Modelling the Filtration Stage of a Pulp Washer

By K Kovasin
Rauma-Repola, Finland
and H V Nordén & L Ahonen
Helsinki University of Technology, Department of Chemical Engineering, Finland

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The thickening stage in the Pro-Feed washer was assumed to consist of two parts: mat formation and compression. The filtration model during thickening was based on the Kozeny-Carman equation. In the washing stage, submerged displacement takes place while, simultaneously, the pulp mat thickness increases. The modelling in this stage was based on continuity equations.

For this example, softwood kraft pulp was studied. Diagrams are presented for the consistency profile during thickening as well as for the impact of varying inlet consistency on the final consistency after the thickening stage. For the washing stage, typical displacement velocity and consistency profiles are given.

Considering filtration phenomena in washers and other filtration equipment for pulp suspensions, rigorous theoretical considerations imply the formulating of equations of continuity for a three-dimensional, two-phase flow problem. In general, solving the system equations is not impossible but usually a laborious task. It is, however, possible to assume constant average flow and bed properties over a differential layer of a porous pulp mat which remarkably simplifies the calculations. Equations of this kind have been derived(1, 2) and it is convenient to formulate the filtration problem by using the Kozeny-Carman equation for the pressure loss in a porous bed and incorporate Darcy's law into that model. It is reasonable to assume laminar flow for pulp washing(3) which justifies the use of Darcy's law in this connection.

In the consistency range normally applied in pulp washing, the porosity of the pulp bed is rather high. Correlations for the Kozeny factor in the high porosity region have been developed(4, 5).

A previous filtration model(1) has been further extended(6), and by choosing a few dimensionless and dimensional quantities properly, it is possible to make the integration of the Kozeny equation remarkably easier.

In this paper, the mathematical modelling of the filtration phenomena in the Pro-Feed pressure washer will be presented. The treatment consists of two separate entities: thickening and displacement washing. The theory applied is based on the previous method(6).

The Pro-Feed washer

A schematic diagram of the Pro-Feed washer* is presented in Fig 1. It is a drum filter where the required pressure difference is generated by a blower which circulates gases and vapours from the inside of the drum to the washer hood. The gauge pressure in the hood is typically 1-1.5mH$_2$O. Pulp suspension of 2-4% consistency is pumped into the mat formation zone, where the stock is thickened by means of free mat formation and thereafter by mechanical compression of the formation plate (Fig 2). After the mat formation, one- or two-stage submerged displacement washing takes place. The final thickening to 12-14% discharge consistency follows the washing stage. Due to the pressure washer principle, the maximum operation temperature is close to 100°C.

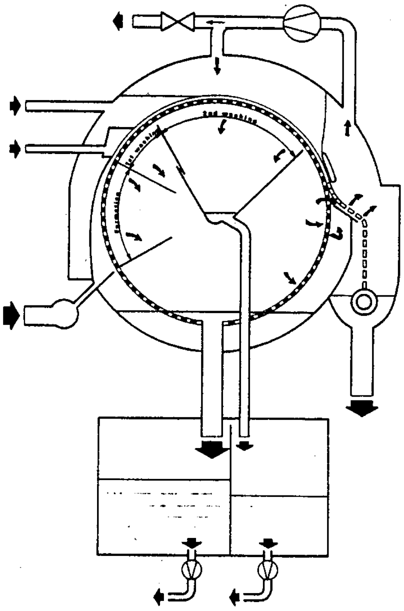

Fig 1. Principle scheme of the Pro-Feed washer

Considering the drum inside, the pressure washer principle also makes it possible to collect the second stage filtrate separately by means of a filtrate trough. This facilitates two-stage washing and, in bleaching applications, separation of, eg acidic and alkaline filtrates.

The washers can be used equally well in brown stock washing and bleaching. Typical results obtained with the machines have been presented(7).

