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## Publication II

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## MEASUREMENT OF APERTURE AREAS USING AN OPTICAL COORDINATE MEASURING MACHINE

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*The use of an optical coordinate measuring machine (CMM) for the diameter measurement of optical apertures is described. The traceability and mechanical stability of the aperture areas are of importance for accurate photometric and radiometric measurements. Detailed evaluation of the measurement uncertainty for the aperture diameter is presented. High-accuracy mechanical CMM was used to confirm the validity of the optical CMM results. The difference between the contact and non-contact measurement was 0.1 μm for the mean diameter result. If the required standard uncertainty for the mean diameter is of the order of 1 μm, the optical CMM provides an efficient method for aperture area measurements.*

### 1. INTRODUCTION

Apertures are used in optical radiometry\* to define a precisely known area of an incoming radiation field in front of a detector. When the detector has a calibrated optical power responsivity, the known aperture area allows to determine such quantities as illuminance or irradiance which describe optical power divided by area. An ideal aperture would have zero thickness in order to avoid shadowing of light rays entering in other than exactly perpendicular direction to the aperture plane. Reliable area measurements of apertures with thin edges are especially important to primary scale realizations, as otherwise it is not possible to get access to many essential radiometric quantities.

The area of a nominally round aperture can be measured via determination of its effective diameter. A straightforward method for diameter measurement is to determine in different directions the largest distance between the edges of the aperture. The edges can be observed either by a microscope in an optical coordinate measuring machine (CMM) or by a physical contact in a mechanical CMM. The main advantage of the non-contact optical CMM method is that it does not damage the thin aperture edge. Furthermore, the measurement setup and alignment can be made

\*Optical radiometry is the field of science which studies the measurement of electromagnetic radiation, including visible light. Light is also measured using the techniques of photometry that deal with brightness as perceived by the human eye.

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## NOMENCLATURE

$D$	diameter	$\delta e$	error for measurement of $D$ due to optical parameters and edge sharpness
$\bar{D}$	mean diameter	$\delta r$	error for measurement of $r$ due to edge roundness of aperture
$k$	coverage factor	$\delta x$	repeatability error for measurement of $x$ -coordinate
$i, j, m$	index variables	$\delta y$	repeatability error for measurement of $y$ -coordinate
$n$	number of measured points		
$R, r$	Radius		
$x, y$	Cartesian coordinate		

relatively simple and it is easy to get a large number of diameter values in different directions. For mechanical contact measurements, utmost care is needed to protect the thin aperture edge and to achieve correct alignment of the aperture plane relative to the probe and probe motion. As a whole the contact measurement, per aperture diameter value, takes considerably longer time than the non-contact measurement. However, the measurement uncertainty of the contact method can be lower than that of the non-contact method since in the latter case uncertainty is limited by the difficulty in reliable determination of the aperture edge position in the microscope image.

Several contact (Martin et al. 1998) and non-contact (Fowler et al. 2000; Fowler et al. 1998; Lassila et al. 1997; Ikonen et al. 1998; Stock and Goebel 2000; Razet and Bastie 2006; Hartmann et al. 2000; Fowler and Litorja 2003) methods have been used for measurement of aperture areas. Some non-contact methods (Lassila et al. 1997; Ikonen et al. 1998; Stock and Goebel 2000) can measure the area directly and are thus not sensitive to the shape of the apertures, but still require long measurement time. The reported relative standard measurement uncertainties are typically  $10^{-4}$  or less. However, even the best measurements of illuminance responsivity (Köhler et al. 2004) and spectral irradiance (Woolliams et al. 2006) have relative standard uncertainties larger than  $10^{-3}$ . Therefore, in some cases an increase in measurement uncertainty for area is acceptable to improve the speed and efficiency of the aperture area measurements. Such an efficiency improvement is especially important for a method (Kubarsepp et al. 2000) where more than ten separate detectors (filter radiometers) are used to realize the spectral irradiance scale, as each of these filter radiometers would need a dedicated 3-mm-diameter aperture with known area. Another need for straightforward aperture area measurement comes from the study of mechanical stability of aperture diameter over time scales of several months after the drilling. The non-contact optical CMM method is a good candidate for an efficient aperture area measurement. However, if the aperture area uncertainty starts to approach other uncertainty components in the spectral irradiance uncertainty budget, the need for a reliable uncertainty evaluation of aperture diameter measurement with the optical CMM is emphasized.

During the last 10 years, coordinate measurement machines fitted with CCD cameras and machine vision software have been developed. These optical CMM's are nowadays used widely especially in the electronic industry, because of their ability for fast automated and accurate non-contact measurements. Typical claimed

accuracies for optical CMM's range from  $0.8\ \mu\text{m}$  to  $6\ \mu\text{m}$ . (Lazzari et al. 2004; Kiviö et al. 2004). These accuracies apply for one length measurement only, but with these machines complicated measurements, for example flatness, roundness, cylindricity, coaxiality, etc., are often carried out without knowledge of the task specified uncertainty. Moreover, the uncertainties written on manufacturers brochures are quite seldom realistic and therefore the apparent estimated uncertainty of measurements may be far too small (Kiviö et al. 2004).

