Department of Mathematics and Systems Analysis

Robust reliability and resource allocation - Models and algorithms

Antti Toppila

Aalto University

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Antti Toppila

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Abstract

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Abstract

Organizational decision makers (DMs) such as companies, institutions and public sector agencies rely on mathematical models for decision support. Often these models have parameters such as probabilities of events and outcomes of actions, which typically are epistemically uncertain due to the lack of historical data or other information. In such cases, DMs often need to understand how this epistemic uncertainty impacts the decision recommendations.

This Dissertation considers models for supporting allocation decisions in settings where epistemic uncertainty is modeled explicitly through incomplete information. The resulting decision recommendations that account for epistemic uncertainty are derived through dominance: Alternative A dominates alternative B if A is at least as good as B for all parameters that are compatible with the available incomplete information, and moreover, strictly better for some. A dominated alternative should not be selected, because there exist at least one alternative that is not worse for any parameters and is strictly better for some. Thus, the decision recommendation to select an alternative that is non-dominated (ND) is robust with respect to the epistemic uncertainty.

In the models considered in this Dissertation, generating the ND alternatives leads to a computationally challenging combinatorial optimization problem. Several exact algorithms and approximative methods for computing the ND alternatives are developed. The exact methods are based on classical dynamic programming and branch-and-bound algorithms, as well as binary decision diagrams, which have recently been used in solving challenging optimization problems. The simplification methods, on the other hand, are more ad hoc in nature and based on problem specific approaches.

This Dissertation contributes by providing ways for analyzing the impact of epistemic uncertainty with incomplete information in application areas which are central in the fields of risk analysis and decision analysis, namely (i) probabilistic risk analysis based on importance measures, (ii) allocation of resources to reliability enhancing actions, (iii) project portfolio selection, and (iv) resource allocation to standardization activities. The developed methods are generic in that they could likely be adopted with small refinements even in other application areas.

Keywords Epistemic uncertainty; incomplete information; combinatorial optimization; importance measures; binary decision diagrams; project portfolio selection

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Tiivistelmä

Organisaatiot, kuten yritykset, instituutiot ja julkiset toimijat, tukeutuvat usein matemaattisiin malleihin tehdessään päätöksiä. Nämä mallit sisältävät lähes aina parametreja, kuten tapahtumien todennäköisyyksiä tai arvioita vaikutuksista, jotka ovat episteemisesti epävarmoja, koska tarkkojen estimaattien muodostamiseen ei löydy riittävästi historiallista dataa tai muita tietolähteitä. Tällöin päätöksentekijät usein tarvitsevat tukea ymmärtääkseen, miten episteeminen epävarmuus vaikuttaa päätössuosituksiin.

Tämä väitöskirja tarkastelee päätöksentekomalleja, joissa episteemistä epävarmuutta kuvataan eksplisiittisesti epätäydellisellä informaatiolla. Episteemisen epävarmuuden huomioivat päätössuositukset johdetaan dominanssin avulla: Vaihtoehto A dominoi vaihtoehtoa B jos A on vähintään yhtä hyvä kuin B kaikilla parametreilla, jotka ovat yhteneväisiä annetun epätäydellisen informaation kanssa ja lisäksi aidosti parempi joillakin parametrien arvoilla. Näin ollen episteemisen epävarmuuden kannalta robusti päätössuositus on valita ei-dominoitu (ND; Non-Dominated) vaihtoehto.

Tässä väitöskirjassa tarkastelluissa malleissa ND-vaihtoehtojen generoiminen edellyttää laskennallisesti haastavien kombinatoristen optimointiongelmien ratkaisua. Ongelmien ratkaisemiseen kehitetään useita tarkkoja ja approksimatiivisia menetelmiä. Tarkat menetelmät perustuvat klassisiin dynaamisen optimoinnin ja hajoita-ja-hallitse (B&B; Branch-and-Bound) algoritmeihin, sekä binäärisiin päätöskaavioihin, joita viime aikoina on käytetty haastavien optimointiongelmien ratkaisemiseen. Approksimatiiviset menetelmät puolestaan ovat enemmän ad hoc-luonteisia ja perustuvat ongelmaspesifeihin lähestymistapoihin.

Tämän väitöskirjan tulosten avulla voidaan episteemistä epävarmuutta analysoida epätäydellisen informaation avulla eri sovellusaloilla, jotka ovat keskeisiä riskianalyysin ja päätösanalyysin kentillä. Tässä väitöskirjassa sovellusalueina ovat (i) todennäköisyysperustainen riskianalyysi (PRA; Probabilistic Risk Analysis), (ii) resurssien allokointi luotettavuutta parantaville toimenpiteille, (iii) projektiportfolion valinta ja (iv) standardointiresurssien allokointi. Kehitettyjä menetelmiä voidaan vähäisin muutoksin hyödyntää myös muilla sovellusaloilla.

Avainsanat Episteeminen epävarmuus; epätäydellinen informaatio; kombinatoorinen optimointi; tärkeysmitat; binääriset päätöskaaviot; projektiportfolion valinta

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Publications

This Dissertation consists of a summary article and the following papers.

I A. Toppila and A. Salo. A computational framework for prioritization of events in fault tree analysis under interval-valued probabilities. *IEEE Transactions on Reliability*, Vol. 62, No. 3, pp. 583-595, 2013.

II A. Toppila and A. Salo. Selection of risk reduction portfolios under interval-valued probabilities. *Manuscript*, 30 pages, 2016.

III A. Toppila and A. Salo. Binary decision diagrams for generating and storing non-dominated project portfolios with interval-valued project scores. *Manuscript*, 30+10 pages, 2016.

IV A. Toppila, J. Liesiö and A. Salo. A resource allocation model for R&D investments – A case study in telecommunication standardization. *In:* A. Salo, J. Keisler and A. Morton (eds.)*, Portfolio Decision Analysis: Improved Methods for Resource Allocation*, pp. 281–258, Springer, New York, 2011.

Contributions of the author

Toppila is the primary author of Papers I-IV. He also developed and implemented the models and algorithms, and produced and reported the numerical results.

The topic and initial idea for the optimization algorithm in Paper I was proposed by Salo. Toppila developed it to a computer implementable algorithm and wrote the paper. Salo provided comments.

The topic of Paper II was jointly developed by Toppila and Salo. The algorithm and the numerical examples were developed by Toppila. Toppila wrote the paper and Salo provided comments.

The topic and the computational methods in Paper III were suggested by Toppila. The contribution was scoped and positioned to the literature jointly by Toppila and Salo. Toppila wrote the paper and Salo provided comments.

The topic and model in Paper IV was proposed by Salo and Liesiö. Toppila carried out the numerical experiments, organized the elicitation of expert assessment together with the client company of the case study, and proposed simplification methods. Toppila, Liesiö and Salo planned and prepared the material for the expert assessments. Toppila was the primary author. Salo participated in writing the final version of the paper.

Preface

While writing this Dissertation, I have had the privilege to work with brilliant minds and great personalities. This has shaped my thinking, motivated me to find scientific knowledge, and probably made me a better person in general.

I wish to thank Professor Ahti Salo for supervising and instructing my Dissertation. Ahti's broad view over the relevant themes in operations research helped me in completing this Dissertation, and I am glad for all his support and guidance.

Thanks go also to my other co-author, Professor Juuso Liesiö, for all the ideas, comments and other effort to our joint research. Juuso's high quality research has been a source of inspiration for me. I am grateful to Professor Enrico Bartolini for our deep discussions about optimization related topics, which helped me to think clearly about development of algorithms. I also wish to acknowledge the Finnish Research Programme on Nuclear Power Plant Safety SAFIR 2011-2014, through which a part of this research was funded. I also thank Timo Ali-Vehmas and his team for their collaboration in our Nokia-project.

All of the work has been done in the Systems Analysis Laboratory, which has been a terrific place for all these years with. I thank Professor Emeritus Raimo P. Hämäläinen, the founder and long time director of the laboratory, for his devotion for making the laboratory such a good place to work.

