Impulse method to calculate the frequency response of the electromagnetic forces on whirling cage rotors

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Abstract

The paper presents an impulse method to calculate the frequency response of the electromagnetic forces acting between the rotor and stator of a cage induction motor when the rotor is in whirling motion. Time-stepping finite element analysis is used for solving the magnetic field and the forces are calculated from the air gap field based on the principle of the virtual work. The impulse response method is applied to the finite element analysis by moving the rotor from its central position for a short period of time. This displacement excitation disturbs the magnetic field and, by doing this, produces forces between the rotor and stator. Using spectral analysis techniques, the frequency response function is calculated using the excitation and response signals. The forces are calculated from the frequency response function. The forces calculated by impulse response method are compared with the forces calculated by a conventional computation. The results show very good agreement. The use of impulse method to calculate the forces in electrical machines is also discussed.

1. Introduction

This paper presents a new way to define the forces acting between the rotor and stator when the rotor is performing cylindrical circular whirling motion with respect to the stator. This means that the rotor is aligned with the stator but the geometrical centreline of the rotor travels around the geometrical centreline of the stator in a circular orbit with a certain frequency, called whirling frequency, and certain radius, called whirling radius. The presented impulse method calculates the frequency response function of the electromagnetic forces on whirling cage rotors. The frequency response function presents the electromagnetic forces on a wide whirling frequency range, including zero, static eccentricity, and synchronous, dynamic eccentricity, frequencies.

An eccentric rotor creates a non-symmetric flux distribution that causes the unbalanced magnetic pull. The non-ideal field may induce circulating currents in the rotor cage and parallel paths of the stator winding. These currents tend to equalise the flux distribution and they may significantly reduce the radial force.

Conventionally, the forces acting between the rotor and stator have been studied by analytical means. There are lot of papers in which the effects of the rotor eccentricity on the unbalanced magnetic pull, i.e. forces, are studied analytically [1-4]. The problem with the analytical models is how to evaluate the equalising currents induced in the windings by the asymmetric flux distribution. The effects of saturation and stator and rotor slotting are also difficult to model by analytical means.

Numerical field calculation methods have only rarely been used for analysing eccentric rotors. DeBortoli et al. [5] used a time-stepping method for studying the equalising currents set up by an eccentric rotor in the parallel circuits of the stator windings. They also presented some results of the force calculation. Tenhunen [6] used time stepping analysis for studying the equalising currents and the forces when the rotor is not aligned with the stator. Arkkio and Lindgren [7] studied the forces in a high-speed motor using also time-stepping finite element analysis. Their study was focused on static eccentricity and the method of analysis was verified by measurements.

The references given above focus on the two special cases of whirling motion i.e. the static and dynamic eccentricity. However, the whirling motion of the rotor can also occur at other frequencies. Früctenicht et al. [8] developed analytical tools to study the cage induction motor in a more general whirling motion. Arkkio et al. [9] used numerical method to study the electromagnetic force for a general circular whirling motion. They modelled the whirling motion by forcing the centre point of the rotor to move along a circular path at constant speed i.e. they used a harmonic excitation to create the forces. They verified their calculations by measurements with active magnetic bearings and results showed excellent agreement. In Reference [9], the forces for a whirling motion are presented using frequency response function $K(i\omega_w)$:

$$F(i\omega_w) = K(i\omega_w)\varepsilon(i\omega_w) \tag{1}$$

where F is the force, ω_w is whirling frequency and ε is the displacement. According to Equation (1), the forces are linear functions of the displacement.

The time-stepping calculation of the forces between the stator and rotor by forcing the centre point of the rotor to move along a circular path at constant speed requires huge calculation times. This is the case especially if the interest is in the forces as a function of whirling frequency, because the forces have to be calculated separately for each whirling frequency.

The starting point for this study was to find out a way to calculate the forces for a wide whirling frequency range quickly keeping up the accuracy at the same level it has been in previous studies [6,9].

In mechanical engineering, the vibration characteristics of a system are widely studied by defining the frequency response function using different kind of excitations, including harmonic and impulse excitation. The transfer function $K(s)$ is a generalisation of the frequency response $K(i\omega)$. If the system is linear and there are no hidden sources inside the system, the transfer function, defined by impulse excitation, is exactly the same as the frequency response, obtained by harmonic excitation [10].

We propose an impulse response method to calculate the forces between the rotor and stator when the rotor is displaced from the centre point of the stator. The basic idea of the approach is to move the rotor from its central position for a short period of time. This displacement excitation disturbs the flux density distribution in the air gap, and by doing this, produces forces between the rotor and stator. Using spectral analysis techniques, the frequency response functions are defined from the excitation and response signals. The forces are then obtained from the frequency response function.

