Out-of-sample testing on portfolio performance in the Asian equity market: Can optimized portfolio outperformed simpler strategy?

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ABSTRACT OF
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Objectives

The main objectives of this study were to examine the performance of different asset allocation strategy in the Asian market by using out-of-sample testing method. The secondary objective is to determine whether optimized portfolios outperformed other portfolios with simpler strategy such as equally-weighted portfolio (EWP) and value-weighted market portfolio (VWMP).

Summary

This study collected market return, return of assets formed on size, book-to-market and momentum from 1991 to 2018. Optimized portfolios will be formed based on these return, and will be test against the EWP in the period from 2011 to 2018 and few more sub periods. The findings were analyzed by using t-test, f-test, Jobson and Korkie test, and capital accumulation.

Conclusions

Overall, there is no statistical evidence to conclude that optimized portfolios performed better than the EWP, and the other around, there is no statistical evidence to conclude that the EWP outperformed optimized portfolio. This result is mostly due to the estimation error when constructing optimized portfolio. However, the minimum variance portfolio that shrinks the covariance matrix and allows short, in general, delivers better performance in terms of risk-adjusted return. From a practitioner viewpoint, therefore, one should choose this strategy for asset allocation.

Key words: Portfolio theory, out-of-sample testing, minimum variance portfolio, equally weighted portfolio,

Language: English

Grade:

Contents

1. Introduction

Since the beginning of the financial market, portfolio management and performance has long been one of the most important subjects for both investors and academics. With the introduction of “Modern Portfolio Theory” (MPT) by Markowitz (1952), the world of investing has been revolutionized by using a quantitative and analytical approach for the construction of a portfolio instead of an intuitive and feeling approach. However, the mean-variance optimization approach of MPT to create an optimal portfolio has many shortcomings.

Many assumptions of the model and the model itself do not reflect the reality of the investment world. For example, it is illogical and unintuitive to many investors to view expected returns as the measure of reward and standard deviation of returns as the measure of risk. Every investor has a goal for his investment that the return must reach a certain level; therefore, it would be unreasonable for them to view the upside variance based on the determined level of return as a risk instead of potential reward. Furthermore, an investor might have his speculation view on certain assets of the portfolio, and this view might significantly affect the asset allocation of the portfolio because many types of research show that a small change in expected return and standard deviation can lead to a large change in asset allocation (ZHOU, 2009). Over the years, new theory and model have been established to overcome the problem of this assumption such as Post Modern Theory (Rom & Ferguson, 1993). This theory uses downside deviation as the measure of risk. There is also Black-Litterman model (Black & Litterman, 1991), which allows the investors to incorporate their view into the construction of the portfolio. Many research has confirmed that this model can better reflect the reality of the investment world.

Another crucial shortcoming of the model is the ex-post approach to optimization, which is using the past data to formulate the optimal portfolio. Hence, this method begs the question of will these mean-variance optimal portfolios continue to be “optimal”? The most common methods to test portfolio performance out-of-sample are measurements such as Sharpe Ratio or Certainty Equivalent return. This out-of-sample method is producing many significantly different result among academics. There are studies claim that using these portfolio optimization model is unnecessary since a naïve portfolio can have a better performance. In contrast, many researchers denounce this view claiming that these optimization model can produce a better result, and some think that a specific model of minimum variance portfolio optimization produces the best result. Therefore, it is clear that is no definite answer to the question of whether optimized portfolio has better performance the simpler strategy such as an equally-weighted portfolio or investing in the market index. Furthermore, there seems to be a lack of research in the Asian equity market, as most of the research focuses on developed markets such as the US and Europe.

Seeing these gaps and problems, this research, therefore, aims to answer these questions of the effectiveness of these optimization model using the data from the Asian market:

- What is the performance of the naïve portfolio in the Asian market?
- What is the performance of the mean-variance efficient portfolio in the Asian market?
- What is the optimal asset allocation strategy in the Asian market?

2. Literature review

2.1. Modern portfolio theory and capital asset pricing theory

2.1.1. Concept

For any investor, the first and foremost idea that they must learn can be explained in one idiom: “Don’t put all your eggs in one basket.” This idiom represents the concept of diversification, which is vital for every investor to learn as it has been proven to reduce risk and increase the return of one’s investment (Jackson, 2013; Sharma, 2017). This topic of diversification was widely discussed among market practitioners; however, Markowitz (1952) is the one who brings the most impactful discussion to this topic through his paper “Portfolio Selection” in 1952. This paper set the foundation for the development of MPT, which was later improved by many of his colleagues and helped him won the Nobel Prize in Economics thirty-eight years later. His avant-garde discovery (Markowitz, 1952) and his sublime portfolio management theory and practice (Markowitz, 1959) provide us with a precise and analytics scope for the financial market.

It has been general agreement among academics and financial sector that more significant return on investment implies greater risk (Markowitz, 1952). Hence, the ultimate goal for an investor is to maximize the profit he can receive while minimizing the risk (Markowitz, 1991). Consequently, the purpose of Markowitz for creating the model is finding a collection of portfolios that maximize the return for a given level of risk or minimize risk for a given level of performance. This group of portfolios is called the "efficient frontier," and each portfolio in the collection is an "efficient portfolio." On the opposite, a portfolio is "inefficient" if there is another portfolio that can achieve a lower risk for the same level of return or the higher level return for the same level of risk (Markowitz, 1991).

According to Sharpe (1964), the risk of a portfolio is divided into two categories: systematic risk and unsystematic risk. Systematic risk is the risk of investing in the stock market in general, so it cannot be reduced through diversification. Systematic risk is, namely, the risk of interest rate change, inflation, economic turmoil is almost unavoidable. Unsystematic risk, however, can be minimized through the process of diversification through the framework of MPT. The example of unsystematic risk is the risk closely related to the operation of the company itself such as losing contracts, marketing or PR failure, and mismanagement of inventory. The strategy of Markowitz is to mix securities that correlate less than one to cancel out the loss of securities, thus reduce the risk of the whole portfolio. Sharpe (1963; 1964) adopted the strategy and devised an optimization model with a risk-free asset to formulate an optimal portfolio for investors.

2.1.2. Application

The concept of quantifying the trade-off between risk and return for financial decision-making is a breakthrough for two reasons. First, it changed the traditional financial analysis, which focused on the valuation of a single investment. In other words, an investor should invest in investments that offer the highest future value compare to the current price. This strategy means that the investor would construct the portfolio purely based on his “subjective statistical belief” about the return of the stocks (Markowitz, 1991). Hence, the portfolio would be constructed by adding the shares whose price is believed to increase without any further consideration. Second, it changed the financial decision-making process into an optimization problem that can be solved (Kolm et al., 2014). This method of MPT would analyze the relationships between stocks through the return, risk, and correlation and optimize these relationships through diversification and allocation.

Due to its revolutionary nature, MPT has a significant impact on the literature of financial industry and academic research. Many papers aimed to either exploit or improving the model, and they brought many valuable insights to both the concept of diversification and also the model itself. There is two different school of thoughts on diversification arise after the emergence of the MPT, one supports and one against diversification. Looking at ex-post data, international diversification is feasible; however, analysis of ex-ante data shows that diversification is difficult because correlations between assets will change throughout time. Furthermore, some countries are more heavily correlated to each other than other which makes the practice more difficult. Nonetheless, there is substantial evidence found proving that internationally diversified portfolio can reduce risk without affecting the return (Shawky, et al., 1997). Many papers confirmed the benefit of diversification, which increases gain and reduces risk, through the implementation of MPT (Grujić, 2016; Biswas, 2015; Zaimovic, et al., 2017). Rao et al. (2012) found that firms of different sectors behave in risk-reward relationships, which suggest that there is information asymmetry in the market or investor’s bias influence this relationship. Zaimovic et al. (2017) stated that regionally optimized portfolio in South East Europe could increase performance for the portfolio.

2.1.3. Model

2.1.3.1. Assumptions

According to Markowitz (1952) and (Vaclavik & Jablonsky, 2012), to mathematically formulate the model, there are several assumptions must be made:

- The expected return of the portfolio or the weighted average of the expected return of individual assets is considered as the measure of reward. Standard deviation is used as the measure of risk, which means that both positive and negative variation is regarded as a risk
- There will be no transaction cost involved such as taxes, commission for broker or money exchange fee for trading globally.
- The historical returns are normally distributed.
- The correlations between the assets are constant.
- The financial instruments are homogenous.
- The user of the model is risk-averse, and the market is efficient.
- The user of the model is price taker instead of a price maker.
- The user of the model can only enter a long position.

2.1.3.2. Mathematical model

An investment portfolio of n assets is analyzed with their respective future returns $r_1, r_2, \ldots, r_n$, which are denoted by a vector $r = [r_1, \ldots, r_n]^\top$. The other component of the portfolio is the weight of each asset is demonstrated by n-dimensional vector $\omega = [\omega_1, \ldots, \omega_n]^\top$ the sum of this weight is equal to 1. Accordingly, the return of the portfolio is represented by a multivariable linear function:

$$r_P(\omega) = \omega_1 r_1 + \cdots + \omega_n r_n = \omega^\top r$$

The covariance matrix of the portfolio is denoted as follow:

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{bmatrix}$$

Where $\sigma_{ii} = \sigma_i^2$ is the variance of the asset i or the square of the standard deviation of the asset I, and $\sigma_{ij} = \sigma_{ji} = \rho_{ij}\sigma_i\sigma_j$ is the covariance between two assets which equal the product of two asset's standard deviation and correlation. The standard deviation of the portfolio is used in the MPT model as the measure of risk (Markowitz, 1991), and is calculated accordingly with the formula

$$\sigma(\omega) = \sqrt{\omega^\top \Sigma \omega}$$

As shown in the formula, the standard deviation of the portfolio depends on the standard deviation, correlation and weight of each asset in the portfolio. Therefore, adding more stocks to the portfolio alone is not enough to reduce the risk of the portfolio but also asset allocation and examining the individual risks and correlations. This view has been substantiated by researchers proving that economic factors can significantly change return of the portfolio because of the correlation between assets and naïve portfolio would not reduce the effect of diversification as shown in figure 2.1 (Alekneviciene, et al., 2012).

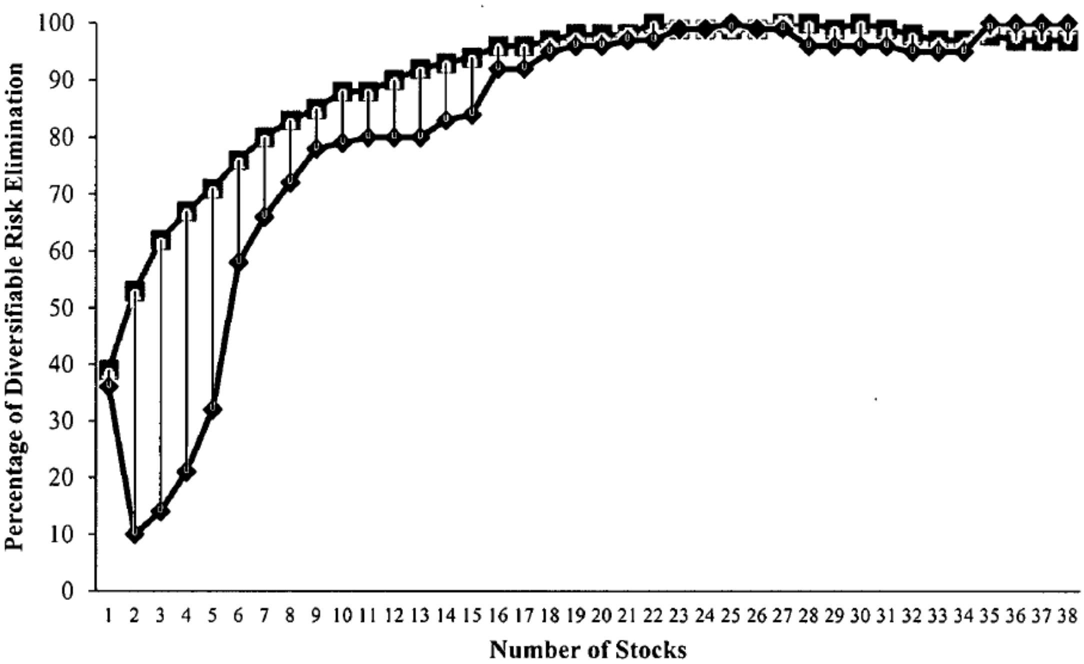

Figure 2.1. Percentage of diversifiable risk elimination in Naïve and differently-weighted portfolio

Expected returns of the securities

$$\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix}$$

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Where $\mu_i = E(R_i)$ for all $i = 1, \ldots, n$. Since the performance of the portfolio is estimated based on the return and the weight of individual assets, it is necessary to analyze the return of the individual return of the asset for the portfolio. Regarding expected return, there are two ways of estimating it. One way is to determine the possible return with a certain level of uncertainty. However, this method is tough to achieve as it related to speculation of the return of the asset. Thus, the other logical way of estimate return is to look at the historical return of the asset (Wilford, 2012).

Assuming that *Ω is the set of possible portfolios*, the mathematical representation of the Markowitz model is the following form (Kolm, et al., 2014):

$$\max_{\omega \in \Omega} \mu^\top \omega \quad \omega^\top \Sigma \omega \leqslant \sigma^2_{\max} \qquad \text{or} \qquad \min_{\omega \in \Omega} \omega^\top \Sigma \omega \quad \mu^\top \omega \geqslant R_{\min}$$

2.1.3.3. Efficient frontier:

Through the above formula, the efficient frontier is the set of the portfolio that gives the investors the best risk and return relationship. It has been agreed among the literature that risk and return are connected. Higher returns require a higher level of risk (GEAMBASU, 2013). The trade-off relationships are captured through the efficient frontier with one axis is the risk or the standard deviation of the portfolio, and the other axis is the return of the portfolio (Markowitz, 1959). The curve or the "Markowitz bullet" is the graphical presentation of collections of portfolios that give the highest return for the given level of risk, and each point in the curve represents one portfolio in the collection that offers the highest return for a given level of risk. The bottom point of the line is the portfolio that gives the lowest possible risk, and the end point is the portfolio that offers the highest level of return.

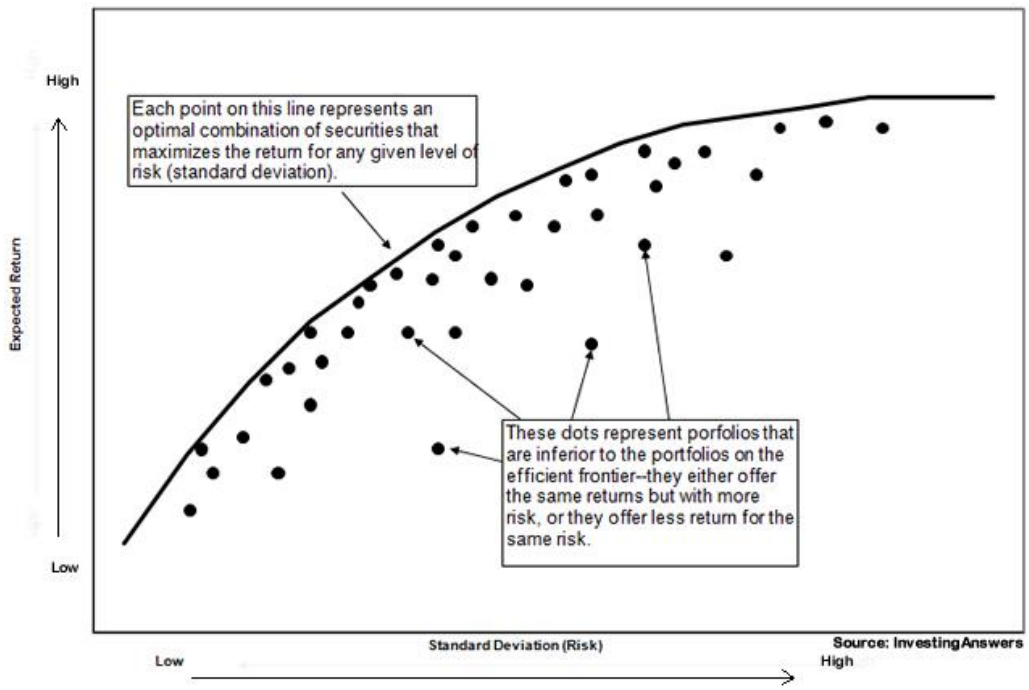

Figure 2.2. The efficient frontier that shows the relationship between risk and returns

The figure 2 shows that many portfolios could be formed, but the portfolios lie below the efficient frontier is not "efficient," which they give more risk for the same level of return and vice versa compare to the portfolios in the efficient frontier. Furthermore, the figure also shows the relationship between risk and return. If the risk incurred, the potential reward would be higher.

