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An efficient algorithm for bi-objective combined heat and power production planning under the emission trading scheme

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Abstract

The growing environmental awareness and the apparent conflicts between economic and environmental objectives turn energy planning problems naturally into multi-objective optimization problems. In the current study, mixed fuel combustion is considered as an option to achieve tradeoff between economic objective (associated with fuel cost) and emission objective (measured in CO_2 emission cost according to fuels and emission allowance price) because a fuel with higher emissions is usually cheaper than one with lower emissions. Combined heat and power (CHP) production is an important high-efficiency technology to promote under the emission trading scheme. In CHP production, the production planning of both commodities must be done in coordination. A long-term planning problem decomposes into thousands of hourly subproblems. In this paper, a bi-objective multi-period linear programming CHP planning model is presented first. Then, an efficient specialized merging algorithm for constructing the exact Pareto frontier (PF) of the problem is presented. The algorithm is theoretically and empirically compared against a modified dichotomic search algorithm. The efficiency and effectiveness of the algorithm is justified.

Keywords: Combined heat and power production; Multi-objective linear programming; Energy optimization; Energy efficiency, Environmental/economic dispatch.

Nomenclature

Indices

t	Index of a period or a point in time. The period t is between points t-1 and t. In our problem, period length is one hour.
p, q	Super/subscripts or prefixes for power and heat.

Index Sets

J	Set of extreme points of the operating regions of all components including non-generating components (e.g., contracts). ($J = \bigcup_{u\in U} J_u$).
J_u	Set of extreme points of the operating region of component $u \in U$,
U	Set of all components including non-generating components.

Parameters

$(\pi_{j,t}, p_{j,t}, q_{j,t})$	Extreme point $j \in J_u$ of operating region of component $u \in U$ (fuel consumption, power, heat) in MW in period t.
$c_{e,t}$	Emission allowance price in €/ton for period t.
$c_{\phi(j),j,t}$	Price of fuel $\phi(j)$ in €/MW at plant $u \in U$ and the same for $j \in J_u$ in period t.
$c_{p\pm,t}$	Power sales/purchase price in €/MW on the power market in period t.
$c_{q+,t}$	Heat surplus penalty cost in €/MW in period t.
$\eta_{\phi(j)}$	Specific CO_2 emission in ton/MW for fuel $\phi(j)$ at plant $u \in U$ and the same for $j \in J_u$.
P_t	Power demand in MW in period t.
Q_t	Heat demand in MW in period t.

T	Number of periods over the planning horizon.

Decision variables

$x_{j,t}$	Variables encoding the operating level of each component in terms of extreme points $j \in J$ in period *t*.
$x_{p\pm,t}$	Power sales and purchase volume in MW on the power market in period *t*.
$x_{q+,t}$	Heat surplus variable in MW in period *t*.

Notation associated with multi-objective optimization algorithms

MA	Merging Algorithm
MDSA	Modified Dichotomic Search Algorithm
DSA	Dichotomic Search Algorithm
Y_N	Non-dominated set of the problem
$Y_{N,t}$	Non-dominated set of the period t subproblem
Y_N^M	Non-dominated set of the problem generated by MA
Y_N^{MD}	Non-dominated set of the problem generated by MDSA
$Y_{N,\max}$	Max non-dominated set of the problem, $\lvert Y_{N,\max} \rvert = 1 + \sum_{t=1}^{T} (\lvert Y_{N,t}^M \rvert - 1)$.

1. Introduction

The increasing concerns about environmental impacts of energy production have become an integral part of energy policy planning. To combat climate change, the European Union (EU) has launched an emission trading scheme (ETS) since 2005 and has simultaneously promoted clean production technologies with smaller emissions [1]. The EU-ETS is now by far the largest emission market in the world, covering more than 11 thousand power stations and industrial plants in 31 countries, as well as airlines. The emission market utilizes the market force to reduce emission cost-efficiently.

CHP production means the simultaneous production of useful heat and electric power in a single integrated process. It can utilize the excess heat that would be wasted in conventional power production and thus can achieve higher efficiency. For example, the efficiency of a gas turbine is typically between 36-40% when used for power production only, but over 80% if also the heat is utilized. CHP is considered an environmentally beneficial technology due to its high energy efficiency compared to conventional separate heat and power production. This leads to significant savings in fuel and emissions, typically between 10-40% depending on the technique used and the system replaced [2].

Considering the fact that fossil based technologies are currently dominant [3] for supplying heat and power all over the world and CHP is an important technology to improve the energy overall efficiency of heat and power production, we study here using a fuel mix (including biomass) [3, 4] as an option to implement the transition into future sustainable low-carbon energy systems. A suitable fuel mix can achieve tradeoff between economic objective (associated with fuel cost) and emission objective (measured in CO_2 emission cost according to fuels and emission allowance price) [5]. Usually, a fuel with higher emissions is cheaper than one with lower emissions. We have considered using multi-objective linear programming (MOLP) approaches to deal with a medium- or long-term CHP environmental/economic dispatch problem (EED), which can be viewed as a subproblem of long term generation expansion CHP planning problem [6]. It means that the plant characteristics are assumed to be convex. It has been commented by [7] that the convexity assumption is not as limiting as it may seem. Multiple criteria decision making approaches, including MOLP, have for a long time been used in energy planning for both traditional power-only and heat-only systems [8-10] as well as for poly-generation including CHP systems [11]. Some recent research related to applying MOLP for dealing with poly-generation planning can be referred to [12, 13].

In the long term generation expansion planning context [14], for a given investment decision, the operation subproblem, which is used to estimate operating costs, is a long term EED problem when emission impacts need to be considered. The long term EED problem can be simplified into a sequence of single period subproblems without dynamic constraints. The natural period length is typically one hour. This simplification may be necessary for at least two reasons. First, the longer planning horizon (15 or 20 years) means that the size of the problem is large and it is difficult to handle the problem efficiently without simplification. Second, in a broader context of risk analysis where numerous scenarios need to be considered, each scenario corresponds to a deterministic long term planning problem that must be solved efficiently. Simulation based scenario analysis [15-22] is a widely used approach and the computational effort is usually large.

For the single objective case, operating costs of the multi-period planning problem without dynamic constraints can be obtained simply by summing up the results of single period subproblems. However, it is not a trivial problem in the multi-objective optimization context because typically there is no single global optimal solution. The solution process consists of identifying a representation of the Pareto frontier (PF) with a number of non-dominated outcomes in the objective space, which correspond to efficient solutions in the decision space. For the MOLP, the continuity of the PF [23] means that the number of non-dominated outcomes used to represent the PF can be rather large. Therefore, the computational effort can be huge, even though each non-dominated outcome can be obtained in polynomial time. For the bi-objective case, all of the non-dominated outcomes for representing the PF can be obtained by solving a series of weighted-sum functions. One approach is called *dichotomic search* [24] and the other approach is called *parametric simplex method* [23].

