



Aalto University
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Factor-based Investing

Analysing market anomalies in the US
equity market

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Antti Ilvonen
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Author	Antti Ilvonen	
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Abstract

Passive investment strategies can be improved by statistically sorting the market based on various metrics known as factors. By only buying top securities of the market based on the sorting factor, the portfolio can generate higher returns while still maintaining high diversification. I found that two factors, value (sort based on book value to market value -ratio) and momentum (sort based on past performance), generated stable and statistically significant excess-returns based on data from 1926 to 2018.

The aim of this study was to evaluate the factors' past performance and to provide logical reasons for their persistence in the future. Key challenge to using factor sorting in investing is the risk of market pricing the excess-returns out. However, I provide evidence that release of new information has not led to significantly smaller factor returns. Also, I note that slow rebalancing of portfolio betas can explain part of the poor performance of momentum strategies during economic downturns. Furthermore, I analyzed volatilities of portfolios combining both factors with the market portfolio and found that diversifying investment portfolio over multiple sources of risk increases risk-adjusted returns.

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1. Introduction

1.1 Structure

In this thesis I will study factor investing strategies and their reliability in the future. The goal of this thesis is to explain why factor investing can be profitable and validate these reasons with data analysis. First, I will explain the basic fundamental ideas behind factor investing. I will go through different approaches to investing and explain how factor investing differs from traditional investment strategies. In addition, I will present various general methods to evaluate investments. Second, I will review literature that supports factor investing and explaining how factors are detected and validated. Third, I will go through the methodology of my analysis and how I set up my test portfolios along with explanations for my approach. Fourth, I will present my test factors and results. Finally, I will combine the two factors to explain diversification across factor and equity risks and present conclusions as well as topics for further research.

1.2 Research questions and the scope of this thesis

My research question is divided into a series of hypotheses. Are there mispricings in the market than can be measured by sorting the market based on various factors and explained by logical reasons? If yes, is there a way to implement an investment strategy to take advantage of those mispricings? If yes, has that strategy proven to be so efficient and reliable in the past, that it could be used also in the future? If yes, how should such strategy be implemented in practice?

The scope of this thesis is limited to the equity market (stock market) of the United States. I do briefly refer to literature which discusses similar mispricings and strategies in other asset classes and geographical markets as well, but my analysis is based purely on return data of US equity market. I do not cover every factor strategy that I found reliable, but instead limit my analysis of two factors that I found the most reliable.

1.3 Introduction to differences between active and passive investment strategies

I will begin with briefly explaining my perception of concepts related to factor investing based on literature that I have studied. Factor investing is a specific approach to long-term investing, which is closely related to investing in an index consisting of very large number of stocks or other securities. Investment strategies, which involve investing in an index are also known as passive investment strategies. Their fundamental idea is to maximize the investment portfolio's

diversification in order to provide stable returns over very long periods of time, which in this case can be as long as 10 or 20 years. When investor invests in an index, he effectively buys a small share of all the securities within an index, which often means hundreds or even thousands of different securities. This is made possible by different kinds of investment funds. The investors invest in the fund and the fund then purchases the securities in the index. This way, the investor can invest in a very large number of securities without having to pay large transaction fees for purchasing each of the securities independently.

Active investing

The opposite of these passive strategies is active investment management. In the passive strategy the investor buys the securities and then holds them even if the securities lose part of their value during the investment period. The speed of economic growth varies cyclically, which means that sometimes the growth speed is faster and other times it is slower. The changes between economic cycle's phases typically translate to the prices of investment securities as well. A classic example is the stock market, where the stock prices typically increase and decrease over time.

The periods when the stock markets are rising are called bull markets and the periods when the stock market is losing value are called bear markets. Some investors try to profit from these changes by trying to identify securities that are increasing value and only investing in those. This is called active investment management. Its downsides include lower diversification, higher transaction costs and the inevitable difficulty of identifying the right moments for buying and selling.

Investors and academics have argued for years which of the two approaches is superior, active or passive. As David M. Blanchett and Craig L. Israelsen note in their article "Spotlighting Common Methodological Biases in Active-Vs.-Passive Studies" (2007), the answer is not that simple. Both types of strategies have found support over the years, even though the passive strategies are more often found to be better.

In this thesis, I will not be studying whether active or passive investment management is the superior choice. However, I argue that it is important to understand the difference between active and passive investing to fully understand the benefits of factor investing. I will handle a topic that deals with a problem regarding the passive strategies. The reason why many investors choose the route of stock picking is simply the relatively low expected return of the passive investment

strategies. For example, The Helsinki Stock Market has historically returned 12,91 % annually with sample period from 1912 to 2009 according to a study by Peter Nyberg and Mika Vaihekoski "Equity premium in Finland and long-term performance of the Finnish equity and money markets" (2013). This is a nominal rate that includes dividend gains but does not take inflation into account. However, this is irrelevant when simply comparing active and passive strategies.

Although this might seem like a relatively decent rate of return, the sample period always contains lengthy periods of much lower returns as well. If the investors choose to go for active stock picking, they will face higher transaction costs and less diversification. If investor invests in an actively managed mutual/investment fund, they will face high management costs as well.

Why diversification is important?

Diversification is known as the "only free lunch in investing" as it effectively decreases the portfolio risk without lowering the expected rate of return, as mentioned by Antti Ilmanen (2011). According to the theory of efficient markets, which was first published by Eugene Fama (1965) in his article "Random Walks in Stock Market Prices", all the securities should have identical risk to return -ratio, because the market pricing of each security reflects all the information available that is related to the security. This means that if the security poses a higher risk, it also has a higher expected rate of return. If any security would produce higher, or so-called excess returns, the informed investors would take advantage of this arbitrage situation and the profits would be immediately priced out. Thus, there should be no securities in the market that would provide the investor with excess returns compared to other securities.

The developments of prices of different securities do vary, however. This causes negative correlation between the price developments which lowers the portfolio risk without decreasing the expected rate of return, which is known as diversification benefit. If the investor invests in only one security and its market value decreases 10 %, the investor loses 10 % of his total investment. If the investor invests in 10 securities and one of them loses 10 %, the investor only loses 1 % of his total investment. However, because all the securities have identical risk to return -ratio, both portfolios have identical expected rate of return, even though the latter has a significantly lower risk. The risk of a single security is often measured by its volatility or the square root of its variance:

$$\sigma(r) = \sqrt{\sigma^2(r)} = \sqrt{\sum_{i=1}^n p_i [r_i - E(r)]^2}$$

$E(r)$ represents the average expected return of the security, r_s represents the expected return in scenario s and p_s represents the probability of scenario s . To account for negative and positive values offsetting each other out, the values are raised to the power of two to achieve a figure known as variance. Because variance has been raised to the power of two, it cannot be used as such but instead the square root of variance, the volatility (also known as standard deviation), is used.

When we add new securities to portfolio, the combined volatility of the portfolio starts decreasing. As mentioned above, this is the fundamental idea underlying behind passive investment strategies. Because the expected return to risk ratio is identical for all securities, the rational investor should just simply buy all the securities to maximize his diversification and thus minimize his portfolio volatility to achieve the best possible portfolio risk to return -ratio. There might be times when active stock picking will produce higher returns, but in the long run, the passive investor will always make the highest profits.

1.4 Factor investing as a method to increase passive investment returns

Simply buying all the securities in a market does not intuitively seem like the best possible strategy, however. Eugene Fama, together with Kenneth French (1992), are responsible for popularizing an investment approach called factor investing as an extension to the well-known Capital Asset Pricing -model, or CAPM, which was originally created by William Sharpe in 1964. Before diving into the fundamental ideas of factor investing, I briefly present the CAPM.

The CAPM presents the expected return of a single security by comparing its riskiness to the risk-free rate and the expected return of the market portfolio. The risk-free rate varies, but usually government bond yields with the highest credit scores are considered risk free. Examples of these bonds would be the German government bonds in Europe or the US Treasuries in the United States. The riskiness of a single security is measured with beta, which effectively compares the covariance between the stock and the market to the variance of the entire market. Covariance is calculated similarly to the variance:

$$cov(x, y) = \frac{1}{n} \sum_{i=1}^n [x_i - E(x)] * [y_i - E(y)]$$

In this case, the single security return is expressed with x and the market return is expressed with y. The beta (β) is then calculated as follows:

$$\beta_i = \frac{Cov(r_i, r_m)}{Var(r_m)}$$

where m refers to market and i refers to the single security. When the beta, the risk-free return (r_f) and the expected market return [$E(r_m)$] are known, the CAPM takes the shape of:

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f]$$

From this equation we can clearly see, that the expected return of the stock increases when the beta or its riskiness increases. The beta is often seen as a measure of individual security's riskiness as it represents the security's covariance with the market. Beta less than one implies a security that is less volatile than the market, and vice versa.

The CAPM tells us that assuming efficient markets, the individual security's expected return is purely explained by its covariance with the market. It's worth noting, that the CAPM does not take into account any metrics traditionally used by market analysts, such as valuation models like price-to-earnings -ratio or dividend yields. The theory assumes, that this information is distributed to all investors and all the new information is priced into the security's market price immediately meaning that investors cannot find under- or overvalued securities just by analyzing for example firms' income levels or valuation metrics.

