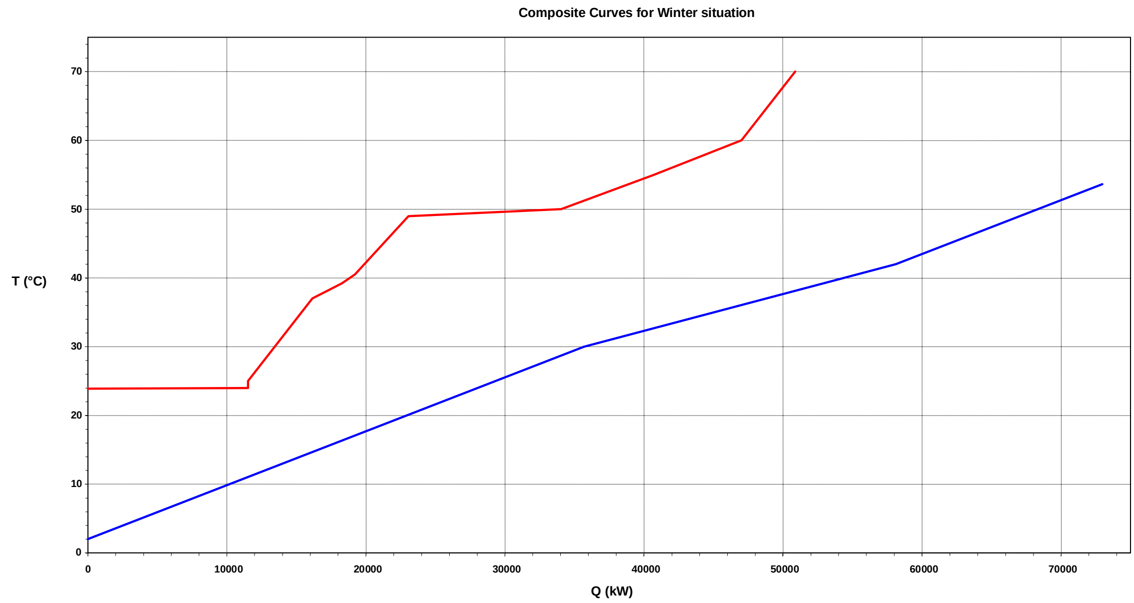

Figure 23. Composite curves for winter situation.

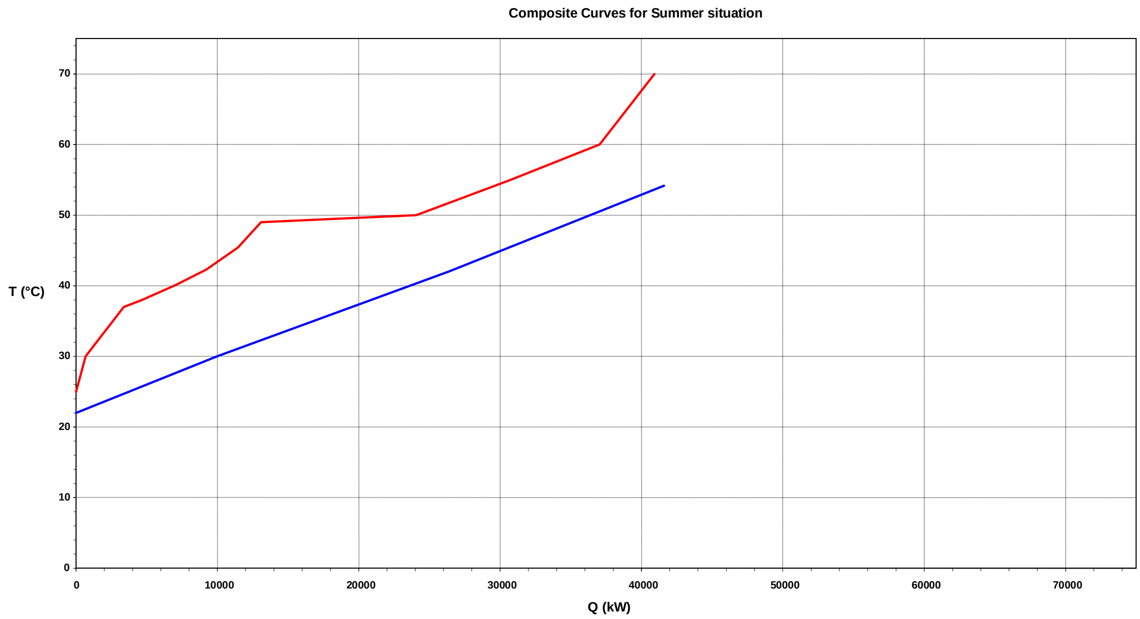

Figure 24. Composite curves for summer situation.