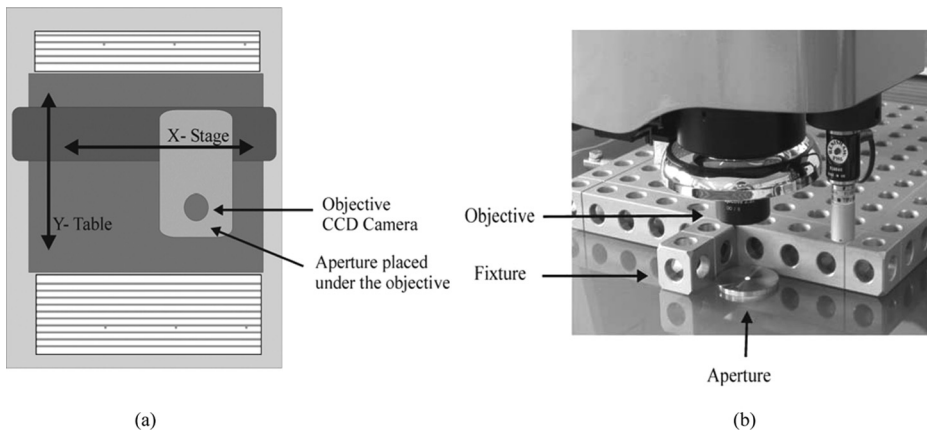
Calculation of the measurement uncertainty for a real measurement task with a CMM is considered to be demanding mainly because there is a large number of uncertainty sources and error components. Also several measurement strategies can be used for the same task. Finally, the measurement task itself may include complex geometries or fitting algorithms (Chan et al. 1996). Additional practical difficulty comes from the object to be measured with optical CMM: depending on the selected light source and magnification, on the edge detection parameters, and on the edge itself, different results may be the outcome of measurements.

In this work, the use of optical CMM is studied as a candidate for an efficient aperture area measurement method. A detailed uncertainty analysis of optical CMM measurements is presented for the first time with uncertainty budgets for diameter measurements to help for a better understanding of the measurement method. Especially, the measurement geometry and measurement strategy together with the effects of the quality of the edges are explained. A specific purpose of the measurements was to study the stability of diameters of aluminium apertures over a time scale of six months after the drilling. After drilling the diameter might change, for example due to oxidation or stresses in the material. The measured diameters showed only minor changes comparable with the uncertainties in the measurements. The standard deviation for the measured diameter for one aperture was 35% or less of the standard uncertainty of the measurement. Measurements with a high-accuracy mechanical CMM were used to confirm the validity of the optical CMM results. The optical method was found to provide a fast way for aperture area measurements at an uncertainty level which is sufficient for most practical applications in photometry and radiometry and satisfies the needs of even the most accurate spectral irradiance measurements.

This article is organized as follows. Section 2 describes the measurement setup and Section 3 the measurement results. To confirm these non-contact measurements, measurements with contacting probe are made and described in Section 4. Before any conclusions can be drawn on the stability of the apertures or the suitability of the measurement method, an uncertainty budget is needed as described in Section 5. In Section 6, conclusions on these two topics are presented.

## 2. OPTICAL APERTURE MEASUREMENTS

The optical diameter measurements were conducted using Mitutoyo Quickvision Hyper CMM at the Centre for Metrology and Accreditation (MIKES) shown in Figure 1. Quickvision CMM is equipped with crystallized glass scale with the resolution of  $0.02\ \mu\text{m}$  and thermal expansion coefficient of  $0.08 \times 10^{-6}/\text{K}$ . The best specified measurement uncertainty in one axis is  $0.8\ \mu\text{m}$  (Mitutoyo 2001). As shown in Figure 1, the object to be measured is placed on the motorized Y-table and the



**Figure 1.** A CMM used for the measurement of optical apertures. *a)* Schematic diagram and *b)* measurement of apertures positioned using a fixture.

camera is moved using an X-stage and a vertical Z-stage. The instrument can be used with coaxial light and stage light. Stage light, also called contour illumination, uses a light source beneath the glass-made measuring table, and is useful when the object is too reflective for coaxial light. The apertures have been made of aluminium with a nominal diameter of 3 mm. Properties of the apertures are shown in Table 1. To avoid a situation where both the new apertures and the CMM would be unstable, old, presumably stable apertures were also measured. The new apertures were machined by conventional turning some days before the first measurement. If the aperture dimensions would be unstable immediately after turning, the difference in the behaviour of new and old apertures would reveal this phenomenon.