2.1.4. Capital asset pricing theory

According to Markowitz, any risk-adjusted return investor would prefer to have a portfolio on the efficient frontier. However, when there is a risk-free asset, the efficient frontier turns into a line that is called optimal capital allocation line (CAL) which is the line that is tangent to the efficient frontier starting from the risk-free asset. The tangent point on the efficient frontier is called the tangency portfolio, which is the optimal portfolio of risky-asset when combining with the risk-free asset. From the CAL, every mean-variance efficient investor would invest in a portfolio in CAL, which is a combination of a certain weight in the risk-free asset and certain weight in the tangency portfolio. The expected return of a portfolio p in the CAL can be expressed by the following equation:

$$E[r_p] = r_f + \sigma_p \frac{E[r_{tan}] - r_f}{\sigma_{tan}} \quad (2.1)$$

where r_f is the risk-free rate of return, σ_p is the standard deviation of returns of the portfolio, σ_{tan} is the standard deviation of returns from the tangency portfolio and $E[r_{tan}]$ is the expected return on the tangency portfolio.

Investing in the CAL means that the investor is mean-variance efficient, and wants the best trade-off between risk and return. One way to measure this risk and return trade-off is the Sharpe ratio, which is normally calculated as follow:

$$SR_p = \frac{E[r_p] - r_f}{\sigma_p} \quad (2.2)$$

Looking at equation (2.2) it can be inferred that the optimal Sharpe ratio is the slope of the CAL, and portfolio allocated along the CAL gives the highest Sharpe ratio.

When the risk-free rate is introduced, the efficient frontier is the CAL, and the mean-variance efficient investor would allocate the portfolio base on the utility function shaped by his risk preference. The portfolio chosen will be the intersect between the indifferent curve and the CAL. An investor who is risk-averse will allocate his portfolio close to the risk-free asset. In contrast, an investor with the high-risk preference would choose to move towards the tangency portfolio and can even short the risk-free rate to increase his holding in the tangency portfolio. In both scenarios, the investor would have the portfolio with the highest Sharpe ratio.

One of the most famous financial theory, namely the Capital Asset Pricing Model (CAPM), claims that portfolio returns can be estimate given certain assumptions. The idea of the CAPM was developed by Sharpe (1964), and Lintner (1965). It states that a portfolio's expected return can be expressed as the sum of risk-free rate plus the beta coefficient of a portfolio times the market risk premium. The beta coefficient can be understood as the movement of the portfolio compared to the market. The CAPM can be expressed by the following equation:

$$E[r_p] = r_f + \beta_p(E[r_m] - rf) \quad (2.3)$$

Where β_p is the beta coefficient of portfolio and $E[r_m]$ is the expected return on the market portfolio. One of the most crucial assumption of the CAPM theory is that every investor would invest in the CAL to maximize the Sharpe ratio of their portfolio.

Therefore, when all of the investment accumulate, the market portfolio will be the tangency portfolio with the optimal Sharpe ratio. This market portfolio comprises of all existing assets weighted by their market capitalization.

With this assumption of the CAPM theory and an efficient market, one should invest in a portfolio that is similar the market portfolio to achieve the highest Sharpe ratio. However, the literature suggests otherwise. In fact, many asset-allocation strategies out-perform the VWMP. The following part of this literature review will explore studies testing the out-of-sample performance of many portfolios and prove that VWMP does not offer the highest Sharpe ratio.

2.2. Empirical studies on various portfolio optimization strategies

2.2.1. Out-of-sample testing

There is much research on the effect of different portfolio optimization strategies on the performance of portfolio through out-of-sample testing, and the result is mixed. There are two primary schools of thoughts regarding this subject. Some researches deny the use of sophisticated optimization strategies and argue that the basic strategy of naïve portfolio or putting an equal weight across assets in a portfolio has the best performance regarding return and Sharpe ratio with a significance level of certainty (Duchin & Levy, 2009; DeMiguel, et al., 2009). Others repudiate this claim and prove that various strategies such as using a mean-variance, minimum variance, and Black-Litterman optimal portfolio produces better portfolio performance than the naïve strategy(Clarke, et al., 2006; Kritzman, et al., 2010; IDZOREK, 2004).

In support of naïve portfolio strategy, the most famous work is done by DeMiguel, et al. (2009). Using 14 different portfolio strategy on 7 datasets of monthly return, he discovered that the naïve portfolio strategy consistently produces a better result than many mean-variance optimized models by Sharpe ratio, certainty equivalent and turnover with significant certainty. Some of these mean-variance models were implemented with constraints such as long-only constraint and long-only shrinkage constraint. The performance made by these mean-variance models will be compared with market benchmarks index and the naïve portfolio. The time frame used for the model to allocate the asset composed of rolling 60- and 120-month windows. With the main out-of-sample period from July 1963 to November 2004, the naïve portfolio statistically gives highest Sharpe ratio, follow portfolio with constraints and portfolio with no constraints. In the research, this poor performance is explained by the resistant nature of naïve portfolio to estimation. This research then stretches the finding to find out how large the time frame should be to reduce the effect of estimation error significantly. Based on the U.S. stock market data, a portfolio of 25 assets needs an estimation window of 3000 months or 250 years for the mean-variance portfolio to outperform the naïve portfolio and diminish the effect of estimation error. Since estimation error is one of the main reason that affects the performance of the mean-variance portfolio, a new model that reduce this error is required. This problem can be solved by using a Bayesian approach to reduce estimation error in the mean-variance model. Several types of research use this approach to reduce estimation error, disprove and find flaws in the research of DeMiguel (Tu & Zhou, 2011; Kirby & Ostdiek, 2012).

Another study that supports the use of naïve portfolio is done by Duchin and Levy (2009). They discovered that the average return of naïve diversification is higher than the return of other strategies when the portfolio composed of 15, 20 and 25 assets. However, when the number of assets increases to 30 assets, the naïve portfolio produce an inferior average return. Therefore, it suggests that for a portfolio with few assets, which is a typical portfolio of an individual investor, the best strategy is naïve diversification. In contrast, the best strategy for an institutional investor with many assets in a portfolio is to use optimization model to change the asset allocation of the portfolio actively.

In opposed of a naïve diversification strategy, the most critical research is implemented by Kritzman et al. (2010). The authors give a similar explanation for the inferior empirical result of mean-variance portfolio compare to the explanation of DeMiguel, which is the short sample interval for mean estimation. They criticize the conventionally used of 60- or 120- month historical samples for portfolio optimization, which can lead to an implausible result. They overcome this shortcoming by computing the parameter for portfolio optimization in the rolling window time frame of 50 years. They then compute the out-of-sample Sharpe ratio for 8 datasets with the out-of-sample period from 1978 to 2008. The result shows the Sharpe ratio of mean-variance and minimum variance portfolio is superior to the Sharpe ratio naïve portfolio.

However, there was no statistical test for the Sharpe ratio to back up the result of research, which can render the research entirely false.

2.2.2. Minimum variance portfolio

A common sub-branch of literature on the performance of portfolio optimization strategy focuses on minimum variance portfolio. This particular strategy put a high emphasis on low-volatility assets or less risky assets to lower the volatility of the portfolio. The strategy receives many acclaims from academics for three reasons. Firstly, there are many types of research empirically proves that minimum variance portfolio gives better out-of-sample risk-adjusted return than any mean-variance efficient portfolio. This finding contradicts the risk-reward relationship of a security under the CAPM theory that portfolio with higher beta has higher expected return than portfolios with low beta. Secondly, since the construction of minimum variance portfolio is independent of expected return forecast and only need an estimation of the covariance matrix, estimation errors can be significantly reduced. Thirdly, after many harsh and severe financial crash, there is an increasing risk-aversion mentality among investors, which leads to a need in low-volatility financial products.

Another research that supports more sophisticated asset allocation technique is done by Nielsen & Aylursubramanian (2008). They found that most studies regarding minimum-variance portfolios have emphasized on domestic or regional markets. Thus in this research, they aim to look at portfolio strategy in a global context. Using the MSCI global MV Index as the representation for minimum variance portfolio, it shows a better result compared to a market benchmark such as the MSCI World Index and the long duration FI index. Also, analysis of its characteristics confirmed that the MSCI MV World index performance profile is consistent with earlier studies of minimum variance portfolios for US and European markets. The MSCI MV World index experienced approximately 30% lower volatility than the MSCI World Index over the period June 1995 to December 2007. Its performance, measured by Sharpe ratios, was superior relative to the MSCI World Index--0.67 vs. 0.45.17 Its overall excess return (above 1-month T-Bill) was 6.5% compared to 6% for MSCI World Index.

Clarke, et al. (2006), who also is opposed to a naïve diversification strategy, conduct research focusing on minimum variance portfolio in the US equity market. The author tested this strategy using different covariance estimation methods and restricting the strategy with certain constraints. The author uses a covariance matrix enhancement method by Bayesian shrinkage procedure and the rolling 5-year monthly method. They also restrict short sales, and an upper limit of 3% for all of the securities tested. Furthermore, he also tested the minimum variance method using a rolling 1-year daily method and removed the short sale constraints, but forces market neutrality constraints to make the result produce the same ex-ante characteristic of the market. The authors found that a minimum variance portfolio has lower volatility than the S&P500 with the same level of return. They conclude that it is possible to predict the variance and covariance of minimum variance portfolio, which contradicts the general opinion that variance and covariance are unpredictable. Furthermore, the research also finds that minimum variance portfolio adds value over the S&P500.

2.3. Estimation error

Since the actual mean and variance of security are unknown, the construction of mean-variance efficient portfolio for out-of-sample testing is dependent on estimation of those parameters. The conventional method of estimating these parameters is using historical information or expectations about the future. The result of this method is evidently imprecise and can introduce many errors. Therefore, it can be inferred that performance of optimized portfolio out-of-sample may not be optimal due to these errors, specifically, the difference between the estimated return and covariance. This problem has been addressed by many literatures, and several solutions have been introduced.

The first literature to acknowledge this estimation error in portfolio optimization is Michaud (1989). The author report that unconstrained mean-variance optimization can lead to suboptimal portfolios. Thus, he opts for the equal-weighted strategy in asset allocation. He refers the term “error maximization” to mean-variance optimization because small estimation errors can eventually create large output errors and decimate the whole positive effect of portfolio optimization. The paper tries to solve this problem by adding several constraints to portfolio construction such as, adding short-selling constraints and shrinkage estimator.

Another paper addresses the problem of estimation error but emphasizes on the estimation of the covariance matrix (Ledoit & Wolf, 2004). They claim that a sample covariance matrix formed based on historical information has fatal errors due to periodical variations and especially extreme observations. To solve this problem, a new method of estimating the covariance matrix by shrinking the sample covariance matrix towards the constant-correlation matrix is suggested by the author. They tested the out-of-sample performance of the portfolios using their shrinkage method and found that portfolios using shrink the covariance matrix perform empirically better than other portfolios. The paper by DeMiguel et al. (2009) also uses this method and concludes that portfolio built with long-only constraints and shrinkage covariance matrix reduces estimation and gives better Sharpe ratio than the equally-weighted portfolio in 5 out of 7 datasets.

2.4. Conceptual framework

To sum up, the topic of whether or not to use sophisticated portfolio construction strategy is hotly debated among literature. There are two opposite opinions on the asset allocation strategy: one is in favor of the naïve portfolio, one is in opposed to this strategy. Additionally, on the opposing side, there is a branch namely minimum variance portfolio, which receives a lot of attention for multiple reasons. Supporters of these two views overall give sufficient explanation for their choice and back their opinion by reliable testing method and statistical measurements such as out-of-sample testing and Jobson and Korkie test, which will be used in this thesis.

The main argument for applying naïve portfolio strategy is the adverse effect of estimation error on other portfolio optimization strategy and simplicity of implementing. In response to estimation error, an opponent of naïve strategy circumvents this problem by introducing shrinkage estimation of covariance matrix or adding short constraints. Another option to reduce estimation error is reducing the parameters that are used to compute portfolio. The minimum variance portfolio strategy satisfies this option because it eliminates the need for estimating the expected mean return of an asset. Overall, the stream of literature have a different method of testing their ideas, but the general framework for constructing and evaluating portfolio strategy is described in figure 2.3. However, the central gap of research is on the Asian market, as there are a paucity of research using the above method to test portfolio performance in this geographical area. The findings, methodology, and gap of literature are crucial in formulating the methodology in this research paper, and give a more explicit direction in answering the research question of the thesis, which remains the same and will be stated as below:

- What is the performance of the naïve portfolio in the Asian market?
- What is the performance of the mean-variance efficient portfolio in the Asian market?
- Which asset allocation strategy is better in the Asian market: naïve or mean-variance efficient?

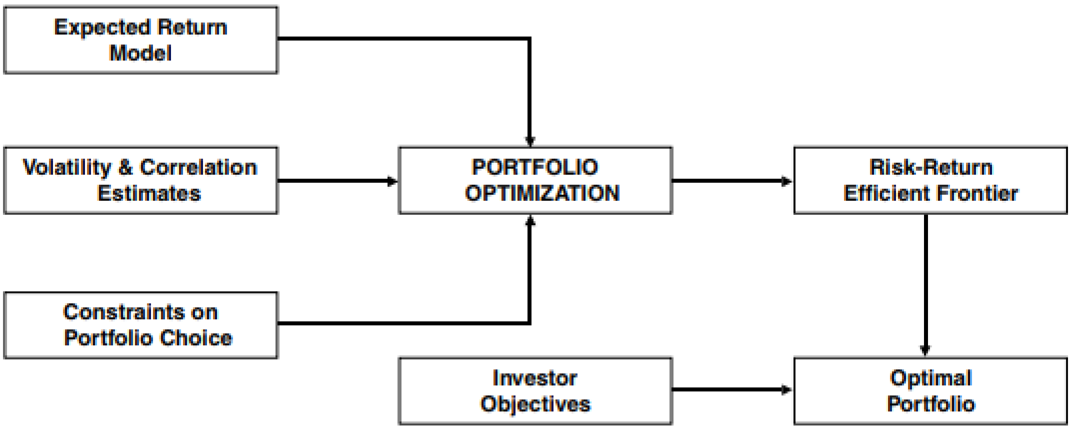

Figure 2.3. The theoretical framework for constructing a portfolio

3. Methodology

In this sections, the detail of the construction of portfolios based on different strategy will be presented with the method of measuring portfolio performance and the relevant statistical test. The strategies that will be used in this research are provided below with their abbreviation:

- naïve strategy (EWP)
- Value weighted market portfolio (VWMP)
- minimum variance portfolio (MinVP)
- minimum variance portfolio allow shorting (MinVPshort)
- mean-variance portfolio(MeanVP)
- minimum variance portfolio using shrinkage method (MinVP_shrink)

- minimum variance portfolio using shrinkage method allow shorting (MinVPshort_shrink)
- mean-variance portfolio using shrinkage method(MeanVP_shrink)

The goal of this thesis is to replicate two most important studies regarding the subject of out-of-sample testing of DeMiguel et al. (2009) and Kritzman et al. (2010) using different datasets of the Asian area. Therefore, it is inevitable that the methodology of this thesis is very similar to their study. The construction of the portfolios will be made using the free programming language and R, developed by the R Core Team.

3.1. Description of models and estimations

The foundation for the any Markowitz portfolio optimization strategy is based on the assumption that an investor will maximize his utility and will base his utility assessment on the ratio between risk and return. Thus, it suggests that investors will prefer portfolios with the higher Sharpe ratio. Based on the assumption of CAPM, which states that all of the investors are utility-maximizing and will invest in the tangency portfolio which translate into the market capitalization portfolio or the VWMP. There are many methods to translate the utility maximizing mentality of an investor into a mathematical function, but for the scope of this thesis, a quadratic function indifferent curve for portfolio utility is used:

$$\max_{w} U(r_p) = E[r_p] - \frac{\gamma}{2} Var[r_p] \quad (3.1)$$

where γ is a scalar that represents risk aversion level of the investor. The portfolio that an investor would choose is the tangent between the capital allocation line and this indifferent curve, which optimize the investor risk tolerance and performance of the portfolio.