To the best of the authors´ knowledge, no research is reported to deal specifically with the bi-objective multi-period CHP planning problem with no dynamic constraints. A possible reason for this may be that it is the simplest multi-period planning problem and most people think that a general solution approach can handle it. However, it is not true. An efficient solution approach to the problem is demanding in the context of risk analysis and generation expansion planning and it is not a trivial task to solve it efficiently if the planning horizon is large.

The contributions of the current study can be summarized as follows. First, we have defined a fuel mix setting for the bi-objective CHP EED problem. Second, we have presented an efficient iterative merging algorithm (MA) for constructing the exact PF for the bi-objective LP CHP planning problem on the basis of the PF for the single period subproblem. The MA utilizes the convexity of the PF by arranging slopes of two consecutive non-dominated outcomes in each period in a non-decreasing order. Third, we have conducted theoretical time complexity analysis for the MA and for a traditional algorithm to justify the efficiency of the MA. Finally, we have done numerical experiments using both real and artificially derived plants to show the applicability of the MA in practice. It is worth mentioning that the current research is a new extension of our specialized efficient algorithms for single objective optimization [25, 26] to the multi-objective context and to achieve sufficient efficiency for dealing with environmental impacts taking emission costs explicitly as an objective.

The paper is organized as follows. Section 2 describes the model of the individual CHP plant as well as the model of the bi-objective CHP planning problem considering fuel mix. Section 3 presents two algorithms. The first one is a modified dichotomic search algorithm (MDSA) for a general bi-objective LP problem and the second one is a specialized merging algorithm (MA) for constructing the exact PF for the problem in the current study. Then, these two algorithms are compared theoretically through time complexity analysis. Section 4 reports the computational results with both real and artificially derived CHP plants. A comparison is made between MDSA and MA in terms of representation of the PF and solution efficiency to validate the theoretical analysis.

2. Problem description

In addition to generating units (CHP plant, power-only plant, heat-only plant), a CHP system may consists of non-generating components such as various bilateral purchase and sales contracts. All components (plants and contracts) can be modeled based on a unified technique as discussed below. In the subsequent discussion, “plants” refer to generating units while “components” include both generating units and non-generating components. For the system under study, different types of fuels with different specific CO_2 emissions are combusted at plants but it is required that one plant should only combust one fuel to facilitate emission calculation. Usually a fuel with higher emissions is cheaper than one with lower emissions. For example, coal is cheaper than natural gas. It means that there is a tradeoff between fuel cost and emission cost.

Under ETS, the CHP planning problem is to simultaneously optimize the overall *net acquisition costs* for power and heat as well as *emission costs* associated with providing power and heat. The emissions for the plant are caused by the fuel combusted at the plant. The emissions for the non-generating component are based on a reference system (e.g., coal-fired condensing power plant for power component or coal-fired boiler for heat component). The net acquisition costs consist of actual production costs (fuel costs), costs for purchasing components subtracted by revenue from selling the produced energy. The planning horizon is usually long (multiple years) in a strategic long-term planning problem.

2.1 CHP plant model

Here we assume, for the sake of simplicity, that the plant characteristics are convex, which allows us to use a linear programming (LP) solver for the environmental/economic dispatch (EED) problem. In addition, the PF is also convex for the MOLP. In the following, we present the convex CHP model in a simplified way to facilitate readers understand the system model in Section 2.2. The detailed description of the plant characteristics was referred to our previous research (e.g. [25-29]). Note that this is the background information for the study and not related to the contributions for the current study as well as for all of our previous studies.

In a CHP plant, power and heat generation are interdependent. Let $P_{u,t}$, $Q_{u,t}$, and $C_{u,t}$= $=C_{u,t}(P_{u,t},Q_{u,t})$ denote the hourly power generation, heat generation and operating cost of plant u respectively. The model of a convex CHP plant be represented by a convex combination (see e.g. [30, 31]) of extreme points ($c_{j,t}$, $p_{j,t}$, $q_{j,t}$) (the coordinate of operating cost, power and heat) for the operating region as follows.

$$
\begin{aligned}
C_{u,t} &= \sum\nolimits_{j\in J_u} c_{j,t} x_{j,t} \ , \\
P_{u,t} &= \sum\nolimits_{j\in J_u} p_{j,t} x_{j,t} \ , \\
Q_{u,t} &= \sum\nolimits_{j\in J_u} q_{j,t} x_{j,t} \ , \qquad (1) \\
\sum\nolimits_{j\in J_u} x_{j,t} &= 1, \\
x_{j,t} &\ge 0 , \qquad j \in J_u.
\end{aligned}
$$

Here the variables $x_{j,t}$ are used for forming the convex combination and J_u is the index set of extreme points of plant u. Note that formula (1) represents a feasible operating region instead of a single point. In the current study, power ($P_{u,t}$) and heat ($Q_{u,t}$) generation of each plant as well as associated cost ($C_{u,t}$) can be determined by the power and heat demand of the system (refer to (5) and (6)) as well as other constraints in the system model (3)-(8). The $x_{j,t}$ can be determined by solving model (3)-(8) and then $C_{u,t}$, $P_{u,t}$ and $Q_{u,t}$ can be determined according to the first three equations of (1) for each plant.

If emissions need to be considered explicitly, it is convenient to directly transform the extreme characteristic points ($c_{j,t}$, $p_{j,t}$, $q_{j,t}$) into fuel characteristic points ($\pi_{j,t}$, $p_{j,t}$, $q_{j,t}$) if a single fuel is combusted in the plant, where $\pi_{j,t}$ is the fuel consumption corresponding to the extreme point.

The cost is mainly determined by fuel consumption, i.e., $c_{j,t} = c_{\phi(j),j,t}\pi_{j,t}$, where $c_{\phi(j),j,t}$ is the price of fuel $\phi(j)$ combusted at plant u and the same for $j \in J_u$. Let $\eta_{\phi(j)}$ denote specific CO_2 emission for fuel $\phi(j)$, $F_{u,t}$ and $E_{u,t}$ denote hourly fuel consumption and emissions associated with $P_{u,t}$ and $Q_{u,t}$. Then

$$F_{u,t} = \sum\nolimits_{j \in J_u} \pi_{j,t} x_{j,t} \,,$$

$$E_{u,t} = \sum\nolimits_{j \in J_u} \pi_{j,t} \eta_{\phi(j)} x_{j,t}. \tag{2}$$

Similarly, the fuel consumption and associated emissions of the plant can be determined according to (2) if $x_{j,t}$ is determined.

The above modeling technique (1) has been used in CHP planning [7, 25-29, 32]. In conjunction with (2), emissions associated with heat and power generation can be considered in planning. Non-CHP components (power-only or heat only) can be modeled as special cases of the CHP plant model (1) with either $q_{j,t} = 0$ (in power components) or $p_{j,t} = 0$ (in heat components). For the non-generation components such as contracts, the fuel characteristics are obtained based on the fuel specified for the reference system as mentioned before.