A measurement program was made, where 120 points at the circumference of the apertures were measured and combined to a circle using the least-squares criteria. During the measurement X-Y-movements are made and a small part of the edge of the aperture is seen by the camera. Using the point measurement tool in the

**Table 1.** Studied apertures and illumination used in the measurements. The percent values indicate the fraction of full light intensity of the Quickvision Hyper

Aperture	Age	Coating	Illumination (%)
HUT-1	new	anodized	Coaxial 80
HUT-2	old	anodized	Coaxial 80
HUT-3	new	anodized	Coaxial 80
HUT-4	new	none	Stage 30
HUT-5	new	anodized	Coaxial 80
HUT-6	old	none	Stage 30
HUT-7	old	anodized	Coaxial 80
HUT-8	new	none	Stage 30
HUT-9	new	none	Stage 30
HUT-10	old	none	Stage 30

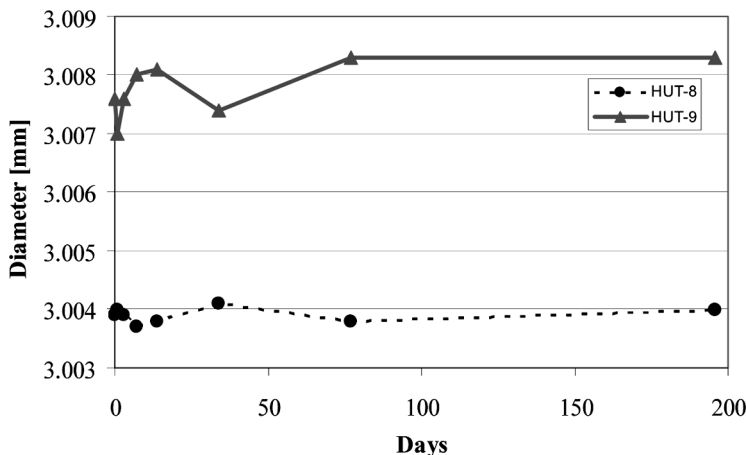
**Table 2.** Results for mean diameters (in mm) of ten apertures for the period of six months

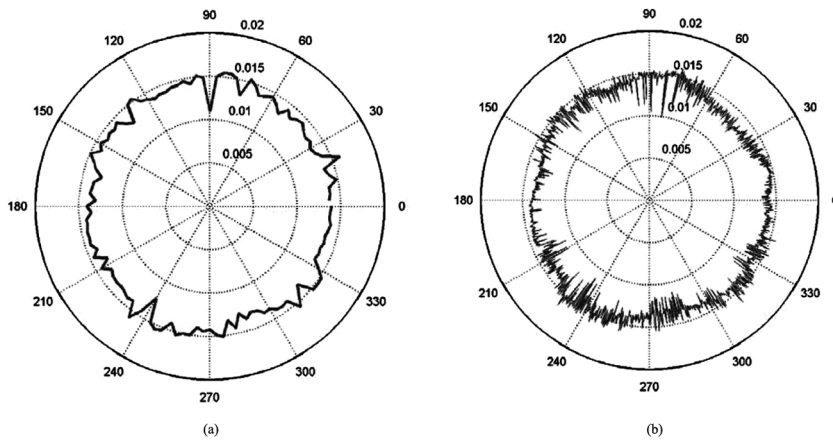
Day	HUT-1	HUT-2	HUT-3	HUT-4	HUT-5	HUT-6	HUT-7	HUT-8	HUT-9	HUT-10
0	3.0082	3.0387	3.0063	3.0084	3.0025	3.0016	3.0336	3.0039	3.0076	2.9992
1	3.0085	3.0387	3.0057	3.0083	3.0034	3.0018	3.0333	3.0040	3.0070	2.9992
3								3.0039	3.0076	
7								3.0037	3.0080	
14	3.0079	3.0389	3.0054	3.0082	3.0033	3.0003	3.0332	3.0038	3.0081	2.9993
34	3.0083	3.0388	3.0056	3.0085	3.0034	3.0019	3.0330	3.0041	3.0074	2.9989
77	3.0085	3.0387	3.0056	3.0085	3.0032	3.0018	3.0334	3.0038	3.0083	2.9994
196	3.0080	3.0388	3.0052	3.0083	3.0036	3.0016	3.0332	3.0040	3.0083	2.9996

Quickvision software, one point on the edge is measured. This sequence is repeated at equal angle steps until a circle is completed. In addition to the calculated diameter, an out-of-roundness estimate was obtained from the circle-fitting function of the QVPak program of the CMM. Some raw measurement data of coordinates ( $x,y$ ) were also saved for verification and plotting using Matlab (MathWorks, Inc.). During the measurements the temperature of the CMM varied from 19.3°C to 20.4°C due to the change of the ambient conditions. To ensure a good reproducibility and to minimize the influence of the systematic errors of the CMM, a fixture was used to position the apertures (Figure 1).

### 3. MEASUREMENT RESULTS

The apertures were named HUT-1, HUT-2... HUT-9. Results of the measurements are shown in Table 2. The variations in diameter for nine apertures were within 1.3  $\mu\text{m}$  and for one old aperture, HUT-6, the maximum variation was 1.6  $\mu\text{m}$ . It is difficult to see any trend indicating a systematic change in any of the apertures. Figure 2 shows the results graphically for apertures HUT-8 and HUT-9, which were randomly selected to be measured also on day 3 and day 7.

