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FINITE-ELEMENT CALCULATION OF UNBALANCED MAGNETIC PULL AND CIRCULATING CURRENT BETWEEN PARALLEL WINDINGS IN INDUCTION MOTOR WITH NON-UNIFORM ECCENTRIC ROTOR

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Abstract - In induction machines, an eccentrically positioned rotor experienced a strong unbalanced pull toward the short side of the air gap. The non-symmetric flux distribution due to the eccentricity induces circulating currents in the rotor cage and parallel branches of the stator winding. These currents equalise the flux distribution and reduce the unbalanced magnetic pull. In this paper, the effects of the rotor eccentricity and equalising currents on the unbalanced magnetic pull are studied by multi-slice, time-stepping, finite element model. The modelling is done for the cases in which the rotor is parallel or diagonal with respect to the stator. The results of series and parallel connected motors are then compared.

I. INTRODUCTION

When designing the construction of a rotating electrical machine one has to know the forces acting on the rotor. In addition to the weight of the rotor and the forces caused by the load machine, the electrical motor itself generates electromagnetic forces that have to be supported by the bearings. For instance, this is the case when the rotor is not concentric with the stator bore. Two basic modes of eccentricity can be distinguished. A static eccentricity occurs when the centre axis of the rotor is displaced from the stator axis, but remains stationary with respect to the stator. In a dynamic eccentricity, the axis of revolution coincides with the stator axis but not with the symmetry axis of the rotor.

An eccentric rotor creates a non-symmetric flux distribution that causes the unbalanced magnetic pull. The non-ideal field may induce circulating currents in the rotor cage and parallel paths of the stator winding. These currents tend to equalise the flux distribution and they may significantly reduce the radial force [1]. According to DeBortoli et al. [1] the mechanism by which the parallel connection of stator windings affects is that the inductance is lower in those circuits located where the air gap is larger than nominal, relative to the inductance of circuits located where the air gap is smaller than nominal. Since all circuits are connected to the same voltage source, the currents in the circuits facing the larger air gap are slightly greater than those facing the smaller air gap. The net effect is that the eccentricity induced distortion of the air gap flux density, and the associated magnetic force waves, is reduced by the difference in the parallel circuit currents.

It has not been very common to use numerical methods for analysing eccentric rotors. Arkkio [2-4] has studied unbalanced magnetic pull for motors with different kind of asymmetry including eccentricity. His study includes both the static and dynamic cases and the effects of equalising currents in stator windings. In reference [5] the electromagnetic force acting on a whirling cage rotor has been studied. The study was focused on the electromagnetic damping on the unbalanced magnetic pull. DeBortoli et al. [1] used a time-stepping method to study the equalising currents set up by an eccentric rotor in the parallel circuits of stator windings. They also presented some results of force calculation.

Analytical methods have been applied widely to the problems associated with eccentricity. However, the effects of saturation and stator and rotor slottings are difficult to model by analytical means.

Freise and Jordan [6] derived equations for the forces caused by eccentricity. They used damping factors for taking into account the force reduction caused by equalising currents. They also noticed that these currents change the direction of the total force from the direction of the shortest air gap. Since then several people have studied unbalanced magnetic pull due to rotor eccentricity, most notably Früchtenicht [7], Belmans et al. [8] and Smith and Dorrell [9].

Belmans et al. [8] showed that in two-pole induction machine an eccentric rotor may cause a significant homopolar flux closing through the shaft, bearings, end-caps and frame. The magnitude of the flux depends on the reluctance of this path. A homopolar flux increases the static pull and generates an additional force component varying at twice the supply frequency.

In both, the static and dynamic eccentricity, the rotor can be parallel or diagonal with respect to the stator axis. Almost all of the studies in the literature handle the eccentric rotor, which is parallel with respect to the stator. Dorrell [10] studied the diagonal eccentricity in an analytical way, but no numerical analysis of this problem has been presented. The modelling of the parallel eccentricity is possible to do by two-dimensional finite element computation. If the rotor is not parallel with respect to the stator, the changes of the magnetic field in the axial direction have to be included some way or other into the model. The use of 3D FEM is one solution, but the size of the problem and computation times limits the use of 3D modelling. Alternative way is a multi-slice model, in which 3D effects are included into the 2D FEM by taking a set of cross sections of the motor perpendicular to the stator shaft. These slices have to be connected together. This can be done by forcing the currents to be continuous from slice to slice.