2. Methods of analysis

2.1 BASIC RESULTS BY ANALYTICAL MEANS

The magnetic field, currents and forces associated with whirling motion were studied in [8] by analytical means. The basic results are briefly summarised below to give background for the discussion of the numerical results.

A rotor in whirling motion produces harmonics in the air gap field. Only the harmonics of order $p\pm1$ interact with the fundamental harmonic of the machine and generate the forces. This is the reason why the effects of the other harmonics are usually neglected in the analysis. The eccentricity harmonics are of the form:

$$b_{p\pm1}(x,t) = B_{p\pm1}\cos\left[\left(p\pm1\right)x - \left(\omega_1 \pm \omega_w\right)t - \left(\varphi_p \pm \varphi_w\right)\right] \tag{2}$$

where p is the number of pole pairs, ω_1 is the fundamental frequency, ω_w is the whirling frequency, φ_p and φ_w are phase angles. The slips of the rotor with respect to these two harmonics are

$$s_{p\pm1} = 1 - \frac{p\pm1}{p}\frac{\omega_1}{\omega_1 \pm \omega_w}\left(1-s\right) \tag{3}$$

where s is the slip of the rotor with respect to the fundamental harmonic. The slips become zero at whirling frequencies

$$\omega_w = \frac{1-s\left(1\pm p\right)}{p}\omega_1 \tag{4}$$

If the slip of an eccentricity harmonic is nonzero, the harmonic induces currents in the rotor cage. The currents modify the amplitude and phase of the eccentricity harmonic and, by doing this, they affect the radial force. In general, the radial component of the force (in the direction of the shortest air gap) is reduced and the tangential component of the force is generated.

2.2. CALCULATION OF THE FORCES

The calculation of the magnetic field and operating characteristics of the induction motor is based on time stepping, finite-element analysis of the magnetic field. The details of the method are presented in Reference [11]. The magnetic field in the core region of the motor is assumed to be two-dimensional. End-winding impedances are used in circuit equations of the windings to model the end effects approximately. The magnetic field and circuit equations are discretized and solved together as a system of equations. The time-dependence of the variables is modelled by the Crank-Nicholson method.

The method presented by Coulomb [12] was used for computing the electromagnetic forces. It is based on the principle of the virtual work, and the forces are obtained as a volume integral computed in an air layer surrounding the rotor. In the two-dimensional formulation, the computation reduces to a surface integration over the finite elements in the air gap. The method was chosen because it has given accurate results when computing the forces of the electrical machines and it is verified by measurements [11]. The forces are calculated at each time step, and as a result, one gets the forces as a function of simulation time.

The motion of the rotor is obtained by changing the finite-element mesh in the air gap. Second order, isoparametric, triangular elements were used. A typical finite-element mesh for the cross section of the test motor contained about 10000 nodes.

Several simplifications have been made to keep the amount of computation to a reasonable level. The magnetic field in the core region is assumed to be two-dimensional. The laminated iron core is treated as a non-conducting magnetically non-linear medium, and the non-linearity is modelled by a single-value magnetisation curve. The homopolar flux, that may be associated with eccentricity, is neglected. This study concentrates on the four pole machines, in which the homopolar flux due to the rotor eccentricity is negligible [3,4]. The method of analysis should model properly the effects of equalising currents, slotting and saturation.

In theory, the transient response signal should be infinitely long. However, the response signal has only a finite length T of data. This is the reason why the frequency response function (FRF) was calculated by dividing the cross-spectral density of the response and excitation signals $S_{\mathrm{fx}}(\omega)$ by the auto spectral density of the excitation signal $S_{\mathrm{ff}}(\omega)$

$$K(\omega) = \frac{S_{\mathrm{fx}}(\omega)}{S_{\mathrm{ff}}(\omega)} \tag{5}$$

where subscript f corresponds to the excitation and subscript x corresponds to the response signal. More information of the signal analysis associated with the frequency response can be found from References [10,13].