Cost information used in the case is:

- the annual costs of unit duties are 115.2 €/(kW·a) and 1.3 €/(kW·a) for hot and cold utility,
- the costs equation for the exchangers is 25000 €/unit + 641.7 €/m^2,
- lifetime used is 3 a and rate of interest 18%,
- overall heat transfer coefficients for all matches are 4 $kW \cdot m^{-2} \cdot K^{-1}$.

4.2.2 Initialization

The MILP target model was used to find upper and lower bounds for the superstructure stages and the number of different units, as is shown in Table 12. The lower bound of each unit type represents the number which allows the MILP model to achieve the solution of the minimum total number of units. The upper bound of certain units is the maximum number required in order to achieve the minimum total number of units. According to Table 12, 13 process to process units are required in order to have a feasible heat exchanger network with 18 total numbers of units. Even if the numbers of the other unit types were in their lower bounds, no more than 17 units are needed. Therefore, these upper bounds on the number of units give reasonable initial limits for the search space of the problem. These bounds are used as corresponding upper bounds in the MINLP model. These bounds were calculated at the threshold situations by using EMAT = 1°C, HU^{UP}_{winter} = 22111kW and HU^{UP}_{summer} = 697kW.

Table 12. The unit limits of the problem for the threshold situation.

Number of	Lower bound	Upper bound
Process to process units	13	17
Hot utility units	1	2
Cold utility units	0	4
Stages in superstructure	3	-

4.2.3 Multiperiod simultaneous MINLP model and LP feasibility test

The search of a pool of local solutions was implemented by solving the MINLP problem with the upper bounds of the units in Table 12, ε = 1°C, and by changing the upper bounds of utility consumptions. In the first stage, only two periods, winter and summer, were used as a multiperiod data of the MINLP model. The best solution from the MINLP model was achieved when HU^{UP}_{winter} = 22500 kW and HU^{UP}_{summer} = 850 kW, and when the number of stages were 4. Despite the fact that the superstructure was divided into 4 stages, the best solution found used only 3 of them. The higher numbers of stages, up to 9, were also tried but the solution did not improve.

The problem contained 1926 single equations, 1039 single variables and 146 binary variables. Total solver times were NLP = 2.78 sec and MILP = 41.43 sec on a (mobile Intel® P III 1GHz) using DICOPT++ (Viswanathan and Grossmann, 1990) with MINOS (Murtaugh and Saunders, 1985) and CPLEX® via GAMS (Brooke et al. 1988). The total annual costs after the MINLP optimization was 2376 k€ when the maximum area was 2779.7 m^2 distributed among the 19 units. The periodical results are shown in Table 13 - Table 15.

The resulting network was checked with the LP feasibility test model to test the feasible operation all year round. The problem was divided into 100 periods containing 41509 single equations and 32409 single variables. The execution time (GAMS, CPLEX®, mobile Intel® P III 1GHz) for the LP model was 1.3 sec yielding zero for all slack variables (sdt_{ijkp}), which means that the HEN is structurally flexible, and also when considering exchanger conductance approximations, can be feasibly operated all year round.

Table 13. Results from MINLP stage, period 1.