Factor investing, however, aims to identify metrics that would provide either risk-based or behavioral reasons for certain securities to be mispriced in the long run. These mispricings could then be used to create investment strategies generating excess-returns. In the foreword of the book "Your Complete Guide To Factor-Based Investing" (2016) by Andrew L. Berkin and Larry E. Swedroe, Cliff Asness defines factor investing as "defining and then systematically following a set of rules that produce diversified portfolios". The key is to find those metrics or factors, and then instead of buying all the securities in the market, buy the best securities sorted by the chosen factor while still maintaining high diversification. By doing this, we maintain the high return-to-risk -ratio, low transaction costs and low management costs (due to no one actively managing the

portfolio) discussed above but attempt to find logical reasons why some of the securities should outperform or underperform the market in the long run.

2. Review of literature explaining the key characteristics of factors

In this section, I will review literature and present findings that are common to factors in general. I present a list of criteria which can be used to evaluate factors and then discuss differences between behavioral and risk-based reasons for mispricing of securities.

2.1 Criteria for true factors

As noted by Berkin and Swedroe (2016), one of the key challenges regarding factor investing is determining which factors are truly a source for returns, that exceed the returns of market portfolio. Market portfolio is considered to be a benchmark for comparison. From this point on, I will simply call these higher-than-market returns excess-returns. To clarify, a portfolio can be profitable, even if it produces lower returns than the market portfolio. The objective, however, is to create a portfolio that produces higher returns than the market portfolio in the long run. The challenge is, that we obviously cannot predict what is going to happen in the future, and simply data about past returns is no guarantee of future profits.

If we would take a look at the past returns exclusively, we could probably figure out hundreds of different portfolios that would have beaten the market portfolio during past decades. But the question remains, that was it simply coincidence or was there a logical reason why the portfolio did outperform the market. Many academics have attempted to find out various factors, which could be used to generate these market-outperforming portfolios, and that have been able to do that in the past. For example, in their paper Campbell R. Harvey, Yan Liu and Heqing Zhu (2015) attempted to find out and evaluate various factors suggested by academia, which had been evaluated in reputable top-level journals and conferences. Even with their strict criteria, they ended up with a total of 316 different factors, which in my opinion is an enormous number even if some of the factors presented were highly correlated with each other.

Campbell et al. (2015) also note, that the “overwhelming majority” of the all the factors they studied produced returns that were statistically significant at 5 % significance. However, not all of them count as true factors, because their existence is only or mostly based on the past returns, they have generated. Even if the past returns have overperformed the benchmark, there might have just simply been a special period of time when the metric has worked, but that is of no use

for investor if it cannot generate excess-returns reliably also in the future. Berkin and Swedroe (2016) present 5 requirements for the metric to be classified as a true factor:

- 1) It has to be persistent: The factor must have been able to generate stable excess-returns in the past for a long time.
- 2) It has to be pervasive: True factors should apply across asset classes, and not only in stocks for example. This criterion decreases the risk of the factor being simply a result of data-mining, because one must study many different markets to classify a metric as a factor.
- 3) It has to be robust: There must be multiple metrics that can be used to measure the fundamental phenomenon behind the factor. As mentioned, each factor should have a logical explanation for its excess-returns, and one must be able to test that logical explanation with various metrics.
- 4) It has to be investable: Many investing strategies might work in theory, but in the real world they might be difficult to execute. For example, a strategy where the investor attempts to buy and sell securities with very short intervals to profit from the securities' intraday price movements may generate some profits, but it also generates massive transaction fees, which might make the strategy not usable.
- 5) It has to be intuitive: As mentioned above, there needs to be a logical, risk or behavioral-based explanation why the factor should exist. I argue that this is the key to understanding why these factors should exist also after they have been discovered and published, and the investors are starting to make investment decisions based on them.

These types of requirements are presented in other academic literature as well. For instance, Koedijk, Slager and Stork (2016) also list five requirements for factors, although they emphasize the ease of explaining the factor over its robustness.

2.2 Differences between risk-based and behavioral explanations for mispricings

I will briefly go through what these terms mean based on explanations provided for example by Berkin and Swedroe (2016), and I will present more concrete examples later when studying the factors in more detail. When studying the risk-based reasons for higher profits, we must first understand the difference between systematic and unsystematic (also known as idiosyncratic) risk. Financial theory states that if the investor carries a higher risk on his investments, he should be awarded with higher returns, or so-called risk premium, as well. However, this only applies to

systematic risk. Systematic risk is the part of the investment's risk which cannot be diversified away. So, as explained above, if the investor decides to invest in only a single security instead of investing in the market portfolio, he carries a higher risk. Yet, he could diversify that risk away at any time and thus should not be awarded a higher risk premium.

After the investor has diversified as much as he can, there is little he can do to increase the amount of systematic risk to gain higher risk premium. Some of the factors attempt to generate higher systematic risk, so the investor could also enjoy the higher premiums. Obviously, higher systematic risk could also lead to losing money in short perspective but in the long run, carrying higher risk should be awarded with higher returns.

Behavioral-based reasons are caused by inefficiencies in the way human beings make investment decisions. Most traditional financial theories assume efficient markets, which includes the assumption that all investors do only rational choices every time they invest. In reality, many investors, for example, like to invest in more "high risk, high return" -type of securities which might cause them to be overvalued, yet it does not stop investors from investing to them. I argue that this can be seen as a one type of gambling, which people do even though it is not rational. Some factors try to abuse these behavioral anomalies to generate excess-returns.

2.2.1 Understanding systematic risk as a source of higher profits

But why does the market not price out the higher systematic risk if we know it is there? Systematic risk of a single security is measured by its covariance with the market (beta) and not by the security's volatility. The underlying logic is a simple adaption of basic economic theory, but important for understanding why systematic risk results in risk premiums. Economic theory states, that with perfect competition, the marginal cost of any product should match the marginal utility that the consumer experiences when consuming the product. In the world of investing, this means that under efficient markets, price of any security should match the experienced utility of the return investor makes from his/her investment.

This is important, because as we know, we cannot forecast any excessively high returns under efficient markets, but the experienced utility of the same profits might vary. For example, let's assume two stocks, A and B, which both have the same expected rate of return. Stock A is highly correlated with the market (high beta), and generates market beating profits during bull markets

and vice versa. The stock B is negatively correlated (low or negative beta) with the market and outperforms the market during bear markets.

If the entire market is winning, the investor does not feel too special about the profits he/she makes with stock A. However, when the markets are crashing, the joy the investor feels from his/her positive return from stock B is a lot higher. Thus, the stock B's return has higher marginal utility which leads to higher price. Now the expected return of any investment is calculated by dividing the expected return by the investment's price.

$$\text{Expected profit of investment} = \frac{\text{Expected return}}{\text{Price}}$$

Thus, as the expected returns are equal for all stocks, the higher price of low-correlation stocks causes them to have lower expected profits, simply because investors value their profits more. In the end, all sources of systematic risk, including the factor risks, lead to higher profits because the irrational investors do not experience as high utility from systematically risky investments. The rational investor can take advantage of this, as in the long run, all the profits are just as valuable. Only problem is, that increasing systematic risk is difficult, but that is exactly what the factors aim to do.

I emphasize, that this applies also to factors backed by behavioral explanations, as irrational investor behavior is explained by the differences in utilities that the investor experiences. For example, the behavioral bias of going for "high risk, high return" is explained by the fact, that investors feel higher utility of the chance of big profits, and lower utility of lower yet steadier profits.

2.2.2 Why is it important to recognize the reason for factor's existence?

I feel that being able to explain the logic behind any factor is possibly the most important thing to do before it can be considered a true factor. When a factor is discovered and discussed in scientific journals, it becomes increasingly well known also among investors. After some time, it would be logical to think that these increased premiums should be priced out because an increasing number of rational investors try to profit from them. The increasing demand increases the price of undervalued securities and decreases the demand for overvalued securities.

It is true, that increase in the use of factor-based strategies naturally decreases their profits, but if there is a fundamental reason for their existence, the anomalies do not simply disappear because

some investors try to profit from them. For example, the overvaluation of systematically risky securities discussed above would not simply stop even if some investors started to implement strategies trying to profit from that overvaluation. Berkin and Swedroe aim to identify more of these reasons behind the excess-returns. Arnott, Beck, Kalesnik and West (2016) raise a similar concern and mention that many research articles promoting new factors fail to provide enough credible reasons why the factor should generate structural, long-lasting excess-returns also in the future and rely mostly on just the past performance of the factor.

3. Methodology: Constructing factor portfolios

In this section, I will go through different methods of measuring factor performance based on literature. I will also present the metrics that I used in my data analysis and the style that I used to create the test portfolios.

When discussing the factors in more detail, I will refer to multiple sources, which may have constructed their test-portfolios differently. Typically, when investors implement these factor-based strategies, it is done by a technique called “factor-tilting”. This means, that the investor begins constructing their portfolio with 100 % of the funds invested into a vast market-index. For example, in many cases this market-index is the Standard & Poor’s 500 -index (S&P500) which is commonly considered to be the benchmark index for US equity market. The investor then chooses his preferred factors, and then increases the portfolio weights of the securities which should provide the largest factor-premiums. In a way, the portfolio is “tilted” towards the chosen factor. This type of investing is also known as “smart-beta” investing, discussed for instance by Arnott et al. (2016)

3.1 Short positions in factor portfolios

One key difference between test-portfolios of different research papers is the use of either long/short or long-only portfolios. I will briefly explain the investment strategy of short-selling to explain this. If an investor wants to profit from securities being overvalued, he could consider lending the security, then selling it instantly. After the overvalued security’s market price has decreased, the investor buys back the security and returns it to the lender. The decrease in the security’s market price is the investor’s profit, net of any transaction fees and interest paid for lending the security. This is known as short-selling and it is important in factor investing as many of the factors are actually based on overvaluation of certain securities, rather than undervaluation.