I also thank current and former colleagues Alessandro, Anssi, Anton, Antti P., Edoardo, Eero, Eeva, Heikki, Ilkka, Jirka, Jouni, Juha, Jussi, Kimmo, Matteo, Mikko, Pekka, Simo, Tuomas, Ville B., Ville M., Vilma, and Yrjänä. Together, we have had great times during lunches, conferences, doctoral education network trips, karonkkas and various other events.

Especially memorable is how I managed together with Kimmo, Eero and Heikki to reinvigorate the orienteering tradition in the laboratory to the extent that we have participated with a seven person lab team in the last six Jukola-relays. I also thank Jussi for our common times as undergraduate students, and our later activities, including those in the board of the Finnish Operational Research Society and out common time as doctoral students.

Finally, I thank my wife Henriikka and my sons Lauri and Oliver for being there for me. I also thank my father Esko and mother Hilkka for their mathematical legacy, which I intend to pass on to my children.

Espoo, October 10, 2016,

Antti Toppila

Contents

1. Introduction

A vast range of models have been proposed for aiding decision making (Keefer et al., 2004). Indeed, the quality of a decision can be significantly improved using structured decision analytic methods, when there are messy, unclear and conflicting objectives, several alternative courses of action characterized by multiple attributes, and estimates about outcomes of alternatives are uncertain (Matheson and Matheson, 1998; Keisler, 2011; Howard, 1988). This is particularly the case when choices among the alternatives are interdependent due to a common pool of resources for implementing the actions; or there are outcome or resource synergies between the alternatives, which call for considering multiple decisions simultaneously (Salo et al., 2011).

Decision analytic models are normative, i.e., they model how decisions should be made according to widely accepted axioms of rational choice (Keefer et al., 2004; Clemen, 1996; von Neumann and Morgenstern, 1947; Savage, 1954). Of these axioms, a particularly debatable is the one which states that a decision maker's (DM) preferences should be complete (Aumann, 1962). According to Nau (2007), this means that given any two alternatives that might be proposed, no matter how complicated or hypothetical or even counterfactual, the DM is always able to state that either she strictly prefers one to the other or else she is exactly indifferent between them. After assessing the parameters of a model that fulfills the completeness axiom, regardless of how uncertain the assessments are, the model will always recommend some alternative (or a set of equivalent alternatives) as the most preferred one. Conversely, *any* model that complies with the completeness axiom cannot satisfactorily model indeterminacy or indecision between preferences of alternatives – which DM's typically experience. This, in turn, can provoke distrust and scepticism towards the results of such models (e.g. Walley, 1991; Salo and Hämäläinen, 2010).

A natural way of relaxing the completeness assumption is to allow the DM to specify the parameters of a conventional decision model incompletely. More specifically, given a model that has n parameters that form a vector $\theta \in \mathbb{R}^n$, incomplete information about the parameters can be expressed using an information set $\Theta \subseteq \mathbb{R}^n$: Here, if the cardinality of Θ is 1, then there is a unique value for θ and the information is complete. The larger the set Θ, the more incomplete is the information about θ. For instance, if one of the parameters is the value of completing a project v, then a DM can express incomplete information about this parameter by giving a plausible lower bound $\underline{v}$ and a plausible upper bound $\overline{v}$ such that (s)he thinks that $\underline{v} \leq v \leq \overline{v}$. In what follows, we refer to parameters θ about which incomplete information is given as *set-valued*.

Based on the experiences from a multitude of applications and case studies, the use of incomplete information can expedite or enhance a decision making process in several ways (for a review, see Salo and Hämäläinen, 2010). For example, the decision process can be sped up by first gathering coarse estimates about the parameters, and then by discarding the least promising alternatives from further consideration with this incomplete information. Further elicitation efforts can be focused on the remaining alternatives. Also, incomplete information can model uncertainty that is difficult to capture using probability distributions, e.g., because of lack of statistical data. The use of incomplete information can increase the DM's commitment to the analysis, because the DM may feel more confident that the parameters are correctly elicited. Furthermore, in cases where the DM is not a single person but a group of stakeholders, incomplete information can capture the joint preferences of all the stakeholders without enforcing them to agree on all model parameters (Vilkkumaa et al., 2014).

The drawback with using incomplete information is that a single "best" decision is typically not identified, because in most applications the decision model's recommendation depends on what value $\theta \in \Theta$ is used. There are two theoretically sound ways to proceed (c.f. Hazen, 1986). If it is assumed that there exists a unique parameter vector which ideally is the correct parameter choice for this model, but because of elicitation difficulties is incompletely defined, then it is reasonable to focus the analysis to those alternatives that are most preferred for some $\theta \in \Theta$. Such alternatives are called *potentially optimal* [1]. On the other hand, if Θ represents irreducible set-valued uncertainty about the parameter – meaning that we cannot imagine ever identifying ideal numerical values for θ (c.f. Walley, 1991) – then it is appropriate to consider alternatives that are *non-dominated* (ND): An alternative A is ND if there is no other alternative B such that B is at least as preferred as A for all $\theta \in \Theta$, and B is preferred over A for some $\theta \in \Theta$. Clearly, the DM should select a ND alternative, and hence we say that the recommendation to select a ND alternative is *robust* with respect to the incomplete information.

In general, a potentially optimal alternative need not be non-dominated and

[1] It should be noted that this sensitivity analysis based interpretation may not be acceptable in all circumstances. Walley (1991) argues that this principle of ideal precision is unwarranted in situations in which little or no statistical evidence is available, and that in such cases interval-valued probabilities are needed to coherently represent knowledge about uncertainty.

vice versa (Weber, 1987), and hence this choice of interpretation has a potential impact on the decision recommendations. In this Dissertation, our application domains (as detailed in Section 1.1) are such that it is more natural to use set-valued parameters to characterize irreducible uncertainty, and consequently we are interested in analyzing ND alternatives.

In the context of selecting a single alternative out of many, there are several well-established methods for eliciting incomplete information and computing the ND alternatives (e.g. Hazen, 1986; Weber, 1987; Salo and Hämäläinen, 1992; Salo and Hämälainen, 2001; Mustajoki and Hämäläinen, 2005; Salo and Hämäläinen, 2010). However, as the field of decision analysis has matured, decision analysis has been applied to more complex decisions at a greater level of detail. In particular, the advent of portfolio decision analysis (PDA; Salo et al., 2011) – which can help the DM make better decisions when selecting a *portfolio* consisting multiple alternatives that are interlinked and interdependent – has made it possible to approach a great variety of new interesting application areas such as infrastructure management (Mild and Salo, 2009), reliability allocation (Kuo and Wan, 2007), and innovation management (Lindstedt et al., 2008).

The added expressiveness of portfolio models has also added to the complexity of computing the decision recommendations, i.e., the ND portfolios. This is because in many applications, every portfolio cannot be explicitly compared to every other portfolio. For instance, if there are n alternatives, then the DM can form a portfolio of these alternatives in 2^n ways. For large n (say $n > 50$), it is computationally too demanding to evaluate each portfolio separately, let alone compare each pair of portfolios for dominance. Consequently, a typical PDA model is formulated and solved as a combinatorial optimization problem so that not all portfolios need to be evaluated explicitly. However, incorporating set-valued parameters to such optimization models make the general purpose solution algorithms for these optimization problems inapplicable. Therefore, the use of incomplete information in PDA calls for the development of methods for computing the ND alternatives so that decision recommendations can be given.

The computational challenge of generating the ND alternatives in PDA has not received much attention in the literature (note that other related forms of dominance have attracted research much more extensively, as detailed in Section 2). A framework for generating all ND portfolios is the Robust Portfolio Modeling-framework (RPM; Liesiö et al., 2007, 2008). RPM considers selection of project portfolios, where each project is scored with respect to multiple attributes, the value of a portfolio is a weighted sum of the scores of the portfolio's projects, and the set of portfolios that can be selected are implicitly defined using a set of linear inequalities. Incomplete information about scores and weights can be expressed using interval-valued scores and (convex) polyhderal-valued weights.