p = 1	Match i.j.k	1 13	2 12	2 13	3 22	4 12	5 13	6 12	7 13	8 12	9 13	1012	1213	1313	1423	1513	1614
$A_{i,j,k,p}$	[m^2]	269.3	452.5	369.2	35.7	60.0	30.9	57.3	129.9	103.3	139.9	364.0	125.1	135.9	43.2	41.9	337.7
$q_{i,j,k,p}$	[kW]	6179	3783	7453	800	900	1000	860	4200	1550	4525	5459	542	928	0	1201	11512
$F^{h}_{i,j,k,p}$	[kW/K]	137.3	249.7	249.7	800.0	90.0	1000.0	86.0	4200.0	155.0	4525.0	545.9	246.4	265.1	0.0	66.7	115120.0
$F^{c}_{i,j,k,p}$	[kW/K]	302.5	384.1	365.0	594.2	91.4	49.0	87.3	205.7	157.4	221.6	554.3	26.5	45.4	0.0	58.8	1274.5
$t^{h}_{i,k,p}$	[ºC]	70.0	70.0	54.9	50.0	60.0	50.0	60.0	50.0	60.0	50.0	60.0	39.2	40.5	-	55.0	24.0
$t^{h}_{i,k+1,p}$	[ºC]	25.0	54.9	25.0	49.0	50.0	49.0	50.0	49.0	50.0	49.0	50.0	37.0	37.0	-	37.0	23.9
$t^{c}_{j,k+1,p}$	[ºC]	11.0	31.5	11.0	30.0	31.5	11.0	31.5	11.0	31.5	11.0	31.5	11.0	11.0	30.0	11.0	2.0
$t^{c}_{j,k,p}$	[ºC]	31.5	41.3	31.5	31.4	41.3	31.5	41.3	31.5	41.3	31.5	41.3	31.5	31.5	30.0	31.5	11.0
$F^{hb}_{i,j,k,p}$	[kW/K]	30.1	151.5	66.7	742.6	56.1	960.0	53.6	4031.7	96.6	4343.8	340.4	227.0	232.8	-	33.4	-
$F^{hin}_{i,j,k,p}$	[kW/K]	107.2	98.2	183.1	57.5	33.9	40.1	32.4	168.3	58.3	181.2	205.5	19.4	32.3	-	33.3	-
$t^{ha}_{i,k,p}$	[ºC]	12.4	31.5	14.1	36.1	33.4	25.0	33.4	25.0	33.4	25.0	33.4	11.2	11.8	-	18.9	-
$dt^{ha}_{i,k,p}$	[ºC]	1.4	0.0	3.1	6.1	2.0	14.0	2.0	14.0	2.0	14.0	2.0	0.2	0.8	-	7.9	-
$F^{cb}_{i,j,k,p}$	[kW/K]	178.2	285.9	162.5	537.4	57.5	19.7	54.9	82.6	99.0	89.0	348.6	7.0	13.9	-	26.5	-
$F^{cin}_{i,j,k,p}$	[kW/K]	124.4	98.2	202.5	56.8	33.9	29.3	32.4	123.1	58.4	132.6	205.8	19.5	31.6	-	32.3	-
$t^{ca}_{j,k,p}$	[ºC]	60.7	70.0	47.8	44.1	58.0	45.2	58.0	45.2	58.0	45.2	58.0	38.8	40.4	-	48.2	-
$dt^{ca}_{j,k,p}$	[ºC]	9.3	0.0	7.0	5.9	2.0	4.8	2.0	4.8	2.0	4.8	2.0	0.4	0.1	-	6.8	-

Table 14. Results from MINLP stage, period 2.

p = 2	Match i.j.k	1 13	2 12	2 13	3 22	4 12	5 13	6 12	7 13	8 12	9 13	1012	1213	1313	1423	1513	1614
$A_{i,j,k,p}$	[m^2]	269.3	452.5	369.2	35.7	60.0	30.9	57.3	129.9	103.3	139.9	364.0	125.1	135.9	43.2	41.9	337.7
$q_{i,j,k,p}$	[kW]	6179	6579	3409	800	900	1000	860	4200	1550	4525	5459	1313	2286	537	1201	0
$F^{h}_{i,j,k,p}$	[kW/K]	137.3	249.7	249.7	800.0	90.0	1000.0	86.0	4200.0	155.0	4525.0	545.9	247.7	272.1	173.2	66.7	0.0
$F^{c}_{i,j,k,p}$	[kW/K]	320.1	535.5	176.6	116.7	73.2	51.8	70.0	217.6	126.2	234.4	444.4	68.0	118.4	116.7	62.2	1249.3
$t^{h}_{i,k,p}$	[ºC]	70.0	70.0	43.7	50.0	60.0	50.0	60.0	50.0	60.0	50.0	60.0	42.3	45.4	40.1	55.0	-
$t^{h}_{i,k+1,p}$	[ºC]	25.0	43.7	30.0	49.0	50.0	49.0	50.0	49.0	50.0	49.0	50.0	37.0	37.0	37.0	37.0	-
$t^{c}_{j,k+1,p}$	[ºC]	22.0	41.3	22.0	34.6	41.3	22.0	41.3	22.0	41.3	22.0	41.3	22.0	22.0	30.0	22.0	22.0
$t^{c}_{j,k,p}$	[ºC]	41.3	53.6	41.3	41.5	53.6	41.3	53.6	41.3	53.6	41.3	53.6	41.3	41.3	34.6	41.3	22.0
$F^{hb}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$F^{hin}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$t^{ha}_{i,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$dt^{ha}_{i,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$F^{cb}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1249.3
$F^{cin}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$t^{ca}_{j,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$dt^{ca}_{j,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 15. Utility data from MINLP stage.

