Aalto-yliopisto
Aalto-universitetet
Aalto University

Master’s Programme in Electrical Engineering

Modelling and analysis of losses in a permanent magnet synchronous motor

Jan Chvojka

Master’s Thesis
2022

Aalto-yliopisto
Aalto-universitetet
Aalto University

Author Jan Chvojka

Title of thesis Modelling and analysis of losses in a permanent magnet synchronous motor

Programme Automation and Electrical Engineering

Major Electrical Power and Energy Engineering

Thesis supervisor Prof. Anouar Belahcen

Thesis advisor(s) D.Sc Martin Floran

Date 04.05.2022 **Number of pages** 44 / 8 **Language** English

Abstract

This thesis was done in the Electromechanics group of Department of Electrical Engineering, Aalto University as a part of electromechanics research. Thesis is concentrated on analysing losses in permanent magnets of concentrated winding, outer rotor with surface mounted magnets of permanent magnet synchronous machine.

The approach to establish permanent magnet losses is done via two-dimensional simulation in FEMM software with post-process analytical calculation. To compare results obtained from this approach, simulation in COMSOL Multiphysics environment, purely analytical computation following Attalah's method and EMDtool simulation is used. EMDtool simulation is taken as a reference, hence it is considering the end effect via hybrid 2D-3D method and solving the electric scalar potential in 3D.

Results obtained from thesis method were compared with all mentioned used methods. With using the end effect coefficient, used method shows effective way how to estimate eddy current losses in permanent magnets.

Keywords Eddy current, PM losses, PMSM, FEMM, MATLAB, FEM, EMDtool.

Contents

Preface ... 6
Symbols and abbreviations ... 7
Symbols ... 7
Operators ... 8
Abbreviations ... 8
1 Introduction ... 9
1.1 Objective ... 9
2 State of the art ... 12
2.1 Basic theory about the PMSM under investigation ... 12
2.1.1 Design of the PMSM ... 12
2.1.2 PM losses sources ... 14
2.2 Methods for computing losses in magnets ... 15
2.3 Analytical models ... 15
2.3.1 Two-dimensional ... 15
2.3.2 Three-dimensional ... 16
2.4 FEA models ... 17
2.5 Hybrid models ... 19
2.6 Coefficient linking losses ... 19
3 Methods used in the thesis ... 20
3.1 Analytical method ... 20
3.2 FEA methods ... 22
3.2.1 COMSOL ... 22
3.2.2 EMDtool ... 25
3.3 Thesis method ... 26
4 Results ... 30
4.1 Analytical method results ... 30
4.2 FEA results ... 31
4.2.1 COMSOL results ... 31
4.2.2 EMDtool results ... 35
4.3 Thesis method results – FEMM + post-processing calculations ... 39
4.4 Comparison of all results obtained ... 43

5 Discussion 46
6 Thesis outcome 48
6.1 Conclusion 48
6.2 Future work 48
7 References 49

Preface

I would like to start off with a quote, “To like people and love people is the whole secret and the only one recipe for happiness.” ~ Jan Werich. Firstly, I would love to express my gratitude to Prof. Anouar Belahcen, the supervisor of my thesis, for giving me the opportunity to do the thesis under his guidance. Also, I have to express how amazed I was by his friendly attitude, trust in others ability and heart-warming kindness. It is very rare, and I appreciate it so much.

I also want to thank D.Sc. Floran Martin for his support during this period. His devotion to work and to help others is something amazing to witness. I am thankful for his technical assistance and practical guidance necessary for this thesis to be completed.

I wish to thank my colleague and friend, D.Sc. Antti Lehikoinen, for always being around with advice. I am grateful for his constant help through all the time he has to spend with not so profound questions of mine. Thank you, Antti, for other discussion we are still having and for the laughter.

Also, I would love to express my gratitude to all my colleagues I had luck to meet in the group of electromechanics. I always felt like the happiest person on earth around them all. Thank you all for becoming my friends.

I need to thank my parents Libor and Helena, my brothers Libor and Petr and my sister Martina, for love and support throughout my studies. I would like to thank my fiancé, Adela, for having the patience with me not only throughout my studies and for the love I can feel every day. Finally, I wish to thank to my son Jan for those few calm moments I needed to write and finish this thesis.

Otaniemi, 4 May 2022

Jan Chvojka

Symbols and abbreviations

Symbols

$\mathbf{A}$	magnetic vector potential
$\mathbf{B}$	magnetic flux density
$\mathbf{E}$	electric field intensity
f	frequency
F_{cn}	correction factor
$\mathbf{H}$	magnetic field intensity
I_m	peak phase current
$\mathbf{J}$	current density
J	eddy current in magnet
J_0	mean value of eddy current in magnet
$K_{\omega n}$	winding factor
L	axial length of magnet
l	length of magnet
m	number of phases in the machine
n	space harmonic order
n	speed in rpm
N_s	number of series per phase
p_r	number of rotor magnet pole pairs
p_s	number of stator winding pole pairs
R_m	magnet radius
R_r	rotor radius
R_s	stator radius
S_{pm}	surface of magnet
w_m	width of magnet
α	magnet pole arc
μ_0	permeability of vacuum
μ_r	relative permeability
ρ	electrical resistivity
σ	electrical conductivity
Ω	angular velocity of the rotor
ω	speed in mechanical angles
ω_n	angular frequency

Operators

$\nabla \times \mathbf{A}$	curl of vector **A**
$\frac{\delta}{\delta t}$	partial derivative with respect to variable *t*
Σ_i	sum over index
$\mathbf{A} \cdot \mathbf{B}$	dot product of vectors **A** and **B**

Abbreviations

AC	alternating current
DC	direct current
FEA	finite element analysis
MEC	magnetic equivalent circuit
MMF	magnetomotive force
PM	permanent magnet
PMSM	permanent magnet synchronous motor
PWM	pulse-width modulation
rpm	revolutions per minute

1 Introduction

Permanent magnet synchronous motors (PMSM) are widely used motors in traction and in applications where high-speed and high efficiency is needed e.g., electrical vehicles. Applications in electrical vehicles is the most investigated area of PMSM [1]. Usage of PMSM in traction and in high-speed applications mainly grows because of improvement in the manufacturing technology of permanent magnets e.g., NdFeB and SmCo [2]. Main advantages of PMSMs are high performance, silent operation, high efficiency, and good dynamic parameters [1].

However, permanent magnets might become demagnetized due to excessive heating. Excessive heating is caused by eddy current losses in permanent magnets of the motor. Eddy currents usually generates thermal heat and that is causing mentioned losses [3]. Temperature issues arises especially in high-speed applications, where significant losses are induced in the rotor. Eddy current losses in permanent magnets mainly occurs due to the design of the motor and PMs. With new PMSM designs emerging, sources of eddy current losses in PMs arises in a form of unsynchronized space harmonics and slotting effect [4], [5]. One of techniques to reduce the amount of eddy current losses created in PMs is called magnet segmentation [3].

The PMSM under investigation in this thesis is an outer rotor, concentric winding, surface permanent magnet in-wheel motor, which geometry can be seen in Figure 1. In PMSM chosen for this thesis, used design of winding is an important contribution to eddy current losses. Concentric winding can have many advantages, starting with manufacture process, reduces stator resistance and related resistive power losses [1]. On the other hand, it is causing more space harmonics, which means higher eddy current losses in the machine.

To prevent PMs from demagnetization, a need has arisen to efficiently predict eddy current losses. Methods to predict eddy current losses in PMs, has been developed since 1970s [6]. Nowadays, losses computation is carried out with finite element method software. To obtain the most accurate result, three-dimensional finite element computations are used. However, these computations require long time. Two-dimensional finite element computations are efficient and accurate enough if some three-dimensional issues are accounted for [1].

1.1 Objective

Main objective of this thesis is to evaluate feasibility of hybrid method approach to calculate losses in permanent magnets of specific PMSM. To be more specific, the analysis of the losses in the permanent magnets will focus on losses created by eddy currents. Thesis method is implemented in two steps:

1. Two-dimensional FEA in FEMM.
2. Post-process analytical calculation of eddy current losses in PMs.

Simulation of the MATLAB-FEMM environment, concerning described motor, has been obtained without implementation of PMs eddy current losses calculation. Calculations of permanent magnet losses will be implemented into obtained simulation script in MATLAB and compared with other computational methods.

Design of the PMSM used in this thesis will be described in Chapter 2. Moreover, origin of eddy current losses in the PMSM will be detailly described and other methods for computing eddy current losses will be gathered, assembled and explained.

Chapter 3 will describe methods used in this thesis, their advantages, limitations, and disadvantages.

In Chapter 4, results of each used method will be presented and compared.

Discussion will be carried out in Chapter 5. Obtained results will be summarized in Chapter 6, Thesis outcome.

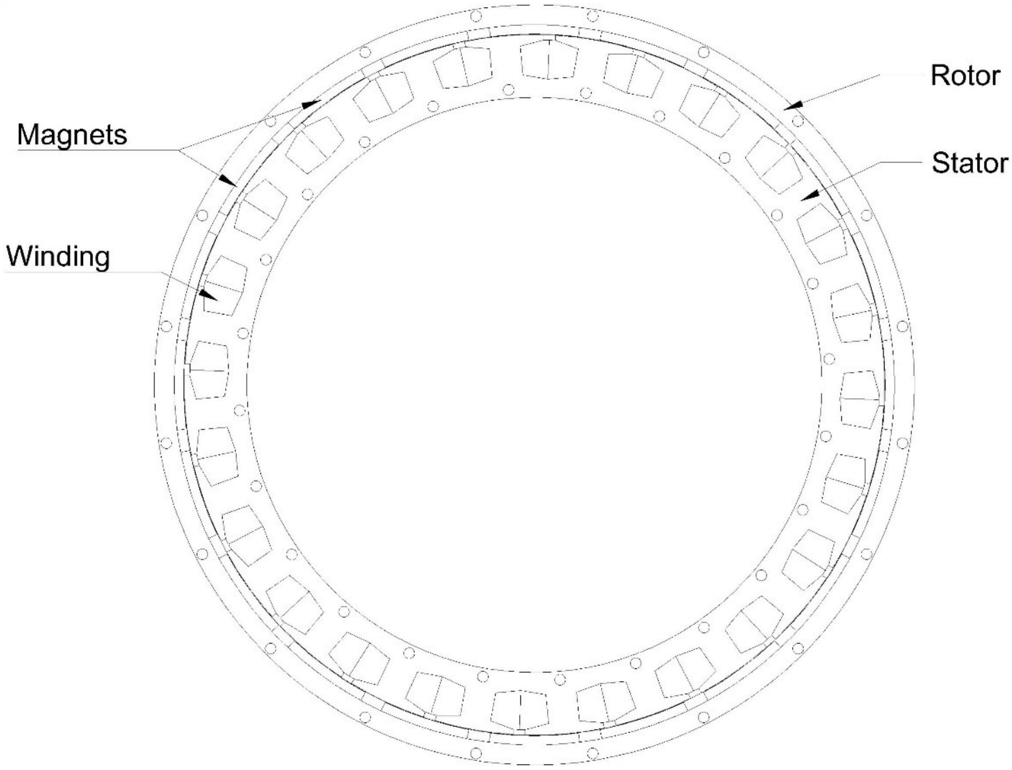

Figure 1: PMSM geometry used in the thesis.

2 State of the art

2.1 Basic theory about the PMSM under investigation

2.1.1 Design of the PMSM

The defined design and chosen application play a huge role in precision of permanent magnet losses estimation. Hence, we are dealing with high-speed application, demands of precise eddy current losses estimation is needed. PMSM chosen for this thesis is a motor with concentric windings, outer rotor with surface-mounted permanent magnets.

The concentric (also toothed) winding is wound in one or more positions on the ferrous or non-ferrous core. In this type of winding, each thread occupies approximately same magnetic flux, so that the same voltage is induced in each thread. The coils of a three-phase toothed winding are always wound around one tooth, and in comparison, with other types of windings it is possible to obtain tight-fitting winding surfaces with very good heat dissipation to the stator. Due to the high filling with copper conductor and small winding surfaces, the motors become more compact. This technique can be better automated than the current slotting technique for distributed winding. For segment stators, there are pre-prepared coils they can slide directly on individual teeth. In the case of one-piece sheet metal bundles, the technique of hairpin winding of conductors is used. In high voltage applications, such as wind generators, concentric winding brings the possibility of omitting the interphase insulation, because the coils have no contact with each other in the winding surfaces and inside the slotting. It is not the case of this thesis, there are needed in this layout, as it has coil-sides from different phases in some slots. Machines with concentric type of winding have higher efficiency and smaller dimensions for the same power than machines with distributed windings. Concentric winding distribution in one quarter of a motor under investigation can be seen on Figure 2.