The RPM framework has been applied in several domains including infrastructure asset management (Mild et al., 2015), air traffic control (Grushka-Cockayne et al., 2008), and strategic portfolio development (Lindstedt et al., 2008). However, this framework cannot be used for all PDA applications. For instance, the assumption that the value of a portfolio is linear, i.e., it is a (weighted) sum of the projects' scores, does not suit all decision making contexts. For example, the reliability of the system is a multilinear function of its components' reliabilities and hence if the reliability of the system is to be maximized, the assumption does not hold. In principle, non-linear functions can be modeled by introducing dummy projects that correspond to interaction terms, but the increased computational effort introduced by increasing the number of decision variables would restrict such approaches to relatively small problems.

In summary, the use of incomplete information in decision analysis has a sound theoretical justification and several practical advantages. Although in theory incomplete information can be adopted to PDA methods simply by allowing parameters to be set-valued, in practice one is faced with the difficulties in computing the decision recommendations from such models. Hence, there is a need to develop better computational methods for supporting PDA under incomplete information about model parameters.

1.1 Objectives, scope and research methods

In view of the above discussion, this Dissertation makes contributions to the following overarching research themes.

RT1: Support modeling in risk analysis and decision analysis by capturing lack of knowledge and epistemic uncertainty with incomplete information.

RT2: Develop exact methods for computing the ND alternatives.

RT3: Use approximative computational methods to solve even larger problems than what the exact algorithms can solve.

A few remarks about these research themes and their relation to the papers are in order. RT1 can be approached in many ways. In this Dissertation the goal is not to cover every conceivable application area. Rather, the focus is on a few application areas which together can be regarded as characteristic for PDA. These application areas are probabilistic risk analysis (Papers I and II), project portfolio selection (Paper III), and innovation management (Paper IV).

With respect RT1, Paper IV is the most concrete, as it describes how uncertainty was modeled using interval-valued parameters in an actual case study. Papers I and II study a realistic example that portrays how uncertainty about the failure probabilities of a residual heat removal system of a nuclear power plant can be captured with interval-valued probabilities. Paper III is concerned with developing improved exact algorithms for computing the ND set for multiobjective project portfolio selection under incomplete information. This paper illustrates the computational improvements using a problem instance reported in the literature.

RT2 is the most fundamental from the viewpoint of any method that uses incomplete information, because if the ND set cannot be solved, the model cannot be used for providing decision recommendations. As mentioned before, typical PDA models are formulated and solved as combinatorial optimization models. The techniques used for solving these optimization models, e.g., implicit enumeration, dynamic programming and branch-and-bound, can be used for solving the ND portfolios, too.

Paper I considers the prioritization of failure events in technical systems using risk importance measures. When the events' probabilities are interval-valued, checking dominance between two events becomes a multilinear optimization problem, for which the Dissertation develops a dedicated dynamic programming algorithm. Paper II develops an implicit enumeration algorithm for solving 0-1 multilinear objective function and linear inequality constraints. Paper III provides a novel method that uses binary decision diagrams (BDDs) for generating the ND portfolios. This method is based on transforming the problem to an interval-valued multiobjective optimization problem. This paper also deals with the problem of how a huge number of ND portfolios can be generated and stored efficiently using BDDs. Paper IV develops an approximative model that is formulated as a mixed integer linear program (MILP). Although the MILP problem is solved exactly (using commercial optimization software), the fundamental approach is based on finding an approximate solution.

RT3 is motivated by that many of the decision models correspond to optimization problems which are hard to solve even with complete information,

Table 1.1. Scope of the papers.

Paper	Objective function	Objectives	Exact	Approximative	Applications
I	Multilinear	Single	✓	(✓)	(✓)
II	Multilinear	Single	✓	✓	(✓)
III	Linear	Multiple	✓		
IV	Linear/Complex	Single		✓	✓

and introducing incomplete information can easily introduce unsurmountable computational difficulties. Hence, given that the state-of-the-art methods for solving the problems are at the moment limited to rather small instances, one may need to resort to approximative methods for solving the ND portfolios.

Approximative methods are developed in Papers I, II and IV. Papers I and II both consider approximative methods for simplifying the multilinear objective function so that the resulting optimization problem would be easier to solve. In Paper IV, the problem is approximated in several ways: Continuous variables are discretized, a heuristic for interactions among alternatives is introduced, and the problem dimension is reduced through bounding, and the ND portfolios are approximated by a set of potentially optimal portfolios. In this paper, the rationale for approximation is not only computational, but also the assessment of the model parameters require some kind of approximation.

The above discussion about the contribution and the scope of the papers is summarized in Table 1.1. The second and third columns characterize the objective function. The last three columns relate the papers to the research themes: A checkmark indicates that the main contribution of the paper is within the corresponding theme. A checkmark in parenthesis indicates that the paper makes a contribution to the theme, but the main contribution of the paper is to the other themes.

The rest of this Dissertation is structured as follows. Section 2 reviews how the research questions have been addressed in the literature. Section 3 presents the contributions of the papers. Section 4 gives a discussion on the impacts of the papers and suggestions on future research avenues.

2. Methodological background

In this Section, we discuss the background of the methods used in this Dissertation. Specifically, in Section 2.1 we explicate how PDA models can be formulated as optimization problems, and discuss the incorporation of incomplete information in such models. In the Sections 2.2, 2.3, and 2.4, we elaborate on these topics in the context of application domains in which the papers have been developed. Finally, in Section 2.5, we discuss methods for exact combinatorial optimization on which our algorithms for computing the ND portfolios are based on.

2.1 Portfolio decision analysis and incomplete information

The common paradigm for the methods in this Dissertation is that of PDA. Briefly, PDA refers to "... a body of theory, methods, and practice which seeks to help decision makers make informed multiple selections from a discrete set of alternatives through mathematical modeling that accounts for relevant constraints, preferences, and uncertainties" (Salo et al., 2011). PDA builds on classical theories in Decision Analysis over single choices (e.g. expected utility theory, von Neumann and Morgenstern, 1947; Savage, 1954), but emphasizes modeling aspects related to the contingencies between the decision alternatives. Due to the contingencies, typical PDA models have natural formulations as optimization problems, as we will discuss next.

Consider selecting a portfolio (i.e. a subset of alternatives) out of n alternatives in total. A portfolio can be described using a vector $x \in \{0,1\}^n$ such that $x_j = 1$ if alternative j is selected and $x_j = 0$ otherwise. The relevant constraints (e.g. an upper bound on how much money can be invested into the alternatives) implicitly define what portfolios are *feasible*, i.e., viable candidates to be selected. This set of candidates form the *set of feasible portfolios* Z_F.

The preferences of the DM are modeled with a real valued value function $V(x): \{0,1\}^n \mapsto \mathbb{R}$ which has the property that $V(x) > V(y)$ if and only if portfolio x is preferred to portfolio y. For instance, if the DM seeks to maximize profits, then his/her preferences can be captured by a value function $V(x) = \sum_{i1}^{n} p_i x_i$, where p_i is the profit of selecting alternative i.

The decision recommendation is to select the feasible portfolio $x^* \in Z_F$ which yields the highest value because it is the most preferred among the portfolios that the DM can select. This portfolio can be found as the solution to the optimization problem

$$\max_{x \in Z_F} \qquad V(x)\,. \tag{2.1}$$

Although conceptually simple, solving the decision recommendation may involve computational challenges because in most practical cases problem (2.1) is NP hard – in other words it belongs to a class of optimization problems that are in theory difficult to solve (for instance the well known knapsack problem belongs to this class, see e.g. Kellerer et al., 2004). However, advances in optimization methods have made it possible to solve a wide variety of problem instances of practical relevance and therefore using such models is computationally viable.