So, for instance, if an investor is able to identify a factor which determines part of the stock market overvalued, he might simply just avoid buying those stocks or he could systematically also short-sell those stocks to increase his profits.

I note that in my analysis short-selling does not actually increase absolute returns of the portfolios, as the short positions generate some profits even if they do underperform compared to market. However, the short-selling helps to isolate market risk from the portfolio and generate higher factor risk, which can be useful when diversifying across different types of risk premia. This is explained and discussed in more detail in section 5.

Differences between long-only and long/short-portfolios

A portfolio which uses a combination of buying stocks (taking the long position) and short-selling stocks (taking the short position) is known as long/short portfolio. This type of portfolio is used by Berkin and Swedroe (2016) for example. If the portfolio does not consist of any short positions, it is known as a long-only portfolio, which is a preferred for example by Koedijk et al. (2016). The choice between taking short positions or not varies rather a lot and researchers have varying opinions over which approach should be taken.

Long/short-portfolios have the apparent benefit of being zero-cost. The cash flow generated by selling the lent stocks short can be used to finance the long positions of the portfolio. Thus, implementing long/short-portfolios does not require any capital, although it does increase the investor's financial leverage due to lending of stocks.

Which portfolio-style should be used?

Roger Clarke, Harindra de Silva and Steven Thorley (2016) discuss long-only portfolios' capability of capturing these factor premiums in their article and discover that long-only portfolios do better when purchasing only several individual stocks. However, when going for a highly diversified factor-portfolio, they capture only 40 % of the premium. Similarly, my results discussed in section 4 of this thesis imply that long/short -portfolios do capture larger premium than long-only -portfolios. Nevertheless, finding out whether the higher costs of short positions offset this larger premium and figuring the general superiority of these approaches is out of scope of this thesis and topics for further research.

However, I will present my findings based on varying literature on why short-selling is sometimes avoided as these are also the reason behind some factor anomalies. First, when purchasing stocks or other similar securities, the investor may only lose his invested capital. When short-selling, the investor may theoretically lose an infinite amount of capital, as the price of the security does not have an upper limit. Thus, the chance of heavy losses increases compared to long positions. Second, short positions usually have fixed loan periods for lending the security. This means that eventually the lent asset must be returned to the owner. Long positions, on the contrary, can be held indefinitely assuming that the asset has any market value left.

Berkin and Swedroe (2016) note that even 10 years might not be a long enough time to prove the efficiency of factor portfolios. Thus, taking short positions is riskier, because the investor cannot know how long he will end up holding the position. Finally, the investor must usually also pay some interest for lending the security which decreases the profits for short positions. Berkin and Swedroe (2016) note that these reasons have led to many investment funds not taking any short positions at all as they are considered too speculative.

3.2 Measuring factor performance

In general, both factors, that I will discuss, do provide a long history of stable and statistically significant excess-returns compared to a benchmark. The statistical significance in most publications is measured with Student's T-test and the excess-returns are measured with simply deducting the benchmark returns from the factor-portfolio returns. This way some researchers such as Berkin and Swedroe (2016) also calculate the historical odds of outperforming the benchmark. One common metric for measuring risk-adjusted excess-returns of a portfolio is also the Sharpe Ratio, which is measured by deducting the risk-free return from the historical returns of the portfolio and then dividing the result with the portfolio's volatility. The risk free -rate used in all of this thesis' calculation is the 1-month US T-bill's interest rate as it is the shortest maturity US government bond available and thus also the least risky.

$$\text{Sharpe Ratio} = \frac{E(r_i - r_f)}{\sqrt{\text{Var}(r_i - r_f)}}$$

Sharpe Ratio compares the portfolio's returns to its riskiness, higher ratio implies better portfolio performance.

4. Theory behind individual factors and analysis results

In this section, I will present two different factors that I found the most credible for producing stable excess-returns. This section is thus divided in two sub-sections 4.1 and 4.2. In both sub-sections, I first discuss literature that is relevant uniquely for the factor and its history. Second, I will present my analysis of the factor's absolute historical performance and its statistical significance. Last, I will review literature explaining logical reasons why the factor should persist to exist also in the future and present my own arguments.

4.1 Value

Value investing has been popularized over the years by investors such as Warren Buffett. The general idea deals with finding undervalued stocks that will generate high profits in the long run. In the world of factor investing, however, the investor does not attempt to pick out stocks that he considers undervalued. Instead, the investor must determine a rule which can be used to sort out all the stocks so that he can then buy for example the best 30 % of all the stocks.

The value factor was popularized by Kenneth French and Eugene Fama (1992), although they cite Dennis Stattman (1980) as well as Barr Rosenberg, Kenneth Reid and Ronald Lanstein (1985) for discovering the phenomenon in the US stock market. French and Fama (1992) took the firms' book values and compared them to the firms' market values and found that the firm's that have the highest book-to-market value -ratios (BtM) outperform the market in the long run.

So, in this case, a long-only portfolio based on the BtM-ratio would include for example the top 30 % of all stocks based on their BtM-ratios. A long/short-portfolio would also short-sell the worst 30 % based on the BtM-ration. As a side note, the number of securities bought or sold is somewhat irrelevant as long as the portfolio is diversified enough. It could be higher or smaller than 30 % but the 30 % is the number used, for example, by Berkowitz and Swedroe. To account for value factor's robustness, I mention that there are other metrics than BtM as well that can be used to capture the same phenomenon. Clifford Asness, Antti Ilmanen, Ronen Israel and Tobias Moskowitz (2015) list a number of metrics including earnings-to-price, cash flow-to-price and sales-to-price ratios.

The top 30 % of stocks are called value stocks and the worst are known as growth or "glamour" stocks. This is because low BtM-ratio is often caused by high market value as the market is pricing in a chance for a smaller company to suddenly generate new innovations that would raise its value significantly. I note that growth companies are often technology-oriented and even though they

rarely end up being highly profitable, there are examples such as Apple or Uber where the owners have generated massive profits.

On the contrary, value companies are often older firms that have already stabilized their positions in their market area. They have steady cash flows and they often generate stable dividend yields for the investors. The market is not pricing in a chance for massive future expansion anymore such as with the growth stocks. It is intuitive, that value companies seem like the safer bet, but on the other hand, based on the efficient-market hypothesis, the higher risk of the growth stocks should also be reflected in their market prices.

4.1.1 Expected returns of value stocks

To demonstrate the historical performance of value premium I used the Kenneth French's data library from CRSP which is also referred to, for example, by Berkowitz and Swedroe (2016). French's data consists of combined returns of all the stocks in the US stock market from the New York Stock Exchange (NYSE), American Stock Exchange (AMEX, nowadays known as NYSE American) and Nasdaq. Their portfolio data combines the needed accounting figures to the respective company's return data and then sorts the return data based on the selected criterion, which in this case is the BtM-ratio.

To account for the pervasiveness criterion mentioned earlier, I simply note that Clifford Asness, Tobias Moskowitz and Lasse Pedersen (2013) studied value premium in 18 different countries' equity markets and found that the premium existed in all of them. Value cannot sensibly be measured in other asset classes as bonds or currencies, but I argue that the diverse geographical evidence is enough to prove value premium pervasive enough to count as a true factor.

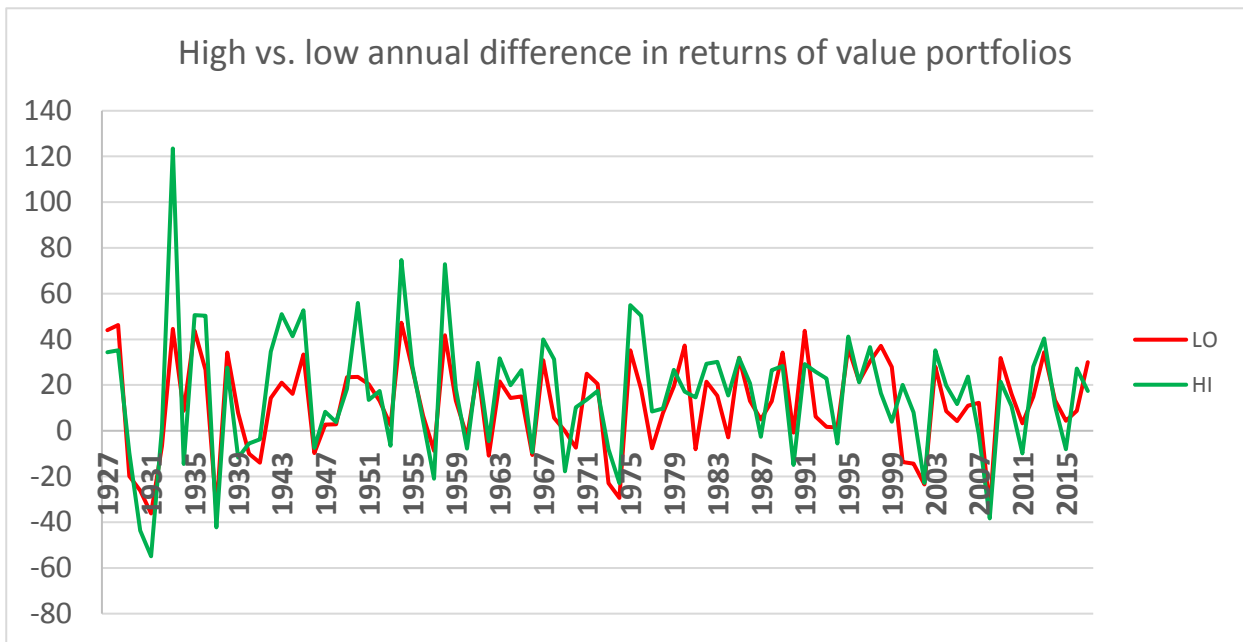


Chart 1. The annual returns (%) of high-BtM stocks (HI, marked with green) compared to low-BtM stocks (LO, marked with red)