Theory

Mathematical modelling of the thickening stage. Thickening is assumed to consist of two stages: mat formation and mat compression (Fig 2). The mathematical formulation is based on the theory presented and further extended(6, 8, 9).

Thickening is thought to take place as a one-dimensional operation in direction X in a cylindrical vessel as shown in Fig 3, although the real thickening takes place as a three-dimensional operation as in Fig 2. The simple model can be assumed to represent the real operation rather well, since during thickening the pulp mat in the real operation

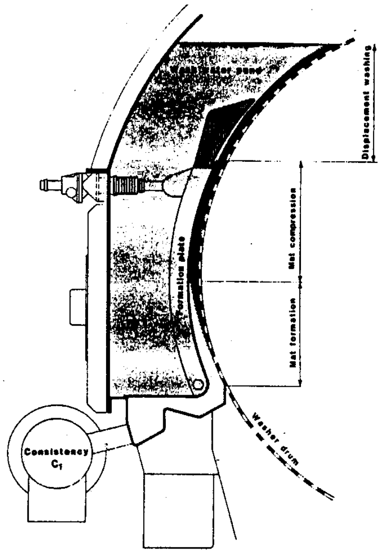

Fig 2. Thickening and washing stages at the Pro-Feed washer

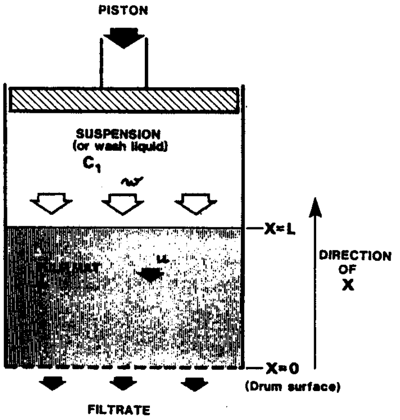

Fig 3. Modelling of thickening and washing stages in the Pro-Feed washer

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*The Pro-Feed multi-stage washer has been developed by Rauma-Repola to provide high quality pressure washing and drum washer reliability, and to give a high capacity over the drum area with wash liquid displacement as close as possible to the ideal.

can be assumed to be formed in one direction perpendicular to the drum surface.

Pressure drop over a differential thickness layer dX is given in the modified Kozeny-Carman equation:

$$\frac{\partial p}{\partial X} = - \frac{S_v^2 \eta w v^2 c^2}{} \frac{\{5.0 + \exp[14.0\,(0.2 - vc)]\}}{(1 - vc)^3} \tag{1}$$

where p = compaction pressure against fibres, Pa.

In Eq (1), the Kozeny factor k is expressed by using the formula$^{(5)}$:

$$k = 5.0 + \exp[14.0\,(0.2 - vc)] \tag{2}$$

When Eq (1) is integrated over the mat thickness, two equations can be obtained. Consistency is first expressed by using the equilibrium equation $^{(10)}$ which takes the compressibility of pulp mat into account:

$$c = Mp^N \tag{3}$$

For the dimensionless mat thickness:

$$\int_{y_1}^{y_2} \frac{(1 - y^N)^3}{y^{2N}\{5.0 + \exp[14.0\,(0.2 - y^N)]\}}\,dy = \frac{L}{X_o} = \Delta x \tag{4}$$

and then for the dimensionless surface loading (pulp amount per surface area):

$$\int_{y_1}^{y_2} \frac{(1 - y^N)^3}{y^N\{5.0 + \exp[14.0\,(0.2 - y^N)]\}}\,dy = \frac{L\,c}{X_o\,c_o} = \Delta W \tag{5}$$

where y = dimensionless pressure = p/p_o (6)

$$p_o = (vM)^{-1/N} \tag{7}$$

$$1/X_0 = \eta w\, S_v^2\, v^2 M^2\, p_o^{\,2N-1} \tag{8}$$

$$c_o = M\,p_o^{\,N} \tag{9}$$

In the above equations:

