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Low complexity space-time MMSE equalization in WCDMA systems

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Abstract—In this paper we propose a low complexity frequency-domain method for finding the minimum mean square error (MMSE) equalizer. Special properties of circulant matrices are exploited in the method. Simulations show that only a minor performance loss is experienced compared to time-domain processing. We also compare joint space-time processing to decoupled equalization and spatial combining. The complexity reduction due to the decoupled processing is evident, while the performance loss remains small. Consequently, the proposed low complexity frequency-domain method can also be used with multiple receive antennas with tolerable performance loss compared to joint space time equalization. Additionally, different MMSE equalizer definitions for multiple receive antennas are considered, and their performance is studied in simulation using estimated channels. Simulations are carried out in high data rate WCDMA system model.

I. INTRODUCTION

Low complexity and fast adaptation to time varying channel coefficients are important receiver design criteria in future wireless communication systems. From the proposed linear receivers for WCDMA downlink, the approximate minimum mean square error (MMSE) equalizer seems to be an appealing alternative. It has improved performance over conventional RAKE receiver and Zero Forcing equalizer, see e.g. [3], [4], [5]. Its drawback is higher computational complexity due to matrix inversion. There are good number of studies on adaptive implementations of the MMSE equalizers. Unfortunately, adaptive MMSE equalizers typically converge slowly and have poor tracking performance in fast fading channels, see e.g. [3], [6]. The faster converging algorithms, e.g. RLS, are naturally more complex, and frequent algorithm updates (at symbol or chip rate) further increase the complexity. As a consequence, direct inversion of the covariance matrix needed in Wiener-Hopf equations appears to be very attractive choice if only the complexity could be significantly reduced. Lower complexity methods, such as the Levinson recursion, can be utilised if the covariance matrix has Toeplitz structure. In case of single receive antenna, the covariance matrix processes such structure.

When there are multiple antennas at the receiver, the space-time covariance matrix has a block Toeplitz structure which causes a clear increase in complexity. The joint space-time equalization is commonly employed. However, the gains obtained via joint processing are rarely considered. In this paper,

we first compare joint space-time equalization to decoupled time domain equalization and simple spatial combining of the antenna outputs. The decoupled processing lowers the computational complexity significantly while the performance loss remains tolerable. Additionally, we consider different methods for estimating the signal covariance matrix and the influence of channel estimation error to the MMSE equalizer performance.

For further complexity reduction we consider the following frequency domain approach. Asymptotically a Toeplitz matrix can be approximated with a circulant matrix [7]. A circulant matrix can be inverted using Fourier transformations. Therefore the inversion may be implemented with $\mathcal{O}(n \log(n))$ complexity in the frequency domain. This method reduces the computational complexity especially when the matrix dimensions are large. We will show by simulations that the circulant approximation does not influence the performance, except at high signal to noise ratios (SNR) where the performance is slightly worse than with direct time-domain inversion. Additionally, the method is not sensitive to channel estimation errors. As a consequence, the proposed equalizer provides a good compromise between complexity and performance. The Toeplitz matrix structure has been employed in finding MMSE equalizer in CDMA systems e.g. in [8], [9], [5]. The approximately circulant structure may be exploited by inverting the covariance matrix in frequency domain, see [7]. This idea was independently used for equalization in HSDPA systems in a simple scenario [8].

This paper is organised as follows: First the system model is introduced. In section III different derivations of the MMSE equalizer are reviewed. In section IV, a low complexity frequency domain method for inverting the sample covariance matrix is derived. Simulation results comparing different MMSE implementations with joint and decoupled spatial processing and the proposed frequency domain method are presented in section V. Finally, section VI concludes this paper.