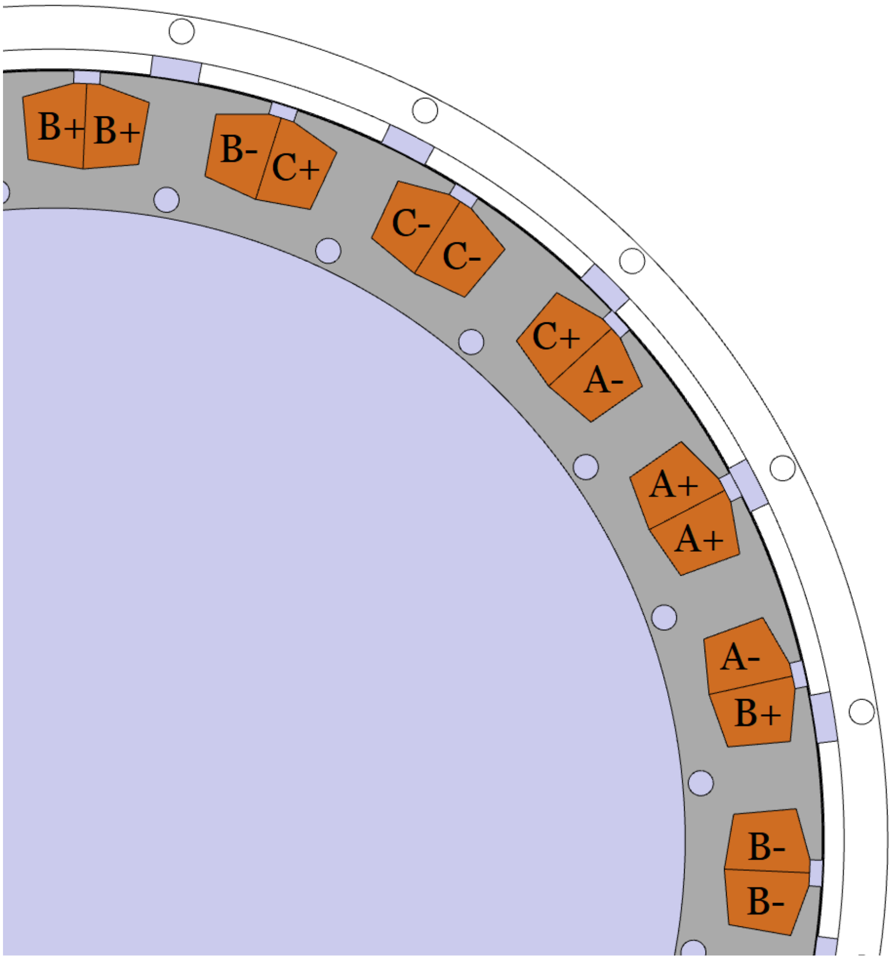

Figure 2: Concentric winding distribution of the motor under investigation.

Moving on to the rotor design, as stated, rotor is designed as an external rotor with surface permanent magnets. Motors with an external rotor are used in wheel drives, where the rotor with permanent magnets surrounds a three-phase stator. This type of design is used in applications in which the motor is integrated directly into the wheel structure. The stator is connected to the wheel axle; the rotor is part of the wheel. The principles of power supply and control of external rotor motors are the same as for internal rotor motors. The advantage of motors with an external rotor compared to the classic design is the higher achievable torque that the motor generates while maintaining the same volume. The disadvantage is the poorer removal of heat losses from the interior of the machine. Removal, or heat losses themselves, are essential for permanent magnets. Once heat losses exceed a predefined heat limit for permanent magnets, demagnetization can occur. This scenario can be considered as a major failure of the motor.

The magnets used in the PMSM under investigation, are surface mounted. This technical solution gives high magnetic induction in the air-gap, hence passing magnetic flux is not affected by different material between PM and the air-gap. Slight disadvantage of this technical solution could be low mechanical strength but wrapping impregnated glass bandages around the rotor with permanent magnets can increase the mechanical strength as well as protect magnets from mechanical point of view.

2.1.2 PM losses sources

PM eddy current losses sources can be divided into these.

- Winding distribution.
- Slotting effect.
- Supply and control of the motor.

The winding distribution plays an important role for eddy current losses creation in permanent magnets. With the variation of the flux, space harmonics of the MMF creates and induces eddy current losses. The amount of eddy current losses can be risen by the fundamental and lower order MMF harmonics [7], [3]. Asynchronous components of the MMF spatial harmonics are directly related to eddy currents [8]. Then, low order of spatial harmonics, which are unsynchronized, causes important amount of eddy current losses [9]. Concentric winding has an advantage for having a small coil-end, however, eddy current losses are becoming large, because of the presence of low order spatial harmonics in armature MMF [10].

The slotting of the stator brings harmonic content in the flux. Slotting can be explained as a permeance variation due to the teeth of the stator, which is the interaction of the magnetic field with the magnets, from which asynchronous field harmonics arise [11].

Supply currents fed into the stator are creating asynchronous components in the air-gap field in all electrical machines [12]. The other part is the control of the motor. PMSM controlled by pulse width modulation (PWM), which is varying stator magnetic field at high frequencies, causes losses in the magnets. The main cause of those losses are carrier harmonics of the PWM inverter [13].

2.2 Methods for computing losses in magnets

Different methods for calculation of PM were developed throughout the years. The first analytical models were developed in the early 1970s. To logically split different methods, firstly analytical models will be discussed, followed by FEA models and at last hybrid models will be described.

2.3 Analytical models

As stated above, the first analytical models were developed early in the 1970s. Authors of [6] calculated the eddy currents in thin conducting plates with using the Ritz and Kantorovich variational methods for solving the field equation and calculating power losses. For applying the stream function, some assumptions need to be satisfied [6]:

- Secondary magnetic field due to eddy currents is negligible in comparison with the exciting flux [6].
- Thickness d of the conducting plate has to be much smaller than the depth of penetration of the electromagnetic wave [6].

In [14], the authors used surface impedance to predict eddy current fields in conductors. Authors of [15] then proposed representation of PM by resistances in the equivalent circuits. The end effects in [15] were neglected. One-dimensional field was considered, and analytical electromagnetic model was developed to predict parasitic losses by authors of [16].

2.3.1 Two-dimensional models

Striving for higher precision, mathematical model to predict eddy current losses, which analysed two-dimensional electromagnetic field in polar coordinates was developed [12]. In [12], authors developed analytical model based on a two-dimensional polar coordinate model and calculation of the magnetostatic field distribution in the slotted airgap and magnet regions [12]. The losses were predicted in both the retaining sleeve and magnets for any specified load. In this paper authors have found that the developed magnetostatic model has a good agreement only at somehow low inducing frequencies. This approach was followed by [3], while assumption of rotor and stator cores being infinitely permeable was made. Extended model, which accounted time harmonics in the stator MMF distribution was then found by authors of [17]. Improvement, by considering the reaction field, was done in [7].

In [18], authors consider the eddy current three-dimensional flow, so correction factor for two-dimensional analytical model was proposed:

$$F_{cn}(\omega_n) = 1 + \frac{2/(aL)}{\coth\left(\frac{\lambda L}{2}\right) + \left(\frac{a}{\gamma}\right)\coth\left(\frac{\lambda L}{2}\right) - 2a/L8y^2} \tag{1}$$

Where L is the axial length of magnets, a, λ, and γ for the n^{th} harmonic are given by [18]:

$$\begin{aligned} a &= \frac{np_s}{R_s}; \\ \lambda &= \sqrt{j\omega_n\mu_0\mu_r\sigma}; \\ \gamma &= \sqrt{a^2 + \frac{\lambda}{g\mu_i}}, \end{aligned} \tag{2}$$

where μ_r and σ are relative permeability of the coil and magnets respectively, ω_n is the angular frequency of the n^{th} harmonic [18].

Using a calculation method which includes a nonlinear magnetic equivalent circuit (MEC) was applied on interior PMSM in [19]. That method considers magnetic saturation. In [20], authors used MEC or reluctance network analysis method for calculating eddy current losses.

Research article [21] is giving information about possibility of under or over-estimation of PM losses. This is presented in such a way that interaction of eddy current harmonics with the same frequency, but different spatial order should not be neglected. Regarding the final permeability of the stator and rotor cores and accurate PM transmittance, eddy current losses were analytically calculated for slotless PMSM, where the eddy current reaction field was neglected, and the induced eddy currents were resistively limited [22].

In [23] subdomain model was shown and was applied to slotted PMSM with emphasis that eddy currents and the diffusion effect were not resistance limited.

2.3.2 Three-dimensional models

Three-dimensional calculation of pm eddy current losses is essential from the seek of precision.

Study of PM eddy current losses based on quasi-three-dimensional analytical model was established in [24].

Then, the end effects were considered in [25], as well as the reaction field. Authors of [25], also gave an insight about the armature reaction and PM slotting effect.

Three-dimensional high frequency eddy current losses for surface mounted PMSM were predicted and proposed via modified image theory in [26].

2.4 FEA models

FEA methods to simulate eddy current losses in PMSM are most commonly used nowadays. FEA can be done in two-dimensional environment as well as in three-dimensional environment.

In [27] and [28] authors used A-q method with double nodes at slits to get overcome the difficulties with these slits of magnets. Those slits are increasing the computational time, so approach of [27] and [28] is giving higher computational efficiency in this sense.

Usually, three-dimensional eddy current losses calculations with consideration of end effects are taking a lot of time. A study [29] is giving an interesting approach to save computational time. Firstly, two-dimensional nonlinear time-domain analysis was applied with PWM voltage waveform. Afterwards, for each remarkable harmonic, three-dimensional frequency domain analysis was used [29]. At last, the total eddy current losses were summarized in two steps, presented in Figure 3.

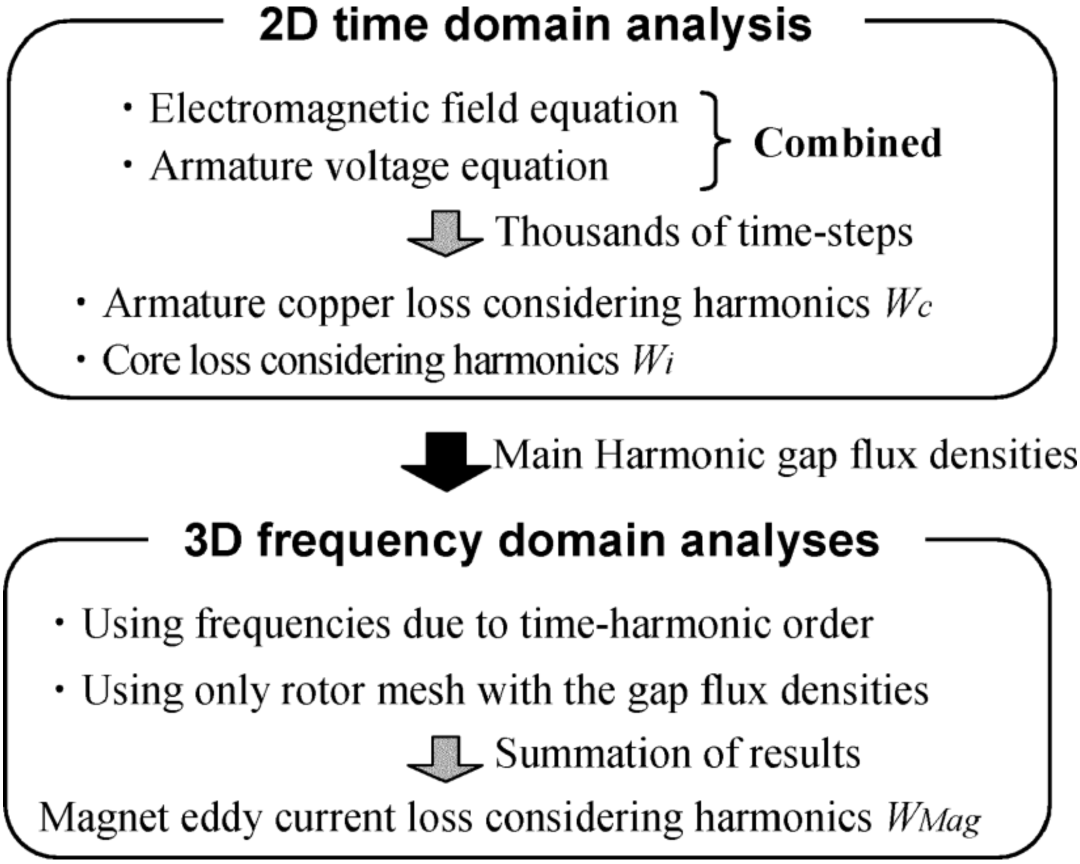

Figure 3: Outline of proposed loss calculation method [29].

Study in [30], gives explanation of three different methods on how to calculate eddy current losses in PMSM, where concentric winding is used:

1. FE magneto-static method with analytical post-processing:
   Losses were calculated analytically from flux density values which were obtained numerically. Eddy current reaction field was neglected, same for inducing effect.
2. FE magneto-static method:
   FEA software calculated directly PMs losses without any additional post-processing calculations. Reaction field was neglected.
3. FE magneto-dynamic method:
   Time-stepping method for solving Maxwell's equation for moving rotor and for any current waveform was used. Reaction field was considered, but it took a longer time.

The research article [31] is combining two-dimensional and three-dimensional methods with considering magnetic saturation of the rotor and harmonics for calculation of eddy current losses.

[32] proposed an approach to calculate eddy current losses with a two-dimensional magnetic vector potential solver coupled to a modified axial *2DTΩ – Solver*, so PM axial length can be included.

2.5 Hybrid models

Hybrid methods for calculation of eddy current losses in PMs are a combination of two different approaches.

Authors of [33] presented a combination of derived current sheet by two-dimensional analytical method and three-dimensional FEA. Nodal method in [34] was developed. Authors of [35] and [36] presented the algorithm for mapping eddy current losses in surface mounted PMSM over a different operating condition. Half analytical and half two-dimensional FEA to determine eddy current losses in PM was proposed in [37].

2.6 Coefficient linking losses

To correct the results obtained from each method, a coefficient accounting for end effects k_L is implemented in [38].

$$k_L = \frac{3L^2}{4(L^2 + w_m^2)}, \tag{3}$$

where L is the length of magnet and w_m is the width of magnet.

3 Methods used in the thesis

As was described in previous chapter, there are many methods to be used for obtaining results of eddy current losses in permanent magnets. In this chapter, more detailed explanation of the used methods in this thesis will be described.