		C1	C2	H11
A^u	$[m^2]$	59.6	22.2	2.2
q^u_1	$[kW]$	15736	6330	0
q^u_4	$[kW]$	743	63	120

4.2.4 NLP improvement model

The year was divided into 5 periods to solve the NLP improvement model. Solving time for the model was 5 sec and included 6451 single equations and 2772 single variables,. Total annual cost after solving the NLP was 2307 k€ when the maximum total area was 2568.0 m^2. The NLP improvement model was executed on a (mobile Intel® P III 1GHz) using MINOS via GAMS. The results are shown in Table 16 - Table 18.

Table 16. Utility data from NLP stage.

		C1	C2	H11
A^u	$[m^2]$	59.6	22.2	2.2
q^u_1	$[kW]$	15736	6330	0
q^u_2	$[kW]$	11903	4764	30
q^u_3	$[kW]$	8127	3197	60
q^u_4	$[kW]$	4406	1630	90
q^u_5	$[kW]$	743	63	120

Table 17. Results from NLP stage, period 1.

p = 1	Match i.j.k	1 1 3	2 1 2	2 1 3	3 2 2	4 1 2	5 1 3	6 1 2	7 1 3	8 1 2	9 1 3	101 2	121 3	131 3	142 3	151 3	161 4
$A_{i,j,k,p}$	[m^2]	283.1	242.3	324.8	35.7	63.1	32.5	60.3	136.5	108.6	147.1	382.5	100.0	142.8	43.2	44.0	337.7
$q_{i,j,k,p}$	[kW]	6179	1486	9751	800	900	1000	860	4200	1550	4525	5459	542	928	0	1201	11512
$F^{h}_{i,j,k,p}$	[kW/K]	137.3	249.7	249.7	800.0	90.0	1000.0	86.0	4200.0	155.0	4525.0	545.9	246.4	265.1	0.0	66.7	115120.0
$F^{c}_{i,j,k,p}$	[kW/K]	474.1	40.7	438.7	594.2	78.1	28.7	73.5	120.6	134.5	129.9	947.8	19.2	31.5	0.0	31.6	1274.5
$t^{h}_{i,k,p}$	[°C]	70.0	70.0	64.1	50.0	60.0	50.0	60.0	50.0	60.0	50.0	60.0	39.2	40.5	-	55.0	24.0
$t^{h}_{i,k+1,p}$	[°C]	25.0	64.1	25.0	49.0	50.0	49.0	50.0	49.0	50.0	49.0	50.0	37.0	37.0	-	37.0	23.9
$t^{c}_{j,k+1,p}$	[°C]	11.0	33.3	11.0	30.0	33.3	11.0	33.3	11.0	33.3	11.0	33.3	11.0	11.0	30.0	11.0	2.0
$t^{c}_{j,k,p}$	[°C]	24.1	69.8	33.3	31.4	44.8	45.9	45.0	45.9	44.8	45.9	39.0	39.2	40.5	30.0	49.0	11.0
$F^{hb}_{i,j,k,p}$	[kW/K]	31.6	189.6	43.2	742.6	52.8	-	50.4	-	90.9	-	331.7	204.2	-	-	-	-
$F^{hin}_{i,j,k,p}$	[kW/K]	105.7	60.1	206.5	57.5	37.3	-	35.7	-	64.1	-	214.2	42.2	-	-	-	-
$t^{ha}_{i,k,p}$	[°C]	11.5	45.3	16.8	36.1	35.8	-	35.9	-	35.8	-	34.5	26.4	-	-	-	-
$dt^{ha}_{i,k,p}$	[°C]	0.5	12.0	5.8	6.1	2.6	-	2.6	-	2.6	-	1.3	15.3	-	-	-	-
$F^{cb}_{i,j,k,p}$	[kW/K]	352.1	-	174.6	537.4	41.5	-	38.6	-	71.5	-	725.9	-	-	-	-	-
$F^{cin}_{i,j,k,p}$	[kW/K]	122.1	-	264.1	56.8	36.6	-	34.9	-	63.0	-	221.8	-	-	-	-	-
$t^{ca}_{j,k,p}$	[°C]	61.7	-	48.0	44.1	57.9	-	57.9	-	57.9	-	57.9	-	-	-	-	-
$dt^{ca}_{j,k,p}$	[°C]	8.3	-	16.1	5.9	2.1	-	2.1	-	2.1	-	2.1	-	-	-	-	-