II. SYSTEM MODEL

The system model considered in this paper is based on the 3rd generation wideband CDMA model for high data rates, see [1], [2]. Due to aperiodic (long) codes, each symbol is spread with different code during the observation period. The p th code for n th symbol is $\mathbf{c}_{np} = [c_{np}(1) \ \dots \ c_{np}(G)]^T$ where G

is the spreading factor. For the frequency selective channel case, we define a code convolution matrix \mathbf{C}_{np} of dimension $G + L - 1 \times L$. Here L is the length of the channel impulse response in chips. Assuming that there are M antennas at the receiver we can model the code matrix as $\mathbf{C}_{np} = \mathbf{I}_M \otimes \mathbf{C}_{np}$, where \otimes denotes the Kronecker product. Codes are scaled such that $|\mathbf{c}_p^H \mathbf{c}_r| = \delta(p - r)$, where δ is the Dirac delta function.

The combined multipath multiple receive antenna channel vector is defined as a $LM \times 1$ vector $\mathbf{h}_n = [\mathbf{h}_{n1}^T \dots \mathbf{h}_{nM}^T]^T$, where the complex channel coefficients at the m th receive antenna are in $\mathbf{h}_{nm}^T = [h_{nm}(1) \dots h_{nm}(L)]^T$. We assume that the coherence time of the channel spans over several symbols. Hence, the channel is considered to be constant during the observation period and the symbol index n may be dropped from the notation.

The received signal is sampled at chip rate and stacked to a vector of length $M(G + L - 1)$. At the receiver the signal transmitted from one base station may now be written as:

$$\mathbf{y}(n) = \sum_{p=1}^P \rho_p \mathbf{C}_{np} \mathbf{h} s_p(n) + \text{ISI} + \mathbf{v}(n), \quad (1)$$

where P is the number of codes, ρ_p is the symbol amplitude, ISI denotes the inter symbol interference and $\mathbf{v}(n)$ is the noise term. It includes both the interference from other base stations and thermal noise. The transmitted symbols $s_p(n)$ are assumed to be independent and identically distributed, such that $E\{s_p(n)s_k(a)^*\} = \delta(p - k)\delta(n - a)$.

III. MMSE EQUALIZATION

In general, the MMSE equalizer in WCDMA systems is time varying due to the aperiodic spreading codes. But in downlink where orthogonal codes and synchronous transmission are used, the time dependency of optimal equalizer is reduced due to code averaging, see [3]. The MMSE equalizer is the solution to the Wiener-Hopf equation, see e.g. [3], [4]:

$$\mathbf{R}_{yy} \mathbf{f} = \mathbf{r}_{yd}, \quad (2)$$

where \mathbf{R}_{yy} is the covariance matrix of the received signal vector and \mathbf{r}_{yd} is the cross-correlation between the received signal and the desired signal.

In order to obtain the MMSE equalizer in fading channels both the covariance matrix and cross correlations need to be estimated using finite sample support, for example by $\hat{\mathbf{R}}_{yy} = \frac{1}{N} \sum_{n=1}^N \mathbf{y}(n)\mathbf{y}^H(n)$, where N is the number of symbols used in averaging. The cross correlation \mathbf{r}_{yd} can be interpreted as a conventional channel estimate, denoted by $\hat{\mathbf{h}}$. It is obtained by correlating the despread pilot symbol estimate with the known pilot symbol, i.e. $\hat{\mathbf{h}}_n = \mathbf{C}_{nc}^H \mathbf{y}(n) s_c^*(n)$, where the index c indicates the pilot signal. Averaging yields $\hat{\mathbf{h}} = \frac{1}{N} \sum_{n=1}^N \{\hat{\mathbf{h}}_n\}$.