A key part of the PDA methodology is to address uncertainties. The classical models deal with *aleatory uncertainty*, which refers to uncertainty arising from the natural variability of a phenomenon (e.g., the outcome of tossing a coin). In typical PDA models such uncertainty is either (i) incorporated into the value function by explicitly modeling the DM's preferences over uncertain outcomes or (ii) modeled implicitly as a constraint e.g., by considering portfolios where outcomes are too uncertain as infeasible (Rockafellar and Uryasev, 2000).

Traditional methods assume that all uncertainty can be quantified in terms of probabilities or probability distributions. However, uncertainty stemming from sources such as ambiguity, conflicting evidence, or lack of knowledge are typically difficult to capture with probability distributions (see, e.g. Walley, 1991; Nau, 2007). This type of uncertainty is referred to as *epistemic uncertainty*.

Epistemic uncertainty can to some extent be captured with incomplete information about the model parameters (see the commentary by Dubois, 2010). More specifically, in most PDA applications, the epistemic uncertainty pertains to lack of knowledge about (i) the preferences of a DM, (ii) probabilities of uncertain events, and (iii) the consequences of alternatives (see, e.g. Weber, 1987), which in typical PDA models are associated with specific model parameters. Consequently, PDA models are usually expressed so that these main sources of epistemic uncertainty can readily be captured with incomplete information about model parameters.

Epistemic uncertainty has been accounted for by various methods. For instance, epistemic uncertainty about probabilities has been tackled with Monte Carlo simulation (Borgonovo, 2008; Modarres, 2006; Zio, 2011; Baraldi et al., 2009; Modarres, 2006), Baysian and robust Bayesian methods (Berger et al., 1994), confidence intervals (Paté-Cornell, 1996), interval-probabilities (Weichselberger, 2000), fuzzy probabilities (Buckley, 2003), coherent lower and upper probabilities (Walley, 1991), imprecise reliability (Utkin and Coolen, 2007) and Dempster-Shafer theory (Dempster, 1967; Shafer, 1976). Epistemic uncertainty about the outcome and the decision maker's preferences have been analyzed using sensitivity analysis, what-if analysis (Mustajoki et al., 2006; Mustajoki and Hämäläinen, 2005), and incomplete preference statements (Salo and Hämälainen, 2001; Ahn and Park, 2008), for instance.

As discussed on page 2, the use of set-valued parameters results in a set of ND portfolios. This set can be analyzed using the *core index* of alternative i (Liesiö et al., 2007, 2008), which is defined as

$$CI_i = \frac{|\{x \in Z_{ND} | x_i = 1\}|}{|Z_{ND}|} ,$$

where Z_{ND} is the set of ND portfolios. Decision recommendations can be based on the core index. An alternative with $CI_i = 1$ is a *core* project and is included in all non-dominated portfolios. Consequently, if this alternative is not selected, then it is not possible to form a ND portfolio. Thus, it is recommended to select all core alternatives. Analogously, projects with $CI_i = 0$ are *exterior* alternatives and consequently they should not be selected. Alternatives for which $0 < CI_i < 1$ are *borderline* projects. They are included in some but not all ND portfolios. Hence no conclusive recommendation can be given for the selection of such alternatives.

We end this section by briefly discussing another related family of methods for dealing with epistemic uncertainty, namely robust combinatorial optimization (Bertsimas and Sim, 2003; Aissi et al., 2009; Ide and Schöbel, 2015; Ehrgott et al., 2014). The concept of robust optimization has been developed in the context of industrial optimization, in which robust solutions with respect to uncertain measurements have been of interest. In particular, in robust optimization problems the aim is to find the optimal solution among those that are feasible with respect to random variation of the parameters. Typically, each alternative is characterized by a single feasible parameter $\theta \in \Theta$ that yields the lowest value for that alternative. Such problems are often computationally challenging (Beyer and Sendhoff, 2007). Also, the robust optimization paradigm consider finding only a single a single ND alternative, and consequently these methods cannot be used for generating all ND alternatives.

2.2 Modeling epistemic uncertainty in probabilistic risk analysis

Probabilistic risk analysis (PRA) is based on a model of how risk events (e.g. component failures, earthquakes) that either alone or together with other risk events may prevent the system performing its operation or cause an accident (e.g. fire or explosion). The probabilities or the occurrence frequencies of the risk events are estimated and used for assessing the reliability of the entire system – i.e. probability of the system being able to perform its function – based on a logical description that relates the risk events to the system (e.g. through the use of fault tree analysis, see e.g. Lee et al., 1985). As a result of this process, and assuming that the risk events are mutually statistically independent, the reliability of a system can be expressed as (see, e.g. Borgonovo, 2010)

$$r(p) = \sum_{I \subseteq \{1,\ldots,n\}} \alpha_I \prod_{j \in I} p_j \,, \tag{2.2}$$

where $\alpha_I \in \mathbb{R}, I \subseteq \{1,\ldots,n\}$ are the multilinear coefficients determined by the structure of the system and and p_j is the probability of event $j = 1,\ldots,n$, and n is the number of events. Mathematically, this function is multilinear, i.e., it is affine in each of its variables.

The probabilities p_i are typically epistemically uncertain, because their estimates are obtained from statistical analyses or measurements, computational simulations or expert assessments which all are typically uncertain (Apostolakis, 1990; Paté-Cornell, 1996). Because this uncertainty can have a significant impact on the results of a PRA, the results of a PRA are not credible if they have not been shown to be robust with respect to epistemic uncertainties (Cheok et al., 1998). We elaborate on the analysis of epistemic uncertainty in the context of two central applications of PRA, namely importance analysis (Kuo and Zhu, 2012b) and reliability allocation (Kuo and Wan, 2007).

Importance measure apportions to each event a part of the risk associated with the entire system (van der Borst and Schoonakker, 2001). For instance, the widely used Fussell-Vesely measure of risk event i is approximately[1] the probability that this event has occurred conditioned on that the system has failed. This measure can be computed by the formula (van der Borst and Schoonakker, 2001)

$$FV_i = \frac{r(p_1,\ldots,p_{i-1},1,p_{i+1},\ldots,p_n) - r(p)}{1 - r(p)} \,. \tag{2.3}$$

[1] The exact definition is technical and it can be found in several sources, e.g., Kuo and Zhu (2012a).

Another widely used importance measure is the Birnbaum measure, which is defined for the risk event i as

$$B_i = \frac{\partial r(p)}{\partial p_i} \tag{2.4}$$

$$= r(p_1,\ldots,p_{i-1},1,p_{i+1},\ldots,p_n) - r(p_1,\ldots,p_{i-1},0,p_{i+1},\ldots,p_n)\,. \tag{2.5}$$

This measure is the difference in system's reliability for the cases where event i occurs and does not occurs for sure, respectively, given that all other events occur with their nominal probabilities.

Importance measures have been used for supporting risk informed decision making in many applications (e.g. Cheok et al., 1998; Zio, 2011; Borgonovo, 2006, 2007; Baraldi et al., 2009; Aven and Nøkland, 2010). Originally, these methods have been used for guiding decisions on how to target component upgrade programs and backfitting activities, and how to prioritize regulatory supervision inspections (Vesely et al., 1983). Nowadays, they have been applied for other purposes, too, including applications such as maintenance planning, precursory analysis and reliability design (Kuo and Zhu, 2012a).

The impact of epistemic uncertainty about probabilities on the results of importance analysis has been approached by various methods. This includes sensitivity analysis (for a review, see Borgonovo and Plischke, 2016) by examining the derivatives of importance measures (Borgonovo, 2010), Monte Carlo simulation (Borgonovo et al., 2003; Vaurio, 2010), and uncertainty importance measures (Pörn, 1997; Borgonovo, 2007; Aven and Nøkland, 2010; Borgonovo and Smith, 2012; Flage et al., 2012). Aproaches that use non-probabilistic methods for modeling epistemic uncertainty are based on the seminal work of Dempster and Kong (1988) and Guth (1991) who studied the propagation of imprecise probabilities through the reliability function. Kozine and Filimonov (2000) and Utkin (2004) analyze the lower and upper bounds for system reliability under interval-valued probabilities. Uncertainty analysis based on imprecise probability is provided by Limbourg et al. (2008); Le Duy et al. (2010, 2011) and Sallak et al. (2012). They compute bounds on traditional importance measures given imprecise event probabilities.

