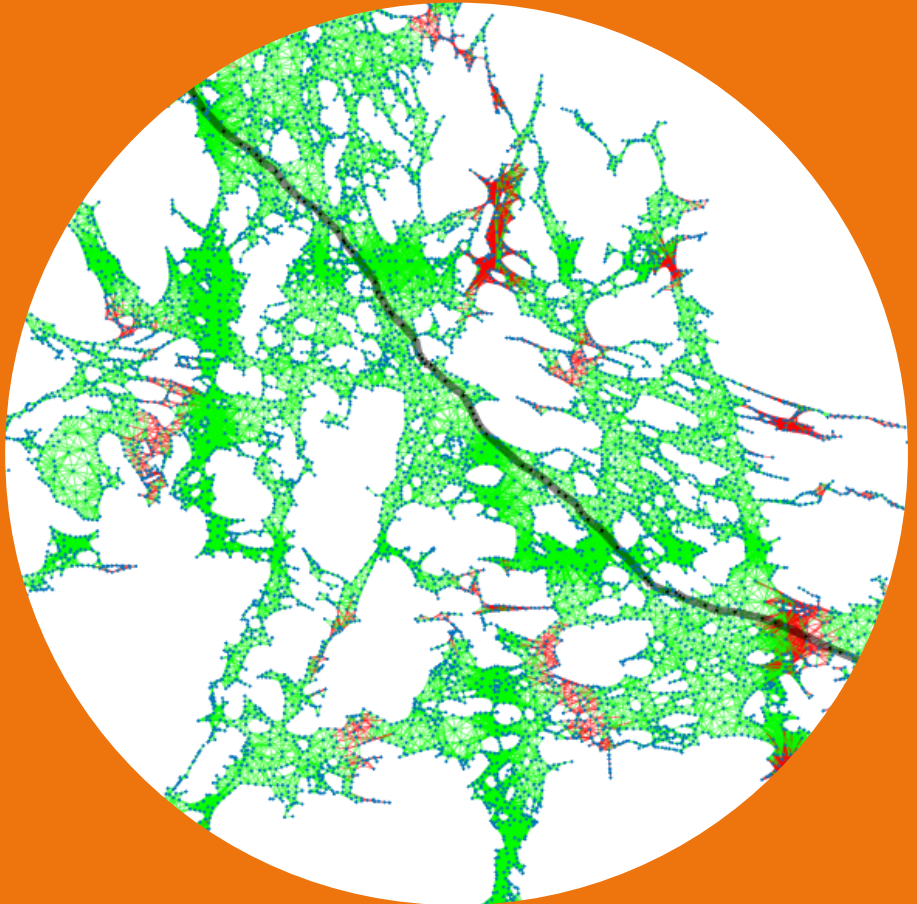


# Computational models for adversarial risk analysis and probabilistic scenario planning

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Juho Roponen



# Computational models for adversarial risk analysis and probabilistic scenario planning

**Juho Roponen**

A doctoral thesis completed for the degree of Doctor of Science (Technology) to be defended, with the permission of the Aalto University School of Science, at a public examination held at the lecture hall H304 (Otakaari 1, Espoo, Finland) of the school on 9 June 2023 at 12 noon.

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People need to make decisions under uncertainty. Both in corporate and public governance, in addition to uncertainty, the decisions can have high costs and far-reaching consequences. Thus, choosing a good decision alternative, or at least avoiding the inferior ones, is crucial. Two sources of uncertainty are especially prevalent in these decision problems: human activity and long planning horizons.

In this dissertation, methods for addressing uncertainties arising from both these sources are developed. By quantifying these uncertainties as probability distributions and preferences over outcomes as utility functions, a well-defined mathematical decision problem can be constructed and then solved using optimization techniques.

First, methods for adversarial risk analysis are developed to model the decision processes of adversarial actors who deliberately try to advance their own interests. The proposed methods facilitate systematic probabilistic analyses with limited knowledge about the adversary's preferences and their available information. This can be especially useful when the exact way the adversary analyzes the situation is difficult to assess or when their goals are deliberately hidden, as is often the case when analyzing military combat or security problems. The dissertation also demonstrates how combat modeling and simulation tools can be applied in adversarial risk analysis. This expands the types of analyses these tools can be used for, making it possible to answer questions such as, how the adversary's actions are impacted by changing circumstances, or how the outcomes of individual battles impact the larger strategic situation.

Second, a new probabilistic cross-impact analysis model is developed to quantify uncertainties associated with future scenarios based on information elicited from subject matter experts. Two different computational approaches are presented for analyzing the elicited cross-impact statements. One takes information about upper and lower bounds on probabilities and then calculates upper and lower bounds on system risk or utility. The other takes the best estimates about probabilities of specific uncertainty factors and their interactions and constructs a joint probability distribution and a Bayesian network. These approaches can be useful when probability information based on statistics or simulations is not available, for example when results need to be produced quickly or the uncertainties are associated with relatively far-off future events or human activity.

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**Tekijä**

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**Väitöskirjan nimi**

Laskennallisia malleja vastakkainasettelulliseen riskianalyyysiin ja todennäköisyyspohjaiseen skenaariosuunnitteluun

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Ihmiset joutuvat väistämättä tekemään päätöksiä epävarmuuden vallitessa. Yritysjohdossa ja julkishallinnossa moniin päätöksiin liittyy epävarmuuden lisäksi myös paljon kustannuksia ja kauaskantoisia seurauksia. Siksi hyvän päätösvaihtoehdon löytäminen, tai ainakin huonojen välttäminen, on ensiarvoisen tärkeää. Näissä konteksteissa epävarmuudet liittyvät yleisimmin joko ihmisten toiminnan tai tulevaisuuden heikkoon ennakoitavuuteen.

