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CONSTITUTIVE MODELLING AND COMPUTATIONAL SIMULATION OF NIP MECHANICS AND WINDING OF PAPER ROLLS

Kilwa Ärölä

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Abstract

In this thesis the behaviour of paper rolls under different loading conditions as well as the winding process are studied using computational simulations. First, the mechanics of a rolling nip contact on a paper stack is investigated. The development of the micro-slip pattern of the contacting surfaces under the rolling nip is elaborated, in particular. It is found that the interlayer slippage between paper layers below the surface of the paper roll substantially influences the events taking place in the nip contact. This implies that a purely elastic, continuous simulation model cannot accurately describe the nip contact phenomena.

Second, the modelling of paper rolls is studied. Due to the immense computational cost of a full contact mechanical analysis of a paper roll, a much more effective continuum model with interlayer slippage for wound rolls of orthotropic material is developed. The constitutive behaviour of the roll is modelled using the theory of plasticity, with plastic shear deformation used to describe the layer-to-layer slippage. The model can be readily implemented in a modern finite element analysis software. The proposed model is used to study the stresses, interlayer slippage and permanent deformations in paper rolls loaded by nip rollers and clamping devices. To validate the model, the calculated nip contact results are compared to experimental findings. The advantages of the new model are computational efficiency as compared to a full contact mechanical model and the ability to effectively simulate the interlayer slippage, permanent deformations as well as hysteresis in repeated loading, any of which cannot be simulated using a purely elastic model.

Finally, a two-dimensional large deformation axisymmetric winding model for wound rolls of hyperelastic orthotropic material is developed. The roll build-up is modelled as an incremental accretion process, where successive pre tensioned hoops are shrunk-fit onto the underlying roll. The model is used to study the effects of the material parameters, winding speed, and tension profiles of the incoming paper web to the stresses in the finished paper roll.

Keywords nip, winding, constitutive model, paper roll

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Tiivistelmä

Tässä työssä tutkitaan laskennallisen simuloinnin keinoin sekä paperirullan käyttäytymistä kuormituksessa että itse rullausprosessia. Aluksi tarkastellaan erillisten paperiarkkien päällä vierivän nippikontaktin mekaniikkaa. Erityisesti keskitytään paperikerrosten välisten luistojen määrittämiseen. Todetaan paperikerrosten välisen luiston vaikuttavan merkittävästi nippikontaktissa havaittaviin ilmiöihin. Tämä osoittaa, että puhtaasti elastisella jatkuva-aineisella simulointimallilla ei pystytä realistisesti kuvaamaan paperirullan käyttäytymistä nippikontaktissa tai suurten paikallisten kuormitusten alaisena.

Seuraavaksi perehdytään paperirullan simulointimallin kehittämiseen. Koska kokonaiseksi rullaksi kierretyn paperirainan kontaktimekaaninen analyysi on laskennallisesti niin raskas tehtävä, että se ei nykyisillä tietokoneilla ole mahdollista, kehitetään työssä plastisuusteoriaan pohjautuva jatkuva-aineinen malli, jossa rullassa tapahtuvat kerrosten väliset liukumat kuvataan plastisena leikkausmuodonmuutoksena. Kehitetty malli on helposti sisällytettävissä nykyaikaiseen elementtimenetelmäohjelmistoon. Mallin avulla tutkitaan paperirullan jännityksiä, kerrosten välisiä luistoja ja rullan pysyviä muodonmuutoksia kuormitettaessa rullaa kantotelaa vasten sekä nostolaitteella. Mallin toiminnan todentamiseksi nippikontaktille laskettuja tuloksia verrataan kokeellisesti saatuihin tuloksiin. Kehitetyn mallin etuina ovat laskennallinen tehokkuus verrattuna täydelliseen kontaktimekaaniseen malliin ja kyky silti kuvata realistisesti kerrosten välistä luistoa, pysyviä muodonmuutoksia ja hystereesiä, joita ei voida kuvata puhtaasti elastisella mallilla.