η = filtrate viscosity, kg/ms
S_v = fibre specific surface, m^2/m^3
v = fibre specific volume, m^3/kg
w = suspension superficial velocity towards the drum surface, m/s
c = fibre consistency, kg/m^3
M,N = compressibility constants, specific for each pulp type – N is dimensionless; M, (kg/m^3)/Pa^N.
L = mat thickness, m

Considering mat formation, a mass balance equation for fibres is also needed:

$$c_1 w = \frac{d}{dt}\Big[\,L\,(\bar{c} - c_1)\,\Big] \tag{10}$$

where c_1 = fibre consistency in the incoming suspension, kg/m^3
$\bar{c}$ = average fibre consistency in the mat, kg/m^3.

By combining Eqs (5) and (10), the mat formation equation is obtained:

$$(L\bar{c})^2 = \frac{2X_o\,w\,c_o\,c_1}{(1 - c_1/\bar{c}_{AV})} \int_o^t \int_{y_1}^{y_2} \frac{(1 - y^N)^3\,dy\,dt}{y^N\,\{5.0 + \exp[14.0\,(0.2 - y^N)]\}} \tag{11}$$

where y_1 = $p_1/p_o = \left(\frac{c_1}{M}\right)^{1/N} \cdot \frac{1}{p_o}$ (on the mat surface)
y_2 = p_t/p_o (on the drum surface)
p_t = total pressure difference over the mat = effective pressure difference
t = time for mat formation, s
$\bar{c}_{AV}$ = average fibre consistency in the mat over the mat formation time, kg/m^3.

The surface load can be calculated from Eq (10), while the average consistency can be obtained by dividing Eq (5) with Eq (4).

Because of the fact that Eqs (4) and (5) were originally derived for the filtration through a pulp bed without any motion of fibres, the dimensionless quantities obtained in Eqs (4) and (5) have to be connected by dividing with derived correction factors$^{(8)}$.

$$f_1 = 1 - (1/L) \cdot \int_O^L \frac{X}{L}\left[\frac{\bar{c}_x/\bar{c} - 1}{\bar{c}/c_1 - 1}\right] dX \tag{12}$$

For dimensionless surface load, divide Eq (5) with:

$$f_2 = 1 - (1/\bar{c}L) \cdot \int_O^L \frac{X}{L}\left[\frac{\bar{c}_x/c - 1}{\bar{c}/c_1 - 1}\right] c\,dX \tag{13}$$

where X = distance from drum surface
$\bar{c}_x$ = average consistency from X = 0 to X = X
$\bar{c}_{AV}$ = average consistency over the entire mat

By using Eqs (11)-(13) the mat formation stage can be calculated if the effective pressure p_t over the pulp mat is known as a function of the location. Here the total pressure difference over the mat or p_t is assumed to decrease linearly in the direction of flow.

Thickening obeys the above equations until the point is reached when the mat thickness becomes the same as the distance between drum surface and formation plate. After that point, thickening continues by means of compression.

Using a simplified model, where the pulp is compressed one-dimensionally in a cylindrical vessel, as in Fig 3, the following theory can be developed. In this case the real mat thickness is not equal any more to that of the model in Fig 3, because the real mat can be compressed or extended in the direction of the drum surface.

Because no new fibres enter the mat from suspension during the compression stage, the surface load keeps constant:

$$L(t)\;\bar{c}\;(t) = L_o\;\bar{c}\;(o) \tag{14}$$

where L(t) = mat thickness in the model of Fig 3 at time t
$\bar{c}(t)$ = average mat consistency at time t
L_o = mat thickness at the beginning of compression
$\bar{c}(o)$ = average mat consistency at the beginning of compression

During compression, – dL/dt = w, therefore, two new equations can be obtained, when Eqs (4) and (5) are multiplied with w:

First from Eq (4):

$$-\frac{dL^2}{dt} = 2\,X_o\,w\,\Delta x \tag{15}$$

From Eq (5):

$$-\frac{dL}{dt} = \frac{X_o\,w\,c_o\,\Delta W}{L\,c} \tag{16}$$

Δx and ΔW are the values of the integral of Eqs (4) and (5), respectively.