3.1 Analytical method

This method formulates the model in two dimensional polar coordinates. The method picked for purely analytical calculation is assuming the stator and rotor cores to be infinitely permeable and was presented in [3].
The equivalent current sheet, which represents the stator winding with respect to the rotor reference system, is given by

$$J_s(\theta_r, t) = \begin{cases} \frac{q}{2}\sum_n J_n \cos(np_s\theta_r + (np_s - p_r)\Omega t), & for\; n = qk + m \\ -\frac{q}{2}\sum_n J_n \cos(np_s\theta_r + (np_s + p_r)\Omega t), & for\; n = qk - m \\ 0, & for\; n \neq qk + m \end{cases} \tag{4}$$

where $m = \pm 1$, n is the space harmonic order and J_n is described as follows,

$$J_n = \left(\frac{2N_s I_m}{\pi R_s}\right) K_{\omega n}, \tag{5}$$

where N_s and I_m are number of series per phase and the peak phase current, respectively, and R_s and $K_{\omega n}$ are the stator radius and winding factor, respectively [3].
The eddy current density induced in one magnet of the motor is given by [3]

$$\begin{aligned} J_m(r, \theta_r, t) &= -\frac{1}{\rho}\frac{\partial A(r, \theta_r, t)}{\partial t} + C(t), \\ C(t) &= \sum_{n=1}^{\infty} C_n(t), \end{aligned} \tag{6}$$

where ρ is electrical resistivity of the PM, A is the magnetic vector potential distribution and C is an integration constant to ensure that at any instant zero net current flows in the PM [3].
To derive eddy current loss in one PM equation 6 is presented as [3]

$$
\begin{aligned}
P &= -\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega}\int_{R_m}^{R_r}\int_{-\alpha/2}^{\alpha/2}\rho J_m^2 r\, drd\theta_r dt \ \ (W/m) \\
P &= \sum_{n=1}^{\infty}(P_{cn}+P_{an}),
\end{aligned}
\tag{7}
$$

where

$$
\begin{aligned}
P_{cn} = \frac{m^2\mu_\sigma^2\alpha}{8\rho}\frac{J_n^2}{n^2p_s^2}(np_s \pm p_r)^2\Omega^2 & \\
\times\Bigg[\left(\frac{R_s}{R_m}\right)^{2np_s} & R_s^2R_m^2F_n \\
+\left(\frac{R_s}{R_r}\right)^{2np_s} & \frac{R_s^2R_r^2}{(2np_s+2)}\times\left(1-\left(\frac{R_m}{R_r}\right)^{2np_s+2}\right) \\
+\left(\frac{R_s}{R_r}\right)^{2np_s} & R_s^2(R_r^2-R_m^2)\Bigg]\Bigg/\left[1-\left(\frac{R_s}{R_r}\right)^{2np_s}\right]^2,
\end{aligned}
\tag{8}
$$

in which

$$
F_n = \frac{\left[\left(\frac{R_s}{R_m}\right)^{-2np_s+2}-1\right]}{(-2np_s+2)},
\tag{9}
$$

for $np_s \neq 1$ and

$$P_{an} = -\frac{m^2\mu_\sigma^2}{\alpha\rho}\frac{J_n^2}{n^4 p_s^4}(np_s \pm p_r)^2\Omega^2 \times \left[\left(\frac{R_s}{R_m}\right)^{np_s} R_s R_m^2 G_n + \left(\frac{R_s}{R_r}\right)^{np_s} \frac{R_s R_r^2}{(np_s+2)}\left(1-\left(\frac{R_m}{R_r}\right)^{np_s+2}\right)\right]^2 \times \frac{\sin^2\left(np_s\frac{\alpha}{2}\right)}{(R_r^2 - R_m^2)\left[1-\left(\frac{R_s}{R_r}\right)^{2np_s}\right]^2}, \tag{10}$$

in which

$$G_n = \frac{\left[\left(\frac{R_r}{R_m}\right)^{-np_s+2} - 1\right]}{(-np_s+2)}. \tag{11}$$

For $np_s \neq 2$ and where n is the space harmonic order, p_s and p_r are the number of pole pairs associated with the stator winding and rotor, respectively [3]. Ω is the rotor angular velocity, R_r and R_m are the rotor and magnet radius, α is the magnet pole arc and m is number of phases in machine.

The used analytical method developed by [3] is giving somewhat higher estimation of induced eddy current losses by MMF stator harmonics. This overestimation is mainly caused by the picked space harmonic order and neglection of end effects in the PMs. End effects were later accounted for by applying the coefficient for end effects, which will be discussed further in this section and presented in the 4 Results section.

3.2 FEA methods

To get additional confirmation of the obtained results and for verification purposes, FEA methods for eddy current losses in permanent magnets were used. Firstly, the whole motor two-dimensional design was constructed in COMSOL software and simulated. Later, EMDtool for MATLAB with Gmsh, an automatic three-dimensional finite element mesh generator, was used to construct PMSM motor design and calculate eddy current losses in magnets.

3.2.1 COMSOL

The COMSOL Multiphysics is a software based on finite element analysis and is used for modelling and simulation of desired physics-based system. In this thesis the first step was to prepare the geometry of the motor. Pre-processing, preparation of model geometry and setting boundary conditions is a time-consuming process. Figure 4 show COMSOL environment prepared for this thesis task.

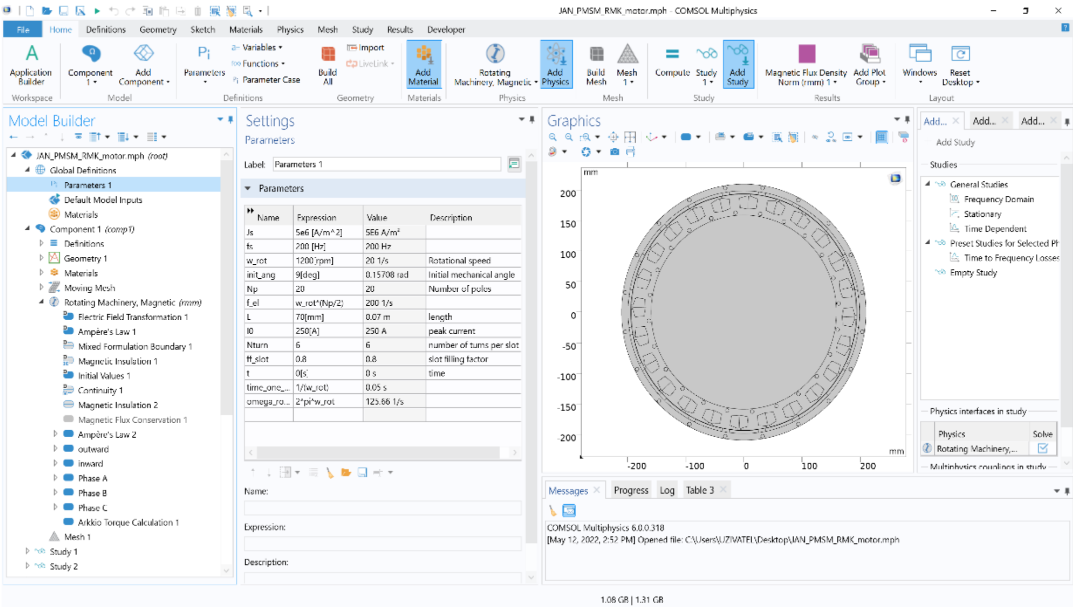

Figure 4: User environment of COMSOL.

After the geometry and parameters has been set, materials are assigned to each part of the motor. Then, boundaries and desired physics for the simulation has been chosen.

In this thesis, AC/DC Module of the COMSOL software has been used. AC/DC module contains simulation of electrostatics, rotating electrical machines, electro-quasistatic approximation, direct current and magneto-quasistatic approximation.

Then, study has been chosen as Rotating Machinery, Magnetic. Firstly, Ampère's Law is settled for air domains. For stationary study, equations used can be expressed as,

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}, \tag{12}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \tag{13}$$

$$\boldsymbol{J} = \sigma \boldsymbol{E}, \tag{14}$$

where $\boldsymbol{H}$ is the magnetic field intensity, $\boldsymbol{J}$ is the current density, $\boldsymbol{B}$ is the magnetic flux density, $\boldsymbol{E}$ is electric field intensity and σ is electrical conductivity.

For time dependent study, additional equation is applied,

$$\boldsymbol{E} = \frac{\partial \boldsymbol{A}}{\partial t}, \tag{15}$$

where $\boldsymbol{A}$ is the magnetic vector potential. Then, magnetic insulation has been defined by assuming equation of,

$$n \cdot \boldsymbol{B} = 0. \tag{16}$$

After, Ampere's law is applied for stator and rotor domains with magnets considered having different remanent flux direction – inward (South), outward (North).

Simulation part consists of three studies. First is stationary, second contains two steps: Step 1: Stationary, Step 2: Time Dependent, and third is used to establish losses, Time to Frequency Losses.

To obtain losses from one magnet, integration of the magnet area is done in pre-processing. Also, global variable which contains magnet integration with energy lost in magnet area is expressed. Integrations can be seen on Figure 5.

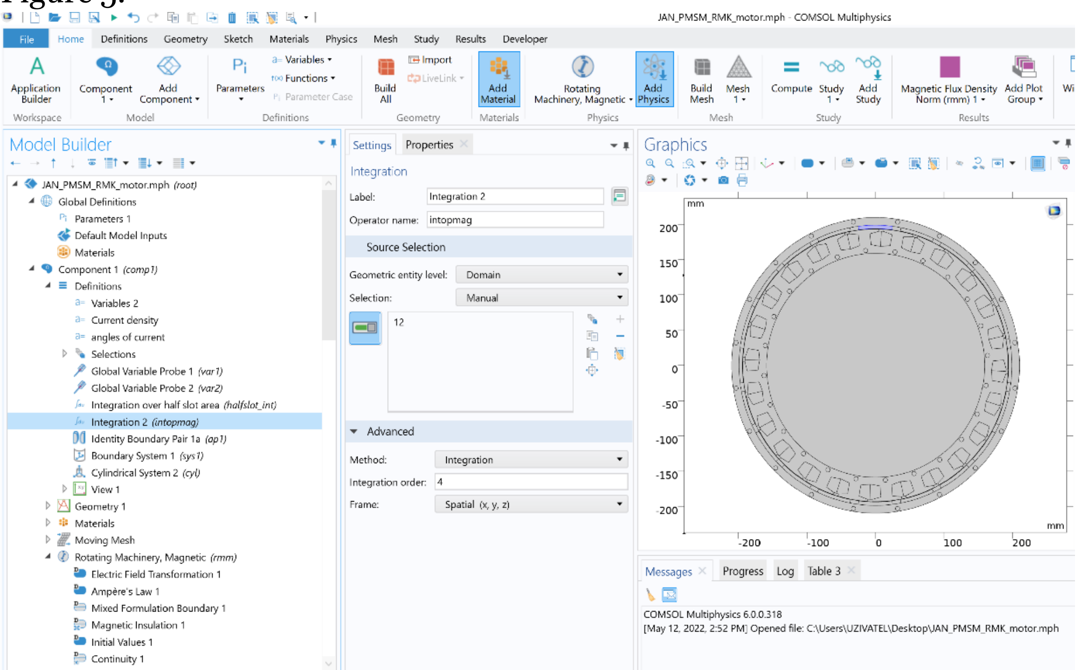

Figure 5: Integration of magnet in COMSOL Multiphysics.

Figure 6 shows global variable, and how exactly losses were established after simulation process.

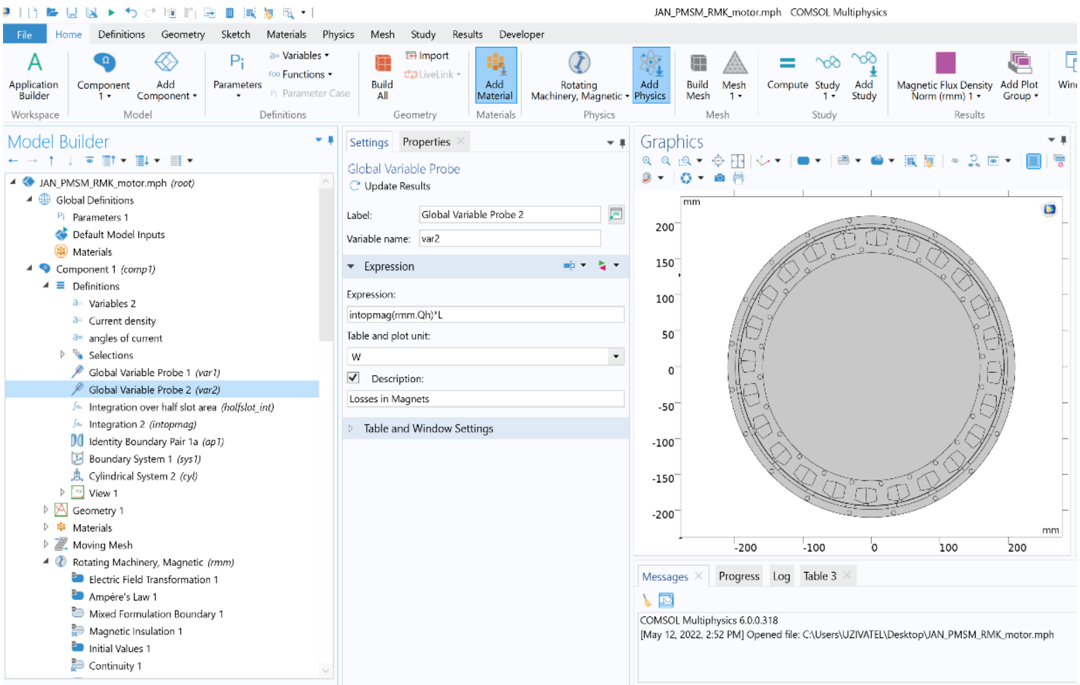

Figure 6: Setting of Global Variable Probe to establish losses in PMs in COMSOL Multiphysics.