Lopuksi kehitetään kaksiulotteinen suurten siirtymien pyörähdyssymmetrinen rullausmalli hyperelastisen ortotrooppisen materiaalin rullauksen mallintamiseksi. Rullan rakentuminen kuvataan vaiheittaisena prosessina, jossa yhä lisää esijännitettyjä kerroksia liitetään jo olemassa olevan rullan ulkopinnalle. Mallia käyttäen tutkitaan materiaaliominaisuuksien, rullausnopeuden ja saapuvan rainan kireysjakaumien vaikutusta valmiin rullan jännitystilaan

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Preface

This thesis is based on the work done at Helsinki University of Technology during 2002–2005.

I wish to thank my supervisor Professor Mauri Määttänen for his support and for providing good working conditions at the Laboratory for Mechanics of Materials that enabled me to concentrate on the present investigation.

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Vantaa, September 2006
Kilwa Ärölä

List of publications

This thesis consists of an introductory report and the following four papers:

[P1] K. Ärölä, R. von Hertzen. Development of sheet tension under a rolling nip on a paper stack. *Int. J. Mechanical Sciences* 47(1)2005, 110–133.

[P2] K. Ärölä, R. von Hertzen. An elastoplastic continuum model for a wound roll with interlayer slippage. *Finite Elements in Analysis and Design* 42(6)2006, 503–517.

[P3] K. Ärölä, R. von Hertzen. Deformation of a paper roll loaded against a nip roller. *J. Structural Mechanics* 38(3)2005, 55–58.

[P4] K. Ärölä, R. von Hertzen. Two-dimensional axisymmetric winding model for finite deformation. Manuscript submitted to be published in *Computational Mechanics*. Printed in *Helsinki University of Technology, Laboratory for Mechanics of Materials Research Reports* TKK-LO-38, 2006.

The author of this thesis is the primary author of papers [P1–P4]. All the development of the simulation models, programming, and numerical calculations have been carried out by the author.

1 Introduction

In everyday life we see paper everywhere around us. During one day we encounter paper when reading news papers, books and magazines. Soft tissue paper is used from drying our hands to wiping the kitchen table. At the office we have piles of paper on our desks, shelves and in our drawers. We fill forms printed on paper every day and pay using banknotes made of paper. In addition, paper and paper board is used extensively as packing material. Some examples of the use of paper (kg/person/year) during year 2004 in some industrialized countries are: Finland 333, Germany 235, Japan 249 and United States 312 [1]. Although the concepts paperless office, paperless desktop and electronic paper come up regularly, it is easy to see that they are still quite a distance away in the future. Until then we have to cope with living with paper. And while so, why not try to make the paper making process as effective as possible? When looking at the huge consumption readings given above, it is easy to understand that even slight improvements in the paper making process, reducing the amount of reject, can lead to significant savings.

Winding of paper is an important intermediate step in the papermaking process. Very large rolls of paper are wound at the dry end of a paper machine. These production rolls can be ten meters long and up to four meters in diameter. The production rolls are then unwound, slitted and rewound into smaller rolls for easier transport and storage. If special paper finishing is required, rolls can be further unwound and rewound at converting machines, for example a coater, calender or printer, used in the converting process. Each winding and unwinding poses a threat of damaging the paper web. In the worst case, inaccurate control of the winding parameters or the lack of knowledge of the appropriate values to use, can ruin the paper roll during the last winding, thus ruining the finished product at the final stage of the manufacturing process. A multitude of different winding defects related to pressure (bursts, baggy lanes, ridges), slippage (crepe wrinkles, dishing, telescoping) and buckling or instability (starring) have been categorized. Although the defects are well known and seen in practice, correcting them often relies on the expertise and working knowledge of the machine operator. In many cases the defects are resolved by trial, in which the input parameters of the winder are varied until a defect free roll is achieved. These trials can be costly, especially if performed on a production winder. In many cases there is a potential for several types of winding defects in a wound roll. Increasing one input parameter to eliminate one defect can give rise to a new different defect. There can be only a small window of opportunity in the winding parameters that will produce a defect free roll. In such situations the use of a mathematical tool for predicting the right input values becomes more efficient than the trial and error approach using a test winder.