In this study, the effect of equalising currents on unbalanced magnetic pull (UMP) for the different types of eccentricity are studied. The used types of eccentricity are the uniform (rotor parallel with respect to the stator), one end eccentricity and eccentricity in opposite directions at each end (types are presented in Fig. 1). The computation is done for both, static and dynamic eccentricity. Unbalanced magnetic pull is computed when stator windings are parallel connected and results are compared with the computation results got from series connected stator when no equalising currents can occur in the stator side.

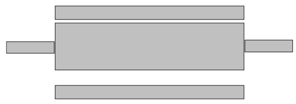
II. DESCRIPTION OF THE MODEL

The calculation of the operating characteristics is based on time-stepping, multi-slice, finite element analysis of the magnetic field. The details of the method have been presented in Ref. [11].

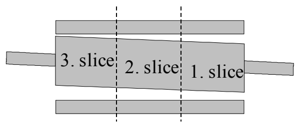
In the multi-slice model, the motor is divided into the slices cut by planes perpendicular to the stator axis. The slices are connected together by forcing the currents in the stator windings and rotor cage to be continuous from slice to slice. The magnetic field in the core region of the slice of the motor is assumed to be two-dimensional, and the two-dimensional field equation is discretised by the finite element method. The effects of end region fields are taken into account approximately by constant end-winding impedances in the circuit equations of the windings. The field equation and the circuit equations are solved together as a system of equations. The time-dependence of the field is modelled by Crank-Nicholson method. The magnetic field, the currents and the potential differences of the windings are obtained in the solution of the coupled field and circuit equations.

The method based on the principle of virtual work presented by Coulomb [12] is used for computing the electromagnetic forces.

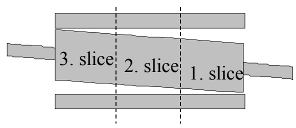
The uniform eccentricity (Case I) is modelled using one slice and modelling of diagonal eccentricity (Case II and Case III) is done using three slices cut by planes perpendicular to the stator axis (Fig. 1). For the one end eccentricity, the first slice is taken from the concentric end, the second one from the middle part of the motor and the third one from the most eccentric end as it is shown in Figure 1. For the case III, first and third slice models the eccentric end, which are in opposite direction, and second one models the middle part of the motor, in which the rotor is concentric with respect to the stator. The unbalanced magnetic pull is computed for each slice and it is presented as a function of slip.



Case I. Uniform eccentricity.



Case II. Eccentricity at one end only.



Case III. Eccentricity in opposite directions at each end.

Figure 1. Different types of eccentricity.

The method of analysis models the effects of equalising currents, slotting and saturation. The homopolar flux that may be caused by the eccentricity is not taken into account in the analysis.

III. RESULTS

The unbalanced magnetic pull is computed for a four-pole three-phase 37 kW induction motor with 48 stator slots and 40 closed rotor slots without skewing. The cross section of the motor is presented in Fig. 2.

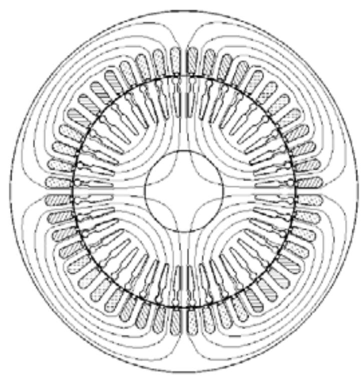


Figure 2. Cross-sectional geometry of the motor studied.

In order to study the effects of equalising (or circulating) currents, the motor is analysed with two types of stator winding; a winding with the coils connected in series and another one having two parallel branches. The parallel paths are obtained by connecting the windings on the two pole pairs in parallel. The numbers of turns in the two windings are chosen to produce equal air gap fluxes at equal supply voltages. In the both cases, the rotor is displaced from the central position by 10 % of the mean air gap.

The direction of the force vector is almost toward the short side of the air gap. For the static eccentricity, it is stationary and for the dynamic eccentricity, the force vector rotates with the rotor. As it is rather difficult to present and compare radial force vectors in two-dimensional plots, the average values of the lengths of these vectors are used to present the forces in the text.

Figure 3 shows the trace of the force vector caused by a rotor with 10% dynamic eccentricity when the rotor is parallel with respect to the stator.