3.2.2 EMDtool

For this thesis purposes, EMDtool for MATLAB is used for a quasi-time-dependent stepping simulation. Simulation has to be prepared by adding path of EMDtool to MATLAB as well as adding path for gmsh mesh generator. Second step is to create the geometry, design, of the machine under study. In the thesis case, it is a surface PM motor with outer rotor. After preparing all the necessities for geometry to be created: dimensions of the machine, number of slots and pole-pairs, winding parameters, materials used, geometry is then plotted as a figure in MATLAB. Once the geometry of the machine is prepared, the meshing can follow, which leads to preparing the whole model for time stepping simulation.

Moving on to the time stepping simulation, operating points, in other words, quantities, such as rotational speed and current supply are settled. In this thesis, quasi-static time-stepping simulation was used with current supply, and without using damping effects of any kind. Then, the problem is initialized as a magnetic problem and field-eddy coupling is disabled, means the circuit is rendered as purely resistive, which can be considered as a limitation. It increases the losses a bit but also improves the simulation speed. Afterwards, parameters such as frequency, speed, steps, etc., are defined and then the source is set – putting current supply into the stator winding geometry as well as specifying coils and phases.

The last part of this method is to solve the given problem. As stated in the beginning of this subchapter, problem solution is in quasistatic time-stepping solution. The torque and PM losses results are plotted in command window. Moreover, magnetic flux density is plotted on a figure and current density in PMs is shown on an animated figure.

EMDtool was the only method used in this thesis, which considered the end effect via a hybrid two-dimensional-three-dimensional method, solving the electric scalar potential in three dimensions.

3.3 Thesis method

Method used in this thesis can be described as FEA with post-processing analytical analysis. In other words, after FEA analysis has been finalized, results of the analysis have been stored in MATLAB and later used in further calculations to obtain the final value of eddy current losses in permanent magnets. For this purpose, geometry and initial values were created in MATLAB environment and simulation was done in FEMM software.

Simulation is prepared as a script in MATLAB, where parameters for the type of simulation have to be set. There are two main types – script performing a computation of the torque as a function of the load angle and script performing a computation of the iron and permanent magnet losses with torque as function of the angular position at a given current of I_rms = 250A. Another script connected to this program is where materials of the PMSM are defined. It is called from the main script to set the material properties in FEMM, prior to the computations. Finally, script containing the motor design in terms of dimensions of the stator and rotor is attached as a text file. The simulation is based on the two-dimensional finite element method in quasi-static regime and with current supply for the windings. Once the simulation environment is set, the main script is opened in MATLAB and simulation can run. After the simulation finishes, the results are shown or plotted in different ways.

Construction of the FEMM model is defined by the geometry in text file script and is fed to the FEMM pre-processing module, which plots this geometry, defines the boundary conditions, the materials, and the load currents all as given by the calling script. Once this is done, a for-loop in the MATLAB script goes through all the simulation cases, updates the geometry by rotating the stator and the currents of the windings of the stator. After each rotation, the torque is computed in the FEMM and retrieved to the MATLAB script. The same goes for the flux densities and the vector potentials. These quantities are essential for computation of the losses.

Once all the computation has been done, MATLAB script makes some post-processing to compute the losses and the flux density harmonics and plots the desired quantities as well as displays them.

For computation of the PM losses, firstly, the magnetic vector potential at desired surface of the magnet is retrieved from the FEMM computations as a function of the rotor position. To make more precise computation, it is retrieved at 310 points on the surface of the magnet. This is done by discretization of the pm surface. Figure 7 shows the logic of the discretization.

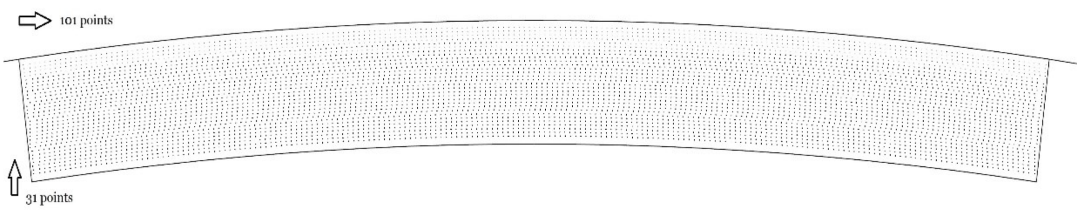

Figure 7: Discretization of the space to find the magnet losses.

After discretization of the desired magnet, the magnetic vector potentials at each point and in z-direction were retrieved from FEMM.

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \tag{17}$$

$$\boldsymbol{A} = \begin{cases} 0 \\ 0 \\ \boldsymbol{A_z}(\boldsymbol{x}, \boldsymbol{y}) \end{cases} => \tag{18}$$

$$\boldsymbol{A} = \begin{cases} \boldsymbol{B_x} \\ \boldsymbol{B_y} \end{cases} => \begin{cases} \dfrac{\partial \boldsymbol{A_z}}{\partial y} \\ -\dfrac{\partial \boldsymbol{A_z}}{\partial x} \end{cases} \tag{19}$$

With retrieved magnetic vector potential, average eddy currents in magnet were calculated by finding time derivative of magnetic vector potential as shown below.

$$J = -\sigma \frac{\partial A}{\partial t} + J_0, \tag{20}$$

where σ is electrical conductivity of magnet, A is magnetic vector potential, J is eddy current in magnet and J_0 is a term which ensures the closure of the eddy current in the cross section of the segmented magnet.

$$\iint_{S_{pm}} J\, dS = -\sigma \frac{\partial}{\partial t} \left[\iint_{S_{pm}} A + A_0 dS \right] = 0, \tag{21}$$

which gives,

$$A_0 = - \iint_{S_{pm}} AdS. \tag{22}$$

By using the function in MATLAB called mo_blockintegral(1), A_0 is retrieved from the magnet in each point shown in the Figure 7. To compute the average eddy current J can be expressed then as,

$$J = -\sigma \frac{\partial A}{\partial t} + J_0 = -\sigma \left[\frac{\partial A}{\partial t} + \frac{\partial A_0}{\partial t} \right]. \tag{23}$$

Hence,

$$J_0 = -\sigma \frac{\partial A_0}{\partial t}. \tag{24}$$

Afterwards, equation (20) is then multiplied with mechanical speed

$$\omega = 2\pi \frac{n}{60}, \tag{25}$$

where ω is the speed in mechanical angles and n is mechanical speed in units of revolutions per minute. Obtaining eddy currents in the magnet have prepared the ground for final computation of losses caused by eddy currents as,

$$p_v(t) = \frac{J^2}{\sigma} = \frac{1}{\sigma}\left[\sigma^2 \frac{\partial A}{\partial t}^2 + J_0^2 - 2\sigma J_0 \frac{\partial A}{\partial t}\right]. \tag{26}$$

Then, finally losses are computed,

$$p(t) = l \iint_{S_{pm}} p_v(t)\, dS, \tag{27}$$

where l is the length of magnet and S_{pm} is the surface of the magnet. As mentioned above, losses in the magnet were computed for each mechanical angle, with given current supply of $I_{rms} = 250A$ and frequency of $f = 200Hz$. Finally, the eddy current losses are calculated by averaging the instantaneous losses over a period.

4 Results

The major aim of this thesis was to estimate losses in permanent magnets of specific PMSM. Results obtained from different approaches will be presented below. All the results, which not consider the end effect will be presented both in their raw form and in the form, which accounts for the end effect. At last, all the results will be confronted together.

4.1 Analytical method results

Losses calculated via analytical method were recalculated for different speeds. Table 1 below, shows first calculation results.

Table 1: Analytical results without and with end effect coefficient.

Without end effects	
Speed(rpm)	Losses(W)
0	0
250	16.559
500	66.237
750	149.034
1000	264.949
1200	381.534
1500	596.135
2000	1059.796
2500	1655.931
3500	3245.625

With end effects	
Speed(rpm)	Losses(W)
0	0
250	8.194
500	32.774
750	73.742
1000	131.097
1200	188.781
1500	294.968
2000	524.387
2500	819.355
3500	1605.935

In another perspective, Figure 8 shows graph of PM segment eddy current losses with respect to rotor speed.

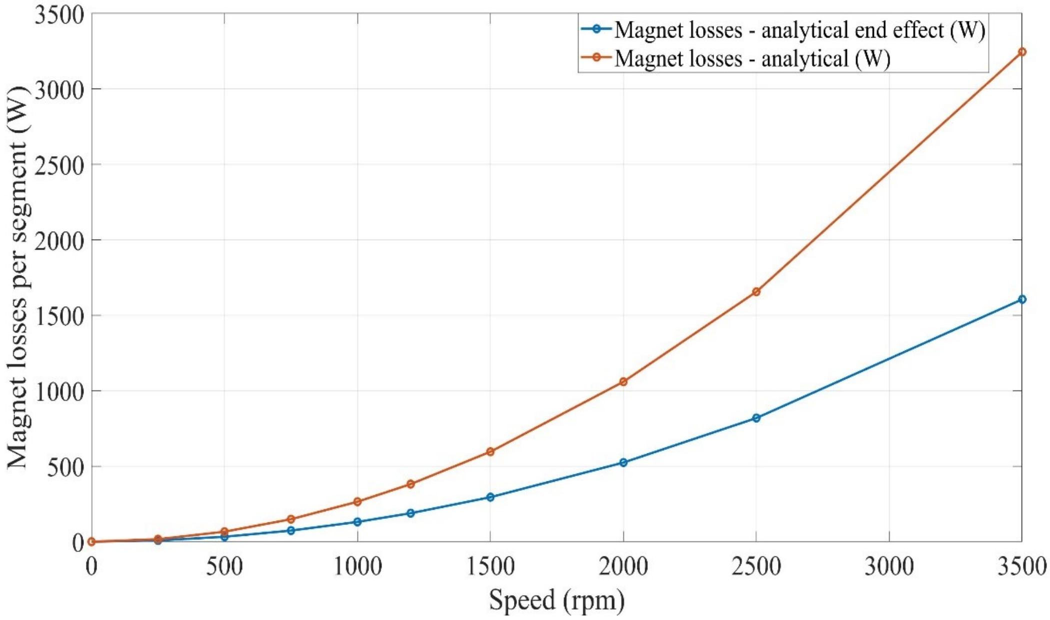

Figure 8: Analytical result of eddy current losses with rotor speed.

Figure 8 presents a graph of results obtained with and without consideration of end effects coefficient at different speeds. These results had to be recalculated for each speed and were calculated with consideration of the 5th space harmonic order. This analytical approach was done in MATLAB by using the thesis method's values obtained from the FEMM simulation. The calculation in MATLAB took only 0.4 seconds.

4.2 FEA results

In this section, results from COMSOL and EMDtool for MATLAB software will be presented. Firstly, the geometry will be shown. Then, the magnetic flux density simulation results will be shown to present the results. Lastly, results of eddy current losses will be presented.

4.2.1 COMSOL results

To start, geometry with the mesh of the motor in COMSOL is presented in Figure 9.

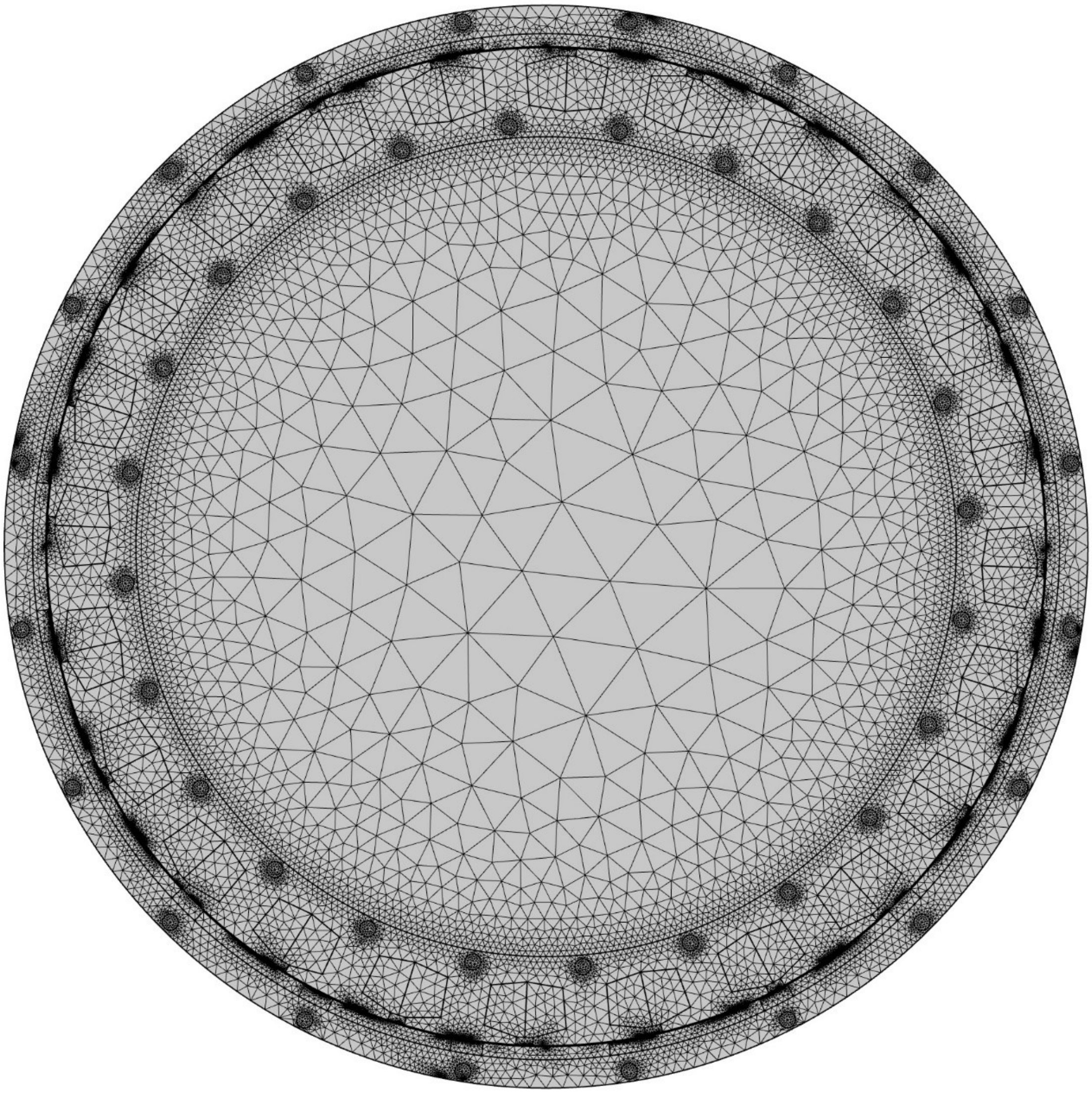

Figure 9: COMSOL geometry of the motor with the mesh.