The finished paper rolls are shipped to customers, for example printing presses, who again unwind the rolls and make the final product. During storage and transport the paper rolls are subjected to various mechanical loads, clamping forces of lifting devices and contact pressures induced due to stacking of the rolls, for example. These localized loads can generate layer-to-layer slippage beneath the surface of the paper roll. Excessive loading can lead to severe permanent deformations and out-of-round rolls. This in turn gives rise to problems when the roll is unwound. Typical problems are vibration of the roll which can lead to variations in the web tension and reduce the quality of the print and, in the worst case, to web breakage.

Numerical simulations can be a valuable aid in the design process of winders and clamping devices. For such simulations the behaviour of the paper roll under different loading conditions must be modelled effectively. A computational model of the paper roll can also be used to calculate properties of a roll in nip contact to be further used in dynamic models of a winder, where the stiffness and damping characteristics of the roll are important input parameters.

In this work computational simulation tools are developed and used to study the mechanical behaviour of paper sheets and paper rolls under loading. A winding model is developed to study the stresses, strains and displacements in a paper roll during and after winding.

2 Outline of the work

This thesis deals with three main themes. First, the mechanics of the nip contact is studied in detail, using computational simulation of a nip roller on a paper stack. The stick and slip regions within the several contact areas are calculated and the tensioning mechanism of the nip is explained. It is found that the high stresses caused by the localized nip load cause slippage, not only between the topmost layer and nip roller, but also between the paper layers deeper in the stack. This implies that when studying the response of a paper roll to a load that introduces local stress concentrations, the interlayer slippage of the paper layers within the roll has to be accounted for by the simulation model. Second, motivated by the first topic, a simulation model capable of effectively taking into account the slippage in the paper roll is proposed. Due to the immense computational cost of a full contact mechanical analysis of a paper roll, a much more effective continuum model with interlayer slippage for wound rolls of orthotropic material is developed and implemented in a finite element program. The model is used to study the stresses, interlayer slippage and permanent deformations in paper rolls loaded by nip rollers and clamping devices. The calculated nip contact results are compared to experimental findings. Third, the winding process is studied. An axisymmetric winding model, taking into account the axial length of the roll, is developed. Large deformation effects are incorporated into the model by using the nonlinear theory of continuum mechanics. The effects of the centrifugal force, induced by the rotation of the roll in a modern high speed winder, as well as the edge effects near the roll ends are studied. Cases with the incoming web tension distributed constantly or linearly in the web width direction are considered. These three main topics are considered in more detail in the following chapters.

2.1 Nip contact

The term nip contact or nip for short arises often when winding is discussed. This refers to the contact region where the roll, paper web and roller meet. An example of a winding configuration with a paper roll riding on top of a nip roller is shown in Fig. 1. Modern winders typically use a nip contact to prevent the entrainment of air into the roll and to control the wound on tension of the web entering the roll. The nip contact constitutes a challenging

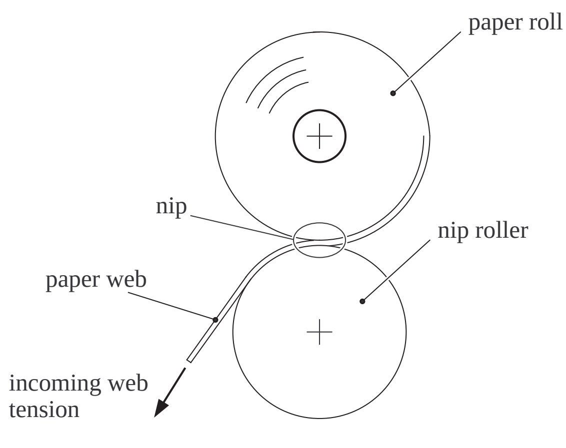

Figure 1: Winding configuration with a paper roll on top of a nip roller.

problem, as it involves the rolling contact of several deformable bodies, free and loaded boundaries with a priori unknown borders, slip-and-stick patterns related to frictional behaviour, and geometrical and possible material nonlinearities.