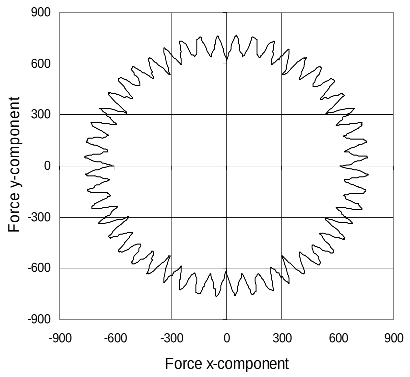


Figure 3. The trace of the magnetic pull vector computed for the 10 % dynamic eccentricity. Rotor is parallel with respect to the stator.

A. Static eccentricity

Figure 4 presents the equalising currents of the parallel branches in stator winding at no-load condition in the case of 10 % parallel eccentricity (case I).

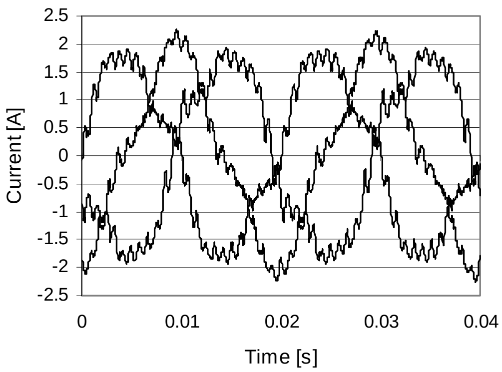


Figure 4. The wave forms of equalising currents of the parallel brances in the stator winding at no-load condition for 10% static parallel eccentricity.

Because the position of the rotor is always the same compared with the stator windings, induced equalising current of the phase windings that face the minimum and maximum air gap has higher amplitude than the equalising current in other phase windings. For slip values bigger than 0.01, the unbalanced magnetic pull start to grow. Dorrell [13] explained the increase of the forces with the load as an effect caused by the higher harmonics of the air-gap flux density. The fundamental component decreases slightly with the increasing load, but many of the higher harmonics are increased.

Figure 5 presents unbalanced magnetic pull as a function of slip for both connections. Thin line is for series connected and thick line for parallel connected motor. When the stator is series connected, the equalising currents can flow only in the rotor side. Parallel connection allows the flow of the equalising currents also in the stator windings. That is the reason why the unbalanced magnetic pull is lower for the parallel connected than series connected motor. At very small slip values, the difference of UMP is big. The reason for that are the closed rotor slots. At small slip values the iron bridges on the top of the rotor bars provide a path along which a part of the non-symmetric flux can flow without inducing equalising currents in the rotor bars. When the motor is loaded the iron bridges become strongly saturated and the unbalanced magnetic pull drops down. The parallel connection of the stator windings allows equalising currents to flow in the stator side also at small slip values and these equalising currents reduce the unbalanced magnetic pull to lower level than in series connected case.

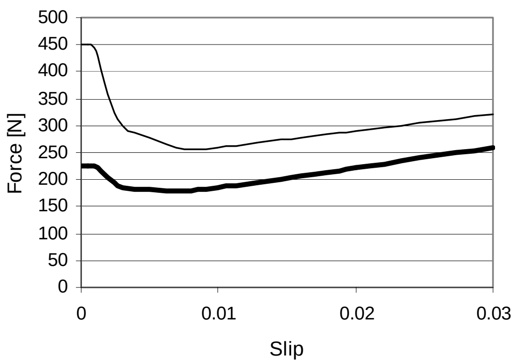


Figure 5. The unbalanced magnetic pull as a function of slip for the induction motor in the case of 10 % static parallel eccentricity. Thin line is for a series connected and thick line for a parallel connected motor.

When the rotor is eccentric only in one end (Case II), the amplitude of the equalising currents is half compared with the parallel eccentricity. Their wave form is approximately the same in both cases. For that reason, the parallel connection of the stator winding has smaller influence than in case I. Because the electromotive force causing equalising currents are mostly induced in the eccentric end of the motor, the equalising currents increase the unbalanced magnetic pull in the concentric end and decrese it in the eccentric end. The unbalanced magnetic pull as a function of slip in this case is presented in Figure 6.