Table 18. Results from NLP stage, period 2.

p = 1	Match i.j.k	1 1 3	2 1 2	2 1 3	3 2 2	4 1 2	5 1 3	6 1 2	7 1 3	8 1 2	9 1 3	101 2	121 3	131 3	142 3	151 3	161 4
$A_{i,j,k,p}$	[m^2]	283.1	242.3	324.8	35.7	63.1	32.5	60.3	136.5	108.6	147.1	382.5	100.0	142.8	43.2	44.0	337.7
$q_{i,j,k,p}$	[kW]	6179	5191	4797	800	900	1000	860	4200	1550	4525	5459	1313	2286	537	1201	0
$F^{h}_{i,j,k,p}$	[kW/K]	137.3	249.7	249.7	800.0	90.0	1000.0	86.0	4200.0	155.0	4525.0	545.9	247.7	272.1	173.2	66.7	0.0
$F^{c}_{i,j,k,p}$	[kW/K]	289.9	443.9	235.0	116.7	82.7	49.2	79.0	206.7	142.3	222.7	501.4	72.0	115.5	116.7	58.2	0.0
$t^{h}_{i,k,p}$	[ºC]	70.0	70.0	49.2	50.0	60.0	50.0	60.0	50.0	60.0	50.0	60.0	42.3	45.4	40.1	55.0	-
$t^{h}_{i,k+1,p}$	[ºC]	25.0	49.2	30.0	49.0	50.0	49.0	50.0	49.0	50.0	49.0	50.0	37.0	37.0	37.0	37.0	-
$t^{c}_{j,k+1,p}$	[ºC]	22.0	42.4	22.0	34.6	42.4	22.0	42.4	22.0	42.4	22.0	42.4	22.0	22.0	30.0	22.0	22.0
$t^{c}_{j,k,p}$	[ºC]	43.3	54.1	42.4	41.5	53.3	42.3	53.3	42.3	53.3	42.3	53.3	40.2	41.8	34.6	42.6	22.0
$F^{hb}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$F^{hin}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$t^{ha}_{i,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$dt^{ha}_{i,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$F^{cb}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1249.3
$F^{cin}_{i,j,k,p}$	[kW/K]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.0
$t^{ca}_{j,k,p}$	[ºC]	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

The final optimized heat exchanger network between waste heat streams and the fresh water and circulating water in debarking station is shown in Figure 25.