Alternatively, the covariance matrix can be written as:

$$\mathbf{R}_{yy} = \sigma_d^2 \mathbf{H} \mathbf{H}^H + \mathbf{R}_{vv}, \quad (3)$$

where \mathbf{R}_{vv} is the noise covariance matrix and σ_d^2 is the variance of the transmitted signal, which is in this paper

scaled as $\sigma_d^2 = 1$. By assuming the noise is white Gaussian, $\mathbf{R}_{vv} = \sigma_v^2 \mathbf{I}$ where σ_v^2 is the noise power, we can find the MMSE equalizer as follows:

$$\mathbf{f}_h = (\hat{\mathbf{H}} \hat{\mathbf{H}}^H + \sigma_v^2 \mathbf{I})^{-1} \hat{\mathbf{h}}_D, \quad (4)$$

where, \mathbf{h}_D is the D th column of the channel convolution matrix \mathbf{H} defined as $\mathbf{H} = [\mathbf{H}_1^T \dots \mathbf{H}_M^T]^T$. The channel convolution matrix of dimension $F \times F + L - 1$ may be written for m th antenna as follows:

$$\mathbf{H}_m = \begin{bmatrix} h_m[L] & \dots & h_m[1] & \dots & \dots & 0 \\ \vdots & \ddots & \vdots & \dots & \ddots & \vdots \\ 0 & \dots & \dots & h_m[L] & \dots & h_m[1] \end{bmatrix}, \quad (5)$$

where F is the equalizer length. The elements in matrix $\mathbf{H} \mathbf{H}^H$ are channel auto- and cross-correlations with different lags. Moreover, if two receive antennas are used, the matrix has following block Toeplitz structure:

$$\mathbf{H} \mathbf{H}^H = \begin{bmatrix} \mathbf{H}_1 \mathbf{H}_1^H & \mathbf{H}_1 \mathbf{H}_2^H \\ \mathbf{H}_2 \mathbf{H}_1^H & \mathbf{H}_2 \mathbf{H}_2^H \end{bmatrix}. \quad (6)$$

We denote the MMSE equalizer defined in equation (4) by HHH and the equalizer obtained using the covariance matrix estimate $\hat{\mathbf{R}}_{yy}$ and channel estimate as RYH. Equation equalizer obtained using RYH is:

$$\mathbf{f}_r = (\hat{\mathbf{R}}_{yy})^{-1} \hat{\mathbf{h}}_D. \quad (7)$$

If the equalizer coefficients are separately calculated for each receive antenna, the off-diagonal blocks in expression (6) are zero and the matrix becomes block diagonal matrix. Similar derivation for RYH can be easily obtained. In the decoupled approach, the above matrices to be inverted have Toeplitz structure with dimension $F \times F$. With joint processing the matrix dimension is $MF \times MF$. In this paper, we mainly calculate the equalizer coefficients using the decoupled approach, but results obtained by joint processing are given as reference. The spatial combining is done using simple addition. Alternative combining methods could be considered, e.g. maximum likelihood combining.

With finite sample support the RYH and HHH approaches have different performance due to channel and covariance matrix estimation errors. In the simulation it will be clearly shown that the HHH approach gives better performance with estimated channels, especially with low sample support. Consequently, the use of the channel estimates provides faster convergence compared to RYH method.

In this paper we have only considered white Gaussian noise, resulting diagonal noise covariance matrix. In case of severe inter-cell-interference the noise is correlated and the noise covariance matrix needs to be more carefully estimated. At cell border this can be done, for example, using the pilot signal of the interfering cell with $\mathbf{R}_{vv} \approx \sigma_i^2 \hat{\mathbf{H}}_i \hat{\mathbf{H}}_i^H + \sigma_v \mathbf{I}$, where the index i denotes the interfering cell.

IV. FREQUENCY DOMAIN MATRIX INVERSION

Finding the MMSE equalizer directly from the Wiener-Hopf equation (2) requires matrix inversion. The matrix inversion, in general, has complexity of order $\mathcal{O}(n^3)$. In the case of joint space-time equalization the matrix dimension is $n = MF$. If the inversion needs to be performed frequently, the computational burden may become intolerable. Depending on the channel coherence time, the updating rate can be lowered substantially without compromising the performance.