Tässä väitöskirjassa kehitetään menetelmiä kummankinlaisten epävarmuuksien käsittelyyn. Kuvaamalla epävarmuuksia todennäköisyysjakaumilla ja seurauksien haluttavuutta hyötyfunktioilla on mahdollista rakentaa hyvin määriteltä matemaattinen päätösongelma ja ratkaista se optimointimenetelmillä.

Ensiksi kehitetään vastakkainasettelullisen riskianalyysin (adversarial risk analysis, ARA) menetelmiä, joiden avulla tavoitteellisten omaa etuaan ajavien vastustajien päätösprosesseja voidaan mallintaa. Kehitetyt menetelmät mahdollistavat todennäköisyyksiin pohjaavan analyysin myös silloin, kun vastustajan käytettävissä olevaa tietoa ja hänen tavoitteitaan ei tunneta hyvin. Tästä on hyötyä tilanteissa, joissa vastustajan tavat analysoida ongelmaa ovat vaikeita arvioida ja hänen tavoitteensa jopa tarkoituksellisesti hämärän peitossa, mikä on yleistä esimerkiksi sotilastoimintaa analysoitaessa. Väitöskirjassa myös näytetään, miten taistelumallinnukseen kehitetyt mallinnus- ja simulaatiotyökalut voivat toimia osana vastakkainasettelullista riskianalyysiä. Näin olemassa olevilla työkaluilla voidaan vastata uuden tyyppiin kysymyksiin, kuten kuinka omat päätökset voivat vaikuttaa vastustajan toimintaan tai kuinka yksittäisten joukkojen kohtaamiset voivat vaikuttaa taistelun kulkuun laajemmin.

Toiseksi kehitetään uusi ristivaikutusanalyysimalli (cross-impact analysis, CIA) tulevaisuuden epävarmuuksien jäsentelyyn ja ennakkointiin asiantuntija-arvioiden perusteella. Näiden ristivaikutusarvioiden kanssa käytettäväksi on luotu kaksi laskennallista lähestymistapaa. Yksi käyttää arvioita todennäköisyyksien ja ristivaikutuksien ylä- ja alarajoista muodostamaan arvion koko systeemin riskistä tai odotusarvoisesta hyödystä. Toinen laskee saatavilla olevien todennäköisyys- ja ristivaikutusarvioiden perusteella yhteisjakauman eri epävarmuustekijöiden tapahtumien yhdistelmille sekä epävarmuustekijöitä kuvaavan Bayes-verkon. Tällaiset asiantuntija-arvioihin pohjaavat lähestymistavat ovat hyödyllisiä silloin, kun tilastollista tai simuloitua aineistoa ei ole saatavilla. Näin voi käydä esimerkiksi silloin, kun tuloksia tarvitaan nopeasti tai kun epävarmuuksia on vaikea arvioida, koska ne liittyvät verrattain etäiseen tulevaisuuteen tai ihmisten toimintaan.

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# Preface

Working on this dissertation has been a great adventure, and one that I would have never embarked upon if it had not been for my supervisor Ahti Salo. Without him, this dissertation would almost certainly have never been written, and I feel that he deserves my utmost gratitude. The journey has not always been easy, but Ahti for his part has always made sure his students have had all the tools necessary for success.

I feel like I should also thank Esa Lappi and Bernt Åkesson. If they had not given me the first push all those years ago, I am not sure I would have ever ended up doing work related to military operations research. Had that not happened, this dissertation would most likely have ended up looking quite different. Of course, that was only the start of a very long path, and along the way, I have met so many wonderful collaborators, coworkers, and students. They have made my work so much more enjoyable and interesting, and I extend my gratitude to all of them.

Finally, I want to thank my mother Outi, my father Terho, and my grandparents Sirkka and Lasse. I have always been able to count on your support, and that has been invaluable.

Espoo, April 26, 2023,

Juho Roponen





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# List of Publications

This thesis consists of an overview and of the following publications which are referred to in the text by their Roman numerals.

**I** Juho Roponen and Ahti Salo. Adversarial risk analysis for enhancing combat simulation models. *Journal of Military Studies*, 6(2), 82-103, December 2015.

**II** Juho Roponen, David Ríos Insua, and Ahti Salo. Adversarial risk analysis under partial information. *European Journal of Operational Research*, 287(1), 306-316, November 2020.

**III** Ahti Salo, Edoardo Tosoni, Juho Roponen, and Derek W. Bunn. Using cross-impact analysis for probabilistic risk assessment. *Futures & Foresight Science*, 4(2), e2103, September 2021.

**IV** Juho Roponen and Ahti Salo. A probabilistic cross-impact methodology for explorative scenario analysis. Submitted to *Futures & Foresight Science*, December 2022.



# Author's Contribution

## **Publication I: “Adversarial risk analysis for enhancing combat simulation models”**

Roponen is the primary author. He wrote most of the manuscript and implemented the computational model for the case example. Salo helped finalize the manuscript.

## **Publication II: “Adversarial risk analysis under partial information”**

Roponen is the primary author. He designed and implemented the methods and wrote most of the manuscript. Ríos Insua helped with structuring and writing the methodology. Salo helped finalize the manuscript.

## **Publication III: “Using cross-impact analysis for probabilistic risk assessment”**

Salo is the primary author. Roponen helped write the section on the relationship between cross-impact statements and scenario probabilities, created the optimization model for the case study, and calculated the results. Tosoni provided the data and wrote the section on the case study. Bunn helped finalize and revise the manuscript.

## **Publication IV: “A probabilistic cross-impact methodology for explorative scenario analysis”**

Roponen is the primary author. He designed and implemented the methods, collected the data and carried out the computations for the case study, and wrote most of the manuscript. Salo helped finalize the manuscript.