Several works considering the nip contact have been published. Bentall and Johnson [2] considered an elastic strip passing between identical rollers. Their results provided details of the contact stresses and deformations, the indentation of the strip by the rollers, the contact width, and the speed at which the strip passes through the nip. No net tractive force was transmitted in the process. Soong and Li [3] studied the steady rolling contact with friction of two freely rolling dissimilar cylinders covered by bonded elastic layers and driving a thin sheet in the nip. The sheet was incompressible in its thickness, had extensional elasticity, but no bending stiffness. They obtained the stresses and the deformations as well as the surface speeds for the cylinders and the sheet in a series form using a stress function formulation. Later Soong and Li [4] accounted for a pushing or pulling force acting at the tail end of the sheet. They studied the effect of the normal load and tail force on the speed ratios of the two cylinders and the sheet, and also the slippage and shear stress in the contact arc. In both papers Soong and Li restricted their treatment to an isotropic elastic material. Batra [5] studied the plane strain problem of a rubber covered roll indented by a rigid roll. Later Hinge and Maniatty [6] extended the solution to the problem of steady rolling contact between rubber-layered rolls with thin media in the nip. The contact interface was assumed to be largely in stick and the bearing in the lower roll offered a negligible resisting torque. Their treatment was also restricted to an isotropic elastic material law and the thin media was assumed to be inextensible, implying a constant thin media velocity through the nip. Kalker [7] considered the rolling contact of two parallel rigid cylinders covered with a number of homogeneous, isotropic and linearly elastic or viscoelastic layers. The layers were completely bonded to each other and to the cylinders so that no interlayer slippage could occur. Partial or complete slip could occur in the interface between the top layers of the cylinders. Friction was assumed to behave according to Coulomb's law with a constant friction coefficient.

A landmark investigation of the effect of a rolling nip upon a pile of separate layers was performed by Pfeiffer [8]. He reported experimental results on the strain-inducing mechanism of a rolling nip on a paper stack. This simulated the winding of a roll with an infinite radius. In this paper, the first quantitative data, displaying the effect of nip load, drum diameter, and the number of sheets in the stack on the amount of nip induced tension, was presented. Pfeiffer's observations, however, accounted for external nip behaviour only, and neither stress or strain distributions nor slip-stick patterns within the nip interface, were considered. Good and Wu [9] considered the mechanism by which a nip roller can increase the wound-in tension in the outer layer of a wound roll. Although Good and Wu provide the first basic understanding of the elongating strain in machine direction, their model comprises only one web layer and does not properly account for the rolling contact with friction, since they employ a classical Hertzian pressure distribution with no shear stress at the upper surface of the web. Mc Donald and Menard [10] studied roll defects associated with interlayer movement experimentally. They considered, in particular, the formation of crepe wrinkles during reeling and winding. Recently, a rigorous contact mechanical model of the winding nip was presented by Jorkama and von Hertzen [11, 12]. Their model consisted of the wound roll, the winding drum and the intervening sheet. The roll and drum were modelled as linearly elastic, orthotropic, homogeneous cylinders with a rigid core, and the sheet was modelled as an orthotropic material as well. They presented a novel stick-and-slip mechanism, which explained the generation of the nip induced tension in the incoming sheet. They also properly described the conditions of the incoming sheet after the nip, which is a distinctive feature of winding compared to calendering. They utilized, however, a solid elastic model for the wound roll. The real layered structure of the roll with possible interlayer slippage can lead to a significantly different strain behaviour, especially in the vicinity of the nip.

In paper [P1] the stresses, strains and displacements in the stack due to a rolling nip are calculated and the slip-and-stick behaviour at all contact interfaces is presented. The layered structure of the paper stack is fully accounted for. A detailed description of the interlayer movement of the paper sheets is given, and the mechanism of the nip induced tension, as a result of the shear stresses and opposing frictional forces, is identified. The studied system is very close to that used by Pfeiffer [8] in his experiment and a qualitative comparison between the calculated and experimental results is given.

2.2 Modelling the paper roll

Calculation methods and computer programs to solve problems of solid mechanics are well established and in every day use by mechanical engineers. Typical engineering calculations encountered in machine design or structural mechanics involve finding the displacements and stresses of solid bodies, which can typically be machine parts, structural components or structures of civil engineering. The calculation methods, of which the finite element method is the most common tool used in practical engineering, are based on numerically solving the governing partial differential equations. The modelling of the material behaviour of individual paper sheets as well as the values of the material parameters for paper have been discussed by many authors [13, 14, 15, 16, 17, 18, 19]. However, these results are not directly applicable to complete paper rolls. Use of the finite element method requires that the studied domain is continuous, or has a limited number of discontinuities. When thinking of the structure of a paper roll, one can quickly see the problem. The number of paper layers in a paper roll, although finite, can be very high. This poses the difficulty that to be able to model the roll as a continuous medium, the element size is dictated by the thickness of the paper web. All the interfaces between the layers are discontinuities. If the physical behaviour of the roll is to be modelled, the finite elements cannot span over these discontiniuities. Because the thickness of paper($\sim 0.1\,\mathrm{mm}$) is very small compared to the dimensions of the complete roll, for example 1000 mm in diameter, the amount of elements to be used is extremely high and the model becomes practically impossible to solve using modern computers. Thus another form of solution has to be sought.