Assume that the investor wants to allocate his investment in a portfolio of N assets with a certain amount of wealth. Under the condition that the investor must invest all of that wealth, the portfolio weights must sum up to 1. The formula for the return of the portfolio can be expressed as:

$$r_p = \sum_{i=1}^{N} w_i r_i \, , subject\ to \sum_{i=1}^{N} w_i = 1, \quad (3.2)$$

where r_p is the return on the portfolio, w_i is the weight in allocated to asset i, and r_i is the return from asset i. Since the future returns are unknown, expectedthe value must be used:

$$E[r_p] = \sum_{i=1}^{N} w_i E[r_i]\,, subject\ to \sum_{i=1}^{N} w_i = 1 \quad (3.3)$$

where $E[r_i]$ is the expected value of the return on asset i. The variance of the return of the portfolio can be formulated as follow:

$$Var[r_p] = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov[r_i, rj], \quad (3.4)$$

where $Var[r_p]$ is the variance of the portfolio return and $Cov[r_i, rj]$ is the covariance of asset i and asset j. Since each portfolio model will be weighted differently, the formula (3.3) and (3.4) will be the same for all portfolios of only risky asset.

Moreover, based on the literature review of the model, moment estimation or the time frame to calculate covariance matrix is needed for the portfolio. On the other hand, the EWP and VWMP will not be dependent on moment estimation for the construction of the portfolio. Table 3.1 is the summary of the needed inputs for each of the strategies.

Portfolio strategy	**Abbreviation**	**Input estimates**
Minimum-variance portfolio	MinVP	Covariance matrix
Minimum variance portfolio allow shorting	MinVPshort	Covariance matrix
Mean-variance portfolio	MeanVP	Mean returns and covariance matrix
Minimum variance portfolio using shrinkage method	MinVP_shrink	Covariance matrix
Minimum variance portfolio using shrinkage method allow shorting	MinVPshort_shrink	Covariance matrix

mean-variance strategy using shrinkage method	MeanVP_shrink	Mean returns and covariance matrix
Equally-weighted	EWP	None needed
Value-weighted market	VWMP	None needed

Table 3.1. Different portfolio strategies and needed input for construction

3.1.1. Moment estimation and shrinkage estimator

The MinVP and the MeanVP are dependent on the covariance matrix to be created. However, the right covariance matrix of a portfolio is not known, so it must be estimated from a certain period of the dataset. The period is called in-sample or look back periods and can have a different length. These can be any number based on the availability of the data and the purpose of the research. In this research, several in-sample periods with different length will be used to estimate the covariance matrix and calculate the out-of-sample outcome for each of the cases. The purpose of choosing many sample period is to see if the out-of-sample results will be changed due to differences in in-sample length.

There are two approaches to out-of-sample testing: rolling-window approach and expanding window approach. The rolling-window approach that will be used in this can be explained as followed. The covariance matrix is estimated over a rolling window of T months, where T is set to 60, 120, 240 which is similar to the T of Kritzman et al. (2010). If T=60 (5 years) the portfolio weight for investment will be calculated based on the covariance matrix of the portfolio from January 1991 to January 1996. The next covariance-matrix estimate will be rolled one month forward, which means that it will take the data from month T+1 while disregarding the data of the first one. So in this example, the covariance matrix will be taken from the data from February 1991 to February 1996. Based on this process, portfolio weights will be updated every month because a new covariance matrix is created every month.

When T= all time the approach is called expanding window. The principle is the same except that when a new covariance matrix is computed, it will not exclude the first month from the calculation. This means that the portfolio starts from January 1996 will take all the data from December 1995 to January 1991, and the portfolio starts from

February 1996 will take all the data from January 1996 to January 1991 to compute covariance matrix for constructing the portfolio.

When considering a sample for covariance matrix, the effect of estimation error can have negative influences on the result. Therefore, several methods can be used to reduce the effect of estimation error. In this thesis, the shrinkage of the covariance matrix will be implemented when constructing both the MinVp and the MeanVP. This method of will pull extremely high coefficient elements in the matrix downwards and pull extremely low coefficient element downward because high coefficients tend to be estimated with positive error, while low coefficients tend to be estimated with negative error. Therefore, this method will make the covariance matrix more constant throughout different time periods. The shrinkage estimator used in this thesis is based on Ledoit and Wolf (2004) can be expressed as

$$\hat{\Sigma}_{Shrink} = \hat{\delta}^{*}F + \left(1 - \hat{\delta}^{*}\right)\hat{\Sigma} \quad (3.5)$$

where $\hat{\Sigma}_{Shrink}$ is the shrunk estimated covariance matrix, $\hat{\delta}^{*}$ is the estimated optimal shrinkage constant which is a number between 0 and 1, F is the sample constant-correlation matrix and $\hat{\Sigma}$ is the sample covariance matrix.

In general, estimation error in this research are expected to be small because of these reasons

- The investable universe only comprises the maximum of 25 different assets
- The shrinkage estimator above will be implemented

3.1.2. Minimum-variance portfolio

The minimum variance portfolio will be the constructed by solving this minimization problem with covariance matrix as input, expressed in matrix notation:

$$\min_{w} \sigma_p^2 = \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}, subject\ to\ \boldsymbol{w}'\boldsymbol{1} = 1 \quad (3.6)$$

Where **1** is an N x 1 vector of ones, **w** is an N x 1 vector of portfolio weights, **Σ** is an N x N covariance matrix and ½ is added for convenience in calculation. Solving this minimization problem returns the vector of weights for the portfolio

$$w^{MinVP} = \frac{\boldsymbol{\Sigma}^{-1}\boldsymbol{1}}{\boldsymbol{1}'\boldsymbol{\Sigma}^{-1}\boldsymbol{1}} \quad (3.7)$$

The MinVPshort will be the MinVP set with short constraint and the MinVPshort_shrink, or the MinVP_shrink will be formulated by replacing the covariance matrix in the equation (3.7) with the shrinkage covariance matrix.

The minimum variance portfolio is the portfolio the gives the lowest variance possible for all possible allocation choice. In the construction of this portfolio, no constraint on wealth allocation is set. When no constraint is set, the portfolio can have extremely high weight in certain asset in a portfolio that does not have many assets. However, this not a significant problem, since the datasets used consists of many stocks which are sorted into a certain amount of assets. This means that this strategy will invest in asset pool that each of the assets is equivalent to a diversified portfolio. Therefore, the minimum portfolio constructed will hold a certain degree of diversification.

3.1.3. Mean-variance portfolio

The mean-variance model of Markowitz (1951) will be used to maximize investor utility function (3.1) while optimizing the risk-return trade-off. The computation for the weight of this portfolio will require estimation of mean and covariance matrix. The problem of maximizing the investor's utility function can be formulated in matrix notation as:

$$\max_{w} U(\mu_p) = \left(\boldsymbol{w}'\left(\boldsymbol{\mu} - \mathbf{1}r_f\right) + rf\right) - \frac{\gamma}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad (3.8)$$

The weights of the MeanVP are chosen that the portfolio has the lowest mean for the return μ_p computed by the equation (3.8). This can be expressed by this minimization problem.

$$\min_{w} \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}\,, subject\ to\ \boldsymbol{w}'\left(\boldsymbol{\mu} - \mathbf{1}r_f\right) + r_f = \mu_p \quad (3.9)$$

The weights of assets after solving this equation are:

$$w^{MeanVP} = \frac{\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \mathbf{1}r_f)}{\mathbf{1}'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \mathbf{1}r_f)} \quad (3.10)$$

As the right mean and covariance matrix is unknown, the sample mean and sample covariance matrix will be used as replacement. This means that there will be estimation error, which might negatively affect the performance of the portfolio out-of-sample. The weight of the MeanVP_shrink will be a function (3.10) using shrinkage estimator of the covariance matrix.

3.1.4. Value-weighted market portfolio and Equally-weighted portfolio

As the market portfolio is unachievable in the real financial world, the conventional method to get an estimation of this portfolio is holding large value-weighted indices such as the Russell 2000 or the S&P500 in the US. According to the CAPM theory, the market portfolio is the portfolio that delivers the highest Sharpe ratio, and every investor should have the mixture between the market portfolio as a risky asset and the risk-free asset. In this thesis, the market portfolio will be constructed by using the market returns provided by Kenneth French library.

The equally weighted portfolio is self-explanatory, as the weight will be allocated equally among assets. For example, a portfolio consists of N assets will have each asset weight 1/N.

3.2. Measuring portfolio performance

3.2.1. Mean return and standard deviation

The computation for the mean of the portfolio will be the excess return taken from the time series of each portfolio of different strategies, and the standard deviation will be computed accordingly based on the time series. Both of the results will be converted from monthly return and standard deviation into yearly excess return and standard deviation. The annualized return and standard deviation of a portfolio will be computed as follows

$$\bar{\mu}_p = \hat{\mu}_p 12$$

$$\bar{\sigma}_p = \hat{\sigma}\sqrt{12}$$

Both of these measures will be given t-test for the mean and f-test for the variance to compare to the naïve portfolio. Thus the null hypothesis for both the mean and the standard deviation can be express as follows

$$H_o: \mu(naive) = \mu(strategy)$$

$$H_0: \sigma(naive) = \sigma(strategy)$$

The null hypothesis will be rejected if p-value < 0.05, which can be used to prove if the excess return of the strategy portfolio is larger than the return of the naïve portfolio or the standard deviation of the strategy portfolio is lower than the return of the naïve portfolio.

3.2.2. Sharpe ratio

The performance of the out-of-sample portfolio can also be evaluated by the Sharpe ratio. Admittedly, there are many limitations using Sharpe ratio as a performance measure; this thesis will not be concerned with that problem. Furthermore, under the CAPM assumption, which is the basis of this thesis, a mean-variance optimizing investor would prefer the portfolio that gives the highest Sharpe ratio. Hence, the use of Sharpe ratio for this study is crucial. The Sharpe ratio computed will be annualized and will be tested against the naïve portfolio to ensure the statistically distinguishable of the result. The formula for computing the out-of-sample Sharpe ratio is calculated as follows:

$$\widehat{SR}_p = \frac{\hat{\mu}_p - r_f}{\hat{\sigma}_p}$$

where $\hat{\mu}$ is the monthly out-of-sample mean return and $\hat{\sigma}$ is the out-of-sample standard deviation of the portfolio's monthly excess return. The Sharpe ratio will then be annualized as the following equation:

$$\overline{SR}_p = \widehat{SR}_p\sqrt{12}$$

The annualized Sharpe ratio will be tested against by the statistical test of Jobson and Korkie test for Sharpe ratio (Jobson & Korkie, 1981). The null hypothesis that $H_0: \widehat{SR}_1 = \widehat{SR}_2$ is test with the formula to calculate Z-score

$$\hat{z} = \frac{\widehat{SR}_1 - \widehat{SR}_2}{\sqrt{\frac{1}{n}[2(1-\hat{\rho}^2) + \frac{1}{2}\left(\widehat{SR}_1^2 + \widehat{SR}_2^2 - 2\widehat{SR}_1\widehat{SR}_2\hat{\rho}^2\right)]}}$$

The z-score will be converted to p-value, and if the p-value<0.05, it can be declared that the two Sharpe ratio is statistically distinguishable. Then, it can be inferred that whether the Sharpe ratio of the strategy is larger than the Sharpe ratio of the naïve portfolio

3.2.3. Capital accumulation

Another performance measure that will be used in this paper is capital accumulation. The capital accumulation will be assessed by how the portfolio returns will change the growth of $1 invested at the start of the out-of-sample period. This method will show which strategy gives the highest return regarding money. According to the CAPM theory, a mean-variance investor would focus on maximizing Sharpe ratio; however, in real life investing, this method maybe unfit because not every investor has the capability to leverage his investment to matches his preferred level of expected return. Therefore, it would be reasonable to add this factor to evaluate portfolio performance.

3.2.4. Testing periods

Due to the limitation of data, the nature of replication process of previous studies, and to the direction for the result of the choice of the out-of-sample period, the empirical testing will be repeated for several time periods. The first period tested will be the full sample period from January 2011 to January 2018. This is the longest time period, which covers 7 years, allowed by all 8 sample after setting aside for the first 20 years as the largest T = 240 for in-sample parameters for estimation and portfolio construction. Admittedly, this might be considered too short for any long-term behavior.

A solution to this problem is to create few subsamples that are longer, which are from January 1996 to January 2018 and January 2001 to January 2018. However, due to the limitation of data, extending the timeframe of subsample must be recompensed by smaller T. The two subsamples are also the longest time period after setting aside for the first 5 years and 10 years for estimation as T = 60, T=all, and T = 120. The table 3.2 is an overview of the implemented periods.

Time period classification	**Time period by date**
Full sample period	01/2011 – 01/2018
Sub-period for T=120	01/2001 – 01/2018
Sub-period for T=60	01/1996 – 01/2018
Sub-period for T=all	01/1996 – 01/2018

4. Data

The data from the Keneth French online data library will be taken as the basis for this study. As the goal of this research is to focus on the Asian area, this thesssis will utilized 8 different data sets as the investment universe for 7 different strategies, each consisting of monthly return. The 8 different datasets are from 2 area of Asia Pacific and Japan to better represent the geographical area. The time frame taken from the data is from 1991 to 2018. However, due to the availability of data and the design of this research only, the period from 2011 to 2018 will be the main time frame taken to analyze, and few more sub-period will be analyzed to consolidate the result. The data taken from the library are the returns sorted by size, book-to-market and momentum. Since all of the data is retrieved from the same source, it is consistent and comparable. The detail construction of each dataset will not be discussed because it is irrelevant to the purpose of this study. For more information on the data, the reader can look at the website at the footnote[1]

Additionally, another dataset used is the Fama/French 3 factors of Japan and the Asia-Pacific area, which will provide the risk-free rate of return and the value-weighted market premium. The value-weighted market premiums are used as the VWMP as mentioned in the methodology, and the risk-free rate is used to obtain the excess return. The relevant datasets along with their purpose, available time periods are summed up in Table 4.1.

Dataset	**Purpose**	**Available time period**
6 Asia Pacific ex Japan Portfolios Formed on Size and Book-to-Market	Investable universe	01/1991-01/2018
6 Japanese Portfolios Formed on Size and Book-to-Market (2 x 3)	Investable universe	01/1991-01/2018
25 Japanese Portfolios Formed on Size and Book-to-Market (5 x 5)	Investable universe	01/1991-01/2018
25 Asia Pacific ex Japan Portfolios Formed on Size and Book-to-Market	Investable universe	01/1991-01/2018

1 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/six_portfolios_developed.h01/1991-01/2018tml

6 Japanese Portfolios Formed on Size and Momentum	Investable universe	01/1991-01/2018
6 Asia Pacific ex Japan Portfolios Formed on Size and Momentum	Investable universe	01/1991-01/2018
25 Japanese Portfolios Formed on Size and Momentum	Investable universe	01/1991-01/2018
25 Asia Pacific ex Japan Portfolios Formed on Size and Momentum	Investable universe	01/1991-01/2018
Fama/French Japanese 3 Factors	Rm and Rf	01/1991-01/2018
Fama/French Asia Pacific ex Japan 3 Factors	Rm and Rf	01/1991-01/2018

Table 4.1. Dates utilized for the study

5. Empirical result and findings

This section will present and compare the results of the empirical study of different asset-allocation strategies described in Section 4. Results from the main period from 2011-2018 will be presented first, and the results from other sub-period will be presented afterward. The empirical results will be discussed further in Section 6. All of the measures including Sharpe ratio, mean return, and standard deviation reported from now on would be annualized.

5.1. Portfolios performance in the period of 2011-2018

The first empirical testing will be the out-of-sample testing which covers the time period from January 2011 to January 2018. Due to the large amount of computations, only the tables that are representative of the overall results will be presented in this section to improve readability, all of the other tables will be shown in the Appendix.

5.1.1. Asia-pacific

Asia-pacific illustrates many distinctive features in the results of its out-of sample-testing. Therefore, only two tables will be presented here along with the graphical representation of cumulative return of two datasets: 25 portfolios formed from size and book-to-market with T = 10 years, and 6 portfolio formed on size and momentum with T = 5 years.