Strictly speaking, the use of importance measures is not in the realm of portfolio decision analysis in that the possible interactions are not explicitly modeled. However, in early phases of analysis, they provide a prioritization of the risk events which are suitable for screening of risk events so that a more detailed analysis of the remaining risk events can be done (Borgonovo, 2008). Under incomplete information, the prioritization can be characterized by ranking intervals which can establish what alternatives are among the K most preferred ones (i) for all, (ii) for some, and (iii) for no combinations of feasible parameters simultaneously for all K (Punkka, 2012).

Importance analysis is explorative in that different measures[2] typically yield somewhat different results, and no set of importance measures can provide a theoretically exhaustive analysis that would cover all aspects of the risk analysis (van der Borst and Schoonakker, 2001; Kuo and Zhu, 2012b). For specific applications, the insights of importance analysis can be deepened by formulating an explicit objective function to be optimized. An important class of such applications are redundancy and reliability allocation problems, which have attracted a significant amount of research since 1960's (for reviews, see Kuo and Wan, 2007; Kuo and Prasad, 2000; Tillman et al., 1977).

In reliability and redundancy allocation problems, the objective is to maximize the reliability of the system by adding (redundant) back-up components or choosing what quality (in terms of reliability) components to use in the system. Another widely used alternative objective is to minimize the cost of the system's components given that the reliability of the system must exceed a given threshold. In general, these problems are formulated as integer non-linear and non-convex programming problems (e.g. Kuo and Wan, 2007), which are then solved with exact algorithms such as branch-and-bound (Ha and Kuo, 2006) and implicit enumeration (Prasad and Kuo, 2000). This problem has also been studied for particular types of systems, which has allowed more tailored solution methods (e.g. Sung and Cho, 1999; Elegbede et al., 2003).

2.3 Project portfolio selection under incomplete information

Multiattribute value theory (Keeney and Raiffa, 1976) is the prevailing paradigm for normative decision making with multiple attributes. If the attributes are mutually preferentially independent, the DM's preferences can be captured with an additive value function

$$v(c) = \sum_i w_i v_i(c_i) \,, \tag{2.6}$$

where $v_i(c_i)$ is the value of the consequence c_i of selecting the alternative with respect to the i-th attribute. Here, the quantity $v_i = v_i(c_i)$ is termed in the literature as the attribute value or the attribute *score* of selecting the alternative.

[2] It should be noted that many measures are equivalent or almost equivalent in that in most instances that occur in practice, the prioritization given by these measures are (almost) identical.

The weights w_i and scores v_i are interlinked in that the appropriate weights depend on the measurement scale (see, e.g. Punkka and Salo, 2014).

In project portfolio selection, the alternatives are project candidates and the decision is which project candidates to select. The selected candidates constitute a project portfolio. When (i) the DM's preferences are project symmetric, (ii) the attributes are weak difference independent, (iii) each set of attributes measuring the criterion specific portfolio performance is weak difference independent, and (iv) each set of attributes measuring the performance of a single project is difference independent, then the DM's preferences over portfolios can be captured with an additive-linear value function (Liesiö, 2014; Golabi et al., 1981)

$$V(x) = \sum_j \sum_i w_i v_{ji} x_j \ , \tag{2.7}$$

where $V(x)$ is the multiattribute value of project x, v_{ji} is the value or *score* of project j with respect to attribute i and w_i is the weight of attribute i. We have that $\sum_i w_i v_{ji}$ is the multiattribute value of selecting alternative j, and thus the value of a portfolio is the sum of the those project's multiattribute values that are included in the portfolio.

In practice, DMs are often unable to completely specify their value functions even if they are able to give incomplete statements about it (e.g., "weight of attribute i is greater than the weight of attribute j"). Preference programming methods (see, e.g. Salo and Hämäläinen, 2010; Salo and Hämäläinen, 1992) make it possible to infer partial decision recommendations based on incomplete information even early on in the analysis. For instance, the incompletely specified information may be sufficient to exclude some of the decision alternatives from further consideration. Such early discarding helps reduce the elicitation effort, and helps to focus on the alternatives that are most likely the best ones.

When uncertainty about weights is modeled using linear inequalities, Liesiö et al. (2007) showed that the ND portfolios of problem (2.1) with a value function given in (2.7) can be solved as the Pareto efficient portfolios of a related multiobjective combinatorial optimization problem (MOCO). MOCOs have been extensively studied in the literature and efficient algorithms for solvig them exist (Ehrgott and Gandibleux, 2000; Lokman and Köksalan, 2012). However, if the value function in (2.7) has interval-valued scores, problem (2.1) has not received much attention. In such cases, the ND set can be solved with the algorithms by Liesiö et al. (2007, 2008). These algorithms are based on breadth-first dynamic programming and they use various methods for detecting which parts of the portfolio space cannot contain ND portfolios. The detection methods help prevent the dynamic programming algorithm from enumerating all 2^n possible portfolios, and hence makes the algorithms computationally viable.

Although the algorithms in Liesiö et al. (2007, 2008) are in theory capable of generating all ND portfolios, there are two challenges: First of all, finding all ND portfolios can be computationally demanding, which has restricted the application of these algorithms to relatively small scale instances (Mild and Salo, 2009). Second, in principle there can be an exponential number of ND portfolios with respect to n, which can cause problems when storing and generating the ND portfolios. In this context, Mild et al. (2015) presents a method for computing a subset of the ND portfolios. In many applications, also a subset of the potentially optimal portfolios are used for approximating the set of all ND portfolios.

2.4 Innovation management of R&D activities

Leading high-technology companies have large Research & Development (R&D) project portfolios to mange. Managing these portfolios well is crucial, because R&D activities require substantial investments in terms of capital and human resources – often the financial investment is proportional to the sales of the company (see, e.g. Coad and Rao, 2010). However, the management of R&D project portfolios is complicated by the facts that the projects generate impacts after a long delay and can differ significantly in scope, relevance, novelty, phase, risk and return, among others (Henriksen and Traynor, 1999; Brummer et al., 2011). Also, the projects may have significant interactions such as (i) cost or resource utilization interaction; (ii) outcome, probability or technical interaction; and (iii) benefit, payoff or effect interaction (Kleinmuntz, 2007; Stummer and Heidenberger, 2003; Liesiö et al., 2008; Fox et al., 1984).

Interactions are often tedious and time consuming to model in detail (Stummer and Heidenberger, 2003; Fox et al., 1984). Yet, even if interactions can in principle alter the decision recommendations, Phillips and Bana e Costa (2007) argue that this does not usually occur except for the strongest interactions. They conclude that typically there are only a few strong interactions that will influence decisions. Moreover, often their impact can be accounted for sufficiently well by starting with an analysis in which interactions are not accounted for and then using judgemental trade-offs to provide a decision recommendation that takes into account these interactions.

A typical way to support R&D portfolio management with PDA is to consider the problem of allocating resources to the projects (see e.g. Kleinmuntz, 2007;

Henriksen and Traynor, 1999). The projects that are initiated will require resources for completion, and hence the allocated resources lead to the starting of the project. Managers may also wish to pick the best parts of each projects. Such “cherry-picking” could be implemented by preparing, e.g., four alternative variants of each project, such as (i) a current plan, (ii) a buy-up option, (iii) a buy-down option, and (iv) a liquidation plan (Sharpe and Keelin, 1997).