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# 1. Introduction

## 1.1 Interpretation of probability

Humankind has always been surrounded by uncertainties. Yet, quantifying that uncertainty with probabilities is a relatively recent innovation (Bernstein, 1996). Still, people have long thought about uncertainty, for already ancient Roman and Jewish laws of evidence include degrees of proof and presumptions to deal with uncertainty in the court of law (Franklin, 2001). Yet, the first written record of probability calculus comes from the 1560s, when Gerolamo Cardano wrote about the sum totals when rolling three dice and how the odds of those totals arose from the combinations of dice rolls that could produce them (Bellhouse, 2005).

All the earliest examples of probability calculus focus on games of chance, including writings of Cardano, Pierre de Fermat, Blaise Pascal, Christiaan Huygens, and even Galileo (Hacking, 2006). Because fair dice or decks of cards provide a discrete set of possible outcomes which can all be assumed equally likely, the underlying probabilities are easily understood. More challenging mathematical problems arise when the number of cards or dice increase, but this does not require substantial changes in problem framing. However, as events with equally likely outcomes rarely appear outside games of chance these early advances in probability theory found little practical use (Bernstein, 1996).

Probabilities cannot be observed or measured directly, which may in part explain why it took so long for probability theory to rise to prominence. Advances in the field of statistics eventually led to new applications for probability theory, finding uses in insurance pricing and policy decisions (Bernstein, 1996). Statistics and probability seemed like such a perfect match, that for a time, probability was widely interpreted as the frequency of a specific outcome when a trial was repeated infinitely many times. Whilst this frequentist interpretation explains well the probabilities of dice rolls and variations found in statistics, it is not particularly helpful

for determining the probabilities of future events (French, 1986). Modern simulation models have made it possible to conduct statistical analysis about the future in well-understood physical systems by varying the initial conditions or simulation parameters (Hammersley, 2013)—an approach that is used in meteorological forecasting (Wilks and Wilby, 1999) and estimating the effects of weapon systems (Brandstein and Horne, 1998), for example. Yet, not all systems are easy to simulate, least of all human behavior.

Because of the difficulty or sheer impossibility of estimating the frequency or propensity of future events (or human activity) in many contexts, probabilities can instead be treated as degrees of beliefs in an event (Corfield and Williamson, 2001; Howard and Abbas, 2016; French, 1986; Raiffa, 1968). This interpretation is called Bayesian probability in honor of 18th-century mathematician Thomas Bayes, who first presented a method for updating beliefs about probabilities that is now called Bayesian inference (Bernstein, 1996). The subjective probability interpretation itself, however, should perhaps be more accurately attributed to Pierre-Simon Laplace (2012). Nowadays, subjective Bayesian probabilities have become the norm in fields like game theory and decision analysis, which deal with human decision-making in particular (Corfield and Williamson, 2001).

It is a matter of philosophical debate, whether the true nature of probability is statistical, subjective, or simply some hidden physical property (Kyburg and Smokler, 1980). Still, all these interpretations agree that probabilities can be treated in a rigorous mathematical manner following the rules of probability theory, a branch of mathematics with a well-defined set of axioms governing probabilities (Kallenberg, 1997). Thus, the exact probability interpretation rarely affects the validity of mathematical methods but can have implications on how the probabilities should be assessed and interpreted (Kyburg Jr, 2012). The author of this dissertation subscribes to the subjective Bayesian school of thinking.

## 1.2 Decision theory

The primary reason that probability theory is so widely applicable is that humans live in a world full of uncertainties. This is not an artifact of the modern world but has existed throughout history (Bernstein, 1996). Humans even seem to have evolved to have some innate understanding of probabilities (Fontanari et al., 2014). While the specific problems faced by humans have changed with the transition from hunter-gatherers to modern societies, uncertainty has not disappeared.

The formal answer guiding choices between uncertain alternatives has been known "ever since mathematicians first began to study the measurement of risk" (Bernoulli, 2011). Laplace (2012) called it mathematical hope.

Nowadays it is known, among other names, as mathematical expectation or expected value. In its simplest form, it is the sum of possible outcomes  $x_i$  weighted by their probabilities  $p_i$

$$E[X] = x_1 p_1 + x_2 p_2 + \dots + x_n p_n, \quad (1.1)$$

but extensions also exist for countably and uncountably infinite sets of possible outcomes (Kallenberg, 1997).

Maximizing expected profits is at the core of most of risk analysis, insurance mathematics, and business optimization. However, for the maximization of mathematical hope to become a truly universal answer to decision problems under uncertainty, one more innovation was needed. In their book *Theory of Games and Economic Behavior* (von Neumann and Morgenstern, 1947), John von Neumann and Oskar Morgenstern laid the foundation for the field of decision theory (as well as game theory). Introducing what is today known as von Neumann–Morgenstern (VNM) utility theorem they postulated a set of axioms describing a rational decision-maker. When faced with risky outcomes this rational decision-maker should choose an alternative that maximizes the expected utility, defined as

$$E[U] = u(x_1) p_1 + u(x_2) p_2 + \dots + u(x_n) p_n, \quad (1.2)$$

where the utility function  $u$  provides a measure for the decision-maker's preferences over the possible outcomes  $x_i$ .

The VNM utility theory has been the target of criticism since its inception, sometimes even quite unjustly (Fishburn, 1989). Still, the theory has been very influential at the heart of the field of decision theory (Peterson, 2017), and even some of the critics are advocating for improving or evolving the theory instead of abolishing it (Kahneman and Tversky, 1979; Schoemaker, 1982; Fishburn, 2013). There is a wide range of literature on the study of utility functions and their role in human decision-making, but will not be discussed further here, because this dissertation's primary focus is on the other part of equation (1.2), that is, the probability distribution.

### 1.3 Objectives of the dissertation

This dissertation develops mathematical methods for characterizing probability information about uncertainties in order to support decision-making. Publications I and II focus primarily on the analysis of uncertainties arising from the competing activity of other decision-makers, whereas publications III and IV focus on quantifying the uncertainties based on expert judgments.