2.2 Problem formulation

When dynamic constraints are ignored, the multi-period CHP planning problem considering fuel mix is simply represented as the sum of independent period subproblems. The bi-objective planning problem under study is represented as a vmin optimization problem. The operator vmin means vector minimization. The vmin problems arise when more than one objective is to be minimized over a given feasible region.

$$\text{vmin}\left(\sum_{t=1}^{T}\left(\sum_{j\in J}\pi_{j,t}c_{\phi(j),j,t}x_{j,t}+c_{p-,t}x_{p-,t}-c_{p+,t}x_{p+,t}+c_{q+,t}x_{q+,t}\right),\sum_{t=1}^{T}\sum_{j\in J}\pi_{j,t}\eta_{\phi(j)}c_{e,t}x_{j,t}\right) \quad (3)$$

subject to

$$\sum_{j\in J_u}x_{j,t}=1, \qquad u\in U, \qquad t=1,\ldots,T, \qquad (4)$$

$$\sum_{j\in J}p_{j,t}x_{j,t}+x_{p-,t}-x_{p+,.t}=P_t, \qquad t=1,\ldots,T, \qquad (5)$$

$$\sum_{j\in J}q_{j,t}x_{j,t}-x_{q+,t}=Q_t, \qquad t=1,\ldots,T, \qquad (6)$$

$$x_{j,t}\geq 0, \qquad j\in J, \qquad t=1,\ldots,T, \qquad (7)$$

$$x_{q+,t}\,,\ x_{p\pm,t}\geq 0, \qquad t=1,\ldots,T. \qquad (8)$$

The above model (3)-(8) is a bi-objective LP model for the CHP planning. The first objective in (3) is to minimize the overall net acquisition costs over the planning horizon, which consists of actual total production costs (fuel costs), i.e., the sum of $C_{u,t}$ (the first equation of (1)) for all components, costs for purchasing components subtracted by revenue from selling the produced energy. It also includes the penalty for the heat surplus. The second objective is to minimize the emission costs of components, i.e., the sum of $E_{u,t}$ (the second equation of (2)) for all components. The minimum and maximum power and heat generation limits of the components are implicitly reflected in the component characteristics. In this formulation, the convex combination for each plant in each period is encoded by a set of $x_{j,t}$ variables, indicating the operating level of each plant in terms of extreme points of the operating region, whose sum is one (4) (the last equation of (1)) and that are non-negative (7) (the constraint of (1)). Constraints (5) and (6) define the power and heat balances. The first terms in left-hand sides of (5) and (6) indicate power (the second equation of (1)) and heat (the third equation of (1)) generation quantities for all components, respectively. Since the power can be freely bought ($x_{p-,t}$) and sold ($x_{p+,t}$) on the market at price $c_{p-,t}$ and $c_{p+,t}$, the power demand (5) can always be satisfied. The model can be infeasible only when the heat production capacity is insufficient. The heat balance (6) states that that the demand Q_t in each period t must be satisfied and if the acquisition of heat exceeds the demand, the surplus $x_{q+,t}$ lead to penalty cost $c_{q+,t}$ in the first objective of the objective function (3).

For the above formulation, the power market can be treated as a power plant with large enough capacity. For the single objective problem with the above first objective as the objective, the problem can be solved by Power Simplex algorithm [25]. If the power transaction cost is ignored and electric power can be freely traded (bought or sold) on the market, then the model can be simplified to the formulation in [26]. Then the efficient envelope-based algorithm presented there can be used to solve the problem. Note that emission costs associated with the power market are not explicitly reflected in the formulation. They are implicitly considered in the power price. If the emission allowance price is a constant, the formulation is equivalent to simultaneously minimizing net costs and emissions. This is the traditional way to model the EED problem [33].

3. Solution approach

In this section, the optimality concept for multi-objective optimization is reviewed first. Then, a modified dichotomic search algorithm (MDSA) for solving a general bi-objective LP problem is presented and the time complexity of the algorithm is given. Next, the procedure for merging algorithm (MA) for solving problem (3)-(8) is presented and the time complexity of the algorithm is also given. Finally, MA and MDSA are compared theoretically.

3.1 Optimality concept for multi-objective optimization

Let X denote the set of feasible solutions in the decision space and Y their images in the objective space. The image of solution $x \in X$ is $f(x) = (f_1(x),\ldots,f_r(x))$, where $r \geq 2$. Solving multi-objective optimization problem here is interpreted as generating its efficient set X_E in the decision space and corresponding image $Y_N = f(X_E)$ in the decision space R^r, called *Pareto frontier* (PF) or *non-dominated set*.

The dominance relations are defined based on the componentwise ordering of R^r, for $y^1, y^2 \in R^r$,

$$y^1 \leq y^2 \Leftrightarrow y_k^1 \leq y_k^2, k = 1,\ldots,r \text{ and } y^1 \neq y^2$$

$$y^1 < y^2 \Leftrightarrow y_k^1 < y_k^2, k = 1,\ldots,r$$

The relations $\geq$ and $>$ are defined accordingly.

For the vmin problem, $f(\bar{x}) \in R^r$ is dominated by $f(x) \in R^r$ if $f(x) \leq f(\bar{x})$.

$$X_E = \{x \in X: \text{there exists no } \bar{x} \in X \text{ with } f(\bar{x}) \leq f(x)\}.$$

For the MOLP, the PF is convex and continuous. In principle, the extreme efficient solutions (EESs) are sufficient to characterize the PF because all the efficient solutions of the problem can be obtained by the convex combination of EESs. The image of the EES in the objective space corresponds to the extreme point of the PF, called extreme non-dominated outcome. Accordingly, the images of the non-extreme efficient solutions are called non-extreme non-dominated outcomes. The PF for the bi-objective vmin LP problem is a piecewise linear convex curve as shown in Figure 1, where point ‘ ● ’ represents an extreme non-dominated outcome while point ‘ ○ ’ represents a non-extreme non-dominated outcome.

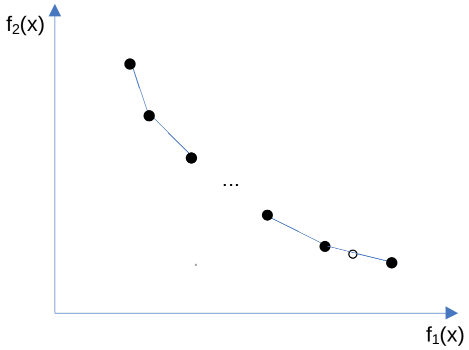

Figure 1 The PF profile of bi-objective vmin LP problem.

Now we introduce the concept for slopes of the PF, where PF:= $\{ (y_1^k, y_2^k), k = 1,\ldots,|Y_N| \}$. Assume that the elements in PF are arranged according to an increasing order of the first objective, i.e. $y_1^1 < \cdots < y_1^{|Y_N|}$. It means that $y_2^1 > \cdots > y_2^{|Y_N|}$. The slopes $\gamma(k, k+1)$ of the PF are defined according to two consecutive non-dominated outcomes

$$\gamma(k,k+1) = \frac{y_2^{k+1} - y_2^k}{y_1^{k+1} - y_1^k}, k = 1,\ldots,|Y_N| - 1 \tag{9}$$

The slopes of the PF assume a non-decreasing profile according to the convexity of the PF.