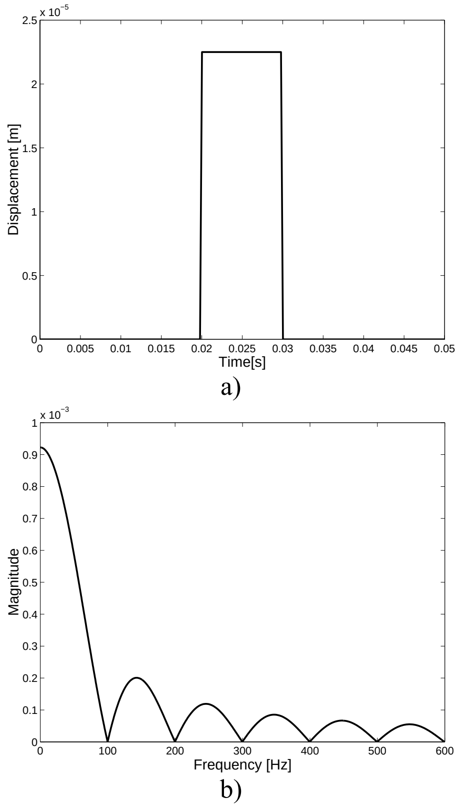
The idea of the approach is to move the rotor from its central position for a short period of time. This displacement excitation produces harmonics into the air-gap field, which in their turn create forces between the rotor and stator. Using spectral analysis techniques, the frequency response function (FRF) is calculated from the excitation and response signals. Now the frequency response function gives the forces per whirling radius for a studied frequency range. Hence, the forces can be calculated from FRF for a certain rotor eccentricity.

The direction of the pulse is fixed in the stator co-ordinate system and the forces are also calculated in the same co-ordinate system. There are several possibilities to choose the type of the excitation pulse. Two representative examples are a rectangular pulse and a cosine pulse, which is defined $\varepsilon(t) = 1\text{-}\cos(2\pi t/T)$, $t_1 < t < t_2$, where T is the length of the pulse. Figures 1 and 2 show the used displacement pulses in time domain and in frequency domain. Regardless of the type of the pulse, the parameters describing the pulse can be derived from the excitation requirements. First of all, it is required that all frequencies at the studied frequency range are excited. Then the upper bound frequency has to be smaller than the first frequency f_1, at which the response of the pulse is zero. It can be shown that there is a direct relationship between the frequency f_1 and the maximum duration of the pulse, $T_1 = t_2 - t_1$ [13]. This relationship can be written as

$$T_1 = \alpha_1 \cdot \frac{1}{f_1} \tag{6}$$

where α_1 is a constant depending on the type of the pulse and on the required flatness of the excitation force as a function of frequency. The value of α_1 is 1.0 for a rectangular pulse and 2.0 for a cosine pulse.

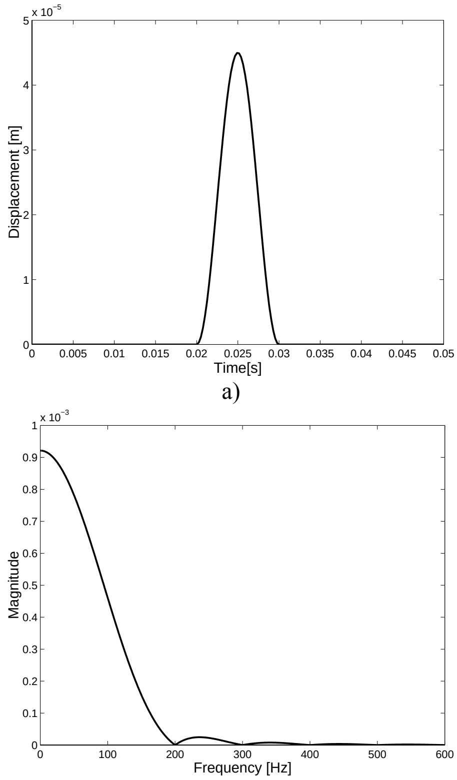
The cosine pulse is quite a good low-pass filter because, according to Figure 2, it gives almost all its energy to the demanded frequency range. The rectangular pulse, instead, excites also the frequencies higher than the frequency f_1 (Figure 1). Anyway, the both excitation pulses give almost identical frequency responses at the studied frequency range. One should keep in mind that this study is focused on the forces, created by eccentricity, the frequency of which is quite often smaller than the frequency of the fundamental field.



a)

b)

Figure 1. The rectangular displacement pulse a) in time domain and b) in frequency domain.



a)

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Figure 2. The cosine displacement pulse a) in time domain and b) in frequency domain.

In the time-stepping, finite element force calculation, there is always some numerical error because of the discretization. This white noise in the response signal might cause error in FRF. The duration and amplitude of the pulse define the amount of energy, which is given to the system. The bigger the energy, the better the signal to noise ratio. The frequency f_1 limits the duration of the pulse. Then, the amplitude of the pulse should be optimised somehow. Unfortunately, if the displacement pulse is too big, the forces are not anymore linear function of displacement [8]. Also the numerical accuracy of the finite element analysis suffers due to the changes in the air gap mesh. The amplitude of the displacement pulse used in the analysis is 10 – 20 % of the air gap.