Results of the magnetic flux density can be seen in Figure 10.

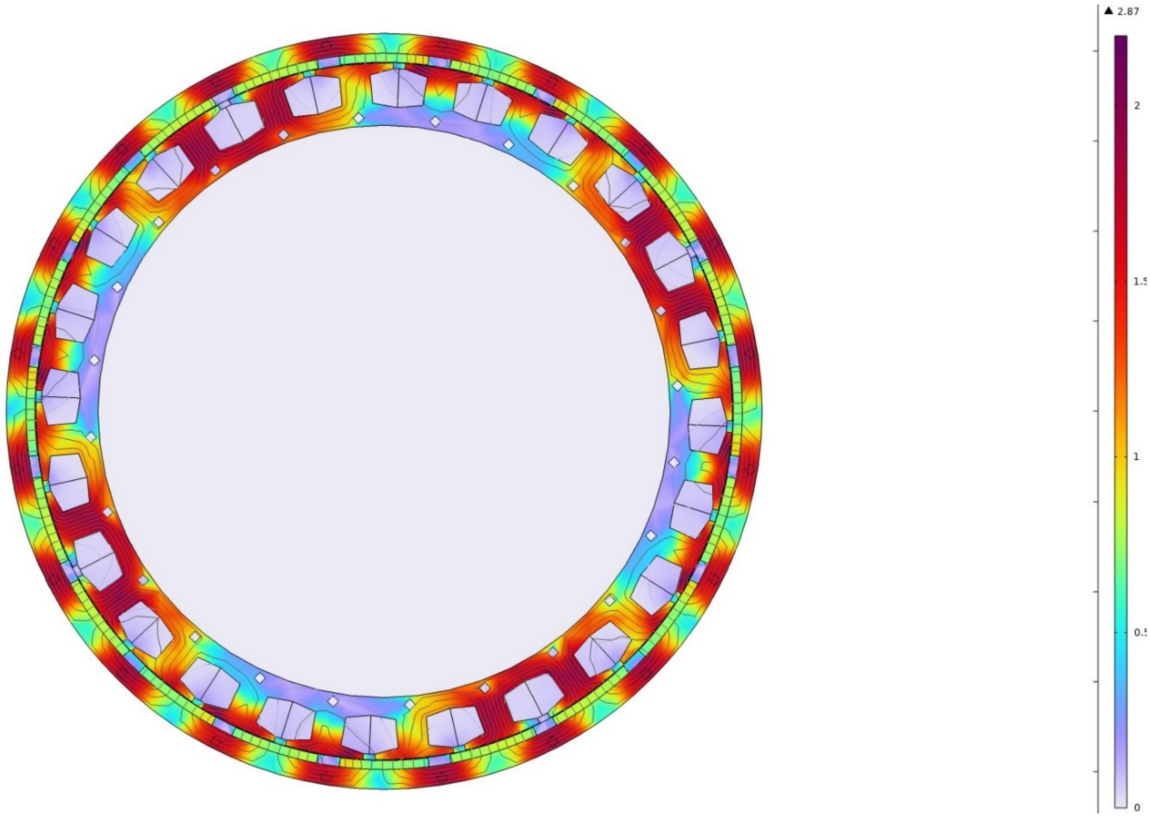

Figure 10: Magnetic flux density result plot.

Results of eddy current losses in magnets obtained from COMSOL can be seen in Table 2 both without and with consideration of end effect.

Table 2: COMSOL results without and with end effect coefficient.

Without end effects		**With end effects**	
Speed(rpm)	Losses(W)	**Speed(rpm)**	Losses(W)
0	0	0	0
250	2.885	250	1.428
500	25.743	500	12.738
750	85.574	750	42.342
1000	194.081	1000	96.031
1200	313.954	1200	155.342
1500	466.803	1500	230.973
2000	828.461	2000	409.922
2500	1259.402	2500	623.151
3500	2349.141	3500	1162.335

Moving from numbers to graphic presentation, graphs of COMSOL results are presented below. Figure 11 shows the results both with and without end effects.

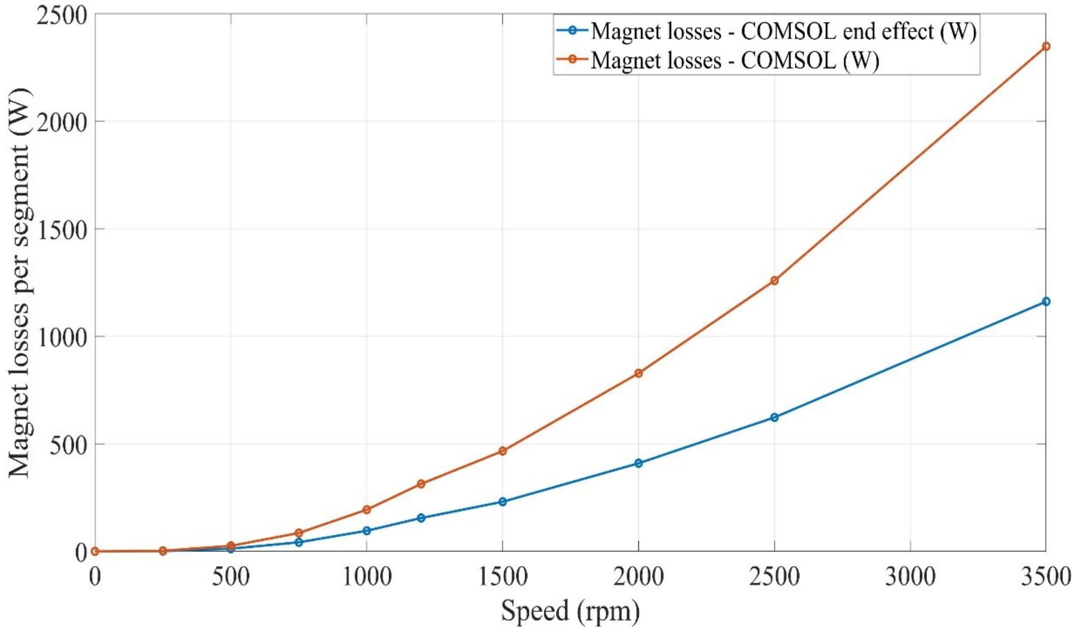

Figure 11: COMSOL results of eddy current losses with rotor speed.

The results are overestimated mainly since the study is a frequency analysis, so some simplification is done. Another aspect of the overestimation is that the eddy current in each magnet do not close properly. Lastly, the Faraday's law is not included in this study. Only magneto-dynamic analysis in COMSOL software could handle the problem. Also, equation

$$\nabla . J = 0, \tag{28}$$

is not respected in each magnet. Hence, the simulation consider that the current could enclose over one magnet pole pair as if they were connected by an end ring, similarly as in the cage of an induction motor.

To present even current density in COMSOL, Figure 12 is shown below.

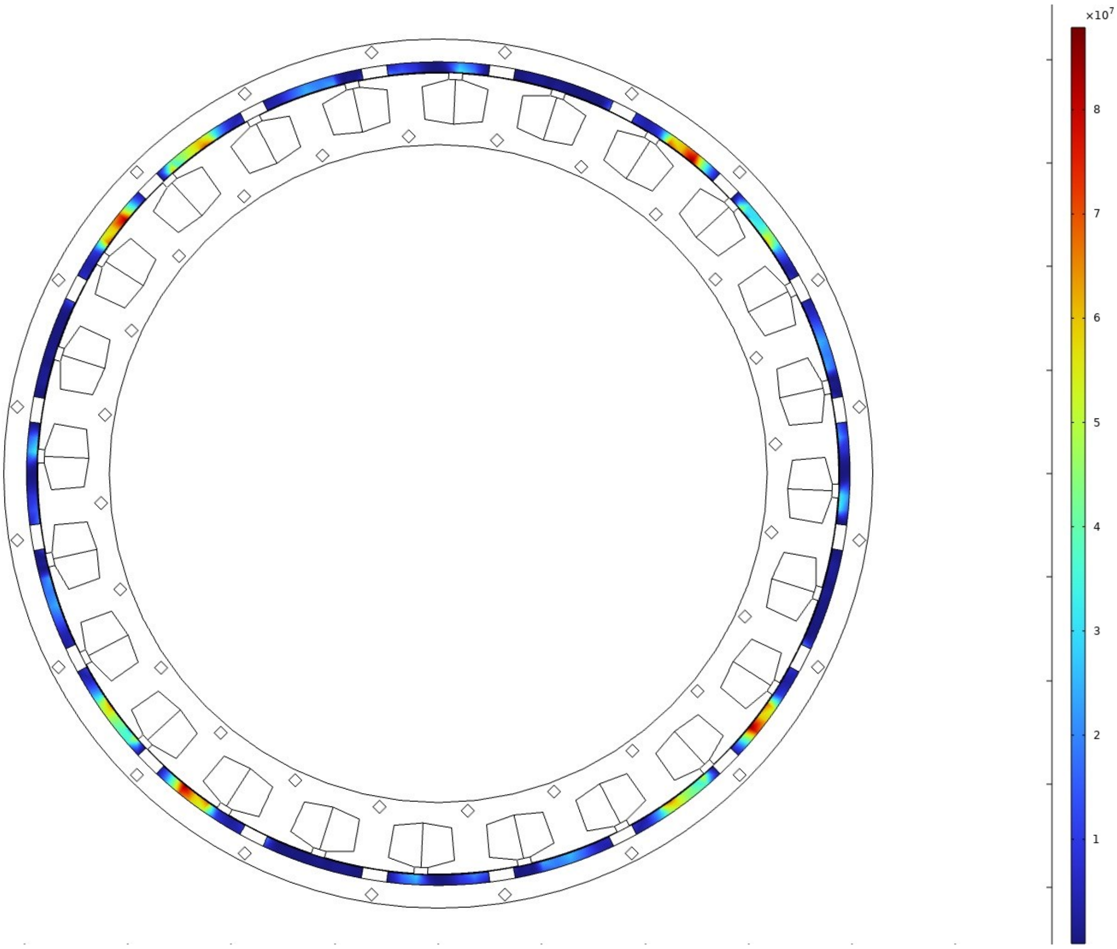

Figure 12: Current density in PMs.

COMSOL simulation took 214 seconds. The losses are then calculated for each speed separately.

4.2.2 EMDtool results

As was stated in previous chapters, EMDtool is the only approach, which is taking in consideration the end effects so no post process application of the Equation (3) is needed.

The geometry obtained from EMDtool simulation can be seen in Figure 13.

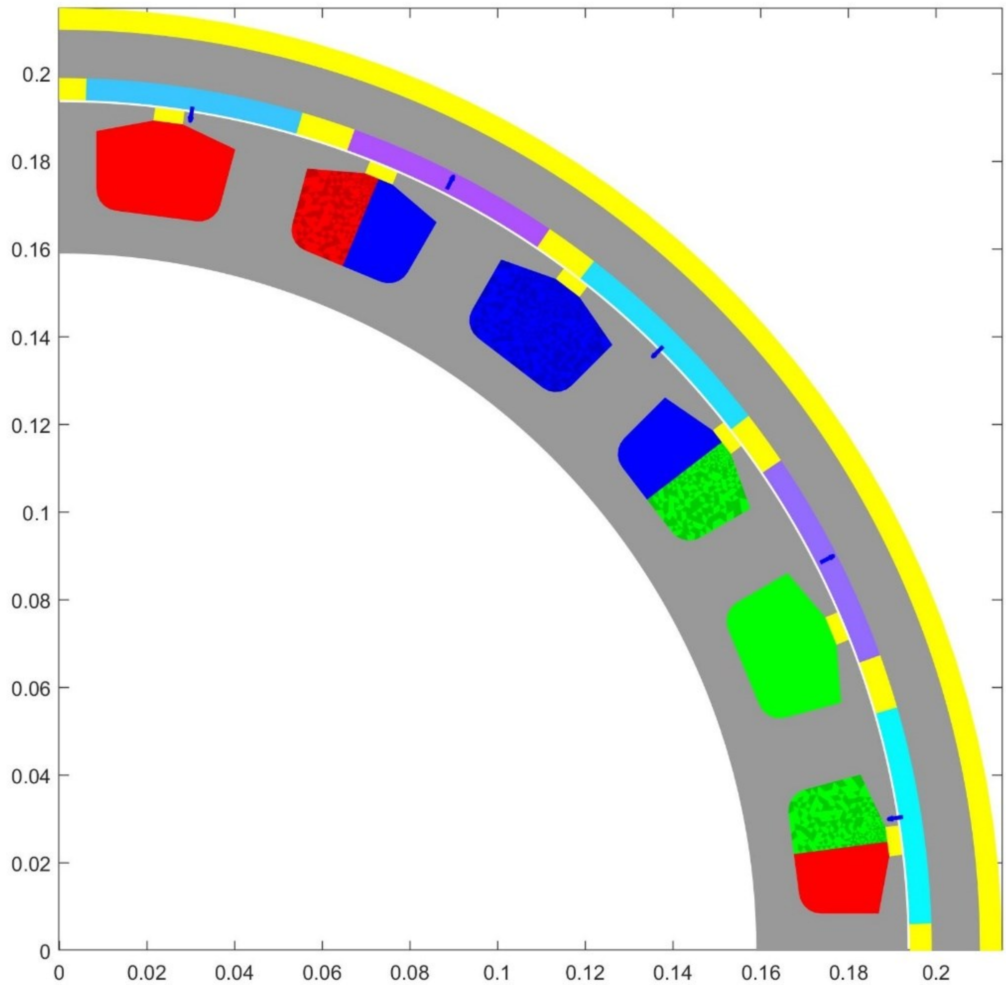

Figure 13: EMDtool geometry of the motor.