Publication I explores how adversarial risk analysis (ARA) could be used to model strategic and tactical decision-making in military combat modeling, where uncertainty arising from the decision processes of different

actors has traditionally not been incorporated in the analysis. Typically, these decisions are modeled based on either very simplified game theoretical equilibriums or just expert opinion, leaving little to no room for uncertainty.

Publication II presents new computational methods for performing ARA without assigning probability distributions for all the involved uncertainties. This is useful especially in security contexts because it avoids assigning probability distributions to adversaries' actions and utilities. Because it is difficult to estimate the utility function of even a cooperative decision-maker, it can be almost impossible to accurately estimate the utilities of an adversarial decision-maker.

Publication III develops a new approach to cross-impact analysis by mapping expert judgments into corresponding probability bounds to different system outcomes. These probabilities are then used to establish upper and lower bounds for the system risk and other performance indicators. This approach makes it possible to form conservative estimates about system safety even if precise information about associated probabilities is not readily available.

Publication IV presents a computational approach to using CIA expert judgments, which may be imprecise and contradictory, to establish a probability distribution for possible system outcomes. This can facilitate probabilistic analysis based on future events and other difficult-to-model systems, like the ones often found in ARA.

## 2. Methodological foundations

The language used to describe decision problems is continuously evolving and quite diverse (Keith and Ahner, 2021), so it is necessary to first explain some of the terminology used throughout this chapter. *Probability* is taken as representing a degree of belief in an event, and an *event* can be any statement about the state of reality, for example, "Tomorrow it will rain." or "Galileo died in December.". A *random variable*<sup>1</sup> is a division of reality into multiple possibilities. These possibilities, called (the random variable's) *outcomes*, are mutually exclusive and jointly exhaustive events, so exactly one of them is guaranteed to always happen. A random variable could be for example "Tomorrow's highest temperature" with outcomes {" < 0°C", " ≥ 0°C and ≤ 20°C", " > 20°C"}. The random variables included in the analysis should always be chosen to be useful for characterizing the decision problem (Howard and Abbas, 2016).

### 2.1 Probability theory

The definitions here broadly follow (French, 1986) and (Kallenberg, 1997), although some of the terms and notation used are different. To start out, let  $(\Omega, \mathcal{F}, P)$  be a probability space. For the sake of simplicity, we assume that the sample space  $\Omega$  is countable i.e. finite or countably infinite. Because  $\Omega$  is countable, all of its subsets can be included in the event set, and thus the event set  $\mathcal{F} = 2^\Omega$ . The set of outcomes  $\mathcal{X}$  for random variable  $X$  is defined as a countable partition of  $\Omega = \bigcup_{x \in \mathcal{X}} x$ . This means that  $\mathcal{X} \subseteq \mathcal{F}$ , and that the outcomes  $x \in \mathcal{X}$  are mutually exclusive and collectively exhaustive. Because  $\mathcal{X} \subseteq \mathcal{F}$ ,  $P(x)$  is defined for all  $x \in \mathcal{X}$ , and also for unions and

---

<sup>1</sup>Various names exist for the same concept in the literature, for example, random variable (Kallenberg, 1997), random event (Harsanyi, 1967), key factor (Bunn and Salo, 1993), lottery (Raiffa, 1968; Myerson, 1997), distinction (Howard and Abbas, 2016), and uncertainty factor (Seeve and Vilkkumaa, 2022). Ultimately, the term random variable is used here to keep the terminology as familiar to most readers as possible. Notably, Publications III and IV primarily use the term uncertainty factor instead.

intersections of outcomes.

Because the outcome sets are countable, the random variables are discrete and defined as a function  $X : \Omega \rightarrow \mathcal{X}$  such that the preimages  $\mathcal{X}^{-1}(x) = x$ , for all  $x \in \mathcal{X}$ . Therefore, random variables describe the possible states of the world by dividing them into specific outcomes. Usually, the focus of the analysis is on the induced distribution  $P \circ X^{-1}$ , and the choice of  $\Omega$  plays little to no role. (Kallenberg, 1997).

It is convenient to also define the sets of possible decision outcomes  $\mathcal{D}$  as countable partitions of  $\Omega = \bigcup_{d \in \mathcal{D}} d$ . Informally, this means that making decision  $d$  implies that we then live in a world where event  $d$  happens. This ensures that for example conditional probabilities such as

$$p_D(\omega|d) = \frac{p_D(\omega \cap d)}{p_D(d)}. \quad (2.1)$$

are defined when  $x$  is an outcome of a random variable and  $d$  is a decision alternative. We use  $P_D$  to denote the beliefs specific to decision-maker  $D$  when the distinction is necessary.

From the perspective of decision-maker  $D$ , there is normally no uncertainty about the outcome, so  $P_D(d) = 1$  if they choose the decision alternative  $d$ . However,  $P_A(d)$  may not be certain from the perspective of another decision-maker  $A$ , if they are unable to directly observe the decision. Defining the decision alternatives as events in the probability space means that all decision-makers use the same  $\Omega$  and  $\mathcal{F}$  and only different  $P$ . It also provides an easy way to define mixed decision strategies if necessary.

## 2.2 Decision theory

The basic decision problem examined in this dissertation is the expected utility maximization for a rational decision-maker  $D$

$$\max_{d \in \mathcal{D}} \sum_{\omega \in \Omega} p_D(\omega|d) u_D(\omega), \quad (2.2)$$

where decision set  $\mathcal{D}$  contains all of  $D$ 's decision alternatives.  $D$ 's probability estimate of  $\omega$  given decision  $d$  is denoted with  $p_D(\omega|d)$ .  $D$ 's utility is represented with a VNM utility function  $u_D : \Omega \rightarrow \mathbb{R}$ .