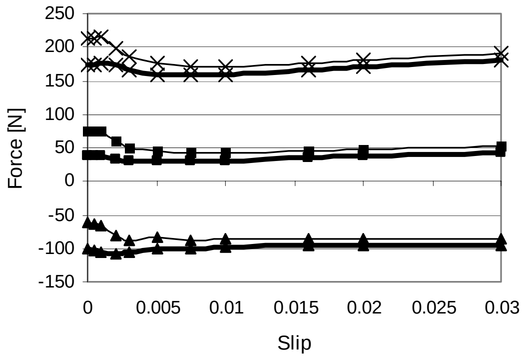


Figure 6. The unbalanced magnetic pull as a function of slip for the motor in the case of 10 % static eccentricity at one end. Thin line is for a series connected and thick line for a parallel connected motor. ▲ is concentric end ■ is middle part of the motor and X most eccentric end.

Third computed case is the eccentricity in the opposite directions at each end (case III). In this case the equalising current induced in one end of the motor has opposite direction compared with the equalising current induced in the other end of the motor. As a result no notable equalising current can occur in stator winding or rotor cage. Only some very small equalising currents occur in the stator windings and the unbalanced magnetic pull is approximately the same for both kinds of connections.

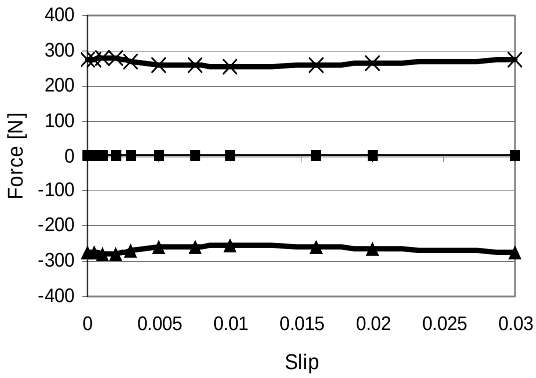


Figure 7. The unbalanced magnetic pull as a function of slip for the motor in the case of 10 % static eccentricity in opposite directions at each end.

B. Dynamic eccentricity

The type of the stator winding and the mode of the eccentricity have significant effects on the unbalanced magnetic pull. The differences in the forces can be explained by the equalising currents induced in the rotor cage and parallel branches of the stator winding. The largest force is obtained for the dynamic eccentricity combined with the series connected stator winding. The series connection eliminates the equalising currents from the stator winding, and the geometric non-symmetry rotates synchronously with the magnetic field. Then, only minor additional currents are induced in the rotor cage. As there are no currents to equalise the non-symmetric flux-distribution, the force is at its maximum. If the motor is loaded, the rotor does not rotate synchronously with the field and the force decreases. As a result, the unbalanced magnetic pull decreases when slip increases. For the parallel connected case, the currents induced in the stator windings equalise the flux-distribution and the force is notably smaller than in the series connected case. This can be seen form Figure 8, in which the unbalanced magnetic pull is presented for a dynamic eccentricity when the rotor is parallel with respect to the stator. For a parallel connected motor, the amplitude of the total force does not decrease with load. Instead, the effect of higher harmonics becomes notable and the force start to increase with load.

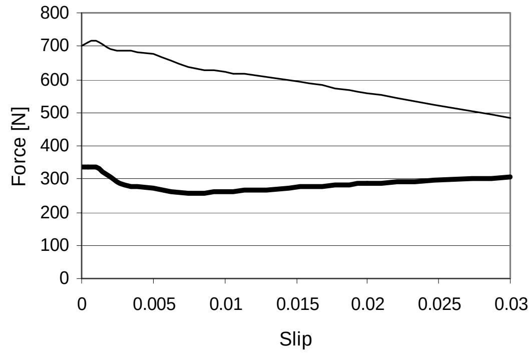


Figure 8. The unbalanced magnetic pull as a function of slip for the motor in the case of 10 % dynamic parallel eccentricity. Thin line is for a series connected and thick line for a parallel connected motor.

