Estimation of daily batch balances of perishable products in retail

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Abstract

Retailers often use automatized replenishment information systems to calculate the optimal amount of goods to order to store shelves. This optimal amount is calculated by simulating inventory given future sales forecasts and incoming deliveries. In case of perishable products, the inventory simulations have to take into account the amount of goods that have to be discarded because they were not sold before their expiration date. For perishables, simulations are restrained by non-existent store-level data on how sales divide to different batches with differing expiration dates. Subsequently the balance data for each batch of a product in a shelf does not exist.

In this thesis, a process for estimating batch balances of perishable products is proposed. The method uses historical store data and runs checks on it to examine whether it is usable for further operations. Solving a system of equations, for some products definite values for batch balances are calculated for the last day of the time period of the given data set. In addition, and if able, the definite values for historical batch balances are calculated for all products in the data set.

Using different depletion models that divide the sales to different batches the inventory is simulated into the future for every product in the data set starting from every point of time when the previous operations have been able to calculate all the definite batch balances. The performance of each depletion model is determined on how well they are able to estimate spoilage.

The best performing depletion model of the inventory simulations is chosen for simulation of inventory for products, for which all definite batch balances could not have been calculated earlier. This simulation is constrained by the real life balance, sales and spoilage data in order to achieve a result that is feasible in real life. The inventory is simulated until the end of the time period given in the data set. At the end of the complete process, we have acquired either definite batch balances or a feasible estimate for almost all the products in the given data set.

The established process was experimented by using a data set acquired from an European grocery retailer. The data set consisted of 386 product-location-combinations of bread products. The process resulted in either definite values or feasible estimates of the batch balances for almost all the product-locations that had consistent enough data to be used in the analysis.

Keywords inventory management, batch balance, inventory simulation, depletion model, ordering policy

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Tiivistelmä

Vähittäisruokakaupan alalla on yleistä käyttää automatisoituja IT-ratkaisuja laskemaan optimaali kaupan hyllyyn tilattava määrä tuotetta. Tämä optimaali määrä lasketaan simuloimalla kaupan varastoa tulevaisuuteen käyttämällä mm. myyntiennustetta ja tietoa kauppaan jo matkalla olevista tuoteyksiköiden määristä. Pilaantuvien tuotteiden tapauksessa simulaation on otettava huomioon päivittäin pilaantuneena poisheitettävät tuoteyksiköt. Pilaantuvien tuotteiden varastosimulointia rajoittaa se, että usein kaupoilla ei ole olemassa dataa siitä, kuinka paljon hyllyssä on saman tuotteen mitäkin pilaantumispäivältään eriävää tuote-erää jäljellä.

Tässä diplomityössä esitetään prosessi, joka estimoi pilaantuvien tuotteiden eräsaldoja. Metodi käyttää historiallista myymälädataa, mutta ajaa ensiksi tarkistusoperaatiot kyseisen datan käytettävyyden varmistamiseksi. Operoimalla historiallista dataa, joillekin tuotteille pystytään analyyttisesti ratkaisemaan eräsaldot käytettävän datasetin viimeiselle aika-askeleelle. Lisäksi jos mahdollista, kaikille tuotteille pyritään analyyttisesti ratkaisemaan myös historialliset eräsaldot.

Käyttämällä erilaisia tyhjennysmalleja, jotka jakavat päivittäiset myynnit eri erille, kaupan varastoa simuloidaan tulevaisuuteen jokaiselle tuotteelle, joille aikaisemmin pystyttiin muodostamaan varmat eräsaldot jollekin datasettiin kuuluvalle ajanhetkelle. Tyhjennysmalleja arvioidaan niiden kyvyllä simulaatiossa estimoida pilaantuneiden tuotteiden määrää mahdollisimman tarkasti.

Parhaaksi arvioitua tyhjennysmallia käyttäen hyllyvarastoja simuloidaan datasetin viimeiselle aika-askeleelle saakka niille tuotteille, joille ei aikaisemmin pystytty ratkaisemaan eräsaldoja kyseiselle ajanhetkelle. Tämä simulaatio on rajoitettu oikean elämän saldo-, myynti- ja pilaantumisdatalla, jotta lopuksi saavutetaan tosielämässä mahdollinen lopputulos. Prosessin päätteeksi tuloksena on varmat eräsaldot, tai sitten tosielämässä mahdolliset estimaatit eräsaldoille lähes kaikille datasetin tuotteille.

Luotua prosessia kokeiltiin käyttäen eurooppalaisen vähittäisruokakauppaketjun dataa. Datasetti koostui 386 eri tuote-myymälä-kombinaatiosta leipätuotteita. Prosessi johti joko varmoihin eräsaldoihin, tai sitten tosielämässä mahdollisiin estimaatteihin eräsaldoista melkein kaikille tuote-myymälä-kombinaatioille, joille oli olemassa prosessiin käyttökelpoista dataa.

Avainsanat varastonhallinta, eräsaldo, varastosimulointi, tyhjennysmalli, tilauskäytäntö

Preface

Writing is hard. I have been in the Finnish school system for a combined amount of 18.5 years and the thought of having to write actual, comprehensible sentences still terrifies me. After all the tests, writing assignments and essays I have had to write in my life, I am now at the end of the line. This Master's Thesis is the last thing I ever had to write that has to be submitted for evaluation of school personnel.

But the truth is, I cheated. I used the language of mathematics in order to avoid writing tens of pages of actual sentences. Maybe that is one reason why I started studying mathematics in the first place. The stuff just speaks for itself. All statements are absolute and there is no room for interpretation. However, this attribute brings a great risk and responsibility. When writing a mathematical statement, there is a chance you are absolutely wrong, and there is no way to weasel out of it. Mathematics is all about grinding until you get the one correct result - patience is everything. Once you eliminate the impossible, whatever remains must be the truth.

I must express my gratitude for everyone who has ever supported me in my studies. Most importantly this includes my family and every single teacher I have had in my life. For all the help in writing this thesis I must say thank you to my advisor Janne and supervisor Lasse for guiding me through the whole process, Leo Norilo for giving invaluable feedback for the first draft of this thesis, my co-workers for giving me great advice, and Relex Solutions for giving me the chance to work on a subject so deeply related to my field of study.

Finishing this thesis brings an end to a very important and memorable part of my life. During my university years I have had the time of my life, mostly due to all the amazing student activities I have been able to take part in. I'd like to thank The Guild of Physics and especially boards 2015 and 2016 for all the amazing times I have had in my years in Otaniemi.

Looking back and reflecting on what I have learned in these past six years in university, all I can say is nothing, and everything. It's a cliché to say that in university especially Engineering Physics and Mathematics students mostly just learn how to learn, but there is very much truth in it. Studying at Aalto and being surrounded by the brightest minds of Finland has had a massive positive impact for my mental growth and made me unafraid of any complex task ahead of me. Now new challenges lie ahead of me, and I have never been more prepared in my life.

Otaniemi, 22.11.2019

Hannu Pantsar

Contents

Symbols and abbreviations

Symbols

$\mathbf{B}$	Batch balance matrix
$\mathbf{E}^s$	Simulated batch balance matrix

Abbreviations

DM	Depletion Model
KPI	Key Performance Indicator
SKU	Stock Keeping Unit
PL	Product-Location
BB	Batch Balance
VOI	Value-Of-Information
FIFO	First-In-First-Out
LIFO	Last-In-First-Out
EOQ	Economic Order Quantity
WTP	Willingness-To-Pay

1 Introduction

1.1 Motivation

In the world of grocery retail the problem of ordering the right amount of perishable products to store shelves is an age-old problem from both environmental and economical point of view. Because product units that have not been sold before the end of their expiration date are wasted, the store loses both in the price it paid for the unit and also in costs related to managing the waste. Food waste is a massive issue in today's world, where third of food products for humans are wasted, and many of these due to inefficiencies in replenishment of grocery retailers (Gustavsson et al., 2011). Combatting food waste is most important at the end of its value chain since at that point it has already been transported and set for display, adding to its economical and environmental impacts. In only Finland, 120–160 million kg of food waste is generated yearly in households and 65–75 million kg in retail sector (Katajajuuri et al., 2014).

From a purely economical point of view, in the U.S retailers' and consumers' food waste is measured in 165.6 billion USD every year (Buzby, 2013). Total sales of perishables in supermarkets in U.S. amount to approximately 217 billion USD every year but the net profit of the entire industry of grocery retail is a mere 2.2% (Dudlicek. J, 2018; Biery and SageWorksStats). This clearly shows that grocery retail is a very competitive industry where even the slightest advantage in cutting costs can either make or break a retailer. Having a cost-effective supply chain management is therefore most essential for retailers and can give a substantial competitive advantage.

Because retailers want to minimize their costs in planning and executing their supply chain strategy, an optimal and automatized replenishment logic for stores is required. This replenishment logic can take into account future demand forecast and simulate future inventory. Based on these simulations, an optimal amount of goods can be ordered to the store shelves from a central distributor.

In case of perishable products, what makes inventory simulation more complicated is the fact that product units spoil after a certain amount of time and it is difficult to project how many units spoil on a certain day. This is often because of store shelves containing many product units from different batches that have arrived on different times. An optimal replenishment logic would have to take into account the batch from which each unit originates. The essential difference between different batches is their expiration date, which can have a significant effect on the purchase decision of a customer. From a store's perspective the optimal consumer behaviour would be choosing the batch in a First-In-First-Out (FIFO) type of way, where customer would always choose the product with the nearest upcoming expiration date (Parlar et al., 2011). However, customers do not always act like this since they can have the tendency to choose the product with the latest expiration date in order to maximize the freshness of the product. Due to customers choosing newer batches, older products stay in the shelves and in the end get discarded when they reach their expiration date, causing more food waste in stores.

From the perspective of supply chain management, the problem is that typically

there is no information on how many products from which batch are left in the shelf. Most of the product units could be up for spoilage tomorrow without the replenishment logic taking this into account, causing availability issues and increased spoilage over time. An automatized replenishment logic could greatly benefit from having accurate batch balance information from stores, but since it is often not available, even an estimate of the current batch balances of a product in store shelf could be beneficial. In this thesis, a method for estimating these batch balances using store-level data is proposed. The presented result of this thesis could be used in enhancing an automatized replenishment logic, which itself is beyond the scope of this thesis.

1.2 Research question

The goal of this thesis is to find a process, that uses the usual type of data retailers are able to share with a third party software company, and is able to give some feasible simulated estimate for store batch balances at the end of the time period specified in the data set. The resulting process therefore has to be applicable for this particular type of data set. In addition, we want the process to be able to simulate batch balances when assuming different types of depletion models, i.e. models that describe from which batch the sold product is chosen from. Since there is no actual historical data of batch balances, the measuring of actual accuracy of the produced estimates is left out of the scope of this thesis.