In the following, we introduce notation for the current problem. Let x_t and x denote the decision variable vector in period t and over the entire planning horizon, respectively.

$$y_{1,t} = f_1(x_t) = \sum\nolimits_{j \in J} \pi_{j,t} c_{\phi(j),j,t} x_{j,t} - c_{p+,t} x_{p+,t} + c_{p-,t} x_{p-,t} + c_{q+,t} x_{q+,t} \tag{10}$$

$$y_{2,t} = f_2(x_t) = \sum\nolimits_{j \in J} \pi_{j,t} \eta_{\phi(j)} c_{e,t} x_{j,t} \tag{11}$$

$$y_1 = f_1(x) = \sum\nolimits_{t=1}^{T} f_1(x_t) \tag{12}$$

$$y_2 = f_2(x) = \sum_{t=1}^{T} f_2(x_t) \tag{13}$$

The weighted-sum function with a weight vector $\lambda = (\lambda_1, \lambda_2)$ is defined as

$$f_\lambda(x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) \tag{14}$$

3.2 Modified dichotomic search algorithm (MDSA)

The dichotomic search algorithm (DSA) was a general approach for solving the bi-objective LP problem. It was first developed by [24] for solving the bi-objective LP transportation problem. In the multi-objective combinatorial optimization context, it was mainly used to find the supported non-dominated outcomes for the problem [34, 35]. The supported non-dominated outcomes of the problem can be obtained by solving a series of weighted-sum functions while the unsupported non-dominated outcomes cannot be reached by any weighted-sum function [36]. To facilitate discussion, we call the algorithms presented in [24], [34] and [35] DSA1, DSA2 and DSA3, respectively. These algorithms are the same in the basic principle that attempts to enumerate all possible new non-dominated outcomes between two known non-dominated outcomes. There are slight differences in the structure of the algorithm and in determining whether a new outcome is dominated or not. DSA1 and DSA3 adopt an iterative procedure while DSA2 adopts a recursive procedure. For determining whether a new outcome is dominated or not, the new outcome is compared with only two known non-dominated outcomes on which the weight vector is based for DSA1 and DSA3 while the new outcome is compared against all known outcomes explicitly for DSA2.

For our problem, it is found that the comparison scheme to determine whether the new outcome is dominated or not for DSA1 and DSA3 is not sufficient to guarantee that the algorithm work properly because it is possible that new outcome coincides with other known non-dominated outcomes. The reason behind this originates from the fact that it is possible for DSA to generate non-extreme non-dominated outcome. A modified DSA (MDSA) proposed on the basis of DSA3 is given below.

Algorithm 1. Modified dichotomic search algorithm (MDSA) for solving the bi-objective vmin LP problem.

Step 1. Compute the lexicographic minimal (lexmin) solutions x_1 and x_2 with respect to f_1 and f_2, respectively. Let $x_1 \in \arg\operatorname{lex}\min\{(f_1(x), f_2(x) : x \in X)\}$ and $x_2 \in \arg\operatorname{lex}\min\{(f_2(x), f_1(x) : x \in X)\}$ .Let $y^1 := f(x_1)$, $y^2 := f(x_2)$, $V := \varnothing$ and $k := 2$.

Step 2. Let $R := \{y^1, \ldots, y^k\}$ with $y_1^1 < y_1^2 < \cdots y_1^k$. If $R \backslash V = \{y^k\}$, then stop; otherwise let $y^i \in \arg\min\{y_1 : y \in R \setminus V\}$.

Step 3. Let $\lambda_1 := y_2^i - y_2^{i+1}$ and $\lambda_2 := y_1^{i+1} - y_1^i$, form weighted-sum function (14).

Step 4. Compute the single objective optimal solution $\overline{x}$ with respect to (14). If $y_1^i < f_1(\overline{x}) < y_1^{i+1}$ and $y_2^{i+1} < f_2(\overline{x}) < y_2^i$, then $y^{k+1} := f(\overline{x})$ and $R := R \cup y^{k+1}$; otherwise let $V := V \cup y^i$. Let $k := k+1$ and go to Step 2.

At the end of the procedure, Set R corresponds to Y_N, i.e., $|R| = |Y_N|$ and the non-dominated outcomes are arranged in an increasing order of the first objective in the set.

It can be seen from Algorithm 1 that the main modification lies in how to determine whether the new outcome is dominated or not at Step 4. The comparisons remain restricting to two known non-dominated outcomes but comparison scheme changes from directly comparing with the two non-dominated outcomes of DSA3 to locating the position of the new outcome. This scheme originates from the convexity property of the PF, i.e., if the new outcome is located between the two consecutive non-dominated outcomes on which the weight vector is based, then it is not dominated; otherwise, it is dominated (coincides with the known non-dominated outcomes). This is due to the fact that DSA allows multiple (more than two) outcomes with the same slope to coexist, i.e., the coexistence of the extreme and the non-extreme non-dominated outcomes. The remaining modification is just an adaption of DSA3 from solving vmax to solving vmin problem. For example, maximal→minimal, lexmax→lexmin and arglexmax→arglexmin as well as the ranking order at Step 2.

Lemma 1. The time complexity of Algorithm 1 for solving a general bi-objective LP problem is $O(h(n,m)\ |Y_N|)$, where $h(n,m)$ is the time complexity of solving the corresponding single objective LP problem and n and m are number of variables and number of constraints for the problem.

Proof: To generate Y_N, the number of weighted-sum functions (single objective problems) to solve is $|R|+|V| = |Y_N|+|Y_N|-1=2|Y_N|-1$ according to the terminating condition at Step 2 of Algorithm 1. The time complexity of solving one single objective problem is $h(n,m)$. Thus, the time complexity of Algorithm 1 for solving a general bi-objective LP problem is $O(h(n,m)|Y_N|)$.

□

Corollary 1. The time complexity of solving problem (3)-(8) is $O(g(n_s,m_s)T|Y_N|)$, where $g(n_s,m_s)$ is time complexity of solving a single period subproblem of (3)-(8) and $n_s = |J|+3$ and $m_s = |U|+2$ are number of variables and number of constraints for the single period subproblem.

3.3 Merging algorithm (MA)

The idea of the merging algorithm (MA) is based on the convexity of the PF for the MOLP. If non-dominated outcomes are arranged in an increasing order of the first-objective, then, for the vmin problem, slopes of the PF assume a non-decreasing order profile as mentioned in Section 3.1. This profile is true for both the PF of the single period subproblem and the PF of the multi-period problem. If the single period subproblem is independent of each other, then slopes of the PF for the single period subproblem should be maintained in slopes of the PF for the multi-period problem as illustrated in Figure 2. Consequently, the PF of multi-period problem is the accumulative results of the single period subproblem in terms of slopes.