The waveform of equalising current for a 10 % dynamic parallel eccentricity at no-load condition is presented in Figure 9. According to the basic theory of the electromagnetic fields [7], the magnetic flux density b as a function of time t and position θ can be written in a following form

$$\begin{aligned} b(\theta,t) = B_{\mathrm{p}} \{ & \cos(p\theta - \omega t - \varphi_{\mathrm{m}}) \\ & + \frac{\varepsilon}{2}\cos\left((p+1)\theta - (\omega+\omega_{\varepsilon})t - (\varphi_{\mathrm{m}}+\varphi_{\varepsilon})\right) \\ & + \frac{\varepsilon}{2}\cos\left((p-1)\theta - (\omega-\omega_{\varepsilon})t - (\varphi_{\mathrm{m}}-\varphi_{\varepsilon})\right) \} \end{aligned} \quad (1)$$

in which B_{p} is the amplitude and ω the angular frequency of the fundamental component of flux density, p number of pole pairs, ε the relative eccentricity and φ_{m} and φ_{ε} the angles of the magnetising current and the eccentricity field. The angular frequency ω_ε is defined for the dynamic eccentricity [7] as follows

$$\omega_\varepsilon = (1-s)\frac{\omega}{p} \qquad (2)$$

Because the frequency of the supply is 50 Hz, the frequencies of the main harmonics got from equation (1) are then 25 Hz and 75 Hz, the periodicity of the curve is 0.04 seconds.

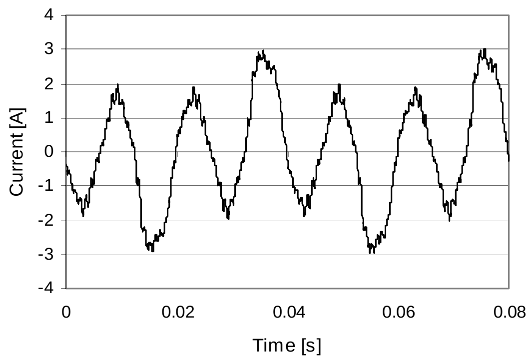


Figure 9. The wave form of an equalising current flowing in the parallel branches of the stator winding at no load condition.

When the rotor is eccentric only in one end of the motor (case II), the amplitude of equalising current is about 50 % compared with the parallel eccentricity. The waveform is approximately the same in both cases. That is the reason why the parallel connection of the stator winding has smaller influence than in case I. Because the equalising currents of the stator winding are mostly induced in the eccentric end of the motor, these currents increase the unbalanced magnetic pull in the concentric end and decrease it in the eccentric end. The unbalanced magnetic pull as a function of slip in this case is presented in Figure 10.

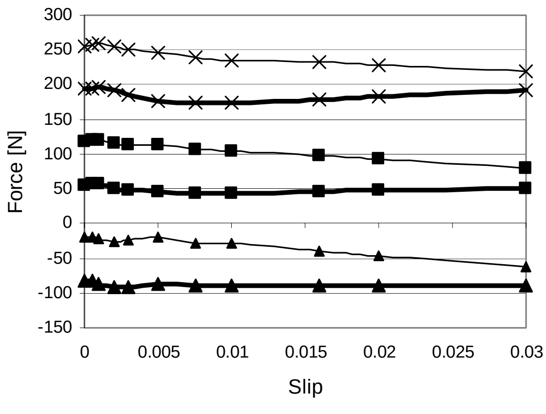


Figure 10. Unbalanced magnetic pull for a one end dynamic eccentricity as a function of slip. Thin line series and thick line parallel connected stator windings. ▲ is concentric end ■ is middle part of the motor and X most eccentric end.

The last type of eccentricity considered in this study is dynamic eccentricity at opposite directions at each end (case III in Figure 1). Just like for a static eccentricity, the equalising currents induced in one end of the machine has opposite direction compared with the currents induced in the other end of the machine and no notable equalising currents occur in rotor bars or in parallel connected stator windings. The unbalanced magnetic pull for both, series and parallel connections is the same as for a static eccentricity presented in Figure 7.

IV. CONCLUSIONS

The computation of the unbalanced magnetic pull and equalising currents for the induction motor with eccentric rotor is briefly presented in this paper. As a result of the study, the equalising currents and unbalanced magnetic pull are computed for different types of rotor eccentricity as a function of slip. The effect of equalising currents on the unbalanced magnetic pull is analysed from the computation results.
According to the analysis, the mode of the eccentricity and the type of the connection of the stator winding have a strong influence on the unbalanced magnetic pull. Parallel connection of the stator winding reduces the unbalanced magnetic pull clearly when the eccentric rotor is parallel with respect to the stator. Decrement is smaller but notable for the one end eccentricity, but when the eccentricity is in opposite directions at each end, the connection has no influence on unbalanced magnetic pull.

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