I deducted the monthly returns of the worst 30 % of stocks based on the BtM-ratio from the best 30 %. The data dates back to July 1927, but unlike French's and Fama's (1992) original studies, it also contains the latest data all the way to September of 2018. This means that there were 1107 data points, which cover data from 92 years and 3 months. The combined excess-return when comparing high-BtM -stocks to low-BtM stocks was 375,21 % or 4,07 % when annualized, which represents the long/short-portfolio's excess-return in this case. I also calculated the total return of the stock market by calculating the weighted average return of the top and bottom 30 % of stocks based on BtM and the "core" 40 %. When I compared the high-BtM returns to the returns of the entire stock market, the premium was still 233,40 % or 2,53 % annualized. This represents the long-only portfolio's excess returns.

To repeat and clarify, the excess-returns do not represent the annual total returns of the portfolios. The long/short-portfolio's, the long-only portfolio's and the total stock market's average annual returns were 16,6 %, 15,1 % and 12,6 % respectively. For the long/short-portfolios, the total return is calculated by simply adding the excess-returns to the market portfolio's returns. I do this simply to illustrate the size of the premium, as I will present practical solutions to investing with long/short-portfolios in more detail in section 5.

Are the excess-returns statistically significant?

A question arises that what are the odds, that the excess-returns that I've presented are simply random? I will discuss shorter time-series later, but for the sake of proving that the historical higher returns are not simply an accident, I calculated the so-called t-statistics for the entire dataset. T-tests, that provide the t-statistics, are statistical tests which compare the average return to a null hypothesis and the standard deviation of the difference in returns. I used so-called paired two-sample t-tests, which are used when the same dataset is measured twice with some changes between the two measurements. The idea is to find out, what is the probability that the factor portfolio's excess-returns compared to the null-hypothesis are not just a coincidence.

In this case, the null-hypothesis, which the factor-portfolio is compared, is the entire stock market, which had standard deviation of 5,81. The BtM-sorted long/short-portfolio's difference to the stock market had standard deviation of 4,02. The long/short-portfolio's total standard deviation was 8,53. The same measurements for the long-only-portfolio were 2,16 and 7,18 respectively. The t-statistic is calculated as follows:

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

Where \bar{X} is the factor portfolio's average monthly return, μ_0 is the stock market's average monthly return (the null-hypothesis), s is the standard deviation of the excess-returns and n is the number of observations (1107 with all portfolios). The t-statistic is compared to the t-distribution to find out the probability. As a side note, the t-distribution is used instead of normal distribution as we are calculating the odds based on a sample rather than the entire "population" (as it is not sensible to attempt to gather data for the complete set of all the data of the stocks' historical returns). T-distributions vary slightly depending on the degrees of freedom, which are calculated as *degrees of freedom* = $n - 1$. However, when the degrees of freedom exceed 30, the t-distribution highly resembles the normal distribution. In most publications, as noted by Berkin and Swedroe (2016), the t-statistic that is found high enough for the excess-returns to be statistically significant is 2. Anything higher than 2 indicates even stronger significance. To prove this, I can easily compare the t-statistic of 2 with varying degrees of freedom to the t-distribution and see that the t-statistic of 2 implies roughly 95 % statistical certainty that the excess-returns are not random. The degrees of freedom in this case are always $1107-1 = 1106$.

I calculated that the long/short-portfolio's t-statistic is 2,81 (99,7 % probability) and the long-only-portfolio's t-statistic is 3,25 (99,9 % probability) meaning that both portfolios generate statistically significant excess-returns.

Returns after publication of Fama and French

Before addressing logical reasons behind the value factor, I will present similar statistics as above for time series beginning from the publication of the factor by Fama and French (1992) to this date. The reason for this is to prove that the value factor still exists even after wider audience has become aware of it and the market does not just simply price the value premium out. As mentioned earlier, the value factor was presented by other researchers already before Fama and French, but as their 1992 publication is such widely referenced (1717 citations in EBSCOhost Business Source Complete database) article, it is a natural choice for a benchmark. The article was released in June 1992, so the following calculations are based on data beginning from July 1992 to September 2018.

The results were not quite as impressive as with the longer time-series though. The annual premium of long/short-portfolio was only 0,91% with t-statistic of only 0,44. The annual premium of long-only-portfolio was 0,64 % with t-statistic of only 0,60. Thus, neither of these premia are statistically significant. The dataset is relatively long, yet it does contain for example the two longest bull markets of US equities (October 1990 to March 2000 and March 2009 to August 2018) which might cause the results to be somewhat distorted. It appears, that the value factor has indeed underperformed compared to the total market, first total of -17,4 % from 10/1990 to 3/2000, and then total of -5,1 % from 3/2009 to present date. Despite the long bull markets lasting roughly two thirds of the entire post-6/1992 dataset, the value premium was still positive after all, which could indicate that it has not disappeared even though there have been multiple academic publications related to it. However, whether the poor performance was related to value factor simply underperforming during bull markets in general, or due to the value premium getting priced out by the market, remains a subject for further research.

Summary of statistics

Table 1.1 and 1.2 based on data series from July 1927 to September 2018

Based on Book-to-Market -ratio				
Sorting	Total Market	Hi 30 %	Lo 30 %	Long/Short -premium + total
Average annual returns	12,6 %	15,1 %	11,0 %	16,6 %
Variance	33,7	51,5	28,3	72,7
Standard deviation	5,81	7,18	5,32	8,53

Based on Book-to-Market -ratio		
Portfolios	Long/Short	Long-only
Annual excess-return	4,07 %	2,53 %
Standard deviation	4,02	2,16
t-statistic	2,81	3,25
Probability	99,7 %	99,9 %

Tables 1.3 and 1.4 based on data series from July 1992 to September 2018

Based on Book-to-Market -ratio				
Sorting	Total Market	Hi 30 %	Lo 30 %	Long/Short -premium + total
Average annual returns	11,5 %	12,1 %	11,2 %	12,4 %
Variance	17,0	23,5	17,3	33,1
Standard deviation	4,12	4,85	4,16	5,75

Based on Book-to-Market -ratio		
Portfolios	Long/Short	Long-only
Annual excess-return	0,91 %	0,64 %
Standard deviation	4,02	2,16
t-statistic	0,44	0,60

4.1.2 Logical reasons for value premium's existence

Risk-based

I will first go through risk-based reason for higher returns of value stocks. In the chart 1, one can roughly observe that the High-BtM -stocks generate higher returns during economically good times but also higher losses during bad times. This is also represented by the higher standard deviations of the high-BtM -portfolios in tables 1.1 and 1.3. This already suggests that the returns of high-BtM stocks are more volatile and thus riskier than low-BtM stocks. And as noted earlier, higher risk should also be rewarded with higher return as well.

Nai-Fu Chen and Feng Zhan (1998) discuss the reasons for higher risk of value companies. One of the key reasons for the higher risk is higher financial leverage used by value companies and the higher financial distress caused by it. Value companies have more stable cash flows and they can handle the periodical interest payments with more certainty than growth companies. Interest is

usually tax-deductible and due to the fact, that issuing new equity often reveals that the management considers the stock overpriced, as mentioned by Stewart Myers and Nicholas Majluf (1984), debt is generally preferred over external equity for financing when available. This means that in efficient markets all companies should prefer debt over external equity, but value companies are more likely to be able to withstand the financial distress that comes with it. Debt causes financial distress as interest must be paid out whereas dividends are optional. Thus, value companies generally have higher debt/equity -ratio (financial leverage) than growth companies, and thus they are also generally riskier.

Robert Peterkort and James Nielsen (2005) go even further than Chen and Zhan (1998) and argue that the entire BtM-premium is explained by higher leverage. They found that in firms that have no long-standing debt there is no excess-returns associated with higher BtM-ratio. However, I argue that from investor's point of view, this is not highly relevant as most value companies have at least some level of debt. They also note, that the leverage does not explain the entire BtM-premium in companies that are at least partly financed with debt.