Asia pacific 2011-2018, T=10 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08	0.00	0.18	0.00	0.42	0.00	1.55
Market	0.06	0.57	0.16	0.16	0.37	0.91	1.41
Markowitz	0.16	0.21	0.19	0.71	0.82	0.00	2.71
Minimum variance	0.11	0.38	0.18	0.39	0.60	0.20	1.91
Minimum variance short	0.17	0.17	0.20	0.77	0.88	0.14	3.00
Markowitz shrinkage	0.16	0.20	0.20	0.77	0.83	0.01	2.79
Minimum variance shrinkage	0.10	0.39	0.17	0.33	0.59	0.23	1.88
Minimum variance shot shrinkage	0.19	0.18	0.27	1.00	0.69	0.53	2.92

Table 5.1. Portfolio performance of Asia pacific 2011-2018, T=10 years, 25 portfolios formed on size and book-to-market

Asia pacific 2011-2018, T = 5 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naïve	0.05	0.00	0.17	0.00	0.30	0.00	1.31
Market	0.06	0.46	0.16	0.40	0.37	0.91	1.41
Markowitz	0.09	0.36	0.21	0.98	0.41	0.49	1.60
Minimum variance	0.09	0.31	0.16	0.29	0.60	0.01	1.80

Minimum variance short	0.25	0.01	0.14	0.04	1.79	0.00	5.44
Markowitz shrinkage	0.09	0.36	0.21	0.97	0.42	0.44	1.62
Minimum variance shrinkage	0.09	0.31	0.16	0.27	0.60	0.00	1.81
Minimum variance shot shrinkage	0.23	0.02	0.17	0.48	1.37	0.00	4.58

Table 5.2. Portfolio performance of Asia pacific 2011-2018, T = 5 years, 6 portfolios formed on size and momentum

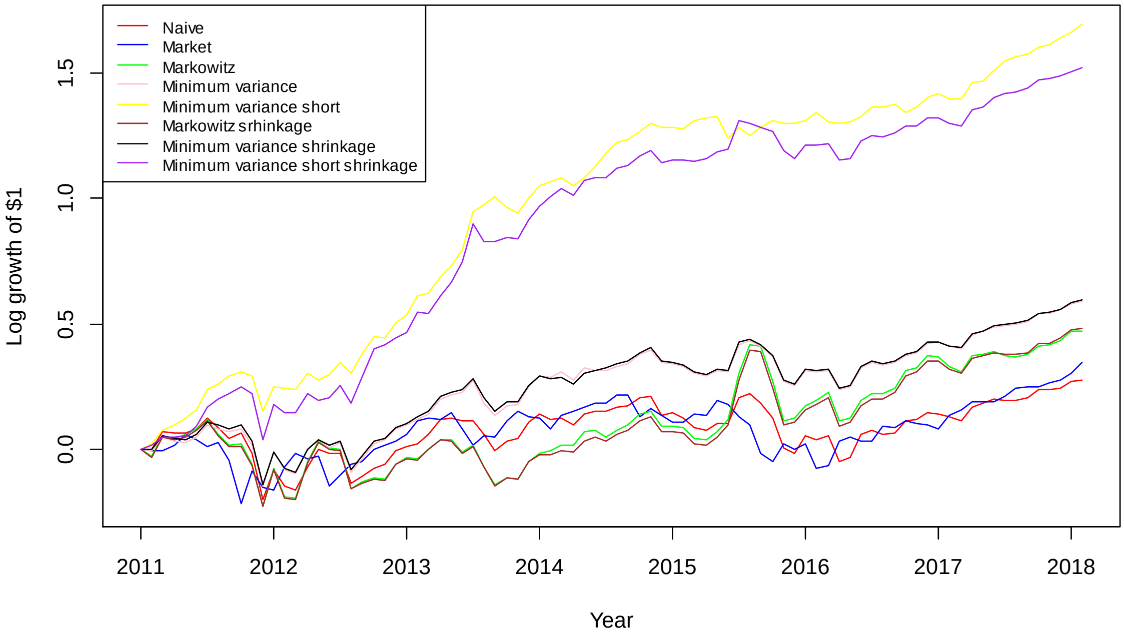

Figure 5.1. Capital accumulation of Asia pacific 2011-2018, T=10 years, 25 portfolios formed on size and book-to-market

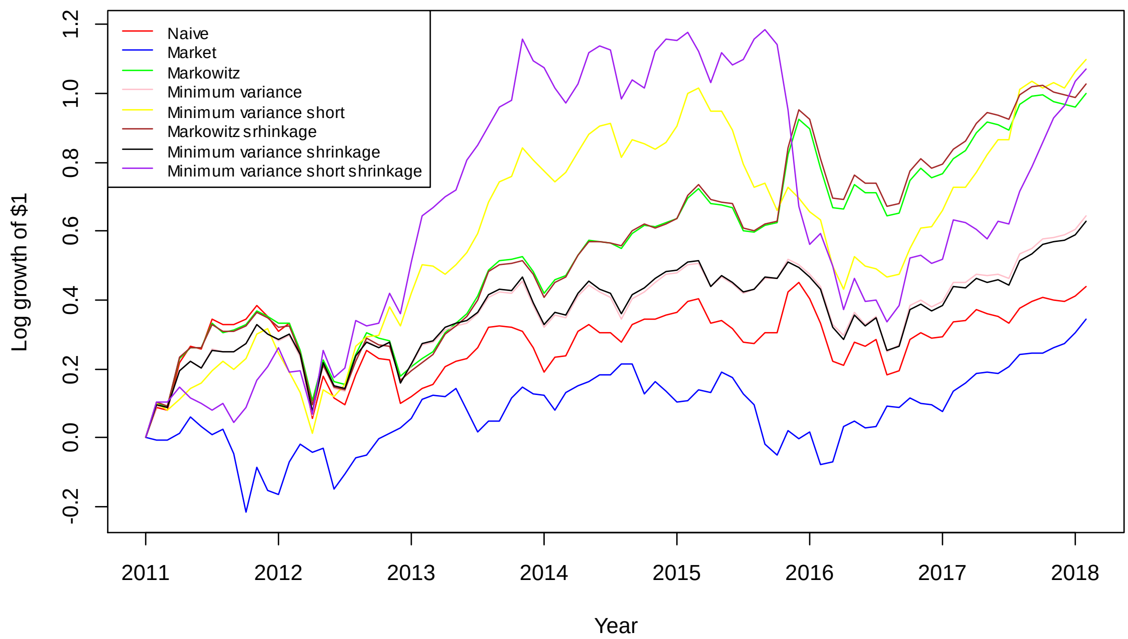

Figure 5.2. Capital accumulation of Asia pacific 2011-2018, T = 5 years, 6 portfolios formed on size and momentum

5.1.1.1. Sharpe ratio

When observing the empirical data reported in the Asia-pacific area during the period of 2011 to 2018, there are certain characteristics that are prominent. First of all, the numerical value of the Sharpe ratio of different portfolio strategies will be dissected, followed by an evaluation on statistical differentiability. The first noticeable element of the result from the data of this geographical area is that the minimum variance that allows short (MinVPshort and MinVPshort_shrink) appears consistently and delivers a higher Sharpe ratio. This results seems to suggest that an estimation error from shorting might have a beneficial effect on the performance of minimum variance portfolio. Additionally, a second finding displays that optimized portfolios do seem to outperform EWP and VWMP. From this result, one can argue that, optimized portfolio do outperform simpler strategies, such as the EWP and the VWMP. The final observation suggests that EWP might deliver the lowest Sharpe ratio of all strategies. Consequently, this result might imply that holding the EWP portfolio is not an optimal decision for risk-return efficient investors. However, statistical significance should be taken into consideration because there is one out of four investable universes where the EWP underperforms the VWMP, the 6 portfolios formed on size and book-to-market. Furthermore, even when the EWP was less effective than the VWMP, the VWMP still showed to be underperforming when compared with the other optimized portfolios. This might indicate that investors should refrain from choosing the market portfolio. Also, portfolios that shrink the covariance matrix seems to outperformed non-shrink portfolio, this results implies that shrinking the covariance matrix can bring beneficial result.

In terms of statistical significance, the MinVPshort and the MinVPshort_shrink deliver statistically significantly higher than the EWP in the 6 portfolios formed on size and momentum and 25 portfolios formed on size and momentum with all T. Moreover, the MeanVP appears to have statistically higher Sharpe than EWP in most of the cases in the 6 portfolios formed on size and book-to-market and 25 portfolios formed on size and book-to-market. Besides the two cases above, there are no strong evidence that suggests that any other portfolio could outperform the EWP. However, this should still be regarded as a reliable proof that explains how optimized portfolios perform better than EWP from both a numerical and statistical standpoint.

5.1.1.2. Other portfolio measures

In term of standard deviation, the portfolio that has the lowest standard deviation across dataset is the MinVPshort_shrinkage. This result is very intuitive alone and is consolidated by the f-test when compared against the EWP. At this period in the Asia-pacific area, the portfolios deliver the highest mean return and capital accumulation are the MinVPshort and MinVPshort_shrink. This result is astounding as according to risk-reward relationship, the portfolio that offers the lowest standard deviation should be compensated by lower return. The statistical test furthers validate this observation as the p-value for difference in mean and f-test for differences in standard deviation of MinVPshort is lower than 0.05 across the portfolio formed in size and momentum dataset. This means that the mean and standard deviation of MinVPshort is statistically higher than the EWP. The other noticeable result is that the MeanVP and MeanVP_shrink deliver the second highest mean return and capital accumulation. This suggests that when the investors focus on monetary value alone, the MeanVP and MeanVP_shrink can be a good option.

5.1.2. Japan

In this section, only two of the most representative data table of measure and graphical representation of accumulated return will be presented. The table and figure will be followed by a detailed analysis of the full dataset.

Japan 2011-2018, T = 10 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.98	0.00	2.28
Market	0.09	0.67	0.13	0.50	0.74	0.01	1.85
Markowitz	0.15	0.35	0.13	0.52	1.17	0.19	2.71
Minimum variance	0.11	0.58	0.12	0.37	0.91	0.31	2.09
Minimum variance short	0.10	0.66	0.12	0.29	0.81	0.43	1.89
Markowitz shrinkage	0.15	0.35	0.13	0.52	1.17	0.18	2.72
Minimum variance shrinkage	0.11	0.57	0.12	0.40	0.90	0.20	2.10
Minimum variance shot shrinkage	0.09	0.68	0.12	0.38	0.75	0.21	1.83

Table 5.3. Portfolio performance of Japan 2011-2018, T = 10 years, 6 portfolios formed on size and book-to-market

Japan 2011-2018, T = 5 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.07	0.00	2.47
Market	0.09	0.73	0.13	0.51	0.74	0.56	1.85
Markowitz	0.16	0.33	0.12	0.48	1.31	0.18	3.02
Minimum variance	0.13	0.52	0.12	0.45	1.06	0.93	2.42
Minimum variance short	0.11	0.62	0.15	0.94	0.75	0.40	2.06
Markowitz shrinkage	0.17	0.31	0.12	0.42	1.36	0.10	3.12

Minimum variance shrinkage	0. 13	0.51	0.12	0.49	1.0 6	0.88	2.43
Minimum variance shot shrinkage	0. 09	0.75	0.11	0.17	0.8 2	0.32	1.86

Table 5.4. Portfolio performance of Japan 2011-2018, T = 5 years, 25 portfolios formed on size and momentum

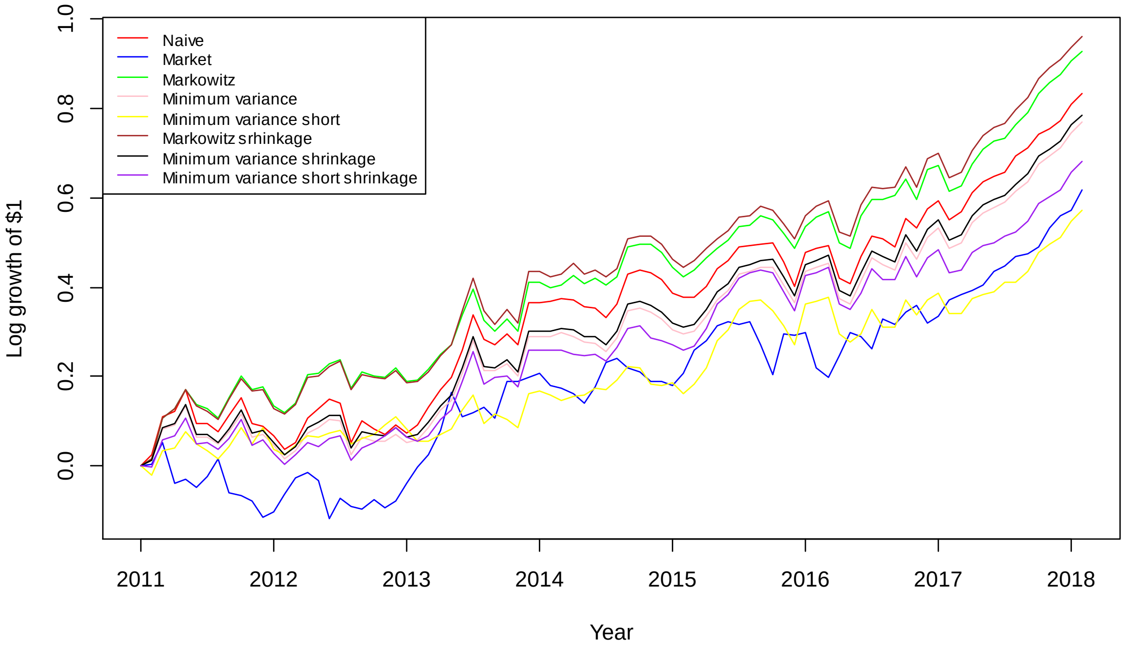

Figure 5.3. Capital accumulation of Japan 2011-2018, T = 10 years, 6 portfolios formed on size and book-to-market

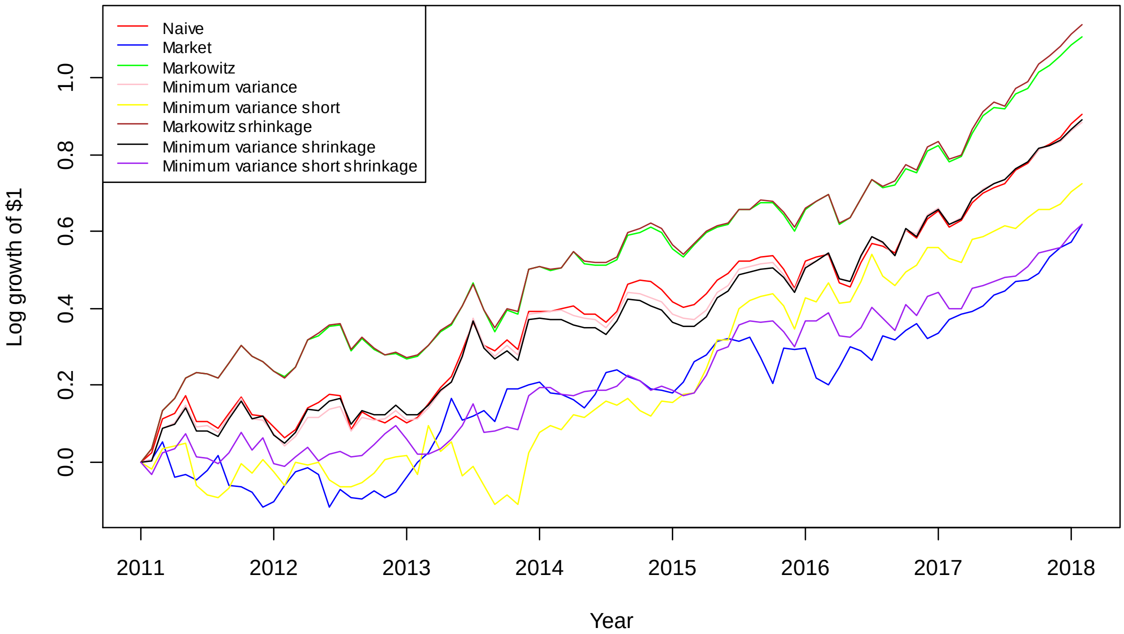

Figure 5.4. Capital accumulation of Japan 2011-2018, T = 5 years, 25 portfolios formed on size and momentum

5.1.2.1. Sharpe ratio

The result of Sharpe ratio in this geographical area also has many distinctive features, and they are very different to the in nature compare the result of Sharpe ratio from the Asia area. Firstly, the VWMP is the portfolio with the performance in term of numerical value, and there is one case where it has a statistically lower Sharpe ratio than EWP. This observation corroborates the notion that VWMP is not a good portfolio strategy to invest in from previous analysis of the Asia-pacific area. Secondly, the MeanVP_shrink and the EWP seem to outperform other strategies in most of the cases. Therefore, this dataset seems to be in favor of the naïve portfolio strategy. Nonetheless, since the MeanVP_shrink as an optimized portfolio have equally good performance compare to the EWP, one cannot conclude that naïve portfolio performs better than the optimized portfolio, especially when the Sharpe ratios are not statistically different. In fact, there is only one dataset of 6 portfolios formed on size and book-to-market with T = all that the EWP has the statistically higher Sharpe ratio. Also, as the MeanVP_shrink appears to outperform the MeanVP, this suggests a positive effect from shrinking the portfolio in the mean-variance portfolio strategy.