Alternatively, there are several efficient algorithms which can be used for inverting the matrix. Moreover, the inverse is not always needed since the equalizer is the solution to equation (2). This can be solved, for example, with Levinson recursion with $\mathcal{O}(n^2)$ complexity, if \mathbf{R}_{yy} is Toeplitz.

Using the properties of Toeplitz matrices the matrix inversion can also be performed in the frequency domain. A finite dimensional Toeplitz matrix can be approximated with a circulant matrix, see [7]. Since a circulant matrix can be diagonalized using FFT, the inversion can be performed separately for each element. This method provides only an approximate solution to the original matrix inversion problem, whereas the time domain methods such as Levinson recursion, solve the problem exactly.

The channel autocorrelation matrix $\mathbf{H}_1\mathbf{H}_1^H$ (for one antenna) is a finite order Toeplitz, with first row: $[a_0 \ a_1 \ \dots \ a_{L-1} \ \dots \ 0]$ and first column: $[a_0 \ a_{-1} \ \dots \ a_{-L+1} \ \dots \ 0]^T$. Here a_l denotes channel auto-correlation with lag l . The matrix has finite order since $a_l = 0, \forall l > L - 1$. Asymptotically a finite order Toeplitz matrix is equivalent to a circulant matrix, where the upper right and lower left corners are filled with appropriate entries to make the matrix exactly circulant, see [7]:

$$\begin{bmatrix} a_0 & a_1 & \dots & 0 & \dots & a_{-2} & a_{-1} \\ a_{-1} & a_0 & \ddots & \ddots & & a_{-3} & a_{-2} \\ \vdots & \ddots & \ddots & & & & \\ a_{-L+1} & a_{-L+2} & & & & & \\ & \ddots & \ddots & & & & \\ 0 & \dots & a_{-L+1} & \dots & a_{L-1} & \dots & 0 \\ & \ddots & \ddots & \ddots & & & \\ a_2 & & & \ddots & \ddots & a_0 & a_1 \\ a_1 & a_2 & \dots & 0 & \dots & a_{-1} & a_0 \end{bmatrix}. \quad (8)$$

Using FFT to diagonalize a circulant matrix and element-wise inversion, the total complexity of the matrix inversion is of order $\mathcal{O}(n \log(n))$. The inverse can be derived using any row of the circulant matrix. We will use the row which has the zero-lag auto-correlation in the middle, i.e. $[0 \ \dots \ 0 \ a_{-L+1} \ \dots \ a_0 \ \dots \ a_{L-1} \ 0 \ \dots \ 0]$. This vector is denoted as \mathbf{a} and the index of the term a_0 is denoted as τ . The number of zeros can be chosen such that length of \mathbf{a} is power of 2 to obtain an efficient FFT. In simulations we have tested different number of zeros needed for adequate approximation depending on the filter length. In general, the

approximation improves as the number of zeros increase, but this also increases the complexity. For now we assume there are enough zeros padded at both ends. Following equation (4) we first add noise variance to the term $\mathbf{a}(\tau)$, i.e. $\mathbf{a}(\tau) = a_0 + \sigma_v^2$. Then the following algorithm is used:

$$\Phi = \text{FFT}(\mathbf{a}) \quad (9)$$

$$\Psi(i) = \frac{1}{\Phi(i)}, \quad \forall i \quad (10)$$

$$\psi = \text{IFFT}(\Psi) \quad (11)$$

$$(\mathbf{H}\mathbf{H}^H + \sigma_v^2)^{-1} \approx \text{Toeplitz}(\psi(\tau : \Gamma)). \quad (12)$$

Here $(\tau : \Gamma)$ denotes indexes from τ to Γ . We choose Γ to match the matrix dimensions. If necessary, zeros are added to the end of the vector ψ .