The goal of this thesis is to produce the best possible result with the given data. It has to be emphasized that in thesis we approach problems with data and its capabilities in mind first. There are most likely different and more optimal ways of performing the estimations with different type of data sets, but in this thesis we are constrained by what we are capable of doing with the given data set.

The research question posed can easily raise suspicions as whether there would be easier and more simple means of finding out batch balances than making estimates based on data. At the time of writing this thesis there are methods for finding out exact batch balances, but none of them are currently widely implemented. This can be due to either economical or technical reasons, or just for the simple reason of stores not seeing the value of implementing them. For instance, modern barcode standards such as GS1 DataMatrix are able to contain batch balance information unlike the by far the most used barcode standards EAN/UPC that can contain very limited amount of information (GS1, b,a). Another tool for tracking batch balances could be the use of RFID-tagging, but the technology has not been widely implemented mostly due to cost-related reasons and risks in implementation (Owunwanne.D, 2010). Maybe the most obvious way of finding out batch balances is having the store personnel count every product unit at the end of the day. However, is very labour demanding and with all other labour costs, adding a substantial amount of working hours is most likely not tempting for retailers.

In making of this thesis we acknowledge that in upcoming years technology will most likely make any result of this thesis obsolete for batch balance calculations. What we do however want to find out is whether the process we are looking for

can be right now be made in a way that can be reasonably easily implemented in a third party software responsible for automatic replenishment. This thesis has been done with the support of Relex Solutions, a producer of such software and also the provider of data for this thesis. The choice of methods in this thesis is thus not made only with the goal of finding the optimal solution, but also the implementability of the solution.

2 Background

2.1 Inventory system basics

A basic discrete-time inventory state equation for a particular perishable product can be written as

$$X_t = X_{t-1} + O_t - D_t - S_t \tag{1}$$

where X_t is the inventory balance at the end of the day t, O_t is the delivered order amount at the beginning of the day, D_t is the amount of sales and S_t is the amount spoiled goods at the end of the day. The inventory balance X_t of a product can consist of multiple different batches. We denote the balances of these batches at the end of the day t by $x_{i,t}$, with $i-1$ being the remaining lifetime of a batch and generally $i = 1, ..., L$ being the batch identifier of the product with $i = 1$ being the oldest batch, that is the one that will go to waste next and L the newest. Assuming that every batch has a unique expiration date and arriving batches always have the same fixed life-cycle of L days, the maximum amount of simultaneous batches in the inventory is L. The total balance can now be stated as

$$X_t = \sum_{i=2}^{L} x_{i,t} \tag{2}$$

The reason why $x_{1,t}$ is not in the sum because it is the oldest batch at the end of the day t and is therefore spoiled. By definition spoiled goods of the day t are not included in the inventory balance X_t. When taking into account the fact that the sales of a product are divided between different batches, the definition of sales D_t can be expanded as

$$D_t = \sum_{i=1}^{L} d_{i,t} \tag{3}$$

In the case of perishable goods it is common to give a discount for the products on the last day of their life-cycle. We denote these *special sales* by

$$A_t = d_{1,t} \tag{4}$$

The amount of spoiled goods is the number of product units that is left in inventory after L timesteps of the batch arriving. We can determine that the amount of spoilage S_t at the end of day t is

$$S_t = x_{1,t} \tag{5}$$

and

$$S_t = O_{t-L+1} - \sum_{i=1}^{L} d_{i,t+1-i} \tag{6}$$

In Equation 6 the spoilage S_t is depicted as the difference between the delivered amount of the batch and all the sales it has had in its life-cycle.

In case of an inventory system as stated in previous equations, having knowledge of historical sales, balances and spoilages leaves as only unknown variables left the batch balances $x_{i,t}$ and the batch sales $d_{i,t}$. A model that describes how sales D_t are divided to batch sales $d_{i,t}$ is in the following sections referred to as a *depletion model* (DM). In literature these DMs are also called depletion policies, issuance rules and issuing policies (Nahmias, 1977; Deniz et al., 2009; Huq et al., 2005).

2.2 Inventory management and simulation for perishables

In retail, ordering the right amount of product at the right time is essential for maximizing availability and minimizing stock-outs, especially in case of perishable products. Different types of *ordering policies* with possibly differing *ordering parameters* suited for the restrictions of the inventory and supply chain as a whole are used in determining the right amount to order at the right time. These ordering policies usually use inputs such as lead time, batch size, current balance, future sales forecasts, future deliveries, and for perishables, projected future spoilage. In an automatized replenishment logic, the inventory is simulated into the future using this ordering policy in order to determine if more products should be ordered to a store shelf. In case of perishables, for simulation it is also a requirement to have an assumption of a depletion model that splits the sales D_t to different batches (Broekmeulen and van Donselaar, 2009; Chiu, 1995).

In general, simulating inventory is a very common way of seeking for best possible inventory management practices. Performing basic inventory simulations typically requires some type of estimate for demand stream and an ordering policy driving the replenishment. Most studies regarding inventory simulation are performed to compare different types of ordering policies in order to find the most profitable of them. Usually the optimal ordering policy has to find the right mixture of minimizing the amount of inventory but maximizing product availability for customers (Kapuscinski and Tayur, 1999).

In an ideal situation a retailer always has perfect information on its current stock in its inventory management systems. Maintaining perfect information can be very challenging, since there are multiple things that can make the reported number differ from real life count, such as theft, incorrect deliveries and misplaced items. A study by Raman et al. (2001) found out that in retail, for over 65% of stock keeping units (SKU) the inventory data was inaccurate with the difference to real life inventory being 35% in average. In the case study these inventory inaccuracies accumulated up to 10% of reduction in profits. Fleisch and Tellkamp (2005) studied the effect of inventory inaccuracy and came to the result that getting rid of inventory inaccuracy does reduce supply chain costs and the amount of stock-outs. The lowest level of inventory inaccuracy this effect was observed was as low as 2%.

Since an inventory of perishable products can be modeled as a discrete-time system as in Equation 1, the life-cycle L is also required for simulation of perishable inventory in addition to ordering policy and demand stream. For inventory simulation of perishables a notable problem retailers can have is that the expiration date of a product is not known for certain when the retail receives the product from the supplier.

Ferguson and Ketzenberg (2006) studied the effect of VOI (Value of Information) of these expiration dates and came to the conclusion that having this information has a notable positive effect on managing inventory. The retailer benefited from this life-cycle information the most when the demand of the product had a high variance, product lifetimes were short and the cost of the product was high.

Retailers can also be completely lacking information that could potentially be beneficial for inventory management, such as batch balances of products in their own store shelves. In most cases, stores do not track how many units from which batch are left in the shelf at the end of the day, with the exception of discounting products that are about to spoil in near future or discarding spoiled products. The discounting of products at the end of their life-cycle can have a massive business impact, such as in the case of Finland's biggest retailer S Group, that sells over 70 million product units discounted in this manner every year (S-ryhmä, 2019). Discounting products in this way also has an enormous reducing effect on food waste produced by retailers (Korhonen et al., 2019).

2.3 Depletion models for inventory

2.3.1 Industry standards

A simple and commonly assumed DM in inventory simulation for fixed shelf-life is FIFO (First-In-First-Out) type of depletion. In FIFO, the customers choice of batch is always the oldest one (Eilon, 1961). In case of multiple product-ages on the shelf, for store it would be most beneficial not to allow the depletion of the freshest products. For this reason stores usually try to force the choice of FIFO to customers. Stores usually attempt to minimize the depletion of newest product units by strategically keeping some of the units in the backroom. Since the space in stores can be very limited, the backroom treatment can not be applied for most products. This leaves for stores the second best option of filling the shelves from the back, which is a well established practice especially in case of milk cartons. This still leaves the customers the ability to choose between batches and in general stores just have to live with the fact that FIFO can not be forced to customers. Not choosing from the oldest batch leads to the display shelf containing units of same product from different batches with differing expiration dates. In order to make older batches more inviting, it can be beneficial to give less fresh products a lower price in order to avoid spoilage (Herbon, 2017).

The customer purchasing behaviour completely opposite to FIFO is named LIFO (Last-In-First-Out). In LIFO the customer always chooses the product from the freshest batch available. Since it is the exact opposite type of depletion to FIFO, it causes more spoilage due to lack of demand for older batches. From purely environmental perspective it has to be remembered that even though LIFO maximizes waste in stores, household wastes are double compared to retail (Katajajuuri et al., 2014). It has to be remembered that choosing the freshest batch the customer might try to minimize his/her personal accumulation of waste. Due to to this reason it can not be directly stated that LIFO-minded customer causes more food waste.

Parlar et al. (2011) did an extensive study on comparing whether FIFO or LIFO would be the optimal choice of DM for inventory policy. The result was that with all combinations of unit revenue, penalty cost due to unsatisfied demand and intensity of demand the FIFO-model was the better model when measuring expected profit. The LIFO-model had a bigger expected profit only in a situation when the inventory holding cost of a product was very high or the purchase cost was very low. The reason for this is stated to be the fact that when FIFO-depleted, items stay longer in the shelf.

It can be said that FIFO is somewhat a standard due to the amount of inventory management literature using it, and the fact that automatized replenishment logic providers such as Relex Solutions assume it in their simulations. The problem with simulating an inventory with FIFO is that since it underestimates the amount of spoilage, it can lead to more stock-outs in real life.

2.3.2 Alternative models

Even though FIFO and LIFO are the clearly most used DMs in literature, different authors have used several approaches in trying to find optimal inventory policies. Lowalekar et al. (2015) used an uniform distribution model where all the units of product in a shelf had the same probability to be chosen by the customer. However, using real data from a blood bank, no evidence of this type of randomness was found in blood banks. Ferguson and Ketzenberg (2006) used a proportional distribution model where the probability of a batch unit to be sold was the amount of units in the batch divided by the total amount of product in the shelf. With these two proportional models the customers did not prefer newer units of product in any way, but since typically the newest batch makes up most of the inventory it was depleted the most, making the models lean more to LIFO than FIFO.

Avinadav and Arponen (2009) developed an EOQ (Economic Order Quantity) model for perishables which follows a demand rate, which is deterministic and declining over time, has a polynomial function form and is dependent on the remaining normalized time to product expiry. You (2005) applied a demand model for fresh produce where the demand of a perishable product decreased exponentially in time.