Thirdly, it is surprising that MinVPshort, with a superior performance, appears to have an inferior performance compared to other portfolio strategy, as its Sharpe ratio only higher than that of the VWMP in the size and momentum dataset. Nevertheless, there is little statistical evidence support this observation as there is only one case where the MinVPshort has the statistically lower Sharpe ratio than the EWP. The MinVP and the MinVPshrink, on the other hand, statistically delivers lower Sharpe ratio than the EWP. This finding suggests that setting a short constraint may not be beneficial to portfolio performance as suggested by the literature.

5.1.2.2. Other portfolio measures

Firstly, the result regarding standard deviation remains the same as the minimum variance portfolios deliver the offers the lowest standard deviation. The second noticeable result is that the MeanVP and MeanVP_shrink overall gives a higher mean return and capital accumulation. This observation validates the claim from the previous analysis that when an investor aims for return alone, MeanVP and MeanVP_shrink are lucrative options.

5.2. Portfolio performance in sub-periods

This part will discuss the performance of portfolios performance in several sub-period to further test the conclusion from the previous section.

5.2.1. Asia Pacific

The result in the sub-periods will be reported in the same manner as the main period, in which only the most representative table and graphical presentation will be shown here.

Asia Pacific 2001-2018, T=10 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Minimum variance short	0.15	0.21	0.21	0.31	0.72	0.08	11.19
Markowitz	0.15	0.21	0.23	0.72	0.68	0.00	11.15
Markowitz shrinkage	0.16	0.20	0.23	0.78	0.68	0.00	11.26
Minimum variance shrinkage	0.11	0.41	0.20	0.12	0.54	0.10	5.62

Minimum variance	0.11	0.41	0.20	0.18	0.53	0.12	5.53
Market	0.10	0.44	0.20	0.08	0.52	0.77	5.20
Minimum variance shot shrinkage	0.12	0.38	0.25	0.97	0.47	0.83	5.43
Naive	0.09		0.22		0.43		3.99

Table 5.5. Portfolio performance of Asia pacific 2001-2018, T=10 years, 25 portfolios formed on size and book-to-market

Asia pacific 2001-2018, T =10 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Minimum variance short	0.25	0.03	0.17	0.00	1.46	0.00	64.07
Minimum variance shot shrinkage	0.26	0.02	0.18	0.01	1.43	0.00	74.41
Markowitz shrinkage	0.20	0.14	0.22	0.64	0.92	0.00	24.55
Markowitz	0.19	0.18	0.22	0.66	0.86	0.00	20.16
Minimum variance shrinkage	0.15	0.32	0.20	0.18	0.76	0.00	11.61
Minimum variance	0.15	0.34	0.20	0.22	0.74	0.00	10.93
Naive	0.12		0.21		0.56		6.43
Market	0.10	0.59	0.20	0.15	0.52	0.91	5.20

Table 5.6. Portfolio performance of Asia Pacific 2001-2018, T =10 years, 25 portfolios formed on size and momentum

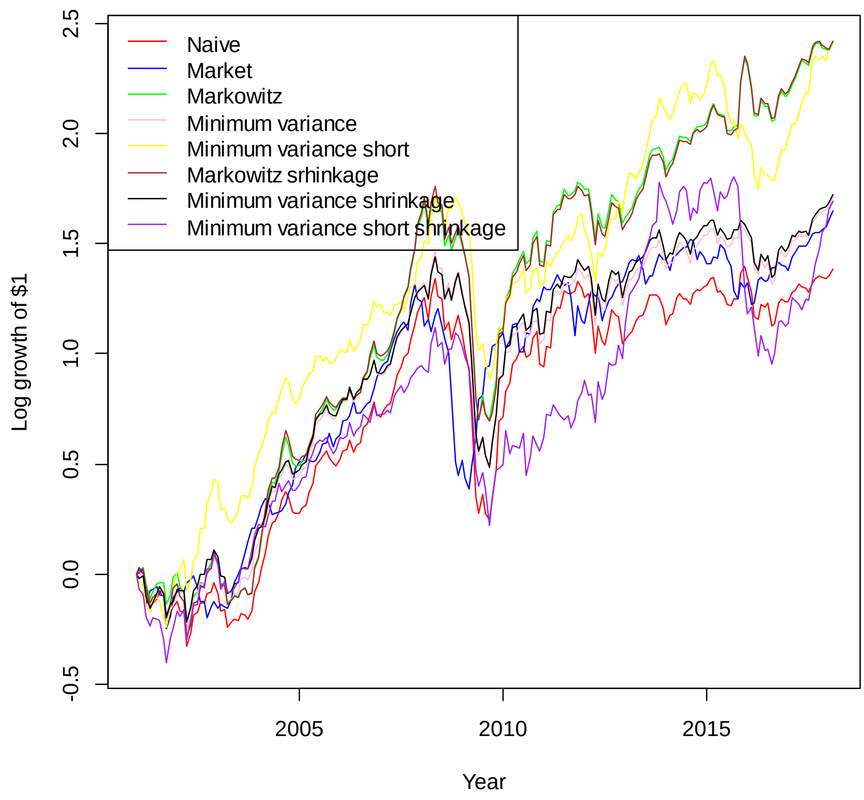

Figure 5.5. Capital accumulation of Asia Pacific 2001-2018, T=10 years, 25 portfolios formed on size and book-to-market

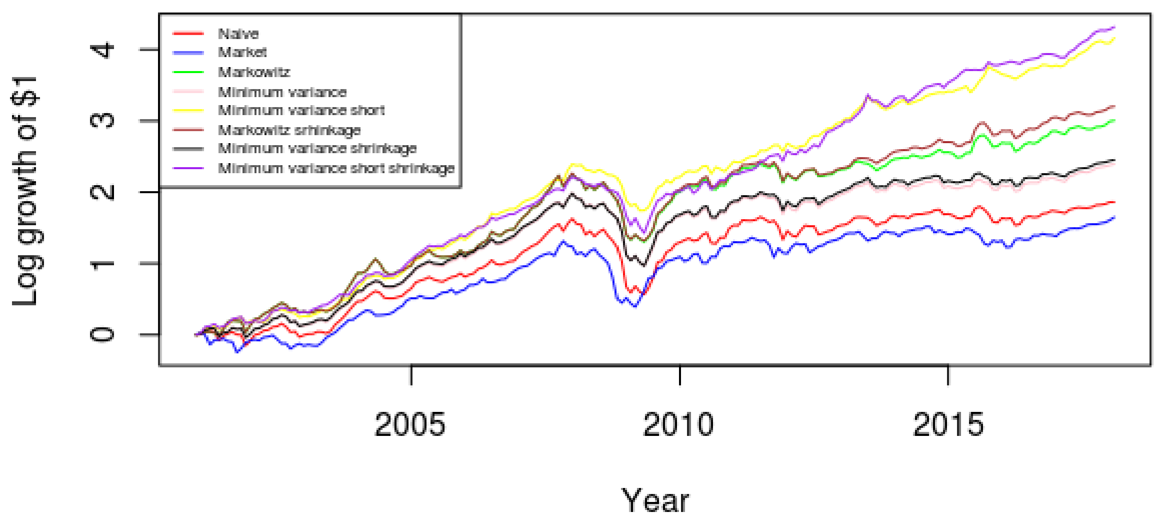

Figure 5.6. Capital accumulation of Asia Pacific 2001-2018, T =10 years, 25 portfolios formed on size and momentum

5.2.1.1. Sharpe ratio

According to the result of this period, the optimized portfolio, namely, the MinVPshort, MeanVP and MeanVP_shrink, outperformed the EWP in both numerical value and statistical test. The result is much more evident in the size and momentum dataset, as shown in table 5.6, where most of the optimized portfolio outperformed the EWP, especially the MinVPshort. On the other hand, the book-to-market portfolio shows the better performance of both MinVPshort and MeanVP, but the result is only statistically significant to the MeanVP. The VWMP continues to be the portfolio delivers the lowest Sharpe ratio numerically but not statistically.

5.2.2. Japan

The result in the sub-periods will be reported in the same manner as the main period, in which only the most representative table and graphical presentation will be shown here

Japan 1996-2018, T = all, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.03		0.19		0.17		2.23
Market	0.02	0.60	0.18	0.10	0.10	0.82	1.73
Markowitz	0.04	0.44	0.19	0.43	0.22	0.77	2.72
Minimum variance	0.02	0.56	0.17	0.07	0.14	0.69	1.96
Minimum variance short	0.03	0.55	0.20	0.84	0.12	0.83	1.77
Markowitz shrinkage	0.04	0.44	0.19	0.31	0.22	0.76	2.73
Minimum variance shrinkage	0.03	0.55	0.17	0.03	0.15	0.81	2.07
Minimum variance shot shrinkage	0.02	0.59	0.17	0.04	0.11	0.79	1.80

Table 5.7. Portfolio performance of Japan 1996-2018, T = all, 25 portfolios formed on size and book-to-market

Japan 2001-2018, T =10 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.06		0.16		0.39		2.94
Market	0.04	0.63	0.16	0.36	0.29	0.76	2.20
Markowitz	0.06	0.53	0.16	0.47	0.37	0.83	2.76
Minimum variance	0.05	0.58	0.15	0.14	0.35	0.46	2.51
Minimum variance short	0.06	0.53	0.14	0.01	0.43	0.82	2.91
Markowitz shrinkage	0.06	0.52	0.16	0.51	0.38	0.89	2.84
Minimum variance shrinkage	0.05	0.56	0.15	0.13	0.37	0.66	2.63
Minimum variance shot shrinkage	0.06	0.54	0.14	0.02	0.41	0.88	2.81

Table 5.8. Portfolio performance of Japan 2001-2018, T =10 years, 6 portfolios formed on size and momentum

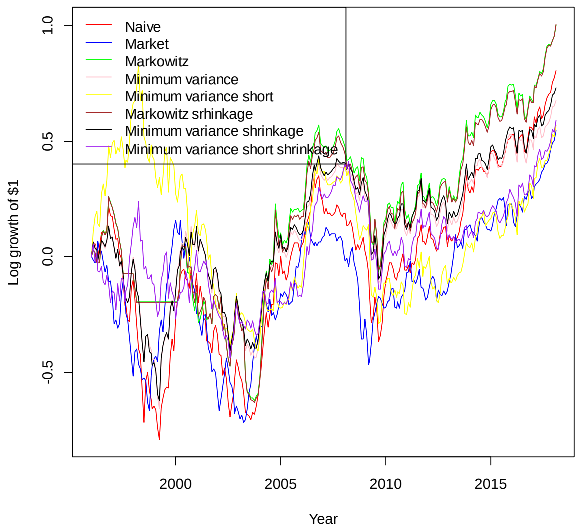

Figure 5.7. Capital accumulation of Japan 1996-2018, T = all, 25 portfolios formed on size and book-to-market

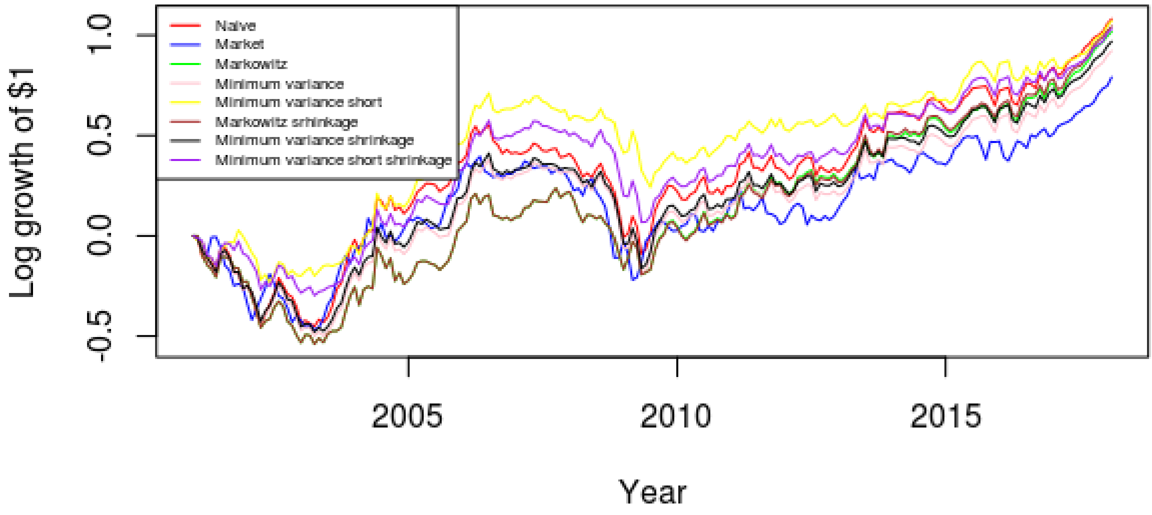

Figure 5.8. Capital accumulation of Japan 2001-2018, T =10 years, 6 portfolios formed on size and momentum

5.2.2.1. Sharpe ratio

The result of Sharpe ratio from this sub-period strengthens some of previous views from the test of the main period. Firstly, the notion that the VWMP is apparently consistently underperformed when compared to other portfolio is verified and even supported by statistical evidence in some cases. Secondly, the other striking result from this sub-period is that the MinVPshort and MinVPshort_shrink appear to be the superior portfolio strategy as they have, in most case, the highest Sharpe ratio. This finding contradicts the main period testing in this geographical area. However, there is little statistical evidence to support this claim. Nonetheless, this result can still be used as evidence to support optimized portfolio and specifically, the minimum variance strategy. The third observation is that the EWP, in most cases, is underperformed by the MinVP and MinVPshort_shrink, and when it is superior to the two strategies, it is underperformed compared to the MeanVP and MeanVP_shrink. This fact suggests that the EWP, in general, is inferior to other optimized portfolios.

5.2.2.2. Other portfolio measures

In this dataset, the result regarding portfolio volatility remains unchanged, as the minimum variance portfolio, this result is backed by a statistical test. The result of this statistical test is impactful as there is not enough statistical evidence to support the underperformance of Sharpe ratio of MinVPshort_shrink and MinVPshort, the statistical significance of the f-test is concrete proof to the superior performance of MinVPshort_shrink and MinVPshort. The conclusion of mean and capital accumulation of portfolio remains unchanged as the MeanVP and MeanVP_shrink remain the portfolio with highest mean and capital accumulation. Nonetheless, there are still some cases where the MinVPshort_shrink and MinVPshort have better performance. However, the t-test of mean does not support this conclusion, as there is no cases where the portfolios are statistically different compared to the EWP.

6. Discussion

6.1. The general performance of the implemented portfolios

Based on numerical value of Sharpe ratio alone, the minimum variance portfolios strategy delivers the best performance, especially the MinVPshort and the MinVPshort_shrink. This result align with literature on the performance of minimum-variance portfolios (Clarke, et al., 2006). However, this result is not fully backed by statistical test, as there are many cases where the Sharpe ratio of the minimum variance strategy is not statistically distinguished from the benchmark based on the Jonson and Korkie test, especially in the book-to-market dataset and in the Japan market. Moreover, the MinVPshort and the MinVPshort_shrink usually deliver better performance than the MinVP and MinVP_shrink, this suggests that setting a constraint in short might not beneficial for portfolio performance. Additionally, there is evidence showing that MinVPshort_shrink has a better performance than the MinVPshort. Hence, one can safely assume that shrinking the covariance matrix might not have a positive impact on the portfolio performance. This notion is enforced by a fact that MeanVP_shrink also outperformed MeanVP in some cases. Another interesting observation from the MinVPshort and the MinVPshort_shrink is that they also deliver the highest mean return and capital accumulation in some datasets. This result might contradict the portfolio theory that low risk should be compensate by lower return (Markowitz, 1991).