Real-life problems, however, are not always modeled with just a single conditional probability distribution, but often involve multiple decisions and other random variables (Raiffa, 1968). The utility function can also be expressed as a function of random variable outcomes instead of sample space  $\Omega$  to better tie it to the problem structure (French, 1986), resulting in a utility function of form

$$u(\omega) = f(X_1(\omega), X_2(\omega), \dots, X_N(\omega)). \quad (2.3)$$

In fact, this is often preferable in practice because it is prohibitively difficult to establish a utility function over a very large  $\Omega$  otherwise.

## 2.3 Games

Decision problems with multiple decision-makers whose interests do not align are often modeled as games. In this context, a game is a collection of rules that describes the decision-makers' decision alternatives, available information, and random variables (Myerson, 1997). Incorporating another decision-maker changes the decision-maker  $D$ 's expected utility to

$$\mathbb{E}[U_D] = \sum_{a \in \mathcal{A}} \sum_{\omega \in \Omega} p_D(\omega|a, d) p_D(a|d) u_D(\omega), \quad (2.4)$$

where  $a \in \mathcal{A}$  represents the decision made by the other decision-maker, henceforth referred to as Adversary or  $A$ .

Finding the best decision alternative  $d$  now requires determining how  $D$  believes  $A$  will react to the changing environment as represented by  $p_D(a|d)$ . Without detailing the game, very little can be said about  $p_D(a|d)$ , because it depends on the information the decision-makers act on. In adversarial risk analysis, the problem is solved by assigning probabilities to the possible Adversary types represented by pair  $T_A = (u_A, p_A)$ , corresponding to  $A$ 's utility function and beliefs about probabilities respectively (Banks et al., 2015).

Whilst it would be technically possible to include the Adversary's type in the same probability space as the decision problem, for the sake of simplicity its probability space will be denoted here with  $(\mathcal{T}_A, F^T, P_D^T)$ , where the sample space  $\mathcal{T}_A$  is the set of possible Adversary types  $T_A$ . The probabilities are denoted with  $P_D^T$  to emphasize that these are  $D$ 's subjective beliefs about the Adversary type.

Assuming that the Adversary is also a rational decision-maker, their decision can now be determined for each possible Adversary type

$$a(T_A) = \operatorname{argmax}_{a \in \mathcal{A}} \sum_{\omega \in \Omega} p_A(\omega|a) u_A(\omega). \quad (2.5)$$

Here, it is assumed that the Adversary's decision  $a(T_A)$  has a unique solution, but that is not always true. Analyzing the Adversary's decision problem may produce one or multiple optimal decisions  $a$ , or there may not be an optimum at all if set  $\mathcal{A}$  is not finite.

The original decision problem can now be solved as

$$\max_{d \in \mathcal{D}} \sum_{T_A \in \mathcal{T}_A} p_D^T(T_A) \sum_{\omega \in \Omega} p_D(\omega|a(T_A), d) u_D(\omega), \quad (2.6)$$

assuming that mixed (randomized) decision strategies are disallowed. Incorporating mixed strategies would change the decision alternatives of all decision-makers into probability distributions over specific actions  $d$  and  $a$ , but otherwise, the problem would remain similar.

In practice however, it is difficult to assign well-founded probability distributions for  $p_D^T(T_A)$ ,  $p_D(\omega|a(T_A), d)$  and especially  $p_A(\omega|a)$ , because a



rational Adversary should be expected to perform an analysis of their own, effectively mirroring what is done in (2.5) and (2.6). This translates the problem into a Bayesian game (Harsanyi, 1967), which cannot be solved without exploring the information upon which the decisions are based on. Thus, complex games are often not studied purely algebraically, but also incorporate graphical models that show what information is available at each stage of the decision process (Myerson, 1997).

## 2.4 Graphical models

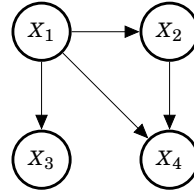
Visual models are an integral part of analyzing complex decision problems. They provide an easy-to-read representation of the information and interdependence structure of the decision problem and are easier to construct and interpret than purely algebraic representations. Some of the simplest and most widely used visual representations are decision trees (see, for example French, 1986; Raiffa, 1968), and in the case of multiple decision-maker systems, game trees (Myerson, 1997), but they grow in size exponentially as the problem complexity increases. Here we opt to use graph-based models instead that provide largely the same information as decision trees but do not grow impractically large as quickly.

These networks use directed acyclic graphs to represent dependencies between random variables. The random variables are often chosen corresponding to some physical systems or easily observable system outputs. In more abstract problems, like those that concern warfare, counter-terrorism, or foresight, the random variables will be less concrete, but they are still chosen in a way that supports analyzing the decision problem at hand (Howard and Abbas, 2016).

### 2.4.1 Bayesian networks

A Bayesian network (Pearl, 1985) consists of a graph  $G = (\mathcal{V}, E)$  that is a pair consisting of nodes (vertices)  $\mathcal{V}$  that correspond to random variables and edges  $E$  that describe the conditional dependencies between the variables. With a slight abuse of notation, we will use  $X_i$  to denote both the random variables and the associated nodes of the network, so we write  $\mathcal{V} = \{X_1, X_2, \dots, X_N\}$ . The set of edges  $E$  consists of ordered pairs of distinct nodes  $E \subseteq \{(X, Y) | (X, Y) \in \mathcal{V}^2 \text{ and } X \neq Y\}$ . Because we only discuss directed (simple) graphs, any mentions of edges refer to directed edges, and the existence of  $(X, Y) \in E$  implies  $(Y, X) \notin E$ .

Figure 2.1 shows a simple Bayesian network. The circles represent nodes and the connecting arrows represent edges. The edges indicate probabilistic dependencies between the random variables, i.e. an edge from node  $X_1$  to node  $X_2$  implies that the conditional probability  $P(X_2|X_1)$



**Figure 2.1.** A Bayesian network.

differs in some way from probability  $P(X_2)$  for some events in  $\mathcal{X}_1$  and  $\mathcal{X}_2$ . Conversely, the lack of a connecting edge implies conditional independence, i.e. the conditional probability distributions of the two random variables conditioned on each of their respective incoming edges are independent. The conditional independence relation is context-specific and depends on which random variables are included in the network and also on the direction of the edges.