Often the key parameters that characterize the projects are obtained through expert assessments. This is because each project is unique and consequently experiences from other related projects can offer only limited guidance on what parameter values should be used. However, the model parameters elicited by the experts may be biased because of vested interests that cause some projects to be unfairly evaluated (e.g. Salo and Liesiö, 2006; Santiago and Vakili, 2005). These biases can be partly mitigated by eliciting incomplete information about numerical assessments, because disagreement can be captured by eliciting interval-valued or ordinal information. That is, even if it is not possible to reach a consensus on what the probability of a given scenario is, it may be that all assessments or opinions satisfy the ordinal statement “the probability of scenario 1 is greater than that of scenario 2”, which then forms a piece of incomplete information (e.g. Salo and Hämäläinen, 2010; Salo and Hämäläinen, 1992; Arbel, 1989; Liesiö et al., 2007).

One particular instance of R&D is that of contributing to technical standardization. The outcome is a (technical) standard, defined as “... a recording of one or more solutions to one or more problems of matching persons, objects, processes or any combination thereof, and which is intended for common and repeated use in any technical field” (Lea and Hall, 2004). The key benefits for companies that participate in technical standardization are technical compatibility and interoperability for meeting consumer expectations (see e.g. Glimstedt, 2001). It should be noted that the benefits of standardization are contingent on the success of standardization and the exploitation of the standard. Hence, the most significant benefits related to standardization are inherently uncertain, because they are realized after a long period of time and tend to be intangible.

One way to analyze the exploitation of standardization is based on *widely adopted technologies*, that is architectures (product and/or process) that become widely accepted as industry standards (also known as dominant designs; see e.g., Anderson and Tushman, 1990). Standardization contributes to the development of widely adopted technologies although they may also emerge independently as *de facto* standards (Koski and Kretschmer, 2007). The long term survival of the company is supported by participating in the development of widely adopted technologies (Suarez and Utterback, 1995). Widely adopted technologies also yield revenues to the holder of proprietary rights to the technology, because these rights can be used for establishing a natural monopoly (Schilling, 2005). Furthermore, proprietary technologies can be made available to other parties. This can lead to a *bandwagon* effect in which the proprietary technology quickly becomes adopted by the industry and this can lead to larger revenues when the size of the market increases (Farrell and Saloner, 1985). However, allowing other participating parties to enter the same markets can also reduce the market share of the company.

2.5 Exact combinatorial optimization

The computation of the ND portfolios is based on the classical combinatorial optimization algorithms. These methods have been treated in several textbooks (e.g. Papadimitriou and Steiglitz, 1998; Bertsimas and Tsitsiklis, 1997). In this section, we give an overview of these basic methods and review some recent extensions which help put the algorithmic contributions of this Dissertation in place.

Dynamic programming solves an optimization problem by splitting it into simpler subproblems that are solved either (i) by splitting them even further or (ii) if the subproblem is so simple that it has a trivial solution, then this solution is given. The power of dynamic optimization stems from the recognition of identical subproblems: If the splitting yields two or more identical subproblems, then by storing the solutions to the subproblems, one needs to solve each subproblem by splitting it only once. Dynamic programming algorithms differ in the order in which subproblems are solved (most notably depth-first or breadth-first), and the ways in which the solutions of the subproblems are stored so that it is possible to efficiently retrieve the subproblem's optimal solution if it has already been solved. Furthermore, even if two subproblems are not identical, one subproblem may *dominate* another in that solving the dominated subproblem cannot lead to the identification of a better solution than that obtained by solving the subproblem that dominates it. Hence, dominated subproblems need not to be solved, which can lead to significant savings in computational effort.

Branch-and-bound (B&B) is similar to dynamic programming in that the problem is solved by splitting the problem into subproblems. However, in B&B the problem is typically formulated with mathematical programming methods in the form

$$\begin{aligned} \max_x \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \leq 0 \; i = 1,\ldots,k \\ & x_i \text{ is integer for all } i = 1,\ldots,n, \end{aligned}$$

where $g_i, i = 1,\ldots,n$ and f are real valued functions, k is the number of constraints, and n is the number of integer-valued variables (the rest being continuous). The subproblems are generated by *branching* which is typically done by imposing an additional constraint that divides the feasible set into two parts. For example, branching with respect to a binary[3] variable x_i can be done by considering two problems: One in which $x_i = 0$ and another in which $x_i = 1$ in addition to the constraints in the original problem.

The efficiency of B&B algorithms stems from the *bounding* phase, which is obtained by relaxing the original problem formulation by allowing the integer-valued variables to have continuous values as well (e.g. a binary variable is allowed to have any value in the range $[0,1]$). This *relaxation* is typically much easier to solve than the original problem. For instance in Mixed Integer Linear Programming (MILP) problems the functions f and g are linear, and consequently the relaxed problem is a Linear Programming (LP) problem, which can efficiently be solved with e.g. the Simplex-algorithm.

If the solution to the relaxed problem is a feasible solution of the original subproblem as well, then the solution of the relaxed problem is an optimal solution of the original subproblem, too. Otherwise, the relaxation provides an upper bound on the optimal value of the objective function. If this bound is worse than the best known value of some feasible solution, then solving this subproblem cannot lead to a better feasible solution, and hence the subproblem need not be solved in search for the optimal solution.

The state-of-the-art algorithms for combinatorial optimization problems typically take advantage of the best properties of dynamic programming and B&B. For instance, dynamic programming algorithms can be equipped with a bounding phase (e.g. through the use of state space relaxation by Christofides et al., 1981). B&B algorithms, on the other hand, can be improved by detecting dominated subproblems (e.g. by the dominance procedure by Fischetti and Toth, 1988; Fischetti and Salvagnin, 2010).

A method which has attracted growing interest from the view-point of combinatorial optimization is that of using binary decision diagrams (Bergman et al.,

[3] i.e. an integer-valued variable with constraints $0 \leq x_i \leq 1$.

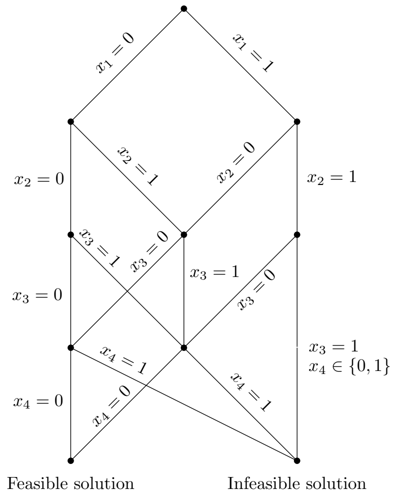

Figure 2.1. Example of a BDD. The diagram is read from top to bottom. Each arc correspond to a decision about a variable $x_i, i = 1, \ldots, 4$. The nodes are arranged in layers so that all arcs that emanate from a given layer will correspond to the same variable. There exists one layer for every variable which implies that every $x \in \{0,1\}^4$ corresponds to a path from the top node to one of the terminal nodes at the bottom. If the path ends in the leftmost terminal node, then x is feasible and otherwise it will lead to the rightmost terminal node and x is infeasible. For example, $x = (1,0,0,0)$ is a feasible solution and $x = (1,1,1,0)$ is an infeasible solution.

2013; Behle, 2007) or decision diagrams in general (Bergman et al., 2016). Binary decision diagrams (BDDs; Bryant, 1986; Brace et al., 1991; Knuth, 2011) are directed acyclic graphs that can be used for storing the set of feasible solutions of combinatorial optimization problems involving binary variables (see, e.g. Minato, 1993; Behle, 2007; Bergman et al., 2013). Figure 2.1 shows an example of a BDD. Once the BDD corresponding to the feasible set has been constructed, then any optimization problem with a separable objective function can be solved as a shortest path problem over the BDD using dynamic programming with $\mathcal{O}(N)$ computational complexity, where N is the number of nodes in the BDD (see, e.g. Behle, 2007).