The EWP is statistically superior to the VWMP in several datasets. This result implies that holding the market portfolio is not Sharpe optimal, and an investor who is Sharpe maximizing should not choose the market portfolio. This notion contradicts the assumption of the CAPM theory that the market portfolio should be the portfolio that offers the highest Sharpe ratio.

The minimum variance portfolios, in general, have higher share ratio than the MeanVP and MeanVP_shrink. This observation suggests that estimating mean returns in optimization process might not add any advantage to the out-of-sample performance than estimating covariance matrix. Hence, when there is no optimal solution to estimate the mean, a minimum variance strategy should be used. Furthermore, when looking at the performance between optimized portfolio and naïve portfolio, the negative effect of estimation can be seen. In fact, the in-sample Sharpe ratio of optimized portfolios are reduced to the same as naïve portfolio or in some case lower. This implies that the effect of estimation error is so detrimental that, in some cases, all gain from optimization is lost. This conclusion is in line with the conclusion of DeMiguel et al. (2009). However, in some cases, the MeanVP and MeanVP_shrink delivers good performance even when they are supposed to offer a poor result due to estimation error.

Another interesting result is that the in-sample Sharpe ratio of MeanVP and MeanVP_shrink are the highest. Hence, optimization can be verified when the means and the covariance matrix are known. The implication of this result is that naïve strategy is not a recommended strategy when the mean, and the covariance matrix are accurately estimate. This conclusion is also reach by many researches (DeMiguel, et al., 2009; Kritzman, et al., 2010).

6.2. Implications of choice of dataset and out-of-sample time period

When implementing the out-of-sample testing in the size and momentum dataset in Asia pacific, it is apparent that both of the optimized strategies deliver a statistically higher Sharpe ratio than the EWP. This conclusion is unchanged even when the dataset is tested in a different time period. For instance, the MinVPshort and the MinVPshort_shrink in this data statistically outperformed the EWP with all T. However, when comparing to other dataset, the performance of MinVPshort and MinVPshort_shrink is not that superior. This shows the importance of analysing data across datasets, as some strategy can perform better in one dataset but poorly on others.

The differences of portfolio performance is not only shown in the choice of datasets but also shown in the time period of testing. There are some periods when Sharpe ratio of the portfolio strategies are higher than others. However, this arguments apply for most of the portfolios collectively, which means that when MeanVP delivers a higher Sharpe ratio, other portfolios also do. This shows that there is not time periods where a single strategy would outperformed the others, as they are always correlate with each other to a certain extend. The only period and dataset where the MinVPshort and the MinVPshort_shrink underperform compared to the EWP is the main period from 2011-2018 in the Japan dataset. Also, there is no evidence suggests that there is any period where a strategy consistently and surprisingly returns a statistically significantly higher Sharpe ratio when compared to the EWP.

6.3. Explanation for performance of minimum variance portfolio

Under the common assumption that lower risk must be compromised by lower expected the return, it is astonishing that the minimum variance portfolio appears to offers the higher Sharpe ratio compare to other strategy. In the comparison between the mean-variance strategy and the minimum variance, a reasonable explanation for the superior performance of minimum variance strategy can be the estimation error of estimating expected return of mean-variance strategy is detrimental enough to decrease all the possible gain from optimization. This result is very clear when the in-sample ratios of MeanVP and MeanVP_shrink are compare to the out-of-sample Sharpe ratios, and is align with some research on out-of-sample testing (DeMiguel, et al., 2009; Kritzman, et al., 2010). Furthermore, as the estimation of covariance are more likely to be correct with the help of shrinking method, the out-of-sample performance of MinVPshort_shrink is less affected by estimation error and receive more gain from optimization process (Ledoit & Wolf, 2004).

Another explanation for the superior performance of minimum variance portfolio is that investors with leverage constraints and leverage aversion will overweight high beta securities in their portfolios. This overweighting behaviour will make the riskier assets to be overpriced and become more volatile, while the opposite happens for the less risky assets as they become undervalued and offers higher risk-adjusted return (Blitz & Vliet, 2007).

7. Conclusion

7.1. Research process

Through review in the academic literature on the performance of different asset allocation strategies, there seems to be a debate on whether or not optimized portfolios perform better than simpler strategy such as naïve portfolio or market portfolio? To answer this question, this thesis synthesize the methodology of two most important thesis representing two opposing view on this subject from DeMiguel, et al. (2009) and Kritzman, et al. (2010) to test the performance of equity market in Asia, since there is little or no research that test portfolio performance in this geographical area. Moreover, as the main drawback of optimized portfolio has been noticed as estimation error, this thesis tries to shrink the covariance matrix to reduce estimation error according to Ledoit & Wolf (2004). The performance measures of these portfolio are mean with t-test for statistical significant, standard deviation with f-test for statistical significant, Sharpe ratio with Jobson & Korkie test for statistical significant and capital accumulation.

7.2. Main findings and implication for international business

Overall, in terms of numerical value alone, the MinVPshort and the MinVPshort_shrink delivers the highest risk-adjusted return. However, this results does not hold statistically, as both the above portfolio and other optimized portfolio does not consistent statistically beat the EWP benchmark. Therefore, it also suggests that the EWP does consistently outperform optimized portfolio. Hence, it can be concluded that the question of whether or not optimized portfolios perform better than simpler asset allocation strategy does not have a definite answer. Furthermore, across dataset the optimized portfolios using shrinkage estimator for covariance matrix delivers higher Sharpe ratio. This suggests that when one constructs an optimized portfolio, one should use shrink the covariance matrix for better portfolio performance. All in all, despite the fact that MinVPshort and MinVPshort_shrink do not consistently outperform the benchmark when statistical significant is concern, from a practitioner's point of view, the overall superior numerical value of Sharpe ratio alone justifies investing in those two portfolios.

Other apparent result is the VWMP underperforms compared to the other portfolio strategies during many out-of-sample periods and across most of the datasets. This result is also statistically distinguishable, which suggests that investing in the market portfolio is suboptimal for investors.

7.3. Limitation

The first limitation of this study is the time-frame for out-of-sample testing is only 7 years, which is too short for long-term decision and conclusion. However, this is due to the limitation of the data and not the methodology. Secondly, this study lacks the geographical diversity, as there are only two countries from this geographical area, which can potentially lead to bias.

7.4. Suggestions for further research

The future research should consider other asset allocation strategy such as the Black-Litterman model, which can incorporate investors view about future movement of the assets into the construction of the portfolio. This model, therefore, can help an investor have better portfolio performance if the predictions are correct.

The other improvement that can be made is not consider standard deviation as a measure of risk as it poses several limitations and does not reflect a rational view on risk. From a rational viewpoint, any investor would have a certain goal for their return, and only the return that bellows that goal should be consider risk, and the return exceed that goal be consider as chance for gain. The standard deviation, however, treat both of this downward and upward deviation as risk.

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Appendix 1: Portfolio performance of main period in Japan

Asia pacific 2011-2018, T=5 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05	0.00	0.16	0.00	0.28	0.00	1.27
Market	0.06	0.43	0.16	0.46	0.37	0.29	1.41
Markowitz	0.07	0.41	0.16	0.48	0.41	0.31	1.47
Minimum variance	0.08	0.34	0.16	0.49	0.51	0.03	1.66
Minimum variance short	0.19	0.05	0.18	0.75	1.10	0.01	3.58
Markowitz shrinkage	0.07	0.39	0.16	0.47	0.43	0.24	1.51
Minimum variance shrinkage	0.08	0.34	0.17	0.56	0.49	0.07	1.64
Minimum variance shot shrinkage	0.15	0.13	0.19	0.91	0.79	0.04	2.60

Asia pacific 2011-2018, T=10 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05	0.00	0.16	0.00	0.28	0.00	1.27
Market	0.06	0.43	0.16	0.46	0.37	0.29	1.41

Markowitz	0.05	0.47	0.16	0.32	0.34	0.46	1.35
Minimum variance	0.07	0.40	0.16	0.41	0.42	0.33	1.49
Minimum variance short	0.15	0.14	0.18	0.72	0.83	0.07	2.54
Markowitz shrinkage	0.06	0.46	0.16	0.33	0.35	0.45	1.37
Minimum variance shrinkage	0.07	0.40	0.16	0.41	0.42	0.33	1.49
Minimum variance shot shrinkage	0.13	0.19	0.20	0.97	0.65	0.21	2.23

Asia pacific 2011-2018, T=20 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05	0.00	0.16	0.00	0.28	0.00	1.27
Market	0.06	0.43	0.16	0.46	0.37	0.29	1.41
Markowitz	0.07	0.40	0.17	0.50	0.42	0.08	1.50
Minimum variance	0.06	0.44	0.16	0.47	0.36	0.51	1.39
Minimum variance short	0.09	0.30	0.17	0.51	0.55	0.23	1.76
Markowitz shrinkage	0.07	0.39	0.17	0.56	0.43	0.08	1.52
Minimum variance shrinkage	0.06	0.44	0.16	0.45	0.37	0.48	1.41
Minimum variance shot shrinkage	0.07	0.38	0.18	0.74	0.42	0.54	1.53

Asia pacific 2011-2018, T=all, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05	0.00	0.16	0.00	0.28	0.00	1.27

Market	0.06	0.43	0.16	0.46	0.37	0.29	1.41
Markowitz	0.08	0.36	0.17	0.54	0.46	0.03	1.58
Minimum variance	0.06	0.45	0.16	0.47	0.35	0.53	1.38
Minimum variance short	0.08	0.33	0.16	0.43	0.52	0.26	1.67
Markowitz shrinkage	0.08	0.36	0.17	0.58	0.46	0.03	1.59
Minimum variance shrinkage	0.06	0.44	0.16	0.46	0.36	0.52	1.39
Minimum variance shot shrinkage	0.07	0.40	0.17	0.64	0.41	0.54	1.49

h							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08	0.00	0.18	0.00	0.42	0.00	1.55
Market	0.06	0.57	0.16	0.16	0.37	0.91	1.41
Markowitz	0.13	0.30	0.18	0.43	0.72	0.04	2.23
Minimum variance	0.10	0.42	0.18	0.44	0.54	0.30	1.78
Minimum variance short	0.18	0.16	0.22	0.95	0.84	0.29	3.09
Markowitz shrinkage	0.12	0.31	0.18	0.39	0.70	0.06	2.18
Minimum variance shrinkage	0.10	0.39	0.18	0.50	0.57	0.21	1.87
Minimum variance shot shrinkage	0.20	0.13	0.23	0.99	0.86	0.25	3.43

Asia pacific 2011-2018, T=10 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative

Naive	0.08	0.00	0.18	0.00	0.42	0.00	1.55
Market	0.06	0.57	0.16	0.16	0.37	0.91	1.41
Markowitz	0.16	0.21	0.19	0.71	0.82	0.00	2.71
Minimum variance	0.11	0.38	0.18	0.39	0.60	0.20	1.91
Minimum variance short	0.17	0.17	0.20	0.77	0.88	0.14	3.00
Markowitz shrinkage	0.16	0.20	0.20	0.77	0.83	0.01	2.79
Minimum variance shrinkage	0.10	0.39	0.17	0.33	0.59	0.23	1.88
Minimum variance shot shrinkage	0.19	0.18	0.27	1.00	0.69	0.53	2.92

5							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08	0.00	0.18	0.00	0.42	0.00	1.55
Market	0.06	0.57	0.16	0.16	0.37	0.91	1.41
Markowitz	0.15	0.23	0.19	0.68	0.79	0.02	2.61
Minimum variance	0.10	0.40	0.18	0.47	0.57	0.16	1.86
Minimum variance short	0.17	0.18	0.19	0.63	0.89	0.04	2.93
Markowitz shrinkage	0.17	0.19	0.20	0.78	0.84	0.02	2.84
Minimum variance shrinkage	0.10	0.41	0.18	0.44	0.55	0.34	1.81
Minimum variance shot shrinkage	0.12	0.33	0.20	0.74	0.62	0.48	2.09

Asia pacific 2011-2018, T=all, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative

Naive	0.08	0.00	0.18	0.00	0.42	0.00	1.55
Market	0.06	0.57	0.16	0.16	0.37	0.91	1.41
Markowitz	0.15	0.22	0.19	0.71	0.80	0.01	2.64
Minimum variance	0.11	0.38	0.18	0.47	0.59	0.06	1.90
Minimum variance short	0.16	0.19	0.18	0.42	0.91	0.02	2.84
Markowitz shrinkage	0.17	0.19	0.20	0.78	0.84	0.02	2.84
Minimum variance shrinkage	0.10	0.41	0.18	0.44	0.55	0.32	1.82
Minimum variance shot shrinkage	0.12	0.32	0.18	0.56	0.67	0.32	2.15

Asia pacific 2011-2018, T = 5 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05	0.00	0.17	0.00	0.30	0.00	1.31
Market	0.06	0.46	0.16	0.40	0.37	0.91	1.41
Markowitz	0.09	0.36	0.21	0.98	0.41	0.49	1.60
Minimum variance	0.09	0.31	0.16	0.29	0.60	0.01	1.80
Minimum variance short	0.25	0.01	0.14	0.04	1.79	0.00	5.44
Markowitz shrinkage	0.09	0.36	0.21	0.97	0.42	0.44	1.62
Minimum variance shrinkage	0.09	0.31	0.16	0.27	0.60	0.00	1.81
Minimum variance shot shrinkage	0.23	0.02	0.17	0.48	1.37	0.00	4.58

Asia pacific 2011-2018, T = 10 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative

Naive	0.05	0.00	0.17	0.00	0.30	0.00	1.31
Market	0.06	0.46	0.16	0.40	0.37	0.91	1.41
Markowitz	0.11	0.28	0.19	0.88	0.56	0.09	1.89
Minimum variance	0.07	0.40	0.16	0.31	0.46	0.22	1.55
Minimum variance short	0.25	0.01	0.14	0.04	1.85	0.00	5.63
Markowitz shrinkage	0.12	0.25	0.19	0.89	0.60	0.05	2.00
Minimum variance shrinkage	0.08	0.38	0.16	0.29	0.49	0.10	1.60
Minimum variance shot shrinkage	0.19	0.06	0.17	0.49	1.13	0.00	3.45

Asia pacific 2011-2018, T = 20 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05	0.00	0.17	0.00	0.30	0.00	1.31
Market	0.06	0.46	0.16	0.40	0.37	0.91	1.41
Markowitz	0.10	0.29	0.18	0.74	0.57	0.02	1.86
Minimum variance	0.08	0.36	0.15	0.21	0.53	0.00	1.65
Minimum variance short	0.22	0.02	0.12	0.00	1.81	0.00	4.54
Markowitz shrinkage	0.10	0.28	0.18	0.75	0.57	0.02	1.87
Minimum variance shrinkage	0.08	0.37	0.15	0.23	0.52	0.01	1.64
Minimum variance shot shrinkage	0.17	0.07	0.14	0.04	1.25	0.00	3.17

Asia pacific 2011-2018, T = all, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative

Naive	0.05	0.00	0.17	0.00	0.30	0.00	1.31
Market	0.06	0.46	0.16	0.40	0.37	0.91	1.41
Markowitz	0.11	0.26	0.18	0.78	0.62	0.01	1.98
Minimum variance	0.09	0.34	0.15	0.19	0.56	0.00	1.70
Minimum variance short	0.23	0.01	0.12	0.00	1.89	0.00	4.75
Markowitz shrinkage	0.11	0.26	0.18	0.78	0.60	0.02	1.95
Minimum variance shrinkage	0.08	0.35	0.15	0.20	0.54	0.00	1.67
Minimum variance shot shrinkage	0.18	0.06	0.13	0.02	1.32	0.00	3.29

Asia pacific 2011-2018, T = 5 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.06	0.00	0.17	0.00	0.35	0.00	1.38
Market	0.06	0.49	0.16	0.45	0.37	0.97	1.41
Markowitz	0.11	0.32	0.20	0.97	0.51	0.32	1.84
Minimum variance	0.10	0.30	0.16	0.30	0.66	0.02	1.92
Minimum variance short	0.19	0.07	0.17	0.55	1.11	0.09	3.41
Markowitz shrinkage	0.10	0.33	0.20	0.96	0.50	0.33	1.78
Minimum variance shrinkage	0.11	0.28	0.15	0.26	0.70	0.01	1.99
Minimum variance shot shrinkage	0.34	0.00	0.16	0.38	2.10	0.00	9.70

Asia pacific 2011-2018, T = 10 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative

Naive	0.06	0.00	0.17	0.00	0.35	0.00	1.38
Market	0.06	0.49	0.16	0.45	0.37	0.97	1.41
Markowitz	0.12	0.25	0.17	0.64	0.69	0.01	2.12
Minimum variance	0.09	0.37	0.16	0.36	0.55	0.13	1.71
Minimum variance short	0.26	0.01	0.15	0.23	1.68	0.00	5.64
Markowitz shrinkage	0.14	0.18	0.17	0.59	0.83	0.00	2.48
Minimum variance shrinkage	0.09	0.35	0.15	0.26	0.60	0.06	1.78
Minimum variance shot shrinkage	0.31	0.00	0.16	0.44	1.88	0.00	7.83

Asia pacific 2011-2018, T = 20 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.06	0.00	0.17	0.00	0.35	0.00	1.38
Market	0.06	0.49	0.16	0.45	0.37	0.97	1.41
Markowitz	0.14	0.18	0.17	0.58	0.82	0.00	2.44
Minimum variance	0.10	0.30	0.15	0.26	0.66	0.00	1.91
Minimum variance short	0.25	0.01	0.13	0.01	1.93	0.00	5.43
Markowitz shrinkage	0.14	0.18	0.17	0.58	0.82	0.00	2.44
Minimum variance shrinkage	0.10	0.32	0.15	0.25	0.63	0.01	1.84
Minimum variance shot shrinkage	0.25	0.01	0.12	0.00	2.07	0.00	5.65

Asia pacific 2011-2018, T = all, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative

Naive	0.06	0.00	0.17	0.00	0.35	0.00	1.38
Market	0.06	0.49	0.16	0.45	0.37	0.97	1.41
Markowitz	0.14	0.18	0.17	0.58	0.82	0.00	2.44
Minimum variance	0.10	0.31	0.15	0.23	0.66	0.00	1.90
Minimum variance short	0.24	0.01	0.12	0.00	1.94	0.00	5.21
Markowitz shrinkage	0.14	0.18	0.17	0.58	0.82	0.00	2.44
Minimum variance shrinkage	0.10	0.32	0.15	0.22	0.65	0.00	1.87
Minimum variance shot shrinkage	0.25	0.01	0.12	0.00	2.08	0.00	5.55

Appendix 2: Portfolio performance of main period in Japan

Japan 2011-2018, T = 5 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.98	0.00	2.28
Market	0.09	0.67	0.13	0.50	0.74	0.01	1.85
Markowitz	0.13	0.45	0.13	0.67	1.00	0.90	2.41
Minimum variance	0.11	0.56	0.13	0.50	0.90	0.31	2.12
Minimum variance short	0.08	0.72	0.12	0.44	0.67	0.30	1.73
Markowitz shrinkage	0.14	0.42	0.13	0.62	1.05	0.66	2.50
Minimum variance shrinkage	0.11	0.56	0.12	0.47	0.91	0.41	2.13
Minimum variance shot shrinkage	0.10	0.64	0.12	0.46	0.79	0.29	1.93

Japan 2011-2018, T = 10 years, 6 portfolios formed on size and book-to-market

	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.98	0.00	2.28
Market	0.09	0.67	0.13	0.50	0.74	0.01	1.85
Markowitz	0.15	0.35	0.13	0.52	1.17	0.19	2.71
Minimum variance	0.11	0.58	0.12	0.37	0.91	0.31	2.09
Minimum variance short	0.10	0.66	0.12	0.29	0.81	0.43	1.89
Markowitz shrinkage	0.15	0.35	0.13	0.52	1.17	0.18	2.72
Minimum variance shrinkage	0.11	0.57	0.12	0.40	0.90	0.20	2.10
Minimum variance shot shrinkage	0.09	0.68	0.12	0.38	0.75	0.21	1.83

Japan 2011-2018, T = 20 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.98	0.00	2.28
Market	0.09	0.67	0.13	0.50	0.74	0.01	1.85
Markowitz	0.11	0.56	0.14	0.90	0.77	0.15	2.07
Minimum variance	0.09	0.69	0.12	0.47	0.71	0.03	1.80
Minimum variance short	0.07	0.80	0.12	0.33	0.56	0.06	1.55
Markowitz shrinkage	0.11	0.55	0.14	0.84	0.81	0.21	2.11
Minimum variance shrinkage	0.09	0.68	0.12	0.45	0.74	0.03	1.84
Minimum variance shot shrinkage	0.08	0.76	0.12	0.35	0.63	0.07	1.64

Japan 2011-2018, T = all, 6 portfolios formed on size and book-to-market

	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.98	0.00	2.28
Market	0.09	0.67	0.13	0.50	0.74	0.01	1.85
Markowitz	0.10	0.61	0.15	0.97	0.66	0.07	1.92
Minimum variance	0.09	0.71	0.13	0.49	0.69	0.02	1.76
Minimum variance short	0.06	0.82	0.12	0.46	0.50	0.04	1.48
Markowitz shrinkage	0.10	0.60	0.15	0.93	0.70	0.08	1.95
Minimum variance shrinkage	0.09	0.69	0.13	0.49	0.71	0.02	1.79
Minimum variance shot shrinkage	0.07	0.79	0.12	0.41	0.57	0.05	1.58

Japan 2011-2018, T = 5 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.00	0.00	2.37
Market	0.09	0.70	0.13	0.43	0.74	0.63	1.85
Markowitz	0.13	0.47	0.15	0.90	0.91	0.56	2.42
Minimum variance	0.11	0.60	0.13	0.54	0.86	0.14	2.09
Minimum variance short	0.17	0.27	0.16	0.98	1.10	0.76	3.17
Markowitz shrinkage	0.15	0.41	0.15	0.89	0.99	0.93	2.61
Minimum variance shrinkage	0.12	0.57	0.13	0.41	0.93	0.44	2.17
Minimum variance shot shrinkage	0.08	0.75	0.14	0.78	0.58	0.18	1.68

Japan 2011-2018, T = 10 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.00	0.00	2.37
Market	0.09	0.70	0.13	0.43	0.74	0.63	1.85
Markowitz	0.15	0.37	0.14	0.75	1.11	0.48	2.78
Minimum variance	0.10	0.64	0.13	0.61	0.78	0.04	1.98
Minimum variance short	0.12	0.57	0.14	0.68	0.86	0.64	2.16
Markowitz shrinkage	0.15	0.36	0.14	0.74	1.11	0.45	2.79
Minimum variance shrinkage	0.11	0.60	0.13	0.53	0.86	0.14	2.09
Minimum variance shot shrinkage	0.09	0.70	0.13	0.48	0.73	0.29	1.84

Japan 2011-2018, T = 20 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.00	0.00	2.37
Market	0.09	0.70	0.13	0.43	0.74	0.63	1.85
Markowitz	0.13	0.47	0.14	0.72	0.98	0.87	2.44
Minimum variance	0.10	0.67	0.13	0.62	0.74	0.05	1.90
Minimum variance short	0.11	0.58	0.14	0.80	0.81	0.48	2.12
Markowitz shrinkage	0.14	0.46	0.13	0.65	1.01	0.96	2.46
Minimum variance shrinkage	0.10	0.66	0.13	0.59	0.76	0.06	1.93
Minimum variance shot shrinkage	0.09	0.70	0.13	0.50	0.72	0.25	1.83

Japan 2011-2018, T = all, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.00	0.00	2.37
Market	0.09	0.70	0.13	0.43	0.74	0.63	1.85
Markowitz	0.13	0.51	0.14	0.82	0.89	0.43	2.30
Minimum variance	0.09	0.71	0.13	0.67	0.67	0.02	1.79
Minimum variance short	0.13	0.51	0.15	0.94	0.83	0.54	2.27
Markowitz shrinkage	0.13	0.48	0.14	0.74	0.96	0.69	2.39
Minimum variance shrinkage	0.09	0.69	0.13	0.65	0.70	0.03	1.84
Minimum variance shot shrinkage	0.10	0.68	0.13	0.59	0.73	0.30	1.89

Japan 2011-2018, T = 5 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.99	0.00	2.30
Market	0.09	0.68	0.13	0.51	0.74	0.66	1.85
Markowitz	0.13	0.50	0.12	0.45	1.01	0.91	2.32
Minimum variance	0.13	0.49	0.12	0.29	1.06	0.29	2.34
Minimum variance short	0.11	0.61	0.12	0.22	0.92	0.77	2.04
Markowitz shrinkage	0.13	0.45	0.12	0.37	1.09	0.51	2.44
Minimum variance shrinkage	0.13	0.49	0.12	0.33	1.05	0.35	2.34
Minimum variance shot shrinkage	0.12	0.51	0.12	0.48	0.99	0.99	2.29

Japan 2011-2018, T = 10 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.99	0.00	2.30
Market	0.09	0.68	0.13	0.51	0.74	0.66	1.85
Markowitz	0.14	0.42	0.12	0.33	1.14	0.30	2.53
Minimum variance	0.11	0.56	0.12	0.26	0.98	0.85	2.16
Minimum variance short	0.09	0.74	0.11	0.07	0.80	0.45	1.77
Markowitz shrinkage	0.14	0.39	0.12	0.44	1.15	0.26	2.61
Minimum variance shrinkage	0.12	0.55	0.12	0.26	1.00	0.91	2.19
Minimum variance shot shrinkage	0.10	0.64	0.11	0.16	0.90	0.56	1.98

Japan 2011-2018, T = 20 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.99	0.00	2.30
Market	0.09	0.68	0.13	0.51	0.74	0.66	1.85
Markowitz	0.13	0.44	0.14	0.78	0.99	0.99	2.45
Minimum variance	0.10	0.63	0.12	0.35	0.85	0.11	1.99
Minimum variance short	0.08	0.74	0.11	0.12	0.76	0.39	1.76
Markowitz shrinkage	0.14	0.43	0.13	0.73	1.02	0.83	2.50
Minimum variance shrinkage	0.11	0.61	0.12	0.32	0.89	0.20	2.04
Minimum variance shot shrinkage	0.09	0.70	0.11	0.16	0.81	0.37	1.84

Japan 2011-2018, T = all, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12	0.00	0.13	0.00	0.99	0.00	2.30
Market	0.09	0.68	0.13	0.51	0.74	0.66	1.85
Markowitz	0.12	0.50	0.13	0.53	0.98	0.91	2.29
Minimum variance	0.10	0.65	0.12	0.41	0.81	0.07	1.94
Minimum variance short	0.08	0.74	0.11	0.18	0.72	0.32	1.73
Markowitz shrinkage	0.13	0.47	0.12	0.45	1.04	0.67	2.38
Minimum variance shrinkage	0.10	0.63	0.12	0.35	0.85	0.11	1.99
Minimum variance shot shrinkage	0.09	0.71	0.11	0.21	0.77	0.29	1.81

Japan 2011-2018, T = 5 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.07	0.00	2.47
Market	0.09	0.73	0.13	0.51	0.74	0.56	1.85
Markowitz	0.16	0.33	0.12	0.48	1.31	0.18	3.02
Minimum variance	0.13	0.52	0.12	0.45	1.06	0.93	2.42
Minimum variance short	0.11	0.62	0.15	0.94	0.75	0.40	2.06
Markowitz shrinkage	0.17	0.31	0.12	0.42	1.36	0.10	3.12
Minimum variance shrinkage	0.13	0.51	0.12	0.49	1.06	0.88	2.43
Minimum variance shot shrinkage	0.09	0.75	0.11	0.17	0.82	0.32	1.86

Japan 2011-2018, T = 10 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.07	0.00	2.47
Market	0.09	0.73	0.13	0.51	0.74	0.56	1.85
Markowitz	0.18	0.27	0.13	0.72	1.32	0.14	3.28
Minimum variance	0.11	0.62	0.12	0.35	0.95	0.17	2.16
Minimum variance short	0.09	0.72	0.13	0.51	0.75	0.25	1.87
Markowitz shrinkage	0.17	0.28	0.13	0.67	1.33	0.11	3.26
Minimum variance shrinkage	0.12	0.62	0.12	0.38	0.95	0.12	2.16
Minimum variance shot shrinkage	0.10	0.72	0.11	0.16	0.86	0.42	1.92

Japan 2011-2018, T = 20 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.07	0.00	2.47
Market	0.09	0.73	0.13	0.51	0.74	0.56	1.85
Markowitz	0.18	0.24	0.14	0.85	1.32	0.18	3.47
Minimum variance	0.11	0.66	0.12	0.42	0.87	0.07	2.03
Minimum variance short	0.10	0.70	0.13	0.62	0.76	0.27	1.92
Markowitz shrinkage	0.18	0.25	0.14	0.84	1.31	0.20	3.42
Minimum variance shrinkage	0.11	0.66	0.12	0.40	0.88	0.06	2.05
Minimum variance shot shrinkage	0.10	0.73	0.12	0.25	0.82	0.34	1.89

Japan 2011-2018, T = all, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.13	0.00	0.13	0.00	1.07	0.00	2.47
Market	0.09	0.73	0.13	0.51	0.74	0.56	1.85
Markowitz	0.18	0.24	0.14	0.84	1.31	0.18	3.45
Minimum variance	0.10	0.67	0.12	0.47	0.84	0.06	2.00
Minimum variance short	0.11	0.63	0.14	0.81	0.80	0.35	2.06
Markowitz shrinkage	0.18	0.25	0.14	0.80	1.31	0.17	3.39
Minimum variance shrinkage	0.10	0.67	0.12	0.43	0.85	0.05	2.01
Minimum variance shot shrinkage	0.10	0.70	0.12	0.36	0.83	0.38	1.95

Appendix 3: Portfolio performance of sub period in Asia Pacific

Asia pacific 1996-2018, T =5 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.22		0.35		5.06
Market	0.08	0.48	0.21	0.17	0.39	0.34	5.87
Markowitz	0.09	0.45	0.23	0.68	0.38	0.68	5.95
Minimum variance	0.07	0.54	0.20	0.09	0.34	0.96	4.76
Minimum variance short	0.10	0.37	0.21	0.15	0.47	0.39	8.78
Markowitz shrinkage	0.09	0.44	0.23	0.63	0.38	0.64	6.06
Minimum variance shrinkage	0.07	0.52	0.20	0.10	0.36	0.87	5.07
Minimum variance shot shrinkage	0.06	0.60	0.21	0.26	0.29	0.68	3.76

Asia pacific 2001-2018, T=10 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.10		0.21		0.48		4.78
Market	0.10	0.49	0.20	0.19	0.52	0.35	5.20
Markowitz	0.12	0.40	0.22	0.65	0.55	0.30	6.40
Minimum variance	0.12	0.42	0.20	0.25	0.58	0.20	6.29
Minimum variance short	0.14	0.27	0.20	0.25	0.72	0.10	10.24
Markowitz shrinkage	0.12	0.41	0.21	0.62	0.55	0.39	6.18
Minimum variance shrinkage	0.11	0.46	0.20	0.16	0.56	0.32	5.74
Minimum variance shot shrinkage	0.11	0.44	0.21	0.63	0.52	0.80	5.70

Asia pacific 2011-2018, T=20 years, 6 portfolios formed on size and book-to-market, using shrinkage							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.05		0.16		0.28		1.27
Market	0.06	0.43	0.16	0.46	0.37	0.29	1.41
Markowitz	0.07	0.40	0.17	0.50	0.42	0.08	1.50
Minimum variance	0.06	0.44	0.16	0.47	0.36	0.51	1.39
Minimum variance short	0.09	0.30	0.17	0.51	0.55	0.23	1.76
Markowitz shrinkage	0.07	0.39	0.17	0.56	0.43	0.08	1.52
Minimum variance shrinkage	0.06	0.44	0.16	0.45	0.37	0.48	1.41

Minimum variance shot shrinkage	0.07	0.38	0.18	0.74	0.42	0.54	1.53

Asia pacific 1996-2018, T= all, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.22		0.35		5.06
Market	0.08	0.48	0.21	0.17	0.39	0.34	5.87
Markowitz	0.11	0.31	0.25	0.97	0.45	0.15	9.74
Minimum variance	0.07	0.53	0.21	0.24	0.34	0.89	4.76
Minimum variance short	0.07	0.55	0.21	0.23	0.33	0.86	4.51
Markowitz shrinkage	0.11	0.31	0.25	0.97	0.45	0.16	9.74
Minimum variance shrinkage	0.07	0.54	0.21	0.17	0.34	0.89	4.72
Minimum variance shot shrinkage	0.04	0.71	0.21	0.31	0.19	0.19	2.38