Iship (1992) modelled demand with two types of customers, one of which buys only the newest unit of product and the other that buys either one. His model did however also include price as one demand variable. Later the same author with the help of another author introduced a demand model, where a customer is either type LIFO or FIFO, as in the customer can only prefer the newest or the oldest unit of product (Iship and Noseb, 1996). Deniz et al. (2009) also considered a case of two separate demand streams where all demand for newer units is fulfilled from the amount of older units and vice versa.

Goh. C (1993) compared two managerial policies for its two-stage perishable inventory model where an unit of product can be either new or old. As its demand model the policies used two independent Poisson processes, where queries for newer or older units occurred at their own independent rates.

For most of these studies what was in common was that the DM itself was not

under inspection but it was rather just one assumption to be made before focusing analysis on something else. Comparisons of DMs other than FIFO or LIFO are a rarity in literature. A problem in making these comparisons is that simulating an inventory system is a complex task with many variables affecting the outcome. A recent Master's Thesis by Montojos (2018) compared LIFO,FIFO, half-FIFO-half-LIFO, Proportional depletion and a model which estimated parameters for demands for newer and older product. This comparison used simulated demand data with two simultaneous batches that had a lifetime of two timesteps. The ordering policy driving these simulations had a substantial effect on results, but nevertheless FIFO caused least waste, LIFO the most waste and other models were somewhere between the two but closer to FIFO-levels of waste.

2.4 Consumer purchasing behaviour

2.4.1 Substitution

In the context of customer purchasing decision *substitution* means that a customer who wants to buy a product with a certain time left in its life-cycle is at some rate willing to choose another or the same product with a different time left in its life-cycle if his/her original preference is unavailable. The term downward substitution can be used to denote the case in which a new product is sold to a customer that demands old; upward substitution is the reverse.

In case of perishable foods a *complete substitution* means that even if a customer prefers his/her product with long/short shelf life, the equivalent unit of same product with short/long amount of shelf life gets chosen if the original preference is not available.

Substitution does not limit itself with the different amount of shelf life left of the same product, but between different products of same type also. Miranda and Kónya (2006) pointed out from their survey made for 473 randomly selected grocery shoppers across Melbourne that when given the choice, consumers often choose a same type of competing product which has a longer expiration date.

In a study by Van Woensel et al. (2007) 3,805 customers over three different stores were interviewed on their substitution habits regarding bakery breads. On average 84% of respondents that they purchase another bread in the sames store if their original preference is out of stock. However, a comparison was made to Gruen et al. (2002) which primarily looked into non-perishable products. What was reported that for European customers the substitution rate is near 50 percent. Van Woensel et al. (2007) states the nature of bakery breads as products of which freshness is highly esteemed for the big difference. In the same study it is also mentioned that regarding substitution the difference between substitution preferences of customers revealed in their interviews and actual behaviour was very high. Due to this reason it was stated that the interview-reported behaviour can be actually set as the lower bound for the actual substitution. However, the biggest takeaway is that in case of bakery breads the substitution rate is staggeringly high. Taking this substitution into account in the holistic inventory policy of a store could lead to better performance (Bakker et al.,

2012). This can be a very hard task considering that there are multiple different types of products that a customer can substitute with and the choice might not always seem rational.

2.4.2 Consumers and waste

Some amount of research has been conducted regarding how consumer behaviour leads to more waste. It is reasonable to assume that consumers prefer fresher products over less fresher ones at some rate. However, there is not much consumer behaviour research conducted on the effect of remaining days until expiration date on the actual purchase decision of consumers. There is even the possibility that consumers do not even use the expiration date of a product as an indicator of freshness, since there can be alternative ways of measuring the freshness of a product such as in case of white breads, the amount they can be squeezed (Callejo et al., 1999).

Some influencing factors driving consumer behaviour regarding perishables have been mentioned in research. Online survey answered by 848 Danish consumers in study made by Aschemann-Witzel et al. (2017) found out out that consumers who pay more attention to prices tend to waste less food than those who don't pay much attention to prices. What was also discovered is that consumers don't buy discount-priced food near its expiration date if they have the possibility of leading to more waste at home, as consumers saw this as a waste of money. The notion that consumers see wasted food first and foremost as a waste of money was also supported by Graham-Rowe et al. (2014), a qualitative study based on interviews of UK household food purchasers who state their discomfort in wasting food being mostly due to wasting money.

There is a lack of using real life store data when researching consumer tendencies in store regarding purchase of perishable foods. Research on the subject mostly uses consumer self-reporting as in Koivupuro et al. (2012). In case of self-reported behaviour regarding perishables this does not always reflect the actual behaviour perfectly (Van Woensel et al., 2007).

One survey-based research by Neff et al. (2015) was conducted on 1010 US consumers and one key finding was that 35% of US consumers perceive that they put a lot of current effort to minimize waste of food and 91% of respondents said they pay attention to date labels.

Big cultural differences between countries do however exist. In Harcar and Karakaya (2005), survey made for 687 people in the three countries found out that 62% of US respondents but only 42% of Turkish respondents check expiration dates of products before purchasing them.

Maybe the most thorough research on the subject of actual purchasing decision by Tsiros and Heilman (2005) had six different perishable product categories with a visible expiration date on their packaging presented to 270 consumers. The categories included prewashed/precut lettuce, prewashed/precut carrots, milk, yogurt, chicken, and beef. Using these products, the consumers' WTP (Willingness To Pay) was evaluated by distributing three different versions of survey with three different shelf lifes (1, 3 and 7 days) for the product categories and asking for the consumers' WTP

for these categories.

Depending on product category, 69%-84% of respondents believed that product quality deteriorates over time in shelf. As seen in Figure 1, consumer WTP is linearly decreasing as a function of remaining shelf life for lettuce, yoghurt, carrots and milk. For chicken and beef the consumer WTP appears to be exponentially decaying. The authors argue that the reason for milk having the least perceived quality at the end of its shelf life can be explained by the fact that its aging process can not be halted by freezing. Overall the conclusion is made that WTP is higher for products for which the consumers look to stop the aging process either by using the product as an ingredient for cooking or freezing it.

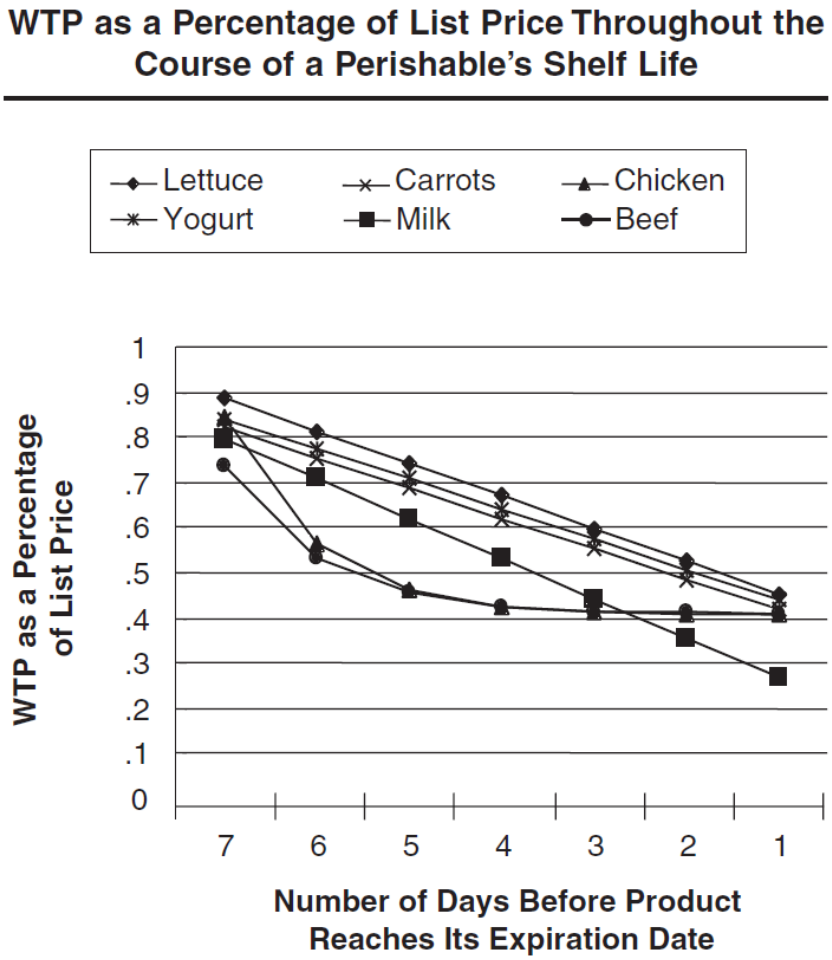

Figure 1: Results of Consumer Behaviour study by Tsiros and Heilman. Retrieved from The Effect of Expiration Dates and Perceived Risk on Purchasing Behavior in Grocery Store Perishable Categories, 2006, Journal of Marketing, 69(2), p.126.

3 Data approach and base assumptions

3.1 Data description

A data set acquired from an European grocery retail chain was used for the analysis. The data set consisted of store data of 386 Product-Locations (PL) over $T = 22$ days. With a PL we denote a certain product in certain store location. The same product can be available in several locations, and each one of these instances are their own PL. All of the PLs in the data set are from same product category, which is breads, and their fixed spoiling times L differ between 3-7 days. The store data contained variable information described in Table 1, which will be referred to as *transactions* from now on. For simplicity the total sales data D_t also include negative store balance adjustments. These adjustments mostly happen due to missing or faulty product units. In the used data set the amount of adjustments was approximately 1.4% of total sales.

Table 1: Transaction data variables in the given data set.

	Notation
Total Sales	D_t
Special Sales	A_t
Normal Sales	R_t
End Balance	X_t
Deliveries	O_t
Spoiled	S_t
Spoiling Time	L
Time Horizon	T

Here t denotes the date, and $t \in [1, T]$ is the range it can gain values from.

3.2 Issues with data and consistency checks

The used data set contained some quality issues which had to be taken into account. Due to errors in data, many PL had *inconsistent* data in a sense that adding up transactions from the beginning of t until T led to impossible situations such as negative end balances or units spoiling at times when they should not. A key requirement for this thesis is that the data used is *consistent*, which means that for a single PL there can not be a single of the mentioned impossible situations in its historical data. In order to ensure this, a series of *consistency checks* for the data had to be built. The following checks were made:

1. Negative end balance check
    - Any PL with $X_t < 0$ at any t was removed from data set.
2. Sum of sales check

- Any PL with $A_t + R_t \neq D_t$ at any t was removed from data set.