The probability information associated with a Bayesian network is expressed as conditional probabilities. The probability of every outcome is conditioned on other random variables connected by an incoming edge. For example, in the Bayesian network from Figure 2.1 random variable  $X_2$  would have its probability distribution encoded as  $P(X_2|X_1)$ . The outcomes of  $X_3$  are not included, because it does not share an edge with  $X_2$ , and neither are the outcomes of  $X_4$ , because the edge between the two nodes is directed from  $X_2$  to  $X_4$ .

Following from the law of total probability and the definition of conditional independence, these conditional probabilities are sufficient to specify the probability of any combination of outcomes  $(x_1, x_2, \dots, x_N)$ . Using the example from Figure 2.1 again, we get

$$P(x_1, x_2, x_3, x_4) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2)P(x_4|x_1, x_2, x_3) \quad (2.7)$$

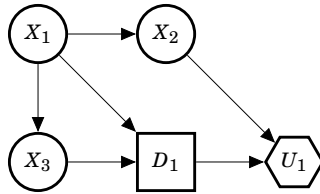
$$= P(x_1)P(x_2|x_1)P(x_3|x_1)P(x_4|x_1, x_2). \quad (2.8)$$

From the computational perspective, these conditional probabilities are also convenient because they require storing far less information than probabilities of outcome combinations separately, assuming the network is sparse enough.

#### 2.4.2 Influence diagrams

Influence diagrams (Howard and Matheson, 2005) can be treated as an extension of Bayesian networks despite predating them conceptually (Howard and Matheson, 1984; Verma and Pearl, 1988; Pearl, 2005). Like a Bayesian network, an influence diagram also consists of a graph  $G = (\mathcal{V}, E)$ , but unlike in a Bayesian network some nodes of the graph represent decisions  $\mathcal{V}_D \subset \mathcal{V}$  and decision-maker utility  $\mathcal{V}_U \subset \mathcal{V}$ . In other words, an influence diagram with neither decision nor utility nodes is just a Bayesian network

(Kjaerulff and Madsen, 2008).



**Figure 2.2.** An influence diagram.

Figure 2.2 depicts an example influence diagram. The circles represent random variables as they do in Bayesian networks, the squares represent decisions, and the hexagons represent utility to the decision-maker. The incoming edges to decisions indicate that the outcomes of these connected nodes are known at the time of the decision, whilst incoming edges to utility nodes indicate which decision and random variable outcomes are used to calculate the utility of the decision-maker.

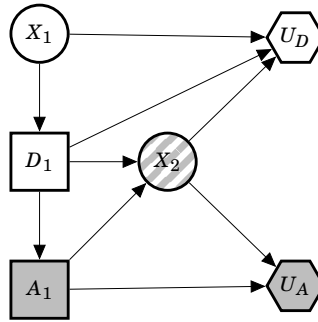
The graphical representation is accompanied by a description detailing if the incoming edges affect the decisions beyond providing information, for example, if they limit the available decision alternatives in some way. Typically, with influence diagrams, it is assumed that all the information the decision-maker had access to during earlier decisions, as well as the decision outcomes, are known when making later decisions (Shachter, 1986; Tatman and Shachter, 1990), but this assumption can be expressly omitted in some cases (Mauá et al., 2012; Kjaerulff and Madsen, 2008).

Unlike random variables and decisions, utility nodes do not involve any uncertainty. They are defined as VNM-utility functions over random variables and decisions that are connected to the utility node. Typically, an influence diagram will have exactly one utility node, but sometimes multiple are used to represent separable components of the utility function (Tatman and Shachter, 1990).

### 2.4.3 Multi-agent influence diagrams

Multi-agent influence diagrams (MAIDs) are influence diagrams that can be used to represent games by including decisions, uncertainties, and utilities of multiple decision-makers in a single graph (Koller and Milch, 2003). This provides a more compact visual representation of a game than a game tree would, whilst still making it possible to denote the order of decisions and uncertain events as well as the flow of information.

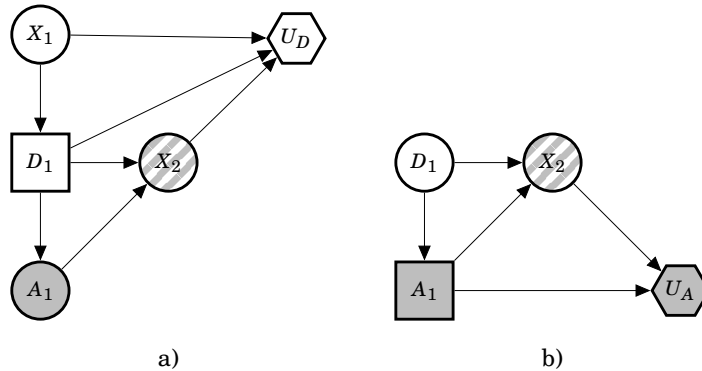
In the MAID in Figure 2.3 colors are used to differentiate between agents. Here, decision-maker D's decisions as well as utilities and uncertainties only relevant to them are colored white, whereas vertices associated with Adversary A are colored gray. Random variable  $X_2$  affects the utility of both D and A, so it is colored with white and gray stripes. Random variable



**Figure 2.3.** A multi-agent influence diagram.

$X_1$  on the other hand is entirely irrelevant to  $A$ 's decision-making, so it is colored white.

Sometimes when using MAIDs, coloring the random variables is omitted because the same relevant information can be deduced from the network structure and the accompanying descriptions of uncertainties, decisions, and utilities. However, coloring all nodes helps separate the decision problems of different actors. Figure 2.4 shows the decision problems of the two agents separately. Producing these influence diagrams is very straightforward when the original MAID is colored (Ortega et al., 2019). The other agent's decisions are replaced with random variables, and the random variables and the utilities associated only with other agents are removed.