If the layer-to-layer slippage in the roll can be modelled in an average sense using elements spanning several physical paper layers, the computation is much more effective. Computational methods for simulation of the plastic behaviour of metals are widely used. By introducing a material model that allows plastic deformation by shearing, the behaviour of layered solids can be simulated effectively. In paper [P2] the layer-to-layer slippage is included in the constitutive equation governing the material response. The starting point is that, with low stress levels, the material behaviour is elastic and all strains are recoverable. If the shear stress in the plane of the paper web overcomes the frictional forces, slippage between the layers begins. This slippage is modelled as plastic shear deformation. Incorporating the slippage between the layers in the material model also has a definite practical advantage. Sophisticated modern commercial finite element programs used in engineering, provide an interface for adding user programmed material models [20]. Thus, the model developed in [P2] can be easily implemented and used by an analyst familiar with the software package. Also the other features available in the software (meshing tools, contact analysis, etc.) can then be taken advantage of. This significantly raises the usability of the model in practical analysis.

Similar methods have been used in the study of jointed rock masses [21] and the collapse of metal coils [22]. However, the highly orthotropic material behaviour has not been introduced in the similar models used before. In paper [P3] it is found that using the jointed orthotropic material model for the paper roll, good correspondence between simulated and measured data can be achieved. It is also demonstrated that the use of a purely elastic model leads to highly exaggerated stiffness of the paper roll. This can be easily understood, since the elastic model allows arbitrarily high shear stresses to develop, without slippage of the paper layers. Also the hysteresis, *i.e.*, the difference in the load-deformation paths during loading and unloading, cannot be found using the elastic model. The proposed model is highly effective, since it allows the calculation of permanent deformations in the roll, for example due to extensive clamping forces. It is demonstrated in paper [P2], that using too high clamping forces severe permanent deformation in the roll can be generated. Such deformations can lead to several difficulties during the unwinding of the roll [23].

2.3 Winding models

The purpose of a winding model, when given the paper grade, caliper and other winding parameters, is to computationally simulate the winding process and give information about the finished roll, typically the stresses. During winding the stresses and strains in the roll are built up in an incremental manner. The resulting stresses determine to a large extent the quality of the roll, and provide the most important piece of information for the evaluation of the future durability and functionality of the finished roll. As the trend in winding technology is towards thinner media and faster winding speeds [24], the mechanical stability of the rolls becomes more and more critical. Although experimental data through the sandwiched pull strip method and some other methods can be obtained in laboratory conditions, practical non-destructive techniques to measure the internal state of stress of a roll on a production line are still lacking. This applies to finished rolls as well as to rolls still in a wind up. To evaluate the quality of the winding, there is an obvious need for a physical model of the winding process. Since the stress build up in a winding roll is a complex process, a reliable roll model is needed to quantify the contribution of various factors influencing the roll quality. In this way design criteria for the roll's state of stress can be developed.

Although winding is a continuous process, it has generally been modelled as an incremental process where successive pretensioned circular hoops are shrunk-fit onto the underlying roll. Thus, the roll is assumed to be a collection of single concentric hoops stuck together. This is an example of an accretion problem of solid mechanics or of the mechanics of growing bodies. All such bodies consist of mass elements that became part of the growing body at different times and at different initial stresses. It is typical of growing bodies that a configuration in which the elements of the body would be connected in a stress free state does not exists.