**Figure 2.** Measurement results of mean diameter for two apertures.



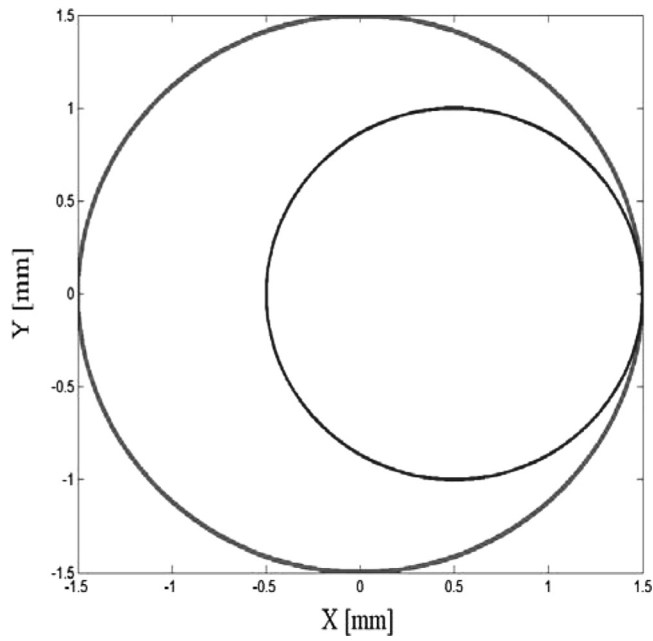
**Figure 3.** Roundness polar plot of aperture HUT-9 measured with 120 points: *a*) measured with 2000 points; *b*) dashed circles indicate 5  $\mu\text{m}$  scale grid in the polar plot.

In diameter measurements, the roundness plots are helpful when judging the quality of the data. From Figure 3, it can be seen that there are rather large local out-of-roundness variations in the measurement results. In Razet and Bastie (2006), this variation is referred to as edge scatter. The reason is probably actual geometrical form and roughness of the apertures combined with the effects of illumination. These variations should be included in the uncertainty analysis as reproducibility errors because the position of the aperture cannot be exactly the same in every measurement.

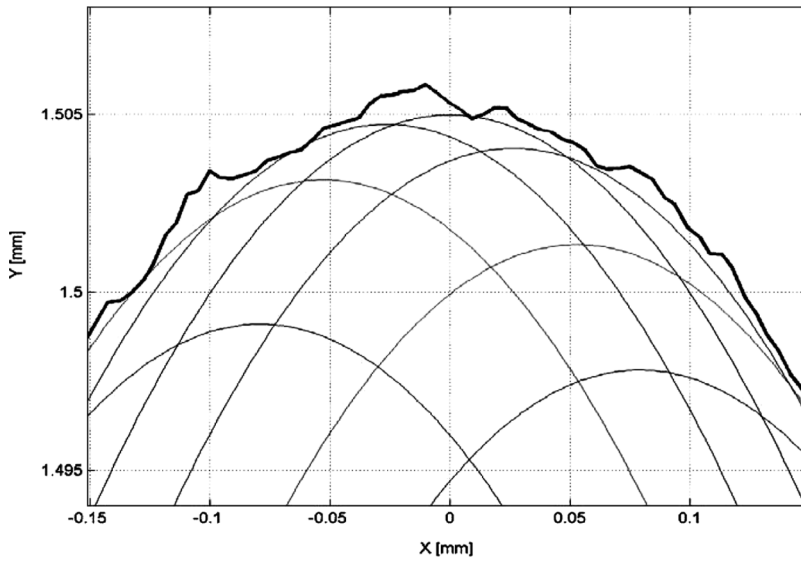
#### 4. COMPARISON TO THE RESULTS OBTAINED WITH MECHANICAL CMM

Verification measurements with a probing CMM Mitutoyo Legex were made for the aperture HUT-1. The nominal maximum permissible error (MPE<sub>e</sub> error) defined in ISO 10360-2 (for this high-accuracy CMM is  $(0.35 + L/1000) \mu\text{m}$ ) where  $L$  is the measured length in millimetres. The MPE<sub>e</sub> error is verified to be less than  $0.2 \mu\text{m}$  for measurement distances shorter than 100 mm using zerodur gauge blocks. The diameter of the probe was 2 mm and the measuring force was 0.045 N (Figure 4). Although the measurement force is quite small, the thin edges of optical apertures may be damaged by the mechanical measurement, and therefore this verification using probing CMM was done for only one aperture. The average measured diameter from 120 points was 3.0061 mm, as determined with mechanical CMM.

Because the probe is large compared to the roughness of the aperture, the measured diameter is decreased by the contact error. During the measurement of each point, the probe approaches the aperture in a radial movement starting approximately from the aperture center. The first contact between the probe and the aperture is registered as a measurement point. Because there are roughness peaks on the aperture, most measurement points will be registered from these peaks. Thus, the mechanical contact between the probe and aperture edge was simulated using optical



**Figure 4.** Basic geometry when measuring a 3-mm inner diameter using a 2-mm touch probe (scales in millimetres).



**Figure 5.** Simulation of the contact between the aperture and the probe at several positions. The center of the aperture is at (0, 0).



CMM profile data. A Matlab script was written to adjust a radial offset to the theoretical probe to match roughness peaks in a 2000-point aperture profile (Figure 5). The result of this simulation was an estimate of the contact error of  $1.1\ \mu\text{m}$  in radius. The surface roughness of the probe was measured with a form and surface roughness measuring instrument. The roughness value  $R_a$  (arithmetical mean roughness) is less than  $0.05\ \mu\text{m}$ , and therefore errors due to roughness effects of the probe are negligible.