3. Stepwise inventory check
    - Any PL with $X_t \neq x_{t-1} + O_t - D_t - S_t$ at any t was removed from data set.
4. Spoiling time check
    - Any PL with $S_t > O_{t-L+1}$ in $t \in [L, T]$ was removed from data set.

Check number 1 simply checks if the End Balance of a PL goes below zero. Check 2 makes sure that the sum of normal sales and special sales adds up to total sales. Check number 3 ensures that the basic inventory formula requirement given in Equation 1 is satisfied. The most strict check is number 4, which effectively removes PLs which have had spoilage transactions on days when no batch could have spoiled due to the assumption that no product unit gets spoiled before it has been available for L periods. This assumption of fixed Spoiling Time is crucial for the analysis, since otherwise it would become impossible to determine from which batch the spoiled units come from.

The listed consistency checks do not yet guarantee that the remaining data is fully consistent. There are some special cases where the inconsistency of data is revealed only after performing necessary steps in the estimation of batch balances. Additional consistency checks are therefore necessary and are thoroughly described in the Methods-part of this thesis. These four consistency checks are bare necessity that should be performed before any further analysis is done.

3.3 Base assumptions

Before proceeding with applying any methods to our data set, few vital assumption have to be asserted. These assumptions have to do with either the characteristics of the data set or how customer behaviour is to be modelled further on in this thesis.

1. **Substitution**
    - Perfect down- and upward substitution of products is assumed. This means that all demand for a product must be satisfied by the product itself. Downward substitution means that if a customer prefers to purchase a product that has a long time left in its life cycle and a batch of that sort is not available, the customer chooses to purchase the same product with a shorter time left in its life cycle instead of choosing another product. Upward substitution is the same concept but with a customer preferring older batch picking a newer batch when an old batch is not available. Together these substitutions lead to customers always picking a certain product whenever it is available.

2. **Aggregation**

   We assume that same type of products such as breads or milks are similar enough so that a same kind of DM can be applied to all products of that type and that the results from each PL from same product group can be aggregated. For example, when performing an inventory simulation for multiple PLs we can sum over all PLs the metric values that measure the performance of the simulation.

3. **Special Sales** A_t

   Due to information received from the provider of the data set it can be assumed that a special sale A_t can deplete only the batch that is about to spoil at the end of day t. However the lack of a special sale transaction A_t does not implicate that the batch expiring at t did not have any sales at all, since there can be instances when a batch was not discounted during its last life-cycle date.

4 Methods

We seek to reach an estimate for batch balances at the end of day $t = T$ and in order to achieve this we approach the problem in several steps. First we aim to analytically find as many definite values for batch balances in the time period $[1, T]$ as we can. This is achieved by inspecting the given transaction data and calculating whether there are days t when it is mathematically impossible for a batch to have other than one definite value. After finding the definite values we try to find a DM that supports the given transaction data and the definite batch balance values derived from it as well as possible. Last, we use the DM that is considered best, constrain it with transaction data, and simulate the inventory system until T.

All operations performed with data are executed using scripting language R. The used methods are fairly algorithmic and chosen based on ability to reach the end goal and implementability. For this reason, the end result may not be mathematically optimal, but rather a good enough heuristic estimate.

4.1 Batch balance matrix

For a single PL, we approach the problem by first defining a *batch balance matrix*, which can be stated in upper-triangular form as

$$\mathbf{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & b_{1,3} & \cdots & \cdots & \cdots & \cdots & \cdots & b_{1,T} \\ & b_{2,2} & b_{2,3} & \cdots & \cdots & \cdots & \cdots & \cdots & b_{2,T} \\ & & \ddots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ & & & \ddots & \cdots & \cdots & \cdots & \cdots & \vdots \\ & & & & \ddots & \cdots & \cdots & \cdots & \vdots \\ & & & & & \ddots & \cdots & \cdots & \vdots \\ & & & & & & \ddots & \cdots & \vdots \\ & & & & & & & \ddots & b_{M-1,T} \\ & & & & & & & & b_{M,T} \\ & & & & & & & & \end{bmatrix} \tag{7}$$

where $b_{m,t} \in \mathbb{N}^{M \times T}, \mathbb{N} \in \{0, 1, 2, ..\}, M = T$. Even though $\mathbf{B}$ is as square matrix we now on use M and T separately for added clarity. The row m depicts the date of arrival of a single batch, and column t is time in days. Therefore on rows we have all the batches with $b_{m,t}$ depicting the amount that batch delivered on day m has left at the end of the day t. Since we have M rows, there are M batches in $\mathbf{B}$. However, these rows can be populated by zeros in case that the delivery O_t for day $m = t$ is zero, or additionally when the newly delivered batch $b_{m,m}$ has been completely depleted on its first day in assortment.

Using notations given in Section 1 with $i - 1$ denoting the remaining lifetime of a

batch, and the batch having a balance of $x_{i,t}$ the elements of $\mathbf{B}$ are defined as

$$b_{m,t} = \begin{cases} x_{L-t+m,t}, \text{ when } m \leq t \leq m+L-1 \\ 0, \text{ else.} \end{cases} \tag{8}$$

Inserted in $\mathbf{B}$, the matrix reads

$$\mathbf{B} = \begin{bmatrix} x_{L,1} & x_{L-1,2} & x_{L-2,3} & \cdots & x_{1,L} & 0 & \cdots & \cdots & 0 \\ & x_{L,2} & x_{L-1,2} & \ddots & \ddots & x_{1,L+1} & 0 & \cdots & 0 \\ & & x_{L,3} & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ & & & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ & & & & \ddots & \ddots & \ddots & \ddots & x_{1,T} \\ & & & & & \ddots & \ddots & \ddots & x_{2,T} \\ & & & & & & \ddots & \ddots & \vdots \\ & & & & & & & x_{L,T-1} & x_{L-1,T} \\ & & & & & & & & x_{L,T} \\ & & & & & & & & \end{bmatrix} \tag{9}$$

where $i \in [1, L]$ and $t \in [1, T]$. Here $b_{t,t} = x_{L,t}$ depicts the amount of units left after first day of sales for the batch arrived on day t. At each column t the batch balances $x_{i,t}$ can be read with $x_{1,t}$ being the oldest batch in assortment and and $x_{L,t}$ the newest batch in assortment. $L-1$ days after the delivery of a batch it has left the amount $b_{m,m+L-1} = x_{1,L+m-1}$, which is the same as the amount of spoiled goods for that batch on that day: $b_{m,m+L-1} = S_{m+L-1}$. The balance of a batch after its expiration date is always 0. Sum of a column t depicts end balance before the amount of spoiled goods is removed. This holds true only for $t \in [L, T]$, because in our data set at starting point $t = 1$, there exists still older batches in balance X_t which can not be found from matrix $\mathbf{B}$. Only after point $t = L$ it is certain that no other batches exist in the inventory than the ones on rows m of $\mathbf{B}$. Therefore when we sum columns of $\mathbf{B}$ we have

$$\sum_{m=1}^{M} b_{m,t} = X_t - S_t \tag{10}$$

when $t \in [L, T]$. Further analysis performed with $\mathbf{B}$ mostly uses only columns $t \in [L, T]$ in order to avoid usage of these hidden batches. The reason for two different notations for batch balance information of a PL is that some operations in this thesis are much clearer to express with elements $b_{m,t}$ and others, especially in case of simulation, with batch balances $x_{i,t}$.

4.2 Solving B, finding definite values

After the creation of batch balance matrix as in Equation 7 we want to solve as many of its elements $b_{m,t}$ as possible. This means that given the data being consistent, an element is solved if there is no possibility of having any other than one definite value. When solving the batch balance matrix, we denote the information whether an element $b_{m,t}$ has a definite value with $c_{m,t}$. More precisely, we define

$$c_{m,t} = \begin{cases} 1, \text{if there is a definite value for } b_{m,t}. \\ 0, \text{else.} \end{cases} \tag{11}$$

In order to solve as many elements as possible, we introduce a set of operations that can be used to our advantage. In these operations we either use raw transaction data or already solved elements of $\mathbf{B}$ to solve more elements $b_{m,t}$. These solved elements include the zero-elements introduced in the definition of $\mathbf{B}$. Since there are operations that use already solved values $c_{m,t} = 1$, the operations have to be performed iteratively for a sufficient number of times in order to ensure that maximum solvable amount of elements are solved.

For the purpose of demonstration we use a $M \times T = 4 \times 4$ version of $\mathbf{B}$ with $L = 3$.

$$\underset{\mathbf{4\times4}}{\mathbf{B}} = \begin{bmatrix} * & * & * & 0 \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \tag{12}$$

Unsolved values are denoted with $*$ and it is assumed that before $t = 1$ the PL has had zero balance. This means that there is no other units in the inventory than those depicted in the matrix. Since $L = 3$, batch $m = 1$ has a batch balance for three days, with $b_{1,3}$ being the amount that is spoiled from the batch, that is $b_{1,3} = S_3$.

The following operations are iteratively performed for a sufficient number of times in order to solve elements of $\mathbf{B}$:

1. Addition of spoilage transactions.

   - The amount of spoilage for a PL is the amount left in the last day of the life cycle of the PL:

     $$b_{m,m+L-1} = S_{L+m-1} \tag{13}$$

     when $m \in [1, T - L + 1]$. When $S_3 = 5, S_4 = 3$, we update the batch balances for the last day of lice-cycle for batches $m = 1$ and $m = 2$:

     $$\begin{bmatrix} * & * & * & 0 \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} * & * & \mathbf{5} & 0 \\ 0 & * & * & \mathbf{3} \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

2. Addition of zero-deliveries.

- If there was no delivery on day t, batch balance for the arrived batch on day t is zero.

$$O_t = 0 \Rightarrow b_{m,t} = 0$$

3. Batch balance can not rise, i.e. $\mathbf{B}$ has nonincreasing rows.

- If since its delivery a batch has had the same value between two different dates t and $t+n$, it has to have had the same value in every period in between these two dates:

$$\begin{aligned} b_{m,t} &= b_{m,t+n} \\ \Rightarrow b_{m,t+a} &= b_{m,t} \end{aligned} \tag{14}$$

when $n \in [0, T-t], a \in [0, n]$. For instance, in the following situation for rows $m = 1$ and $m = 2$ by this rule we can set

$$\begin{bmatrix} 2 & * & 2 & 0 \\ 0 & 3 & * & 3 \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & \mathbf{2} & 2 & 0 \\ 0 & 3 & \mathbf{3} & 3 \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

4. Batch balances already fulfil the end balance.

- If the definite values for column t sum up to the difference of end balance and spoilage, all uncertain values on that day t have to be zero:

$$\begin{aligned} \sum_{m=1}^{M} (b_{m,t} \cdot c_{m,t}) &= X_t - S_t \\ \Rightarrow b_{m,t} | (c_{m,t} = 0) &= 0 \end{aligned} \tag{15}$$

for all $m \in [1, M]$ when $t \in [L, T]$.