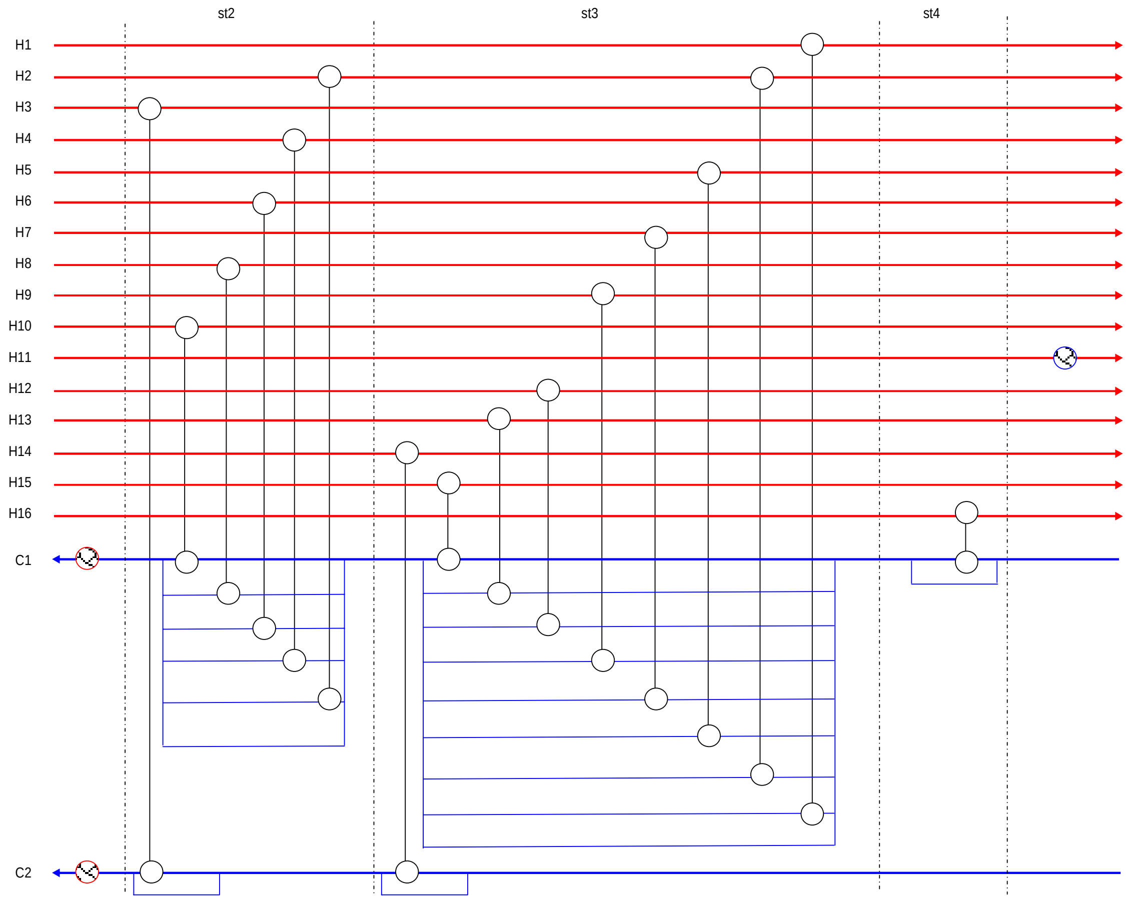

Figure 25. Final network configuration for case Manninen *et al.* (2000).

Three bypasses are installed on the process water stream and two on cold stream 2. The set up values for each possible bypass for each period can be found in Table 17. The set up values are calculated for both cases; (i) bypass in hot stream and (ii) bypass in cold stream. The optimization framework does not consider which one, hot or cold, should be used. In this case, the bypasses are placed with split streams and with larger bypass flows to maximize the allowable pressure drops in exchangers.

After the heat recovery the maximum cold utility load required is practically zero instead of 51 MW, this being the maximum energy content of all the hot streams. The savings in hot utility capacities are 51 MW in winter and 41 MW in summer, while the network investment is 2140 k€.

4.3 Discussions

The flexible heat exchanger network synthesis framework has been applied to two industrial problems. Both problems have the following common characteristics:

- (i) Fully correlated variations and not more than a few short term disturbances take place,
- (ii) Cooling down the waste heat streams with low start temperature is considered,
- (iii) Cold streams with relatively large heat capacity flow rates and low start temperatures are involved.

Both optimized examples result in network configurations with near to theoretical limits for the minimum number of units and utility consumptions, corresponding to a relatively small HRAT = 3-6°C. Thus, the heat recovery is extremely efficient with only a few units installed. However, this still leaves the question of the required exchanger area; what is the ideal trade off between the installed exchanger area, units and operational costs? Is the low price for the heat transfer area leading to the trade off where the number of units and utility consumption are near to minimum? The final answer could be found if the problem was solved to the global optima, but at the present moment only local solutions, such as is mentioned above, are possible.

Further, many of the split streams involved in both resulting configurations are explained by the cold streams having a much larger heat capacity flow rate than the matching candidates (hot streams). This difference between heat capacity flow rates in one cold and several hot streams being coupled in a series, leads to a higher driving force on one match. This, in turn, forces a lower driving force on some of the other matches. Therefore, the first match with a high driving force is a bad choice from the view point of the overall network area whereas splitting a large stream is essential. Splitting the large stream may also increase the possible achievable amount of recovered heat, so that the utility loads can be minimized.