In addition, a force correction should be applied to the mechanical CMM result. The force correction of  $-0.05\ \mu\text{m}$  (in radius) was calculated from formulas for elastic compression for the case of the sphere in contact with the internal cylinder (Puttock and Thwaite 1969). The depth of the cylindrical land in the aperture is about  $150\ \mu\text{m}$ . After applying the force correction and the contact error correction, the resulting diameter is  $3.0081\ \text{mm}$ . The mean value of the measurements, conducted with the optical CMM, was  $3.0082\ \text{mm}$  (Table 2), so there is good agreement between the results.

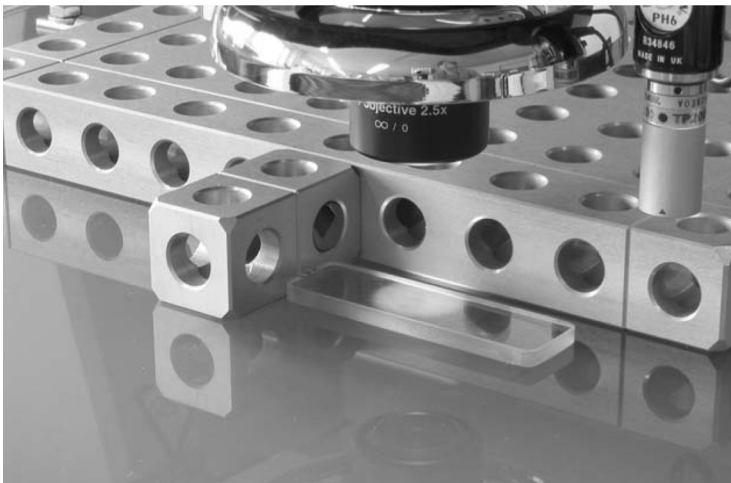
## 5. UNCERTAINTY ANALYSIS FOR MEASUREMENT OF MEAN DIAMETER

### 5.1. Auxiliary Measurement Results

The formal definition of uncertainty of a measurement is “parameter associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand” (GUM 2004). The compilation of the uncertainty budget requires additional studies of general stability of the instrument and repeatability and reproducibility of the measurements. In this article, especially where the stability of the apertures has been studied, the reproducibility of the measurements and the stability of the optical CMM are critical. As previously mentioned, a relative aperture area uncertainty of  $10^{-3}$  would be sufficient for most needs in photometry and radiometry. An aperture with a nominal diameter of  $3\ \text{mm}$  corresponds to a standard uncertainty of  $1\ \mu\text{m}$  for the mean diameter. In the following, an uncertainty analysis is made to show that the accuracy of the measurements of the mean diameter is sufficient for these needs. Also, to make reliable conclusions on the stability of the apertures a similar accuracy is needed.

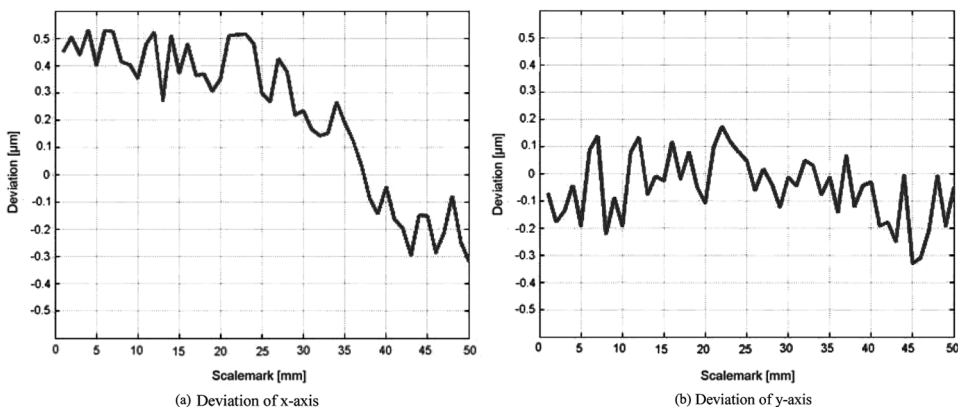
The best measurement capability for Quickvision Hyper optical CMM is  $0.8\ \mu\text{m}$  ( $k = 2$ ,  $k$  is the coverage factor with which the standard uncertainty is multiplied to get an uncertainty at 95 % confidence level) in one axis for a short distance measurement. There is also an additional length-dependent component of uncertainty, but it is negligible in our case because the measured dimension is small. This measurement capability has during recent years been verified by several calibrations and verification measurements of glass scales. It is assumed that the uncertainty consists mainly of the uncompensated error of the scale ( $0.6\ \mu\text{m}$ ,  $k = 2$ ) and of the repeatability error ( $0.5\ \mu\text{m}$ ,  $k = 2$ ). To ensure the accuracy in current measurements, additional verification measurements were performed using a 50-mm glass scale positioned close to the area where the apertures were measured (Figure 6).