5. Solving for a sole uncertain batch balance.

- If there is only one uncertain batch balance for day t, all the information for solving it exists:

$$\begin{aligned} \sum_{m=1}^{M} c_{m,t} &= M - 1 \\ => b_{m,t} | (c_{m,t} = 0) &= X_t - S_t - \sum_{m=1}^{M} (b_{m,t} \cdot c_{m,t}) \end{aligned} \tag{16}$$

when $t \in [L, t]$. When $X_3 = 6$ and $\sum_{m=1}^{4} c_{m,3} = 3$, we can solve $b_{3,3}$ by subtracting $X_3 - b_{2,3}$. Value $b_{1,3}$ is not taken into account here because it is the amount of spoilage for $t = 3$ and thus not in X_3.

$$\begin{bmatrix} 7 & 3 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} 7 & 3 & 3 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \mathbf{5} & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

6. Zero sales equals zero changes.

   - If there are no sales for day t, all balances for batches $i \in [2, L]$ have the same value as on day $t-1$ and the batch arrived on t has the same balance as its delivery O_t.

$$\begin{aligned} & D_t = 0 \\ \Rightarrow & x_{i,t} = b_{t-L+i,t} = x_{i+1,t-1} = b_{t-L+i,t-1}, && (17) \\ & x_{L,t} = b_{t,t} = O_t && (18) \end{aligned}$$

   when $i \in [1, L-1], t \in [2, T]$.

7. Delivery equalling spoilage implies nothing sold from batch.

   - If the amount of a delivery equals the amount of spoiled goods of the same batch, no units from the batch have been sold on delivery day:

$$\begin{aligned} & O_t = S_{t+L-1} \\ \Rightarrow & b_{t,t} = O_t && (19) \end{aligned}$$

   when $m \in [1, M-L+1]$, $t \in [1, T-L+1]$. These ranges are set this way because there is no spoilage data for batches $m \in [M-L, T]$. Obviously having delivery equal spoilage also means that the batch has never had any sales, but we let Operation 3 to manage this after this operation.

8. Batch balance is not higher than end balance of previous day.

   - If a batch balance has a value that is equal to the end balance of the day before, the batch balance has to have had the same value in previous day:

$$\begin{aligned} & X_{t-1} = x_{i,t} = b_{t-L+i,t} \\ \Rightarrow & x_{i+1,t-1} = X_{t-1} = b_{t-L+i,t-1} && (20) \end{aligned}$$

   when $i \in [1, L-1]$. When $X_2 = 3$ demonstrated by

$$\begin{bmatrix} 9 & * & 3 & 0 \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} 9 & \mathbf{3} & 3 & 0 \\ 0 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

   Now $b_{2,2}$ could not have had a value smaller than 3 since batch balance can not rise. It could not have either had value bigger than 3 because the end balance for the day was 3.

The following operations use the special sales data A_t which is assumed to have characteristics as explained in Section 3.3:

9. All sales of an expiring batch are special sales.

   - If there are special sales on a given day t, it is assumed that all special sales are sales from the batch about to expire at the end of that day t. Therefore by summing spoilage and special sales we find the batch balance for the previous day:

$$\begin{aligned} A_t > 0 \\ \Rightarrow b_{t-1-L+2,t-1} = x_{2,t-1} = S_t + A_t \end{aligned} \tag{21}$$

   when $t \in [2, T]$. When $A_3 = 2, A_4 = 3$ we add these values to known batch balances from their final day of life-cycle:

$$\begin{bmatrix} * & * & 3 & 0 \\ 0 & * & * & 1 \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} * & \mathbf{5} & 3 & 0 \\ 0 & * & \mathbf{4} & 1 \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

10. Special sales equals total sales means that there are no sales for batches that are not expiring.

    - If special sales for a day t equal total sales, all batches except for the one expiring at the end of the day t and the one delivered at t have the same balance as at $t-1$ and the batch delivered at t has a balance equal to its delivery:

$$\begin{aligned} A_t = D_t \\ \Rightarrow x_{i,t} = b_{t-L+i,t} = x_{i+1,t-1} = b_{t-L+i,t-1}, \end{aligned} \tag{22}$$

$$b_{t,t} = x_{L,t} = O_t \tag{23}$$

   when $i \in [2, L-1], t \in [L, T]$.

4.3 Additional consistency checks

As mentioned in Section 3.2 all possible inconsistencies in data can not be found with the checks made before starting the analysis. Additional inconsistencies appear when solving for elements of $\mathbf{B}$. A big reason for this is store personnel failing to mark batches properly as spoiled at the end of their life-cycle.

Some of the inconsistencies are seemingly caused by the use of special sales transactions A_t. There are several human errors that can cause special sales data to not work as intended. When a store employee fails to either mark all of the units from the batch expiring next day to be discounted or marks wrong batches to be discounted, assumptions for special sales mentioned in Section 3.3 do not hold true and therefore performing Operations 9 and 10 break the consistency of $\mathbf{B}$. For this reason those PLs for which consistency breaks due to use of special sales, the batch balance matrix $\mathbf{B}$ is recalculated without performing special sales-specific operations.

The following consistency checks are performed for $\mathbf{B}$ for which all possible elements have been solved:

1. Fully solved batch balance matrix columns t must sum up as in Equation 10.

   - Any PL with $\sum_{m=1}^{M} b_{m,t} \neq X_t - S_t$
     when $\sum_{m=1}^{M} c_{m,t} = M$ for any $t \in [L, T]$ is removed from data set.

2. Batch balance can't reach the amount it should spoil.

   - Any PL with $b_{t,t+L-2} - D_{t+L-1} > S_{t+L-1}$
     when $c_{t,t+L-2} = 1$ at any $t \in [L, T - L + 1]$ is removed from data set. For example in

     $$\begin{bmatrix} * & 8 & 2 & 0 \\ 0 & * & * & 1 \\ 0 & 0 & * & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

     with $D_3 = 5$ the batch balance $x_{1,3}$ should be at least 3 but it is 2. This is an impossible situation, thus the data for the PL is deemed inconsistent.

3. Batch arrived during the day can not have balance bigger than its delivery

   - Any PL with $b_{t,t} > O_t$
     when $c_{t,t} = 1$ at any $t \in [1, T]$ is removed from data set.

At the end of these final consistency checks we have acquired batch balance matrices $\mathbf{B}$ with some solved elements for all PLs that have consistent enough data for analysis. At this point meeting the condition

$$\sum_{m=1}^{M} c_{m,T} = M \tag{24}$$

implies that we have analytically solved all the values for batch balances $x_{i,T} = b_{T-L+i,T}$ at final moment T with absolute certainty. This is the best case scenario since decisions on future ordering of the product can now be based on definite values. For those PLs that do not meet this condition, we are required to *estimate* the values for $x_{i,T}$ given the limitations of now the partially solved $\mathbf{B}$.

4.4 Batch depletion models

As mentioned in Section 2.1, in our inventory system the unknown variables are how sales are distributed between different batches. In order to estimate batch balances at T, we need to find a DM that reflects the transaction data as well as possible. For this analysis we have chosen seven different kind of DMs for comparison. These models are based on literature and expert views.

When distributing sales to different batches an important assumption has to be made. It is we assume that all sales happen at once instead of one at a time as in real life. In one at a time-scenario the use of a probabilistic model would have been required, which in turn would have yielded different results with every simulation. As we are building a heuristic model that should be computationally reasonably simple, we are satisfied with the simplification of all sales happening at once.

4.4.1 FIFO

In FIFO-depletion the oldest batch is always depleted first:

1. Assign sales D_t to smallest i with $x_{i,t} > 0$ until either $x_{i,t} = 0$ or sales run out.
2. Repeat 1 until all sales have been assigned.

4.4.2 LIFO

In LIFO-depletion the newest batch is always depleted first:

1. Assign sales D_t to largest i with $x_{i,t} > 0$ until either $x_{i,t} = 0$ or sales run out.
2. Repeat 1 until all sales have been assigned.

4.4.3 Proportional depletion

In our version of Proportional depletion we distribute sales to different batches according to the proportion of their individual balance from the total balance. For this a method called *The Largest Remainder Method* is used. This method is mostly known for allocation of seats in elections using party list voting systems (O'Doherty, 1998).

In this thesis we use the method to distribute sales for batches in following way:

1. All batches i receive their integer proportion of sales from $floor(\frac{x_{t,i}}{x_t} \cdot D_t)$
2. Remaining sales are distributed to batches in order of their remainder quotas $\frac{x_{t,i}}{x_t} \cdot D_t - floor(\frac{x_{t,i}}{x_t} \cdot D_t)$ from the highest quota to the lowest. In case of even remainder quotas, older batch or batches are prioritized over newer ones and are assigned the sale.

The proportional depletion model is a very intuitive one. If a store shelf has a large quantity of one batch compared to another, a customer is more likely to pick the unit that is from the larger batch. In case of large inventories it can feel very tiresome for customer wanting to choose the freshest batch to go through expiration labels of several product units.

4.4.4 Parametric LIFO, rest Proportional depletion

This model assumes that portion $p \in [0, 1]$ of sales are first LIFO-depleted and the rest of sales are assigned proportionally:

1. Assign $round(p \cdot D_t)$ amount of Sales to batches according to LIFO as in 4.4.2
2. Assign remaining Sales to batches according to Proportional model as in 4.4.3

The idea behind this model is that there is some percentage of customers that always try to choose the newest possible batch. The remaining customers do not care for expiration date labels and just pick a product unit from shelf without much thought.

4.4.5 Parametric LIFO, rest FIFO

This model is almost the same as 4.4.4, but instead of proportional depletion after LIFO-depletion, we perform FIFO-depletion after LIFO-depletion:

1. Assign $round(p \cdot D_t)$ amount of sales to batches according to LIFO as in 4.4.2
2. Assign remaining sales to batches according to FIFO as in 4.4.1

This model compared to Model 4.4.4 assumes that the store tries to force customers into FIFO by arranging oldest batch units to be first within customers grasp in a store shelf. Therefore all customers who do not seek to choose the newest batch unit now choose the oldest batch unit.