The reduction in computational complexity depends on the channel length which influences the equalizer length. If the channel length is only a few taps, the proposed FFT method does not provide much benefits, but when the length increases the benefit becomes significant. A commonly used rule of thumb states that the equalizer length should be at least twice the channel length, i.e. $2L$. The length of the vector \mathbf{a} used in the circulant approximation is studied in simulations in section V with two different equalizer lengths.

The proposed implementation is for SISO model since the Toeplitz structure is required. For multiple receive antennas, decoupled processing for each antenna must be used. Due to decoupled processing the complexity increases only linearly with the number of receive antennas.

V. SIMULATIONS

In this section, simulations are carried out to study the performance of the proposed MMSE equalizer in SIMO channels. The used performance measure is the bit error rate (BER) obtained with both the known and estimated channel coefficients.

The channel coefficients are generated using the METRA channel model software based on Vehicular A specifications, see [10], [11]. The delay spread is 11 chips and the average power of the channel taps are: $[0 \ -1 \ -9 \ -10 \ -15 \ -20]$ dB. We have combined the channel with transmit and receiver filters, i.e. with two root raised cosine filters with roll off factor 0.22. After filtering the sampling is performed once per chip and 10 chip long channels are used. The equalizer length for each antenna was set to either $F = 20$ or $F = 30$. The equalizer delay was set to $D = \lfloor (F + L - 1)/2 \rfloor$, see [4]. Two antennas are employed at the receiver with $d = \lambda$ spacing. All the shown results are averaged over 256 random channel realizations.

The spreading code is a combination of orthogonal variable spreading factor (OVSF) codes and long Gold codes (complex). The spreading factor of 16 is used for the HSDPA codes. Additionally control signal is simulated with $G_C = 256$ and 10% of the total transmitted power, [2], [1]. Interfering users are modelled as speech signals with spreading factor $G_I = 128$. Number of HSDPA signals is denoted by P and number of speech signals by P_I .

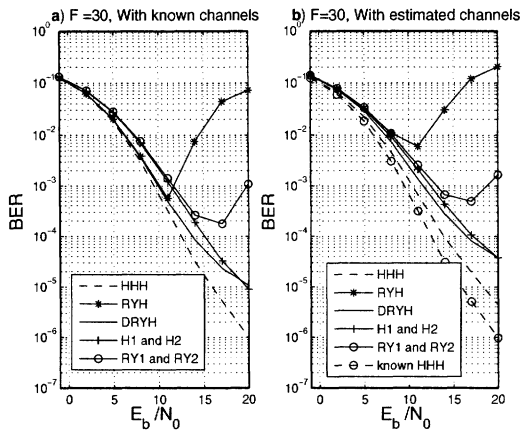


Fig. 1. Raw BER for $M = 2$, $F = 30$, $P = 8$, $P_I = 56$. The performance of the MMSE equalizer obtained with known (figure a) and estimated channel coefficients (figure b). The pilot power was 10 %, the channel and the covariance matrix estimates are averaged over 20 pilot symbols.

In Figure 1 BER results for different implementations of MMSE equalizer are shown. Time averaging over 2 slots is used. This means that the channel and sample covariance matrix estimates are averaged over twenty pilot symbols. For comparison we show in Figure 1a) the BER results assuming perfect channel knowledge and in Figure 1b) results when the channel is estimated using a correlator and the pilot signal. The figure shows the sensitivity of the RYH method to channel estimation errors even with rather long sample support. Using the MMSE equalizer derived with equation (4) the performance difference between the known channel and the estimated channel is about 1 dB at BER 10^{-2} . But if the RYH method of (7) is used with known and estimated channel, the difference is about 3 dB. The performance loss due to decoupled equalization (denoted as H1 and H2) of the signals in two antennas is about 1.5 dB with both known and estimated channels. The decoupled processing using the signal covariance matrices (denoted as RY1 and RY2) is clearly less sensitive to channel estimation errors than joint processing RYH. At high SNR, the inversion of the sample covariance matrix is prone to errors due to ill conditioning of the matrix, especially in joint space-time processing. This problem can be reduced using a small diagonal loading in the covariance matrix $\hat{\mathbf{R}}_{yy}$, denoted here by DRYH. DRYH is also less sensitive to channel estimation errors but still the HHH with estimated channels has better performance at BER of 10^{-2} . Additionally, when using HHH, we don't have the problem of defining a proper factor of diagonal loading.