The position of each scale mark on the glass scale is taken as the middle between the two edges of a scale line. The reference values for the glass scale were



**Figure 6.** Verification measurements using a reference glass scale in the same position as the aperture.

measured using the MIKES line-scale interferometer (Lassila et al. 1994). The uncertainties of the reference values are almost negligible in this case ( $0.09\ \mu\text{m}$ ,  $k = 2$ ) and by repeating the measurement of this glass scale at the optical CMM, the error of the CMM scale and repeatability error can be evaluated. Averaging a large number of measurements (point by point) shows a systematic contribution below  $\pm 0.6\ \mu\text{m}$  (Figure 7), and from these measurements the standard deviation for each point was  $0.2\ \mu\text{m}$ . The systematic contribution is assumed to come from the error of the CMM scale. These results are in agreement with the above mentioned division of the measurement uncertainty into a component of  $0.6\ \mu\text{m}$  for the CMM scale and  $0.5\ \mu\text{m}$  for the CMM repeatability (at coverage factor  $k = 2$ ).



**Figure 7.** Difference between the values measured with MIKES line scale interferometer and Mitutoyo Quickvision in the direction of the *a*) *x*-axis and *b*) *y*-axis.

## 5.2. The Simplified Measurement Model

When estimating measurement uncertainty according to the guidelines presented in GUM (1993), the first task is to write a formula for the measurement model. The basic measurement model for a simple two-point diameter ( $D$ ) measurement in the direction of the  $x$ -axis is

$$D = x_2 - x_1 + \delta x_1 + \delta x_2 + \delta r_1 + \delta r_2 + \delta e, \quad (1)$$

where  $x_1$  and  $x_2$  are the first and second measured coordinates,  $\delta x_1$  and  $\delta x_2$  are the repeatability errors,  $\delta r_1$  and  $\delta r_2$  are the errors from edges of the measured object (roundness errors), and  $\delta e$  is the edge detection error for diameter. The errors  $\delta r_1$  and  $\delta r_2$  are due to roundness errors in the horizontal plane of the aperture, and the edge detection error  $\delta e$  depends on user selected illumination, focus and sharpness of the edge. In one measurement, parameter  $D$  is calculated only from the estimates  $x_1$  and  $x_2$  but the measurement model contains also contributions from errors. The principle of two-point measurement is shown in Figure 8.

The measurements were actually made of 120 points and the resulting diameter can be interpreted as the mean  $\bar{D}$  of 60 diameter measurements. Therefore, the effect of the repeatability error ( $\delta x_1$  and  $\delta x_2$ ) and the roundness error, including roughness, ( $\delta r_1$  and  $\delta r_2$ ) should be decreased by the factor  $1/\sqrt{60}$ . The position of the found edge depends on user selected illumination, focus, and sharpness of the edge. The related error  $\delta e$  is of systematic type and cannot be reduced by averaging and thus the measurement model of repeated measurements is rewritten as

$$\bar{D} = \frac{1}{60} \sum_{m=1}^{60} [x_{2,m} - x_{1,m} + \delta x_{1,m} + \delta x_{2,m} + \delta r_{1,m} + \delta r_{2,m}] + \delta e. \quad (2)$$

The contribution of the uncertainty component  $\delta e$  was studied by changing illumination. It should be noted that this uncertainty component is much more complicated in the detection of the aperture edge than, for example, in the detection

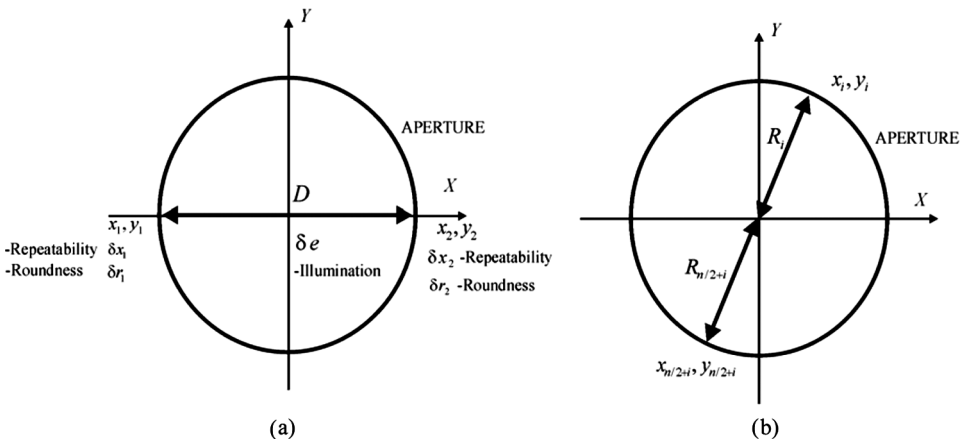


Figure 8. a) Two-point measurement of diameter and b) multi-point measurement of diameter.

of the scale mark distances on the reference glass scale. When using stage light, the aperture is between the camera and the light source. The aperture appears as dark area and at the edge there is a transition of illuminance level. When illumination is increased, the amount of light coming to the CCD camera increases, and finally the whole image would be saturated to bright signal. Before this happens, it can be seen that the transition range between dark and bright seems to move away from the bright area. This phenomenon increases the diameter of bright holes, and decreases the width of dark lines on glass scales. However, the distance between the centers of the scale marks remains unaffected when the position of the scale marks is taken as the center of the dark lines.