**Figure 2.4.** The decision-maker's problem a) and the adversary's problem b) as separate influence diagrams.

## 2.5 Cross-impact analysis

Cross-impact analysis (CIA) encompasses several methods built on the ideas presented in the seminal work of Theodore Gordon and Olaf Helmer

in the 1960s (Gordon, 1994). CIA methods are crafted for the purpose of examining and characterizing interdependencies that exist between random variables, also referred to as uncertainty factors. By having experts rate these dependencies' magnitudes and directions on a numerical scale, it is then possible to draw conclusions about their joint probabilities.

Whilst there are almost as many ways of measuring cross-impacts as there are cross-impact methods (see e.g. Alter, 1979; Amer et al., 2013; Bishop et al., 2007), this dissertation exclusively uses the definition first presented in Publication III, referred to as the cross-impact multiplier definition. Cross-impact multiplier for outcomes  $x_1$  and  $x_2$  of random variables  $X_1$  and  $X_2$  respectively is defined as

$$C_{x_1x_2} = \frac{P(x_1 \cap x_2)}{P(x_1)P(x_2)}. \quad (2.9)$$

It is called the cross-impact multiplier, because it describes the relative change in probability of outcome  $x_1$  when  $x_2$  is known to happen compared to when nothing is known about  $x_2$ . This is because

$$C_{x_1x_2} = \frac{P(x_1 \cap x_2)}{P(x_1)P(x_2)} = \frac{P(x_1|x_2)}{P(x_1)}. \quad (2.10)$$

Compared to other cross-impact approaches, the cross-impact multipliers have some distinct advantages.

- i They facilitate estimating numerical probabilities and are thus compatible with risk and decision analysis methods as well as graphical probability models.
- ii They are symmetric by design, i.e.  $C_{x_1x_2} = C_{x_2x_1}$  as seen from (2.9), which means that dependencies do not need to be evaluated twice for every outcome pair.
- iii They avoid interacting directly with conditional probabilities, which can be difficult for non-experts to estimate (Pollatsek et al., 1987).

## 3. Research Contributions

This dissertation presents new methods for supporting decision-making under uncertainty, especially in problems relating to safety and security. Publications I-II focus on using adversarial risk analysis (ARA) to model the decision processes of adversarial actors. Publications III-IV develop probabilistic cross-impact methods to support risk evaluation and probability estimation based on expert judgments. The contributions are summarized in Table 3.1.

### 3.1 Publication I

ARA combines methods of statistical risk analysis and game theory to help evaluate risks and choose countermeasures against threats posed by intelligent and potentially malignant actors. Many of the earliest ARA applications have focused on counterterrorism. However, despite the long history of game theory and computational models in the military, ARA has not been widely applied in combat modeling (at least publicly). Publication I identifies ways to combine ARA methods with existing combat modeling tools to broaden the scope of analyses that can be performed. Specifically, Publication I discusses how ARA could serve to combine results from low-level simulations to form a picture of how the success of individual units could affect the wider conflict. The publication also offers a simple example of how ARA can be used to evaluate the value of information and the importance of operational secrecy.

### 3.2 Publication II

Evaluating the rationale of other decision-makers poses a persistent challenge in applying ARA and other methods based on game theory to real-world problems. Evaluating the utility function of a cooperative party is challenging, but finding reliable information about the beliefs and pref-

**Table 3.1.** Summary of publications.

Publication	Objectives	Methodology	Results
I	To explore and demonstrate the potential of ARA in military applications.	Adversarial Risk Analysis, Combat Modeling, Simulation	ARA can be combined with existing combat models to analyze questions that would be outside the original model's scope, including decision problems, impacts on the conflict on a larger scale, and the value of hidden information.
II	To develop new ARA methods to analyze problems in which probabilities about the adversary or some other aspect of the system are not known.	Adversarial Risk Analysis, Stochastic Dominance, Combat Modeling, Simulation	The developed method enables solving all ARA problems represented by regular influence diagrams using stochastic dominance and decision rules when exact utility functions or probability distributions are not available. A case study demonstrates the use of the developed method for military planning.
III	To provide a cross-impact interpretation founded on probability theory for use with risk analysis.	Cross-impact Analysis, Risk Analysis, Quadratic Programming	A new definition for probabilistic cross-impacts founded on probability theory is presented. Applicability to risk analysis is demonstrated with an optimization method and a case study.
IV	To develop a method for computing scenario probability distribution based on cross-impact information.	Cross-impact analysis, Scenario Analysis, Least Squares Approximation	A new optimization method, which utilizes cross-impact information to compute joint probability distributions for random variables. A Bayesian network can also be constructed based on the computed probabilities. Applicability to real-world problems is demonstrated with a case study.

erences of adversaries can be near impossible. To avoid having to make unrealistically precise predictions about adversaries' thought processes, Publication II develops methods for characterizing their likely actions based on more general assumptions. Publication II shows how the concepts of partial information, stochastic dominance, and decision rules can be used instead of some or all of the probability distributions and utility functions to analyze adversarial risks. The contributions are demonstrated with a realistic case study about choosing and deploying countermeasures to unmanned aerial vehicles.

### **3.3 Publication III**

Risk analysis of complex systems calls for the identification, characterization, and analysis of numerous possible future events and developments that may negatively impact the system. The task is further complicated by the fact that these uncertainties can also depend on one another. However, looking individually at every possible scenario that can be formed as a combination of these outcomes quickly becomes infeasible when the number of random variables increases. Cross-impact analysis offers a tool for estimating how the perceived probabilities of random variables change based on the outcomes of others. Publication III offers a cross-impacts definition that is founded on probability theory and admits several kinds of probabilistic statements about dependencies between the uncertainty factors. The publication also describes how the statements can be transformed into optimization constraints and used to calculate upper and lower bounds for the overall risk level of the system. The approach is illustrated with an example case about the risk analysis of nuclear waste repositories.