Then, the magnetic flux density results from EMDtool simulation can be seen in Figure 14.

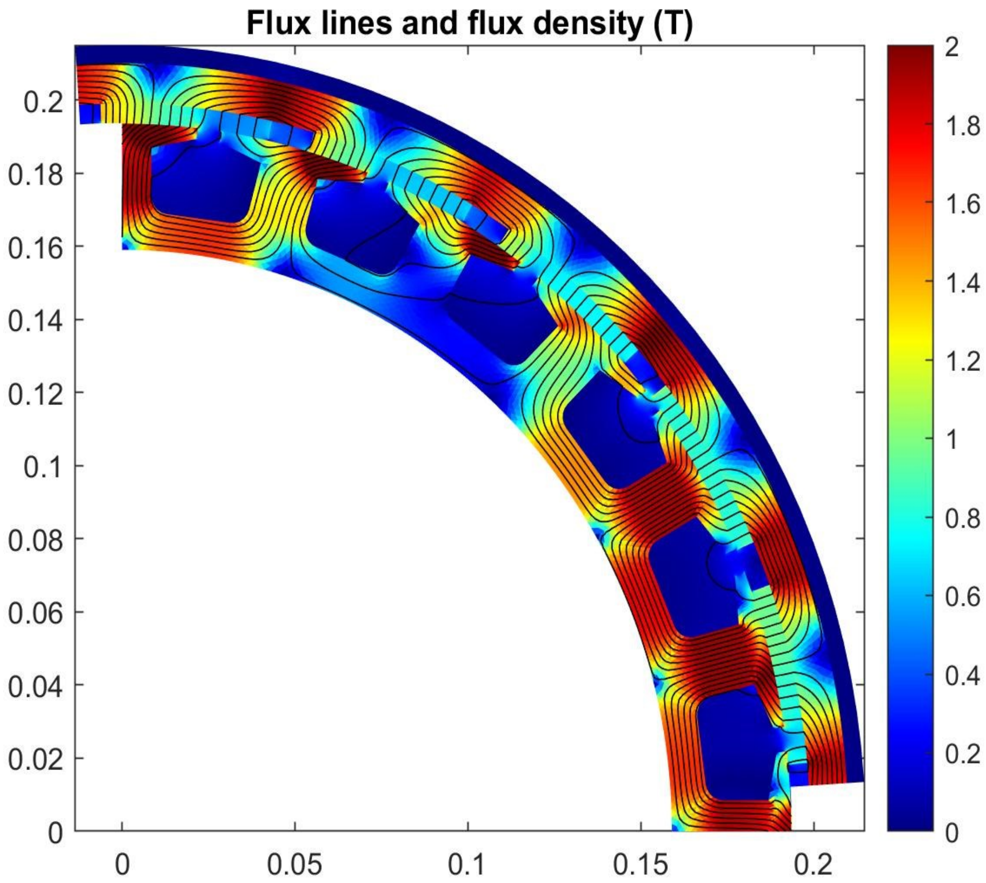

Figure 14: EMDtool magnetic flux density result.

Table 3 shows results obtained from the simulations in MATLAB with using the EMDtool.

Table 3: EMDtool results.

Speed(rpm)	Losses(W)
0	0
250	4.775
500	18.570
750	40.564
1000	69.936
1200	98.231
1500	147.838
2000	247.009
2500	363.214
3500	633.817

Putting these numbers into the graph, gives us Figure 15.

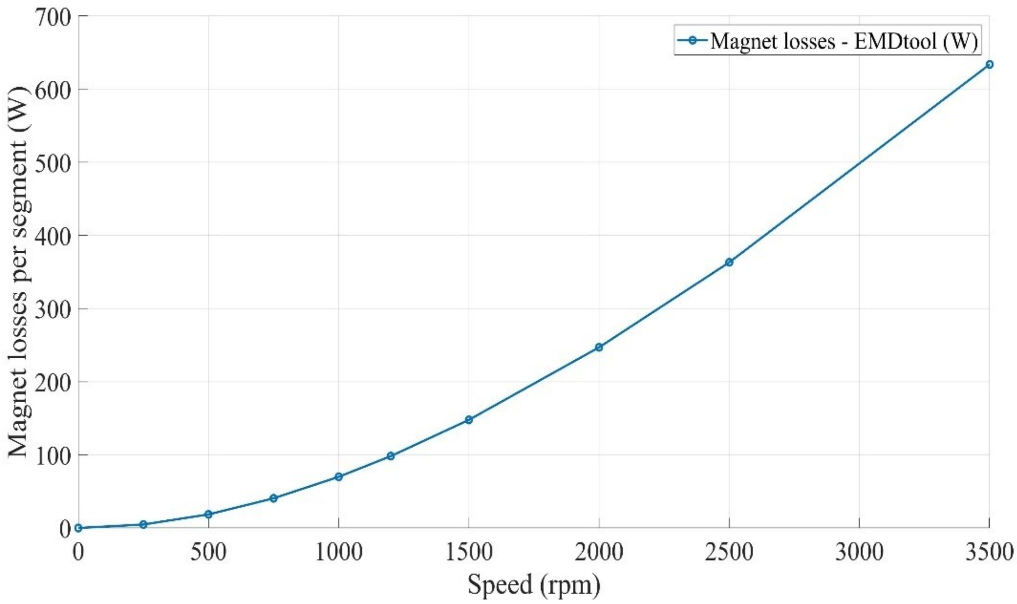

Figure 15: EMDtool results of eddy current losses with rotor speed.

Current density in PMs obtained from EMDtool are presented in Figure 16.

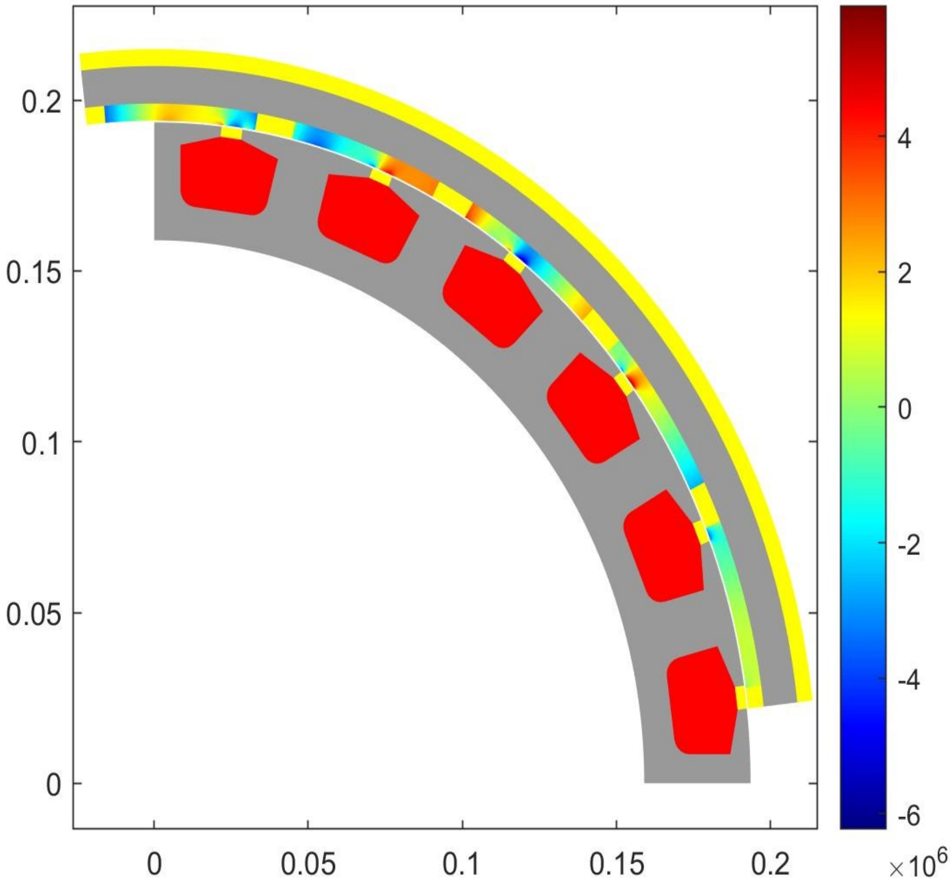

Figure 16: EMDtool current density in PMs results.

EMDtool results considers the feedback coupling from magnetic flux density to eddy currents. Time consumed for this method was 87 seconds. The losses are then calculated for each speed separately as a new simulation.

4.3 Thesis method results – FEMM + post-processing calculations

The eddy current loss model presented in section 3.3 leads to eddy current losses in the magnet computed from the FEMM solution of the magnetic vector potential. The end effects are considered thanks to equation (3).

To show geometry with mesh obtained from FEMM, Figure 17 is presented below.

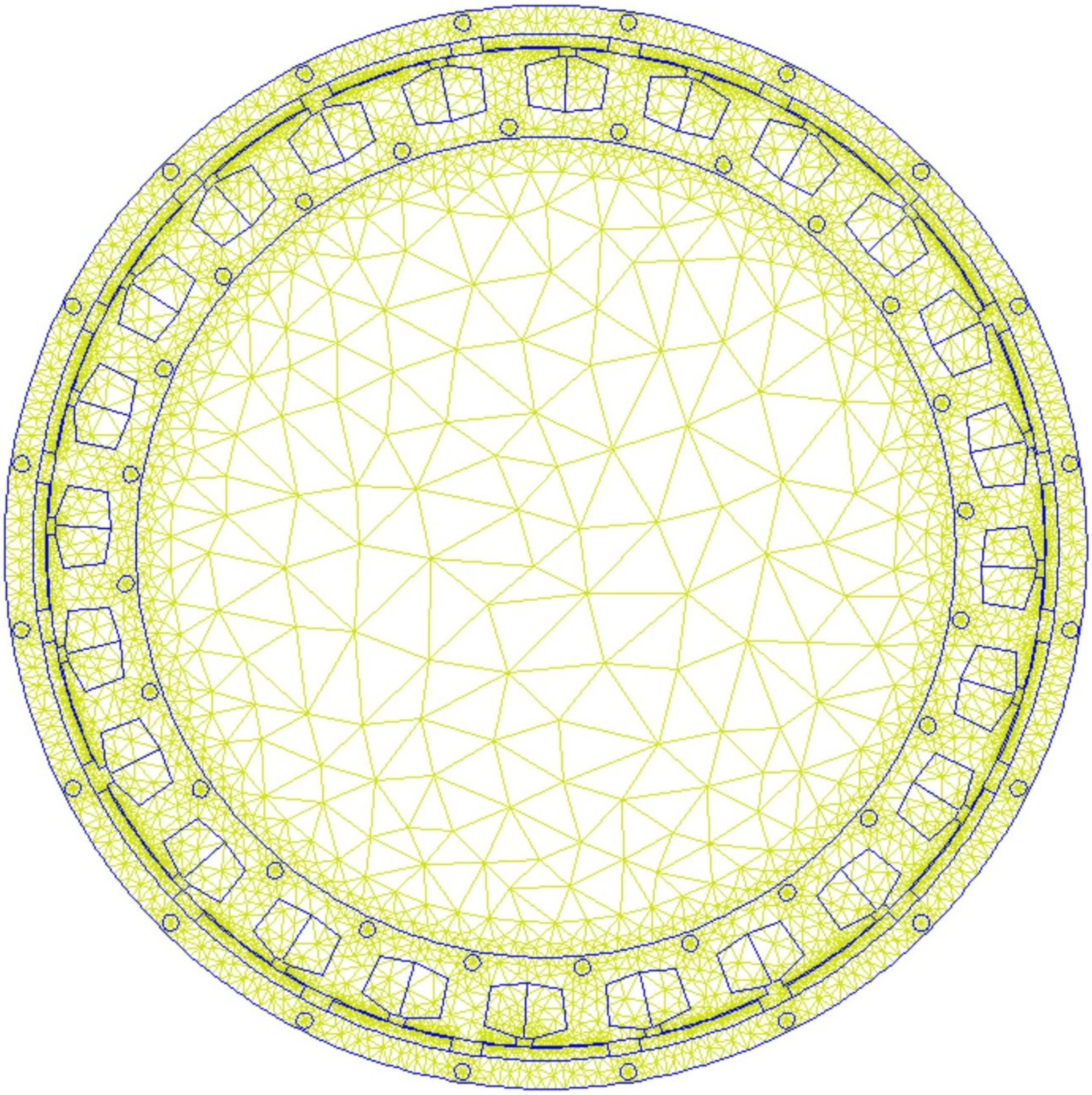

Figure 17: FEMM motor geometry with mesh.