One matte aperture (HUT-1) and one bright aperture (HUT-9) were measured with a large range of combinations of coaxial and stage light. The standard deviation of obtained diameters was  $0.86 \mu\text{m}$  for the matte aperture and  $0.61 \mu\text{m}$  for the bright aperture. The results for the illumination experiment for the matte aperture are given in Table 3. It is seen that strong illumination (50% column for stage light) leads to roughly  $4 \mu\text{m}$  larger diameter than with moderate or zero stage light. However, the user also receives a “saturation warning” so these diameter results are excluded from the data when calculating the above mentioned standard deviations. The standard uncertainty related to  $\delta e$  is estimated to be  $1 \mu\text{m}$ , from Table 3. The value is conservative and rounded upwards to include the contribution of effects which can not be addressed by varying illumination conditions.

Next, the error of roundness type is dealt with. When the diameter measurement was immediately repeated, the variation in results was typically only  $0.1 \mu\text{m}$ . However, if the aperture was removed and measured again, with 120 points, in a slightly different position, the change in diameter was typically  $0.1 \mu\text{m}$ – $0.5 \mu\text{m}$ , indicating a considerable reproducibility error. The reason is seen in Figure 3; i.e., the edge appears to be rough and uneven. The roundness error was typically within  $\pm 4 \mu\text{m}$ . The related standard uncertainty, assuming a rectangular distribution, is  $\delta r_1 = \delta r_2 = (4 \mu\text{m})/\sqrt{3} = 2.31 \mu\text{m}$  (GUM 1993).

**Table 3.** Effects of illumination on the diameter of a matte aperture (HUT-1), in millimetres. The illumination selected for measurements of matte apertures was 80% for coaxial light and 0% for stage light

Coaxial light illumination magnitude [%]	Stage light illumination magnitude [%]				
	0	20	30	40	50
0		3.0071	3.0074	3.0084	3.0115
10					
20		3.0073	3.0075	3.0084	3.0115
30					
40					
50	3.0064	3.0076	3.0076	3.0087	3.0116
60	3.008				
70	3.0081				
80	3.0079	3.0079	3.0076	3.0096	3.0121
90	3.008				
100	3.008	3.008	3.0084	3.0105	3.0128

**Table 4.** Uncertainty budget for an averaged diameter measurement

	Uncertainty Component	Standard uncertainty [ $\mu\text{m}$ ]
$x_1$	Uncompensated error of CMM scales	0.30
$\delta x_1/\sqrt{60}$	Repeatability for one point	0.03
$\delta r_1/\sqrt{60}$	Local error of edge of measured object	0.30
$x_2$	Uncompensated error of CMM scales	0.30
$\delta x_2/\sqrt{60}$	Repeatability for one point	0.03
$\delta r_2/\sqrt{60}$	Local error of edge of measured object	0.30
$\delta e$	Edge detection	1.00
	Combined standard uncertainty ( $k = 1$ )	1.17
$D$	Expanded uncertainty ( $k = 2$ )	2.33

The uncertainty budget for the simplified model is shown in Table 4. The expanded measurement uncertainty for the average diameter is  $2.33 \mu\text{m}$  ( $k = 2$ ) and the corresponding relative standard uncertainty is  $8 \times 10^{-4}$  for a 3-mm-diameter aperture.

For analysis of the stability of the apertures, the uncertainty component  $\delta e$  is zero assuming no changes in ambient illumination or ageing of light bulbs in the optical CMM. It may also be assumed that the effect of the uncompensated systematic error of the CMM scales can be neglected in a reproducibility analysis. With these assumptions, the expanded diameter uncertainty of Table 4 is reduced to  $0.85 \mu\text{m}$  ( $k = 2$ ) for analysis of temporal drift of the aperture diameters. From data in Table 2, it can be calculated that the standard deviation of one measured aperture was on the average  $0.29 \mu\text{m}$ .

During the measurement series the variations of temperature were less than  $\pm 0.55 \text{ K}$ . The thermal expansion of the glass scale of the optical CMM is very low ( $0.08 \times 10^{-6}/\text{K}$ ) and the dimension of the aperture is small. Therefore, variation in the average temperature of the measurement is not significant and is not included in the uncertainty analysis. Another temperature-related error source is the rapid change of temperature of the CMM body structure during one measurement. In an experiment using a 2000-W heat-fan warming at one side of the CMM, the sensitivity of  $0.6 \mu\text{m}/\text{K}$  was found for the diameter measurement. During the measurements, the slow changes of temperature in the measurement laboratory can therefore be neglected.

### 5.3. Measurement Model for Actual Geometry and Monte-Carlo Simulations

The measurement model of Eq. (2) contains only  $x$ -axis components and is therefore not entirely correct. The next step is to rewrite the equation to match the actual measurement of  $n$  points giving  $n/2$  diameters (Figure 8) along the circumference.

$$D_i = R_i + R_{n/2+i} + \delta e, \quad i = 1 \dots n/2, \quad (3)$$

where

$$R_j = \sqrt{(x_j + \delta x_j)^2 + (y_j + \delta y_j)^2} + \delta r_j, \quad j = 1..n \quad (4)$$

Equation (3) can be interpreted as the sum of 60 pairs of opposite 120 radii giving 60 diameters.