### **3.4 Publication IV**

Estimating the probability distributions of the different random variables is one of the main challenges in producing probabilistic forecasts. Simulation models, such as the ones used in weather forecasting and military combat modeling, can be used to quantify future uncertainties governed by chance. However, modeling uncertainty stemming for example from human activity with simulations is often not feasible, and creating detailed simulation models is challenging and time-consuming. Thus, eliciting experts for their estimations about future uncertainties is often the only feasible approach. Still, eliciting information about systems with multiple interdependent random variables poses a challenge, because when the number of variables increases the number of possible interactions with them grows exponentially. Cross-impact methods manage this complexity



by focusing only on the pairwise impacts between two random variables. Publication IV presents a new optimization approach that can synthesize these pairwise cross-impact statements to produce a joint probability distribution for the random variables. When combined with conditional independence information, the calculated probabilities can also be used to construct a Bayesian network to aid what-if analyses. The Publication also includes a case study focusing on the future of 3D-printing in military use.

## 4. Discussion

This dissertation develops new methods to account for uncertainties in support of decision-making in adversarial risk analysis (ARA) and probabilistic scenario analysis. Publications I and II focus on ARA methodology to quantify uncertainties caused by adversarial decision-makers with competing interests. Publications III and IV, on the other hand, present novel approaches for using cross-impact analysis (CIA) to quantify uncertainties associated with future events.

Despite having been developed relatively recently, ARA has already found numerous applications in counter-terrorism and cyber security. Still, much of the military combat modeling research does not use ARA or any other game-theoretic models for adversarial activity (Washburn et al., 2009), despite the great impact that adversaries' actions have on the effectiveness of tactics and weapon systems. As discussed in Publication I, ARA methods can be used to expand the range of analyses that can be performed using pre-existing combat modeling tools, incorporating small-scale encounters as a part of the bigger picture and evaluating the value of secrecy and information.

The shared common knowledge assumptions required by many game-theoretical models have been problematic in attempts to adapt game theory to combat modeling. Bayesian Nash equilibrium developed by Harsanyi (1967) provides the necessary tools for finding robust solutions for facing different types of adversaries, but coming up with a probability distribution over *adversary types* (representing their utility functions and available information) can be onerous. In Publication II, we show that even simple assumptions about adversaries (such as wanting to minimize casualties) can serve as a foundation for a game-theoretic analysis when interpreted as partial preference order relations over outcomes. Whilst this type of analysis cannot be used to predict the adversaries' actions precisely, some of their decision alternatives can be excluded as irrational. Thus, it is possible to find risk mitigation strategies that work against rational adversaries, even if the adversary's precise type or type's probability is not known.

Compared to ARA, CIA approaches quantifying uncertainty very differently. Cross-impact information elicited from experts describes how the likelihood of specific outcomes changes when an outcome of another random variable is known. There exist cross-impact methodologies that differ in almost everything except that basic idea (Alter, 1979; Amer et al., 2013; Bishop et al., 2007). Most of the earliest methods worked similarly to Monte Carlo simulation, drawing random events from the possible list of outcomes and then adjusting the probabilities of remaining outcomes based on the associated cross-impacts (e.g. Gordon, 1994; Dalkey, 1971; Helmer, 1981). More recently, several CIA methods have been developed that eschew probabilities entirely, and only measure the likelihood of outcomes appearing together in terms of how consistent their cross-impacts are (e.g. Weimer-Jehle, 2006; Seeve and Vilkkumaa, 2022). Whilst both of these approaches have their uses in exploring the future, developing scenarios, and fostering managerial thinking, they are not very compatible with either risk or decision analysis.

To facilitate risk analysis based on cross-impacts, Publication III presents a new cross-impact interpretation founded on probability theory. Called cross-impact multipliers, this new cross-impact interpretation, together with information about marginal probabilities of the associated random variables, can be used to determine upper and lower bounds for system risk. Thus it is useful, for example, in demonstrating compliance with regulatory risk bounds as well as in comparing different risk mitigation alternatives.

Whilst the primary focus in Publication III is on risk analysis, Publication IV takes the same cross-impact definition and presents methods for calculating a joint probability distribution for different scenarios formed as combinations of outcomes of random variables. It is also demonstrated, how the computed probabilities together with conditional independence information can be used to construct Bayesian networks, offering a useful tool for what-if type analyses.

This dissertation opens up new research directions as well. First, the presented methods could be tested with more empirical case studies. It would be interesting to try how compatible ARA and CIA are together. ARA is often the preferred method for modeling uncertainty from human activity, and CIA is good for estimating long-term technological and other developments. The two together could be used to analyze safety and security problems with long time horizons, such as investments into new weapon systems or the design of long-term nuclear waste repositories.

There is room for new methodological extensions as well. Expanding further on the methods presented in Publication II, it would be interesting to explore how the partial information approach could be expanded to also include non-sequential games, i.e. games with decisions whose outcomes are not observable before the next decision of the adversary. Although it is

possible that the increased uncertainty would make it impossible to draw any useful conclusions about these games (Fishburn, 1978). Another potential research topic would be examining how different types of ambiguous preference models, such as the ones used by Danielson et al. (2014) or Salo and Punkka (2005), could be applied in ARA.

Expanding on the CIA methods presented in this dissertation, some work has already been done in using the computed probabilities or risks to choose scenarios for more detailed examination (Elfving, 2023). This way probabilistic and narrative scenario methods could be used together to combine some of the best aspects of both traditions. The analytical models help contextualize the scenarios and the narrative approaches provide depth and approachability. Selecting the right scenarios to focus on is also important in many modeling or simulation studies and offers another potential avenue for future research.



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