If the feasible set is defined by a set of linear inequalities over binary variables, then the BDD of the feasible set can be generated using well-established algorithms (Behle, 2007). The generation of the BDD has a worst case computational complexity of $\mathcal{O}(2^n)$ and the resulting BDD can in the worst case have $\mathcal{O}(2^n)$ nodes (here n is the number of binary variables). This can cause computational difficulties, although in many instances of practical relevance, the BDD can be generated with reasonable computational resources (see Hosaka et al., 1997, and references therein). For problem instances where the exact BDD is too large, approaches based on relaxed BDDs have been proposed (see review in Bergman et al., 2013).

3. Results

In this Section, we discuss the contributions of the Papers I-IV, which are summarized in Table 3.1.

3.1 Paper I

Paper I presents a computational framework for prioritizing of risk events in PRA using commonly used importance measures – such as the Fussell-Vesely, Birnbaum, and critical importance measures – under interval-valued probabilities. The prioritization is formed by comparing each event with every other event for dominance. First, it is shown that dominance between two events can be established by checking the sign of a multilinear function over a hyperrectangle, which is an NP-hard problem (Barmish et al., 2009; Ross and Barmish, 2007). Then the paper presents a B&B algorithm for solving this problem. For large fault trees, the paper presents an approximation based on the rare event approximation, which is appropriate to use when the risk events' probabilities are small. The pairwise dominance relations are visualized using a dominance graph. In this graph every event is represented by a node and directed arcs represent dominance relations such that an arc emanates from the dominating event and end in the dominated event (arcs that can be inferred by transitivity are suppressed).

The use of the method is illustrated by the analysis of a fault tree which represents a residual heat removal system of a nuclear reactor. The lower and upper bound of the probabilities were obtained from the 90 % standard confidence intervals of these probabilities, which are typically readily available from standard PRA software. We show that even a large relative difference in the values of the importance measures cannot guarantee that the priority is not sensitive to small deviations in the probabilities. For instance, in the case study, there were two events 1 and 5 with point-valued probability based Fussell-Vesely importances 0.57 and 0.06, respectively, and yet both were ND after introducing the interval-valued probabilities. However, even if both are ND, the structure of the dominance graph helped conclude that event 1 is more important than event 5. In effect, the ND events were categorized further into two groups; those which have high priority regardless of epistemic uncertainty and those events that have a potential of being highly prioritized even if further

Table 3.1. Contributions of the papers.

Paper	Research objectives	Methodology	Main contribution
I	Support importance measure based risk analysis under epistemic uncertainty about risk events' probabilities	Multilinear optimization; branch-and-bound; dominance graphs	Algorithm for solving importance measure based dominance under interval-valued probabilities.
II	Develop methods for reliability allocation under epistemic uncertainty about risk events' probabilities	Implicit enumeration; linear programming; total order interactions	Algorithms for computing the ND portfolios for 0-1 reliability allocation problems under interval-valued probabilities.
III	Develop improved algorithms for computing ND portfolios Propose a methodology for generating and storing a very large number of ND portfolios.	Binary decision diagrams; dynamic programming	Improved algorithms for generating the ND portfolios A novel algorithm for constructing a BDD that represents the set of all ND portfolios when the set of ND portfolios is very large.
IV	Support resource allocation for standardization activities	Case study; mixed integer linear programming; decision trees	A model for resource allocation to standardization activities which takes into account the high uncertainties associated with standardization

information (which results in that the interval become narrower) is obtained.

3.2 Paper II

Paper II extends the use of interval-valued probabilities from importance measure based prioritization used in Paper I to reliability allocation problems. This makes it possible to model portfolio level constraints and to account for interactions that occur when the probabilities of multiple risk events are impacted simultaneously. The ND portfolios are computed with implicit enumeration (which can be seen as one type of B&B algorithm). A bound for the implicit enumeration algorithm is derived by formulating the bounding problem as a 0-1 optimization problem with a multilinear objective function and a linear constraint. Then a continuous knapsack (i.e. linear programming problem with a single constraint) relaxation of this problem is derived so that a sufficient bounding condition can be efficiently computed. This extends the results of Hoeffding (1956), which were used by Liesiö (2014) to bound the value of a symmetric multilinear function in an implicit enumeration algorithm to non-symmetric multilinear functions as well.

Paper II examines the same fault tree of a residual heat removal system as Paper I. This system is analyzed from the perspective of which risk events should be eliminated if at most K risk events can be eliminated. We compare the resulting ND portfolios to the heuristic where the K most important events (with respect to an importance measure) are eliminated. We observe that the heuristic method provides rather similar results to the portfolio model. However, we are able to give stronger recommendations using the portfolio model in that we can detect more core events to be eliminated in comparison with the budget independent partial prioritization given by the heuristic method.

Yet, solving the ND portfolios is computationally challenging and only relatively small problems can be solved. For that reason, several approximative methods are introduced. The basic methods are dimension reduction by shortlisting a smaller subset of events that can be eliminated and shrinking the intervals so that the ND set becomes smaller and hopefully easier to solve. A more advanced technique was based on simplifying the reliability function using the method of total order interactions (TOI; Borgonovo, 2010; Borgonovo and Smith, 2011). This method helps point out which product terms – that is the terms that correspond to interactions – in the multilinear reliability function are significant. Thus, by eliminating the insignificant interaction terms or prioritizing branching with respect to terms that are involved in significant interaction terms, we were able to solve the original problem in much shorter time. Based on our computational experiments, the approximative method based on TOI was able to solve a significant share of all ND portfolios: For some instance all ND portfolios of the original problem was found.

3.3 Paper III

Paper III considers the computation of the ND portfolios in multiattribute portfolio project selection under interval-valued project scores (c.f. Liesiö et al., 2007). First, we formulate the problem of finding the ND set as an interval-valued multiobjective shortest path problem in a BDD. In this formulation, a path in the graph corresponds to a feasible portfolio. The shortest path problem is solved by propagating partly fixed portfolios portfolios (PFPs) – i.e. portfolios which content is fixed with respect to a subset of the projects – over the BDD. All PFPs that are propagated to the same node are checked for partial dominance (similar to detecting dominated subproblems in dynamic programming algorithms). We consider two ways of propagating the PFPs, namely top-down and bottom-up. In the computational experiments, we find that bottom-up propagation is much more efficient than the previous method by Liesiö et al. (2007, 2008) and the top-down propagation method. This computational efficiency of the bottom-up propagation stems from efficient detection of dominated PFPs.

Second, the paper considers how to generate and store ND sets of problem instances that have a large number of ND portfolios – so large that generating and storing them is likely to cause computational difficulties. We develop a method for generating the ND set through dynamic programming. This method is based on the detection of equivalent feasible improvements (c.f. Fischetti and Toth, 1988): A feasible improvement for a portfolio $x \in \{0,1\}^n$ is $(\Delta^+, \Delta^-) \in \{0,1\}^n \times \{0,1\}^n$ such that $y = x + \Delta^+ - \Delta^- \in Z_F$ and y dominates x. The paper develops a method for generating the BDD of the ND portfolios when the selection problem has a special structure which corresponds to the problem of identifying resource efficient portfolios (c.f. Kangaspunta and Salo, 2014). Based on the computational experiments, this algorithm can efficiently generate the BDDs of ND sets involving tens of thousands of portfolios, and is therefore up to hundreds of times faster than previous methods.

3.4 Paper IV

Paper IV presents a model and a case study for allocating resources to standardization activities in a large telecommunication company. The company had interest in hundreds of telecommunication standardization activities related to different technologies and it had to decide which activities it would participate in. Consequently, making an informed decision made it necessary to consult numerous experts including technical standardization experts, road mappers and project managers. Also, the amount of time that the experts could devote to this resource allocation problem was limited. These practical considerations prevented face-to-face meetings with most of the experts (c.f. Burk and Parnell, 2011, p. 350), made it necessary to build a sufficiently simple model so that only a relatively few parameters would have to be assessed, and minimal training would be required for being able to give assessments. Simplicity was also encouraged by that the model was intended for repeated application, permitting adjustments to the portfolio over the years (c.f. Mild et al., 2015).