3. Results

The test motor is a 15 kW four-pole cage induction motor. The main parameters of the motor are given in Table 1 and its cross-sectional geometry is shown in Figure 3.

Table 1: Parameters of the motor

Parameter	
Number of poles	4
Number of phases	3
Number of parallel paths	1
Outer diameter of stator [mm]	235
Core length [mm]	195
Inner diameter of stator [mm]	145
Airgap length [mm]	0.45
Number of stator slots	36
Number of rotor slots	34
Connection	Delta
Rated voltage [V]	380
Rated frequency [Hz]	50
Rated current [A]	28
Rated power [kW]	15

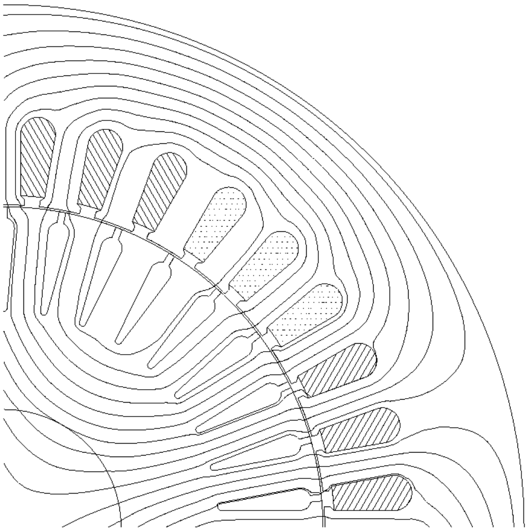

Figure 3. Cross-sectional geometry of the test motor.

At first, the forces were calculated at no-load condition using rectangular pulse excitation. According to Equation 4, the slips associated with the eccentricity harmonics are simultaneously zero when the whirling frequency is equal to the rotation frequency.

The forces are divided into a radial component in the direction of the shortest air gap and a tangential component perpendicular to the radial one. Fig. 4 shows the computed radial and tangential components of the force with respect to the direction of the pulse.

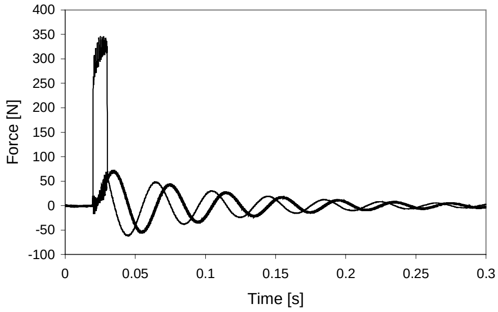

Figure 4. Calculated electromagnetic forces in a time domain for a 15% rectangular pulse. Thin line the radial component and thick line the tangential component of the force.

In the analysis, the duration of the horizontal rectangular displacement pulse was 0.01 s and the amplitude was 15% of air gap length. Total simulation time was 1 s with constant time-step of 0.05 ms. The excitation pulse was given in the stator co-ordinates because the forces were calculated in the stator co-ordinates.

The discrete excitation and force signals were then transformed into the frequency domain by the fast Fourier transform without filtering or windowing. The number of sample points was 8192, and the length of the signal was extended to be 2 s by adding zeros to the end of the sample in order to obtain a frequency resolution of 0.5 Hz. FRF, calculated by Equation (5), is presented in Figure 5.

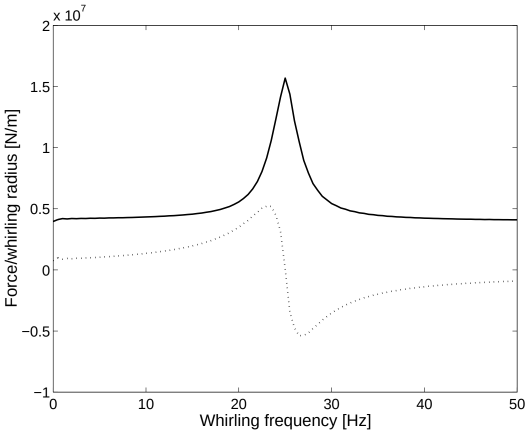

Figure 5. The frequency response function at no load condition. Thick line, the radial and dashed line the tangential component.