Another risk-based explanation by Lu Zhang (2005) deals with my earlier observation of value stock returns decreasing more than growth stock returns during difficult economic times. According to Zhang (2005), value companies in general have higher amounts capacity based on long-term fixed assets. It is difficult to reduce this excess-capacity during economically bad times because the investments to fixed assets can rarely be reversed. Thus, value companies cannot adapt to the lower demand as well as growth companies and have higher risk. During economically good times, it's relatively easy for both value and growth companies to raise capital and expand capacity, so in conclusion value companies end up being riskier. Again, higher risk should be rewarded with higher premium.

Behavioral

Next, I will go through behavioral reasons for value premium's existence. The basic behavioral concept is mostly related to the overvaluation of growth stocks. Joseph Piotroski and Eric So (2012) researched these behavioral mispricings in their study. They found that the firms with low BtM-ratio and weak overall financial condition are generally overvalued where as the high-BtM and fundamentally strong firms were undervalued. In growth stocks, the investors tend to overestimate the future outlook even if the firm in reality has very weak financial state. In a similar

manner, value stocks might be undervalued, because investors overestimate the risk caused by higher financial leverage and tend to be too pessimistic about them. This leads to increase in the value premium.

Another mispricing phenomenon is known as anchoring and is described by Berkin and Swedroe (2016). Anchoring in this case means investors' tendency to hold on to losing investments as they do not want to sell them for loss. Instead, they rather hold the investment waiting for it to reach at least break-even level even if the investment does not seem profitable on its own anymore. Growth stocks might have very high valuation levels when measured with metrics such as price-to-earnings (P/E) simply because their earnings levels are very small compared to the future expectations. The investors might anchor themselves to expecting the high return suggested by the P/E even though the future expectations might decrease after a while. So even if the P/E or other valuation metric keeps falling, the investors still hold on to their investments wanting to believe for the higher returns suggested by the previous high P/E would ultimately realize.

4.2 Momentum

Momentum refers to a strategy where the investor simply buys securities that have performed well in the past and optionally also short-sells securities that have performed poorly. Even though this strategy seems logical, it does not comply well with the basic logic of the efficient market theory. If the markets are efficient, the market price of the security should reflect all the relevant information about the security and any price changes should only be caused by new information becoming available. Thus, it does not really make sense that the past good (bad) performance would imply higher (lower) returns in the future.

Yet it still is regarded as one of the most persistent factor strategies. One of the groundbreaking articles regarding momentum was published by Narasimhan Jegadeesh and Sheridan Titman (1992) who noticed that momentum strategies generated high excess-returns. Importantly, they also argued that such high returns cannot be explained purely by systematic risk and that investors must have some sort of consistent behavioral bias that causes this effect.

The momentum has then been discussed in multiple papers over the years. Mark Carhart (1997) combines the factor created by Jegadeesh and Titman to the original model by Fama and French from 1992 which I cited earlier in the value-section. Like the book-to-market -ratio, momentum is

calculated as “winners minus losers (WML)” by deducting the returns of top ranked securities from the returns of worst ranked securities. The sorting criterion in this case is simply the past performance of a chosen time period. Jegadeesh and Titman (1992) used past returns and holding periods of 6 months in most of their calculation, but they also studied different time periods and noticed that the momentum effect reached its maximum after 12 months. Most publications seem to agree, as for example Berking and Swedroe (2016) use 12-month periods for calculating the past performance. Nevertheless, I argue that the fact that different time periods have performed well for different researchers accounts for the robustness criterion for classifying momentum as true factor.

I should note that there are two distinctive ways to measure momentum. The method explained above is known as cross-sectional momentum and the past performance of a single security is measured *relative* to the entire market. As a hypothetical example, this could lead to buying securities that have lost value over past months if the market has been losing even more on average. Another way to measure momentum is known as time-series momentum, which means that the *absolute* past performance of a security is used. This concept might be easier to grasp, as only securities that have gained value are bought and only securities that have lost value are sold short.

There has been some discussion about which measurement should be used, and for example, Ron Bird, Xiaojun Gao and Danny Yeung (2017) compared the time-series momentum to the original cross-sectional momentum of Jegadeesh and Titman (1992). They argue, that even though both strategies generate excess-returns, the time-series momentum is the superior choice. This is interesting as the cross-sectional momentum has been clearly the most studied factor of the two. Despite the findings of Bird et al. (2017) I will use the cross-sectional momentum in my calculations as it has been the consensus choice in most literature and as I will show, there is high evidence of its strong performance.

I used the same data source as with value, and the test portfolios were created exactly the same for easy comparison. Again, I will present long/short-portfolio which buys the top 30 % of stocks in the US stock market based on the past performance of 12 months and short-sells the worst 30 %, and long-only portfolio which only buys the top 30 %. The portfolios do not have specified holding periods like some of the portfolios in articles such as the one by Jegadeesh and Titman (1992), but

instead the portfolios are rebalanced on a monthly basis. This again makes the comparison between value and momentum easier.

Momentum strategies can naturally be implemented in other asset classes as well, although that is out of scope of this thesis. For pervasiveness criterion of momentum, I refer to the same article as with value by Asness et al. (2013) which studied momentum in 18 different countries but also in other asset classes such as government bonds and commodities. The study found statistically significant excess-returns across all asset classes in all markets except Japan.

4.2.1 Expected return of momentum stocks

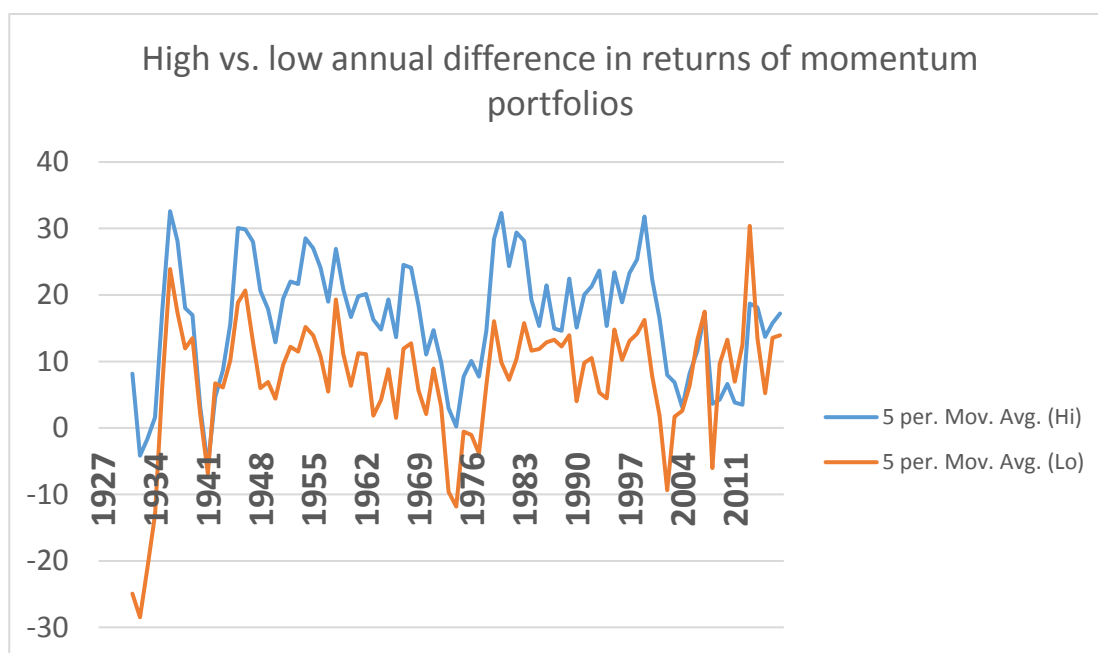


Chart 2. The annual returns (%) of winning momentum stocks (HI, blue) compared to losing momentum stocks (LO, orange) smoothed with 5-year moving average.

The excess-returns generated by momentum were even more clear than with value. The long/short -portfolio generated total average annual return of 19,51 % and the average excess-return to the total market was 8,25 %. The long-only portfolio's return was also impressive, the total average annual return was 15,4 % and the average excess-return was 4,15 %. It should be noted, that the dataset that was used to calculate these returns differs slightly from the dataset used to calculate value premiums. The criteria for including a single stock's data in the dataset are marginally different as the value dataset has criterion for both market equity data and book equity data, whereas the momentum dataset has also criterion for price data. Some missing data values might cause a small number of stocks to be excluded from either of the portfolios and the

momentum dataset also begins 6 months later than the value dataset (January 1927). Thus, the total average market return is lower at 11,25 % which amplifies the nominal values of momentum premiums. However, as the statistical significance of these results is very strong (as I will present next), I argue that this marginal difference does not affect the credibility of the results.

The t-statistic for the long/short -portfolio's excess-returns is 4,20 and for the long-only portfolio the t-statistic is 4,33. The probability for the excess-returns being only a result of randomness is less than 0,003 % for the long/short-portfolio and less than 0,001 % for the long-only portfolio.

Momentum was discovered around the same time as value so for comparability, I will use the same starting date for the short-term calculations as with value, the July 1992. The long/short-portfolio still performed well as the average annual return was 16,41 % and the excess-return compared to total market was 5,55 %. The long-only portfolio generated average annual return of 13,48 % and excess-return of 2,62 %. However, as with value, neither of the excess-returns were statistically significant at 5 % significance (t-statistic of 2) as the t-statistics were 1,52 for long/short-portfolio and 1,54 for the long-only portfolio.