In Figure 18, magnetic flux density results are presented.

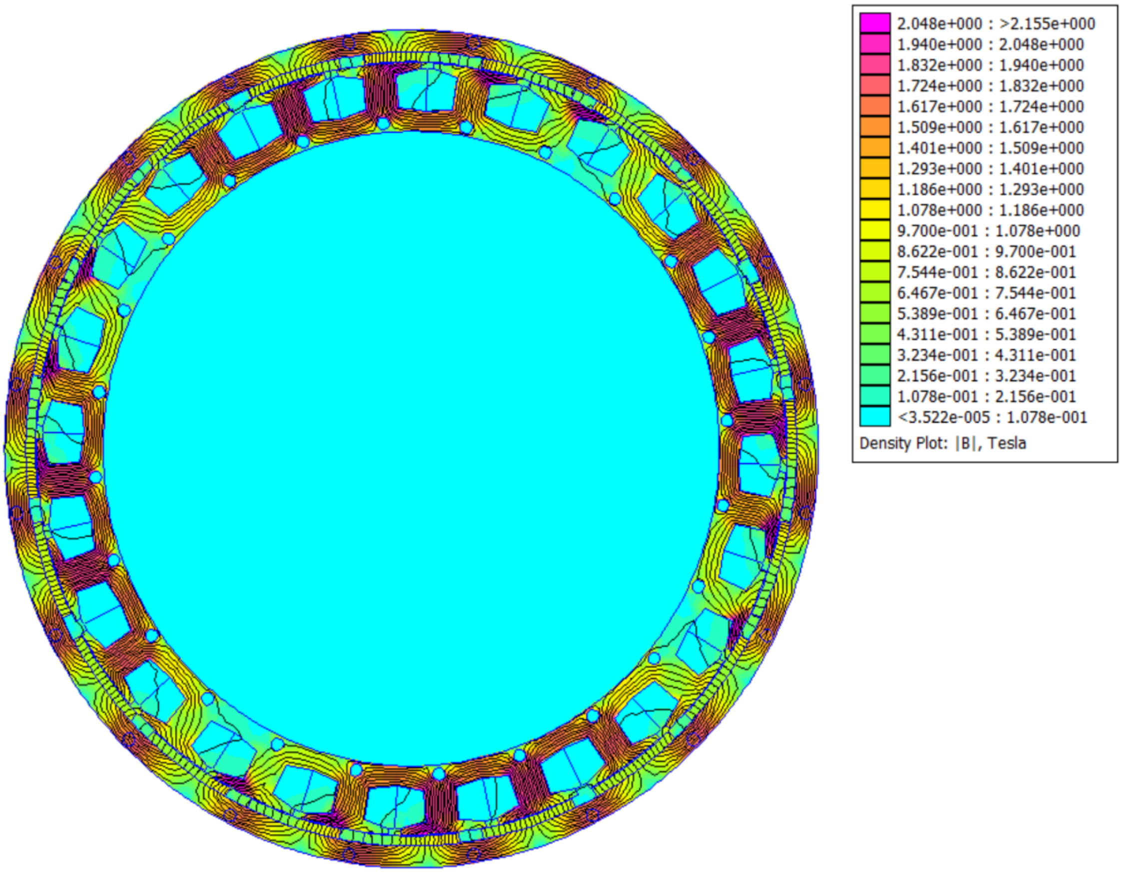

Figure 18: FEMM magnetic flux density results.

Table 4 is showing eddy current losses with speed both without and with end effect coefficient.

Table 4: Thesis method eddy current losses without and with end effects coefficient.

Without end effects		With end effects	
Speed(rpm)	**Losses(W)**	**Speed(rpm)**	**Losses(W)**
0	0	0	0
250	8.841	250	4.375
500	35.366	500	17.499
750	79.573	750	39.373
1000	141.463	1000	69.996
1200	203.711	1200	100.796
1500	318.292	1500	157.491
2000	565.852	2000	279.983
2500	884.143	2500	437.474
3500	1732.921	3500	857.449

Alternatively, presenting these numbers in graphs is being shown below in Figure 19 both without and with end effects coefficient.

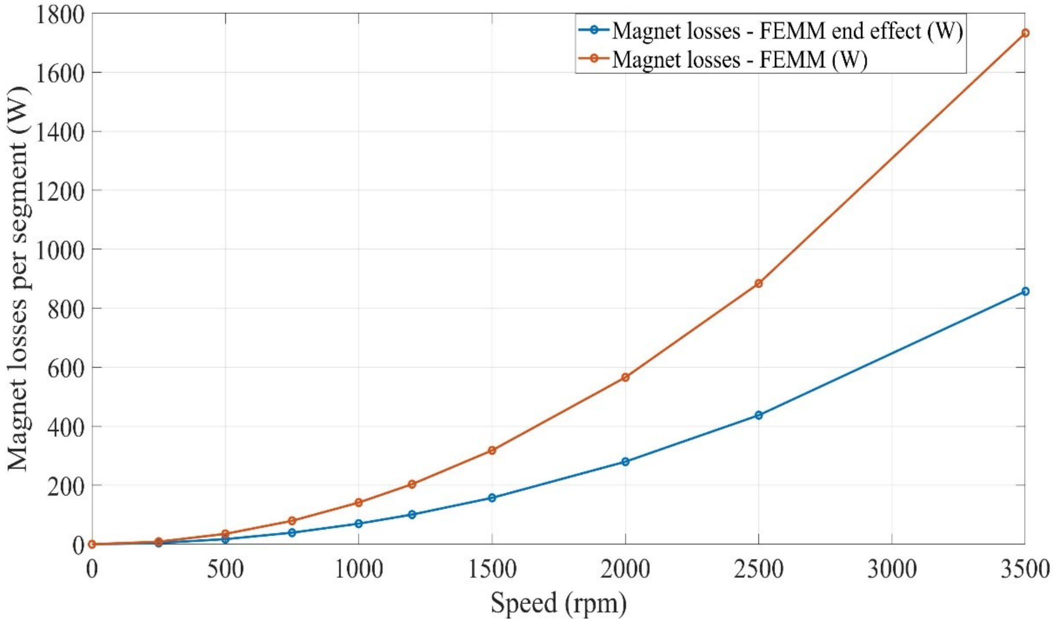

Figure 19: Thesis method eddy current losses with rotor speed.

To show how the eddy current losses in one magnet are developing throughout the change of rotor position, Figure 20 is introduced.

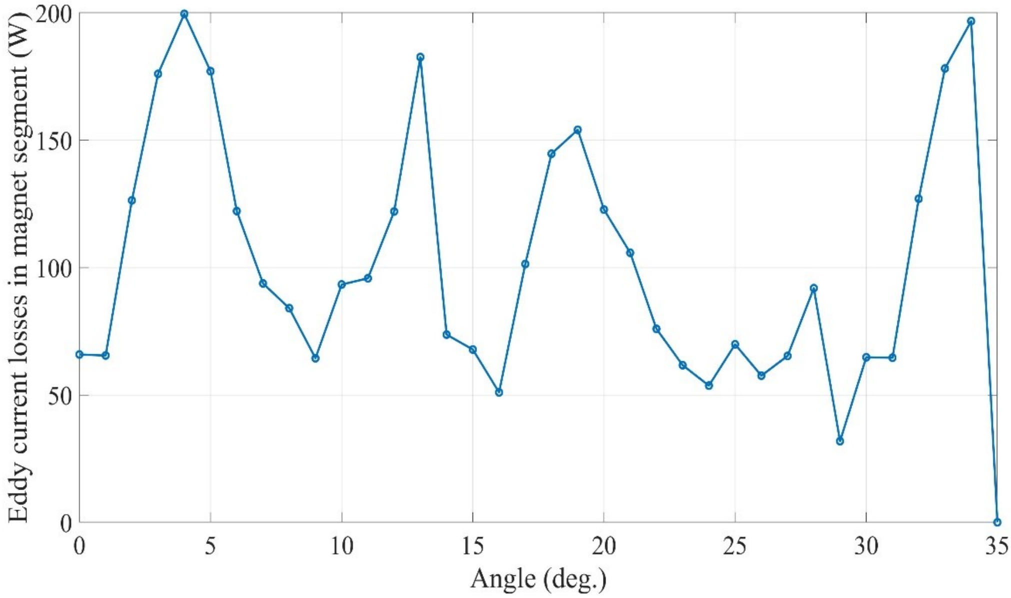

Figure 20: Example of the result of eddy current losses in one magnet as a function of rotor position.

The implementation of Figure 20 is carried out for different stator angles -Ωt, and the stator rotates at constant speed Ω.

The post process calculation of eddy current losses took about 2 seconds, mainly due to the fact that all complicated calculations were done in FEMM simulation part. Also, the losses were calculated for all the speeds as post processing.

4.4 Comparison of all results obtained

Obtained results from different approaches must have some logical connection between each other. In other words, if one method gives certain result, next method must have somewhat close results to the other results as well. Logically, there will be differences in the precision, hence all methods have their own approaches to find eddy current losses in PM. However, results cannot vary too much, inasmuch as the input quantities stays for all methods the same.

To show the variation of different approaches with respect to rotor speed, Figure 21 is presented below. This figure presents all the results without the end effect consideration.

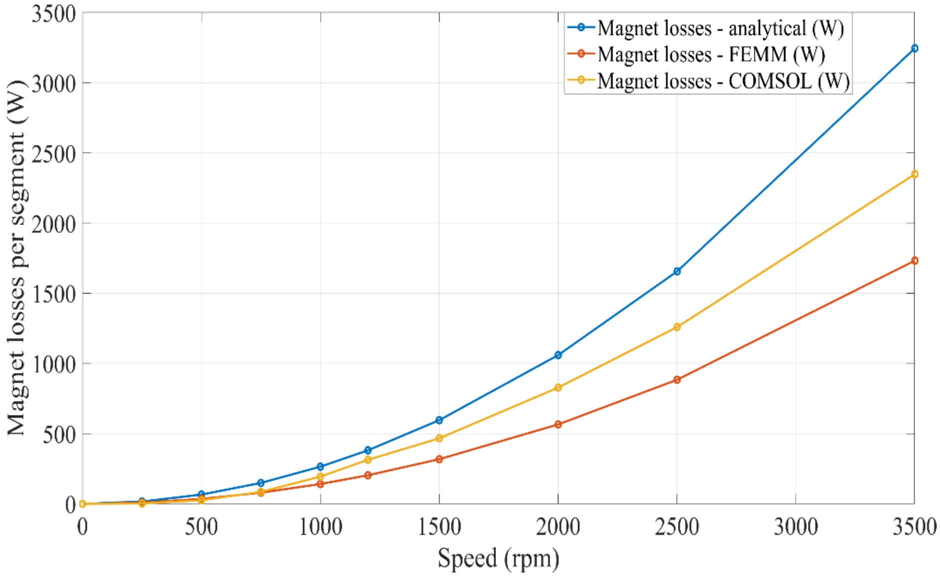

Figure 21: Comparison of different methods results without end effect.

Figure 22 presents results from different methods with the end effect consideration.

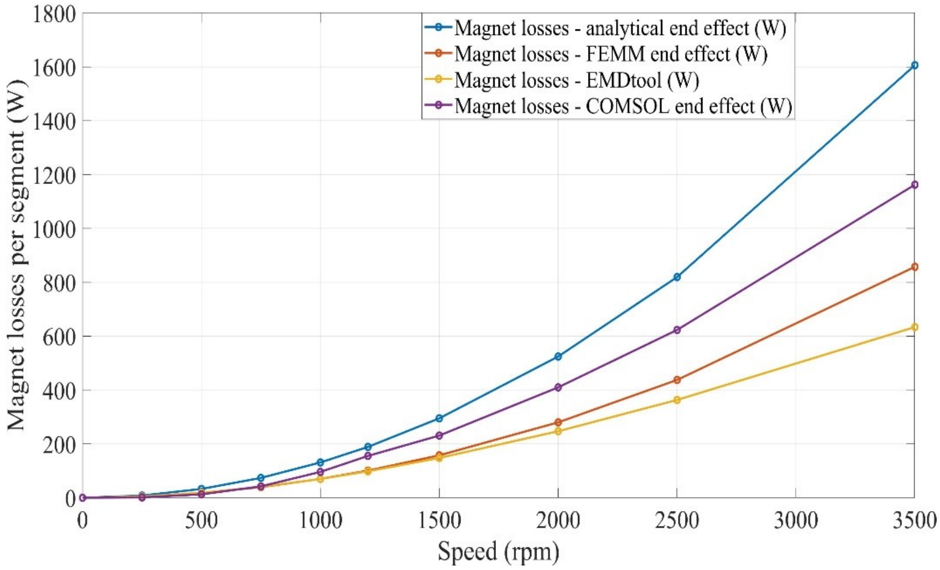

Figure 22: Comparison of different methods results with end effect.

Table 5 is presented below to show, how much computational time it took to obtain the result from each method applied in this thesis. In case of FEM and time-stepping simulations, computation of the losses at each speed separately is needed. In the FEMM+ MATLAB method, only one field solution is needed, and the losses can be computed for all the speeds as post processing.

Table 5: Comparison of the computational time of each method used.