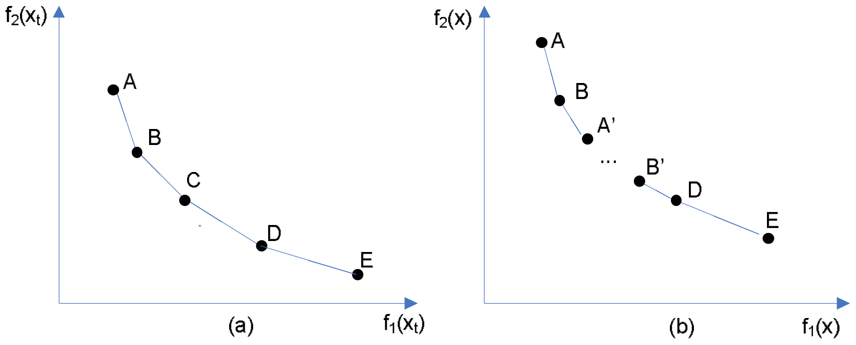

Figure 2. The PF of a single period subproblem and PF of the multi-period problem

In Figure 2, sub-figures (a) and (b) are the PF of a single period t subproblem and the PF of the multi-period problem including period t, respectively. All slopes in the single period subproblem will appear in the multi-period problem. For example, $\gamma(A,B)$ and $\gamma(D,E)$ in (b) are the same as $\gamma(A,B)$ and $\gamma(D,E)$ in (a). $\gamma(B,A')$ and $\gamma(B',D)$ in (b) come from other periods than *t*. $\gamma(B,C)$ and $\gamma(C,D)$ in (a) should be located between points A' and B' in (b). However, the absolute coordinates of the points in (b) should be the sum of the coordinates for the single period subproblems.

In the following, the algorithm for merging the PF of the two-period problem is first given. Then the algorithm for generating the PF of the problem (3)-(8) is presented.

Let $Y_{N,t}$ denote the set of non-dominated outcomes for period *t* subproblem. If $|Y_{N,t}| = 1$, then it is a trivial case to merge, it is simply to add each non-dominated outcome of the other period with $(y^1_{1,t}, y^1_{2,t})$. In the following assume that $|Y_{N,t}| \geq 2$ and non-dominated outcomes $\{(y^k_{1,t}, y^k_{2,t}),\ k = 1,\ldots,Y_{N,t}\}$ are arranged in an increasing order of the first objective. The slopes of the PF for two periods *t*1 and *t*2 are sequentially chosen according to a non-decreasing order to obtain the PF of the two-period problem. The algorithm is given below.

Algorithm 2. Procedure for merging the PF of two periods

```
Step 1. Initialization. $k$:= 1, $i$:=1, $j$:=1.
Step 2.
        if ($|Y_{N,t1}| = 1$ or $|Y_{N,t2}| = 1$)
            if ($|Y_{N,t1}| = 1$)
                for ($k = 1$ to $|Y_{N,t2}|$)
                    $y_1^k = y_{1,t1}^1 + y_{1,t2}^k, y_2^k = y_{2,t1}^1 + y_{2,t2}^k$.
```

```
            end for
        else
            for ($k$= 1 to $|Y_{N,t1}|$)
                $y_1^k = y_{1,t1}^k + y_{1,t2}^1, y_2^k = y_{2,t1}^k + y_{2,t2}^1$.
            end for
        end if
    else
        while ($i <|Y_{N,t1}|$ or $j<|Y_{N,t2}|$ )
            while ($i <|Y_{N,t1}|$ and $j<|Y_{N,t2}|$)
                $y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j$.
                $k$:= $k$+1.
                if ( $\gamma_{t1}(i,i+1) < \gamma_{t2}(j,j+1)$
                    $i$ := $i$+1..
                else if ( $\gamma_{t1}(i,i+1) = \gamma_{t2}(j,j+1)$
                    $i$ := $i$+1, $j$ := $j$+1.
                else
                    $j$ := $j$+1
            end while
            while ($i <|Y_{N,t1}|$)
                $y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j$.
                $k$:= $k$+1, $i$ := $i$+1.
            end while
            while ($j <|Y_{N,t2}|$)
```

```
                $y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j.$
                $k := k+1,\ \ j := j+1.$
            end while
        end while
        $y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j.$
    end if
```

At the end of Algorithm 2, k is the number of non-dominated outcomes for two periods. k= max(|$Y_{N,t1}$| , |$Y_{N,t2}$|) if |$Y_{N,t1}$| =1 or |$Y_{N,t2}$| =1 and $k \leq |Y_{N,t1}| + |Y_{N,t2}| - 1$ otherwise. It is clear that the time complexity of Algorithm 2 is O(k). The output of Algorithm 2 is $\{(y_1^i, y_2^i), i = 1,...k\}$

Algorithm 3. Merging algorithm (MA) for generating the PF of problem (3)-(8).

```
Step 1. t:= 1, call Algorithm 1 to generate the PF:= $\{(y_{1,t}^i, y_{2,t}^i), i = 1,...|Y_{N,t}|\}$ of period t
        subproblem, t2:= t; t:= t+1.
Step 2.
    while (t<T+1)
        Call Algorithm 1 to generate the PF:= $\{(y_{1,t}^i, y_{2,t}^i), i = 1,...|Y_{N,t}|\}$ of period t subproblem;
        t1:= t.
        Call Algorithm 2 to generate PF:= $\{(y_1^i, y_2^i), i = 1,...k\}$ by merging PF:=
        $\{(y_{1,t1}^i, y_{2,t1}^i), i = 1,...|Y_{N,t1}|\}$ and PF:= $\{(y_{1,t2}^j, y_{2,t2}^j), j = 1,...|Y_{N,t2}|\}$.
        if (t<T)
            $|Y_{N,t2}| := k,\ y_{1,t2}^i := y_1^i,\ y_{2,t2}^i := y_2^i,\ i = 1,\ldots,k.$
```

```
        end if
        $t := t+1$.
    end while
```

Lemma 2. $|Y_N| = O(T)$ and the time complexity of Algorithm 3 for solving problem (3)-(8) is $O(g(n_s,m_s)T)$, where $g(n_s,m_s)$ is time complexity of solving a single period subproblem of (3)-(8) and $n_s = |J|+3$ and $m_s = |U|+2$ are number of variables and number of constraints for the single period subproblem.

Proof: Assume that slopes of the PF in period $t = 1,\dots,T$ are unique, then $|Y_{N,\max}| = 1 + \sum_{t=1}^{T}(|Y_{N,t}| - 1)$, where $|Y_{N,t}| \le M$ and M is a constant. Then $|Y_N| \le |Y_{N,\max}| \le 1 + (M-1)T$. Thus, $|Y_N| = O(T)$.

The time complexity of generating the PF of a single period subproblem is $g(n_s,m_s)$ and the time complexity of Algorithm 2 is $O(|Y_N|)$. According to Algorithm 3, the accumulative effect of T is fully reflected in $|Y_N|$. Thus, the time complexity of Algorithm 3 for solving problem (3)-(8) is $O(g(n_s,m_s)|Y_N|) = O(g(n_s,m_s)T)$.□

3.4 Theoretical comparisons of MDSA and MA

Let $|Y_N^{\mathrm{MD}}|$ and $|Y_N^{\mathrm{M}}|$ denote the size of the non-dominated set of problem (3)-(8) generated by MDSA and MA respectively. Both MDSA and MA generate the exact PF for problem (3)-(8). $|Y_N^{\mathrm{MD}}| \ge |Y_N^{\mathrm{M}}|$ because MDSA has chance to generate the non-extreme non-dominated outcomes while MA only generates extreme non-dominated outcomes. Based on the results of numerical experiments, for the single period problem, it seems that MDSA does not generate non-extreme non-dominated outcomes. The number of non-extreme non-dominated outcomes generated by MDSA increases as the planning horizon increases.