5 CONCLUSIONS AND SIGNIFICANCE

The heat exchanger network synthesis presents an extremely challenging task for modelers, since it is a potentially explosive combinatorial problem that includes nonlinearities. This is true even if the thermal-hydraulic properties of the fluids and units, dynamic behavior, discontinuities in cost structure and many other properties related to the practical heat exchanger network design are strongly simplified or even excluded. Therefore, the basic condition for the modeling of the task is to make compromises between precise representations and practical considerations. The most important objectives in this work have been to develop a method to help engineers in industry to systematically find out the general view of the HEN before the detailed design stage, and also to make feasibility studies of the heat recovery investments more reliable. When developing such a method the main goal must be, not as much the detailed presentation of the HEN, but create the ability to solve larger scale problems.

For the flexible HEN synthesis these kinds of methods for solving larger scale problems have been developed already. However, in previous studies decomposition of the problem has been used. Because the benefits of the simultaneous HEN synthesis approaches versus decomposed ones have been demonstrated, and while the sizes of the simultaneously solved flexible HEN synthesis problems have been very small, a method with the characteristics of simultaneous optimization has been proposed to solve larger size problems. The proposed framework synthesizes the flexible HEN and is able to find its set points for different operating conditions, so that annual costs will be minimized. This framework consists of two optimization levels. The first level involves an interactive procedure for synthesizing the HEN configuration and identifying its critical conditions, whereas the second level overcomes limitations related to the first level.

It has been shown that the simplified superstructure presentation proposed by Yee and Grossmann (1990a) can be applied for generating flexible heat exchanger networks. Furthermore, a scheme has been presented which eliminates the modeling of bypasses, so that the nonlinear heat balances, binary variables, temperature variables and flow variables related to each bypass in the superstructure are no longer needed in the model. Such a scheme being combined to the simplified superstructure is essential as the problem size increases, since the primary bottleneck in the simultaneous synthesis of the flexible HEN is the size of the MINLP formulation. The elimination of bypass modeling, a stage-wise superstructure presentation and isothermal mixing assumption, allows the feasible space in the MINLP model to be defined by a set of linear constraints. These simplifications make the MINLP model more robust and efficient to solve, so that industrial size heat exchanger network problems can be solved simultaneously.

The proposed flexible HEN configuration synthesis ignores pinch considerations, and it does not rely on a sequential decomposition of the problem. This synthesis method is able to take into account weighted periods, so that the most common operating condition can dominate while the uncommon ones are still considered.

Finally, the proposed HEN synthesis strategy has been successively tested with two industrial problems. In these two situations, waste heat streams have been cooled down, forming a local and site level energy integration, to gain savings on steam consumption and to avoid cooling tower investment. The energy integration is formed via a flexible heat exchanger network, so that annual costs will be minimized.

In short, this work introduces a new way to model simultaneously flexible heat exchanger network synthesis whilst also presenting substance for further methodological development. Additionally, the work seeks to inspire the energy intensive branch of industry to choose more systematic methods when assessing or designing heat integration within their own branch or in cooperation with other branches. In industry there is still lot of potential to make an energy system more efficient and thereby reduce the waste heat available. On the other hand there is an option to export the waste heat to another industry or to society. When the use of heat exchanger network is considered for these tasks the proposed optimization framework can be implemented to find out the cost optimal investments.

6 FUTURE WORK

One of the future challenges in the proposed framework is to reduce the complexity and search space of the problem, introducing tighter bounds at the initialization level, so that the MINLP model can be solved for larger size problems. For smaller size problems, the challenge is to make the model more realistic, so that factors such as allowable pressure drop, exchanger type, fouling and controllability can be taken into account. Stream repiping and exchanger reassignments should also be considered at the retrofit cases.

Further work should concentrate on applying methods other than mathematical programming ,e.g. evolutionary algorithms, to minimize total annual costs in a HEN based on maximum areas, units and operational costs. This could also be the way out of the stage-wise superstructure, so that the more realistic superstructure can be used. Then it would be possible to develop a one stage optimization routine taking account of all costs and operability simultaneously, whilst still allowing for larger problems.

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