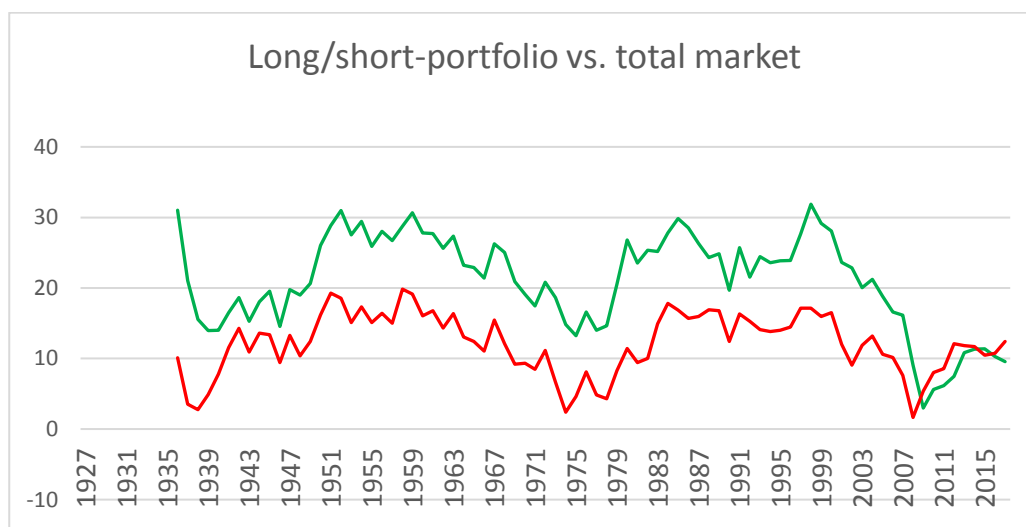


Chart 3. Long-short portfolio's 10-year moving average (green) and total market portfolio's 10-year moving average (red)

The above chart visualizes the long/short-portfolio's performance relative to the total market. I smoothed both time-series by calculating the 10-year moving average to clear some of the high volatility seen in chart 2. Interestingly, we can see that the momentum performance has faded noticeably over the past years, but the poor performance begins relatively late after the momentum performance was discovered. In fact, the excess-return even increases towards the

end of the century and then diminishes just when the financial crisis started to affect the markets in 2007.

However, before financial crisis, the momentum premium has been very stable over the years and changes in overall market performance have not affected the size of the premium. This suggests, that the poor performance of momentum over past 10 years would not be due to information effects or markets pricing the premium out. Jegadeesh and Titman in their subsequent article from 2001 come to the same conclusion and argue that the stable premium over 1990's despite multiple academic articles regarding the subject prove, that momentum premium is not simply a result of data mining.

Summary of statistics

Tables 2.1 and 2.2 based on data series from January 1927 to September 2018

Based on momentum				
Sorting	Total Market	Hi 30 %	Lo 30 %	Long/Short -premium + total
Average annual returns	11,3 %	15,4 %	7,15 %	19,5 %
Variance	36,2	30,8	64,0	34,2
Standard deviation	6,02	5,55	8,00	5,85

Based on momentum		
Portfolios	Long/Short	Long-only
Annual excess-return	8,25 %	4,15 %
Standard deviation	5,43	2,65
t-statistic	4,20	4,33

Tables 2.3 and 2.4 based on data series from July 1992 to September 2018

Based on momentum				
Sorting	Total Market	Hi 30 %	Lo 30 %	Long/Short -premium + total
Average annual returns	10,9 %	13,5 %	7,94 %	16,4 %
Variance	20,0	19,0	44,7	26,2
Standard deviation	4,48	4,36	6,69	5,12

Based on momentum		
Portfolios	Long/Short	Long-only
Annual excess-return	5,55 %	2,62 %
Standard deviation	5,37	2,52
t-statistic	1,53	1,54

4.2.2 Logical reasons for momentum premium's existence

Risk-based

When comparing statistics of value and momentum portfolios, one can clearly note an interesting difference. The high-BtM value portfolios have significantly higher volatility than the market portfolio which logically suggests higher returns. However, the top 30 % -momentum portfolio based on past performance has roughly the same volatility as the market portfolio. As a matter of fact, the low 30 % -momentum portfolio has the highest volatility of all momentum portfolios despite its relatively poor performance. I argue that this suggests that simply higher risk is not behind the high historical premiums of momentum stocks.

One risk-based explanation to momentum is, however, the fact that momentum securities have generally high expectations loaded on them. The good past performance might be due to investors' increased hopes for future profits and thus the tail-risk of these hopes not getting fulfilled increases. This increased risk could justify the higher returns. I argue that this might cause major losses of momentum strategies during financial downturns such as the one seen during 2007-2008 in chart 3.

Behavioral

Most academic researchers agree with this as momentum is mostly considered to be a behavioral phenomenon, as noted by Berking and Swedroe (2016). Simple behavioral explanation is that momentum is caused by slow reaction of the markets. To recall, efficient markets -hypothesis states that any new relevant information about a security causes its price to react *immediately*. It is true that many professional investors might react almost immediately when new information arises but there are still many non-professional, or so-called uninformed investors, who do not follow the news as accurately. This causes the price to react slower than what one could expect based on efficient markets -hypothesis.

This slow reaction causes temporary over- and undervaluation of securities. The reason why the financial markets do not reverse these anomalies is their relatively slow development. Zhi Da, Umit Gurun and Mitch Warachka (2013) compared how investors react to slow price developments relative to fast price movements. Their study suggests that momentum is simply caused by uninformed investors not reacting to small and gradual information signals even though larger moves cause appropriate price movements. This leads to short-term excess-returns captured by momentum portfolios.

Is momentum still viable strategy despite the recent underperformance?

As I mentioned, it does not seem like markets have simply managed to price out the momentum premium due to increased knowledge about it. Multiple publications have been made already in the 1990's about momentum stocks' good performance yet the premium remained stable over the decade. I analyzed a recent explanation by Kent Daniel and Tobias Moskowitz (2016). They argue that momentum strategies tend to fail when the market volatility is high which leads to panic in the markets and major economic downturns, and the poor performance after such downturn is explained by relative betas of the long and short positions within the portfolio. For example, during a bull market when stock prices are rising, the stocks that are bought to the portfolio generally have high betas. This is because the entire stock market is rising, and the best performers just simply rise even more than the average. Correspondingly, the stocks that are sold short have relatively low betas.

When the stock market then crashes, the high beta stocks tend to crash even harder than the market. When the portfolio is rebalanced after the downturn, the best performers during the downturn were the stocks with lowest betas because they suffered the least from the market crashing. This leads to low beta stocks getting bought to the portfolio even though they might not be the winners during the next bull market that follows the downturn. Thus, the momentum not only crashes harder than the stock market during downturns, but the poor performance also lasts long afterwards until the portfolio finally gets properly rebalanced. I studied this by calculating timeseries of portfolio betas for both top 30 % and worst 30 % portfolios based on momentum from beginning of year 2000 to September 2018. I calculated rolling covariances and market variances based on previous 100 days to illustrate the changes in the portfolio betas, based on daily data. This is somewhat shorter time period than Daniel's and Moskowitz's 126 days, but shorter time period illustrates the effect more clearly.

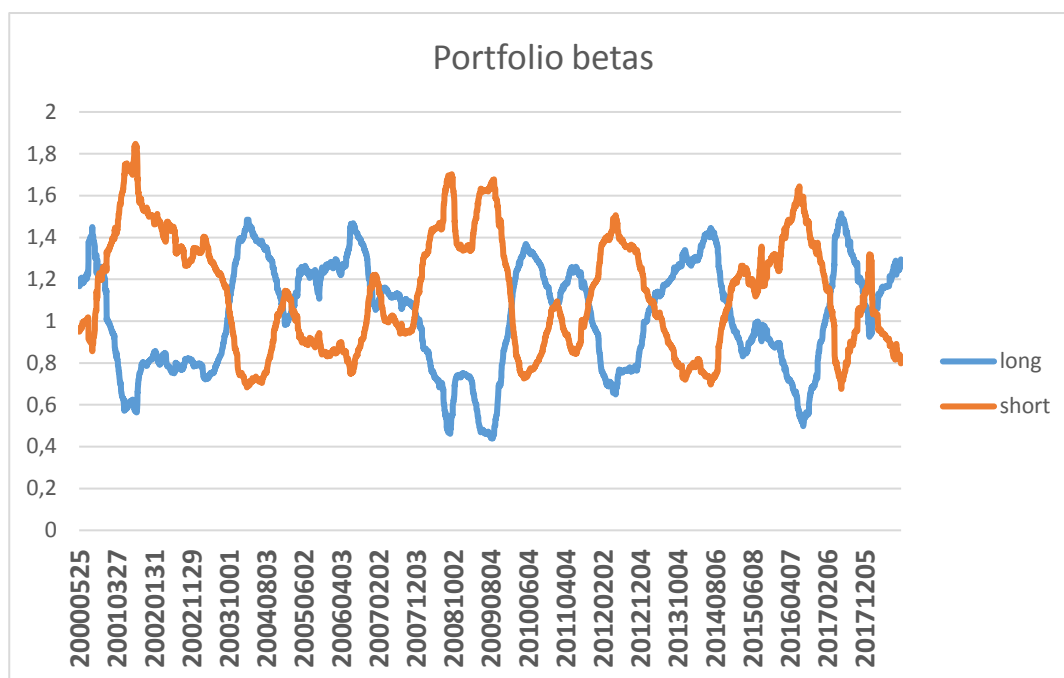


Chart 4. Rolling observations of betas of long (blue) and short (orange) portfolios (top and bottom 30 % based on momentum) from January 2000 to September 2018.