Moreover, MA is more efficient than MDSA according to Lemma 2 and Corollary 1. According to $|Y_N| = O(T)$, the time complexity of MDSA for solving problem (3)-(8) is $O(g(n_s,m_s)\, T^2)$ while the time complexity of MA is $O(g(n_s,m_s)\, T)$. If T is much larger than n_s and m_s, then $g(n_s,m_s)$ can be treated as a constant and the time complexity of MDSA is reduced to $O(T^2)$ while the time complexity of MA is $O(T)$.

4. Computational experiments

To evaluate the efficiency and effectiveness of the merging algorithm (MA), the modified dichotomic search algorithm (MDSA) was used as a benchmark. In addition, to verify the correctness of MDSA, a general dichotomic search algorithm (DSA) was also implemented, where the new outcome is compared against all known non-dominated outcomes explicitly at Step 4 of Algorithm 1. The on-line envelope based (ECON) algorithm developed by [26] was used an LP solver for solving the single objective (weighted-sum function) hourly subproblem. For handling the small-size problem, on the average, ECON is on the average 467 times faster than CPLEX 9.0 according to instances in [26]. CPLEX is general commercial software for solving large-scale mathematical programming problems. To facilitate comparison, we here provide the relative performance of these two solvers according to test instances in the current experiment based on the latest version of CPLEX 12.5 [37].

All algorithms (MDSA, DSA and MA) were implemented in C++ in the Microsoft visual studio 2003 environment. All experiments were carried out on a 2.49 GHz Pentium PC (Dual core CPU) with 2.9 GB RAM under Windows XP operating systems.

4.1 Test problems

Our test problems were adapted from non-convex problems [38] ignoring the non-convexity characteristics. In practice, the non-convexity characteristics may be ignored in some strategic planning where capacities of plants are main concerns. The original test problems consist of six plants, where there are three real plants and three artificially derived plants modified according to real plants. Among the three real plants, one is backpressure (BP) plant (A1) and the other two are combined steam and gas cycle (CSG) plants (B1 and C1). Three artificially derived plants (A2, B2 and C2) were constructed by perturbing extreme points and restricting real plants (A1, B1 and C1) to operate within certain regions. In the current study, the fuel combusted at each plant needs to be specified explicitly since emission cost is explicitly considered as an objective. It is assumed that plants A1 and A2 are coal -fired, plants B1 and B2 are gas-fired and plants C1 and C2 are oil-fired. Table 1 summarizes the properties of six plants relevant to the current study. Tables 2 and 3 give the fuel characteristics (π, p, q) of three real plants (A1, B1 and C1) and three artificially derived plants (A2, B2 and C2), respectively.

Table 1. Properties of CHP plants

Plant	Type	Points	Fuel
A1	BP	28	coal
B1	CSG	27	gas
C1	CSG	28	oil
A2	BP	16	coal
B2	CSG	16	gas
C2	CSG	16	oil

Table 2 Fuel characteristic (π, p,q) of three real plants A1, B1 and C1.

A1			B1			C1		
28			27			28		
160	48	0	63.636	21	0	127.273	42	0
247.276	81.6	0.001	130.306	43	0.001	187.276	61.8	0.001
309.097	102	0.002	173.34	52	0.002	260.526	85	14
350.009	115.5	0.003	208.743	62.62	0.003	306.818	105	30
234.957	81.6	21.781	203.03	64	3	115.362	42	37.6
309.455	102	21.782	56.824	21	17.64	207.798	85	37.601
386.623	129	21.783	110.256	43	17.641	277.435	103.89	37.602
387.5	129	26	142.129	52	17.643	323.623	114.5	37.603
114.818	48	27.78	185.555	64	17.644	279.616	105	37.604
147.222	48	58	69.333	21	31	140	42	63
229.429	81.6	79	64.286	16	38	128.571	32	76
266.178	102	79.001	125.714	43	45	318.182	129	81
315.155	129	79.002	140.581	52	45.001	248.571	85	89
132.584	30	88	162.69	64	45.002	285.296	105	89.001
281.159	102	92	81.605	21	45.1	325.376	129	89.002
329.852	129	92.001	148.571	52	52	161.829	42	90.7
184.375	48	99.5	170.59	64	52.001	300	105	105
246.667	75	110	116.883	33	57	344.119	129	105.001
155.557	30	110.001	108.642	28	60	233.766	66	114
266.667	80	120	172.603	55	71	225.61	58	127
332.778	108.6	131	143.59	40	72	341.096	109	140
384.507	129	144	197.222	64	78	287.179	79	145
283.125	75	151.5	107.778	16	81	395.833	129	156
302.025	81.6	157	151.852	40	83	213.333	32	160
354.545	100	173	184.416	52	90	303.704	79	167
265.683	60.8	173.001	184.417	52	90.001	374.684	103	193
354.548	100	173.002	165.116	40	102	374.685	103	193.001
311.494	73	198				330.233	79	205

Then six (D1-D6) test problems are generated based on different combinations of above six plants, where D2 consists of three real plants. Table 4 shows dimensions (m_s, n_s) of single period test problems as well as the solution time of CPLEX and ECON. The m_s and n_s represent the number of constraints and variables respectively. As mentioned in the beginning of Section 4, since the ECON algorithm is used as an LP solver, it means that the transaction costs in the market are ignored, i.e., $c_{p+,t} = c_{p-,t}$. Then, the power sales and purchase volume ($x_{p\pm,t}$) can be replaced by one variable $x_{p,t}$ (refer to [26]). Consequently, $n_s = |J|+2$ and $m_s = |U|+2$ respectively. To form a valid test problem, the heat demand is generated based on history data of a Finnish energy company and Figure 3 shows the daily and weekly patterns of heat demand. The power price is generated based on the spot price history of the Nordic power market [39] and the emission allowance price is generated based on uniform distribution within [6.0, 16.0] €/ton according to the discussion in [40]. Following the assumption that the fuel with higher emissions is cheaper, prices of gas, oil and coal are fixed at 20, 15 and 10 €/MW, respectively.

Table 3 Fuel characteristic (π, p, q) of three artifically derived plants A2, B2 and C2.