Using Eqs (14)-(16), it is now possible to create a calculation algorithm by means of which the compression stage can be calculated.

The same correction factors as in the compression stage, ie f_1 and f_2 from Eqs (12) and (13), are assumed to be applicable also during the formation stage. The integrals in Eqs (4) and (5) are corrected in the same way as in the formation stage. This assumption has to be made because the formulating of correction factors separately for the compression stage seems to be difficult. Quite likely the same f_1 and f_2 as in formation hold for some time at the beginning of compression.

Mathematical modelling of the displacement washing stage. In this theoretical analysis, one-stage washing is assumed to take place after the thickening zone (Fig 2). When the thickened pulp mat emerges from below the formation plate, submerged displacement washing takes place.

During the washing stage, wash liquor flows through the mat thus displacing the 'dirty' mat liquor. The evenness of displacement can be expressed by means of washing efficiency, eg with the E-factor$^{(11)}$ or by means of the extension of the original theory$^{(12)}$. Simultaneously with the wash water displacement, the pulp mat is assumed to swell and, therefore, the average mat consistency decreases in the course of washing.

Especially due to the latter assumption, there is no simple way to express the filtration phenomena during the washing stage. The modelling has to be performed by using the equations of continuity. Washing is assumed to take place perpendicularly to the drum surface only. For this reason, it is sufficient to study the situation in one dimension only. X is the dimension perpendicular to the drum surface, X = 0 at the drum surface and grows towards the mat surface.

The equation of continuity for the fibres:

$$\frac{\partial c}{\partial t} - \frac{\partial}{\partial X}(cu) = 0 \tag{17}$$

where u = velocity of the fibres in the mat; in m/s.

In Eq (17) the negative sign of the second term on the left hand side is due to the opposite directions of u and X (Fig 3). The equation of continuity for the entire fibre-liquid system is:

$$\frac{\partial w}{\partial X} = 0 \tag{18}$$

the pressure drop equation:

$$\frac{\partial p}{\partial X} = -\,k\,(c)\,(w - u) \tag{19}$$

and the equilibrium equation:

$$c = Mp^N \tag{20}$$

In addition to the above equations a mass balance equation is also

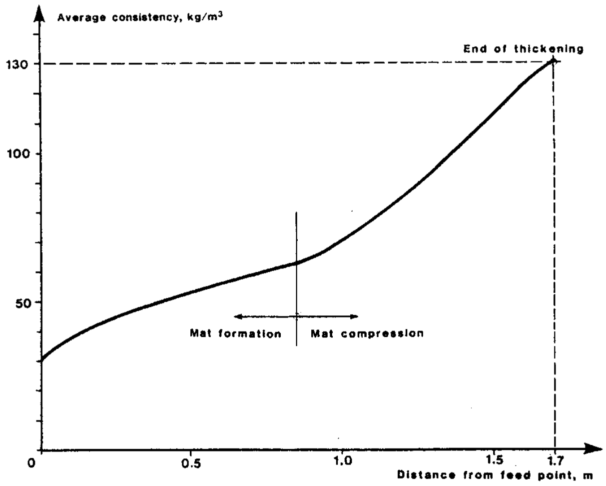
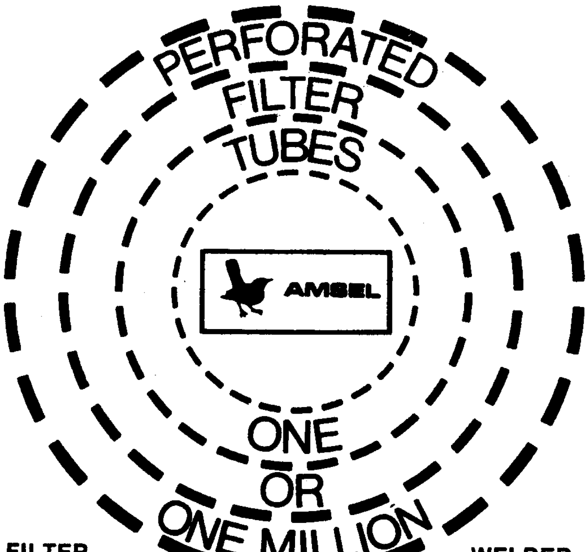