4.4.6 Parametric Special Sales, rest LIFO

This model takes into account the fact that store discounts batches that have one day left on their life-cycle. The parameter $p \in [0, 1]$ now describes how big of a portion of sales are special sales.

1. If $x_{1,t} > 0$, move to Step 2, otherwise skip straight to Step 3
2. Assign $round(p \cdot D_t)$ of sales to expiring batch $x_{t,1}$.
3. Assign remaining sales to batches according to LIFO as in 4.4.2

Presumably, this model describes customer's purchasing behaviour during an expiration day of one batch better than the previous models, which do not take it into account at all. One possible weakness of this model lies in the usage of LIFO for days without a discounted batch.

4.4.7 Mixed

This model is a mixture of previous models with some added complexity. Now parameter $p \in [0, 1]$ is used to describe the portion of special sales from all sales during the days when there is an expiring batch. On other days we use the parameter $q \in [0, 1]$ to describe the amount of LIFO-sales, with the rest of sales being FIFO.

1. If $x_{t,1} > 0$, move to Step 2, otherwise skip to 4
2. Assign $round(p \cdot D_t)$ of sales to expiring batch $x_{t,1}$.
3. Assign remaining sales to batches according to Proportional model as in 4.4.3. End of algorithm without Steps 4 and 5.
4. Assign $round(q \cdot D_t)$ of sales to batches according to LIFO as in 4.4.2.
5. Assign remaining sales to batches according to FIFO as in 4.4.1.

This model aims to combine all the best attributes of previous models. It is also computationally the most complex in both the depletion calculation itself and later in this thesis when seeking for optimal parameters p,q.

4.5 Comparison of depletion models

4.5.1 Comparison metrics

DMs are tested by simulating inventory with them and then comparing the results with historical transaction data. In the simulation we use historical transaction data D_t and O_t. In real life, the store could have an order policy which uses total balance as a parameter. This could lead to the delivered amount to differ from O_t. For this reason we set the length of simulation period to L. With any longer period the simulated values would start to differ more and more from values that would have resulted from simulating inventory with the same order policy than it actually has in real life replenishment.

Since the simulated balance can differ from historical balance, we also have to cap D_t to not exceed the simulated balance. The use of special sales transactions A_t is omitted in simulation due to consistency problems mentioned at Section 3. For the purposes of simulation we now define a *simulated batch balance matrix* $\mathbf{E}^s$, which is a $M \times T, M = T$ matrix with s denoting the starting column for the simulation. Therefore, for one PL there can be multiple simulated matrices depending on how many possible simulation starting points s there are. $\mathbf{E}^s$ has the same structure properties as $\mathbf{B}$ with the only possible difference being the values of matrix elements. For $\mathbf{E}^s$ we denote its elements by $e^s_{m,t}$.

The inventory simulation is performed in the following manner:

1. For the partially solved batch balance matrix $\mathbf{B}$ find all instances s of columns $t \in [L, T-L+1]$ when $(\sum_{m=1}^{M} c_{m,t} = M) \wedge (X_t - S_t \neq 0)$ i.e. there is perfect knowledge of batch balance situation and the inventory is not empty. If not a single instance of s is found, end here.

2. Set column s of $\mathbf{E}^s$ to be the same as column s of $\mathbf{B}$.

3. Simulate inventory in all instances s from s until $s + L - 1$ using the chosen DM with sales D_t and deliveries O_t and update $\mathbf{E}^s$ accordingly. If the amount of sale at day t is higher than the sum of simulated inventory balance at $t-1$ and the incoming delivery at t, deplete the whole inventory and nothing more in order to avoid negative balance.

As an end result for one PL either zero or nonzero number of simulated matrices $\mathbf{E}^s$ are gained. These simulated matrices can now be compared with $\mathbf{B}$ from the equivalent time periods. Comparison can be made on elements already solved in $\mathbf{B}$, that is for elements m, t for which $c_{m,t} = 1$. Most important of all, these solved elements always include the amount of spoiled goods S_t.

Until this point all operations presented have been performed for one single PL. For comparison of different DMs we use all possible PLs that have passed our consistency checks. Thus the comparison of DMs is performed on an aggregate level, which means that we use the transaction data of all PLs in the data set, since what we search for is the best model to be used for an entire product group and not the best model for each PL. For this purpose we now denote $u = 1, 2, ..., U$ to

be the identifier for different PLs with life-cycles L_u and accordingly batch balance matrix ${}^u\mathbf{B}$ and simulated batch balance matrix ${}^u\mathbf{E}^{s_u}$ with elements ${}^u b_{m,t}$ and ${}^u e^{s_u}_{m,t}$. Accordingly for ${}^u\mathbf{B}$ we now have the definite value information ${}^u c_{m,t}$.

In order to compare DMs we establish four different metrics. In metric M1 we sum the differences between definite batch balance values and the simulated values.

$$\sum_{u=1}^{U}\sum_{s_u}\left(\sum_{m=s_u-L_u+1}^{s_u+L_u-1}\sum_{t=s_u}^{s_u+L_u-1}(({}^u(e)^{s_u}_{m,t} -^u (b)_{m,t}) \cdot^u c_{m,t})\right) \tag{M1}$$

In metric M2 we sum the absolute values of differences between definite batch balance values.

$$\sum_{u=1}^{U}\sum_{s_u}\left(\sum_{m=s_u-L_u+1}^{s_u+L_u-1}\sum_{t=s_u}^{s_u+L_u-1}(\left|{}^u(e)^{s_u}_{m,t} -^u (b)_{m,t}\right| \cdot^u c_{m,t})\right) \tag{M2}$$

In metric M3 we sum the differences between spoilage values and simulated spoilage values.

$$\sum_{u=1}^{U}\sum_{s_u}\left(\sum_{m=s_u-L_u+1}^{s_u+L_u-1}({}^u(e)^{s_u}_{m,m+L_u-1} -^u (b)_{m,m+L_u-1})\right) \tag{M3}$$

In metric M4 we sum the absolute values of differences between spoilage values and simulated spoilage values.

$$\sum_{u=1}^{U}\sum_{s_u}\left(\sum_{m=s_u-L_u+1}^{s_u+L_u-1}\left|{}^u(e)^{s_u}_{m,m+L_u-1} -^u (b)_{m,m+L_u-1}\right|\right) \tag{M4}$$

Information term ${}^u c_{m,t}$ is not required in metrics M3 and M4 due to ${}^u b_{m,m+L-1}$ always being populated by spoilage transaction $S_{m,m+L-1}$. Performance of all four metrics is determined by their distance from 0. Metric values nearer to 0 are deemed more optimal. The metrics were chosen for their ability to measure the simulations' ability to reflect the transaction data well enough. Percentage-based metrics were not chosen for measuring the difference in spoilage transactions and simulated spoilages due to most of the spoilage transactions S_t having zero values. Other, more statistics-based metrics could have been chosen but they were deemed off the scope of this thesis.

4.5.2 Finding best parameters for parametric models

Of all DMs under inspection, there are four that use either one or two parameters in determining the distribution of sales to batches. Using the defined metrics, we seek to find for parametric DMs the parameters that support the given transaction data the best. For this purpose, the optimal performance for a set of parameters was chosen to be minimization of metric M4. This is because the best estimation of spoilage is what store owners most likely prefer over best estimations of historical batch balances due to differing spoilages having more actual real life impact. Also, since demand is fixed in our simulations, situations occur when we over-/underestimate the spoilage at one moment of time only to under/overestimate spoilage by the same amount at a later time. This is undesirable from a store's perspective, which is another reason

for choosing metric M4. For a parametric depletion model DM our minimization problem takes the form

$$\min_{p} \sum_{u=1}^{U} \sum_{s_u} \left(\sum_{m=s_u-L_u+1}^{s_u+L_u-1} \left| {}^{u}(e)_{m,m+L_u-1}^{s_u} - {}^{u}(b)_{m,m+L_u-1} \right| \right) \tag{25}$$

$$\text{s.t. } {}^{u}\mathbf{E}^{s_u} \text{ is simulated with a DM using parameter } p \in [0,1]$$

for DMs with one parameter, and form

$$\min_{(p,q)} \sum_{u=1}^{U} \sum_{s_u} \left(\sum_{m=s-L_u+1}^{s+L_u-1} \left| {}^{u}(e)_{m,m+L_u-1}^{s_u} - {}^{u}(b)_{m,m+L_u-1} \right| \right) \tag{26}$$

$$\text{s.t. } {}^{u}\mathbf{E}^{s_u} \text{ is simulated with a DM using parameters } p \in [0,1], q \in [0,1]$$

for a DM with two parameters. The minimization is performed with brute force by evaluating the metric at every $p = 0.00, 0.05, 0.10, ..., 0.95, 1.00$ and accordingly for two-parameter-models also at every $q = 0.00, 0.05, 0.10, ..., 0.95, 1.00$. The interval of 0.05 was chosen because any higher accuracy would not significantly change the value of the minimization given the magnitude of transaction values in the given data set.

4.6 Constraining depletion model with transaction data

In our simulation it was possible for a DM to have a different simulated value for spoilage than the transaction value S_t, leading to simulated total balance values differing from transaction values X_t. For this reason it is not possible to just simulate inventory until $t = T$, since real life data has to be accounted for. This means that the DM used for estimation has to be constrained by transaction data to avoid the occurrence of impossible events that break the consistency of the inventory system.

Since the PLs used in the analysis have consistent data, it is enough to force the simulated spoilage to equal transaction spoilage to keep the entire inventory system consistent. The goal is to force the batch balance simulation to spoil S_t amount of units with as little adjustments as possible so that the characteristics of the chosen DM still drive the batch depletion simulation as much as possible.

Because the goal is to force the amount of spoilage S_t for all t in the simulation period, we have to ensure that batch balances $x_{i,t}$ have such values that the amount of spoilage for a batch is always reachable given future sales transactions. To achieve this, we have to identify situations when a batch i has a balance that can not reach its intended spoilage S_t anymore. When we evaluate the system at every t in the simulation period we ensure that the inventory does not end up in an impossible situation. In general this kind of situation can happen in two occasions. First, a batch can be depleted too much before its spoiling date, which means that a simulated batch balance will have the property

$$x_{i,t} < S_{t+i-1}. \tag{27}$$

In this occasion we adjust the batch balances followingly:

1. For each t in simulation period, find all i which fulfil Equation 27.
2. For all these i add $S_{t+i-1} - x_{i,t}$ to batch balance.
3. Deplete the amounts added in Stage 2 from other batches by starting from the oldest batch that doesn't fulfil Equation 27, depleting it until either running out of units to deplete or when $x_{i,t} = S_{t+i-1}$ for the batch being depleted, in which case move to next oldest batch that doesn't fulfil Equation 27.