A2			B2			C2		
16			16			16		
160	48	0	73.333	22	0	146.667	44	0
247.276	81.6	0.001	160.003	48	0.001	167.245	55.19	0.001
340.007	102	0.002	206.673	62	0.002	274.444	95	28.5
385.01	115.5	0.003	225.01	67.5	0.003	116.926	44	35.51
240.421	81.6	21.781	223.684	73	12	250.983	95	35.511
309.455	102	21.782	57.235	22	16.92	284.494	103.89	35.512
396.797	129	21.783	129.842	48	16.921	347.807	121	35.513
387.5	129	26	183.54	62	16.922	309.091	121	49
114.818	48	27.78	219.324	73	16.923	148	44	67
147.222	48	58	73.333	22	33	365.672	145	100
229.429	81.6	79	142.857	48	52	279.577	95	103.5
266.178	102	79.001	167.649	62	52.001	325.364	121	103.501
315.155	129	79.002	186.57	73	52.002	370.899	145	103.502
281.159	102	92	178.571	62	63	350	121	124
329.852	129	92.001	197.103	73	63.001	389.857	145	124.001
384.507	129	144	222.222	73	87	444.444	145	175

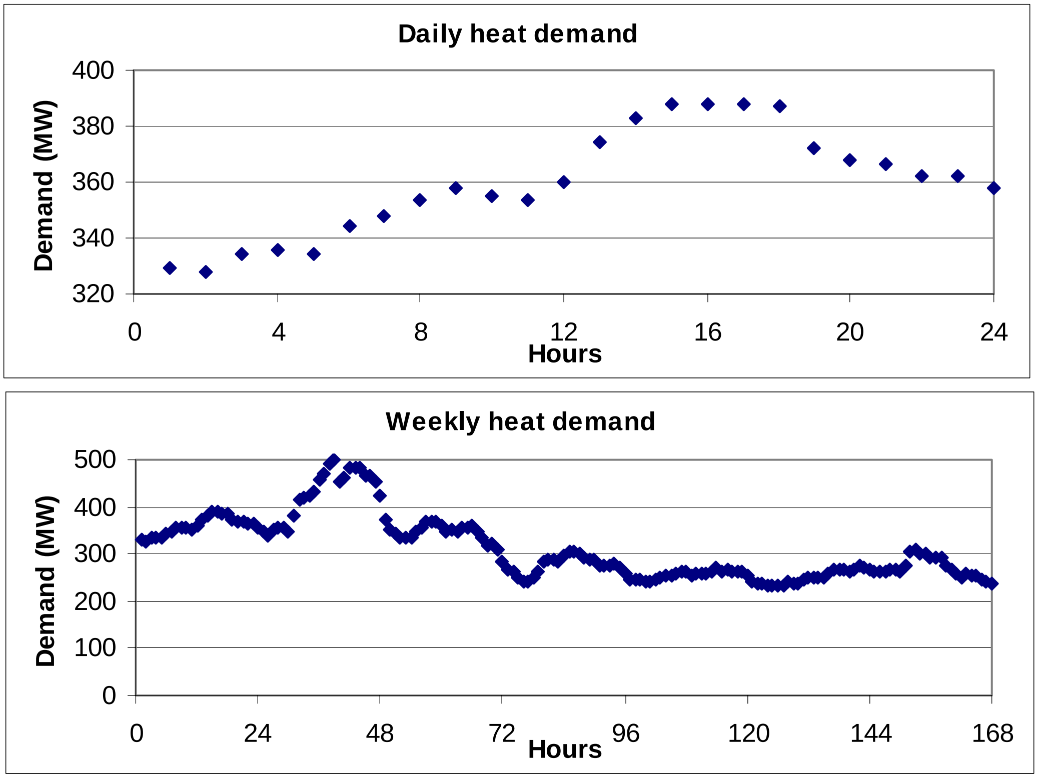

Figure 3 Daily and weekly heat demand patterns

Table 4 Dimensions of single period problems as well as the solution time of CPLEX and ECON

Problem	$\|U\|$	Dimension		Solution time (s)			
		m_s	n_s	CPLEX(0)	CPLEX (1)	CPLEX(2)	ECON
D1	4	6	77	98.734	92.453	93.5	0.0188
D2	3	5	85	95.266	89.25	87.172	0.0187
D3	4	6	101	89.687	90.312	94.391	0.025
D4	5	7	105	89.468	109.25	91.172	0.0265
D5	5	7	117	89.953	97.156	92	0.0282
D6	6	8	103	99.203	100.766	91.812	0.0344
Avg				93.719	96.531	91.675	0.0253

For the solution time in Table 4, we solved in sequence 8760 single objective hourly models of (3)-(8) with the first objective as the objective and the CPU time of both CPLEX and ECON were compared. For CPLEX, we recorded the time according to different values of clock type taking 0, 1 and 2, denoted by CPLEX (0), CPLEX (1) and CPLEX (3), respectively. According to [37], clock type taking 1, 2 and 0 represents the CPU time and the wall clock time as well as the time that CPLEX chooses to record, respectively. The CPU time is the total execution time or runtime for which CPU was devoted to a process. The wall clock time is the total physical time elapsed. For a pure sequential process, the CPU time should be less than the wall clock time. However, this may be not the case if there are parallel processes involved. This can be seen from the difference between CPLEX (1) and CPLEX (2) in the table. Sometimes, the CUP time is larger while other times the wall clock time is larger. Nonetheless, no matter which time that CPLEX takes, the speed ratio of ECON against CPLEX is larger than 2668 (the speed ratio of ECON against CPLEX (2) for D6). The speed ratio of ECON against CPLEX (1) is in range [2929, 4917] with average 3820. This ratio is much larger than that reported by [26]. In the current experiment, the number of characteristic points in the plant is much larger than that in previous experiment. One possible reason is the number of characteristic points on the envelope is much smaller than that in the plant for the current instances and thus ECON can gain much more benefit. According to [26], the points on the envelope are a function of power prices but it seems that the number of points on the envelope does not change much with the power price and is usually smaller than that in the plant. Similarly, for solving the weighted-sum function (14) of the bi-objective problem, the price of emission allowances also affects the points on the envelope. However, the trend should be similar. The other reason may be attributed to the test environment (computational facilities).

4.2 Computational results

We have solved test problems using general DSA, MDSA and MA for different planning horizons *T* (two-week (336 hour), four-week (672), eight-week (1344) and one-year (8760)). If the planning horizon is less than one year, then we have solved multiple non-overlap planning problems for the corresponding horizon within a year for each test problem and the average results of the corresponding horizons are obtained. For example, for an eight-week planning horizon, we can form a total of 6 non-overlap planning problems with six starting periods such as 1, 1345, 2689, 4033, 5377 and 6721. The numerical results showed that MDSA and the general DSA generate the same representation of the non-dominated set for all considered test problems. It means that the comparison scheme at Step 4 of MDSA is correct. In addition, MDSA gains a little advantage over the general DSA in terms of solution time. The average improvement is between 1% and 2% for the considered test problems. This may be due to the fact that solving weighted-sum functions for DSA is more time consuming than determining whether a new outcome is dominated or not.

Table 5 Average number of non-dominated outcomes for MDSA and MA for different planning horizons.