In Figure 2 results are shown for the case when only 5 pilot symbols, i.e. half of one slot, is used for estimation of the channel and sample covariance matrix. The quality of the obtained channel estimate is not very good using simple correlator and averaging over 5 pilot symbols. Yet, also in this case the HHH method clearly outperforms the RYH method. The Figure 2 indicates that with fast fading channels the HHH method should be used instead of RYH. In general, the HHH

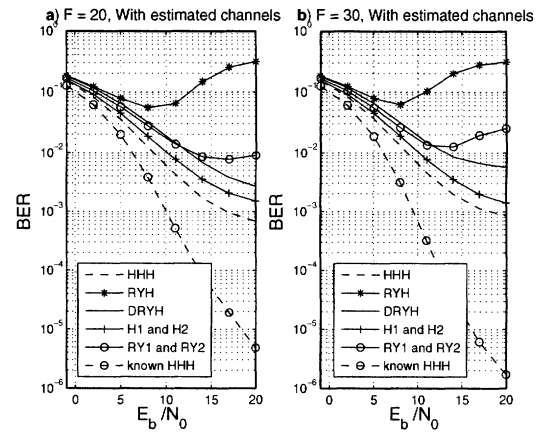


Fig. 2. Raw BER for $M = 2$, $P = 8$, $P_I = 56$. The performance of the MMSE equalizer obtained with two different filter lengths, $F = 20$ and $F = 30$ and estimated channel coefficients. The pilot power was 10 %, the channel and the covariance matrix estimates are averaged over 5 pilot symbols.

method performs in terms of BER better than or equally well in comparison to RYH independent of the sample support. If the noise is colored, e.g. due to interference from neighbouring cell, the noise covariance matrix needed for the HHH method needs to be estimate more accurately.

It is interesting to note that with estimated channels and sample covariance matrix the decoupled spatial processing gives better results than joint processing. The same phenomenon is seen with known channels if the sample support is small and the covariance matrix estimate is erroneous.

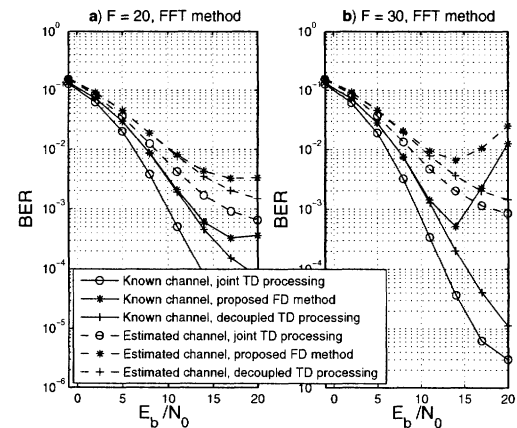


Fig. 3. Raw BER for $M = 2$, $P = 8$, $P_I = 56$. The performance of the MMSE equalizer obtained with known and estimated channel coefficients, and filter lengths $F = 20$ and $F = 30$. The solid lines are derived with known channels and the dashed lines with estimated channels. When the two receive antennas are separately equalized and then combined both the time-domain and frequency domain implementation yield same results. Only at high SNR, the FFT method performance is degraded.