For example, after simulating inventory for $t = 3$ with $S_3 = 4$ and $S_4 = 3$ and a simulated batch balance having a value below its intended spoilage, we have to adjust by

$$\begin{bmatrix} 6 & 5 & 4 & 0 \\ 0 & 7 & 1 & 3 \\ 0 & 0 & 6 & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} 6 & 5 & 4 & 0 \\ 0 & 7 & \mathbf{3} & 3 \\ 0 & 0 & \mathbf{4} & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

After making these adjustments, the second occasion when a batch can not reach its intended spoilage is when the batch balance has a higher value than the spoilage of the batch and there are not enough future sales to decrease the batch balance to its intended spoilage. For this tricky special occasion we first define

$$Z_{i,t} = x_{i,t} - S_{t+i-1} \tag{28}$$

which tells the difference between the batch balance and the intended spoilage of batch when $t \in [1, T - L + 1]$. Then for the sake of computation we define

$$N_{i,t} = \omega \tag{29}$$

for all $i \in [1, L]$ when $t \in [1, T]$ and ω is some large number. With this we mean that every element $N_{i,t}$ has value ω in the beginning of the simulation. Now, also at every t we have

$$N_{1,t} = Z_{1,t} \tag{30}$$

and

$$N_{i,t} = \sum_{a=t+1}^{t+i-1} (D_a - Z_{i,a}) \tag{31}$$

when $i \in [2, \min(L, T - t + 1)]$. The minimization function in the definition of this range is chosen because we do not want to update $N_{i,t}$ for batches i that we do not have a spoilage transaction for. When positive, $N_{i,t}$ depicts the amount that can be added to the batch before it can no longer reach its intended spoilage with given transactions. We first set all values of N to ω because when approaching T there are batches for which there exists no spoilage transactions yet because the spoilage will occur after T. The sales of these batches are therefore not bounded by spoilage transaction data. Thus, the adjusting process will be the following:

1. At every t in a simulation period, update $Z_{i,t}$ and $N_{i,t}$. If $N_{i,t} \geq 0$ for all i, skip Steps 2-5 and end algorithm.

2. Set $V = 0$. If $N_{1,t} < 0$, update $x_{1,t}$ to $x_{1,t} + N_{1,t}$ and set $V = N_{1,t}$.

3. Starting from $i = 2$, if $N_{i,t} < 0$, update $x_{i,t}$ to $x_{i,t} + N_{i,t}$ and add $N_{i,t}$ to V. Update $Z_{i,t}$ and $N_{i,t}$ for all i. Move to $i + 1$.

4. Set $W = |V|$.

5. Starting from the newest batch $i = L$, if $N_{i,t} > 0$, add the amount $min(W, x_{i+1,t-1} - x_{i,t}, N_{i,t}, N_{i+1,t}, .., N_{L,t})$ to $x_{i,t}$, subtract the added amount from W and update $Z_{i,t}$ and $N_{i,t}$ for all i. Move to batch $i - 1$ until $W = 0$.

In Step 1 we stop if there are no batch balances that can not reach their intended spoilage. In Step 2 we simply adjust the batch balance of the PL spoiling at the end of the day to match the day's spoilage transaction. In Step 3 we go through batches and deplete the necessary amount so that the batch can reach its target spoilage. All $N_{i,t}$ have to be updated because they are dependant on values of the $x_{i,t}$. In Step 4 we set to W the amount that we have depleted from batches, so that we can add it to the untouched batches. In Step 5 we add W amount of units to untouched batches, starting from the newest batch.

Including $x_{i+1,t-1} - x_{i,t}$ in min-function ensures that a batch will not receive a higher balance than it had the day before. Having $N_{i,t}, N_{i+1,t}, .., N_{L,t}$ in min-function ensures that by adjusting we do not inadvertently make a batch unable to reach its intended spoilage again. The reason why we don't just adjust only in the case of $N_{1,t} < 0$ is that when simulating inventory we want the batch balances of the entire simulation period to be feasible and not only when $t = T$.

For example, we need to make the following adjustment after simulating a DM at $t = 3$ when $S_t = 3, D_4 = 3$.

$$\begin{bmatrix} 9 & 8 & 5 & 0 \\ 0 & 6 & 4 & 1 \\ 0 & 0 & 6 & * \\ 0 & 0 & 0 & * \end{bmatrix} \longrightarrow \begin{bmatrix} 9 & 8 & \mathbf{3} & 0 \\ 0 & 6 & 4 & 1 \\ 0 & 0 & \mathbf{8} & * \\ 0 & 0 & 0 & * \end{bmatrix}$$

The simulated DM has no effect on the adjustment logic and therefore the same logic can be applied when using any DM. For the sake of simplicity, the adjustment logic was made separately and was not built into DMs. If the adjustments were made using the same logic as in the used DM, the simulation could maintain the characteristics of that particular DM better.

4.7 Estimation of batch balances

We estimate the batch balances at $t = T$ by simulating inventory with the chosen optimal DM that is constrained with transaction data as explained in Section 4.6. Simulation is performed only for products that fulfil $\sum_{m=1}^{M} c_{m,T} \neq M$, i.e. their batch

balances are not fully known at $t = T$. Simulation is also omitted for PLs without a single possible starting point s. If we wanted to simulate PLs without starting point s, we would have had to use a different type of estimation model. This would have led to more questionable accuracy of the end result. For maximal accuracy, the starting point of each PL's simulation is its largest s. The simulation is performed by depleting batches with the chosen DM at every step t and adjusting with transaction data if needed. This is continued until $t = T$. In the end, a feasible estimate for batch balances $x_{i,T}$ is achieved. Further testing with the actual accuracy of these estimates is out of the scope of this thesis and would not have been even possible to conduct with the given data set.

5 Results

5.1 Initial consistency check outcomes

The performance of PLs in each consistency check as mentioned in Section 3.2 can be found from Table 2.

Table 2: Performance of data set in consistency checks. Check 1 was performed first with dropping the inconsistent PLs immediately. The other checks were carried out to all remaining PLs.

Consistency check	Product-locations passed
1. Negative End Balance	306/386
2. Sum of Sales	306/306
3. Stepwise Inventory	301/306
4. Spoiling Time	153/306
All checks OK	151

Only approximately 40% of Product-locations were left for analysis after running the consistency checks, which is a somewhat disappointing result for further analysis. PLs with high volume balances and sales were dropped more likely than PLs with low volume balances and sales, most likely because handling of more units also leads to more errors in logging them. Unfortunately, the data set also had some issues with balance data, as some of it was reported to be negative. These 80 PLs with negative balance were dropped first before proceeding with other consistency checks. However, the biggest issue with the given data set was the inconsistency caused by faulty marking of spoiled units. By inspecting the data it became apparent that many spoilage transactions happened either one day before or one day after a batch was meant to spoil. This means that the spoiling time L of a PL may not have been actually constant, but possibly dependant also on other factors such as day of the delivery.

5.2 Solved batch balance matrices

Batch balance matrix $\mathbf{B}$ was formed for PLs passing initial consistency checks and definite values were solved for it as explained in Section 4.2. After solving for definite values both with and without using special sales A_t additional consistency checks as explained in Section 3 were performed. The results of these operations are reported in Table 3.

After final consistency checks the amount left for further analysis was set to $U = 122$. While this amount is not optimal, it was sufficient for further analysis. The amount of inconsistencies caused by special sales was deduced to be mostly caused by reasons specified in Section 3. This notion was supported by the fact that for many of the dropped PLs there were not enough special sales to reach their supposed amount of spoilage.

Table 3: Results of additional consistency checks with checks ran before and after calculating **B** using special sales A_t. Here BB stands for batch balance.

	Before Special Sales	**After Special Sales**
1. BB-Matrix column Sum	124/151	116/122
2. BB can't reach Spoilage	135/151	122/122
3. BB can't exceed Delivery	139/151	116/122
Total passed	122/151	111/122

Solving for definite values of **B** produced results that can be seen from Table 4. The high amount of completely solved batch balances at $t = T = 22$ can be explained by the fact that in this data set there were usually no more than three batches of the same bread in a shelf, and the amount of units in shelf was not particularly large. For this reason, many of the PLs were prone to stock-outs, which led to more effortless solving for batch balances. The result is a good one, since finding definite values for batch balances is obviously the most desirable situation. However, since most of the solved PLs are slow movers, the business value for solving their batch balances is lower than that of solving batch balances for fast movers.

Table 4: Statistics of solved and unsolved PLs.

	Solved	**Unsolved**
Avg. Balance	3.34	9.04
Avg. Simult. Batches	0.97	2.61
Total amount of PLs	73	49

5.3 Performance of depletion models

For all parametric DMs the optimal parameters were sought with the goal of minimizing metric M4. The results can be found in Table 5.

Table 5: Optimal parameters for parametric DMs and their performance.

	p or p,q	**M4**
LIFO-Proportional	0	519
LIFO-FIFO	0.5	414
Special-LIFO	0.65	556
Mixed	0.25, 0.35	431

A notable result is the optimal value $p = 0$ for LIFO-Proportional-model. Essentially this means that all simultaneous batches are depleted proportionally and that the model is effectively the same as the Proportional model. The best performing model overall was the LIFO-FIFO-model with half of the demand being LIFO and the other half FIFO. How these two models perform as the function of p can be

seen from Figures 2 and 3. From these Figures it is clear that models leaning more towards LIFO clearly have the worse performance. In Figure 2 the value for metric M4 starts to rise with smaller p than in Figure 3, because the Proportional model already depletes quite a lot from the newest batch at $p = 0$. This is explained by the fact that when a new batch is delivered, it most likely has more units than the batches that have already been in shelf for several days. In Figure 3 it can also be clearly seen that a model leaning more heavily on FIFO performs reasonably well. This tells that at least in the case of this data set the retailer does manage to force the customer to FIFO quite well.

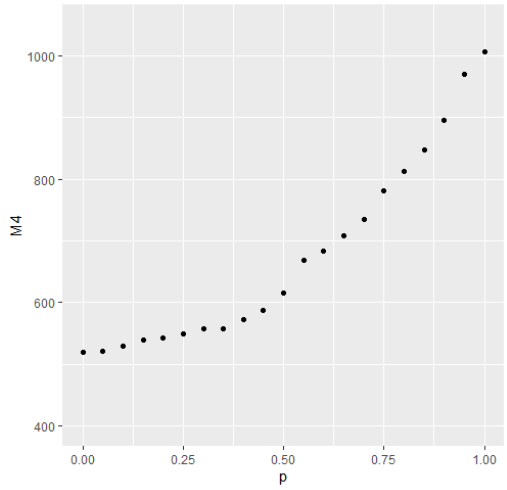

Figure 2: M4 performance in LIFO-Proportional-model.