Problem	MDSA				MA			
	one-year	eight-week	four-week	two-week	one-year	eight-week	four-week	two-week
D1	46114	7163.3	3428.3	1794.9	45475	7144.3	3423.1	1793.5
D2	42170	6480.0	3106.9	1629.4	41463	6437.5	3088.6	1621.8
D3	71603	10809.3	5313.3	2745.4	69648	10747.7	5297.8	2741.2
D4	49352	7674.0	3691.4	1912.0	48561	7651.5	3685.2	1909.9
D5	49778	7679.3	3733.2	1927.0	49075	7653.2	3727.2	1925.1
D6	75649	11487.5	5590.5	2909.6	73297	11397.0	5567.2	2902.7

In the following, the results of MDSA and MA for different planning horizons are reported. Tables 5 and 6 give the size of the non-dominated set and the solution time for MDSA and MA respectively.

Based on Table 5, first, the size of non-dominated set is roughly proportional to *T*. Second, $|Y_N^{\mathrm{MD}}| \geq |Y_N^{\mathrm{M}}|$ and the $|Y_N^{\mathrm{MD}}| - |Y_N^{\mathrm{M}}|$ increases as *T* increases, from 4 for two-week horizon to 1191 for one-year horizon. These results agree with the discussion in Section 3.4. The above first point implies that it may not be a trivial problem to find the exact the PF of the long-term CHP planning problem even though dynamic constraints are ignored due to the large size of the non-dominated set. The second point means that the representation of the non-dominated set based on the results of MA is compact. According to MA algorithm, if slopes for the PF are unique for all single period models, then $|Y_N^M| = |Y_{N,\max}| = 1 + \sum_{t=1}^{T}(|Y_{N,t}^M| - 1)$. $|Y_N^M| / |Y_{N,\max}| \approx 0.8$ for the problems considered in the experiment. It means that about 20% slopes of the PF for different periods coincide.

Table 6 Average solution time (s) for MDSA and MA for different planning horizons.

Problem	MDSA				MA			
	one-year	eight-week	four-week	two-week	one-year	eight-week	four-week	two-week
D1	2349.22	56.64	13.52	3.55	12.59	0.58	0.23	0.10
D2	2114.20	50.77	12.13	3.19	10.97	0.56	0.22	0.099
D3	4607.83	107.78	26.41	6.83	27.42	0.89	0.32	0.13
D4	3321.08	80.77	19.36	5.02	13.72	0.66	0.25	0.11
D5	3764.98	90.88	22.02	5.70	15.11	0.65	0.26	0.11
D6	6905.44	159.99	39.01	10.14	28.66	1.05	0.37	0.15

Based on Table 6, the solution time for MDSA is roughly proportional to T^2 while the solution time for MA is roughly proportional to kT, where $k \leq 10$. It means that the solution time of the single period subproblem is bounded by a constant. This again agrees with the discussion in Section 3.4 and MA is much more efficient than MDSA. It is not difficult for MA to handle problems for long planning horizons (e.g. 15 or 20 years). On the other hand, the efficiency for MA is largely attributed to the efficiency of the solver for the single period weighted-sum subproblem. According to Table 4, it is difficult for MA to handle a yearly planning problem if ECON is replaced by CPLEX. Similarly, the MA is also more efficient than the ε-method where the multi-period problem needs to be solved by a general solver (e.g. CPLEX). It is easy to see that it is even difficult for MDSA to handle a two-week planning problem if ECON is replaced by CPLEX.

Finally, we use MA to investigate the effect of emission allowance price on the size of the non-dominated set and on the solution efficiency according to yearly planning problems. We use the scenario with constant emission allowance price as a benchmark. It is equivalent to contrasting the difference between the traditional EED (EED1) [33] where emissions are treated as the second objective and the current EED (EED2) where emission costs are treated as the second objective. Table 7 shows the results.

Table 7 Effect of the emission allowance price on the size of non-dominated set and on the solution efficiency for yearly planning problems.

Problem	$\sum_{t=1}^{T}\|Y_{N,t}^{M}\|$	EED1			EED2		
		CPU (s)	$\|Y_N^M\|$	$\|Y_N^M\|/\|Y_{N,\max}\|$	CPU (s)	$\|Y_N^M\|$	$\|Y_N^M\|/\|Y_{N,\max}\|$
D1	65898	4.66	17520	0.31	12.59	45475	0.80
D2	59778	4.52	16812	0.33	10.97	41463	0.81
D3	93734	8.28	31043	0.37	27.42	69648	0.82
D4	76585	5.55	21611	0.32	13.72	48561	0.72
D5	72628	6.53	24191	0.38	15.11	49075	0.77
D6	102003	10.86	37849	0.41	28.66	73297	0.79

For both EED1 and EED2, $\sum_{t=1}^{T}|Y_{N,t}^{M}|$ are the same. It means that allowance price does not affect the size of the non-dominated set for a single period subproblem. However, the size of the non-dominated ($|Y_N^M|$) for the EED1 is much smaller because profiles of the PF (slopes for two consecutive non-dominated outcomes) from period to period are similar. Based on $|Y_N^M|/|Y_{N,\max}|$, 60% to 70% slopes of the PF for single period subproblems coincide for the EED1 while about 20% slopes coincide for the EED2. This means that the planning problem under ETS considering emission costs as the second objective is harder than the traditional planning problem considering emissions as the second objective. This also reflects in the solution time (CPU(s)). On the other hand, it can be seen that tradeoff between economic and emission objectives is not sensitive to emission allowance price under the fuel mix setting in the sense that tradeoff (results of EED1) should exist regardless of emission allowance prices. It means that it is less likely that speculative options for single optimization (aggregating economic and emission objective) are recommended to favor the fuel with higher emissions when the emission allowance price is lower. In other words, the multi-objective approach can provide better decision support under uncertainties of emission allowance market.

The results of Table 7 agree with the theoretical results of Lemma 2, i.e., the solution time for MA is proportional to $|Y_N^M|$ (the size of the non-dominated set). Prices of emission allowances mainly affect slopes of the PF. In the extreme case, slopes of PF in each period are unique and $|Y_N^M|/|Y_{N,\max}|=1$. As we mentioned before, D2 is a real instance. We can obtain the worst case (largest) solution time of the MA for D2 as 10.97/0.81=14 (s) for a yearly planning problem according to Table 7 and Lemma 2. It means that the MA is fully applicable to real world planning.

5 Conclusion

In this paper, we have presented an efficient specialized merging algorithm (MA) to find the exact PF for the bi-objective convex CHP planning problem considering a fuel mix setting. If the fuel with higher emissions is cheaper than that with lower emissions, then plants fired by all types of fuels may be put into use. The size of the non-dominated set can be large and is proportional to the planning horizon. For a yearly planning problem, the size can be more than 40 thousand. Such a large size challenges the solution of the problem even though each non-dominated outcome can be obtained by a polynomial algorithm for the traditional dichotomic search algorithm. It is difficult for a general solver such as CPLEX to handle the problem. The efficiency of the MA is justified theoretically and empirically. The MA is applicable to the long term planning problem for risk analysis and generation expansion planning. The MA may lay foundation for integrating multicriteria decision analysis and scenario planning [22].

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