Figure 3 illustrates the performance of the low complexity FFT matrix inversion method used with both true channel and estimated channel. Channel estimate is obtained with a

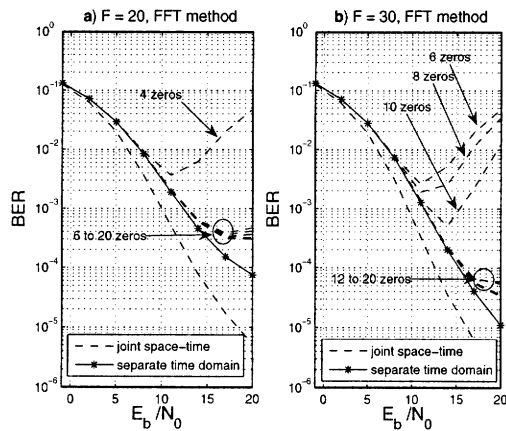


Fig. 4. Raw BER for $M = 2$, $P = 8$, $P_1 = 56$. The performance of the MMSE equalizer with the proposed FFT matrix inversion and known channel. Different number of zeros added to before and after the channel auto-correlation vector prior to inversion with FFT method. Channel length was 10 chips. In figure a) the equalizer length was 20 and the number of zeros varied from 4 to 20. In figure b) the equalizer length was 30, and the number of zeros varied from 6 to 20.

correlator averaged over 5 pilot symbols. Equalizer length was in Figure 3a) $F = 20$ and in 3b) $F = 30$. In this simulation 10 zeros were added to both ends of the channel auto-correlation vector, i.e. length of vector \mathbf{a} was 39. As seen in the Figure, the FFT method experiences only minor performance loss at high SNR regime compared to decoupled processing in the time domain when $F = 20$. Using a longer filter, the approximation error starts to appear at high SNR, see Figure 3b. In this case, more zeros are needed for adequate approximation. At BER 10^{-2} there is no difference between the performance of exact time domain solution and the proposed low complexity FFT method. Additionally the FFT method is not very sensitive to channel estimation errors, as seen with the dotted lines, where a rather poor channel estimate is used. The loss compared to joint space-time equalization is about 1.5 dB at BER 10^{-2} with both known and estimated channels and there is no extra loss in the performance due to FFT processing. With multiple receive antennas, decoupled spatial processing provides a clear complexity reduction compared to joint space-time MMSE equalization. With the FFT method, the complexity can be further reduced. If the loss in BER performance is affordable, the proposed method provides a good compromise between performance and complexity compared to joint space-time MMSE equalizer.

In Figure 4 we have shown BER results with different lengths of the vector \mathbf{a} , assuming known channel. The MMSE equalizers are calculated with the approximative matrix inverse, see equation (12). The equalizer length was $F = 20$ in Figure 4a) and $F = 30$ in Figure 4b). The channel length was 10, so the length of the channel auto-correlation vector is 19. Different number of zeros were added to both ends of this vector yielding different lengths for \mathbf{a} . For example, if 4 zeros we added to both ends the length of \mathbf{a} is 27 and with 6 zeros 31. Figure 4a) shows that with $F = 20$, adding 6

(or more zeros) results nearly identical BER performance to decoupled time domain processing. With a longer equalizer the length of \mathbf{a} needs to be increased by adding more zeros, see figure 4b). With 12 zeros at both ends, the result are close to decoupled time domain processing. With less zeros there is some instability at high SNR values. The BER values for the joint space-time equalizer calculated in time domain are shown for comparison.

VI. CONCLUSIONS

In this paper we have proposed a low complexity FFT based method for implementing the matrix inversion needed for MMSE equalization. The method provides significant benefits especially if the equalizer is long. Only minor performance loss is experienced at high SNR regime compared to time domain approach with single antenna. Additionally, we have compared different approaches for finding the space-time MMSE equalizer for WCDMA systems. The performance of the different implementations depends clearly on the quality of the available channel estimate. We have shown via simulations that if the estimated channel auto-correlation matrix is used instead of the signal covariance matrix for finding the MMSE equalizer, the sensitivity to channel estimation errors is clearly decreased. We have also compared joint space-time equalization with decoupled time domain equalization. The performance loss due to decoupled processing is between 1-2 dB while the complexity reduction is significant.

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