Actually, FRF gives the electromagnetic force per whirling radius as a function of whirling frequency. Assuming the spatial linearity, the force for a specified whirling radius is easy to calculate from the frequency response function FRF by multiplying it by the absolute value of eccentricity. The radial force has a sharp maximum at the synchronous whirling frequency. As described in Section 2.1, the eccentricity produces harmonics with pole numbers $p\pm1$. These harmonics do not induce any damping currents at the synchronous whirling frequency and the maximum radial force occurs.

The load of the motor affects the radial and tangential force distributions. According to Equation 4, the single radial-force maximum is now divided into two force peaks. The eccentricity harmonics p-1 and p+1 of the air gap have their zero slips at whirling frequencies of 25.8 Hz and 22.6 Hz. At these frequencies, the corresponding rotor current is zero, and because the flux density harmonic is not damped by rotor currents, it has its maximum amplitude. As the two eccentricity harmonics make the main contribution to the force, it also has two maxima close to those whirling frequencies at which the harmonics have zero slips. As the used frequency resolution is 0.5 Hz, the calculated force peaks are not exactly at the right frequencies. This inaccuracy causes leakage and the calculated peaks are shifted to the discrete calculated frequencies. Figure 6 shows the radial and tangential components of the force computed by impulse method and forced whirling method as a function of whirling frequency. The motor is running at the rated speed (s=3.2%). The whirling radius is 50 µm. The method, which is called forced whirling method is the traditional way to calculate numerically the forces due to the rotor eccentricity. In this method, the centre point of the rotor is forced to move along the circular path at constant whirling frequency. The calculation is done for 36 different whirling frequencies to reach the results presented in Figure 6. In the impulse response calculation, the length of the cosine displacement pulse was 0.01 s and the amplitude 15% of air gap length. The total simulation time was 1 s with a constant time-step of 0.05 ms. The forces are calculated from the FRF by multiplying it by whirling radius. The results of the two methods show excellent agreement.

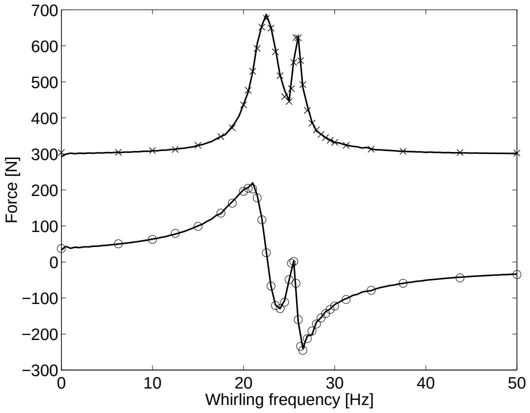

Figure 6. Electromagnetic forces calculated by a whirling and impulse method. The results of the forced whirling method are marked by x the radial and ○ the tangential component.

4. Discussions

The impulse method is computationally very effective requiring less than 5 % of the computational time of the conventional forced whirling method to calculate the forces as a function of whirling frequency at some frequency range. The advantage of the impulse method is that one simulation gives the forces for a demand frequency range. The accuracy of the method seems to be very good. The results are compared with the conventional calculation, which is verified by measurement in Reference [9].

The used spectral analysis is quite simple. We have not used windowing or filtering to improve the response signal. The exponential window might improve the accuracy of the method, but the effects of windowing should be compensated. The correction for exponential windowing can only be done in parametric force model level [13]. The problems arising from the finite length of response data, such as truncating or leakage, cause some error to the calculated forces. According to the results, this error is small. The method is quite sensitive to the accuracy of the time-stepping finite element analysis. The calculated forces at the end of the simulation are small. The discretisation error in the field solution causes an error in the force calculation and then deteriorates the accuracy of the method. However, the results presented show that despite of the potential error sources, the method gives good results.

5. Conclusions

This paper presents the impulse method to calculate the frequency response of the electromagnetic forces acting between the rotor and stator of a cage induction motor when the rotor is in whirling motion. The presented impulse response method is applied to the time stepping finite element force calculation by moving the rotor from its central position for a short period of time. This displacement excitation produces a force between the rotor and stator. Using spectral analysis techniques, the frequency response function of the electromagnetic forces is calculated using the excitation and response signals. The force is then calculated from the frequency response function. The results of the impulse method are compared with the results got by conventional calculation. The two methods yield almost equivalent results. The new method is computationally more efficient requiring less than 5 % of the computational time of the conventional method when studying the forces as a function of whirling frequency. The advantage of the impulse method is that it gives the forces for a wide frequency range, including as special cases the static and dynamic eccentricities, by one simulation.

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