In the model, the impact of the adjusted resource allocation was modeled by a decision tree that captures the uncertainty with respect to successful standardization and successful development of a widely adopted technology given the resources for these tasks. The benefits of developing a widely adopted technology was measured in terms of expected future sales. The sales estimates were obtained from experts. Because the experts felt uncomfortable in giving this parameter a numerical value, future sales were elicited as intervals formed by plausible lower and upper bounds on the future sales.

The individual allocation decisions were considered jointly as a portfolio, because the standardization and related development activities had a common budget for standardization and development, respectively. Also, there were (numerous) interactions among the standardization and development activities. In line with Phillips and Bana e Costa (2007) only those interactions that had most potential of influencing the decision were modeled. This model for interactions is approximative and considers interactions between pairs of portfolios.

The case study applies the model to 100 standardization activities and related development projects from several different areas of technical standardization. Because the computational effort of generating all ND portfolios was too demanding, decision recommendations are derived by approximating the set of ND portfolios with a subset of potentially optimal portfolios. This subset is generated by randomly assigning the interval-valued parameters point values within the intervals and solving the resulting MILP with a commercial solver.

This set is used as a surrogate for the ND set when computing the core indices, which are the primary output of the analysis. The intervals given for the sales are quite wide, which characterize the substantial uncertainty about the future sales. Yet, the model is provided direction on how the current resource allocation should be adjusted.

The experts in the company found that the model was transparent and easy-enough to use and extend. After the case study, the model was adopted by the company to support essentially all their resource allocation decisions about telecommunication standardization.

4. Discussion

4.1 Theoretical and practical implications

The use of incomplete information provides a way of producing decision recommendations which are robust with respect to uncertainty about the model's parameters. Papers I and II make methodological contributions for analysing the robustness of decision recommendations by introducing interval-valued probabilities to standard PRA methods. In practice, these papers develop methods that complement widely used Monte Carlo (MC) simulation methods (e.g. Borgonovo and Plischke, 2016; Borgonovo, 2008; Zio, 2011; Baraldi et al., 2009; Modarres, 2006). These MC-methods derive a distribution for what probability an alternative is most preferred. One question that arises is that how this probability distribution can be converted to a decision recommendation. With the interval-based approach, on the other hand, the uncertainty about the model parameters is simply propagated through the model, and thus this question does not arise. Interval-valued probabilities are appealing also in that they have a well founded theory for describing epistemic uncertainty, whereas probabilistic approaches for modeling epistemic uncertainty about probabilities can be challenging when there is lack of knowledge, for instance (see, e.g. Walley, 1991).

A drawback with set-valued parameters is that they may not be readily available. For instance, in PRA, interval-valued probabilities are not customarily generated as a part of standard PRA analyses. Thus, modeling with interval-valued probabilities requires the use of an appropriate method for computing the intervals (see, e.g. Le Duy et al., 2011; Aguirre et al., 2013, and Section 2.2). This can require a substantial amount of work, especially if the model is large and has many epistemically uncertain probability parameters to be elicited.

In our models, we have used the confidence intervals provided by probabilistic models, which are typically readily available from standard PRA software. Using this data, we have demonstrated what type of insights can be obtained from models with interval-valued probabilities. However, there are theoretical concerns in interpreting confidence intervals as interval-valued probabilities. For instance, the probability of two parameters simultaneously belonging to their, say, 90% confidence intervals is 81% < 90 % (assuming the parameters vary independently of each other). Thus the interpretation of the confidence interval changes when considering the variability of more than one uncertain parameter at a time. Furthermore, the use of confidence intervals requires that a confidence level is specified and how this level should be chosen is also a source of ambiguity. Other fundamental controversies that arise from the use of statistical knowledge in expressing beliefs are thoroughly discussed in Hempel (1962). Thus, although the confidence intervals can provide practitioners a quick starting point for introducing interval-valued probabilities to their models, rigorous analysis should be based on theoretically sound methods for deriving interval-valued probabilities.

In this Dissertation, epistemic uncertainty has been characterized by incomplete information in the form of interval-valued parameters. Dominance is established under the assumption that the parameters can vary independently within their intervals. This assumptions has its limitation regarding the types of incomplete information that can be represented. For instance, an expert might say that ‘two generators are identical, and hence have the same reliability, and the reliability of one generator is in the interval [0.9,0.95]”. This information is not exactly represented by assigning the generators interval-valued reliabilities of [0.9,0.95], because when the parameters can be selected independently of each other, the feasible parameter combinations include cases where the reliabilities are not equal (e.g. reliability of generator 1 is 0.9 and reliability of generator 2 is 0.95).

Even if incomplete information is not represented exactly by intervals, it can still be useful to derive decision recommendations (see also discussion in Paper I). In the previous example, simply disregarding the equality of the generators’ reliabilities gives decision recommendations that are not as strong as they could be, because dominance is established over a superset of the parameters that are considered plausible. Yet, it is simple to prove that these recommendations are valid even for the exact information, because the by reducing the feasible parameter space can only make ND alternatives dominated but dominated alternatives cannot become ND.

If decision recommendations from the inexactly represented information are to be too weak for providing guidance, one could also exactly model the given information by introducing a linear constraint between the parameters (i.e. the reliabilites r_1, r_2 are interval-valued such that $0.9 \le r_1 = r_2 \le 0.95$). This type of extension has been done by Liesiö et al. (2007) in the context of multiattribute project portfolio selection, where decision weights are allowed to vary in a polyhedron defined by linear inequalities. However, given the non-linearity of the reliability function, such an extension could be computationally very demanding and hence require development of novel theory and algorithms for generating the ND portfolios so that decision recommendations can be provided.

Methods that use intervals can also be criticised for being uninformative, especially when the intervals are relatively wide. This is because in such cases, decision models may not yield any definite decision recommendations. In Paper IV, the case study for resource allocation to standardization activities shows that even the use of quite wide intervals can still provide useful guidance. In effect, we have shown that incomplete information expressed as intervals can be used for modeling extreme uncertainty too, without necessarily resulting in uninformative decision recommendations. Hence, from the view point of the analysts, this suggests that decisions that are characterized by high uncertainties may benefit from pooling the decision alternatives and consider them together as a portfolio.

The contribution to the exact computational methods for generating the ND portfolios are generic in that they could be applied for other problems, too. For instance, the algorithm in Paper I is applied to alternatives that are pairwise compared to each other. Even though the paper is presented in the context of using importance measures, any comparison which can be formulated as that of the checking the sign of a multilinear function can use this algorithm. Unlike in the case of a linear objective function, where dominance can be checked in polynomial time, with the multilinear objective function the task of checking dominance is NP-hard. This means that checking dominance is computationally more demanding, and hence the size of problems that can be solved is not that large. In the same vein, the algorithmic developments can be of interest in other application areas too.

4.2 Prospective research directions

This Dissertation opens several avenues for further research. One direction is the continued improvement of the computational methods: Even though Papers I-III present several exact algorithms for computing the ND alternatives, there is a potential and a need for even more efficient approaches. These improvements could, for instance, be based on recent advances in multiobjective optimization (Lokman and Köksalan, 2012). Also, decision diagrams (Bergman et al., 2016) may provide further improvements. In particular, the BDD based approach presented in Paper III could potentially be used for computing the ND portfolios in reliability allocation problems as presented in Paper II. Other related computational methods include the use of relaxed and restricted BDDs (Bergman et al., 2013), and AND/OR multi-valued decision diagrams (Mateescu et al., 2008).

Second, the application domain for the methods presented in this Dissertation could be extended. For instance, these methods could be potentially applied in the sensitivity analysis of Baysian networks (see, e.g. Nielsen and Jensen, 2009). Another interesting topic is analysing the impact of incomplete information about the multiattribute value function, when the DM's preferences are captured by a multiplicative value function (Liesiö, 2014).

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