There is an extensive literature on the mechanics of wound rolls. Most of the presented models are one-dimensional accounting for the radial change of the stress and displacement fields only. The one-dimensional models can be grouped into four categories according to the constitutive law of the modelled roll. These constitutive laws have been linearly [25, 26, 27] or nonlinearly [28, 29, 30] elastic and linearly [31, 32, 33] or nonlinearly [34] viscoelastic and have treated the material as being orthotropic. The roll has been considered in a plane stress or plane strain state corresponding to very short or long rolls, respectively (magnetic tape packs or large paper rolls, for example), in which uniform mechanical properties along the roll's axial direction are assumed.

Recently, a few models treating the roll as a two-dimensional system have been published [35, 36, 37, 38, 39]. In these models, the core and web regions are considered to be of finite width. The stresses and strains are allowed to vary both in the roll's radial and axial directions, and four stress components - radial, circumferential, axial, and shear - as well as two displacements - radial and axial - are included. With these models, nonuniform winding tension, web thickness, elastic moduli, and core stiffness, depending on the axial coordinate, can be treated. In these works, the model has been applied to magnetic tape packs. Zabaras *et al.* [35, 36] have examined the effect of the nonuniform winding tension on the stresses in the tape pack. Lee and Wickert [37, 38] have treated several realistic cartridge hub (core) designs, and the roles of hub compliance and wound-in tension gradients in setting the tape pack's stress field are considered. Lately, Li and Cao have presented a hybrid approach for the winding process of thin-sheet coils [39]. They develop an approximate multi-layer finite element model to study the coil deformation under gravitational loading. They use in their finite element model the stresses calculated by an incremental winding model as initial stresses for the subsequent analysis of the "soft coil" problem and other two-dimensional phenomena. It should be noted, however, that their winding model itself is one-dimensional.

In many winding applications the displacements can be large and the displacement gradients are not small compared to unity. This applies particularly in paper winding, where the radial strains of the paper rolls take typically values of the order $5-10\%$, and even 15% in the case of certain soft paper boards. Therefore, the infinitesimal strain theory, utilized usually for strains well below 1%, is not a good approximation for such rolls. Also, as the winding speeds continuously increase due to larger production demands, centrifugal forces may have a significant effect on the resulting stress distributions. Although Benson [30] has treated large deformations in winding and Olsen [40] the effect of centrifugal forces, their models have been one-dimensional. A unified treatment of large deformations and centrifugal forces in a two-dimensional winding model has not yet been presented.

In paper [P4], a two-dimensional winding model for predicting the stress and strain fields within a wound roll of web material, in which the radial, circumferential, axial, and shear stresses, and the radial and axial strains can vary in both the roll's radial and axial (cross-web) directions, is presented.

The material behaviour of the roll is considered as orthotropically anisotropic, linearly elastic in the circumferential and axial directions and nonlinearly elastic in the radial direction. The roll is built up in an incremental manner and total equilibrium in the roll is required after adding of each single hoop. Here the tensioned hoop is allowed to shrink on the surface of the underlying roll, so that the wound-in tension loss [41] is automatically accounted for. The effect of centrifugal forces is also taken into account. The numerical solution is developed using the finite element method. In particular, finite strains are incorporated in the model using the total Lagrangian formulation. Also the core is modelled by the finite element method so that cores with nonlinear material behaviour can be treated. The model is applied for the analysis of paper rolls. The effect of the finite width of the roll on the stresses inside the roll and near the roll's free ends is studied. Also, the effect of a nonuniform distribution of the incoming winding tension in the roll axial direction is discussed.

3 Conclusions

The main results of this thesis, which are believed to be original, can be summarized as follows:

[P1] The stress and strain patterns as well as stick and slip regions in a paper stack under a rolling nip are presented for the first time. The interaction between the separate sheets is presented and the tightening mechanism of the rolling nip is explained.

[P2] The jointed material model is extended to account for orthotropic materials and used to study paper rolls. The permanent deformations due to interlayer slippage in paper rolls are calculated for several different loading cases.

[P3] The effectiveness of the model presented in [P2] is demonstrated and the model is verified. Results for the compressive nip contact between a winding drum and paper roll, calculated using a paper roll model allowing interlayer slippage, are compared to experimental data.

[P4] The center winding model is extended to take into account large deformations. The edge effects near the roll end are analyzed in detail. The role of high shear stress gradients near the roll core in the development of winding defects is discussed.

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