Asia pacific 1991-2018, T=5 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.23		0.35		5.09
Market	0.08	0.48	0.21	0.10	0.39	0.89	5.87
Markowitz	0.12	0.28	0.24	0.87	0.49	0.07	11.60
Minimum variance	0.09	0.41	0.21	0.09	0.45	0.12	7.72
Minimum variance short	0.14	0.19	0.22	0.46	0.61	0.17	18.76
Markowitz shrinkage	0.12	0.29	0.24	0.86	0.49	0.09	11.26

Minimum variance shrinkage	0.10	0.37	0.21	0.06	0.48	0.05	8.94
Minimum variance shot shrinkage	0.11	0.33	0.22	0.33	0.49	0.46	10.21

Asia pacific 2001-2018, T=10 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.09		0.22		0.43		3.99
Market	0.10	0.44	0.20	0.08	0.52	0.77	5.20
Markowitz	0.15	0.21	0.23	0.72	0.68	0.00	11.15
Minimum variance	0.11	0.41	0.20	0.18	0.53	0.12	5.53
Minimum variance short	0.15	0.21	0.21	0.31	0.72	0.08	11.19
Markowitz shrinkage	0.16	0.20	0.23	0.78	0.68	0.00	11.26
Minimum variance shrinkage	0.11	0.41	0.20	0.12	0.54	0.10	5.62
Minimum variance shot shrinkage	0.12	0.38	0.25	0.97	0.47	0.83	5.43

Asia pacific 2011-2018, T=20 years, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.18		0.42		1.55
Market	0.06	0.57	0.16	0.16	0.37	0.91	1.41
Markowitz	0.15	0.23	0.19	0.68	0.79	0.02	2.61
Minimum variance	0.10	0.40	0.18	0.47	0.57	0.16	1.86
Minimum variance short	0.17	0.18	0.19	0.63	0.89	0.04	2.93

Markowitz shrinkage	0.17	0.19	0.20	0.78	0.84	0.02	2.84
Minimum variance shrinkage	0.10	0.41	0.18	0.44	0.55	0.34	1.81
Minimum variance shot shrinkage	0.12	0.33	0.20	0.74	0.62	0.48	2.09

Asia pacific 1996-2018, T= all, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.23		0.35		5.09
Market	0.08	0.48	0.21	0.10	0.39	0.89	5.87
Markowitz	0.13	0.22	0.24	0.80	0.55	0.01	15.33
Minimum variance	0.09	0.44	0.22	0.25	0.41	0.15	6.69
Minimum variance short	0.09	0.43	0.21	0.08	0.44	0.50	7.26
Markowitz shrinkage	0.14	0.18	0.24	0.88	0.59	0.00	19.83
Minimum variance shrinkage	0.08	0.47	0.21	0.17	0.39	0.41	6.12
Minimum variance shot shrinkage	0.08	0.49	0.21	0.10	0.39	0.78	5.81

Asia pacific 1996-2018, T =5 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.22		0.37		5.79
Market	0.08	0.52	0.21	0.13	0.39	0.96	5.87
Markowitz	0.08	0.50	0.23	0.68	0.36	0.91	5.54
Minimum variance	0.11	0.37	0.21	0.08	0.51	0.02	10.27
Minimum variance short	0.15	0.14	0.19	0.00	0.80	0.01	29.68

Markowitz shrinkage	0.08	0.50	0.23	0.69	0.36	0.91	5.54
Minimum variance shrinkage	0.10	0.38	0.21	0.09	0.50	0.04	9.74
Minimum variance shot shrinkage	0.15	0.17	0.21	0.15	0.70	0.03	24.44

Asia pacific 2001-2018, T =10 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.11		0.21		0.51		5.26
Market	0.10	0.52	0.20	0.18	0.52	0.96	5.20
Markowitz	0.14	0.30	0.21	0.61	0.67	0.04	9.77
Minimum variance	0.13	0.37	0.20	0.20	0.65	0.01	8.04
Minimum variance short	0.22	0.04	0.17	0.00	1.32	0.00	42.21
Markowitz shrinkage	0.14	0.30	0.21	0.57	0.68	0.03	9.85
Minimum variance shrinkage	0.13	0.37	0.20	0.15	0.67	0.00	8.26
Minimum variance shot shrinkage	0.19	0.11	0.19	0.06	1.02	0.00	23.87

Asia pacific 1996-2018, T =all, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.22		0.37		5.79
Market	0.08	0.52	0.21	0.13	0.39	0.96	5.87
Markowitz	0.12	0.31	0.23	0.72	0.51	0.10	11.68
Minimum variance	0.10	0.39	0.21	0.08	0.50	0.00	9.54

Minimum variance short	0.16	0.09	0.18	0.00	0.93	0.00	41.89
Markowitz shrinkage	0.12	0.32	0.23	0.68	0.50	0.13	11.06
Minimum variance shrinkage	0.10	0.40	0.21	0.10	0.48	0.01	8.86
Minimum variance shot shrinkage	0.13	0.21	0.19	0.00	0.70	0.00	20.20

Asia pacific 1996-2018, T = 5 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.09		0.22		0.41		6.78
Market	0.08	0.56	0.21	0.12	0.39	0.95	5.87
Markowitz	0.17	0.13	0.23	0.61	0.74	0.00	35.90
Minimum variance	0.12	0.32	0.21	0.10	0.59	0.01	14.63
Minimum variance short	0.13	0.28	0.20	0.05	0.63	0.30	17.11
Markowitz shrinkage	0.16	0.15	0.23	0.56	0.72	0.00	32.01
Minimum variance shrinkage	0.12	0.33	0.20	0.05	0.59	0.01	14.09
Minimum variance shot shrinkage	0.17	0.10	0.21	0.09	0.84	0.03	45.35

Asia pacific 2001-2018, T =10 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.12		0.21		0.56		6.43
Market	0.10	0.59	0.20	0.15	0.52	0.91	5.20
Markowitz	0.19	0.18	0.22	0.66	0.86	0.00	20.16
Minimum variance	0.15	0.34	0.20	0.22	0.74	0.00	10.93

Minimum variance short	0.25	0.03	0.17	0.00	1.46	0.00	64.07
Markowitz shrinkage	0.20	0.14	0.22	0.64	0.92	0.00	24.55
Minimum variance shrinkage	0.15	0.32	0.20	0.18	0.76	0.00	11.61
Minimum variance shot shrinkage	0.26	0.02	0.18	0.01	1.43	0.00	74.41

Asia pacific 1996-2018, T =all, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.09		0.22		0.41		6.78
Market	0.08	0.56	0.21	0.12	0.39	0.95	5.87
Markowitz	0.19	0.07	0.23	0.63	0.83	0.00	57.47
Minimum variance	0.11	0.37	0.20	0.07	0.55	0.02	11.74
Minimum variance short	0.17	0.09	0.18	0.00	0.97	0.00	49.29
Markowitz shrinkage	0.19	0.07	0.23	0.69	0.83	0.00	60.41
Minimum variance shrinkage	0.11	0.40	0.20	0.05	0.53	0.04	10.57
Minimum variance shot shrinkage	0.17	0.11	0.17	0.00	0.95	0.00	43.39

Appendix 4: Portfolio performance of sup period in Japan

Japan 1996-2018, T =5 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.03		0.19		0.17	0.00	2.23
Market	0.02	0.60	0.18	0.18	0.10	0.28	1.73
Markowitz	0.03	0.52	0.16	0.01	0.18	0.95	2.30

Minimum variance	0.04	0.47	0.17	0.10	0.21	0.61	2.60
Minimum variance short	0.02	0.56	0.18	0.29	0.13	0.79	1.87
Markowitz shrinkage	0.03	0.50	0.16	0.01	0.20	0.85	2.46
Minimum variance shrinkage	0.04	0.47	0.17	0.10	0.21	0.59	2.60
Minimum variance shot shrinkage	0.06	0.28	0.18	0.35	0.35	0.28	4.60

Japan 2001-2018, T =10 years, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.07		0.16		0.44		3.32
Market	0.04	0.68	0.16	0.36	0.29	0.02	2.20
Markowitz	0.09	0.38	0.15	0.32	0.56	0.23	4.43
Minimum variance	0.06	0.56	0.15	0.26	0.40	0.62	2.91
Minimum variance short	0.05	0.64	0.15	0.14	0.34	0.52	2.46
Markowitz shrinkage	0.09	0.38	0.15	0.32	0.56	0.23	4.44
Minimum variance shrinkage	0.06	0.56	0.15	0.28	0.40	0.62	2.93
Minimum variance shot shrinkage	0.06	0.58	0.15	0.18	0.39	0.74	2.82

Japan 1996-2018, T = all, 6 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.03		0.19		0.17		2.23
Market	0.02	0.60	0.18	0.18	0.10	0.28	1.73
Markowitz	0.03	0.49	0.17	0.04	0.20	0.85	2.46

Minimum variance	0. 02	0.57	0.17	0.08	0.1 3	0.65	1.93
Minimum variance short	0. 01	0.68	0.17	0.14	0.0 3	0.43	1.30
Markowitz shrinkage	0. 03	0.49	0.17	0.03	0.2 0	0.82	2.50
Minimum variance shrinkage	0. 02	0.56	0.17	0.09	0.1 3	0.67	1.96
Minimum variance shot shrinkage	0. 04	0.46	0.17	0.10	0.2 1	0.77	2.64

Japan 1996-2018, T =5 years, 25 portfolios formed on size and book-to-market							
	Mea n	t test (p-value)	Standard deviation	f test (p-value)	Sharp e	Jobson and Korkie test (p-value)	Cumulativ e
Naive	0. 03		0.19		0.1 7		2.23
Market	0. 02	0.60	0.18	0.10	0.1 0	0.82	1.73
Markowitz	0. 01	0.63	0.19	0.52	0.0 7	0.43	1.44
Minimum variance	0. 02	0.58	0.17	0.02	0.1 3	0.61	1.92
Minimum variance short	0. 04	0.46	0.23	1.00	0.1 7	0.99	2.16
Markowitz shrinkage	0. 02	0.58	0.19	0.49	0.1 1	0.66	1.74
Minimum variance shrinkage	0. 03	0.50	0.17	0.03	0.1 9	0.81	2.40
Minimum variance shot shrinkage	0. 04	0.43	0.18	0.14	0.2 3	0.76	2.87

Japan 2001-2018, T =10 years, 25 portfolios formed on size and book-to-market							
	Mea n	t test (p-value)	Standard deviation	f test (p-value)	Sharp e	Jobson and Korkie test (p-value)	Cumulativ e
Naive	0. 05		0.17		0.3 1		2.40
Market	0. 04	0.55	0.16	0.12	0.2 9	0.96	2.20
Markowitz	0. 09	0.28	0.19	0.95	0.4 7	0.21	4.18

Minimum variance	0.04	0.60	0.15	0.05	0.25	0.50	1.98
Minimum variance short	0.02	0.69	0.15	0.03	0.17	0.49	1.59
Markowitz shrinkage	0.09	0.28	0.19	0.95	0.46	0.22	4.10
Minimum variance shrinkage	0.04	0.57	0.15	0.05	0.28	0.72	2.12
Minimum variance shot shrinkage	0.03	0.63	0.14	0.00	0.25	0.77	1.93

Japan 1996-2018, T = all, 25 portfolios formed on size and book-to-market							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.03		0.19		0.17		2.23
Market	0.02	0.60	0.18	0.10	0.10	0.82	1.73
Markowitz	0.04	0.44	0.19	0.43	0.22	0.77	2.72
Minimum variance	0.02	0.56	0.17	0.07	0.14	0.69	1.96
Minimum variance short	0.03	0.55	0.20	0.84	0.12	0.83	1.77
Markowitz shrinkage	0.04	0.44	0.19	0.31	0.22	0.76	2.73
Minimum variance shrinkage	0.03	0.55	0.17	0.03	0.15	0.81	2.07
Minimum variance shot shrinkage	0.02	0.59	0.17	0.04	0.11	0.79	1.80

Japan 1996-2018, T =5 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.03		0.18		0.18		2.35
Market	0.02	0.61	0.18	0.27	0.10	0.79	1.73

Markowitz	0.03	0.56	0.19	0.73	0.13	0.73	1.92
Minimum variance	0.05	0.38	0.17	0.09	0.29	0.14	3.55
Minimum variance short	0.04	0.49	0.16	0.02	0.22	0.83	2.66
Markowitz shrinkage	0.03	0.52	0.19	0.69	0.16	0.89	2.17
Minimum variance shrinkage	0.05	0.39	0.17	0.08	0.29	0.14	3.44
Minimum variance shot shrinkage	0.05	0.40	0.17	0.10	0.28	0.54	3.31

Japan 2001-2018, T =10 years, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.06		0.16		0.39		2.94
Market	0.04	0.63	0.16	0.36	0.29	0.76	2.20
Markowitz	0.06	0.53	0.16	0.47	0.37	0.83	2.76
Minimum variance	0.05	0.58	0.15	0.14	0.35	0.46	2.51
Minimum variance short	0.06	0.53	0.14	0.01	0.43	0.82	2.91
Markowitz shrinkage	0.06	0.52	0.16	0.51	0.38	0.89	2.84
Minimum variance shrinkage	0.05	0.56	0.15	0.13	0.37	0.66	2.63
Minimum variance shot shrinkage	0.06	0.54	0.14	0.02	0.41	0.88	2.81

Japan 1996-2018, T = all, 6 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.03		0.18		0.18		2.35
Market	0.02	0.61	0.18	0.27	0.10	0.79	1.73

Markowitz	0.01	0.68	0.17	0.13	0.05	0.42	1.40
Minimum variance	0.04	0.43	0.17	0.14	0.25	0.48	3.01
Minimum variance short	0.04	0.47	0.15	0.00	0.24	0.75	2.83
Markowitz shrinkage	0.01	0.67	0.17	0.12	0.06	0.46	1.47
Minimum variance shrinkage	0.04	0.45	0.17	0.08	0.24	0.47	2.91
Minimum variance shot shrinkage	0.05	0.40	0.16	0.02	0.29	0.53	3.38

Japan 1996-2018, T =5 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.04		0.18		0.22		2.73
Market	0.02	0.66	0.18	0.24	0.10	0.70	1.73
Markowitz	0.05	0.41	0.20	0.91	0.27	0.75	3.43
Minimum variance	0.04	0.49	0.17	0.09	0.24	0.74	2.95
Minimum variance short	0.05	0.47	0.19	0.73	0.24	0.94	2.94
Markowitz shrinkage	0.05	0.42	0.20	0.90	0.26	0.80	3.28
Minimum variance shrinkage	0.04	0.47	0.17	0.08	0.26	0.53	3.17
Minimum variance shot shrinkage	0.04	0.52	0.17	0.08	0.23	0.97	2.76

Japan 2001-2018, T =10 years, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.08		0.16		0.48		3.70
Market	0.04	0.72	0.16	0.34	0.29	0.59	2.20

Markowitz	0.09	0.38	0.17	0.81	0.55	0.56	4.83
Minimum variance	0.06	0.65	0.15	0.13	0.38	0.14	2.73
Minimum variance short	0.06	0.64	0.15	0.32	0.37	0.61	2.71
Markowitz shrinkage	0.09	0.37	0.17	0.80	0.56	0.52	4.91
Minimum variance shrinkage	0.06	0.63	0.15	0.14	0.39	0.16	2.80
Minimum variance shot shrinkage	0.06	0.62	0.14	0.03	0.43	0.81	3.00

Japan 1996-2018, T = all, 25 portfolios formed on size and momentum							
	Mean	t test (p-value)	Standard deviation	f test (p-value)	Sharpe	Jobson and Korkie test (p-value)	Cumulative
Naive	0.04		0.18		0.22		2.73
Market	0.02	0.66	0.18	0.24	0.10	0.70	1.73
Markowitz	0.04	0.50	0.18	0.49	0.22	0.99	2.75
Minimum variance	0.04	0.52	0.17	0.09	0.22	0.97	2.72
Minimum variance short	0.07	0.28	0.18	0.32	0.40	0.38	5.60
Markowitz shrinkage	0.04	0.50	0.18	0.45	0.22	1.00	2.71
Minimum variance shrinkage	0.04	0.54	0.17	0.05	0.21	0.95	2.62
Minimum variance shot shrinkage	0.05	0.44	0.16	0.02	0.29	0.72	3.47