Fig 4. Average consistency between the drum surface and formation plate during thickening

Table 1. Simulation of the Pro-Feed washer: initial values used in modelling

Washer dimensions:	Drum diameter	3.5m	Drum length	5.0m
	Length of thickening stage	1.7m	Length of washing stage	1.1m
	Distance between formation plate tip and drum	0.04m		
Process data:	Production rate	450 ODT/D	Feed consistency	30kg/m^3
	Gauge pressure at the feed point	41,678Pa	Gauge pressure at the end of washing	9,807Pa
	Temperature during thickening	90°C	Temperature during washing	80°C
Pulp properties:	Compression exponent	N = 0.43		
	Compression parameter	M = 2.2 (kg/m^3)Pa^N		
	Specific volume	v = 0.0036m^3/kg		
	Specific surface	S_v = 27,000l/m		



needed which shows that the fibre surface load keeps constant during washing:

$$\int_0^L c\,dX = L_o\bar{c}(o) = \text{constant} \tag{21}$$

Boundary conditions are as follows:

$$\text{at the mat surface: } X = L,\ c(L) = 0 \tag{22}$$

$$\text{on the drum surface: } X = 0,\ c(o) = Mp_t(t)^N \tag{23}$$

$$\text{Moreover: } u = -\frac{dL}{dt},\ \text{when } X = L \tag{24}$$

$$u = 0, \qquad \text{when } X = 0 \tag{25}$$

Initial conditions: At the beginning of the washing stage, where t = 0, it is necessary to know the consistency profile through the mat. The equations presented above for the thickening stage do not, however, produce such a profile. At t = 0 the consistencies are known at both boundaries. In addition to that, the average consistency and mat thickness are also known. With this information and by assuming that a quadratic equation with respect to X gives a reasonable fit for the consistency profile, it is now possible to get an approximative profile.

From the above equations, a set of partial differential equations can be derived for the modelling of the washing stage:

From Eqs (19) and (20):

$$\frac{\partial c}{\partial X} = \frac{1}{A(c)} \cdot (w - u) \tag{26}$$

From Eqs (17) and (26) there follows:

$$\frac{\partial c}{\partial t} = w\frac{\partial c}{\partial X} - \frac{\partial}{\partial X}\left[cA(c)\frac{\partial c}{\partial X}\right] \tag{27}$$

Eq (26) can be applied for both mat surface and bottom. For X = 0 there follows:

$$w = A(c)\Big|_{X=0} \cdot \frac{\partial c}{\partial X}\Big|_{X=0} \tag{28}$$

and for X = L there follows:

$$u = -\frac{dL}{dt} = w - A(c)\Big|_{X=L} \cdot \frac{\partial c}{\partial X}\Big|_{X=L} \tag{29}$$

In the above equations:

$$A(c) = \frac{-(1 - vc)^3 c^{1/N - 3}}{S_v^2 \eta v^2 NM^{1/N}\{5.0 + \exp[14.0(0.2 - vc)]\}} \tag{30}$$

The numerical solution of Eqs (21), (26), (28) and (29), of which the washing stage model is composed, can be performed by transforming Eqs (26), (28) and (29) to difference equations and correspondingly formulating the integral in Eq (21) by means of Simpson's formula. In the same way, the boundary conditions in Eqs (22) and (23) as well as the quadratic initial condition for the consistency profile have to be adapted to the system of finite difference equations. Finally, before starting the computations, the velocity of the thickened pulp mat has to be the same as the peripheral velocity of the drum which implies a 'stretching' of the pulp mat immediately at the beginning of the washing stage.