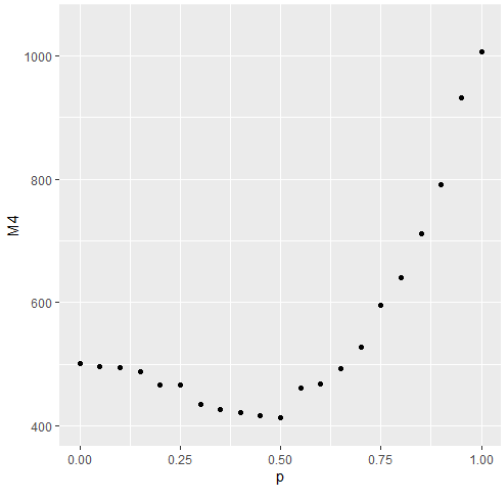

Figure 3: M4 performance in LIFO-FIFO-model.

Comparison of the overall results of non-parametric models and parametric models can be seen in Table 6.

Table 6: Resulting metric values for all DMs after simulation with parametric DMs using parameters minimizing metric M4.

	M1	**M2**	**M3**	**M4**
FIFO	698	1642	-449	501
LIFO	-601	1277	906	1006
Proportional	-48	1122	81	519
LIFO-Proportional	-48	1122	81	519
LIFO-FIFO	261	1203	-166	414
Special-LIFO	138	1438	92	556
Mixed	175	1297	-157	431

The best model for minimizing the difference to actual spoilage was LIFO-FIFO-model. Apart from full LIFO, the difference in the value of metric M4 between different models was not massive. This can be attributed to parametric models leaning more towards FIFO, which leads to substantially better results than LIFO.

The Mixed-model had the second best performance most likely due to its similarity to LIFO-FIFO. Its parameters' values $(p, q) = (0.25, 0.35)$ indicate that customers indeed prefer discounted units to some extent, but on non-discounted days the store manages to force FIFO reasonably well. The difference between Mixed and LIFO-FIFO models was quite marginal, and a possible reason for the Mixed not having the best performance is Proportional depletion on special sales days after the special sales had been depleted. Proportional model leans more to LIFO-type of depletion, which clearly does not perform well in this case.

Unsurprisingly, full FIFO underestimates and full LIFO overestimates spoilage the most. Regarding the performance of metric M3 FIFO underestimate spoilage less than LIFO overestimates it due to the real life spoilage values being closer to zero than the high spoilage values simulated by LIFO. From other models, LIFO-FIFO has the biggest bias which is towards FIFO, but the magnitude of these biases is not even near full FIFO or full LIFO.

Overall, the results gained from analyzing this data set indicate that if a store wants to minimize metric M4, it should lean more towards FIFO-type of depletion with the cost of underestimating spoilage more. Leaning just a bit more to LIFO-type of depletion causes higher value of metric M4 but smaller bias in estimated spoilages. Since $S_t = 0$ for most PLs for almost all t, any model deviating from FIFO causes more spoilage and has a larger likelihood to simulate nonzero spoilage on days when there is no recorded spoilage. For the PLs in this data set it can be stated that it is always a safer choice to simulate batch balances using a DM that leans more towards FIFO.

5.4 Estimated batch balances

Using the LIFO-FIFO-model, an estimate for batch balances at $t = T = 22$ was simulated for 119 PLs using logic specified in Section 4.7 with feasible results. Feasible here means that for all simulated PLs the simulated values for spoilage and total balance matched their respective transactions for the entire simulation period. In addition, the simulated batch balances were monotonically decreasing in t, that is the batch balances did not increase at any point after delivery.

Out of $U = 122$ consistent PLs, definite batch balances were solved for 73 PLs, an estimate was formed for 46 PLs, and for 3 PLs no definite values or estimates were calculated at all due to lack of a starting point s. Average distributions of simulated batch balance values are listed in Table 7.

With the chosen DM only 36 of the PLs had over two simultaneous batches at the end of the time period. This is no surprise since always FIFO-depleting 50% of sales quite effectively depletes the oldest batch to zero balance. However, many of the PLs with bigger balances and more simultaneous batches had to be dropped earlier from the analysis due to consistency issues.

In the end the set goal of the thesis was reached, which was finding a feasible heuristic estimate for batch balances given historical data. From 7 different DMs the best performing one was chosen for this particular data set. It can not be generalized that the best performing DM is the best assumption for a DM for any product group

at any store. Most importantly, the goal of finding a process for estimating batch balances with historical store data was achieved. When applying the process for store data the part of searching for optimal parameters can even be omitted if stores choose a simpler DM or want to plug in their own parameters for parametric models.

Table 7: Average distribution of simulated batch balances for $t = T = 22$.

Simult. batches / Batch oldest to newest	**0**	**1**	**2**	**3**	**4**
1	*	1.00	0.38	0.21	0.17
2	*	*	0.62	0.21	0.25
3	*	*	*	0.58	0.08
4	*	*	*	*	0.50
PLs	8	75	33	2	1

6 Limitations of analysis and possibilities for Future research

6.1 Limitations

For this particular data set the biggest limiting factor for analysis was the amount of inconsistent PL-rows that had to be dropped from the analysis. A fully consistent special sales data A_t could also have raised the amount of solved elements $c_{m,t} = 1$ and helped in further analysis. The biggest limitation for applying our batch balance estimation process is the requirement for consistent data. Maintaining consistent enough data for our process is a large task for retailers since even the smallest of errors can deem a PL inconsistent for the established process. Another big limitation in the established process is that it can not be applied for PLs without a single possible simulation starting point s. Usually these PLs are the ones that have the largest balances and most simultaneous batches. Combined with the fact that these same type of PLs are more prone to having inconsistent data, we are too often unable to use our process for the most business-critical PLs.

When calculating definite values for a batch balance matrix $\mathbf{B}$ we operated under the assumption that a batch could have had special sales only on its last life-cycle day. However, many retailers discount their products even earlier than this, makin the process established in this thesis inapplicable for them.

All DMs simulated assume perfect substitution. As mentioned in Section 3.3 breads have a very high substitution rate and this is not taken into account in our simulations. In our simulations we assumed the demand to be the same as in historical transactions, without considering whether the total balance or the batch balance distribution had any effect on the total demand. A more intricate DM would have also taken into account the fact that customers make purchasing decisions one at a time instead of simplifying customer demand happening all at once as in DMs used in this thesis.

In our simulations we used the transaction values for the size of the deliveries even though in a real life setting with an automatized replenishment logic the size of deliveries would have deviated from these historical values due to the ordering policy of the replenishment logic. Performing inventory simulations using the same ordering policy that the store normally uses, the applicability of the results could potentially be increased.

From the results it was deduced that having a parametric model that separates days with special sales and those without them is possibly not a good approach when searching for a pair of optimal parameters. This is because since we are minimizing difference to spoilage transactions the parameter for special sales day overtly tries to fix the inaccuracy caused by the parametric model for days without special sales. This is why the Mixed-model has the possibility to go badly wrong during longer simulation periods when there are no special sales-days at all. Since we don't have accurate batch balance data it is impossible to measure the performance of DMs during only non-special days.

Constraining the optimal DM with transaction data is performed with a simple FIFO/LIFO type of method, which does not not consider the characteristics of the used DM. With multiple time steps this property can possibly negate the ability of a DM to predict spoilage properly. Constraining a DM in a manner that makes its batch balance adjustments perform as similarly as the underlying DM model as possible could potentially increase the results' accuracy. It also has to be mentioned that because the adjustment process was not mathematically proved to always produce a feasible result, its applicability remains slightly ambiguous even though in this particular data set all the end results ended up feasible.

6.2 Future research

The obvious next step for further research would be to count the batch balances of the used PLs in real life in order to measure the accuracy of result. The resulting optimal DM achieved by the process in this thesis is always highly dependant on the shelving strategy of the store, but if a more holistic result is sought for measuring the customers will to purchase a product in different stages of its life-cycle, a statistical model could be formed with accurate store batch balance data. This analysis could be performed for many different types of product groups in addition to breads, such as meats, dairy products and convenience foods.

The process executed in this thesis could also be performed by differing the period T for the same PLs and inspecting if there is notable differences in the acquired parameter values. This would require that the used PLs would have consistent data for the entire length of the largest tested time period T. With the given data this was discovered to be a challenge and was left out of the thesis.

Instead of using simplistic multiplication-by-parameter-approach for the DMs, more intricate statistical models could be tested. Estimating a probabilistic distribution where a customer has a p chance of picking batch i given the amount of batches and their balances could be the next step when searching for the optimal DM. Using a Monte Carlo-approach with the estimated probabilistic distribution, a distribution-based estimation for batch balances at T could be gained. This type of result could be more beneficial for replenishment since it would provide valuable information for risk management.

To achieve notable business value from the estimates achieved in this thesis, the results have to be combined with future sales forecasts. This way it is possible to project future spoilage and then by replenishment aim to avoid stock-outs. Future study should be performed on the accuracy of these projected spoilages given future sales forecast, and the estimation logic for batch balances established in this thesis. Obtaining a good accuracy for these projections is however a challenging task since both the inaccuracy of sales forecasts and the batch balance estimations stack up. If these projections turned out sufficiently accurate, a retailer could adjust its ordering policy to take into account these spoilage projections. After this point it would be possible to measure critical KPIs such as the amount of stock-outs or inventory costs, and compare these to values gained when not performing batch balance estimation.

In the long term, an interesting study would be calculating the business value of

stores having perfect batch balance information. Measuring critical KPIs when either using or not using accurate batch balance information in replenishment logic could lead to a realistic estimate for business value. If the result was notably positive it could drive retailers to accelerate the transition to intelligent barcodes or equivalent systems counting the batch balances.

7 Summary

Given a data set of consistent historical store data, a parametric LIFO-FIFO depletion model was found to have the best performance out of 7 different alternatives in estimating the amount of spoilage. A process was established, that in most cases found either definite batch balances or a feasible estimate for batch balances of perishable products in store shelves. Feasible estimates were found by simulating inventory with a depletion model deemed optimal, and constraining the simulation with the store data. The process found can be replicated for all data sets with the right type of historical transaction data. In future this batch balance information can be combined with sales forecasts in order to project future store spoilages.

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