As we can see, the beta of the long portfolio has dropped significantly first during the Dotcom-bubble in 2000-2002 and then during the financial crisis in 2007-2009. This suggests that the rebalancing of betas might at least partly explain the momentum's recent poor performance and that the momentum premia might become more evident again in the future.

5. Combining factors to increase risk-adjusted returns

In this section, I will combine the two previously presented factors. My goal is to optimize portfolios which have the highest return *relative* to their risk. This is the key difference between the previous section 4 and the section 5 as previously I have only calculated the portfolios' *absolute* performance. I will first combine only the factors, and second, I will combine the optimized factor portfolio with market portfolio to create an example of a factor portfolio which could easily be implemented also by individual investors. Last, I will examine possibilities of factor diversification during periods when factors are performing poorly relative to the market.

5.1 Combining two factors

The previous statistics have measured absolute excess-returns of value and momentum strategies. All factor strategies rely on good diversification as explained earlier. The factors, however, present

also a new type of diversification, which deals with combining different factor strategies to a single portfolio. Traditionally, diversification has been done across different securities and asset classes as well as geographically. Different markets react differently to varying economic conditions and thus the portfolio's risk can be decreased by diversifying.

Recall, that factors, both the ones backed by behavioral and risk-based explanations, are sources of increased systematic risk. Traditional diversification across securities, asset classes and geographical markets is also diversification across different sources of systematic risk.

And as with traditional diversification, we can observe negative correlation between factor risks, which opens up opportunities for diversifying across the factor risks. For example, I previously argued that value premium has been performing poorly during bull markets and momentum has performed poorly during economic downturns. This implies, that there might be negative correlation between the two factors.

Correlation between two datasets can be measured with Pearson's correlation coefficient, which is calculated by dividing the covariance of the two datasets with the product of their respective standard deviations.

$$\text{Pearson's correlation coefficient} = \frac{\text{Cov}(x, y)}{\sigma(x) * \sigma(y)}$$

The coefficient varies between -1 and 1. A positive value implies positive correlation and vice versa. In this case, negative correlation implies that the two portfolios react differently to the same economic events affecting the market. And indeed, the correlation coefficient between the excess-returns of momentum and value portfolios (deducted of risk-free return) is negative -0,43.

I calculated 11 portfolios with varying weights between value and momentum to study the development of volatility and the average annual returns. To measure the risk-adjusted returns, I calculated the Sharpe Ratios for all portfolios. To calculate the optimal portfolio weights with the highest risk-adjusted return, I created a two-variable optimization model and ran it with Excel's GRG Nonlinear solver engine. The goal was to maximize the Sharpe Ratio of the optimal portfolio.

	<i>Value</i>	<i>Momentum</i>	<i>50/50</i>	<i>Optimal</i>
Variance	16,177	29,514	6,710	6,312
Standard deviation	4,022	5,433	2,590	2,512
Average return (%)	4,094	8,219	6,156	6,007
Sharpe Ratio	1,018	1,513	2,377	2,391

Table 3.1 Statistics for Value, Momentum, 50 %/50 % equal-weighted and optimal portfolios.

Out of the single-factor portfolios, momentum clearly produces the best risk-adjusted returns with Sharpe Ratio of 1,513. As noted earlier, it also produces highest absolute excess-return of these portfolios with annual average of 8,2 %. The equal-weighted and the optimal portfolio, however, produced clearly superior Sharpe Ratios. Despite the apparent superiority of momentum, the optimal portfolio's weights were actually in favor of value with roughly 53,6 % value and 46,4 % momentum.

To illustrate this, I plotted the 11 portfolios (100 % value and 0 % momentum, 90 % and 10 % and so on) along with the highlighted optimal portfolio. The so-called efficient frontier, where the returns cannot be increased without increasing portfolio volatility, is highlighted with red. The y-axis shows the absolute average return and it is compared to the portfolio's standard deviation on the x-axis. The lowest point on the y-axis is naturally the portfolio with 100 % value as value has the lowest absolute return. The momentum's weight is then increased in 10 % intervals.

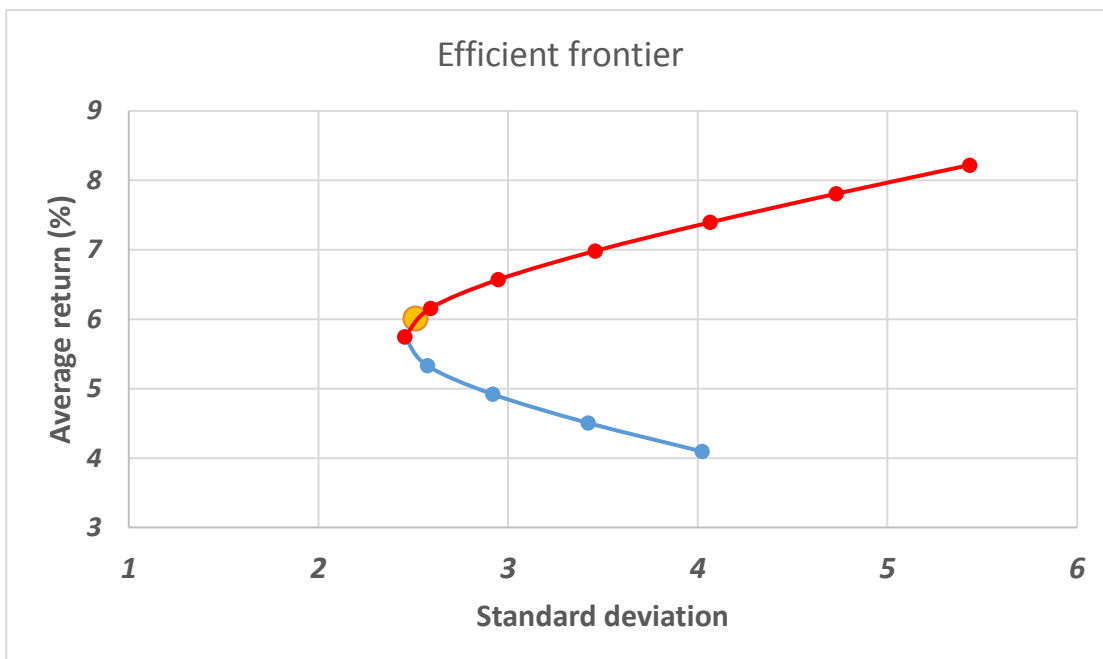


Chart 5. Plotted returns of combined value/momentum -portfolios with highlighted efficient frontier

As the momentum weight is increased, the portfolio's volatility decreases while the average return increases at the same time. The effect is very similar as when comparing to simple stocks except that now I am comparing two portfolios that are already well diversified across the equity market. This proves that both value and momentum portfolios' risk-adjusted returns can be improved by diversifying across factors even if their absolute returns do not increase.

5.2 Combining factor risk with market equity risk

When I calculate the long/short-portfolio's excess-returns, I effectively isolate the excess-risk generated by the factor from the equity market risk. The equity market risk is simply the systematic risk of investing in the total market and it is the source for the total market returns. The factors' excess-returns is based on increased systematic risk generated by the factor. This means, that we can further diversify our portfolio across different types of risk. This is especially emphasized by Asness et al. (2016) who mention that long/short -portfolios generate risk premium that has low or negative correlation with market equity risk premium. When comparing the optimized factor portfolio to market portfolio, I once again noticed low correlation coefficient of -0,001 which implies possibility for diversification benefits.

Brief example of isolating the market equity risk from factor risk

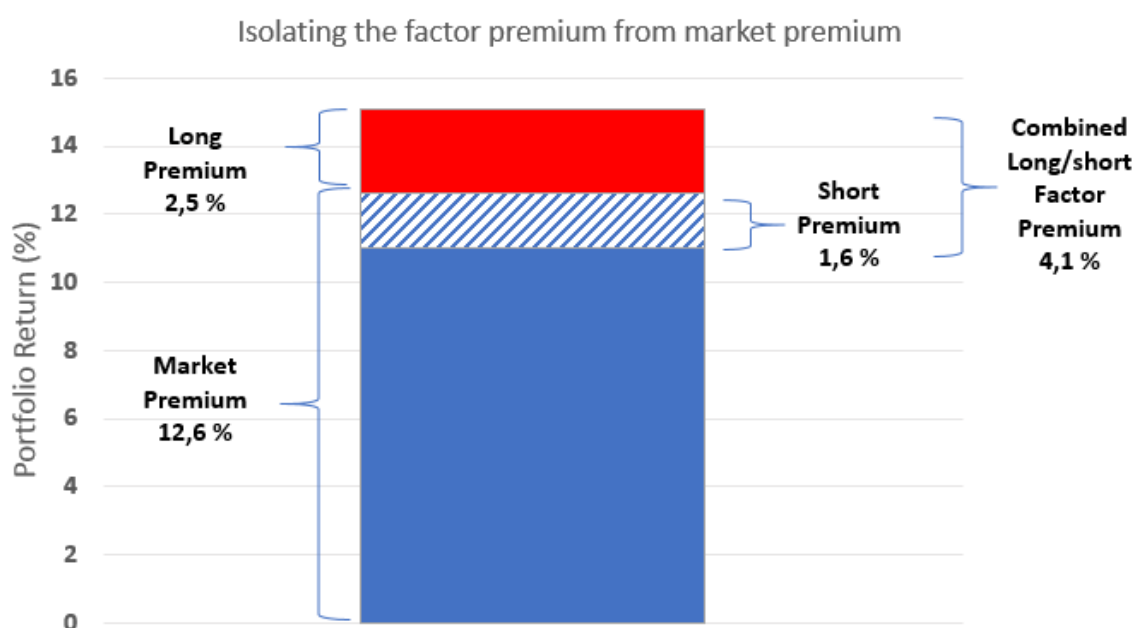


Chart 6. An example of isolating factor premium (red + striped blue) from market premium (blue + striped blue). The percentages represent the value portfolio of section 4.1