Computational time of different methods				
Method	**Analytical**	**COMSOL**	**EMDtool**	**FEMM**
Time (sec.)	0.4	214	87	2

5 Discussion

The aim of this thesis was to calculate in an analytical way, using a simulation from the FEMM software, and thus predict the magnitude of losses caused by eddy currents in the PMSM. The way in which the results were obtained is specified in more detail in the methodology section - thesis method FEMM. These results, by the chosen method, are compared with other methods, in a purely analytical way, by simulation using COMSOL and EMDtool software for MATLAB. These individual methods for calculating eddy current losses differ from each other and therefore the results obtained are different.

The results obtained by simulation in MATLAB using EMDtool are considered as reference, even though it was additionally chosen computation method for this thesis. This is mainly due to the accuracy of the calculations used. The accuracy of the results is given by the fact that closed eddy current paths, end effects are considered.

Another approach, simulation in COMSOL environment, was overestimating the losses from two aspects. First aspect is, that the eddy currents in each magnet do not close properly, furthermore the study is a frequency analysis, so additional simplifications are done. Second aspect can be expressed in such a way, that the Faraday's law is not included, i.e.,

$$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t}, \tag{29}$$

is not respected as in thesis analysis and EMDtool analysis. For adjustment of obtained results, end effect coefficient was used. Model created for this thesis and chosen study in COMSOL does a lot of assumptions, so further work in this software should be done.

Raw analytical approach, following Attalah's method [3], is not adapting slot harmonic, which is essential for this study. It can be employed for harmonic rank 5 and 7, the rank 3 should be eliminated by default, but the winding factor of these harmonic will not be 1, so it is exaggerating the results obtained. This method also uses the end effect coefficient; thus, the exaggeration is a bit milder than without end effect.

At last, method picked for this thesis can be discussed. Results obtained looks promising with applying end effect coefficient. The difference between results from EMDtool and thesis method are not that significant. Time wise, results obtained from thesis method are quick, arguably 2 seconds, which corresponds to a speed up factor of 4 with respect to the EMDtool method.

Also, once the simulation of the motor in FEMM environment is done, there are only post-processing analytical calculations to run.

All the results were put into figures with different rotor speeds to demonstrate the influence of speed on eddy current losses in a PMSM. Figure 22 shows how the accuracy of the analytical model and model in COMSOL reduce significantly with higher rotor speed.

6 Thesis outcome

6.1 Conclusion

The major aim of the thesis has been met by conducting detailed analysis of eddy current losses in permanent magnets of PMSM and implemented in already existing MATLAB code via post-process calculation with FEMM simulation. Observed results have been confronted with direct analytical approach found by [3]. Moreover, two more FEA analysis, for obtaining eddy current losses, have been used to verify obtained data.

From the thesis work, it has been found out that, permanent magnet losses caused by circulating eddy current plays a significant role in particularly designed PMSM (e.g., concentrated winding design). Considering that matter, reducing of those losses must be taken in order.

6.2 Future work

The thesis work has been mainly concentrated on application of the chosen method for eddy current losses calculation into already existing code for already designed PMSM. There has been no further investigation of reducing those losses, which is considered via segmentation of permanent magnets. Further works could be even done in three-dimensional COMSOL or other environment to obtain precise results. Some of the other subsequent works in this thesis could be,

- Implement visualization of eddy current losses into FEMM software.
- Estimate the effectiveness of segmenting the permanent magnets in reducing eddy current losses in magnets.
- Construct more detailed three-dimensional model in FEA software with proper end windings.
- Perform onsite testing on already manufactured motor, which is in correspondence with given design for this thesis project.

7 References

[1] A. Belahcen and A. Arkkio, "Permanent Magnets Models and Losses in 2D FEM Simulation of Electrical Machines;," in *International Conference on Electrical Machines*, Rome, 2010.

[2] J. Pyrhönen, T. Jokinen and V. Hrabovcová, Design of Rotating Electrical Machines, Chichester, West Sussex: John Wiley & Sons, Ltd., 2008.

[3] K. Atallah, D. Howe, P. Mellor and D. Stone, "Rotor loss in permanent-magnet brushless AC machines," *IEEE Trans. Ind. App.,* no. 36, pp. 1612-1618, 2000.

[4] A. Tessarolo, "A survey of state-of-the-art methods to compute rotor eddy-current losses in synchronous permanent magnet machines," *2017 IEEE Workshop on Electrical Machines Design, Control and Diagnosis,* pp. 12-19, 20-21 April 2017.

[5] J. Alexandrova, H. Jussila, J. Nerg and J. Pyrhönen, "Comparison between models for eddy-current loss calculations in rotor surface-mounted permanent magnets," in *19th International Conference on Electrical Machines*, Rome, Italy, 6-8 September 2010.

[6] R. Sikora, J. Purczynski, W. Lipinski and M. Gramz, "Use of variational methods to the eddy-currents calculation in thin conducting plates," *IEEE Trans. Magn.,* vol. 14, pp. 383-385, 1978.

[7] Z. Zhu, K. Ng, N. Schofield and D. Howe, "Improved analytical modelling of rotor eddy-current loss in brushless machines equipped with surface-mounted permanent magnets," *IEE Proc. Electr. Power Appl.,* 2004.

[8] F. Dubas, C. Espanet and A. Miraoui, "Field Diffusion Equation in High-Speed Surface-Mounted Permanent Magnet Motors, Parasitic Eddy-Current Losses," *In Proceedings of the 6th International Symposium on Advanced Electromechanical Motion Systems,* pp. 1-6, 27-29 September 2005.

[9] M. Nakano, H. Kometani and M. Kawamura, "A study on Eddy-current losses in rotors of surface permanent-magnet synchronous machines," *IEEE Trans. Ind. Appl.,* 2006.

[10] M. Nakano and H. Kometani, "A study on eddy-current losses in rotors of surface permanent magnet synchronous machines," *Conference Record of the 2004 IEEE Industry Applications Conference,* pp. 1696-1702, 2004.

[11] D. A. Wills and M. J. Kamper, "Analytical prediction of rotor eddy current loss due to stator slotting in PM machines," *2010 IEEE Energy Conversion Congress and Exposition,* pp. 992-995, 2010.

[12] Z. Q. Zhu, K. Ng, N. Schofield and D. Howe, "Analytical prediction of rotor eddy current loss in brushless machines equipped with surface-mounted permanent magnets. I. Magnetostatic field model," *ICEMS'2001. Proceedings of the Fifth International Conference on Electrical Machines and Systems (IEEE Cat. No.01EX501),* vol. 2, pp. 806-809, 2001.

[13] K. Yamazaki and A.Abe, "Loss Investigation of Interior Permanent-Magnet Motors Considering Carrier Harmonics and Magnet Eddy Currents," *IEEE Transactions on Industry Applications,* vol. 45, no. 2, pp. 659-665, 2009.

[14] K. Davey and L. Turner, "L. Prediction of transient eddy-current fields using surface impedance methods," *IEEE Trans. Magn.,* vol. 25, pp. 4156-4158, 1989.

[15] H. Polinder and M. Hoeijmakers, "Eddy-current losses in the permanent magnets of a PM machine," *In Proceedings of the 1997 Eighth International Conference on Electrical Machines and Drives,* pp. 138-142, 1-3 September 1997.

[16] E. Spooner and A. Williamson, "Parasitic Losses in Modular Permanent-Magnet Generators," *IEE Proc. Electr. Power Appl.,* no. 145, pp. 485-496, 1998.

[17] D. Ishak, Z. Zhu and D. Howe, "Eddy-current loss in the rotor magnets of permanent-magnet brushless machines having a fractional number of slots per pole.," *IEEE Trans. Magn.,* no. 41, pp. 2462-2469, 2005.

[18] J. Wang, F. Papini, R. Chin, W. Arshad and H. Lendenmann, "Computationally efficient approaches for evaluation of rotor eddy current loss in permanent magnet brushless machines," *In Proceedings of the 2009 International Conference on Electrical Machines and Systems,* 15-18 November 2009.

[19] A. Tariq, C. Nino-Baron and E. Strangas, "Iron and Magnet Losses and Torque Calculation of Interior Permanent Magnet Synchronous Machines Using Magnetic Equivalent Circuit," *IEEE Trans. Magn.,* no. 46, pp. 4073-4080, 2010.

[20] Y. Yoshida, K. Nakamura and O. Ichinokura, "A Method for Calculating Eddy Current Loss Distribution Based on Reluctance Network Analysis," *IEEE Trans. Magn.,* no. 47, pp. 4155-4158, 2011.

[21] L. Wu, Z. Zhu, D. Staton, M. Popescu and D. Hawkins, "Analytical Modeling and Analysis of Open-Circuit Magnet Loss in Surface-Mounted Permanent-Magnet Machines.," *IEEE Trans. Magn.,* no.

48, pp. 1234-1247, 2012.

[22] A. Rahideh and T. Korakianitis, “Analytical magnetic field distribution of slotless brushless permanent magnet motors – Part I. Armature reaction field, inductance and rotor eddy-current loss calculations.,” *IET Electr. Power Appl.,* no. 6, p. 628, 2012.

[23] P. Pfister, X. Yin and Y. Fang, “Slotted Permanent-Magnet Machines: General Analytical Model of Magnetic Fields, Torque, Eddy Currents, and Permanent-Magnet Power Losses Including the Diffusion Effect.,” *IEEE Trans. Magn.,* no. 52, pp. 1-13, 2016.

[24] M. Mirzaei, A. Binder, B. Funieru and M. Susic, “Analytical Calculations of Induced Eddy Currents Losses in the Magnets of Surface Mounted PM Machines With Consideration of Circumferential and Axial Segmentation Effects.,” *IEEE Trans. Magn.,* no. 48, pp. 4831-4841, 2012.

[25] M. Paradkar and J. Bocker, “3D analytical model for estimation of eddy-currentlosses in the magnets of IPM machine considering the reaction field of the induced eddy currents.,” *In Proceedings of the 2015 IEEE Energy Conversion Congress and Exposition (ECCE),* pp. 2862-2869, 20-24 September 2015.

[26] S. Nair, J. Wang, L. Chen, R. Chin, I. Manolas and D. Svechkarenko, “Prediction of 3D High-Frequency Eddy Current Loss in Rotor Magnets of SPM Machines.,” *IEEE Trans. Magn.,* no. 52, pp. 1-10, 2016.

[27] Y. Kamiya and T. Onuki, “3D eddy-current analysis by the finite element method using double nodes technique.,” *IEEE Trans. Magn.,* no. 32, pp. 741-744, 1996.

[28] Y. Kawase, T. Ota and H. Fukunaga, “3D eddy-current analysis in permanent magnet of interior permanent magnet motors.,” *IEEE Trans. Magn.,* no. 36, pp. 1863-1866, 2000.

[29] K. Yamazaki and S. Watari, “Loss analysis of permanent-magnet motor considering carrier harmonics of PWM inverter using combination of 2-D and 3-D finite-element method,” *IEEE Transactions on Magnetics,* vol. 41, no. 5, pp. 1980-1983, 2005.

[30] C. Deak, L. Petrovic, A. Binder, M. Mirzaei, D. Irimie and B. Funieru, “Calculation of eddy-current losses in permanent magnets of synchronous machines.,” *In Proceedings of the 2008 International Symposium on Power Electronics, Electrical Drives, Automation and Motion,* vol. 30, pp. 26-31, 11-13 June 2008.

[31] K. Yamazaki and Y. Kanou, “Rotor Loss Analysis of Interior Permanent Magnet Motors Using Combination of 2D and 3D Finite Element Method.,” *IEEE Trans. Magn.,* vol. 45, pp. 1772-1775, 2009.

[32] S. Steentjes, S. Boehmer and K. Hameyer, "Permanent Magnet Eddy-Current Losses in 2D FEM Simulations of Electrical Machines.," *IEEE Trans. Magn.,* vol. 51, pp. 1-4, 2015.

[33] D. Saban and T. Lipo, "Hybrid Approach for Determining Eddy-Current Losses in High-Speed PM Rotors.," *In Proceedings of the 2007 IEEE International Electric Machines & Drives Conference,* vol. 1, pp. 658-661, 3-5 May 2007.

[34] S. Niu, K. Chau, J. Li and W. Li, "Eddy-Current Analysis of Double-Stator Inset-Type Permanent Magnet Brushless Machines.," *IEEE Trans. Appl. Supercond.,* vol. 20, pp. 1097-1101, 2010.

[35] X. Wu, R. Wrobel, P. Mellor and C. Zhang, "A computationally efficient PM power loss derivation for surface-mounted brushless AC PM machines.," *In Proceedings of the 2014 International Conference on Electrical Machines (ICEM),* vol. 105, pp. 17-23, 2-5 September 2014.

[36] X. Wu, R. Wrobel, P. Mellor and C. Zhang, "A Computationally Efficient PM Power Loss Mapping for Brushless AC PM Machines With Surface-Mounted PM Rotor Construction.," *IEEE Trans. Ind. Electron.,* vol. 62, pp. 7391-7401, 2015.

[37] A. Daanoune, R. Lateb and J. da Silva, "Semi analytical model for eddy current losses calculation in high speed permanent magnet synchronous machines.," *In Proceedings of the International Conference on Electrical Machines and Systems (ICEMS),* pp. 1-6, 13-16 November 2016.

[38] S. Ruoho, T. Santa-Nokki, J. Kolehmainen and A. Arkkio, "Modeling Magnet Length In 2D Finite-Element Analysis of Electric Machines.," *IEEE Trans. Magn.,* vol. 45, pp. 3114-3120, 2009.

[39] R. Stoll, The Analysis of Eddy Currents, Oxford, UK: Oxford University Press, 1974.

[40] S. Nair, J. Wang, R. Chin, L. Chen and T. Sun, "Analytical Prediction of 3D Magnet Eddy Current Losses in Surface Mounted PM Machines Accounting Slotting Effect.," *IEEE Trans. Energy Convers.,* no. 32, pp. 414-423, 2017.