Calculation examples

To illustrate the usage of the thickening and washing model presented here, we give a typical example. The initial values are given in Table 1. For the base case, a 3.5m diameter and 5m long Pro-Feed washer is chosen. The production rate is 450 ADMT/d and the specific pulp properties correspond with those of pine kraft pulp at a kappa number close to 30.

Thickening stage. If the feed consistency is 3% (30kg/m^3) the correction factors of Eqs (12) and (13) become: $f_1 = 0.75$ and $f_2 = 0.85$. Fig 4 presents the average mat consistency as a function of the distance from the feed point. The mat formation stage which continues up to the point when the mat thickness becomes equal to the distance between the formation plate and drum surface, takes approximately half of the total thickening length.

In the compression stage, thickening continues to about 13% consistency (130kg/m^3). The effect of the feed consistency is presented in Fig 5. This diagram can be used for the selection of the length of the thickening stage, especially at lower feed consistencies.

Washing stage. Considering the washing stage, the most important thing to know is the velocity of the filtrate through the mat. In Fig 6, average displacement velocities are presented as a function of the average final consistency from thickening. Diagrams like Fig 6 are

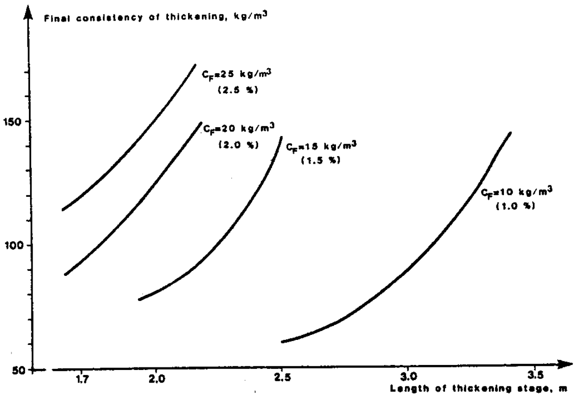


Fig 5. Final thickening consistency as a function of the length of thickening stage and feed consistency

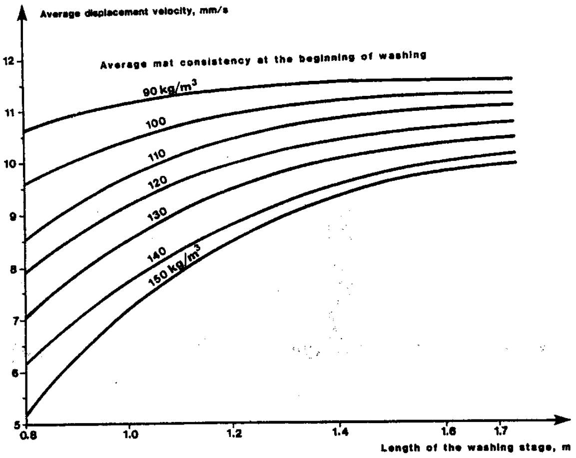


Fig 6. Average wash water displacement velocity during the washing stage

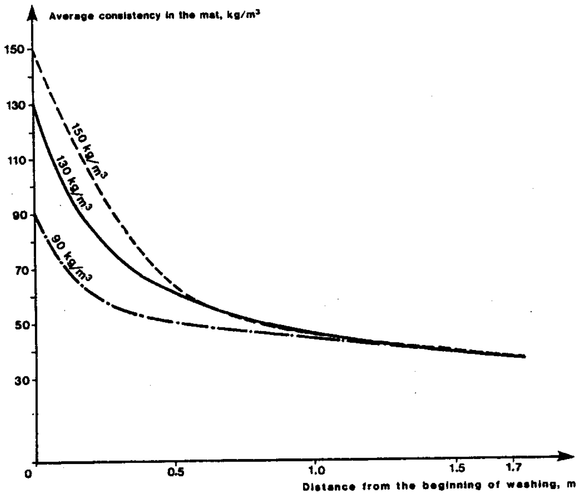


Fig 7. Average mat consistency during the washing stage

very useful in the estimating of wash water amounts after thickening to various consistency levels.