To illustrate isolation of factor risk from market risk, I use a simple bar chart with percentages from the value portfolio of section 4.1. The market premium is marked with blue (including striped blue), and the factor risk exposure from taking the long position with value stocks is marked with red (2,5 %). If we would buy the value stocks and sell short the market, we would isolate the premium marked with red. If we sell short only the low Book-to-Market stocks, we isolate the market risk and gain increased exposure (4,1 %) to the factor risk which is marked with striped blue. The portfolio which combines only the red and striped blue sections of the chart does not contain any market risk and correlates negatively with market risk. After the isolation, we can then look out for negative correlation with other types of risk premia to gain increased risk-adjusted returns by optimizing.

Optimizing the balance of factor risk and market risk

In section 4.3 I diversified the portfolio across two different types of factor risk, value and momentum. Now I recombine that portfolio with total market portfolio to generate the highest possible risk-adjusted returns. I want to emphasize that this kind of diversification strategy could very easily be implemented by any individual investor by investing in factor-based Exchange Traded Funds with weights that I will present next.

Again, I optimized the portfolio weights with Excel and present a 50 % / 50 % portfolio as an example. The following chart 5 plots Sharpe Ratios for all portfolios combining the factor and market portfolio with varying weights using 10 % intervals. This time the factor portfolio is the “Optimal” portfolio presented in section 4.3 with roughly 53,6 % value and 46,4 % momentum.

	Factors	Market	50/50	Market + Factor Optimum
Variance	6,311917	33,86712	9,972909	5,870644317
Standard Deviation	2,512353	5,819546	3,157991	2,422941253
Average Return	6,007304	12,51738	9,262342	7,830995662
Sharpe Ratio	2,391107	2,15092	2,932985	3,232020443

Table 3.2 Statistics for Factor, Market, 50 % / 50 % and Risk optimized portfolios

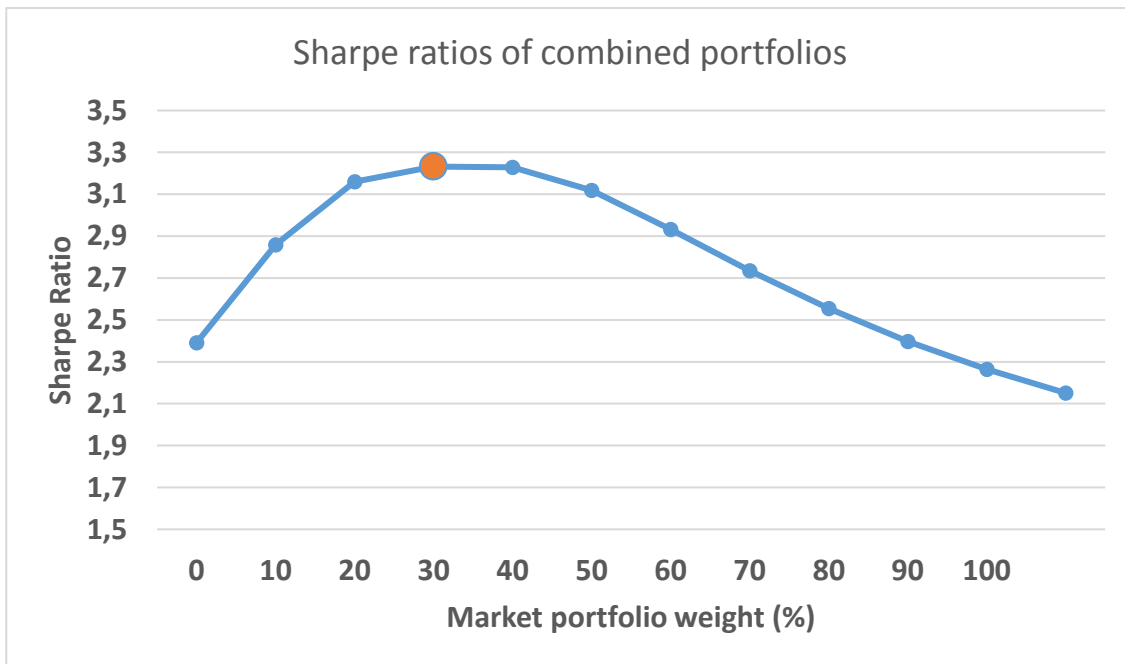


Chart 7. Sharpe Ratios with varying portfolio weights for portfolios combining market and factors, optimum market with orange

As we can see, the new optimal portfolio has lower average return than the market, but it generates highly superior risk-adjusted returns with Sharpe Ratio of 3,23 which is by far the highest of all portfolios presented so far in this thesis. The optimum weights were around 72,0 % factors and 28,0 % market, which leads to final weights of 38,6 % for value and 33,4 % for momentum. This is a simple passive investment strategy which has good diversification across stocks but also different sources of risk, a great example of strategies Asness et al. (2016) described in their papers. And most importantly, it could very easily be replicated by any investor even with low investment capital.

5.3 Factor risk-optimization from 2010-2018

As noted a number of times, both value and momentum have performed poorly since the financial crisis hit in 2007. It seems clear, that diversifying across both factors and market has been the superior strategy when comparing risk-adjusted returns in the long historical perspective. But would similar strategy have been a good choice also, when the absolute performance of the factors has been historically low? I studied this by optimizing all three variables (value, momentum and market) from January 2010 to September 2018 as this represents a time period of historically strong market performance and poor factor performance.

	Market	Optimal
Variance	13,27059	6,658824
Standard Deviation	3,642882	2,58047
Average Return	13,69458	10,96822
Sharpe Ratio	3,759272	4,250473

Table 3.3 Statistics for market portfolio and risk-optimized portfolios over 2010-2018

Surprisingly, simply buying the market has not been the best the strategy despite the poor absolute performance of factors over this time period. Market has indeed been performing exceptionally well, with Sharpe Ratio of 3,76 (compared to 2,15 in 1927-2018) with absolute annual average return of roughly 13,7 %. I argue that this also partly explains the relatively poor performance of factors. The factor risks are not correlated with market equity risk and thus the strong period of market performance does not necessarily lead to high factor returns. In fact, market has been performing so strongly, that the average factor performance in the long run is worse than the recent performance of market, which leads to diminishing of factor premiums.

However, the optimal portfolio still calls for only 68,4 % of market weight and 31,6 % of momentum weight despite the relatively poor performance of momentum seen in Chart 3, as this has generated an outstanding Sharpe Ratio of 4,25. I argue that this is explained by the negative correlation between momentum risk and market equity risk. The key finding here is the use of factor risks as *hedging mechanisms* against market portfolio's volatility. Even when the factors are not outperforming the market and they are not generating *absolute* excess-returns, exposure to factor risks can help to diversify the portfolio risks and thus generate higher risk-to-return ratios as seen in Table 3.3.

6. Conclusions and topics for further research

I showed that both value and momentum factors have generated notable excess-returns in the past and there are several logical reasons for their persistence also in the future. I also provided an example of how diversifying across factors can improve the return to risk -ratio even with portfolios that have already been diversified across the market. The excess-returns have been statistically significant with very high significance levels from 1927 to this date.

The major concern regarding investing in factors is market pricing the premiums out as the strategies become more popular or as informed investors attempt to arbitrage them. However, there seems to be little evidence that the publications of literature revealing the factors would have affected the premiums. For example, the first major crash in momentum premia happened around 15 years after the initial research publications.

Despite this, both value and momentum strategies have been performing poorly over the past years. I found evidence, that value strategies' poor performance is explained by general poor performance during strong bull markets. Similarly, I argue that post-financial crisis underperformance of momentum strategies is explained at least partly by time variability of betas in the momentum portfolio.

Despite diminishing factor premiums, I showed that diversifying across factors can provide higher risk-adjusted returns even when the absolute factor performance is low. Thus, diversifying across factors can act as a hedging mechanism against market equity risk and I argue that this leads to practical implementations of factors even if their absolute performance is lower in the future. Using factors for hedging could be studied more deeply in future research.

Whether the underperformance will persist in the future remains to be seen. In the future, this study could be expanded by adding more factors and especially by doing more analysis with the performance of multi-factor portfolios. The portfolios could also be more diversified across asset classes and geographical areas. Also, the list of behavioral explanations for these market anomalies will most likely never be fully completed as their effect is difficult to measure accurately.

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