The Monte-Carlo method is clearly useful in uncertainty analysis in metrology (Brizzard et al. 2005; Mudronja et al. 2003). Briefly, the idea is to simulate the measurement model  $M$  times using random numbers. According to the law of large numbers the distribution of the  $M$  outputs of the model converge to the actual distribution if the input distributions are reasonably correct. The systematic part of the uncompensated error of the CMM is simulated by a sine function with an amplitude of  $0.5 \mu\text{m}$ , and the repeatability of the CMM is simulated by a normal distribution. The edge detection and local error of the measured object were also simulated by normal distributions based on standard uncertainties described in the previous sections.

The measurement model was implemented in Matlab using random numbers from these distributions. The measurement software of the CMM uses least-squares circle fitting, and least-squares circle fitting was also included in the measurement model in Matlab. Finally, the standard deviation and the 95% confidence interval are calculated from the distribution.

The 95% coverage interval for diameter based on 100,000 simulations from the distribution of the simulated values is  $2.1 \mu\text{m}$  (Figure 9). This is slightly less than the result in Table 4 ( $2.33 \mu\text{m}$ ). A major advantage with the Monte-Carlo approach is the correct simulation of the measurement strategy, and now the number of measurement points  $n$  can be easily changed in the model. Table 5 shows that if the number of measurement points in the model is changed from 120 to 2 points, the expanded uncertainty from the simulation is  $6.7 \mu\text{m}$  which is close to the value  $6.6 \mu\text{m}$  obtained

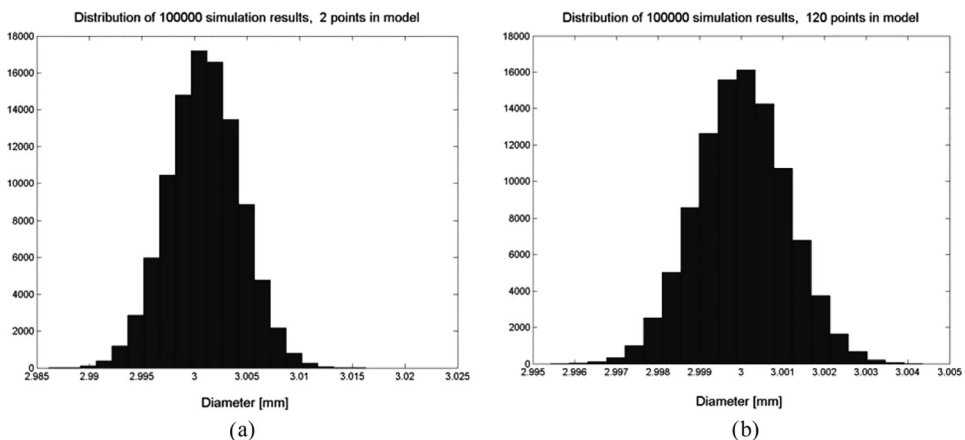


Figure 9. Distribution of the simulated diameter measurements using a) 2 points and b) 120 points.

**Table 5.** Measurement uncertainty and reproducibility using Monte-Carlo simulation

$n$	Uncertainty at 95% confidence level, [ $\mu\text{m}$ ]	Reproducibility at 95% confidence level [ $\mu\text{m}$ ]
2	6.7	6.4
4	4.9	4.5
12	3.3	2.6
36	2.5	1.5
60	2.3	1.2
120	2.1	0.8
360	2.0	0.5

for 2-point measurement with the values in Table 4. The reproducibility according to the simulation with 120 points is 0.8  $\mu\text{m}$ , slightly less than the result of 1.2  $\mu\text{m}$  in the previous section.

## 6. CONCLUSION

The optical CMM was found to be a suitable device for the diameter measurement of optical apertures. The repeatability of the aperture measurement results is very good with a typical standard deviation of 0.2  $\mu\text{m}$  for a point and 0.1  $\mu\text{m}$  for the mean diameter (average from 120 points). The reproducibility, affected by roundness error of the specimen, temperature and ambient light, is much larger, about 1  $\mu\text{m}$  ( $k = 2$ ) for the diameter obtained as an average of 120 points.

Although some of the measured changes in the aperture diameter are close to the estimated reproducibility of the measurement, the nature of these variations suggests that there has not been any systematic growth or shrink, for example, from oxidation or stresses of the apertures. Therefore, it appears that the aperture diameters have been stable during the studied period.

Earlier experiences with the optical CMM and the comparisons with the probing CMM have shown a large operator-dependent factor which probably can be traced to illumination selections. The expanded measurement uncertainty for average diameter is 2.33  $\mu\text{m}$  ( $k = 2$ ). However, in this study the measurements with the probing CMM appear to confirm the optical CMM's result very well. The conclusion is that if the required relative uncertainty in the aperture area is of the order of 0.1% the optical CMM used in this study is useful for aperture diameter measurements.

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