Sometimes it is interesting to know the consistency profile during the course of washing. Fig 7 illustrates the variation of the average mat consistency during the washing stage. The mat consistency seems to decrease rather fast and independently of the initial consistency at the beginning of washing. The same consistency level is obtained if the length of the washing stage is long enough. In practice, the length of the washing stage depends on the surface level of the wash water pond.

Discussion

The mathematical equations presented form the filtration model at the thickening and washing stages of the Pro-Feed washer. The thickening stage was modelled according to the pressure drop over a thin pulp layer. The washing stage was formulated based on the equations of continuity to which the equations for pressure drop and compressibility were incorporated. It is somewhat difficult to estimate the accuracy of the models because experimental data is practically impossible to obtain for the phenomena occurring during thickening and washing in an actual pulp washer. The basic theory applied in the thickening stage has been, however, tested in semi-batch washing by comparing measured pressure losses to those obtained from the theory(13). In most cases, reasonable agreement could be found. For the mat compression and the enlargement during the washing stage, no equations have been found in literature. However, continuity equations have been used for the modelling of the flow of fibre-water suspensions but in most cases for the phenomena occurring at paper machines(14, 15).

Considering some assumptions made for the models, the linear relationship between total pressure difference and the distance from the feed point could be checked by formulating an independent pressure drop equation from the momentum balance of the thickening stage. The analysis, presented elsewhere(16), shows that the assumption of a linear relationship is justified. Another important factor is the introduction of the correction factors (Eqs (12) and (13)), which take the motion of fibres into account during mat formation. In the compression stage, the same correction factors as in formation were used. The numerical values for the correction factors in the compression stage may be somewhat different from those of the formation stage. However, as mentioned earlier, deriving correction factors for compression seems to be difficult. Therefore, as a first approximation using the same correction factors during the entire thickening zone may not cause a big error.

One more thing worth mentioning is that the specific fibre properties, specific volume and surface, as well as the compressibility parameters, have to be known before any calculations can be made. Fortunately, these properties have been determined for some typical pulp types(2). However, for each pulp type the specific fibre properties should be known accurately if accurate modelling results are desired.

Nomenclature

A (c)	defined with equation (30)
c	consistency, kg/m^3
c_o	a constant, defined with equation (9), kg/m^3
c_1	feed consistency, kg/m^3
$\bar{c}$	average consistency in the mat, kg/m^3
$\bar{c}_{AV}$	average fibre consistency in the mat over the mat formation time, kg/m^3
$\bar{c}_X$	average mat consistency from X = 0 to X = X, kg/m^3
f_1, f_2	correction factors, defined with equations (12) and 13), dimensionless
L	mat thickness, m
L_o	mat thickness at the beginning of compression, m
M	compressibility parameter, (kg/m^3)/Pa^N
N	compressibility parameter, dimensionless
p	compaction pressure against fibres, Pa
p_t	total pressure diffeence over the mat, Pa
p_o	a constant, defined with equation (7), Pa
S_v	specific surface of fibres, 1/m
t	time, s
u	superficial velocity of fibres, m/s
v	specific volume of fibres, m^3/kg
w	suspension superficial velocity, m/s
X	distance from the drum surface, m
X_o	a constant defined with equation (8), m
y	dimensionless pressure against fibres, dimensionless
η	filtrate viscosity, kg/ms

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